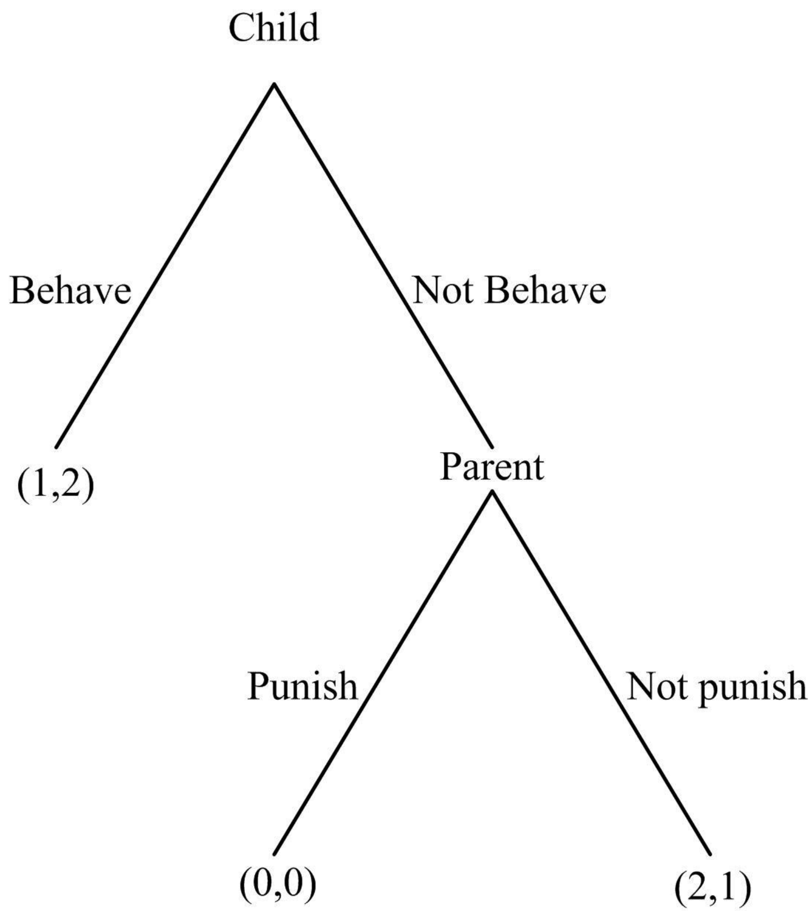
Chapter 8: DYNAMIC GAMES OF COMPLETE INFORMATION: Credibility and Sequential Rationality



Normal form and Nash equilibrium?

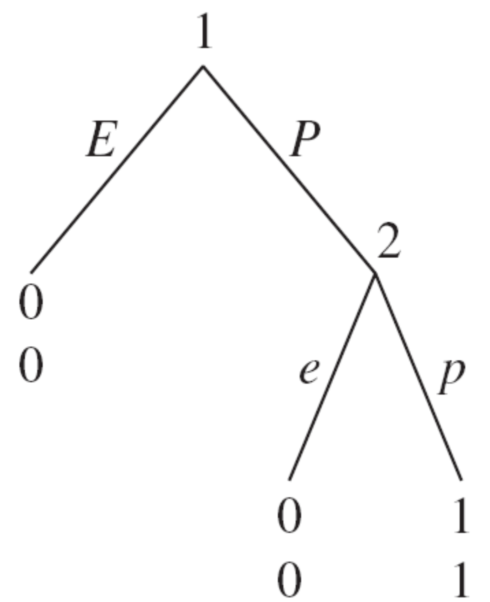


FIGURE 8.1 A coordination game.

Normal form and Nash equilibrium?

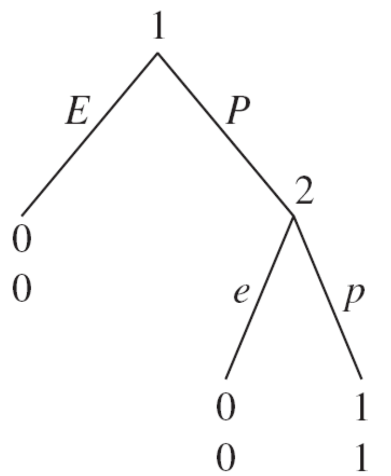


FIGURE 8.1 A coordination game.

	e	p
E	0, 0	0, 0
P	0, 0	1, 1

Consider the simple extensive-form coordination game. Each player can exit (*E* for player 1 and *e* for player 2), or proceed (*P* and *p*, respectively). If the players coordinate on proceeding, then they both receive a payoff of 1 while any other profile of strategies yields each a payoff of 0.

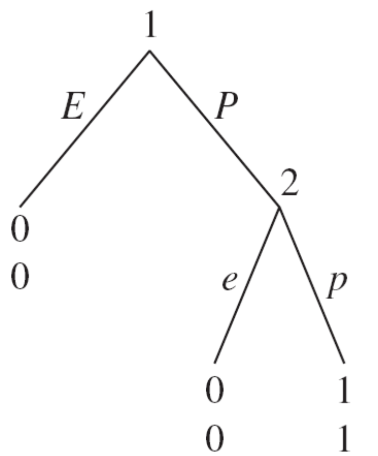


FIGURE 8.1 A coordination game.

	e	p
E	0, 0	0, 0
P	0, 0	1, 1

There is a good reason to assume that the players will successfully coordinate on the Nash equilibrium profile (P, p): the exit strategy is weakly dominated by the choice to proceed.

However, there is another Nash equilibrium: (E,e). The players stick to the equilibrium path of player 1 choosing E because of player 2's threat to choose e off the equilibrium path.

There is therefore something unappealing about the logic of Nash equilibrium in extensive-form games.

The concept asks only for players to act rationally on the equilibrium path given their beliefs about what will transpire both on and off the equilibrium path.

Nevertheless Nash equilibrium puts no restrictions on the beliefs of players off the equilibrium path, nor on how they should consider such beliefs. The normal-form representation of a sequential game is not able to address such a requirement, which adds constraints on what we would tolerate as "rational behavior."

