

Math H110 Definitions.

1. **Endomorphism.** An endomorphism is a group homomorphism from a set to itself (NOTE: does not have to be invertible.)
2. **End  $V$ .** The symbol  $\text{End } V$  is the set of all endomorphisms on  $V$  (and multiplication on  $\text{End } V$  is defined to be function composition).
3. **F-Module.** An  $F$ -module is a generalization of vector spaces over rings.
4. **Subspace.** Let  $V$  be a vector space.  $X$  is a subspace of  $V$  if  $X \subseteq V$  and closed under all relevant operations of  $V$ ,  $X \neq \emptyset$ , and  $X \ni 0$ .
5. **Linear Map / Linear Transformation.** Let  $V$  be a vector space over a field  $F$  with  $v, w \in V$ . Let  $T$  be a map on  $V$  with  $T(v + w) = T(v) + T(w)$  and  $T(\lambda v) = \lambda T(v)$  for all  $\lambda \in F$ . Then,  $T$  is called a linear map or linear transformation.
6. **Linear Operator.** If  $T$  is a linear transformation on a vector spaces  $V$  with  $T : V \rightarrow V$ , then  $T$  is linear operator on  $V$ .
7. **Spans.** The list  $v_1, \dots, v_n$  spans  $V$  iff  $T : F^n \rightarrow V$  is onto.
8. **Linearly Independent.** The list  $v_1, \dots, v_n$  is linearly independent iff  $T : F^n \rightarrow V$  is 1-1. Equivalently, the list  $v_1, \dots, v_n$  is linearly independent if  $\lambda_1 v_1 + \dots + \lambda_n v_n = 0$  implies  $\lambda_i = 0$  for all  $i$ .
9. **Linearly Dependent.** The list  $v_1, \dots, v_n$  is linearly dependent iff  $\lambda_1 v_1 + \dots + \lambda_n v_n = 0$  implies  $\lambda_i \neq 0$  for some  $i$ .
10. **Basis.** The list  $v_1, \dots, v_n$  is a basis of  $V$  if  $\text{span}\{v_1, \dots, v_n\} = V$  and  $v_1, \dots, v_n$  is linearly independent.
11. **Finite-dimensional.**  $V$  is finite-dimensional if  $V$  is spanned by a finite list of vectors.
12. **Sum of Subspaces.** Let  $X_1, \dots, X_t$  be subspaces of  $V$ . Then, we define their sum as  $X_1 + \dots + X_t = \{x_1 + \dots + x_t \mid x_1 \in X_1, \dots, x_t \in X_t\}$ .
13. **Direct Sum of Subspaces.** Let  $X_1, \dots, X_t$  be subspaces of  $V$ . Then, their direct sum,  $X_1 \oplus \dots \oplus X_t$ , is given by a 1-1 linear map  $T$ , with  $T : X_1 \times \dots \times X_t \rightarrow V$ .

14. **Complement of Subspace.** Let  $X, Y$  be subspaces of  $V$ . Then,  $Y$  is a complementary subspace of  $X$  iff  $X + Y = V$  and  $X \cap Y = \{0\}$ .
15. **Rank, Nullity.** The rank of a linear map is the dimension of the range of the linear map. The nullity is the dimension of the null space of the linear map.
16. **Null Space.** The null space is the set of vectors that are mapped to 0.
17. **Isomorphic Vector Spaces.** Two vector spaces  $V, W$  are isomorphic if there exists a linear map  $T : V \rightarrow W$  that is 1-1 and onto.
18. **Quotient Space.** Suppose  $U$  is a subspace of  $V$ . Then, the quotient space  $V/U$  is the set  $V/U = \{v + U \mid v \in V\}$ .