Week3

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Objectives

1. To discover several key signals used to convey a realistic sense of volume and spatial depth in a 2D image.

2. To find out how we represent 3D objects in Direct3D.

3. To learn how we model the virtual camera.

4. To understand the rendering pipeline—the process of taking a geometric description of a 3D scene and generating a 2D image from it.

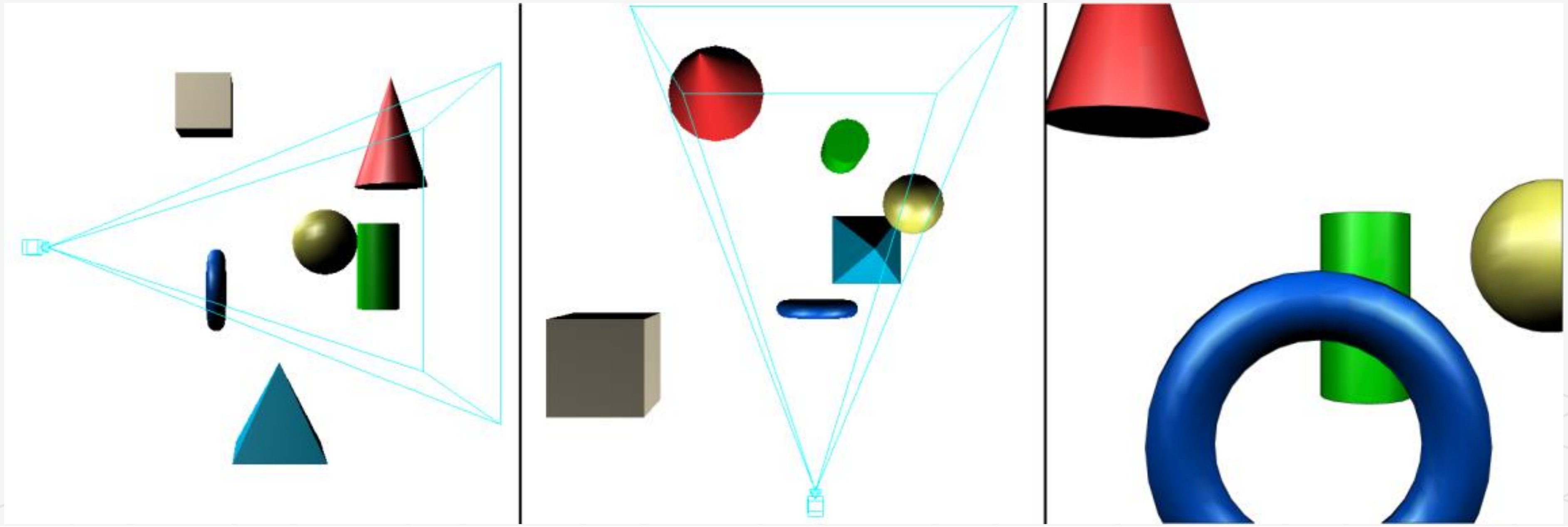
Rendering Pipeline

The rendering pipeline refers to the entire sequence of steps necessary to generate a 2D image based on what the virtual camera sees.

The left image shows a side view of some objects setup in the 3D world with a camera positioned and aimed.

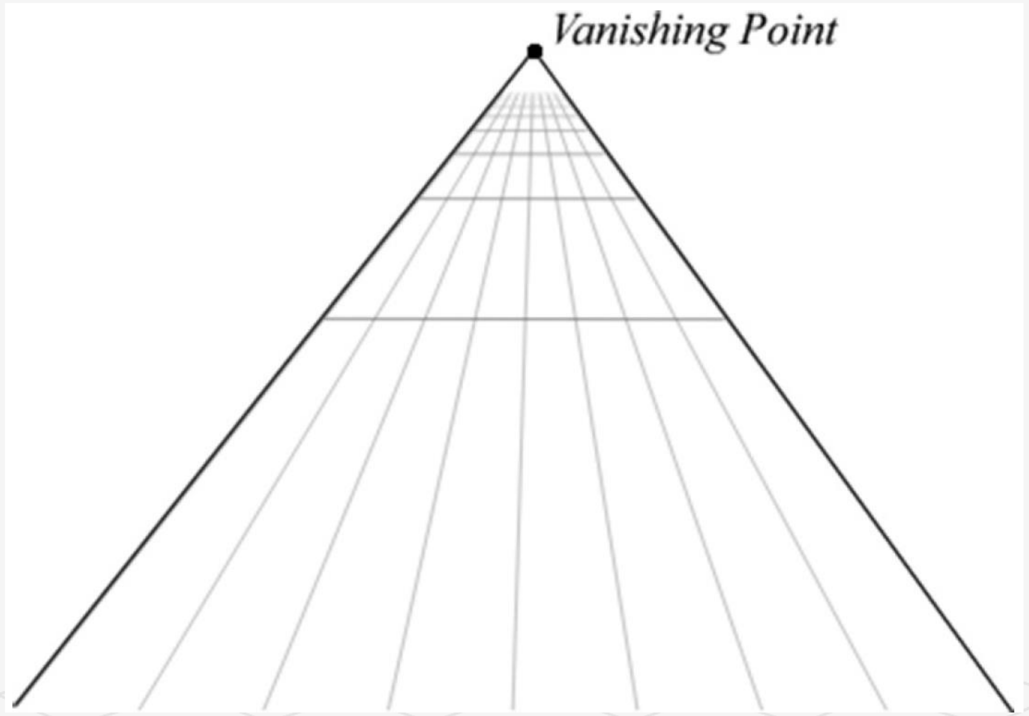
The middle image shows the same scene, but from a top-down view. The “pyramid” volume specifies the volume of space that the viewer can see; objects (and parts of objects) outside this volume are not seen.

The image on the right shows the 2D image created based on what the camera “sees.”