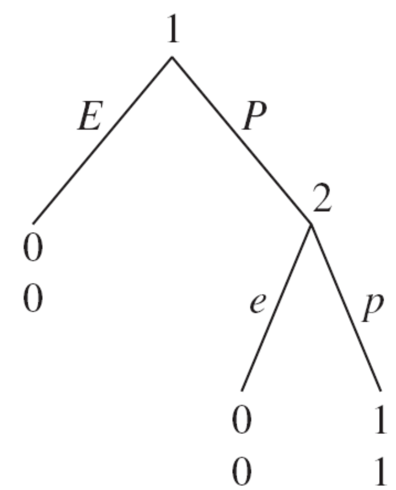


FIGURE 8.1 A coordination game.

We would expect rational players to *play optimally in response to their beliefs whenever they are called to move*. In the simple coordination game in Figure 8.1 this would require that player 2 commit to play *p* when it is his turn to move.

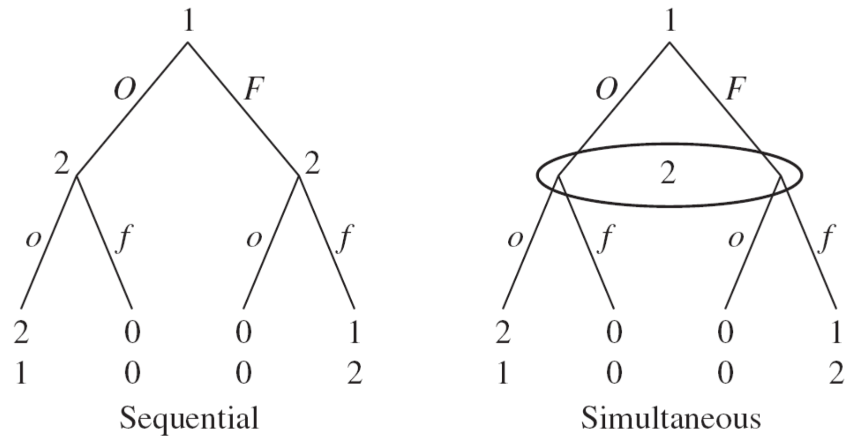


FIGURE 7.11 The Battle of the Sexes game: two versions.

Consider the sequential-move Battle of the Sexes game. Recall that $S_1 = \{O, F\}$ and $S_2 = \{oo, of, fo, ff\}$, where *fo*, for example, means that player 2 plays *f* after player 1 plays *O*, while player 2 plays *o* after player 1 plays *F*.

Player 2

Player 1		*oo*	*of*	*fo*	*ff*
	O	2, 1	2, 1	0, 0	0, 0
	F	0, 0	1, 2	0, 0	1, 2

Applying this reasoning to the sequential-move Battle of the Sexes game suggests that of the three Nash equilibria of the game, two are unappealing.

Both (O, oo) and (F, ff) have player 2 committing to a strategy that, despite being a best response to player 1's strategy, would not have been optimal were player 1 to deviate from his strategy and cause the game to move off the equilibrium path.

In what follows we will introduce a natural requirement that will result in more refined predictions for dynamic games.

8.1 Sequential Rationality and Backward Induction

To address the critique we posed regarding the incredible nature of the equilibria (*O*, *oo*) and (*F*, *ff*) in the sequential-move Battle of the Sexes game, we will insist that a player use strategies that are optimal at every information set in the game tree. We call this principle sequential rationality, because it implies that players are playing rationally at every stage in the sequence of the game, whether it is on or off the equilibrium path of play.

Definition 8.1 Given strategies $\sigma_{-i} \in \Delta S_{-i}$ of i's opponents, we say that σ_i is **sequentially rational** if and only if i is playing a best response to σ_{-i} in each of his information sets.

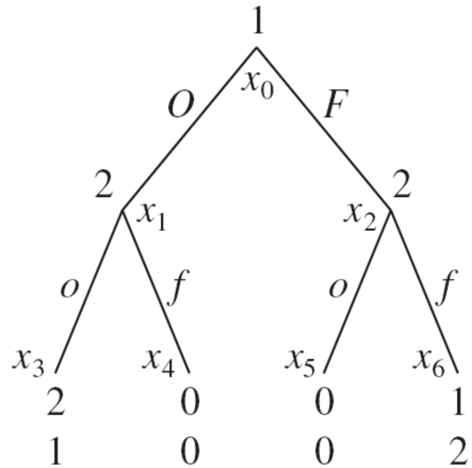


FIGURE 7.2 The sequential-move Battle of the Sexes game.

Using this definition we can revisit the sequential-move Battle of the Sexes game and ask: what are player 2's best responses in each of his information sets?

The answer is obvious: if player 1 played O then player 2 should play o, and if player 1 played F then player 2 should play f . Any other strategy of player 2 is not a best response in at least one of these information sets, and this implies that a sequentially rational player 2 should be playing the pure strategy *of*.

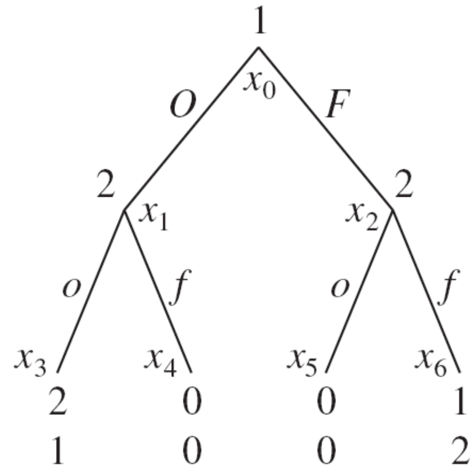


FIGURE 7.2 The sequential-move Battle of the Sexes game.

Taking into account the sequential rationality of player 2, player 1 should conclude that playing *O* will result in the payoffs (2, 1) while playing *F* will result in the payoffs (1, 2). Applying sequential rationality, player 1 should choose *O*. The unique prediction is the path of play that begins with player 1 choosing *O* followed by player 2 choosing *o*. Furthermore the process predicts what would happen if players deviate from the path of play: if player 1 chooses *F* then player 2 will choose *f*. The Nash equilibrium (*O*, *of*) is the unique pair of strategies that survives our requirement of sequential rationality.

By definition a finite game of perfect information has a finite set of terminal nodes. Consider the nodes that immediately precede a terminal node, and call the set of these nodes “level 1” nodes. Select for every player at a level 1 node an action that yields him his highest payoff. (There can be more than one if there are ties in the payoffs.)

Similarly define “level 2” nodes as those that immediately precede a level 1 node, and let the players at these nodes choose the action that maximizes their payoff given that level 1 players will choose their action as previously specified.

This process continues iteratively until we reach the root of the game (because the game has finite length) and results in a specification of moves that are sequentially rational.

