Algebra Definitions

On Rings, Polynomials, and Fields

1. A ring R is a set that is closed under two binary operations, $+$ and $\times$. The following conditions must also be satisfied:
   (a) Additive commutativity.
   (b) Additive associativity.
   (c) Additive identity.
   (d) Additive inverse.
   (e) Multiplicative associativity.
   (f) Multiplicative distributivity 1 & 2.
2. A ring with unity (or with identity) is a ring R that has multiplicative identity.
3. A commutative ring is a ring R that has multiplicative commutativity.
4. An integral domain is a commutative ring R with identity such that for all $a, b \in R$ $ab = 0$ implies $a = 0$ or $b = 0$.
5. A division ring is a ring R that has multiplicative inverse for all nonzero $a \in R$.