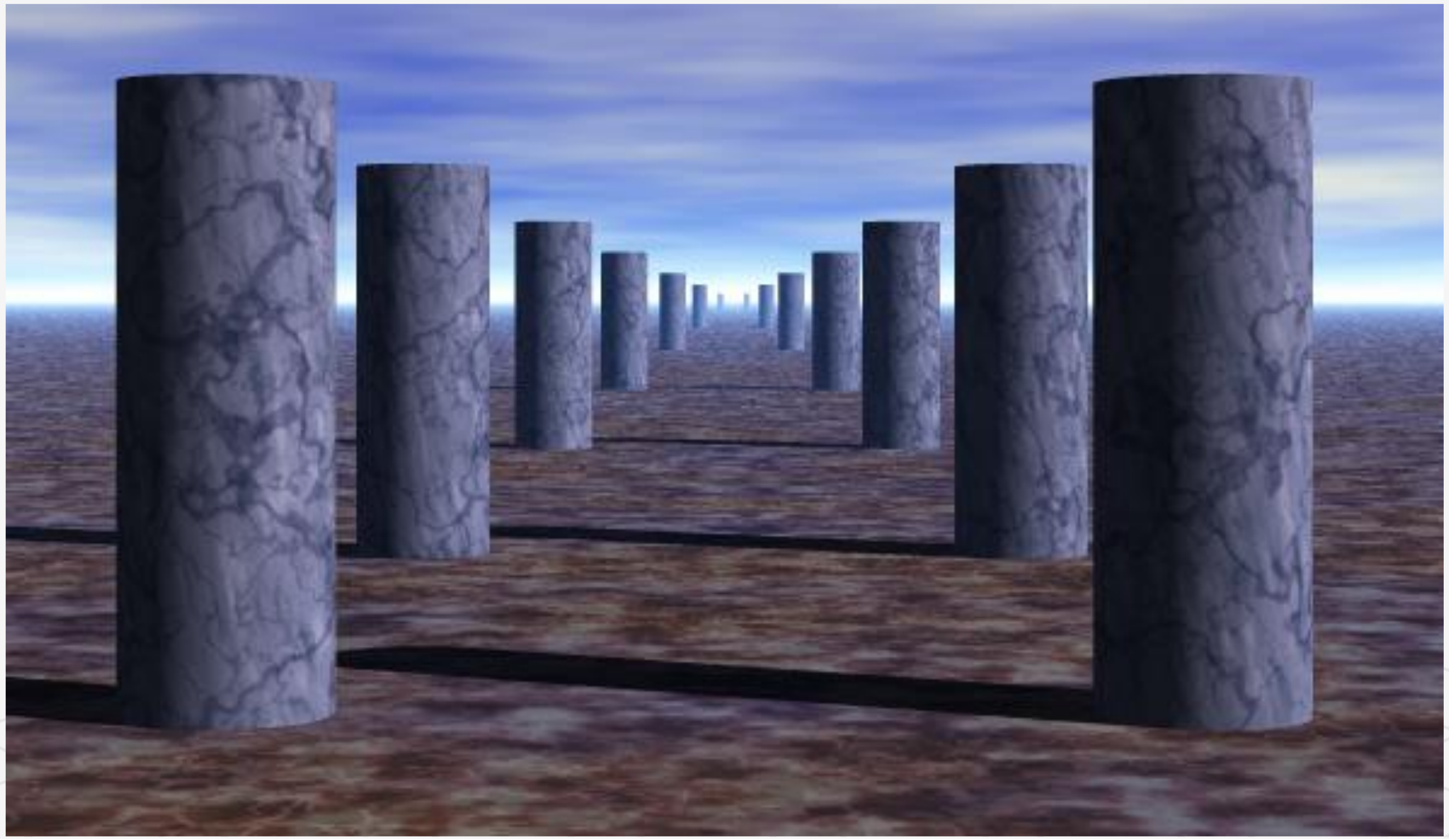
Linear perspective

The railroad rails remain parallel to each other for all time, but if you stand on the railroad and look down its path, you will observe that the two railroad rails get closer and closer together as their distance from you increases, and eventually they converge at an infinite distance. This is one observation that characterizes our human viewing system: parallel lines of vision converge to a *vanishing point*. Artists sometimes call this *linear perspective*.



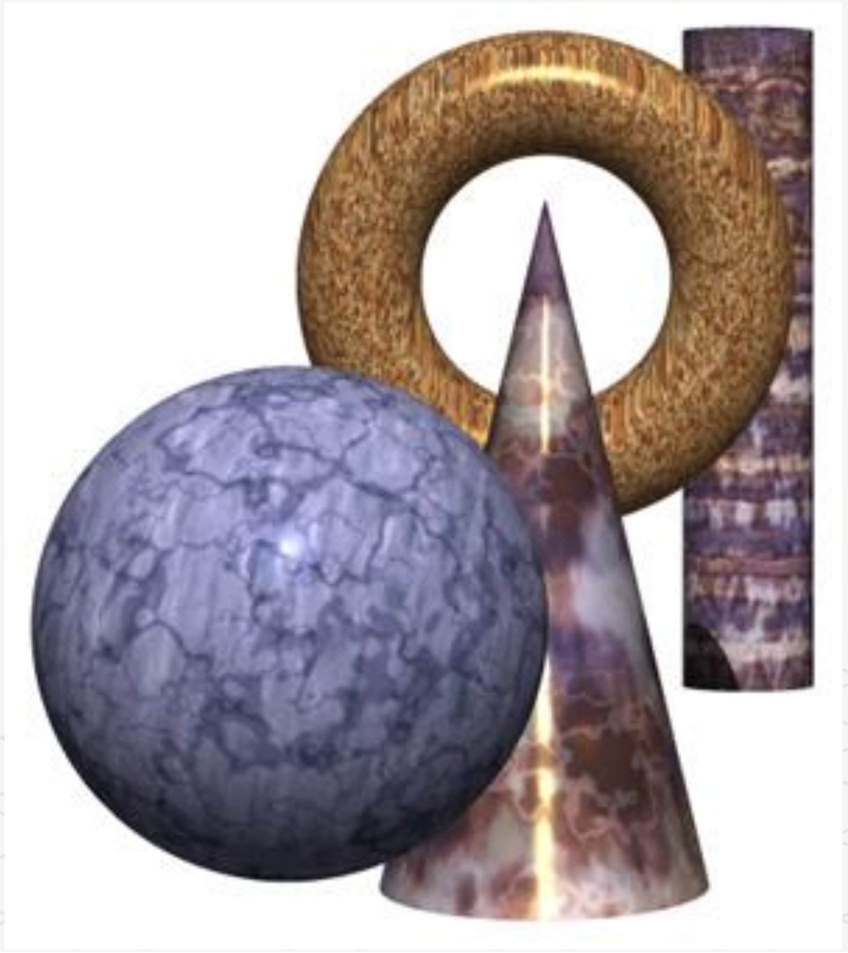
Linear perspective example

Another simple observation of how humans see things is that the size of an object appears to diminish with depth; that is, objects near us look bigger than objects far away. The following figure shows a simple scene where parallel rows of columns are placed behind each other, one after another. The columns are actually all the same size, but as their depths increase from the viewer, they get smaller and smaller. Also notice how the columns are converging to the vanishing point at the horizon.