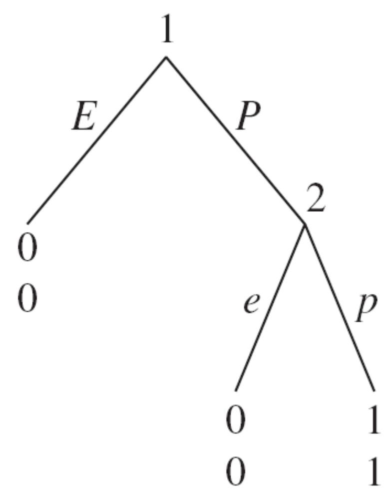


FIGURE 8.1 A coordination game.

This type of procedure, which starts at nodes that directly precede the terminal nodes at the end of the game and then inductively moves backward through the game tree, is known as **backward induction(逆向归纳) in games.** When we apply this procedure to finite games of perfect information it will result in a prescription of strategies for each player that are sequentially rational.

Proposition 8.1 *Any finite game of perfect information has a backward induction solution that is sequentially rational. Furthermore if no two terminal nodes prescribe the same payoffs to any player then the backward induction solution is unique.*

By the construction of the backward induction procedure, each player will necessarily play a best response to the actions of the other players who come after him (the best responses are constructed for every information set).

Corollary 8.1 *Any finite game of perfect information has at least one sequentially rational Nash equilibrium in pure strategies. Furthermore if no two terminal nodes prescribe the same payoffs to any player then the game has a unique sequentially rational Nash equilibrium.*

8.2 Subgame-Perfect Nash Equilibrium: Concept

As the previous section argues, we expect rational players to play in ways that are sequentially rational. Furthermore, backward induction is a useful method to find a sequentially rational Nash equilibrium in finite games of perfect information.

Things become a bit trickier when we try to expand our reach to suggest solutions for games of imperfect information, in which backward induction as previously defined encounters some serious problems.

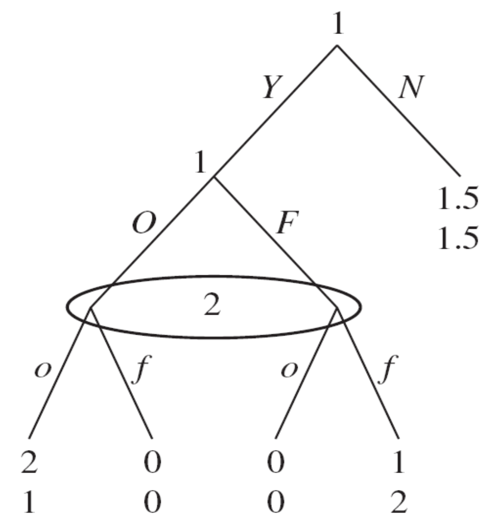


FIGURE 8.2 The voluntary Battle of the Sexes game.

Consider, for example, a game in which player 1 decides whether or not to play a Battle of the Sexes game with player 2. He can decide yes (Y), in which case they play a simultaneous-move Battle of the Sexes game, or no (N), in which case both players get a payoff of 1.5.

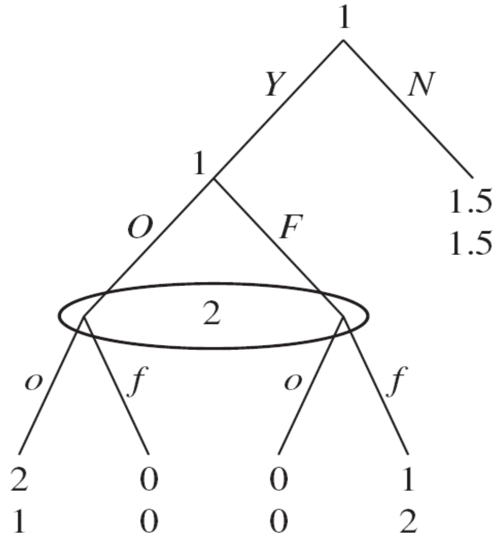


FIGURE 8.2 The voluntary Battle of the Sexes game.

To try to solve this game using backward induction we need to first identify the set of "last players" that precede terminal nodes and then choose actions that would maximize their payoff at this stage. This is not possible, because player 2 has an information set before the terminal nodes that is not a singleton. His best response is therefore not well defined without assigning a belief to this player about what player 1 actually chose to do, and these beliefs are not part of the backward induction process.

Definition 8.2 A proper subgame G of an extensive-form game Γ consists of only a single node and all its successors in Γ with the property that if $x \in G$ and $x' \in h(x)$ then $x' \in G$. The subgame G is itself a *game tree* with its information sets and payoffs inherited from Γ.

The idea of a proper subgame (which we will often just call a subgame) is simple and allows us to “dissect” an extensive-form game into a sequence of smaller games, an approach that will allow us to apply the concept of sequential rationality to games of imperfect information. To be able to do this, we will require that every such smaller game be an extensive-form game in its own right.

In every game of perfect information, every node is a singleton and hence can be a root of a subgame.

In games of perfect information every node, together with all the nodes that succeed it, forms a proper subgame.

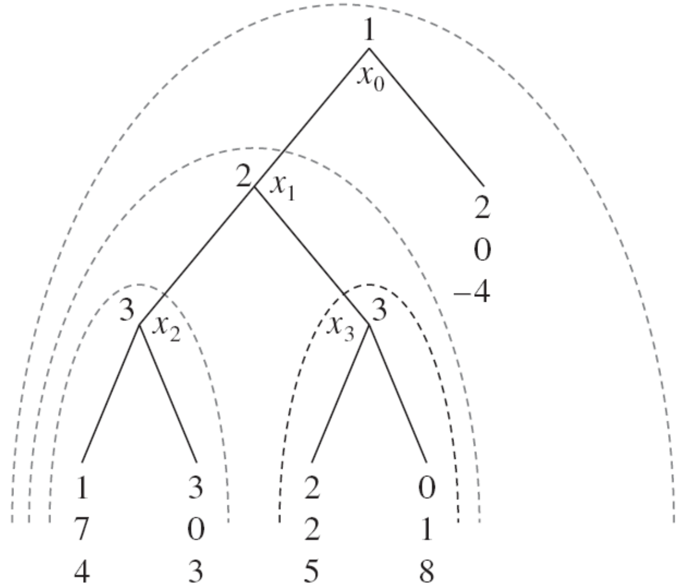


FIGURE 8.3 Subgames in a game with perfect information.

