
村子里的路边上有很多的这样的台子，上的蓝色盒子是收钱的，谁要是想买什么蔬菜就按照标价，将钱放在盒子里。


10
J
Q
K
A
A

# Chapter 7： DYNAMIC GAMES OF COMPLETE INFORMATION: PRELIMINARIE

Consider the Battle of the Sexes game:

| | | Chris O | Chris F |
|---|---|---|---|
| Alex | O | 2, 1 | 0, 0 |
| | F | 0, 0 | 1, 2 |

but with a slight modification.

Imagine that Alex finishes work at 3:00 p.m. while Chris finishes work at 5:00 p.m.

This gives Alex ample time to get to either the football game or the opera and then to call Chris at 4:45 p.m. and announce “I am here.” Chris then has to make a choice of where to go.

Where should Chris go？

|  |  | Chris |  |
| --- | --- | --- | --- |
|  |  | O | F |
| Alex | O | 2, 1 | 0, 0 |
|  | F | 0, 0 | 1, 2 |

If the choice is to the venue where Alex is waiting then Chris will get some payoff. If Chris's choice is to go to the other venue, then he will get 0.

Hence a rational Chris should go to the same venue that Alex did. Anticipating this, Alex ought to choose the opera, because then Alex gets 2 instead of 1 from football.

In the simultaneous-move Battle of the Sexes game neither player knows what the other is choosing, so each player conjectures a belief and plays a best response to this belief.

Here, in contrast, when it’s his turn to make a move, Chris knows what Alex has done, and as a result the notion of conjecturing beliefs is moot.

Furthermore Alex knows, by common knowledge of rationality, that Chris will choose to follow Alex because it is Chris’s best response to do so.

As a result Alex can get what Alex wants. This is a simple yet convincing example of the commonly used phrase “first-mover advantage.” By moving first, Alex gets to set the evening’s venue.

Before we draw far-reaching conclusions about the generality of the first-mover advantage, consider the Matching Pennies game.

If I were to play with you, no matter what my position would be (player 1 or 2), I would be happy to give you the opportunity to show your coin first.

# 7.1 The Extensive-Form Game

In this section we derive the most common representation for games that unfold over time and in which some players move after they learn the actions of other players.

*The innovation is to allow the knowledge of some players, when it is their turn to move, to depend on the previously made choices of other players.*

As with the normal form, two elements must be part of any extensive form game's representation:

**1. Set of players, *N*.**
**2. Players' payoffs as a function of outcomes, $\{v_i(.)\}_{i \in N}$.**

Let's use the Battle of the Sexes example introduced earlier.

In the extensive form the set of players is still $N = \{1, 2\}$ (Alex is 1 and Chris is 2), and their payoffs over outcomes are given as before: $v_1(O, O) = v_2(F, F) = 2$, $v_1(F, F) = v_2(O, O) = 1$, and $v_i(O, F) = v_i(F, O) = 0$ for $i \in \{1, 2\}$.

We need to expand the rather simplistic concept of pure-strategy sets to a more complex organization of actions.

We do this by introducing two parts for actions: *First, what players can do, and second, when they can do it.*

Using our example, we need to specify that player 1 moves first and that it is only then, after learning what player 1 has chosen, that player 2 moves.

Thus in general we need two components to capture sequential play:

**3. Order of moves.**
**4. Actions of players when they can move.**

Because some players move after choices are made by other players, we need to be able to describe the knowledge that players have about the history of the game when it is their turn to move. *Recall that that the simultaneity of the normal form was illustrative of players who know nothing about their opponents' moves when they make their choices.*

To represent the way in which information and knowledge unfold in a game, we add a fifth component to the description of an extensive-form game:

**5. The knowledge that players have when they can move.**

We must account for the possibility that some random event can happen during the course of the game. Stages in a game in which some uncertainty is resolved are called *moves of Nature*. It is useful to think of Nature as a nonstrategic player that has a predetermined stochastic strategy. Thus our sixth element represents Nature as follows:

**6. Probability distributions over *exogenous events*.**

Finally, to be able to analyze these situations with the methods and concepts to which we have already been introduced, we add a final and familiar requirement:

**7. The structure of the extensive-form game represented by 1–6 is common knowledge among all the players.**

# 7.1.1 Game Trees

A game tree will offer a diagrammatic description that is suitable to represent extensive-form games.

Consider a game that is very common and falls under the general category of a “trust game.”

Player 1 first chooses whether to ask for the services of player 2. He can trust player 2 (T) or not trust him (N), the latter choice giving both players a payoff of 0.

If player 1 plays T, then player 2 can choose to cooperate (C), which represents offering player 1 some fair level of service, or defect (D), by which he basically cheats player 1 with an inferior, less costly to provide service.

Assume that if player 2 cooperates then both players get a payoff of 1, while if player 2 chooses to defect then player 1 gets a payoff of −1 and player 2 gets a payoff of 2.

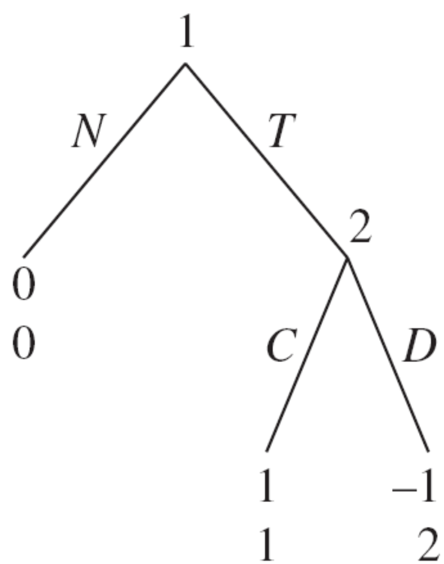



**FIGURE 7.1** A trust game.

*This game represents many real-life trading situations.*

A driver who trusts a mechanic to be honest and perform the right service for his vehicle rather than rip him off.

A buyer on an auction web site like eBay who trusts a seller by paying up front and hoping that the seller will deliver the described item rather than something inferior, or nothing at all.

A local farmer who puts up a roadside produce stand with a jar for money and relies on his customers to pay for the produce they take according to the “honor system.”

This very simple structure is the most elementary form of a game tree. It includes most of the elements we described.

*It still lacks the formal structure that would clearly delineate the "rules" that are used to describe such a game. Most importantly, how do we capture knowledge?*

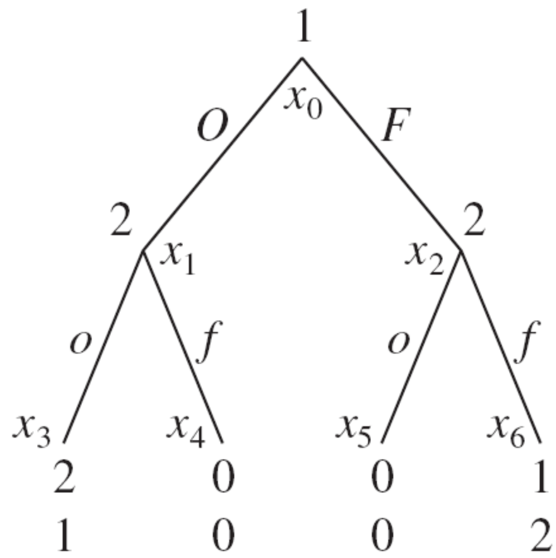



**FIGURE 7.2** The sequential-move Battle of the Sexes game.

For example, going back to the Battle of the Sexes game, in which Alex moved first, we may want to use the diagram in Figure 7.2 to represent it as a game tree. It starts at a node denoted $x_0$, at which player 1 can choose between $O$ and $F$. Then, depending on the choice of player 1, player 2 gets to move at either node $x_1$ or $x_2$ and make a choice between $o$ and $f$ .

How can we distinguish between the sequential case in which player 2 knows the move of player 1 and the simultaneous case in which player 2 moves after player 1 but is ignorant about player 1's move?

To address this concern formally, a certain amount of detail and notation needs to be introduced.

**Definition 7.1** A *game tree* is a set of nodes $x \in X$ with a precedence relation $x > x'$, which means "$x$ precedes $x'$." Every node in a game tree has only one predecessor. The precedence relation is *transitive* ($x > x'$, $x' > x'' \Rightarrow x > x''$), *asymmetric* ($x > x' \Rightarrow$ not $x' > x$), and *incomplete* (not every pair of nodes $x$, $y$ can be ordered). There is a special node called the *root* of the tree, denoted by $x_0$, that precedes any other $x \in X$. Nodes that do not precede other nodes are called *terminal nodes*, denoted by the set $Z \subset X$. Terminal nodes denote the final outcomes of the game with which payoffs are associated. Every node $x$ that is not a terminal node is assigned either to a player, $i(x)$, with the action set $A_i(x)$, or to Nature.

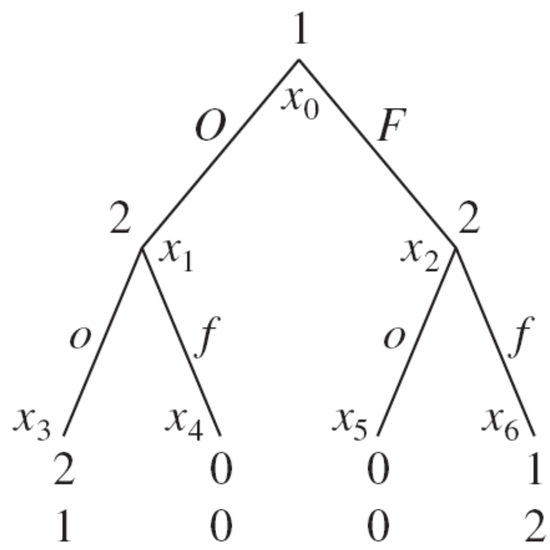



**FIGURE 7.2** The sequential-move Battle of the Sexes game.

Consider the Battle of the Sexes game in Figure 7.2. $x_0$ is where the game begins, and $x_0$ precedes both $x_1$ and $x_2$. Each of these nodes precedes two terminal nodes, each describing a different outcome of the game. Since the terminal nodes are the game's outcomes, payoffs to the two players are noted at the terminal nodes.

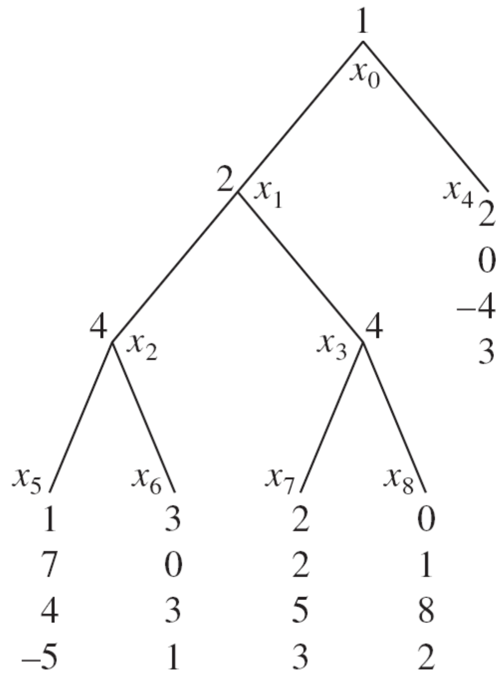



**FIGURE 7.3** A game tree with a "dummy" player.

Another example is given in Figure 7.3. In this game we have payoffs for four players, N = {1, 2, 3, 4}, but only players 1, 2, and 4 have actual moves. We can think of player 3 as a "dummy player." The terminal nodes are $Z = \{x_4, x_5, x_6, x_7, x_8\}$, and payoffs are defined over terminal nodes: $v_i : Z \rightarrow R$, where $v_i(z)$ is $i$'s payoff if terminal node $z \in Z$ is reached. For example, if node $x_5$ is reached, then player 2 gets $v_2(x_5) = 7$ and player 4 gets $v_4(x_5) = -5$.

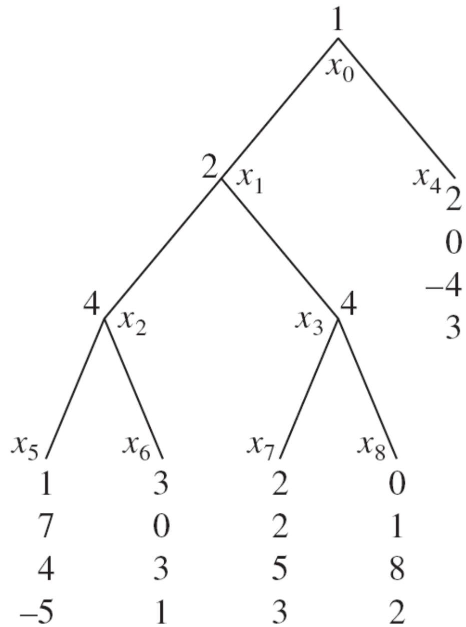



**FIGURE 7.3** A game tree with a “dummy” player.

The precedence relation, together with the way in which players are assigned to nodes, describes the way in which the game unfolds. Player 1 is assigned to the root, so $i(x_0) = 1$. His action set at the root, $A_1(x_0)$, includes two choices that determine whether the game will terminate at node $x_4$ with payoffs $(2, 0, -4, 3)$, or whether player 2 will get to play at node $x_1$. Player 2 then can choose whether player 4 will play at $x_2$ or $x_3$, and at each of these nodes player 4 has two choices that both end in termination of the game. Player 3 has no moves to make.

We proceed to put structure on *the information that a player has when it is his turn* to move. A player can have very fine information and know exactly where he is in the game tree, or he may have coarser information and not know what has happened before his move, therefore not knowing exactly where he is in the game tree. We introduce the following definition:

**Definition 7.2** Every player $i$ has a collection of information sets $h_i \in H_i$ that partition the nodes of the game at which player $i$ moves with the following properties:

1. If $h_i$ is a singleton that includes only $x$ then player $i$ who moves at $x$ knows that he is at $x$.
2. If $x \neq x'$ and if both $x \in h_i$ and $x' \in h_i$ then player $i$ who moves at $x$ does not know whether he is at $x$ or $x'$.
3. If $x \neq x'$ and if both $x \in h_i$ and $x' \in h_i$ then $A_i(x') = A_i(x)$.

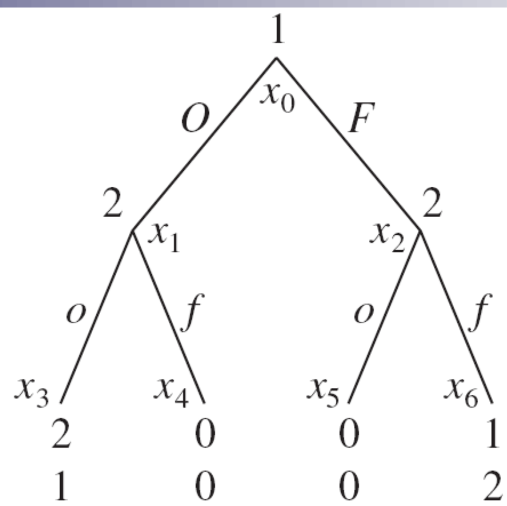



**FIGURE 7.2** The sequential-move Battle of the Sexes game.

Consider the sequential-move Battle of the Sexes game and observe that player 2 moves at $x_1$. We want to describe whether or not he knows that he is at $x_1$. If we write $h_2 = \{x_1\}$, this means that the information set at $x_1$ is a singleton (it includes only the node $x_1$). Hence player 2 has information that says "I am at $x_1$," which is captured by property (1) of the definition. In this case it will follow that player 2 will have another information set, $h'_2 = \{x_2\}$.

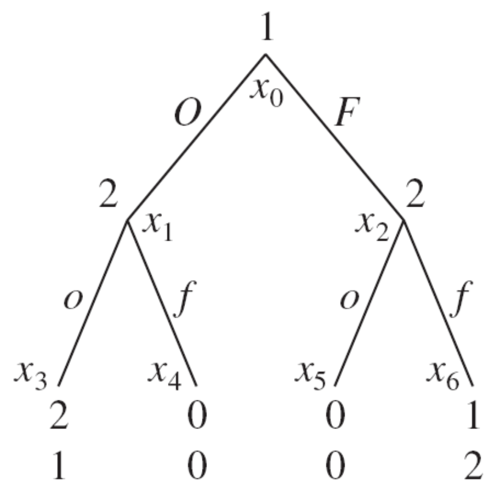



**FIGURE 7.2** The sequential-move Battle of the Sexes game.

If we want to represent a game in which player 2 does not know whether he is at $x_1$ or $x_2$, then it must be the case that his information is "I know that I am at either $x_1$ or $x_2$, but I don't know at which of the two I am." Thus we will write $h_2 = \{x_1, x_2\}$, which exactly means that player 2 cannot tell whether he is at $x_1$ or $x_2$.This is the essence of property (2) of the definition.

Finally, property (3) is also essential to maintain the logic of information. If instead $x \in h_i$ and $x' \in h_i$ but $A_i(x) \neq A_i(x')$ , then by the mere fact that player $i$ has different actions from which to choose at each of the nodes $x$ and $x'$, he should be able to distinguish between these two nodes. It would therefore be illogical to assume that he cannot distinguish between them.

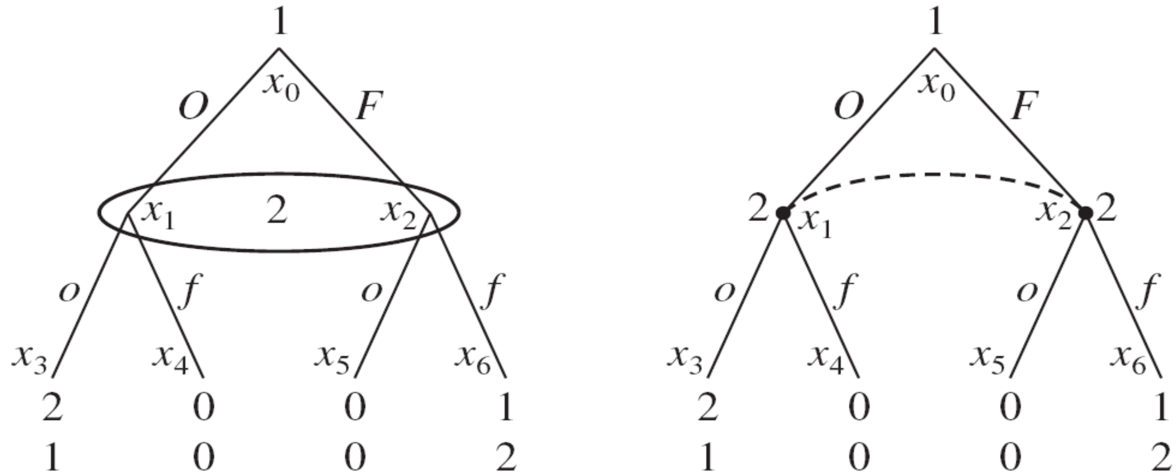



**FIGURE 7.4** The simultaneous-move Battle of the Sexes game.

We now construct a graphical representation to show which nodes belong in the same information set. In Figure 7.4 we present two common ways of depicting this. Player 1 chooses from $A_1 = \{O, F\}$, and player 2 chooses from $A_2 = \{o, f\}$, without observing the choice of player 1. On the left panel we use an ellipse to denote an information set, and all the nodes that are in the same ellipse belong to the same information set. In this example player 2 cannot distinguish between $x_1$ and $x_2$, so that $h_2 = \{x_1, x_2\}$. On the right panel is another common way of depicting information sets, according to which the dashed line connecting $x_1$ with $x_2$ denotes that both are in the same information set.

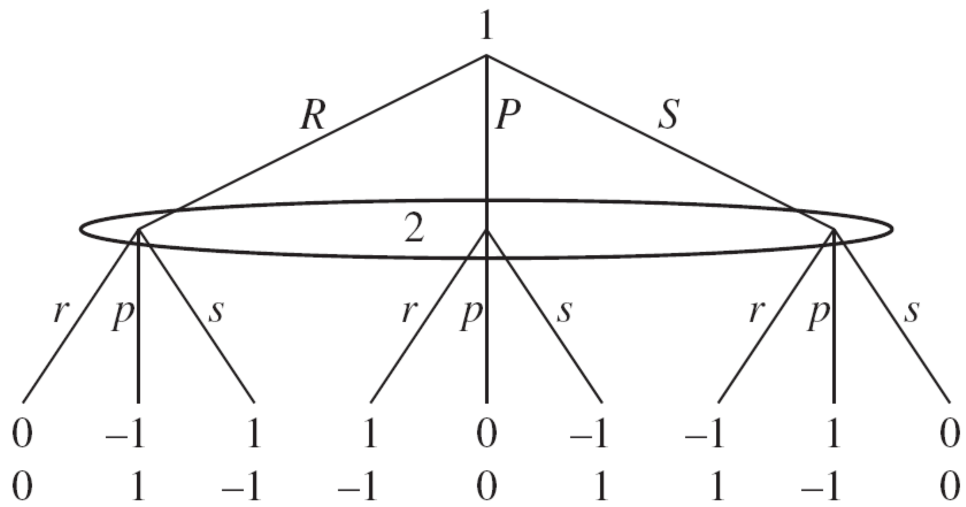



**FIGURE 7.5** Game tree of rock-paper-scissors.

Another example of a simultaneous-move game depicted as a game tree is the game of rock-paper-scissors, depicted in Figure 7.5. In this game player 1 chooses from the set $A_1 = \{R, P, S\}$ while player 2 chooses from the set $A_2 = \{r, p, s\}$, without observing the choice made by player 1.

# 7.1.2 Imperfect versus Perfect Information

For extensive-form games, it is useful to distinguish between two different types of complete-information games:

**Definition 7.3** A game of complete information in which every information set is a singleton and there are no moves of Nature is called a **game of perfect information.** A game in which some information sets contain several nodes or in which there are moves of Nature is called a **game of imperfect information.**

In a game of perfect information every player knows exactly where he is in the game by knowing what occurred before he was called on to move. Examples would be the trust game in Figure 7.1 and the sequential-move Battle of the Sexes game in Figure 7.2.

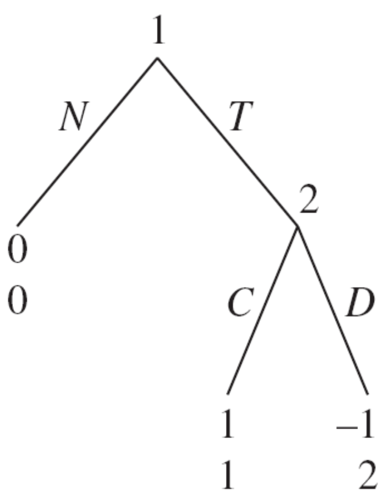



**FIGURE 7.1** A trust game.

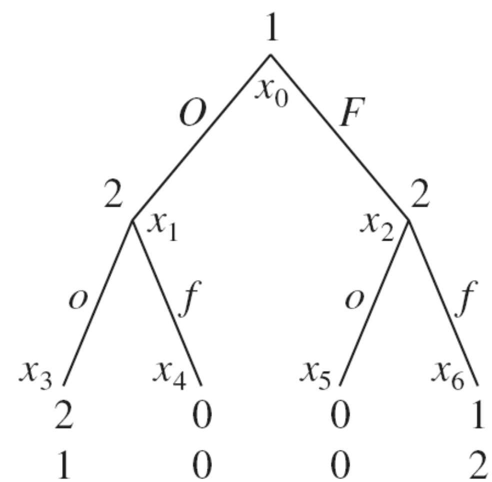



**FIGURE 7.2** The sequential-move Battle of the Sexes game.

In a game of (complete but) imperfect information some players do not know where they are because some information sets include more than one node. This happens, for example, every time they move without knowing what some players have chosen previously, implying that any simultaneous-move game is a game of imperfect information. Examples include the simultaneous-move Battle of the Sexes game shown in Figure 7.4 and the rock-paper-scissors game depicted in Figure 7.5.

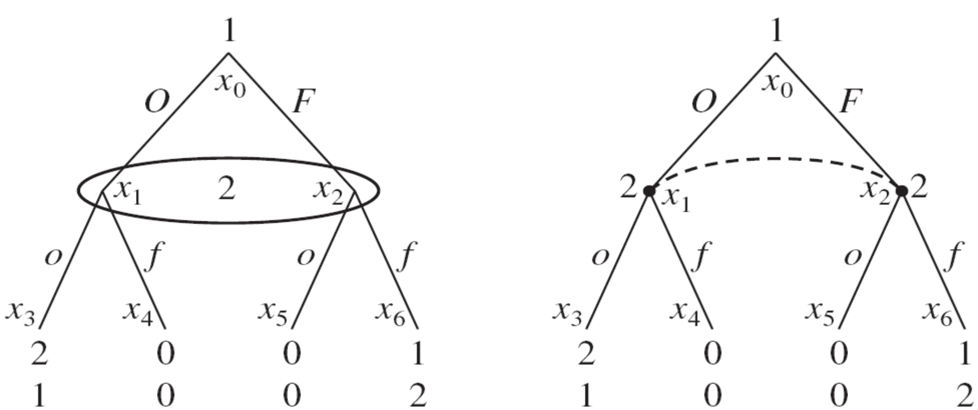



**FIGURE 7.4** The simultaneous-move Battle of the Sexes game.

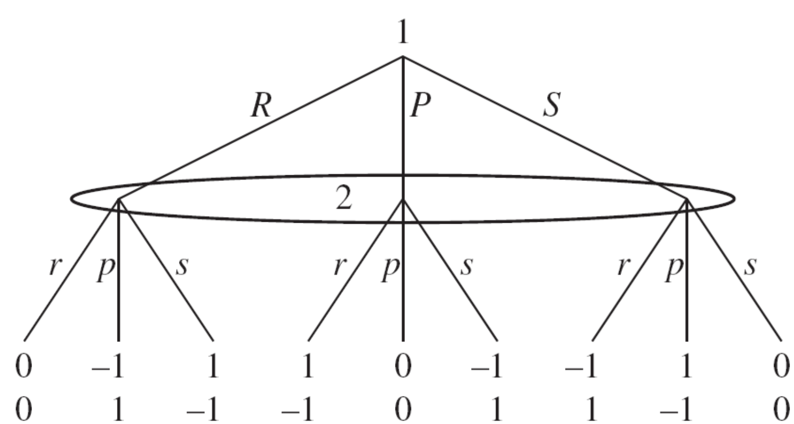



**FIGURE 7.5** Game tree of rock-paper-scissors.

Games of imperfect information are also useful to capture the uncertainty a player may have about acts of Nature.