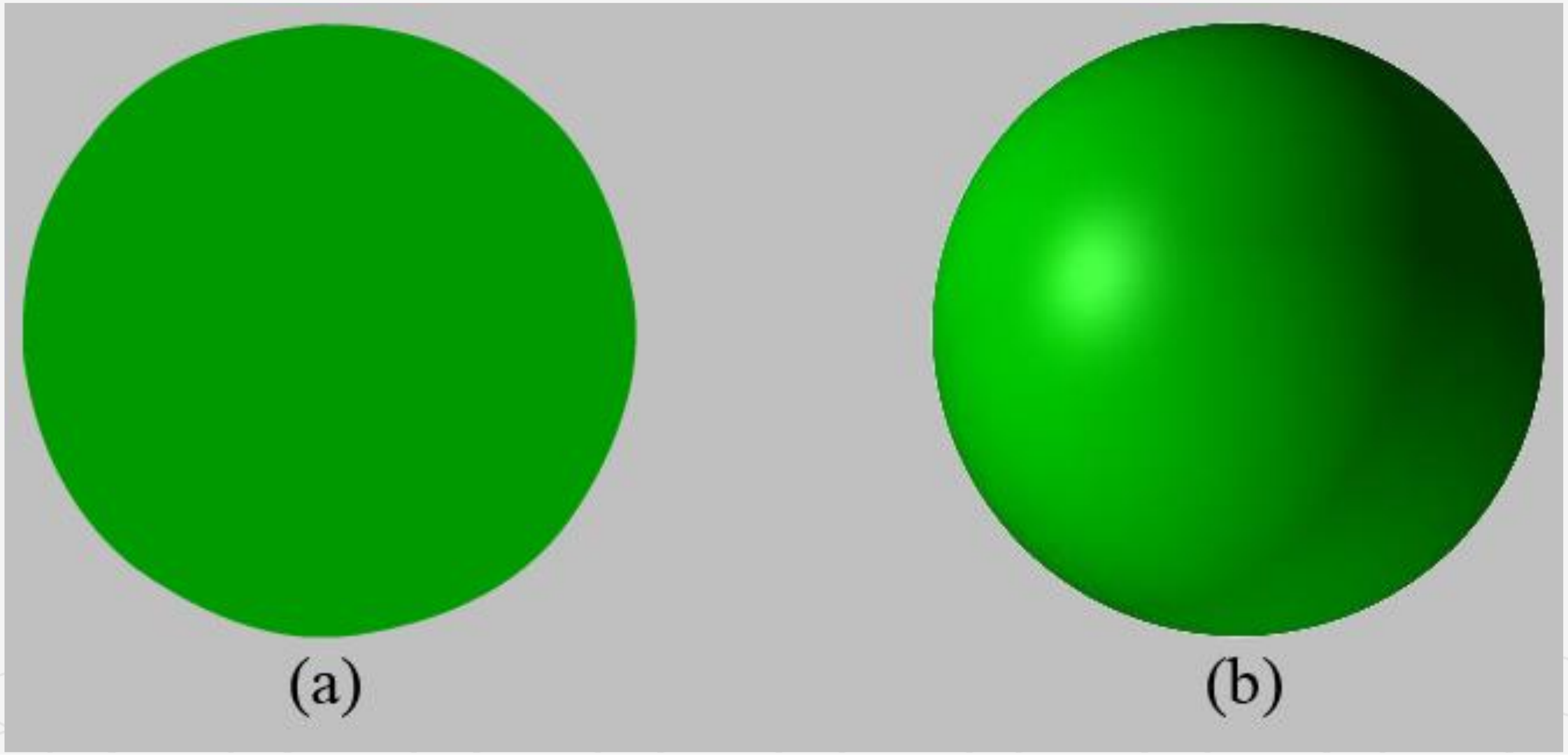
Depth overlap

We all experience *object overlap* which refers to the fact that opaque objects obscure parts (or all) of the objects behind them. This is an important perception, as it conveys the depth ordering relationship of the objects in the scene. Direct3D uses a depth buffer to figure out which pixels are being obscured and thus should not be drawn.



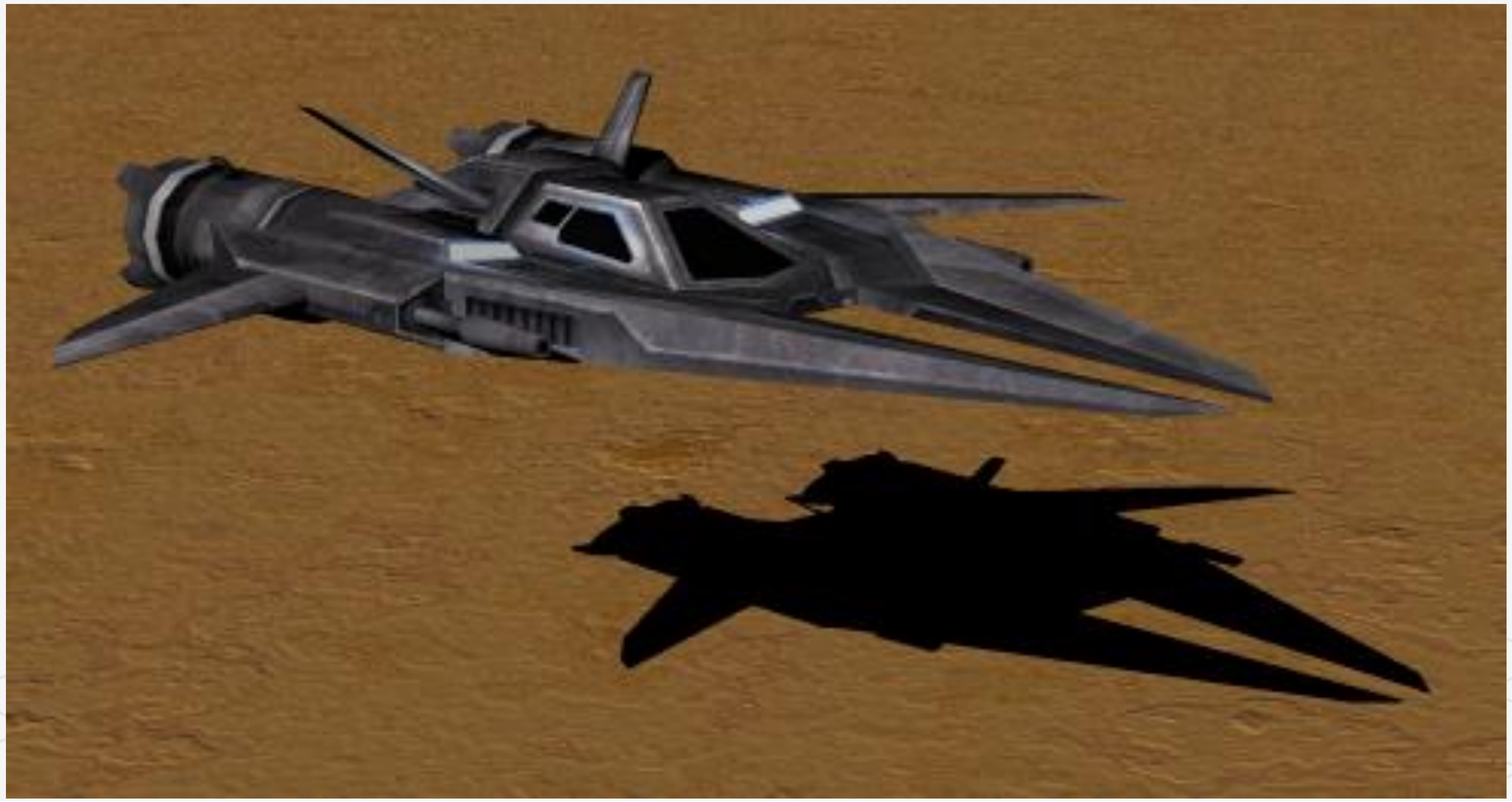
Lighting and shading on a sphere

On the left we have an unlit sphere, and on the right, we have a lit sphere. As you can see, the sphere on the left looks rather flat—maybe it is not even a sphere at all, but just a textured 2D circle! Thus, lighting and shading play a very important role in depicting the solid form and volume of 3D objects.