Consider the game depicted in Figure 8.3.

The two "smallest" subgames start at nodes x_2 and x_3.

A"larger" subgame starts at x_1, and it includes the two subgames that start at x_2 and x_3.

Finally the "largest" subgame starts at the original game's root, x_0, and includes all the other subgames.

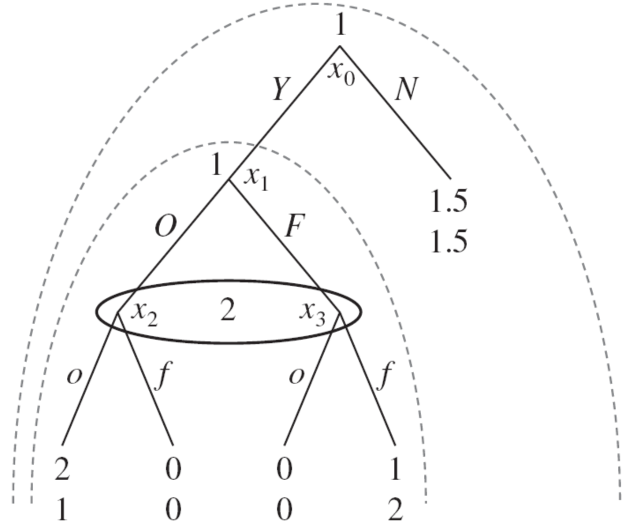


FIGURE 8.4 Proper subgames in the voluntary Battle of the Sexes game.

For the voluntary Battle of the Sexes game, there are only two proper subgames in this game: the whole game and the subgame starting at the node x_1.

Note that x_2 and x_3 cannot be roots of a subgame because they belong to the same information set. If x_2 belongs to a subgame then x_3 must belong to that subgame. Any subgame must begin with a single node, and thus the information set that contains x_2 and x_3 cannot begin a subgame.

Players 1 and 2 put a dollar each in a pot, and player 1 pulls a card out of a deck of kings and aces, with an equal probability of getting a king (K) or an ace (A).

Player 1 observes his card and then decides whether not to play the game (N), and forfeit his dollar to player 2, or proceed with the game (Y).

If player 1 proceeds with the game, then without knowing which card player 1 drew player 2 can fold (F) and forfeit his dollar to player 1 or call (C), in which case each player must add another dollar to the pot.

After this, if player 1 has a king then player 2 wins the pot, while if player 1 has an ace then player 1 wins the pot.

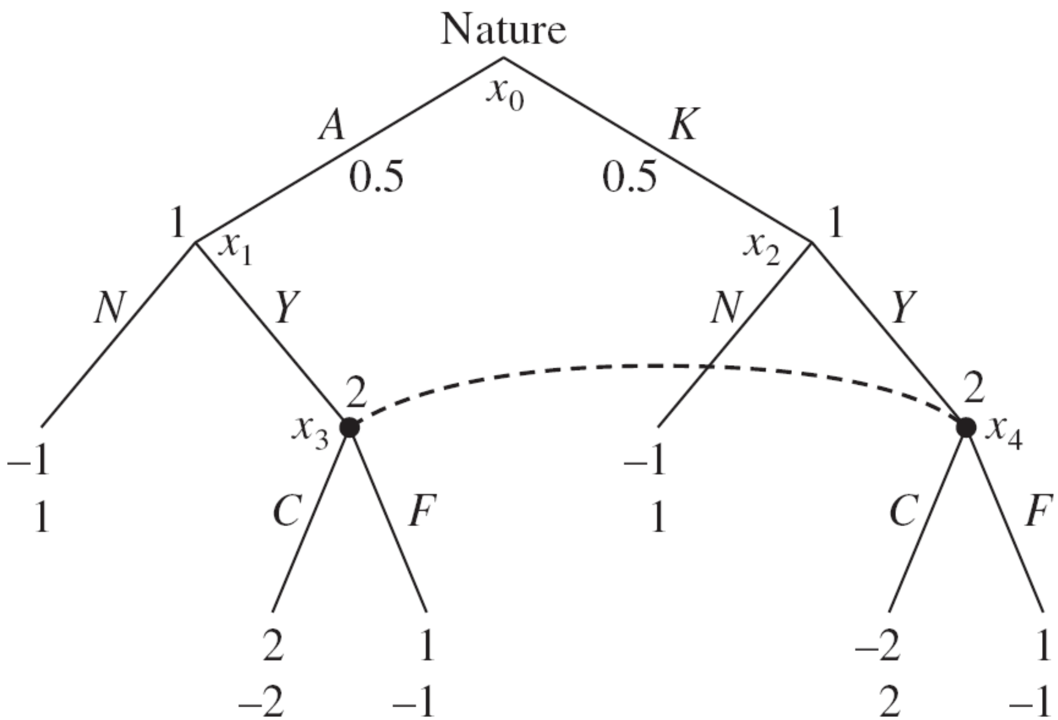


FIGURE 8.5 A game of cards.

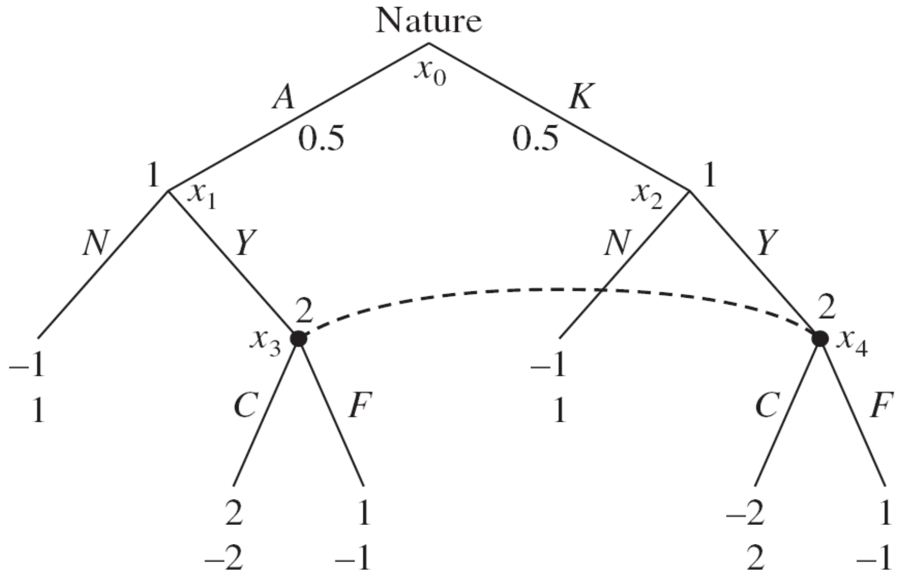


FIGURE 8.5 A game of cards.

