Algebra Definitions

On Rings, Polynomials, and Fields

1. A ring R is a set that is closed under two binary operations, $+$ and $\times$. The following conditions must also be satisfied:

   (a) Additive commutativity.

   (b) Additive associativity.

   (c) Additive identity.

   (d) Additive inverse.

   (e) Multiplicative associativity.

   (f) Multiplicative distributivity 1 & 2.

2. A ring with unity (or with identity) is a ring R that has multiplicative identity.

3. A commutative ring is a ring R that has multiplicative commutativity.

4. An integral domain is a commutative ring R such that for all $a, b \in R$ $ab = 0$ implies $a = 0$ or $b = 0$.

5. A division ring is a ring R that has multiplicative inverse for all nonzero $a \in R$.