

Algebra Theorems

On Rings, Polynomials, and Fields

1 Section 16.2 - Integral Domains and Fields

1. **Prop. 16.15. Cancellation Law.** Let D be a commutative ring with identity. Then D is an integral domain iff for all nonzero elements $a \in D$ with $ab = ac$, we have $b = c$.
2. **Theorem 16.16.** Every finite integral domain is a field.
3. **Lemma 16.18.** Let R be a ring with identity. If 1 has order n , then the characteristic of R is n .
4. **Theorem 16.19.** The characteristic of an integral domain is either prime or zero.

2 Section 16.3 - Ring Homomorphisms and Ideals

1. **Prop. 16.22.** Let $\phi : R \rightarrow S$ be a ring homomorphism. Then:
 - (a) If R is a commutative ring, then $\phi(R)$ is a commutative ring.
 - (b) $\phi(0) = 0$.
 - (c) Let 1_R and 1_S be the identities for R and S , respectively. If ϕ is onto, then $\phi(1_R) = 1_S$.
 - (d) If R is a field and $\phi(R) \neq \{0\}$, then $\phi(R)$ is a field.