Neither x_3 nor x_4 can be a root of a subgame because they belong to the same information set.

Can x_1 be the root of a subgame? If it could be then x_3 must be in its subgame because x_1 precedes x_3. But by definition, if x_3 is in the subgame then $x_4 \in h(x_3)$, and therefore it should be in the subgame. But this would not be a proper subgame because x_1 does not precede x_4. Similarly x_2 cannot be the root of the subgame. The only proper subgame is the whole game.

At any node or information set within a subgame *G*, a player's best response depends only on his beliefs about what the other players are doing within the subgame *G*, and not at nodes that are outside the subgame.

Definition 7.6 A **behavioral strategy** specifies for each information set $h_i \in H_i$ an independent probability distribution over $A_i(h_i)$ and is denoted by $\sigma_i : H_i \to \Delta A_i(h_i)$, where $\sigma_i(a_i(h_i))$ is the probability that player i plays action $a_i(h_i) \in A_i(h_i)$ in information set h_i.

A behavioral strategy is more in tune with the dynamic nature of the extensive-form game. When using such a strategy, a player mixes among his actions whenever he is called to play. This differs from a mixed strategy, in which a player mixes before playing the game but then remains loyal to the selected pure strategy.

Luce and Raiffa (1957) provide a nice analogy for the different strategy types we have introduced.

A pure strategy can be thought of as an instruction manual in which each page tells the player which pure action to take at a particular information set, and the number of pages is equal to the number of information sets the player has. The set S_i of pure strategies can therefore be treated like a library of such pure-strategy manuals.

A mixed strategy consists of choosing one of these manuals at random and then following it precisely.

In contrast a behavioral strategy is a manual that prescribes possibly random actions on each of the pages associated with play at particular information sets.

Definition 8.3 Let Γ be an n-player extensive-form game. A behavioral strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*)$ is a **subgame-perfect (Nash) equilibrium** if for every proper subgame G of Γ the restriction of σ^* to G is a Nash equilibrium in G.

This important concept was introduced by Reinhard Selten (1975), who was the second of the three Nobel Laureates sharing the prize in 1994 for the development of game theory. This equilibrium concept brings sequential rationality into the static Nash equilibrium solution concept.

Subgame perfection requires not only that a Nash equilibrium profile of strategies be a combination of best responses on the equilibrium path, which is a necessary condition of a Nash equilibrium, but also that the profile of strategies consist of mutual best responses off the equilibrium path.

This is precisely what follows from the requirement that the restriction of the strategy profile σ^* be a Nash equilibrium in every proper subgame, including those subgames that are not reached in equilibrium.

By the definition, every subgame perfect equilibrium is a Nash equilibrium. However, not all Nash equilibria are necessarily subgame-perfect equilibria, implying that *subgame-perfect equilibrium refines the set of Nash equilibria, yielding more refined predictions on behavior.*

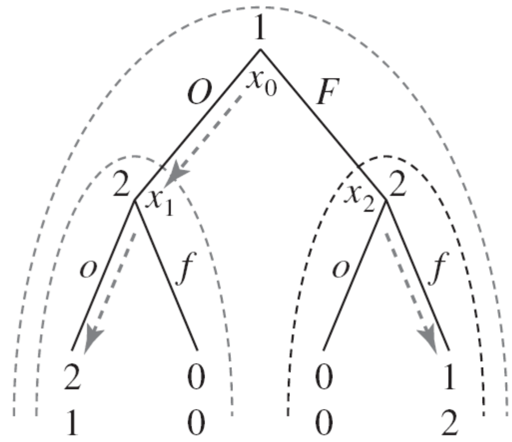


FIGURE 8.6 Subgame-perfect equilibrium in the sequential-move Battle of the Sexes game.

Consider the sequential-move Battle of the Sexes game and its corresponding normal-form, given as:

	oo	*of*	*fo*	*ff*
O	2, 1	2, 1	0, 0	0, 0
F	0, 0	1, 2	0, 0	1, 2

There are three pure-strategy Nash equilibria, (*O*, *oo*), (*F*, *ff*), and (*O*, *of*). Of these three, only (*O*, *of*) is the unique subgame-perfect equilibrium.

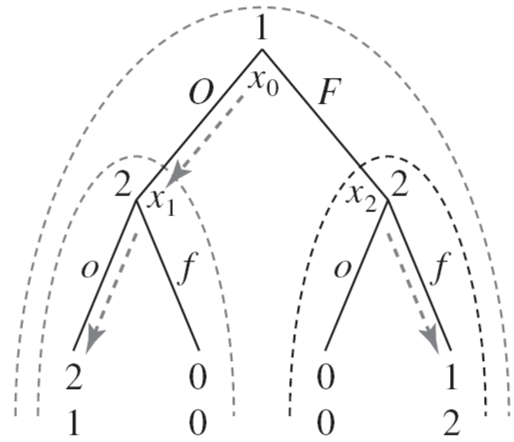


FIGURE 8.6 Subgame-perfect equilibrium in the sequential-move Battle of the Sexes game.

In the subgame beginning at x_1, the only Nash equilibrium is player 2 choosing *o*, because he is the only player in that subgame and he must choose a best response to his belief, which must be "I am at x_1." Similarly in the subgame beginning at x_2, the only Nash equilibrium is player 2 choosing *f* . Anticipating this, player 1 must choose O at x_0. Thus of the three Nash equilibria of the game, only (*O*, *of*) satisfies the condition that its restriction is a Nash equilibrium for every proper subgame of the whole game, and hence (*O*, *oo*) and (*F*, *ff*) are Nash equilibria that are not subgame perfect.

