

# 信息系统分析与设计 Part II Classification



#### >>> Classification: Definition



- Given a collection of records (training set )
  - $\square$  Each record is by characterized by a tuple (x, y), where x is the attribute set and y is the class label
    - x: attribute, predictor, independent variable, input
    - y: class, response, dependent variable, output

#### Task:

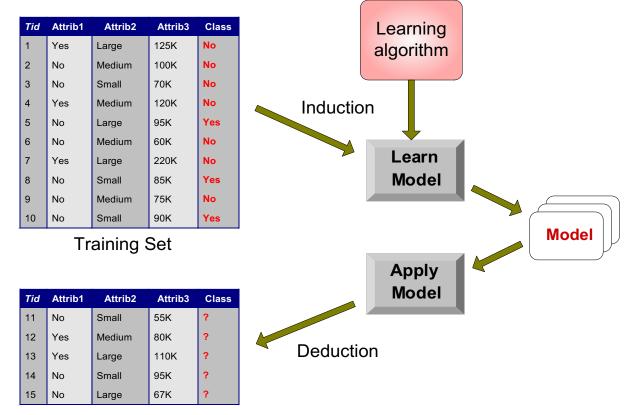
 $\square$  Learn a model that maps each attribute set x into one of the predefined class labels y

# >>> Examples of Classification Task



Task	Attribute set, <b>x</b>	Class label, y
Categorizing email messages	Features extracted from email message header and content	spam or non-spam
Identifying tumor cells	Features extracted from MRI scans	malignant or benign cells
Cataloging galaxies	Features extracted from telescope images	Elliptical, spiral, or irregular- shaped galaxies

# >>> General Approach for Building Classification Model



**Test Set** 

### >>> Classification Techniques



#### Base Classifiers

- □ Rule-based Methods
- Decision Tree based Methods
- Support Vector Machines
- Nearest-neighbor
- Neural Networks
- □ Naïve Bayes and Bayesian Belief Networks

#### Ensemble Classifiers

□ Boosting, Bagging, Random Forests

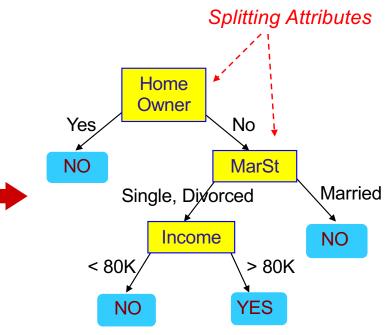
# >>> Example of a Decision Tree





ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

**Training Data** 

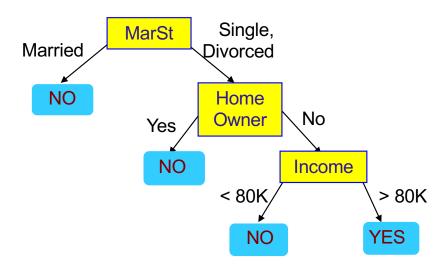


Model: Decision Tree

#### >>> Another Example of Decision Tree



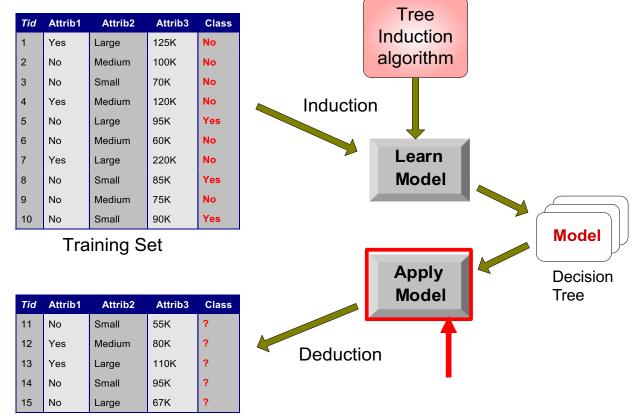
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1	Yes	Single	125K	No
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7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There could be more than one tree that fits the same data!

