## Algebra Definitions

## On Rings, Polynomials, and Fields

## 1 Section 16.1 - Rings

- 1. A ring R is a set that is closed under two binary operations, + and  $\times$ . The following conditions must also be satisfied:
  - (a) Additive commutativity.
  - (b) Additive associativity.
  - (c) Additive identity.
  - (d) Additive inverse.
  - (e) Multiplicative associativity.
  - (f) Multiplicative distributivity 1 & 2.
- 2. A ring with unity (or with identity) is a ring R that has multiplicative identity.
- 3. A commutative ring is a ring R that has multiplicative commutativity.
- 4. An integral domain is a commutative ring R with identity such that for all  $a, b \in R$  ab = 0 implies a = 0 or b = 0.
- 5. A division ring is a ring R that has multiplicative inverse for all nonzero  $a \in R$ .
- 6. A zero divisor of a commutative ring R is an  $a \in R$   $(a \neq 0)$  such that there exists a nonzero  $b \in R$  such that ab = 0.
- 7. The ring of quaternions is the set  $\mathbb{H} = \{a + b\hat{i} + c\hat{j} + d\hat{k} \mid a, b, c, d \in \mathbb{R}\}$ , where  $1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ,  $\hat{i} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ,  $\hat{j} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ ,  $\hat{k} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ .