PROPOSITIONAL EQUIVALENCES

Section 1.2 (Chapter 1.3)

REVIEW EXERCISES

- **Inclusive/Exclusive Or**
  - Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense
    - She has one or two brothers.
- **Conditional and bicondictional propositions**
  - Write it in English in the form “If ... then ... .” or “… if and only of…”:
    - I will buy the tickets only if you call.”
    - To be able to go on the trip, it is necessary that you get written permission.”
    - You need a ticket in order to enter the theater.
    - No shoes, no shirt, no service.
    - It rains exactly when I plan a picnic.
- **Write the negation of “If it rains, I stay home.”**

SECTION SUMMARY

- Tautologies, Contradictions, and Contingencies.
- Logical Equivalence
  - Important Logical Equivalences
  - Showing Logical Equivalence
- Normal Forms
  - Disjunctive Normal Form
  - Conjunctive Normal Form

TAUTOLOGIES, CONTRADICTIONS, AND CONTINGENCIES 永真、矛盾和可能

- A tautology is a proposition which is always true.
  - Example: $p \lor \neg p$
- A *contradiction* is a proposition which is always false.
  - Example: $p \land \neg p$
- A *contingency* is a proposition which is neither a tautology nor a contradiction, such as p

P	$\neg p$	$p \lor \neg p$	$p \land \neg p$
T	F	T	F
F	T	T	F

LOGICALLY EQUIVALENT

- Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.
- We write this as $p \Leftrightarrow q$ or as $p \equiv q$ where p and q are compound propositions.
- Two compound propositions p and q are equivalent if and only if the columns in a truth table giving their truth values agree.
- This truth table show $\neg p \vee q$ is equivalent to $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

TRUTH TABLES FOR COMPOUND PROPOSITIONS

- Construction of a truth table:
- Rows
  - Need a row for every possible combination of values for the atomic propositions.
- Columns
  - Need a column for the compound proposition (usually at far right)
  - Need a column for the truth value of each expression that occurs in the compound proposition as it is built up.
    - This includes the atomic propositions

DE MORGAN'S LAWS

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's *Second* Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Please show proofs of the First Law.

KEY LOGICAL EQUIVALENCES

- Identity Laws: $p \wedge T \equiv p$, $p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$, $p \wedge F \equiv F$
- Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

KEY LOGICAL EQUIVALENCES (*CONT*)

- **Commutative Laws:** $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$
- **Associative Laws:** $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- **Distributive Laws:** $(p \vee (q \wedge r) \equiv (p \vee q)) \wedge (p \vee r)$
  $(p \wedge (q \vee r)) \equiv (p \wedge q) \vee (p \wedge r)$
- **Absorption Laws:** $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$

MORE LOGICAL EQUIVALENCES

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

CONSTRUCTING NEW LOGICAL EQUIVALENCES

- We can show that two expressions are logically equivalent by developing a series of logically equivalent statements.

- To prove that $A \equiv B$ we produce a series of equivalences beginning with *A* and ending with *B*.

$$\begin{gathered} A \equiv A_1 \\ \vdots \\ A_n \equiv B \end{gathered}$$

DISJUNCTIVE NORMAL FORM 析取范式

- Please fill in the compound proposition the meets the truth table.

p	*q*	**?**
T	T	**T**
T	F	**T**
F	T	**F**
F	F	**T**

DISJUNCTIVE NORMAL FORM 析取范式

- A propositional formula is in *disjunctive normal form* if it consists of a disjunction of $(1, \ldots, n)$ disjuncts where each disjunct consists of a conjunction of $(1, \ldots, m)$ atomic formulas or the negation of an atomic formula.
  - $(p \wedge \neg q) \vee (p \wedge q)$

    Yes
  - $p \wedge (p \vee q)$

    No
- Disjunctive Normal Form is important for the circuit design methods.

CONJUNCTIVE NORMAL FORM 合取范式

- A compound proposition is in *Conjunctive Normal Form* (CNF) if it is a conjunction of disjunctions.
- Every proposition can be put in an equivalent CNF.
- Conjunctive Normal Form (CNF) can be obtained by eliminating implications, moving negation inwards and using the distributive and associative laws.
- Important in resolution theorem proving used in artificial Intelligence (AI).
- A compound proposition can be put in conjunctive normal form through repeated application of the logical equivalences covered earlier.

PROPOSITIONAL SATISFIABILITY 命题可满足性

- A compound proposition is *satisfiable* if there is an assignment of truth values to its variables that make it true. When no such assignments exist, the compound proposition is *unsatisfiable*.

- A compound proposition is unsatisfiable if and only if its negation is a tautology.

SUDOKU

- A **Sudoku puzzle** is represented by a 9×9 grid made up of nine 3×3 subgrids, known as **blocks**. Some of the 81 cells of the puzzle are assigned one of the numbers 1,2, …, 9.
- The puzzle is solved by assigning numbers to each blank cell so that every row, column and block contains each of the nine possible numbers.
- Example

	2	9				4		
			5			1		
	4							
				4	2			
6							7	
5								
7			3					5
	1			9				
							6	