#### >>> Decision Tree Classification Task

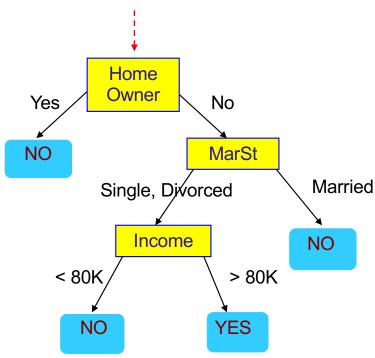




**Test Set** 



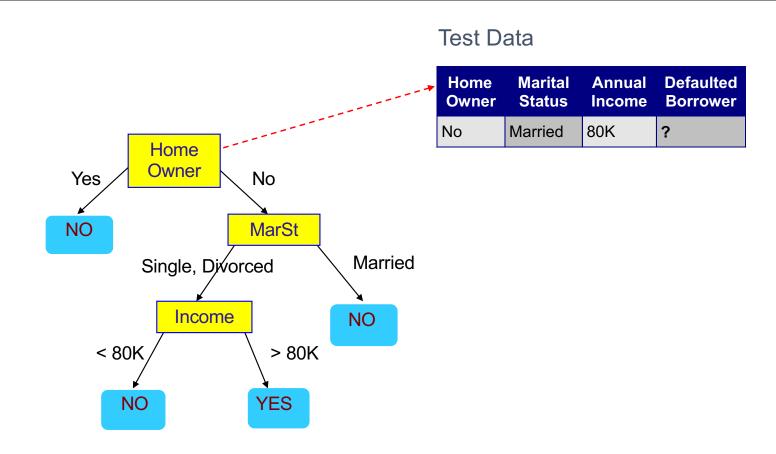




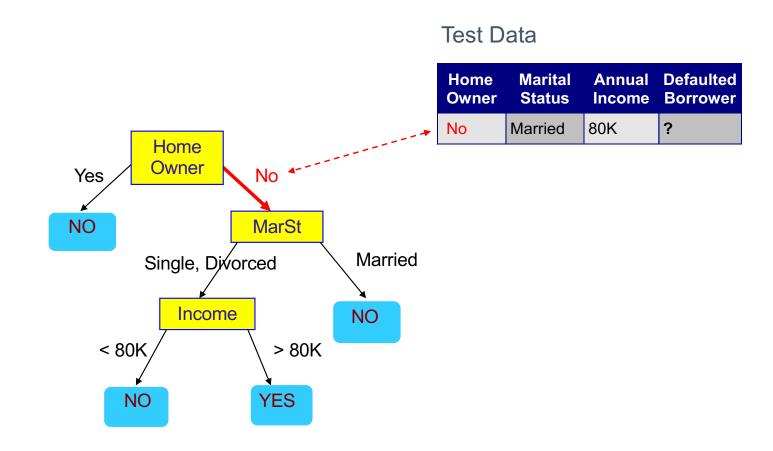
#### **Test Data**

			Defaulted Borrower
No	Married	80K	?

