Math H110 Midterm 1 CheatSheet

1A. (n/a)

1B.

1. **Vector Space.** A vector space V is a set that has scalar multiplication and vector addition defined on it with the following properties:
   (a) Additive commutativity.
   (b) Additive associativity of vectors ($u+(v+w)=(u+v)+w$) and multiplicative associativity for scalars ($(ab)v=a(bv)$).
   (c) Additive identity.
   (d) Additive inverses.
   (e) Multiplicative identity.
   (f) BOTH distributive properties.
2. **V-space (unique additive identity)** A vector space has a unique additive identity.
3. **V-space (unique additive inverses)** Every element in a vector space has a unique additive inverse.

1C.

1. **Subspace.** A subset $U \subseteq V$ is a subspace of V if it is a vector space with the same additive identity, scalar multiplication, and vector addition as defined on V.
2. **Conditions for a Subspace.** A subset $U \subseteq V$ is a subspace of V iff U is closed under vector addition, scalar multiplication, and contains the ”zero” element as in V.
3. **Sums of Subspaces.** Let $V_1,\dots,V_n$ be subspaces of V. Then, we have the sum of subspaces as $V_1+\cdots+V_n=\{v_1+\cdots+v_n \mid v_i \in V_i \text{ for all } i\}$.
4. **Smallest subspace containing each subspace** Suppose $V_1,\dots,V_n$ are subspaces of V. Then, $V_1+\cdots+V_n$ is the smallest subspace of V containing $V_1,\dots,V_n$.
5. **Direct Sum.** Suppose $V_1,\dots,V_m$ are subspaces of V. Then:

(a) The sum $V_1 + \cdots + V_m$ is direct if each element of $V_1 + \cdots + V_m$ can be written uniquely as a sum $v_1 + \cdots + v_m$, where $v_i \in V_i$ for all i.

(b) If $V_1 + \cdots + V_m$ is a direct sum, then we write $V_1 \oplus \cdots \oplus V_m$.

6. **Conditions for a direct sum.** Suppose $V_1, \ldots, V_n$ are subspaces of V. Then, $V_1 + \cdots + V_n$ is direct iff the only way to write 0 from $v_1 + \cdots + v_n$ is by taking $v_i = 0$ for all i.

7. **Direct sum of subspaces.** If U, W are subspaces of V, then $U + W$ is direct iff $U \cap W = \{0\}$.