Math 113 Definitions.

1. **Set.** A set is an unordered collection of elements.

2. **Map.** A map from X to Y is $f : X \to Y$ (a rule that assigns elements to Y to elements in X). So, for any $x \in X$ there exists a unique $y \in Y$ such that $f(x) = y$.

3. **Cartesian Product.** The Cartesian product of X and Y is the set $X \times Y = \{(x,y) \mid x \in X, y \in Y\}$.

4. **Equivalence Relation.** An equivalence relation $R, \sim$ on X is a subset $R \subseteq X \times X$ such that

   (a) Reflexive. ($(x,x) \in R$ for all $x \in X$).

   (b) Symmetric. (if $(x,y) \in R$, then $(y,x) \in R$).

   (c) Transitive. (if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$).

5. **Equivalence Class.** Let X be a set and R be an equivalence relation on X. Then, an equivalence class of $x \in X$ is the set $[x] = [x]_R = [x]_\sim = \{a \in X \mid x \sim a\}$.

6. $\mathbb{Z}/m\mathbb{Z}$. The set of distinct equivalence classes of $\equiv \pmod n$ is $\mathbb{Z}/m\mathbb{Z}$.

7. **Group.** A group G (denote: $(G, \star)$) is a set G with a closed binary operation $\star : G \times G \to G$ such that:

   (a) Associativity: $(a \star b) \star c = a \star (b \star c)$ for all $a, b, c \in G$.

   (b) Identity: There exists an $e \in G$ such that for any $a \in G$, we have $a \star e = e \star a = a$.

   (c) Inverse: For any $a \in G$, there exists an $a^{-1} \in G$ such that $a \star a^{-1} = a^{-1} \star a = e$.