Shadows

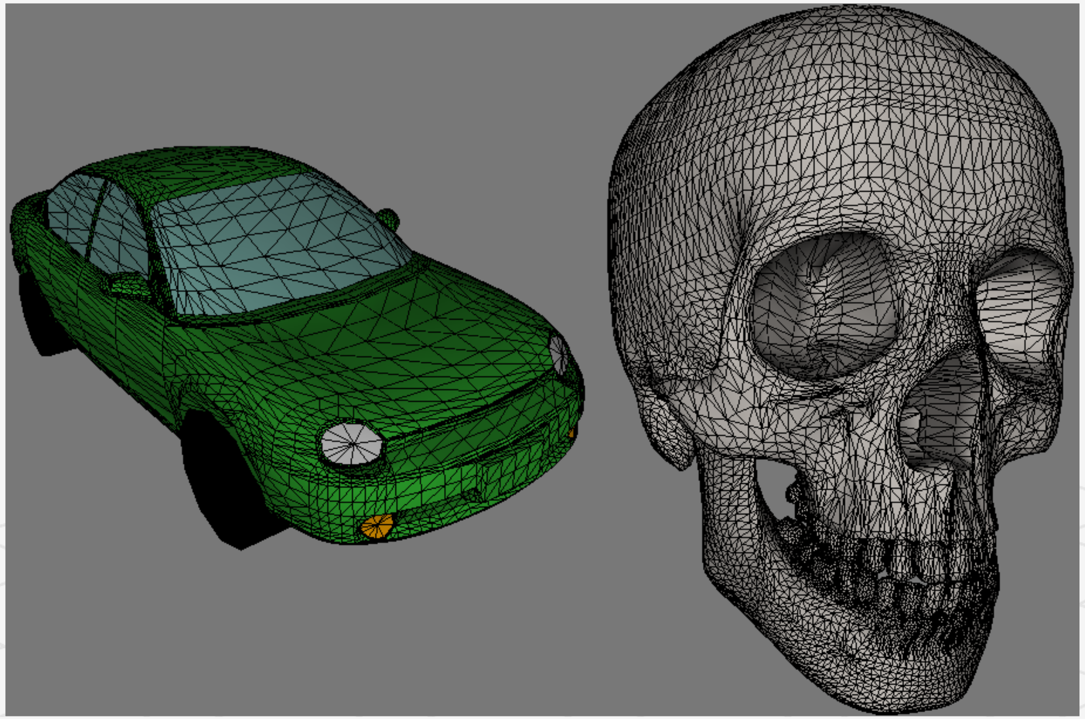
The following shows a spaceship and its shadow. The shadow serves two key purposes. First, it tells us the origin of the light source in the scene. And secondly, it provides us with a rough idea of how high off the ground the spaceship is.



MODEL REPRESENTATION

A solid 3D *object* is represented by a *triangle mesh* approximation, and consequently, triangles form the basic building blocks of the objects we model. An object will have more finer detail with the more triangles you use to build it, but the more triangles we use, the more processing power is needed.

In addition to triangles, it is sometimes useful to draw lines or points. For example, a curve could be graphically drawn by a sequence of short line segments a pixel thick.



Modeler

3D modelers are used to generate and manipulate 3D objects. These modelers allow the user to build complex and realistic meshes in a visual and interactive environment with a rich tool set, thereby making the entire modeling process much easier.

Examples of popular modelers used for game development are:

1. 3D Studio Max (*http://usa.autodesk.com/3ds-max/*)
2. LightWave 3D (*https://www.lightwave3d.com/*)
3. Maya (*http://usa.autodesk.com/maya/*)
4. Softimage|XSI (www.softimage.com)
5. Blender (*www.blender.org/*)

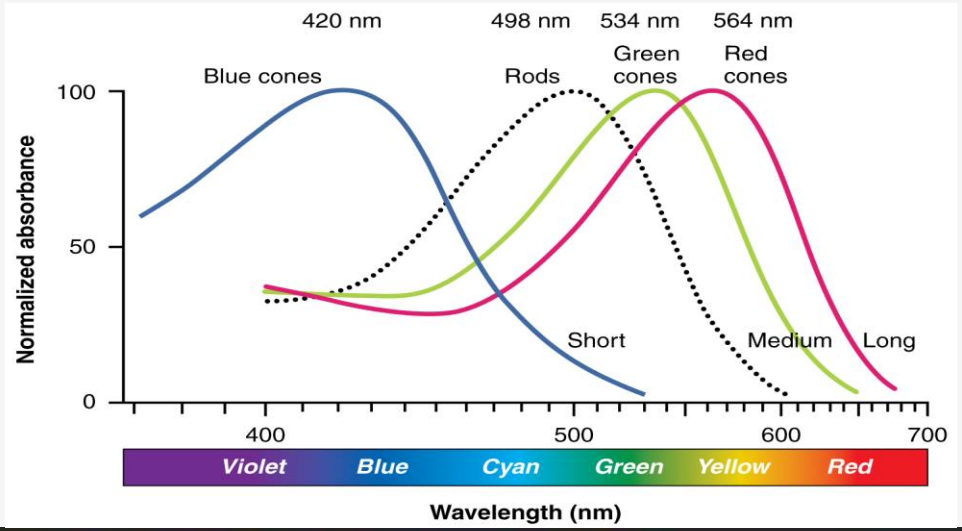
Blender has the advantage for hobbyists of being open source and free.

