Finite Intersection of Open Sets

Proposition. The intersection of finitely many open sets is open.

Proof. Let the open sets be $E_1, \ldots, E_n$. In the first case, if their intersection is $\emptyset$, then the proposition is vacuously true and we are done. Now assume $\cap_{i=1}^n E_i \neq \emptyset$ and put $I := \cap_{i=1}^n E_i$. Let $z \in I$. Since each E_i is open, each point inside E_i is an interior point (for all i). Then, z is an interior point of each E_i. Then, put ∂I as the boundary (contour) of I where z lies inside the region enclosed by ∂I. Put $R := \min\{|z-m|\}$ for each $m \in \partial I$. By assumption, $R > 0$ since if otherwise, we would be in the first case. Then, the open disc $D_{R/2}(z) \subsetneq I$ and so z is an interior point of I. Since z was arbitrary, it follows that every point in I is an interior point and we are done. $\square$.