

Math H110 Definitions.

1. **Endomorphism.** An endomorphism is a group homomorphism from a set to itself (NOTE: does not have to be invertible.)
2. **End V .** The symbol $\text{End } V$ is the set of all endomorphisms on V (and multiplication on $\text{End } V$ is defined to be function composition).
3. **F-Module.** An F -module is a generalization of vector spaces over rings.
4. **Subspace.** Let V be a vector space. X is a subspace of V if $X \subseteq V$ and closed under all relevant operations of V , $X \neq \emptyset$, and $X \ni 0$.
5. **Linear Map / Linear Transformation.** Let V be a vector space over a field F with $v, w \in V$. Let T be a map on V with $T(v + w) = T(v) + T(w)$ and $T(\lambda v) = \lambda T(v)$ for all $\lambda \in F$. Then, T is called a linear map or linear transformation.
6. **Linear Operator.** If T is a linear transformation on a vector spaces V with $T : V \rightarrow V$, then T is linear operator on V .
7. **Spans.** The list v_1, \dots, v_n spans V iff $T : F^n \rightarrow V$ is onto.
8. **Linearly Independent.** The list v_1, \dots, v_n is linearly independent iff $T : F^n \rightarrow V$ is 1-1. Equivalently, the list v_1, \dots, v_n is linearly independent if $\lambda_1 v_1 + \dots + \lambda_n v_n = 0$ implies $\lambda_i = 0$ for all i .
9. **Linearly Dependent.** The list v_1, \dots, v_n is linearly dependent iff $\lambda_1 v_1 + \dots + \lambda_n v_n = 0$ implies $\lambda_i \neq 0$ for some i .
10. **Basis.** The list v_1, \dots, v_n is a basis of V if $\text{span}\{v_1, \dots, v_n\} = V$ and v_1, \dots, v_n is linearly independent.
11. **Finite-dimensional.** V is finite-dimensional if V is spanned by a finite list of vectors.
12. **Sum of Subspaces.** Let X_1, \dots, X_t be subspaces of V . Then, we define their sum as $X_1 + \dots + X_t = \{x_1 + \dots + x_t \mid x_1 \in X_1, \dots, x_t \in X_t\}$.
13. **Direct Sum of Subspaces.** Let X_1, \dots, X_t be subspaces of V . Then, their direct sum, $X_1 \oplus \dots \oplus X_t$, is given by a 1-1 linear map T , with $T : X_1 \times \dots \times X_t \rightarrow V$.

14. **Complement of Subspace.** Let X, Y be subspaces of V . Then, Y is a complementary subspace of X iff $X + Y = V$ and $X \cap Y = \{0\}$.
15. **Rank, Nullity.** The rank of a linear map is the dimension of the range of the linear map. The nullity is the dimension of the null space of the linear map.
16. **Null Space.** The null space is the set of vectors that are mapped to 0.
17. **Isomorphic Vector Spaces.** Two vector spaces V, W are isomorphic if there exists a linear map $T : V \rightarrow W$ that is 1-1 and onto.
18. **Quotient Space.** Suppose U is a subspace of V . Then, the quotient space V/U is the set $V/U = \{v + U \mid v \in V\}$.
19. **Column Rank.** The column rank (rank of the column span of a matrix) is defined to be $\text{rank } T_A$.