For example, imagine the following card game: There is a large deck that includes an equal number of only kings and aces, from which player 1 pulls out a card without looking at it. The probability of getting a king is 0.5, and we can think of this as Nature's move. Hence player 1 moves after Nature and does not know if Nature chose a king or an ace. After drawing the card, player 1 can call (C，跟注) or fold (F，弃牌). If he folds, he pays $1 to player 2. If he calls, he pays $2 to player 2 if the card is a king, while player 2 pays him $2 if the card is an ace.

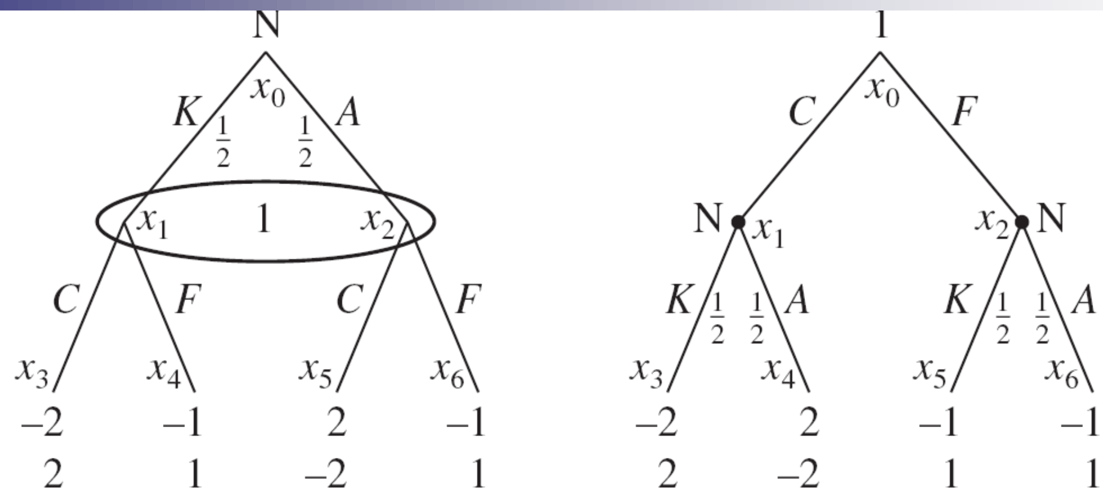



**FIGURE 7.6** A card game.

Consider the game tree in the left panel. The order of appearance is loyal to the story: Nature chose *K* or *A*. Player 1 does not know what happened, but he knows that he is at either node in his information set with probability 1/2 . Then player 1 makes his move and the game ends. The game on the right looks different but is strategically equivalent. Player 1 makes his move (C or F) without knowing which card will be drawn, and then Nature draws the card, K or A, with equal probability.

# 7.2 Strategies and Nash Equilibrium

Recall that in Section 3.1 we argued that “a strategy is often defined as a plan of action intended to accomplish a specific goal.” In the normal form game it was very easy to define a strategy for a player: a pure strategy was some element from his set of actions, $A_i$, and a mixed strategy was some probability distribution over these actions.

As we will now see, a strategy is more involved in extensive-form games.

# 7.2.1 Pure Strategies

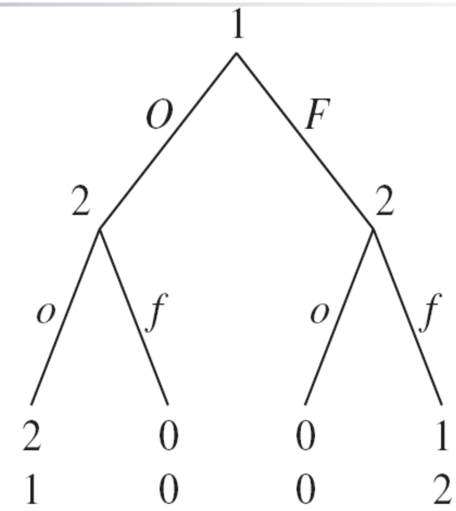



**FIGURE 7.7** The sequential-move Battle of the Sexes game.

Consider the sequential-move Battle of the Sexes game. Player 1 has a single information set with one node, so for him a pure strategy is as simple as "play *O*" or "play *F*." For player 2, things are a bit more involved. Player 2 has two information sets, each associated with a different action of player 1. The two simple statements "play *o*" and "play *f* " do not exhaust all the possibilities for player 2. In particular, player 2 can choose the following rather attractive strategy: "*If player 1 plays O then I will play o, while if player 1 plays F then I will play f* ."

When a player's move follows after the realization of previous events in the game, and if the player can distinguish between these previous events (they result in different information sets), then he can condition his behavior on the events that happened.

A strategy is therefore no longer a simple statement of what a player will do, as in the normal-form simultaneous-move game.

**Pure Strategies in Extensive-Form Games** A pure strategy for player *i* is *a complete plan of play that describes which pure action player i will choose at each of his information sets.*

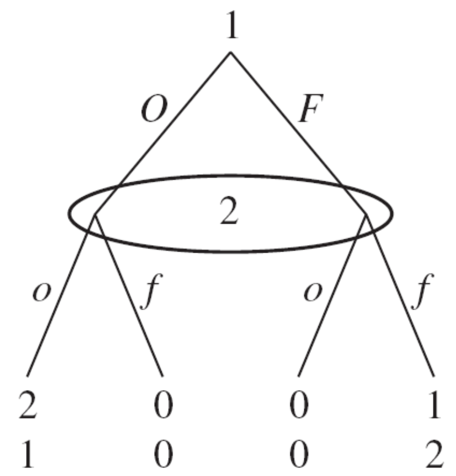



**FIGURE 7.8** The simultaneous-move Battle of the Sexes game.

If we consider the simultaneous-move Battle of the Sexes game, the pure strategies for player 1 are $S_1 = \{O, F\}$, and those for player 2 are $S_2 = \{o, f\}$. Because each player has only one information set, the extensive-form game is identical to the simple normal-form game we have already encountered.