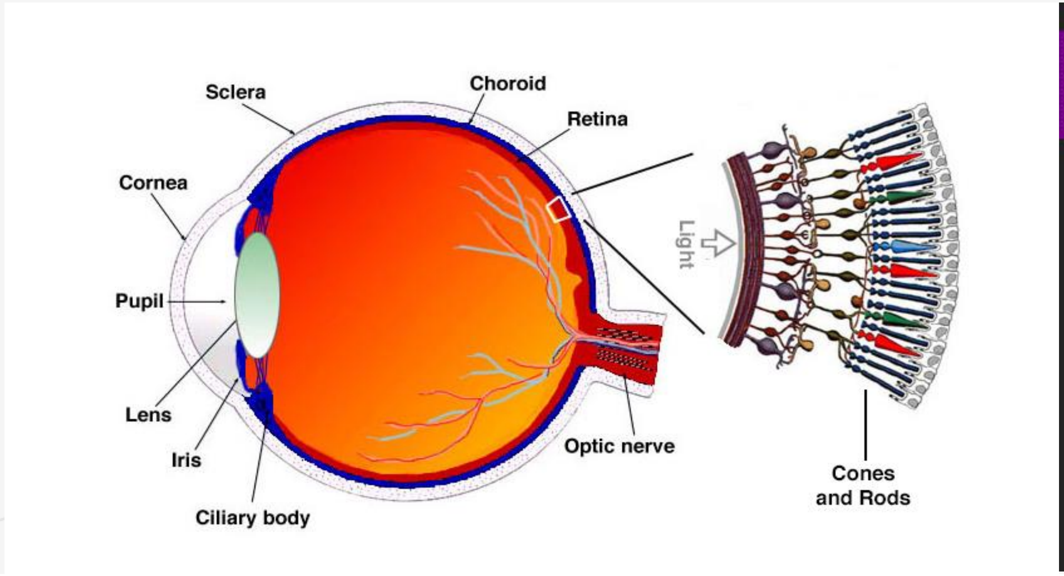
BASIC COMPUTER COLOR

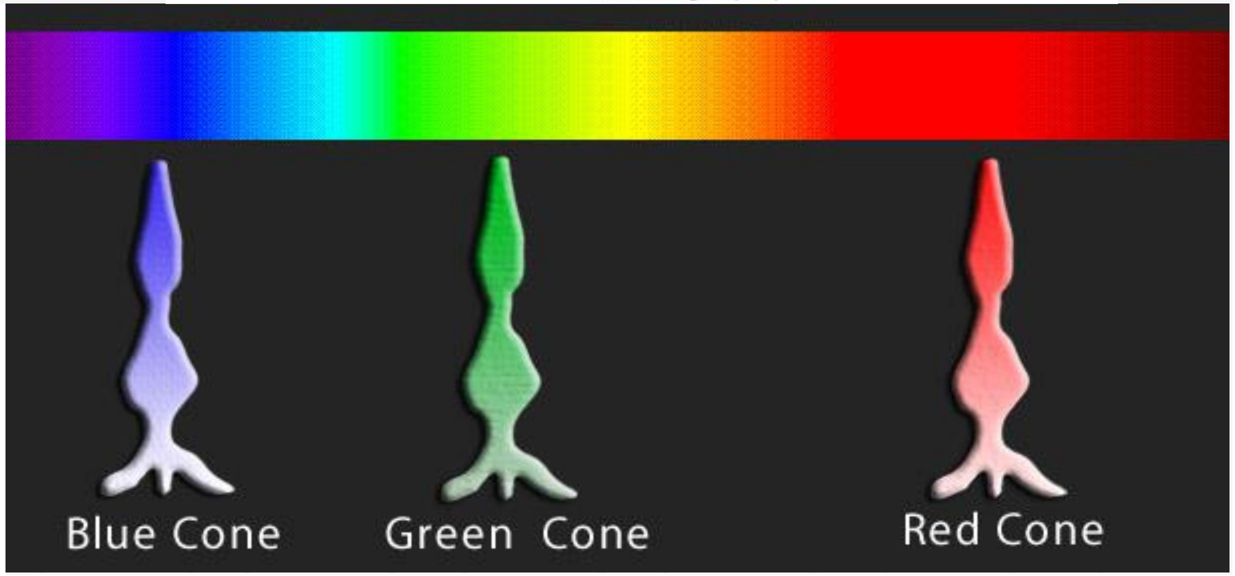
Computer monitors emit a mixture of red, green, and blue light through each pixel.

When the light mixture enters the eye and strikes an area of the retina, cone receptor cells are stimulated and neural impulses are sent down the optic nerve toward the brain.

The brain interprets the signal and generates a color. As the light mixture varies, the cells are stimulated differently, which in turn generates a different color in the mind.







3D color vector

A monitor has a maximum intensity of red, green, and blue light it can emit. To describe the intensities of light, it is useful to use a normalized range from 0 to 1. 0 denotes no intensity and 1 denotes the full intensity. Intermediate values denote intermediate intensities.

For example, the values (0.25, 0.67, 1.0) mean the light mixture consists of 25% intensity of red light, 67% intensity of green light, and 100% intensity of blue light.

We represent a color by a 3D color vector (r, g, b), where $0 \leq r, g, b \leq 1$, and each color component describes the intensity of red, green, and blue light.

we can add color vectors to get new colors: (0.0, 0.5, 0) + (0, 0.0, 0.25) = (0.0, 0.5, 0.25)

By combining a medium intensity green color with a low intensity blue color, we get a dark-green color.

Colors can also be subtracted to get new colors: (1, 1, 1) – (1, 1, 0) = (0, 0, 1)

Scalar multiplication also makes sense. Consider the following: 0.5(1, 1, 1) = (0.5, 0.5, 0.5)

XMColorModulate

Color vectors get their own special color operation called *modulation* or *component-wise* multiplication. It is defined as:

$$(c_r, c_g, c_b) \otimes (k_r, k_g, k_b) = (c_r k_r, c_g k_g, c_b k_b)$$

$$(r, g, b) \otimes (0.5, 0.75, 0.25) = (0.5r, 0.75g, 0.25b)$$

For example, suppose we have an incoming ray of light with color (*r*, *g*, *b*) and it strikes a surface which reflects 50% red light, 75% green light, and 25% blue light, and absorbs the rest. Then the color of the reflected light ray is given by:

```
inline XMVECTOR XM_CALLCONV XMColorModulate
(
    FXMVECTOR C1,
    FXMVECTOR C2
)
{
    return XMVectorMultiply(C1, C2);
}

XMVECTOR color1 = XMVectorSet(1.0f, 0.0f, 0.0f, 1.0f);
XMVECTOR color2 = XMVectorSet(0.5f, 0.75f, 0.25f, 1.0f);
cout << "colorModulation = color1 * color2  = " << XMColorModulate(color1,color2) << endl;
```

DirectX::PackedVector namespace

The DirectX Math library (*#include <DirectXPackedVector.h>*) provides the following structure, in the DirectX::PackedVector namespace, for storing a 32-bit color:

```
namespace DirectX
{

namespace PackedVector
{
struct XMCOLOR
{
    union
    {
        struct
        {
            uint8_t b;  // Blue:    0/255 to 255/255
            uint8_t g;  // Green:   0/255 to 255/255
            uint8_t r;  // Red:     0/255 to 255/255
            uint8_t a;  // Alpha:   0/255 to 255/255
        };
        uint32_t c;
    };

    XMCOLOR() = default;

    XMCOLOR(const XMCOLOR&) = default;
    XMCOLOR& operator=(const XMCOLOR&) = default;

    XMCOLOR(XMCOLOR&&) = default;
    XMCOLOR& operator=(XMCOLOR&&) = default;

    XM_CONSTEXPR XMCOLOR(uint32_t Color) : c(Color) {}
    XMCOLOR(float _r, float _g, float _b, float _a);
    explicit XMCOLOR(_In_reads_(4) const float *pArray);

    operator uint32_t () const { return c; }

    XMCOLOR& operator= (const uint32_t Color) { c = Color; return *this; }
};
```

PackedVector::XMLoadColor

To convert a 32-bit color (XMCOLOR) to a 128-bit color (XMVECTOR) and conversely, the DirectXMath library defines the following functions:

```
XMVECTOR    XM_CALLCONV     XMLoadColor(const XMCOLOR* pSource);

void    XM_CALLCONV     XMStoreColor(XMCOLOR* pDestination, _In_ FXMVECTOR V);
```

Typically, 128-bit colors values are used where high precision color operations are needed (e.g., in a pixel shader); in this way, we have many bits of accuracy for the calculations so arithmetic error does not accumulate too much. The final pixel color, however, is usually stored in a 32-bit color value in the back buffer;

