RCSJ model using python

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I. CURRENT BIASING

We model the Josephson junction in the simplest way possible: A series network of a resistor R, a capacitor C and a nonlinear inductor. The total current running through the network is

$$I(t) = I_s(t) + I_R(t) + I_C(t) \tag{1}$$

$$I(t) = I_c \sin(\phi) + \tfrac{V}{R} + C\frac{\mathrm{d}V}{\mathrm{d}t} \tag{2}$$

$$V(t) = \tfrac{\hbar}{2e}\frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{3}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \tfrac{\hbar}{2e}\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} \tag{4}$$

The equation is simplified with normalized time $\tau = \omega_p t$, where $\omega_p = \sqrt{\frac{2eI_c}{\hbar C}}$ and quality factor $Q = \omega_p RC$ to

$$\frac{\mathrm{d}^2\gamma}{\mathrm{d}\tau^2} + \frac{1}{Q}\frac{\mathrm{d}\gamma}{\mathrm{d}\tau} + \sin(\gamma) = \frac{I}{I_c} \tag{5}$$

We can define two regimes: The **overdamped** case for $Q \ll 1$, and the **underdamped** case for $Q \gg 1$:

$$Q \ll 1: \frac{\mathrm{d}\gamma}{\mathrm{d}\tau} \approx \frac{I}{I_c} - \sin(\gamma) \rightarrow V = R\sqrt{I^2 - I_c^2} \tag{6}$$

$$Q \gg 1: \quad \frac{\mathrm{d}^2\gamma}{\mathrm{d}\tau^2} \approx \frac{2e}{\hbar}V + const. \rightarrow V = RI \tag{7}$$

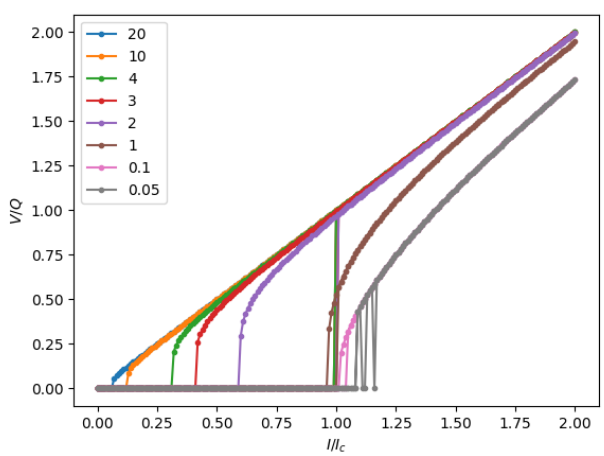

Figure 1. Current-voltage curves for various damping cases. As predicted by theory, high damping ($Q \ll 1$) corresponds to a square-root like IVC with no hysteresis, while low damping ($Q \gg 1$) results in strong hystersis with a almost linear retrapping branch.

II. VOLTAGE BIASING

REFERENCES