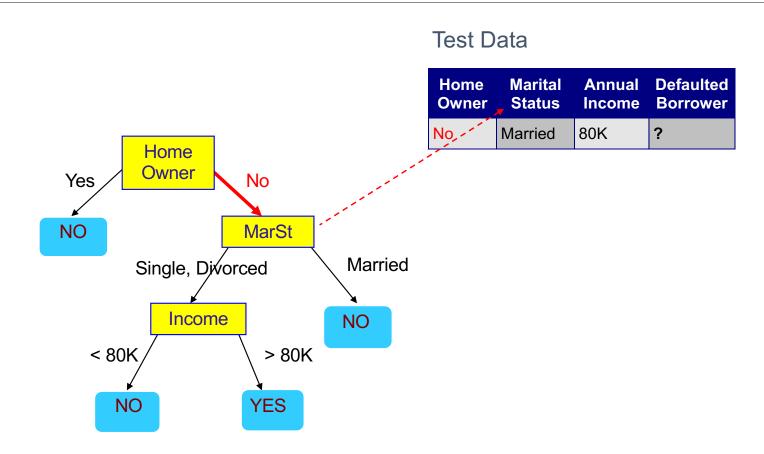




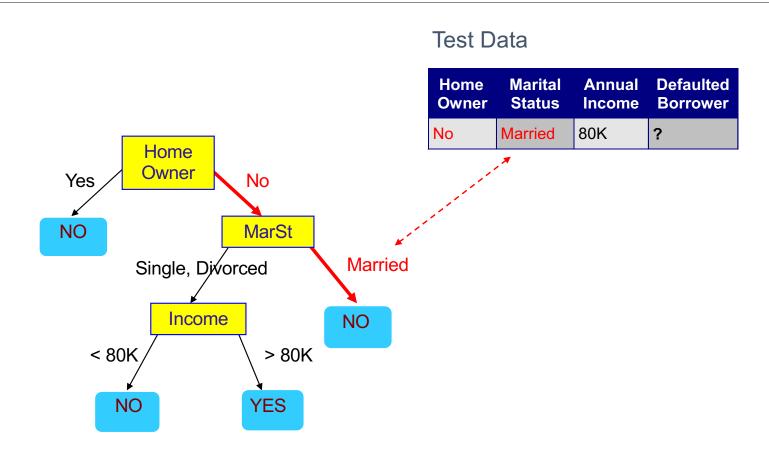




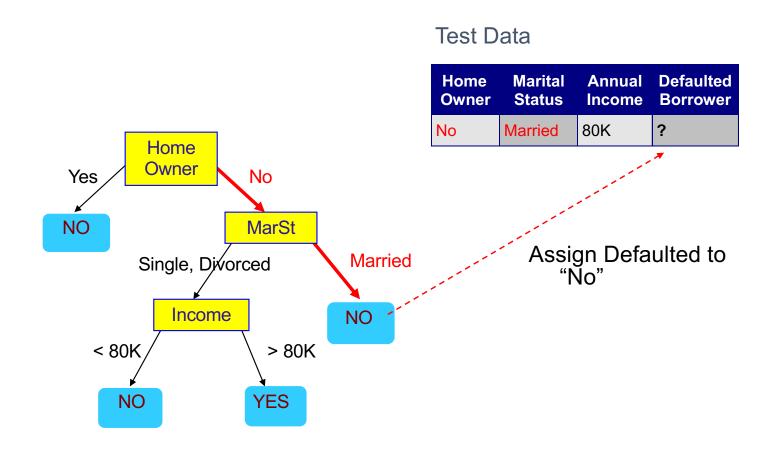






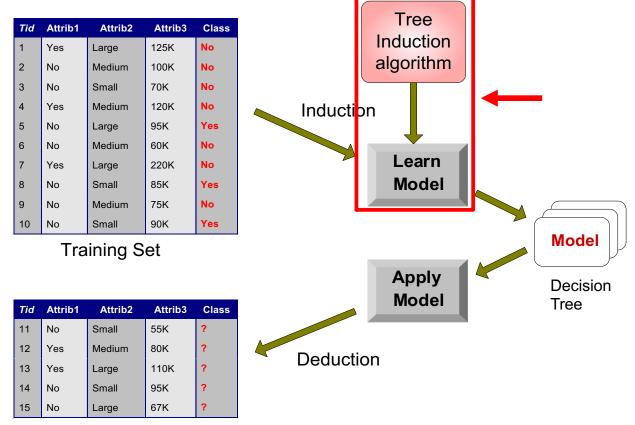






#### >>> Decision Tree Classification Task





**Test Set** 

#### >>> Decision Tree Induction



#### Many Algorithms:

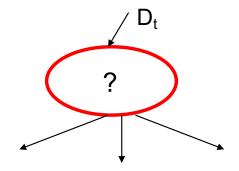
- □ Hunt's Algorithm (one of the earliest)
- CART
- □ ID3, C4.5
- □ SLIQ,SPRINT

#### >>> General Structure of Hunt's Algorithm



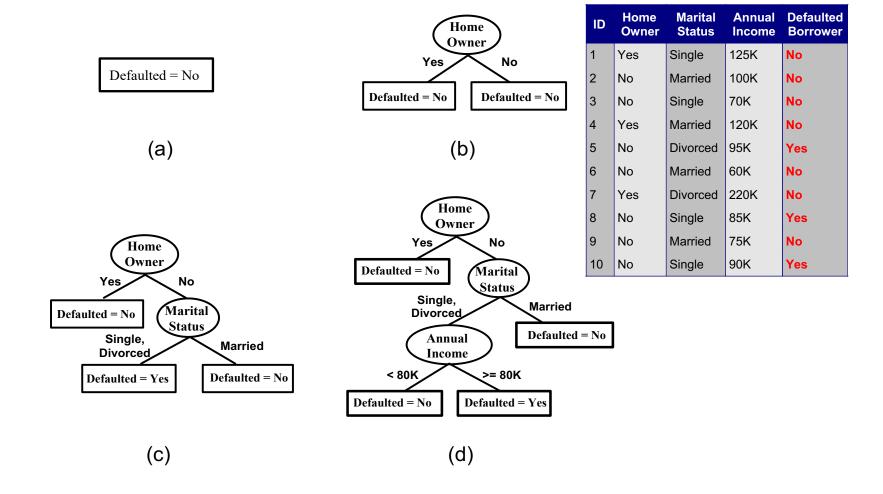
- Let D<sub>t</sub> be the set of training records that reach a node t
- General Procedure:
  - □ If  $D_t$  contains records that belong the same class  $y_t$ , then t is a leaf node labeled as  $y_t$
  - □ If D<sub>t</sub> contains records that belong to more than one class, use an attribute test to split the data into smaller subsets.
     Recursively apply the procedure to each subset.

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
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2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



### >>> Hunt's Algorithm





### >>> Design Issues of Decision Tree Induction



- How should training records be split?
  - Method for specifying test condition
    - depending on attribute types
  - Measure for evaluating the goodness of a test condition
- How should the splitting procedure stop?
  - Stop splitting if all the records belong to the same class or have identical attribute values
  - □ Early termination

# >>> Methods for Expressing Test Conditions



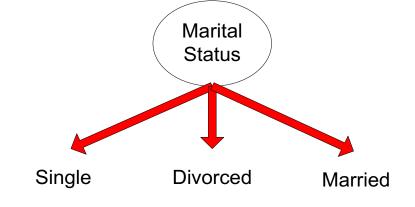
- Depends on attribute types
  - □ Binary
  - Nominal
  - Ordinal
  - Continuous
- Depends on number of ways to split
  - □ 2-way split
  - Multi-way split

#### >>> Test Condition for Nominal Attributes



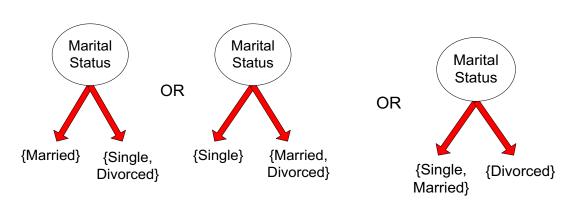
#### Multi-way split:

 Use as many partitions as distinct values.