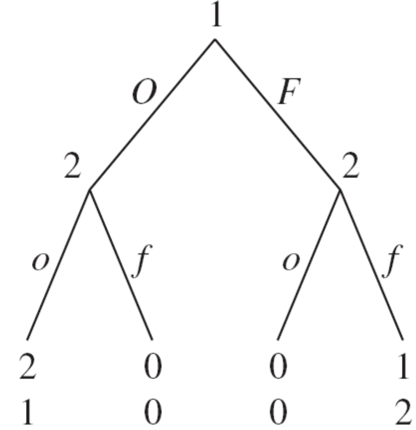



**FIGURE 7.7** The sequential-move Battle of the Sexes game.

In the sequential-move Battle of the Sexes game, player 2 has two distinct information sets in which he can choose *o* or *f*, each information set resulting from the previously made choice of player 1.

*A "complete plan of play" must accommodate a strategy that directs what player 2 will choose for each choice of player 1.* That is, player 2's choice of action from the set {*o*, *f*} can be made contingent on what player 1 does, admitting the possibility of strategies of the form "If player 1 plays *O* then I will play *o*, while if player 1 plays *F* then I will play *f* ."

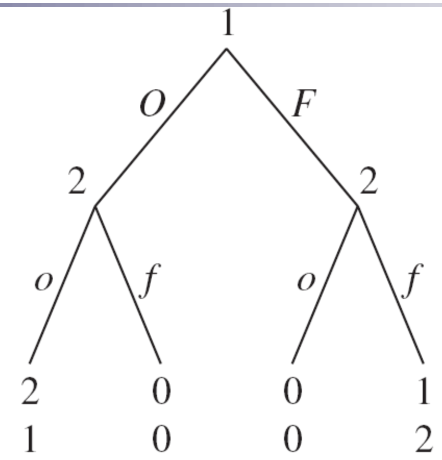



**FIGURE 7.7** The sequential-move Battle of the Sexes game.

For this example, we can describe the set of pure strategies for player 2 as follows:

$S_2 = \{oo, of, fo, ff\}$,

where a pure strategy "*ab*" is shorthand for "I will play *a* if player 1 plays *O* and I will play *b* if he plays *F*." For player 1 the pure strategy set remains $S_1 = \{O, F\}$.

We now introduce some notation that builds on what we have already developed in order to define formally a pure strategy.

Let $H_i$ be the collection of all information sets at which player $i$ plays, and let $h_i \in H_i$ be one of $i$'s information sets.

Let $A_i(h_i)$ be the actions that player $i$ can take at $h_i$, and let $A_i$ be the set of all actions of player $i$, $\mathrm{A_i} = \bigcup_{h_i \in H_i} \mathrm{A_i}(h_i)$ (i.e., the union of all the elements in all the sets $A_i(h_i)$ ).

**Definition 7.4** A **pure strategy** for player $i$ is a mapping $s_i : H_i \to A_i$ that assigns an action $s_i(h_i) \in A_i(h_i)$ for every information set $h_i \in H_i$. We denote by $S_i$ the set of all pure-strategy mappings $s_i \in S_i$.

Even though player 2 has only two actions from which to choose, by moving after observing what player 1 has chosen, his strategy defines actions that are conditional on his information about where he is in the game. In this example the two actions translate into four pure strategies because he has two information sets.

This observation implies that a potentially small set of moves can translate into a much larger set of strategies when sequential moves are possible, and when players have knowledge of what preceded their play. In general assume that player $i$ has $k > 1$ information sets, the first with $m_1$ actions from which to choose, the second with $m_2$, and so on until $m_k$. Letting $|S_i|$ denote the number of elements in $S_i$, the total number of pure strategies player $i$ has is

$$|S_i| = m_1 \times m_2 \times \cdots \times m_k .$$

# 7.2.3 Normal-Form Representation of Extensive-Form Games

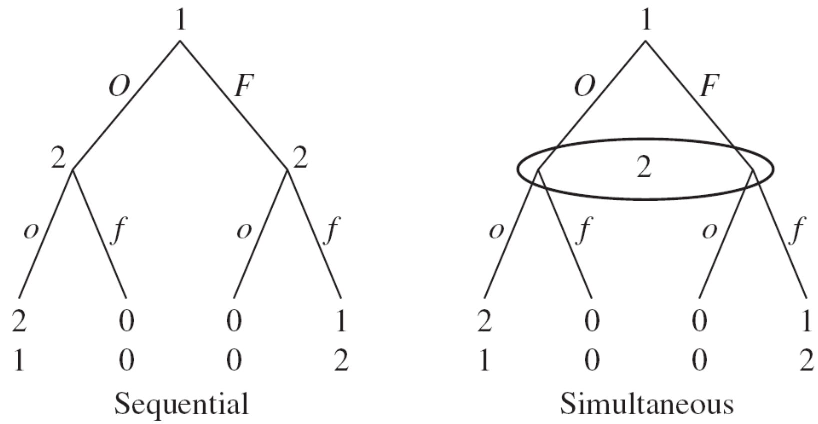



**FIGURE 7.11** The Battle of the Sexes game: two versions.

Consider the two variants of the Battle of the Sexes game. The simultaneous-move version in the right panel is one that we have seen before in its matrix form, as follows:

Player 2

| | | *o* | *f* |
|---|---|---|---|
| Player 1 | *O* | 2, 1 | 0, 0 |
| | *F* | 0, 0 | 1, 2 |

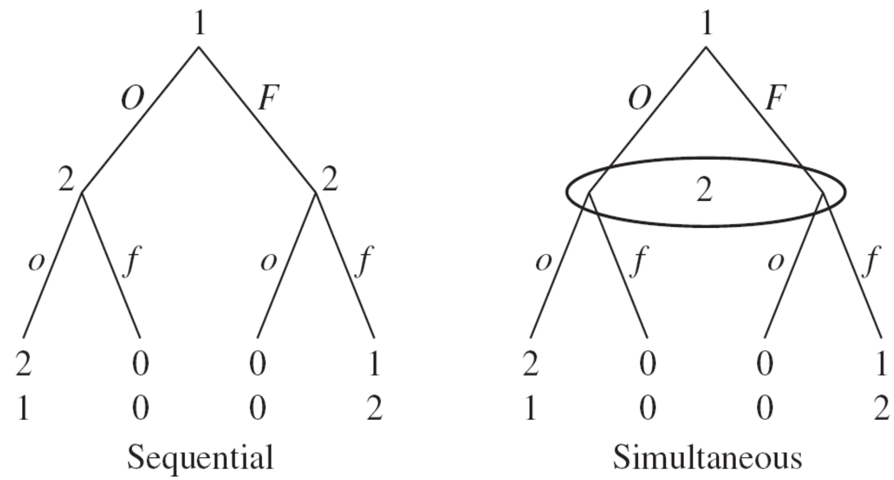



**FIGURE 7.11** The Battle of the Sexes game: two versions.

Consider the sequential-move Battle of the Sexes game. Recall that $S_1 = \{O, F\}$ and $S_2 = \{oo, of, fo, ff\}$, where *fo*, for example, means that player 2 plays *f* after player 1 plays *O*, while player 2 plays *o* after player 1 plays *F*. This game can be represented by a 2 × 4 matrix as follows:

