Math 185 - Quiz 9 Definitions & Theorems

1. **Lemma 0.1.** z_0 is a pole of f iff there is a punctured disc $D_r(z_0) \setminus \{z_0\}$ on which $|f|$ is bounded below (by an $R > 0$) and unbounded above.
2. **Casorati-Weierstrauss.** Let $f : \Omega \to \mathbb{C}$ with $z_0 \in \Omega$. Also let f have an essential singularity at z_0 and f is holomorphic on $D_r(z_0) \setminus \{z_0\}$. Then, $f\left(D_r(z_0) \setminus \{z_0\}\right)$ is dense in $\mathbb{C}$.
3. **Meromorphic.** Let $f : \Omega \to \mathbb{C}$. f is <u>meromorphic</u> if it is holomorphic on Ω except at a discrete set of poles.
4. **Residue Theorem.** Let $\Omega \subseteq \mathbb{C}$ be open and connected and let γ be a simple closed curve in Ω homotopic to a point in Ω. Then, let f be holomorphic on Ω except at a set of points $z_1, \ldots, z_N$ inside γ. Then, we obtain the following:

$$\int_\gamma f(z)dz = 2\pi i \sum_{k=1}^{N} \mathrm{Res}_{z_k}\left(f\right).$$