#### Binary split:

- Divides values into two subsets
- Need to find optimal partitioning.



#### >>> Test Condition for Ordinal Attributes

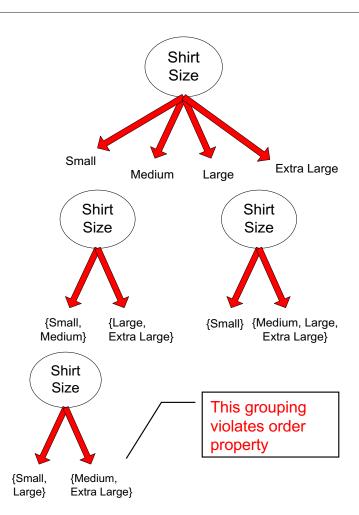


#### Multi-way split:

Use as many partitions as distinct values

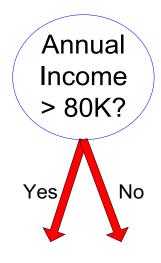
#### Binary split:

- □ Divides values into two subsets
- Need to find optimal partitioning
- □ Preserve the order property among attribute values

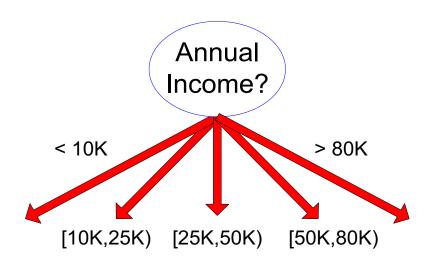


#### >>> Test Condition for Continuous Attributes





(i) Binary split



(ii) Multi-way split

# >>> Splitting Based on Continuous Attributes



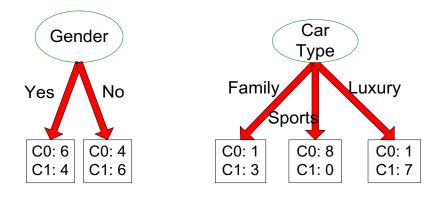
- Different ways of handling
  - □ Discretization to form an ordinal categorical attribute
    - Static discretize once at the beginning
    - Dynamic ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
  - □ Binary Decision: (A < v) or  $(A \ge v)$ 
    - consider all possible splits and finds the best cut
    - can be more compute-intensive

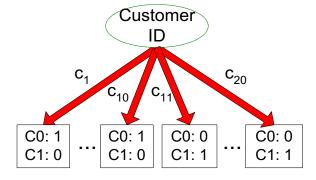
# >>> How to determine the Best Split



Before Splitting: 10 records of class 0, 10 records of class 1

Customer Id	Gender	Car Type	Shirt Size	Class
1	M	Family	Small	C0
2	M	Sports	Medium	C0
3	M	Sports	Medium	C0
4	M	Sports	Large	C0
5	M	Sports	Extra Large	C0
6	M	Sports	Extra Large	C0
7	F	Sports	Small	C0
8	F	Sports	Small	C0
9	F	Sports	Medium	C0
10	F	Luxury	Large	C0
11	M	Family	Large	C1
12	M	Family	Extra Large	C1
13	M	Family	Medium	C1
14	M	Luxury	Extra Large	C1
15	F	Luxury	Small	C1
16	F	Luxury	Small	C1
17	F	Luxury	Medium	C1
18	F	Luxury	Medium	C1
19	F	Luxury	Medium	C1
20	F	Luxury	Large	C1





Which test condition is the best?

### >>> How to determine the Best Split



- Greedy approach:
  - □ Nodes with purer class distribution are preferred
- Need a measure of node impurity:

C0: 5

C1: 5

C0: 9

C1: 1

High degree of impurity

Low degree of impurity

#### >>> Measures of Node Impurity



Gini Index

$$GINI(t) = 1 - \sum_{j} [p(j | t)]^{2}$$

Entropy

$$Entropy(t) = -\sum_{j} p(j \mid t) \log p(j \mid t)$$

Misclassification error

$$Error(t) = 1 - \max_{i} P(i \mid t)$$

### >>> Finding the Best Split



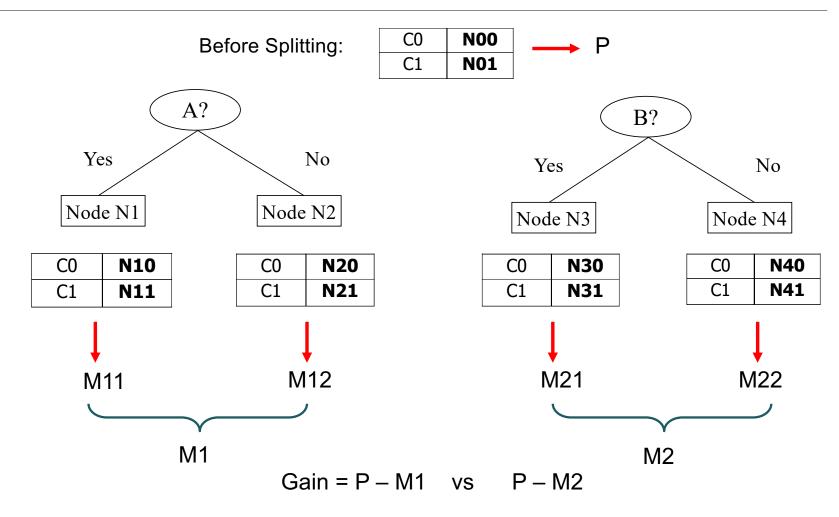
- 1. Compute impurity measure (P) before splitting
- 2. Compute impurity measure (M) after splitting
  - Compute impurity measure of each child node
  - Compute the average impurity of the children (M)
- 3. Choose the attribute test condition that produces the highest gain

$$Gain = P - M$$

or equivalently, lowest impurity measure after splitting (M)

# >>> Finding the Best Split





### >>> Measure of Impurity: GINI



Gini Index for a given node t :

$$GINI(t) = 1 - \sum_{j} [p(j|t)]^{2}$$

(NOTE: p(j | t) is the relative frequency of class j at node t).

- Maximum (1 1/n<sub>c</sub>) when records are equally distributed among all classes, implying least interesting information
- Minimum (0.0) when all records belong to one class, implying most interesting information

C1	0	
C2	6	
Gini=0.000		

C1	1	
C2	5	
Gini=0.278		

# >>> Computing Gini Index of a Single Node



$$GINI(t) = 1 - \sum_{j} [p(j|t)]^{2}$$

C1	0
C2	6

$$P(C1) = 0/6 = 0$$
  $P(C2) = 6/6 = 1$   
 $Gini = 1 - P(C1)^2 - P(C2)^2 = 1 - 0 - 1 = 0$ 

P(C1) = 
$$1/6$$
 P(C2) =  $5/6$   
Gini =  $1 - (1/6)^2 - (5/6)^2 = 0.278$ 

$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Gini = 
$$1 - (2/6)^2 - (4/6)^2 = 0.444$$

# >>> Computing Gini Index for a Collection of Nodes



When a node p is split into k partitions (children)

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

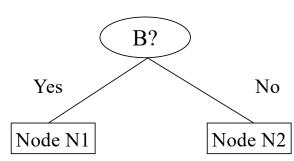
where, ni = number of records at child i, n = number of records at parent node p.

- Choose the attribute that minimizes weighted average Gini index of the children
- Gini index is used in decision tree algorithms such as CART, SLIQ, SPRINT

# >>> Binary Attributes: Computing GINI Index



- Splits into two partitions
- Effect of Weighing partitions:
  - □ Larger and Purer Partitions are sought for.



	Parent
C1	6
C2	6
Gini = 0.500	

Gini(N2)  
= 
$$1 - (2/6)^2 - (4/6)^2$$
  
=  $0.444$ 

	N1	N2	
C1	5	2	
C2	1	4	
Gini=0.361			

#### >>> Categorical Attributes: Computing Gini Index



- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

#### Multi-way split

	CarType									
	Family	Sports	Luxury							
C1	1	8	1							
C2	3	0	7							
Gini		0.163								

Two-way split (find best partition of values)

	CarType								
	{Sports, Luxury}	{Family}							
C1	9	1							
C2	7	3							
Gini	0.468								

	CarType								
	{Sports}	{Family, Luxury}							
C1	8	2							
C2	0	10							
Gini	0.167								

# >>> Continuous Attributes: Computing Gini Index



- Use Binary Decisions based on one value
- Several Choices for the splitting value
  - Number of possible splitting valuesNumber of distinct values
- Each splitting value has a count matrix associated with it
  - $\square$  Class counts in each of the partitions, A < v and A  $\ge$  v
- Simple method to choose best v
  - □ For each v, scan the database to gather count matrix and compute its Gini index
  - Computationally Inefficient! Repetition of work.

