Theorem. Let $K \subseteq \mathbb{C}$ compact, $f : K \to \mathbb{C}$ continuous, f holomorphic on K. Then, $\sup_{z\in K}|f(z)| = \sup z \in \partial K|f(z)|$.

Theorem. Let Ω be open & connected, $f : \Omega \to \mathbb{C}$ holomorphic, $z_0 \in \Omega$, $|f(z_0)| = \sup_{z\in\Omega}$. Then, f constant. Remark: Apply to e^f and $|e^f| = e^{\text{Re}f}$ and get $\text{Re}f(z_0) = \sup_{z\in\Omega}\text{Re}f(z)$ which implies that f is constant. Apply similar to logic to $\text{Im}f$ (replacing f by $-if$).