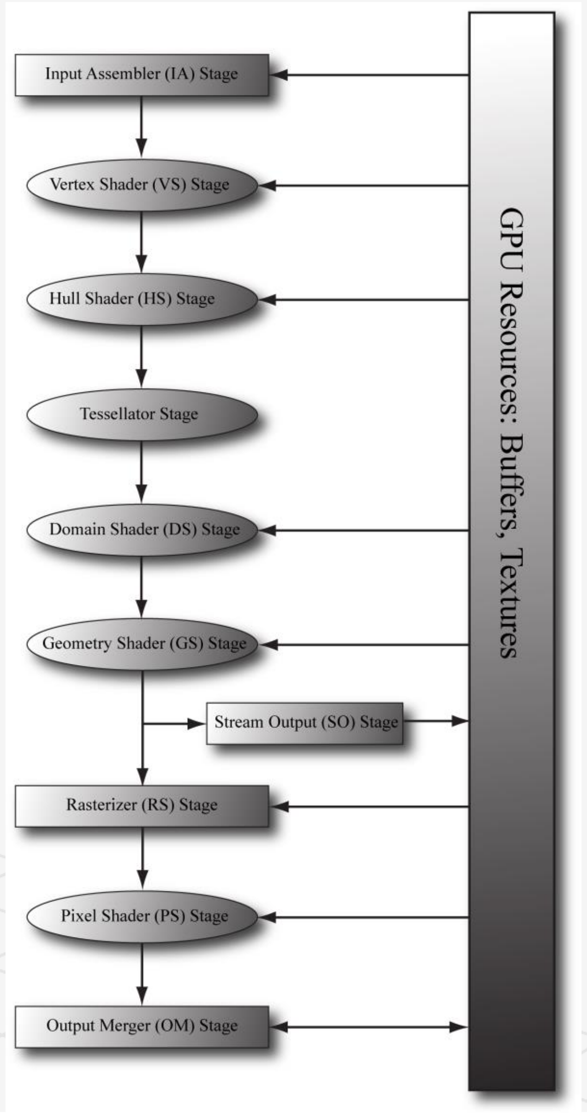
Rendering Pipeline

Most stages do not write to GPU resources Instead, their output is just fed in as input to the next stage .

An arrow going from a stage to memory means the stage writes to GPU resources;

For example, the output merger stage writes data to textures such as the back buffer and depth/stencil buffer. Observe that the arrow for the output merger stage is bidirectional (it reads and writes to GPU resources).

- Input Assembler Stage (IA)
- Vertex Shader Stage (VS)
- Hull Shader Stage (HS)
- Tessellator Stage (TS)
- Domain Shader Stage (DS)
- Geometry Shader Stage (GS)
- Rasterizer Stage (RS)
- Pixel Shader Stage (PS)
- Output Merger Stage (OM)



Input Assembler Stage

. Reads geometric data (vertices and indices) from memory and uses it to assemble geometric primitives (e.g., triangles, lines)

▪ Vertices are bound to the rendering pipeline in a special Direct3D data structure called a vertex buffer

▪ A vertex buffer just stores a list of vertices in contiguous memory.

▪ We tell Direct3D how to form geometric primitives from the vertex data by specifying the primitive topology

```
mCommandList->IASetPrimitiveTopology(D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST);
```

“d3dcommon.h” lists all of existing topologies:

```
typedef enum D3D_PRIMITIVE_TOPOLOGY
    {
        D3D_PRIMITIVE_TOPOLOGY_UNDEFINED= 0, → The IA stage has not been initialized with a primitive topology.
        D3D_PRIMITIVE_TOPOLOGY_POINTLIST= 1,
        D3D_PRIMITIVE_TOPOLOGY_LINELIST= 2,
        D3D_PRIMITIVE_TOPOLOGY_LINESTRIP= 3,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST= 4,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLESTRIP= 5,
```

