Math 113 Theorems.

1. **Prop.** The relation $\equiv \pmod{n}$ is an equivalence relation.

2. **Prop.** $\mathbb{Z}/n\mathbb{Z}$ has exactly n elements.

   (a) **Prop 0.** If $i \in [j]$, then $j \in [i]$ (in $\mathbb{Z}/n\mathbb{Z}$).

   (b) **Prop 1.** If $[i] \cap [j] \neq \emptyset$, then $[i] = [j]$.

   (c) **Prop 2.** If $i \neq j$ and $0 \leq i, j \leq n-1$, then $[i] \cup [j] = \emptyset$.

   (d) **Prop 3.** Every $x \in \mathbb{Z}$ belongs to one of $[0], \ldots, [n-1]$.

3. **Prop.** Addition is correctly (well-defined) defined on $\mathbb{Z}/n\mathbb{Z}$ by $[a] + [b] = [a+b]$.

4. **Prop 3.17.** The identity element in any group is unique.

5. **Prop 3.18.** The inverse is unique for any element g in a group G.

6. **Prop 3.19.** For any $a, b \in G$, where G is a group, $(a \star b)^{-1} = b^{-1}a^{-1}$.

7. **Prop 3.20.** For any $g \in G$, where G is a group, then $(g^{-1})^{-1} = g$.

8. **Theorem 5.1.** S_n is a group with $n!$ elements where the binary operation is the composition of maps.

9. **Prop 5.8.** Let σ and τ be two disjoint cycles in S_X. Then, $\sigma\tau = \tau\sigma$.

10. **Theorem 5.9.** Every permutation in S_n can be written as the product of disjoint cycles.

11. **Prop 5.12.** Any permutation of a finite set containing at least 2 elements can be written as the product of transpositions.

12. **Lemma 5.14.** If the identity is written as the product of r transpositions, $\mathrm{id} = \tau_1 \ldots \tau_r$, then r is even.

13. **Theorem 5.15.** If a permutation σ can be expressed as the product of an even number of transpositions, then any other product of transpositions equaling σ must also contain an even number of transpositions. Similarly, in the case of when σ is odd.

14. **Prop 3.30.** A subset H of G is a subgroup iff:

(a) $e \in G$ also satisfies $e \in H$.
(b) If $h_1, h_2 \in H$, then $h_1 h_2 \in H$.
(c) If $h \in H$, then $h^{-1} \in H$.

15. **Prop 3.31.** Let H be a subset of a group G. Then, H is a subgroup of G iff $H \neq \emptyset$ and if $g, h \in H$, then $gh^{-1} \in H$.

16. **Theorem 4.3.** Take a group G and an element $a \in G$. Consider a cyclic subgroup $\langle a \rangle$. Then, $\langle a \rangle$ is a minimal subgroup of G such that a is in it (minimality: if H is a subgroup of G and $a \in H$, then $\langle a \rangle$ is a subgroup of H).

17. **Theorem 4.9.** Every cyclic group is abelian.

18. **Prop 11.4.** Let $\phi : G \to H$ be a homomorphism. Then:

    (a) $\phi(e_G) = e_H$.
    (b) $\phi(g^{-1}) = (\phi(g))^{-1)}$ for all $g \in G$.
    (c) If $K \leq G$, then $\phi(K) := \{\phi(k) \mid k \in K\}$ is a subgroup of H.
    (d) $\phi(G) := \{\phi(g) \mid g \in G\}$ (the image of ϕ) is a subgroup of H.
    (e) If $M \leq H$, then $\phi^{-1}(M) := \{g \in G \mid \phi(g) \in M\}$ is a subgroup of G.