Math H110 Theorems.

1. **Lemma.** Let F be a field, $\lambda \in F$, V a vector space over F (denoted by V/F), $v \in V$. Then, if $\lambda v = 0$, then $\lambda = 0$ or $v = 0$.

2. **Lemma.** A vector space over a field is a module over a field.

3. **Theorem.** The intersection of a family of subspaces of a vector space V is a subspace of V.

4. **Lemma.** Let $S = \{v_1, \ldots, v_t\}$. Then the subspace of all linear combinations of the elements of S is the span$S$.

5. **Theorem.** Let $L = v_1, \ldots, v_n$ be a list of vectors in a vector space V over a field F and let $T : F^n :\to V$ be linear transformation with $(\lambda_1, \ldots, \lambda_n) \mapsto \lambda_1 v_1 + \cdots + \lambda_n v_n$. Then, we have the following:

   (a) L spans V iff T is onto.

   (b) L is linearly independent iff T is 1-1 iff $\operatorname{nul} T = \{0\}$.

   (c) L is a basis iff T is 1-1 and onto.