Math H110 Definitions.

1. **Endomorphism.** An endomorphism is a group homomorphism from a set to itself (NOTE: does not have to be invertible.)

2. **End V.** The symbol $\operatorname{End} V$ is the set of all endomorphisms on V (and multiplication on $\operatorname{End} V$ is defined to be function composition).

3. **F-Module.** An $F-$module is a generalization of vector spaces over rings.

4. **Subspace.** Let V be a vector space. X is a subspace of V if $X \subseteq V$ and closed under all relevant operations of V, $X \neq \emptyset$, and $X \ni 0$.

5. **Linear Map / Linear Transformation.** Let V be a vector space over a field F with $v, w \in V$. Let T be a map on V with $T(v+w) = T(v) + T(w)$ and $T(\lambda v) = \lambda T(v)$ for all $\lambda \in F$. Then, T is called a linear map or linear transformation.

6. **Linear Operator.** If T is a linear transformation on a vector spaces V with $T : V \to V$, then T is linear operator on V.

7. **Spans.** The list $v_1, \dots, v_n$ spans V iff $T : F^n \to V$ is onto.

8. **Linearly Independent.** The list $v_1, \dots, v_n$ is linearly independent iff $T : F^n \to V$ is 1-1. Equivalently, the list $v_1, \dots, v_n$ is linearly independent if $\lambda_1 v_1 + \cdots + \lambda_n v_n = 0$ implies $\lambda_i = 0$ for all i.

9. **Linearly Dependent.** The list $v_1, \dots, v_n$ is linearly dependent iff $\lambda_1 v_1 + \cdots + \lambda_n v_n = 0$ implies $\lambda_i \neq 0$ for some i.

10. **Basis.** The list $v_1, \dots, v_n$ is a basis of V if $\operatorname{span}\{v_1, \dots, v_n\} = V$ and $v_1, \dots, v_n$ is linearly independent.

11. **Finite-dimensional.** V is finite-dimensional if V is spanned by a finite list of vectors.

12. **Sum of Subspaces.** Let $X_1, \dots, X_t$ be subspaces of V. Then, we define their sum as $X_1 + \cdots + X_t = \{x_1 + \cdots + x_t \mid x_1 \in X_1, \dots, x_t \in X_t\}$.

13. **Direct Sum of Subspaces.** Let $X_1, \dots, X_t$ be subspaces of V. Then, their direct sum, $X_1 \oplus \cdots \oplus X_t$, is given by a 1-1 linear map T, with $T : X_1 \times \cdots \times X_t \to V$.

14. **Complement of Subspace.** Let X, Y be subspaces of of V. Then, Y is a complementary subspace of X iff $X+Y=V$ and $X+Y=X \oplus Y$.

15. **Rank, Nullity.** The rank of a linear map is the dimension of the range of the linear map. The nullity is the dimension of the null space of the linear map.

16. **Null Space.** The null space is the set of vectors that are mapped to 0.

17. **Isomorphic Vector Spaces.** Two vector spaces V, W are isomorphic if there exists a linear map $T: V \to W$ that is 1-1 and onto.

18. **Quotient Space.** Suppose U is a subspace of V. Then, the quotient space V/U is the set $V/U = \{v+U \mid v \in V\}$.

19. **Column Rank.** The column rank (rank of the column span of a matrix) is defined to be $\operatorname{rank} T_A$.

20. **Conjugation.** Let A be an $n \times n$ matrix (over F) and let Q be an $n \times n$ matrix (over F). Then, the conjugation of A by Q is $Q^{-1}AQ$.

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21. **Dual Space.** Let V be an F-vector space. Then the dual space of V is $V' = \mathscr{L}(V, F)$ where the elements of V' are called linear functionals.

22. **Annihilator.** For a subspace $U \subseteq V$, we define the annihilator of U to be $U_0 = \{\phi \in V' \mid \phi(u) = 0 \forall u \in U\}$.

23. **Double Dual.** Let V be a finite-dimensional vector space with dual V'. Then the double dual of V is $(V')' = V'' = V$. Also, $\dim V = n = \dim V' = \dim V''$.