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



#### >>> Continuous Attributes: Computing Gini Index...



- For efficient computation: for each attribute,
  - Sort the attribute on values
  - □ Linearly scan these values, each time updating the count matrix and computing gini index
  - □ Choose the split position that has the least gini index

	Cheat		No		No	)	N	0	Ye	s	Ye	s	Υe	es	N	0	N	lo	N	0		No	
Sorted Values											Ar	nua	l Inc	ome	Э								
	-		60		70	)	7	5	85	5	90	)	9	5	10	00	12	20	12	25		220	
Split Positions	$\longrightarrow$	5	55 65		7	72		80		87		2	97		110		122		172		230		
		۳	>	<=	>	<=	>	<=	<b>^</b>	<b>"</b>	>	<=	>	<b>"</b>	^	<b>\=</b>	<b>^</b>	<b>"</b>	^	<=	>	<=	>
	Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
	No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
	Gini	0.4	120	0.4	00	0.375		0.343		0.4	117	0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

# >>> Measure of Impurity: Entropy



Entropy at a given node t:

$$Entropy(t) = -\sum_{j} p(j \mid t) \log p(j \mid t)$$

(NOTE: p(j | t) is the relative frequency of class j at node t).

- Maximum (log n<sub>c</sub>) when records are equally distributed among all classes implying least information
- Minimum (0.0) when all records belong to one class, implying most information
- Entropy based computations are quite similar to the GINI index computations

# >>> Computing Entropy of a Single Node



$$Entropy(t) = -\sum_{j} p(j \mid t) \log_{2} p(j \mid t)$$

C1	0
C2	6

P(C1) = 
$$0/6 = 0$$
 P(C2) =  $6/6 = 1$   
Entropy =  $-0 \log 0 - 1 \log 1 = -0 - 0 = 0$ 

$$P(C1) = 1/6$$
  $P(C2) = 5/6$   
Entropy =  $-(1/6) \log_2 (1/6) - (5/6) \log_2 (1/6) = 0.65$ 

$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Entropy = 
$$-(2/6) \log_2(2/6) - (4/6) \log_2(4/6) = 0.92$$

# >>> Computing Information Gain After Splitting



Information Gain:

$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy(i)\right)$$

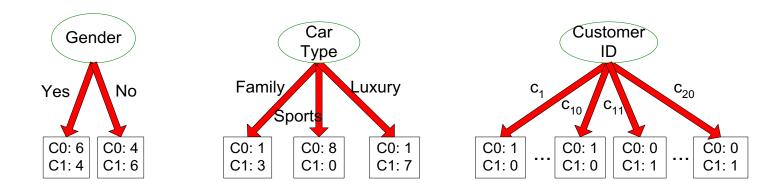
Parent Node, p is split into k partitions; n<sub>i</sub> is number of records in partition i

- □ Choose the split that achieves most reduction (maximizes GAIN)
- □ Used in the ID3 and C4.5 decision tree algorithms

### >>> Problems with Information Gain



Info Gain tends to prefer splits that result in large number of partitions,
 each being small but pure



□ Customer ID has highest information gain because entropy for all the children is zero

#### >>> Gain Ratio



#### Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{split}}{SplitINFO} SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

- Parent Node, p is split into k partitions
- ni is the number of records in partition i
- Adjusts Information Gain by the entropy of the partitioning (SplitINFO).
  - Higher entropy partitioning (large number of small partitions) is penalized!
- □ Used in C4.5 algorithm
- □ Designed to overcome the disadvantage of Information Gain

# >>> Measure of Impurity: Classification Error



Classification error at a node t :

$$Error(t) = 1 - \max_{i} P(i \mid t)$$

- □ Maximum (1 1/n<sub>c</sub>) when records are equally distributed among all classes, implying least interesting information
- □ Minimum (0) when all records belong to one class, implying most interesting information

# >>> Computing Error of a Single Node



$$Error(t) = 1 - \max_{i} P(i \mid t)$$

C1	0
C2	6

$$P(C1) = 0/6 = 0$$
  $P(C2) = 6/6 = 1$   
 $Error = 1 - max(0, 1) = 1 - 1 = 0$ 

Error = 
$$1 - \max(0, 1) = 1 - 1 = 0$$

$$P(C1) = 1/6$$
  $P(C2) = 5/6$ 

Error = 
$$1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

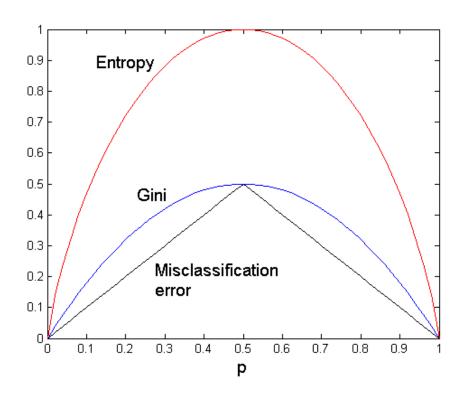
$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Error = 
$$1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

