Math 113 Theorems.

- 1. **Prop.** The relation $\equiv \pmod{n}$ is an equivalence relation.
- 2. **Prop.** $\mathbb{Z}/n\mathbb{Z}$ has exactly *n* elements.
 - (a) **Prop 0.** If $i \in [j]$, then $j \in [i]$ (in $\mathbb{Z}/n\mathbb{Z}$).
 - (b) **Prop 1.** If $[i] \cap [j] \neq \emptyset$, then [i] = [j].
 - (c) **Prop 2.** If $i \neq j$ and $0 \le i, j \le n 1$, then $[i] \cup [j] = \emptyset$.
 - (d) **Prop 3.** Every $x \in \mathbb{Z}$ belongs to one of $[0], \dots, [n-1]$.
- 3. **Prop.** Addition is correctly (well-defined) defined on $\mathbb{Z}/n\mathbb{Z}$ by [a] + [b] = [a+b].
- 4. **Prop 3.17.** The identity element in any group is unique.
- 5. **Prop 3.18.** The inverse is unique for any element g in a group G.
- 6. **Prop 3.19.** For any $a, b \in G$, where G is a group, $(a \star b)^{-1} = b^{-1}a^{-1}$.
- 7. **Prop 3.20.** For any $g \in G$, where *G* is a group, then $(g^{-1})^{-1} = g$.