Notice that the game we just analyzed is a finite game of perfect information. For these games we have an easy and familiar way to find the subgame-perfect equilibria, which is using the procedure of backward induction because

Fact *For any finite game of perfect information, the set of subgame-perfect Nash equilibria coincides with the set of Nash equilibria that survive backward induction.*

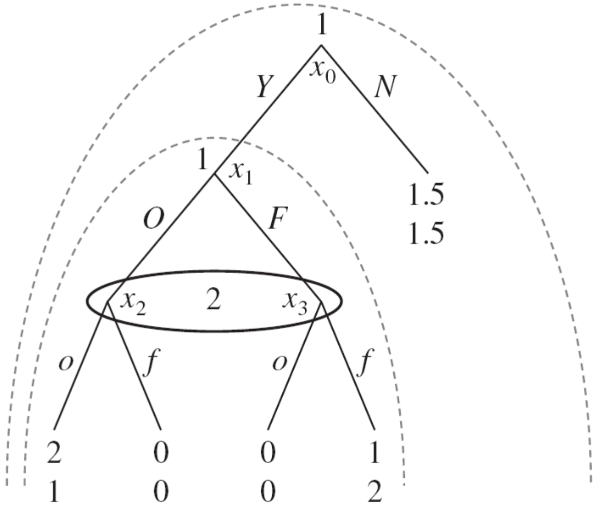


FIGURE 8.4 Proper subgames in the voluntary Battle of the Sexes game.

Nash equilibria?

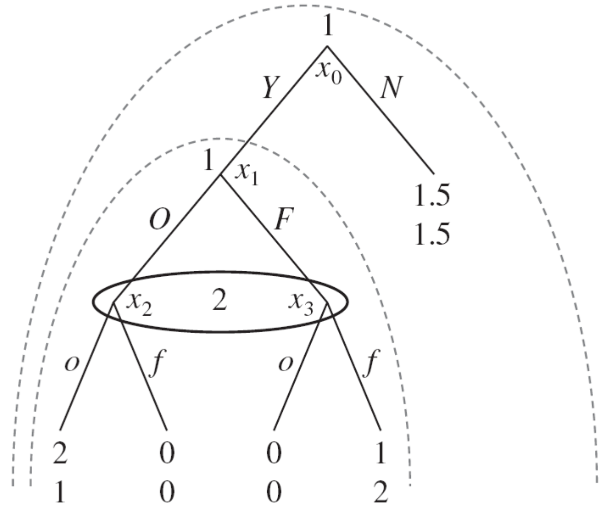


FIGURE 8.4 Proper subgames in the voluntary Battle of the Sexes game.

Consider the voluntary Battle of the Sexes game depicted. In this game player 1 has four pure strategies. The strategy set for player 1 is $S_1 = \{YO, YF, NO, NF\}$, where, for example, *YO* means that player 1 plans to play *Y* at x_0 and *O* at x_1. Note that if player 1 plays either *NO* or *NF* then, regardless of player 2's strategy, the game will end after player 1's choice of *N*. Player 2, on the other hand, has only two strategies, $S_2 = \{o, f\}$, because he must make his choice without knowing the choice of player 1.

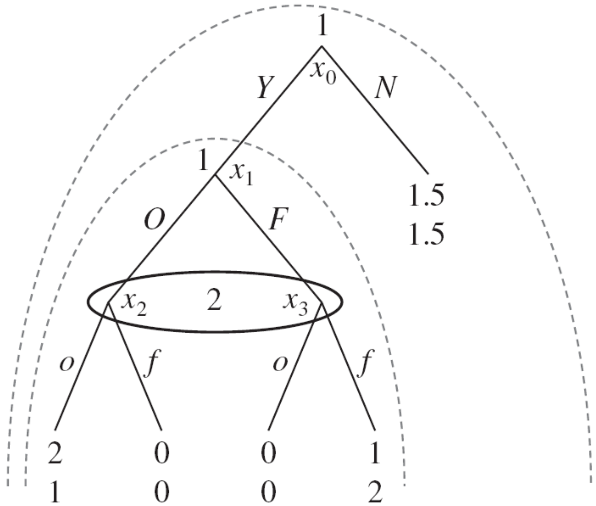


FIGURE 8.4 Proper subgames in the voluntary Battle of the Sexes game.

Player 2

Player 1		*o*	*f*
	YO	2, 1	0, 0
	YF	0, 0	1, 2
	NO	1.5,1.5	1.5,1.5
	NF	1.5,1.5	1.5,1.5

There are three pure-strategy Nash equilibria in this game, given by the set $E^{Nash} = \{(YO, o), (NO, f), (NF, f)\}$. Of these only two pairs of strategies form a subgame-perfect equilibrium, so the set of subgame-perfect equilibria profiles is $E^{SPE} = \{(YO, o), (NF, f)\}$. The reason is that in the subgame that starts at the node x_1, the only pairs of restricted strategies that form a Nash equilibrium are (O, o) and (F, f).

逆向归纳法

定义一个子博弈的长度为此子博弈中最长历史的长度。

逆向归纳法按照下述步骤进行：

- 对于长度为1的子博弈，确定采取行动参与者的最优动作（“最后”的子博弈）。
- 给定这些动作，在长度为2的子博弈中，确定采取行动参与者的最优动作。
- 继续此过程，直到回到博弈的开始。给定在所有更短长度子博弈中参与者的最优动作，在阶段k，对长度为k的子博弈，确定采取行动参与者的最优动作。

在此算法的每个阶段k，采取行动参与者的最优动作很容易确定：给定所有更短长度子博弈中参与者的最优动作，它们只是产生最高收益的那些动作。

8.3 Subgame-Perfect Nash Equilibrium: Examples

8.3.1 The Centipede Game

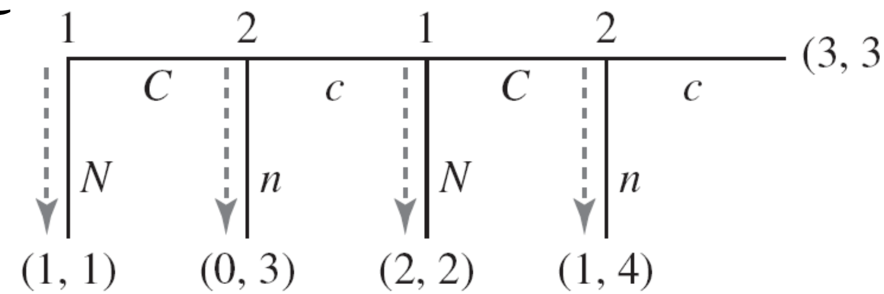


FIGURE 8.8 The Centipede Game.