# >>> Comparison among Impurity Measures

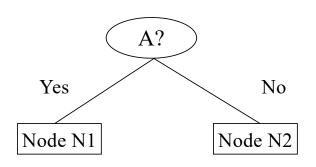


### For a 2-class problem:



## >>> Misclassification Error vs Gini Index





	Parent
C1	7
C2	3
Gini	= 0.42

Gini(N1)  
= 
$$1 - (3/3)^2 - (0/3)^2$$
  
= 0  
Gini(N2)  
=  $1 - (4/7)^2 - (3/7)^2$   
= 0.489

	N1	N2
C1	3	4
C2	0	3
Gin	i=0.3	42

Gini(Children) = 3/10 \* 0 + 7/10 \* 0.489 = 0.342

Gini improves but error remains the same!!

# >>> Evaluation of Classification



#### Confusion Matrix:

	PREDICTED CLASS				
		Class=Yes	Class=No		
ACTUAL	Class=Yes	а	b		
	Class=No	С	d		

a: TP (true positive)

b: FN (false negative)

c: FP (false positive)

d: TN (true negative)





	PREDICTED CLASS					
		Class=Yes	Class=No			
ACTUAL	Class=Yes	a (TP)	b (FN)			
CLASS	Class=No	c (FP)	d (TN)			

Most widely-used metric:

Accuracy = 
$$\frac{a+d}{a+b+c+d} = \frac{TP+TN}{TP+TN+FP+FN}$$

## >>> Problem with Accuracy



- Consider a 2-class problem
  - □ Number of Class 0 examples = 9990
  - □ Number of Class 1 examples = 10
- If a model predicts everything to be class 0, accuracy is 9990/10000 = 99.9 %
  - □ This is misleading because the model does not detect any class 1 example
  - □ Detecting the rare class is usually more interesting (e.g., frauds, intrusions, defects, etc)

### >>> Alternative Measures



	PREDICTED CLASS					
	Class=Yes Class=No					
ACTUAL	Class=Yes	a (TP)	b (FN)			
CLASS	Class=No	c (FP)	d (TN)			

Precision (p) = 
$$\frac{a}{a+c}$$
  
Recall (r) =  $\frac{a}{a+b}$  =  $TPR$   
F - measure (F) =  $\frac{2rp}{r+p}$  =  $\frac{2a}{2a+b+c}$ 

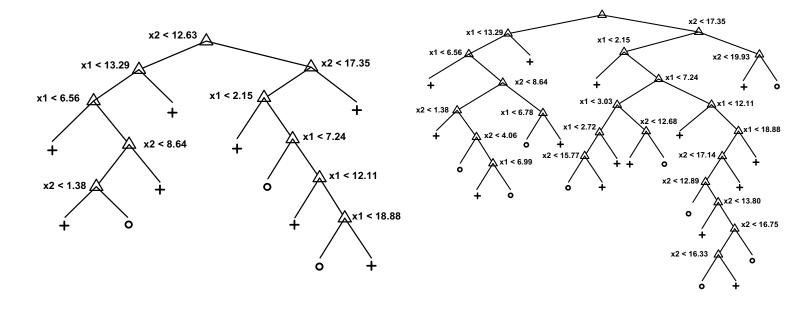
#### >>> Classification Errors



- Training errors (apparent errors)
  - □ Errors committed on the training set
- Test errors
  - □ Errors committed on the test set
- Generalization errors
  - □ Expected error of a model over random selection of records from same distribution

### >>> Decision Tree





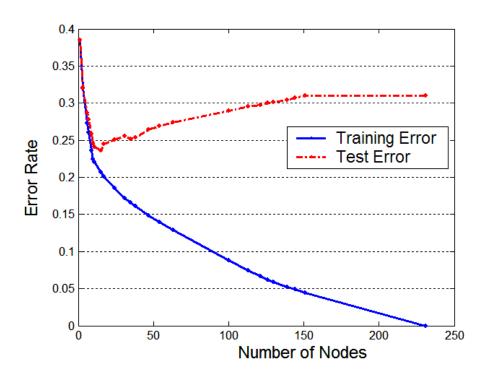
Decision Tree with 11 leaf nodes

Decision Tree with 24 leaf nodes

Which tree is better?