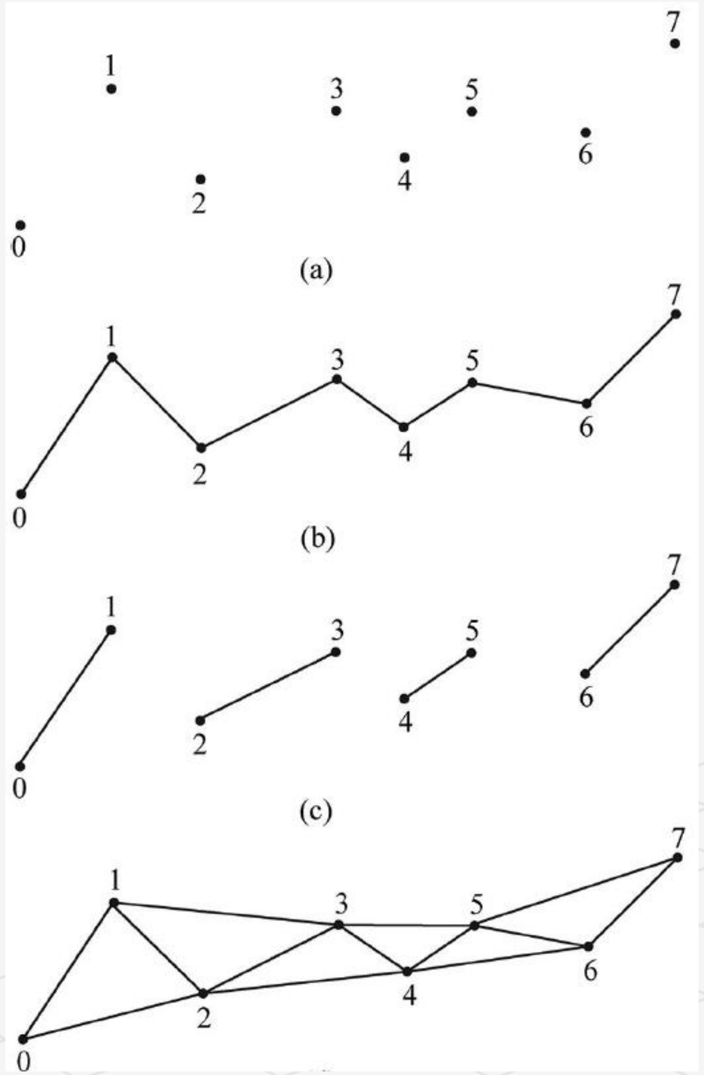
Primitive topologies

a) Point list

b) Line strip

c) Line lists

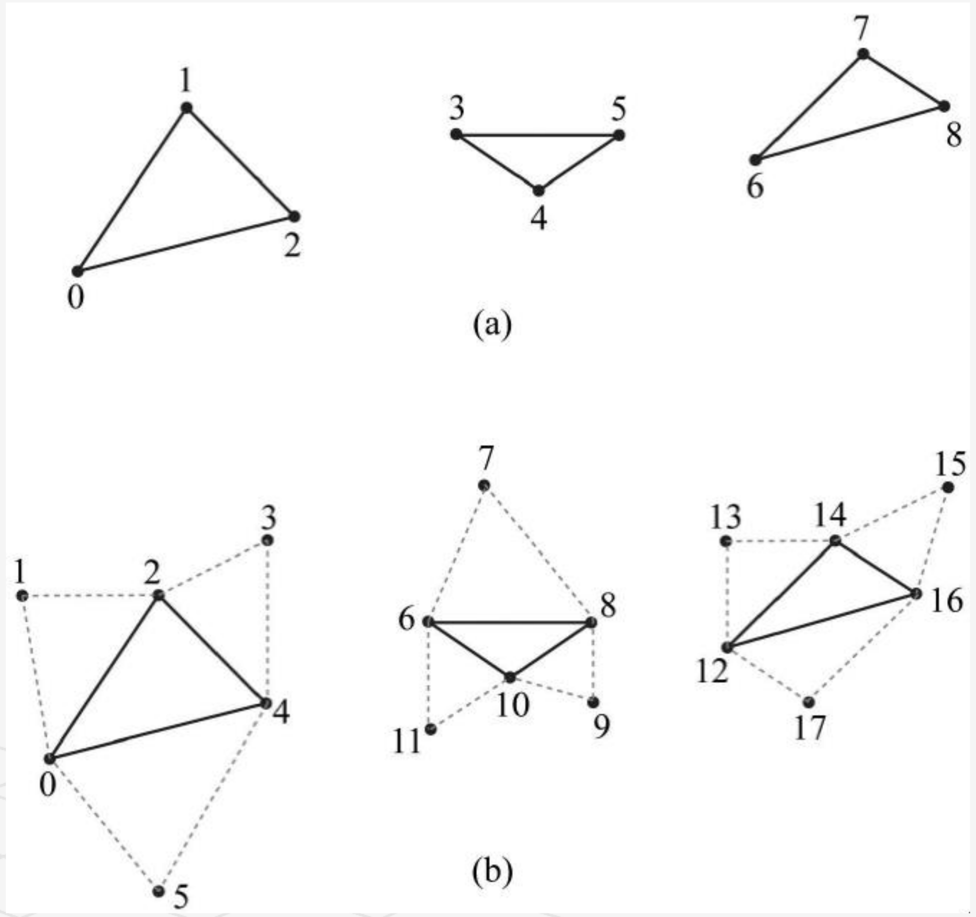
d) Triangle strip



a) Triangle list

b) Triangle list

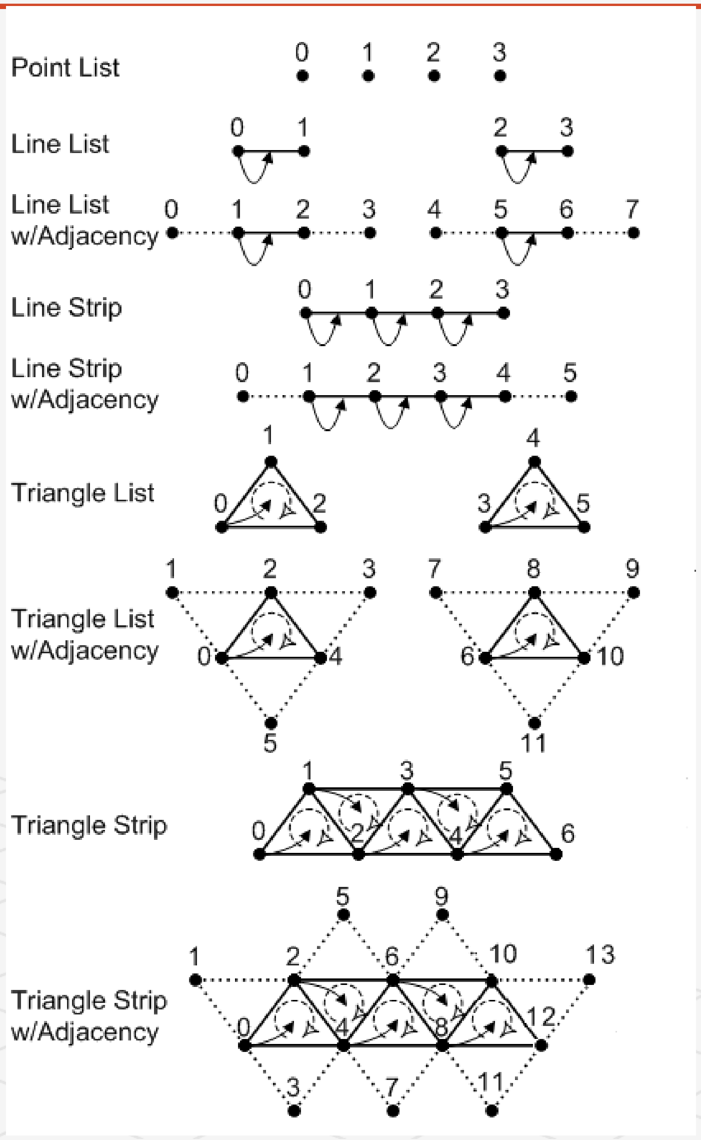
with adjacency



Primitive Adjacency

All Direct3D 10 and higher primitive types (except the point list) are available in two versions: one primitive type with adjacency and one primitive type without adjacency. Primitives with adjacency contain some of the surrounding vertices, while primitives without adjacency contain only the vertices of the target primitive. For example, the line list primitive (represented by the **D3D_PRIMITIVE_TOPOLOGY_LINELIST** value) has a corresponding line list primitive that includes adjacency (represented by the **D3D_PRIMITIVE_TOPOLOGY_LINELIST_ADJ** value.)

```
typedef enum D3D_PRIMITIVE_TOPOLOGY
    {
        D3D_PRIMITIVE_TOPOLOGY_UNDEFINED= 0,
        D3D_PRIMITIVE_TOPOLOGY_POINTLIST= 1,
        D3D_PRIMITIVE_TOPOLOGY_LINELIST= 2,
        D3D_PRIMITIVE_TOPOLOGY_LINESTRIP= 3,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST= 4,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLESTRIP= 5,
        D3D_PRIMITIVE_TOPOLOGY_LINELIST_ADJ= 10,
        D3D_PRIMITIVE_TOPOLOGY_LINESTRIP_ADJ= 11,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST_ADJ= 12,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLESTRIP_ADJ= 13,
```