Player 2

| Player 1 | | *oo* | *of* | *fo* | *ff* |
|---|---|---|---|---|---|
| | *O* | 2, 1 | 2, 1 | 0, 0 | 0, 0 |
| | *F* | 0, 0 | 1, 2 | 0, 0 | 1, 2 |

As this matrix demonstrates, each of the four payoffs in the original extensive-form game is replicated twice. This happens because for any pure strategy of player 1, two of the four pure strategies of player 2 are equivalent.

For example, if player 1 plays *O*, then only the “first component” of player 2’s strategy matters (what player 2 does following player 1’s choice of *O*). Therefore *oo* (player 2 playing o after *O* and o after *F*) and *of* (player 2 playing *o* after *O* and *f* after *F*) yield the same outcome.

Any extensive-form game can be transformed into a normal-form game by using

*the set of pure strategies of the extensive form (see definition 7.4) as the set of pure strategies in the normal form, and*

*the set of payoff functions is derived from how combinations of pure strategies result in the selection of terminal nodes.*

Every extensive-form game will have a unique normal form that represents it, which is not true for the reverse transformation.

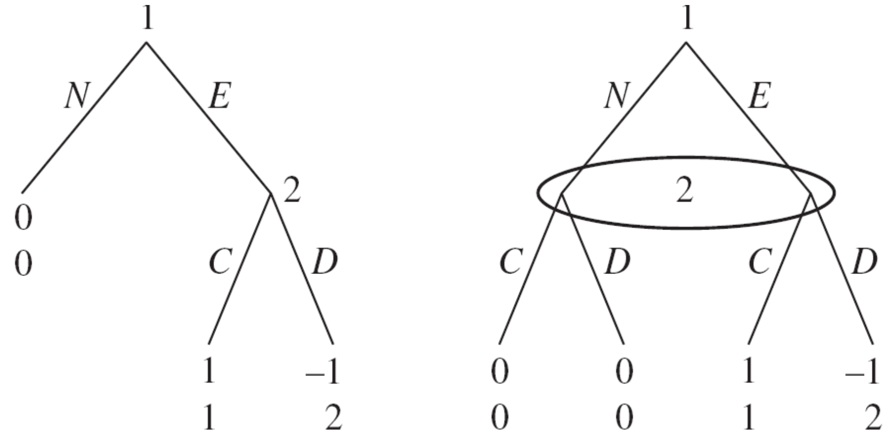



**FIGURE 7.12** Two extensive forms with the same normal form.

Consider the following matrix for a normal-form game:

| | *C* | *D* |
|---|---|---|
| *N* | 0, 0 | 0, 0 |
| *E* | 1, 1 | -1, 2 |

and notice that it is a consistent representation of either of the two game trees depicted in Figure 7.12. The extensive form on the left is a game of perfect information while the game on the right is one of imperfect information.

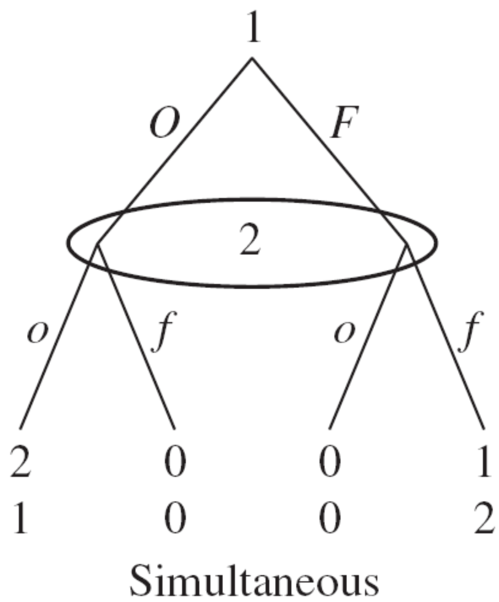
This exercise of transforming extensive-form games into the normal form seems to miss the point of capturing the dynamic structure of the extensive-form game.

Why then would we be interested in this exercise? The reason is that the concept of a Nash equilibrium is static in nature, in that the equilibrium posits that players take the strategies of others as given, and in turn they play a best response.

*The normal-form representation of an extensive form will suffice to find all the Nash equilibria of the game.*

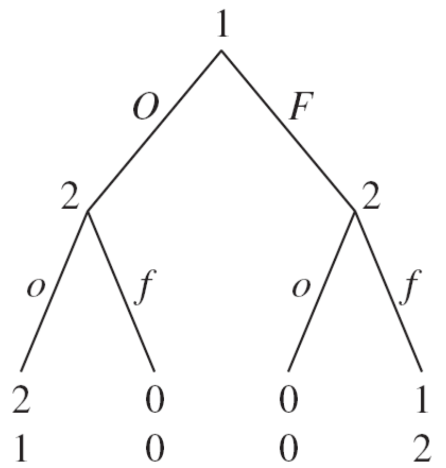
# 7.3 Nash Equilibrium and Paths of Play

By transforming an extensive form into its normal-form representation, we are concisely capturing the strategic essence of the game and can use the normal form to find all the Nash equilibria of the game.




| | | Player 2 | |
|---|---|---|---|
| | | *o* | *f* |
| Player 1 | *O* | 2, 1 | 0, 0 |
| | *F* | 0, 0 | 1, 2 |

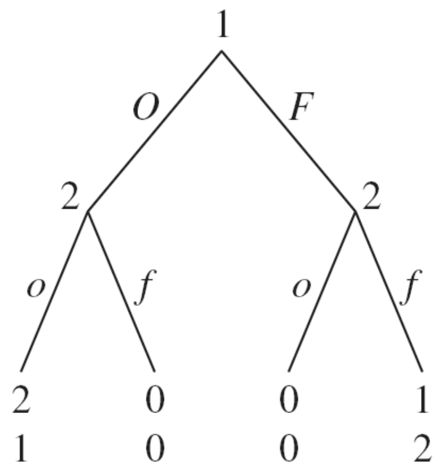
The simultaneous-move Battle of the Sexes game in the right panel of Figure 7.11 is equivalent to the normal form that we had already analyzed. Hence we know that there are two pure-strategy Nash equilibria, ($O$, $o$) and ($F$, $f$).