# >>> Model Overfitting





**Underfitting**: when model is too simple, both training and test errors are large

Overfitting: when model is too complex, training error is small but test error is large

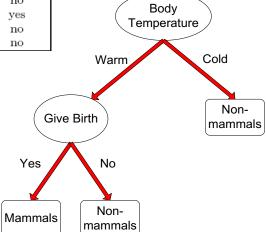
### >>> Mammal Classification Problem



Name	Body	Skin	Gives	Aquatic	Aerial	Has	Hiber-	Mammal
	Temperature	Cover	Birth	Creature	Creature	Legs	nates	
human	warm-blooded	hair	yes	no	no	yes	no	yes
python	cold-blooded	scales	no	no	no	no	yes	no
$_{ m salmon}$	cold-blooded	scales	no	yes	no	no	no	no
whale	warm-blooded	hair	yes	yes	no	no	no	yes
frog	cold-blooded	none	no	$_{ m semi}$	no	yes	yes	no
komodo	cold-blooded	scales	no	no	no	yes	no	no
dragon								
bat	warm-blooded	hair	yes	no	yes	yes	yes	yes
pigeon	warm-blooded	feathers	no	no	yes	yes	no	no
cat	warm-blooded	fur	yes	no	no	yes	no	yes
leopard	cold-blooded	scales	yes	yes	no	no	no	no
shark								
turtle	cold-blooded	scales	no	semi	no	yes	no	no
penguin	warm-blooded	feathers	no	$_{ m semi}$	no	yes	no	no
porcupine	warm-blooded	quills	yes	no	no	yes	yes	yes
eel	cold-blooded	scales	no	yes	no	no	no	no
salamander	cold-blooded	none	no	$_{ m semi}$	no	yes	yes	no

**Training Set** 

**Decision Tree Model** training error = 0%





### >>> Effect of Noise



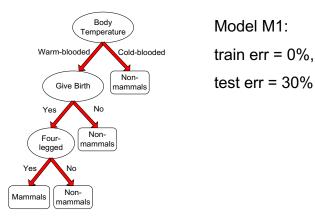
#### **Example**: Mammal Classification problem

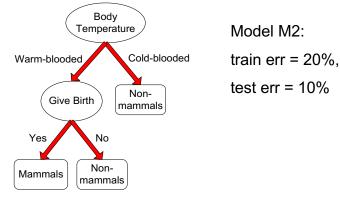
#### Training Set:

Name	Body	Gives	Four-	Hibernates	Class
	Temperature	Birth	legged		Label
porcupine	warm-blooded	yes	yes	yes	yes
cat	warm-blooded	yes	yes	no	yes
bat	warm-blooded	yes	no	yes	no*
whale	warm-blooded	yes	no	no	no*
salamander	cold-blooded	no	yes	yes	no
komodo dragon	cold-blooded	no	yes	no	no
python	cold-blooded	no	no	yes	no
salmon	cold-blooded	no	no	no	no
eagle	warm-blooded	no	no	no	no
guppy	cold-blooded	yes	no	no	no

#### Test Set:

Name	Body	Gives	Four-	Hibernates	Class
	Temperature	Birth	legged		Label
human	warm-blooded	yes	no	no	yes
pigeon	warm-blooded	no	no	no	no
elephant	warm-blooded	yes	yes	no	yes
leopard shark	cold-blooded	yes	no	no	no
turtle	cold-blooded	no	yes	no	no
penguin	cold-blooded	no	no	no	no
eel	cold-blooded	no	no	no	no
dolphin	warm-blooded	yes	no	no	yes
spiny anteater	warm-blooded	no	yes	yes	yes
gila monster	cold-blooded	no	yes	yes	no





# >>> Lack of Representative Samples

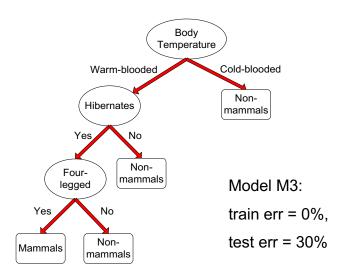


#### **Training Set:**

Name	Body	Four-	Hibernates	Class
	Temperature	legged		Label
salamander	cold-blooded	yes	yes	no
guppy	cold-blooded	no	no	no
eagle	warm-blooded	no	no	no
poorwill	warm-blooded	no	yes	no
platypus	warm-blooded	yes	yes	yes

#### Test Set:

Name	Body	Four-	Hibernates	Class
	Temperature	legged		Label
human	warm-blooded	no	no	yes
pigeon	warm-blooded	no	no	no
elephant	warm-blooded	yes	no	yes
leopard shark	cold-blooded	no	no	no
turtle	cold-blooded	yes	no	no
penguin	cold-blooded	no	no	no
eel	cold-blooded	no	no	no
dolphin	warm-blooded	no	no	yes
spiny anteater	warm-blooded	yes	yes	yes
gila monster	cold-blooded	yes	yes	no



Lack of training records at the leaf nodes for making reliable classification

# >>> Notes on Overfitting



- Overfitting results in decision trees that are <u>more complex</u> than necessary
- Training error no longer provides a good estimate of how well the tree will perform on previously unseen records
- Need new ways for estimating generalization errors

# >>> Using Validation Set



- Divide <u>training</u> data into two parts:
  - □ Training set:
    - use for model building
  - □ Validation set:
    - use for estimating generalization error
    - Note: validation set is not the same as test set
- Drawback:
  - Less data available for training





## Thanks!

# Let's enjoy playing with data?