The game should be read from left to right as follows: Player 1 can terminate the game immediately by choosing *N* in his first information set or can continue by choosing *C*. Then player 2 faces the same choice (using lowercase letters for his choices), and if player 2 chooses to continue then the ball is back in player 1's court, who again can terminate or continue to player 2, at which stage player 2 concludes the game by choosing *n* or *c* for the second time.

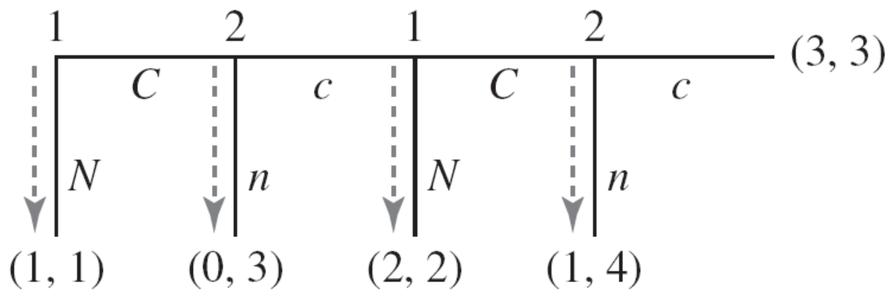


FIGURE 8.8 The Centipede Game.

It would be nice for the players to be able to continue through the game to reach the payoffs of (3, 3). However, backward induction indicates that this will not happen.

At his last information set, player 2 will choose *n* to get 4 instead of 3. Anticipating this a step earlier, player 1 will choose *N* to get 2 instead of 1, and the logic follows until player 1's first information set, at which he will choose *N* and both players will receive a payoff of 1.

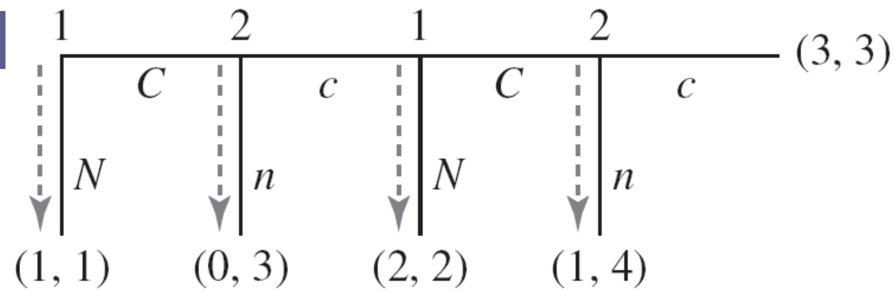


FIGURE 8.8 The Centipede Game.

Notice that this game has an interesting structure: as long as the players continue, the sum of their payoffs goes up by 1. You can easily see that we can continue with the payoffs in this way with (2, 5), (4, 4), (3, 6), . . . and make the payoff from reaching the end of the game extremely large.

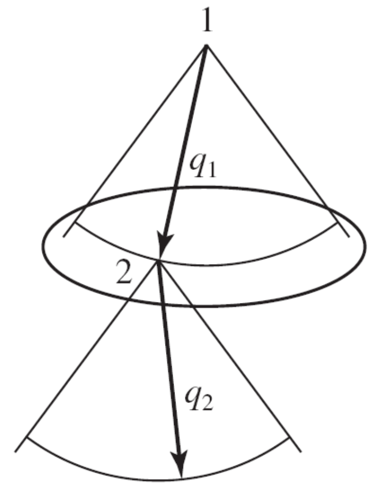
Nevertheless the "curse of rationality," predicts a unique outcome: at the last stage the last player will, by being selfish, prefer to stop short of the payoff that maximizes the sum of the players' payoffs, and backward induction implies that this decision is anticipated and acted on by the player before him, and so on as the actions unravel to the bleak outcome of (1, 1).

Remark The Centipede Game was first introduced by Rosenthal (1981), who used it as an example of a game in which the unique backward induction solution is extremely unappealing and goes against every intuition one might have about how the players will play. Indeed this game has been taken into the lab by letting pairs of players play it, and the experimental evidence of many studies, starting with McKelvey and Palfrey (1992), suggests that players do not play in the way predicted by backward induction. There are at least two reasons that players will not play the backward induction solution. One is that they actually care about each other's payoffs. This explanation may be less convincing for cases in which the players are anonymous, but it is not easy to rule out. The other reason is that players do not share a common knowledge of rationality. Palacios-Huerta and Volij (2009) try to put this second hypothesis to a test by taking the game to a chess tournament and having highly ranked chess players play it. Contrary to previous evidence, their results show that 69% of chess players choose to end the game immediately, and when grand masters are playing, all of them end the game immediately! This striking result suggests that when players are expected to share common knowledge of rationality they indeed play the backward induction solution.

8.3.2 Stackelberg Competition

The Stackelberg duopoly is a game of perfect information that is a sequential-moves variation on the Cournot duopoly model of competition. It was introduced and analyzed by Heinrich von Stackelberg (1934).

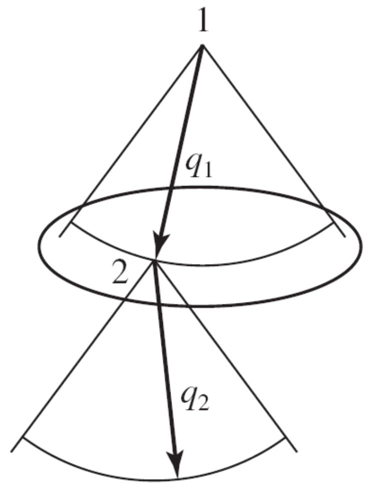
Consider our familiar Cournot game with demand $p = 100 - q$, $q = q_1 + q_2$, and $c_i(q_i) = 10q_i$ for $i \in \{1, 2\}$.



$$(100 - q_1 - q_2)q_1 - 10q_1$$
$$(100 - q_1 - q_2)q_1 - 10q_1$$

FIGURE 8.10 The Cournot duopoly game.

We can heuristically depict the simultaneous-move Cournot game. We draw player 2's information set to include all the choices of q_1 that player 1 can make to try to describe the simultaneous nature of the Cournot game in that player 2 makes its choice without observing the choice of player 1.