Sequential

| | | Player 2 | | | |
|---|---|---|---|---|---|
| | | *oo* | *of* | *fo* | *ff* |
| Player 1 | *O* | 2, 1 | 2, 1 | 0, 0 | 0, 0 |
| | *F* | 0, 0 | 1, 2 | 0, 0 | 1, 2 |

Consider the sequential-move Battle of the Sexes game. It is easy to see that we can "replicate" the outcomes of the Nash equilibria we found for the simultaneous version of this game with the strategies (*O*, *oo*) and (*F*, *ff* ). The first pair of strategies (*O*, *oo*) specify that player 1 chooses *O* and player 2 chooses "I play *o* after *O* and *o* after *F*," which yields the outcome of both players going to the opera. Similarly (*F*, *ff* ) yields the outcome of both going to the football game.



Sequential

Player 2

| | | *oo* | *of* | *fo* | *ff* |
|---|---|---|---|---|---|
| Player 1 | *O* | 2, 1 | 2, 1 | 0, 0 | 0, 0 |
| | *F* | 0, 0 | 1, 2 | 0, 0 | 1, 2 |

The question is whether there are other Nash equilibria in pure strategies.

A third pure-strategy Nash equilibrium exists: (*O*, *of* ). This strategy yields the same outcome as (*O*, *oo*) but with the following strategies: player 1 chooses *O* and player 2 chooses "I play *o* after *O* and *f* after *F*."

Notice also that in the sequential-move Battle of the Sexes example there are two Nash equilibria that result in the exact same outcome of both players going to the opera: (O, of ) and (O, oo).

Is there an important difference between these two predictions? Indeed there is—the difference between these two equilibria is not in what the players actually play *in equilibrium*, but instead what player 2 plans to play in an information set that is *not reached in equilibrium*.

**Definition 7.8** Let $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ be a Nash equilibrium profile of behavioral strategies in an extensive-form game. We say that an information set is **on the equilibrium path** if given $\sigma^*$ it is reached with positive probability. We say that an information set is **off the equilibrium path** if given $\sigma^*$ it is never reached.

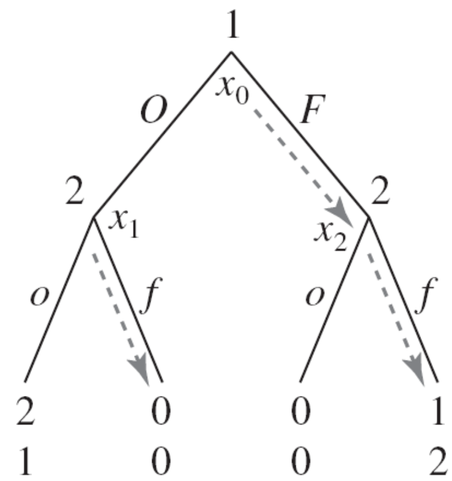



**FIGURE 7.13** Equilibrium paths in the sequential-move Battle of the Sexes game.

*In a Nash equilibrium players choose to proceed on the equilibrium path because of their beliefs about what the other players are doing both on and off the equilibrium path.* Consider the Nash equilibrium ($F$, $ff$). This equilibrium is supported by player 1's correct belief that if he would deviate from the equilibrium path and play $O$ then he would receive 0 because player 2 will proceed to play $f$ in the information set $x_1$. *The "threat" imposed by player 2's strategy of how he will proceed off the equilibrium path is supporting the actions of player 1 on the equilibrium path.*

7.4 **Centipedes:** Imagine a two-player game that proceeds as follows. A pot of money is created with \$6 in it initially. Player 1 moves first, then player 2, then player 1 again, and finally player 2 again. At each player's turn to move, he has two possible actions: grab ($G$) or share ($S$). If he grabs he gets $\frac{2}{3}$ of the current pot of money, the other player gets $\frac{1}{3}$ of the pot, and the game ends. If he shares then the size of the current pot is multiplied by $\frac{3}{2}$ and the next player gets to move. At the last stage at which player 2 moves, if he chooses $S$ then the pot is still multiplied by $\frac{3}{2}$, player 2 gets $\frac{1}{3}$ of the pot, and player 1 gets $\frac{2}{3}$ of the pot.

a. Model this as an extensive-form game tree. Is it a game of perfect or imperfect information?
b. How many terminal nodes does the game have? How many information sets?
c. How many pure strategies does each player have?
d. Find the Nash equilibria of this game. How many outcomes can be supported in equilibrium?
e. Now imagine that at the last stage at which player 2 moves, if he chooses to share then the pot is equally split among the players. Does your answer to part (d) change?

7.6 **Entering an Industry:** A firm (player 1) is considering entering an established industry with one incumbent firm (player 2). Player 1 must choose whether or not to enter the industry. If player 1 enters the industry then player 2 can either accommodate the entry or fight the entry by waging a price war. Player 1's most-preferred outcome is entering with player 2 not fighting, and its least-preferred outcome is entering with player 2 fighting. Player 2's most-preferred outcome is player 1 not entering, and its least-preferred outcome is player 1 entering with player 2 fighting.

a. Model this as an extensive-form game tree (choose payoffs that represent the preferences).
b. How many pure strategies does each player have?
c. Find all the Nash equilibria of this game.

7.7 **Roommates Voting:** Three roommates need to vote on whether they will adopt a new rule and clean their apartment once a week or stick to the current once-a-month rule. Each votes "yes" for the new rule or "no" for the current rule. Players 1 and 2 prefer the new rule while player 3 prefers the old rule.

a. Imagine that the players require a unanimous vote to adopt the new rule. Player 1 votes first, then player 2, and then player 3, the latter two observing the previous votes. Draw this as an extensive-form game and find the Nash equilibria.

b. Imagine now that the players require a majority vote to adopt the new rule (at least two "yes" votes). Again player 1 votes first, then player 2, and then player 3, the latter two observing the previous votes. Draw this as an extensive-form game and find the Nash equilibria.

c. Now imagine that the game is as in part (b), but the players put their votes into a hat—so that the votes of earlier movers are not observed by the later movers—and the votes are counted after all have voted. Draw this as an extensive-form game and find the Nash equilibria. In what way is this result different from the result in (b)?