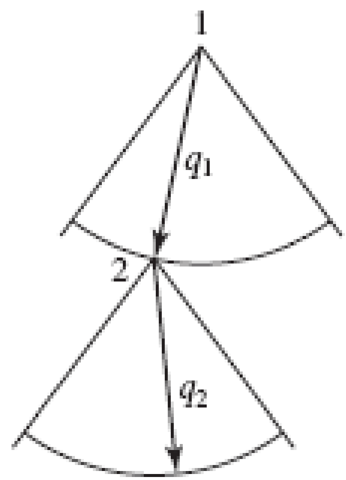
$$(100 - q_1 - q_2)q_1 - 10q_1$$
$$(100 - q_1 - q_2)q_1 - 10q_1$$

FIGURE 8.10 The Cournot duopoly game.

Nash equilibria?

Because every subgame-perfect equilibrium is a Nash equilibrium, and because the whole game is the only unique proper subgame in the Cournot example, then $q_1 = q_2 = 30$ is the unique subgame-perfect equilibrium in the Cournot game.

Assume that player 1 will choose q_1 first, and that player 2 will observe the choice made by player 1 before it makes its choice of q_2. We begin by analyzing the backward induction solution of this game.



From the fact that player 2 maximizes its profit when q_1 is already known it should be clear that player 2 will follow its best-response function, and hence sequential rationality implies that

$$q_2(q_1) = \frac{90 - q_1}{2}. \tag{8.1}$$

This is precisely the backward induction conclusion of considering firm 2 first, because its moves are those that precede the terminal nodes.

Notice that in this game there are infinitely many terminal nodes and infinitely many information sets preceding the terminal nodes.

In particular, every different choice of q_1 is a different information set of player 2.

Nevertheless, because there is perfect information, and because there is a well-defined optimal choice of player 2 for every choice of player 1, the backward induction procedure works.

Now assuming common knowledge of rationality, what should player 1 do? It would be rather naive to maximize its profits based on some fixed belief about q_2, because firm 1 knows exactly how a rational player 2 would respond to its choice of q_1: player 2 will choose q_2 using equation (8.1).

$$q_2(q_1) = \frac{90 - q_1}{2}. \tag{8.1}$$

This in turn means that a rational firm 1 would replace the "fixed" q_2 in its profit function with the best response of firm 2 and choose q_1 to solve

$$\max_{q_1} \left[100 - q_1 - \left(\frac{90 - q_1}{2}\right)\right] q_1 - 10q_1. \tag{8.2}$$

The solution for firm 1 is then given by the first-order condition of (8.2), which is

$$100 - 2q_1 - 45 + q_1 - 10 = 0,$$

yielding $q_1 = 45$. Then, using firm 2's best response, we have $q_2 = 22.5$. The resulting profits are

$$\pi_1 = (100 - 67.5) \times 45 - 10 \times 45 = 1012.5,$$

and

$$\pi_2 = (100 - 67.5) \times 22.5 - 10 \times 22.5 = 506.25.$$

Recall that in the original (simultaneous-move) Cournot example the quantities and profits were $q_1 = q_2 = 30$ and $\pi_1 = \pi_2 = 900$. We conclude that when two firms are competing by setting quantities, if one firm can somehow commit to move first, it will enjoy a first-mover advantage.

There is a simple yet very important point worth emphasizing here. When we write down the pair of strategies as a pure-strategy Nash equilibrium we must be careful to specify the strategies correctly: player 2 has a continuum of information sets, each being a particular choice of q_1. This implies that the backward induction solution yields the following Nash equilibrium:

$$(q_1, q_2(q_1)) = \big(45, (90-q_1)/2\big).$$

Writing down $(q_1, q_2) = (45, 22.5)$ is not identifying a Nash equilibrium because $q_1 = 45$ is not a best response to $q_2 = 22.5$. We must specify q_2 for every information set for it to be a well-defined strategy in the Stackelberg game, and in this case we defined it to be

$$q_2(q_1)= (90-q_1)/2.$$

8.4 **Industry Leader:** Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$, and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The firms choose their quantities dynamically as follows: Firm 1, which is the industry leader, chooses $q_1 \geq 0$. Firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively.

a. How many proper subgames does this dynamic game have? Explain briefly.

b. Is it a game of perfect or imperfect information? Explain briefly.

c. What is the subgame-perfect equilibrium of this game? Show that it is unique.

d. Find a Nash equilibrium that is not a subgame-perfect equilibrium.

8.5 **Technology Adoption:** During the adoption of a new technology a CEO (player 1) can design a new task for a division manager. The new task can be either high level (H) or low level (L). The manager simultaneously chooses to invest in good training (G) or bad training (B). The payoffs from this interaction are given by the following matrix:

		Player 2	
		G	B
Player 1	H	5, 4	−5, 2
	L	2, −2	0, 0

a. Present the game in extensive form (a game tree) and solve for all the Nash equilibria and subgame-perfect equilibria.

b. Now assume that before the game is played the CEO can choose not to adopt this new technology, in which case the payoffs are (1, 1), or to adopt it, in which case the game is played. Present the *entire* game in extensive form. How many proper subgames does it have?

c. Solve for all the Nash equilibria and subgame-perfect equilibria of the game described in (b).

8.9 **Entry Deterrence 2:** Consider the Cournot duopoly game with demand $p = 100 - (q_1 + q_2)$ and variable costs $c_i(q_i) = 0$ for $i \in \{1, 2\}$. The twist is that there is now a fixed cost of production $k > 0$ that is the same for both firms.

a. Assume first that both firms choose their quantities simultaneously. Model this as a normal-form game.

b. Write down the firm's best-response function for $k = 1000$ and solve for a pure-strategy Nash equilibrium. Is it unique?

c. Now assume that firm 1 is a "Stackelberg leader" in the sense that it moves first and chooses q_1. Then after observing q_1 firm 2 chooses q_2. Also assume that if firm 2 cannot make strictly positive profits then it will not produce at all. Model this as an extensive-form game tree as best you can and find a subgame-perfect equilibrium of this game for $k = 25$. Is it unique?

d. How does your answer in (c) change for $k = 225$?

8.10 **Playing It Safe:** Consider the following dynamic game: Player 1 can choose to play it safe (denote this choice by S), in which case both he and player 2 get a payoff of 3 each, or he can risk playing a game with player 2 (denote this choice by R). If he chooses R then they play the following simultaneous-move game:

		Player 2	
		A	B
Player 1	C	8, 0	0, 2
	D	6, 6	2, 2

a. Draw a game tree that represents this game. How many proper subgames does it have?
b. Are there other game trees that would work? Explain briefly.
c. Construct the matrix representation of the normal form of this dynamic game.
d. Find all the Nash and subgame-perfect equilibria of the dynamic game.