Control Point Patch List

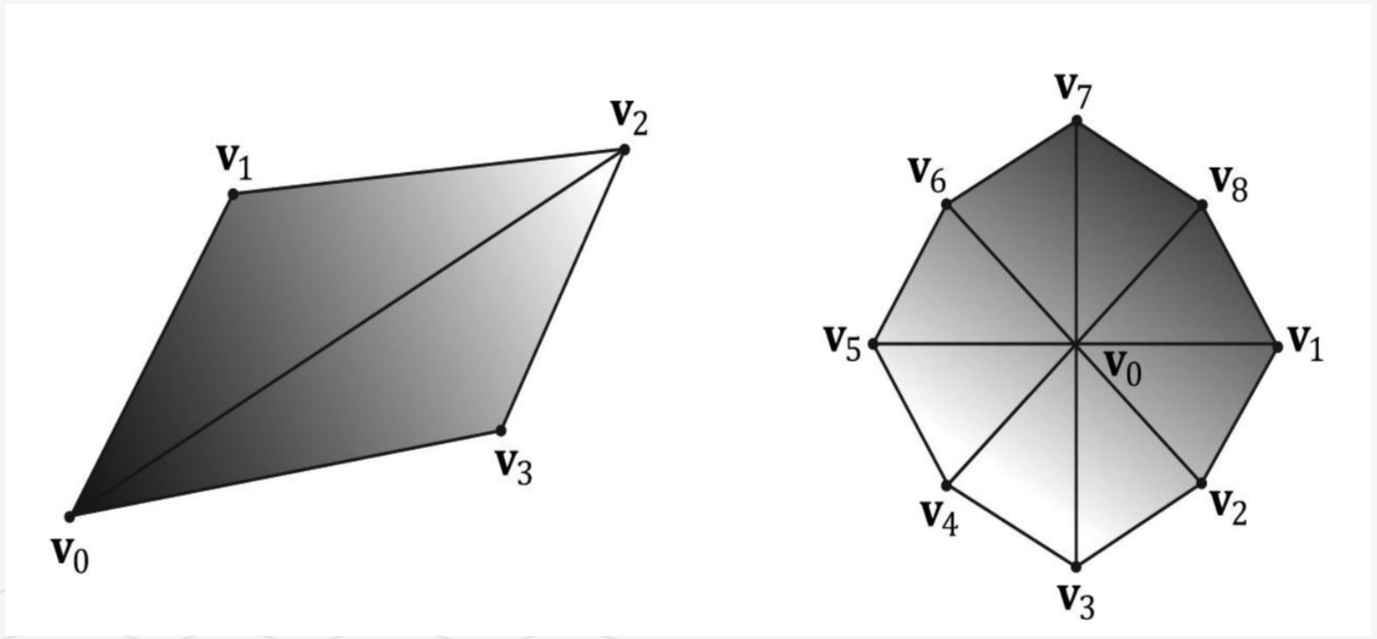
The D3D_PRIMITIVE_TOPOLOGY_*N*_CONTROL_POINT_PATCHLIST topology type indicates that the vertex data should be interpreted as a patch lists with *N* control points. These are used in the (optional) tessellation stage of the rendering pipeline, and therefore, we will postpone a discussion of them

```
typedef enum D3D_PRIMITIVE_TOPOLOGY
    {
        D3D_PRIMITIVE_TOPOLOGY_UNDEFINED= 0,
        D3D_PRIMITIVE_TOPOLOGY_POINTLIST= 1,
        D3D_PRIMITIVE_TOPOLOGY_LINELIST= 2,
        D3D_PRIMITIVE_TOPOLOGY_LINESTRIP= 3,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST= 4,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLESTRIP= 5,
        D3D_PRIMITIVE_TOPOLOGY_LINELIST_ADJ= 10,
        D3D_PRIMITIVE_TOPOLOGY_LINESTRIP_ADJ= 11,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLELIST_ADJ= 12,
        D3D_PRIMITIVE_TOPOLOGY_TRIANGLESTRIP_ADJ= 13,
        D3D_PRIMITIVE_TOPOLOGY_1_CONTROL_POINT_PATCHLIST= 33,
        D3D_PRIMITIVE_TOPOLOGY_2_CONTROL_POINT_PATCHLIST= 34,
        D3D_PRIMITIVE_TOPOLOGY_3_CONTROL_POINT_PATCHLIST= 35,
```

Winding Order

Triangles are the basic building blocks for solid 3D objects. The following code shows the vertex arrays used to construct a quad and octagon using triangle lists (i.e., every three vertices form a triangle). The order in which you specify the vertices of a triangle is important and is called the *winding order*

```
Vertex quad[6] = {
v0, v1, v2, // Triangle 0
v0, v2, v3, // Triangle 1
};
Vertex octagon[24] = {
v0, v1, v2, // Triangle 0
v0, v2, v3, // Triangle 1
v0, v3, v4, // Triangle 2
v0, v4, v5, // Triangle 3
v0, v5, v6, // Triangle 4
v0, v6, v7, // Triangle 5
v0, v7, v8, // Triangle 6
v0, v8, v1 // Triangle 7
};
```



Indices

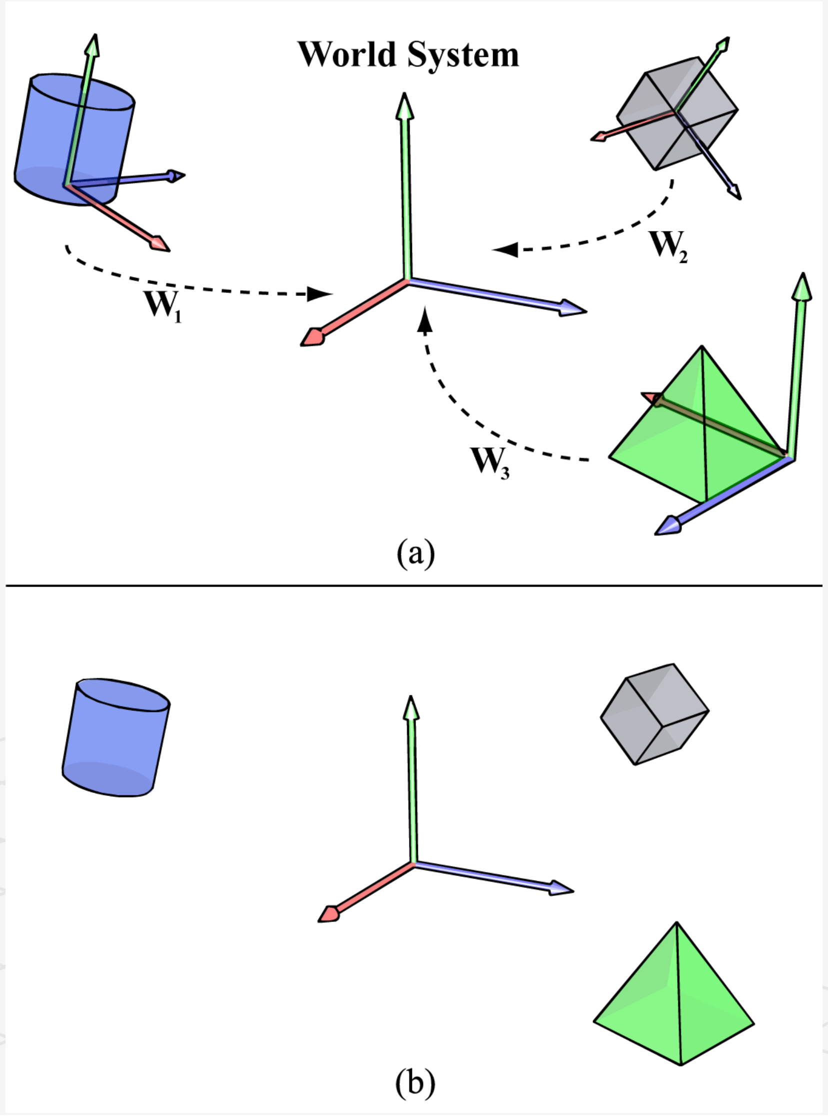
TOO MANY DUPLICATES!!!
Why is that bad?
1- Increased memory requirements
2- Increased processing by the graphics hardware

1- Indices are simply integers and do not take as much memory as a full vertex structure.
2- With good vertex cache ordering, the graphics hardware won’t have to process duplicate vertices.

```
Vertex v[9] = { v0, v1, v2, v3, v4, v5, v6, v7, v8 };
UINT indexList[24] = {
0, 1, 2, // Triangle 0
0, 2, 3, // Triangle 1
0, 3, 4, // Triangle 2
0, 4, 5, // Triangle 3
0, 5, 6, // Triangle 4
0, 6, 7, // Triangle 5
0, 7, 8, // Triangle 6
0, 8, 1 // Triangle 7
};
```

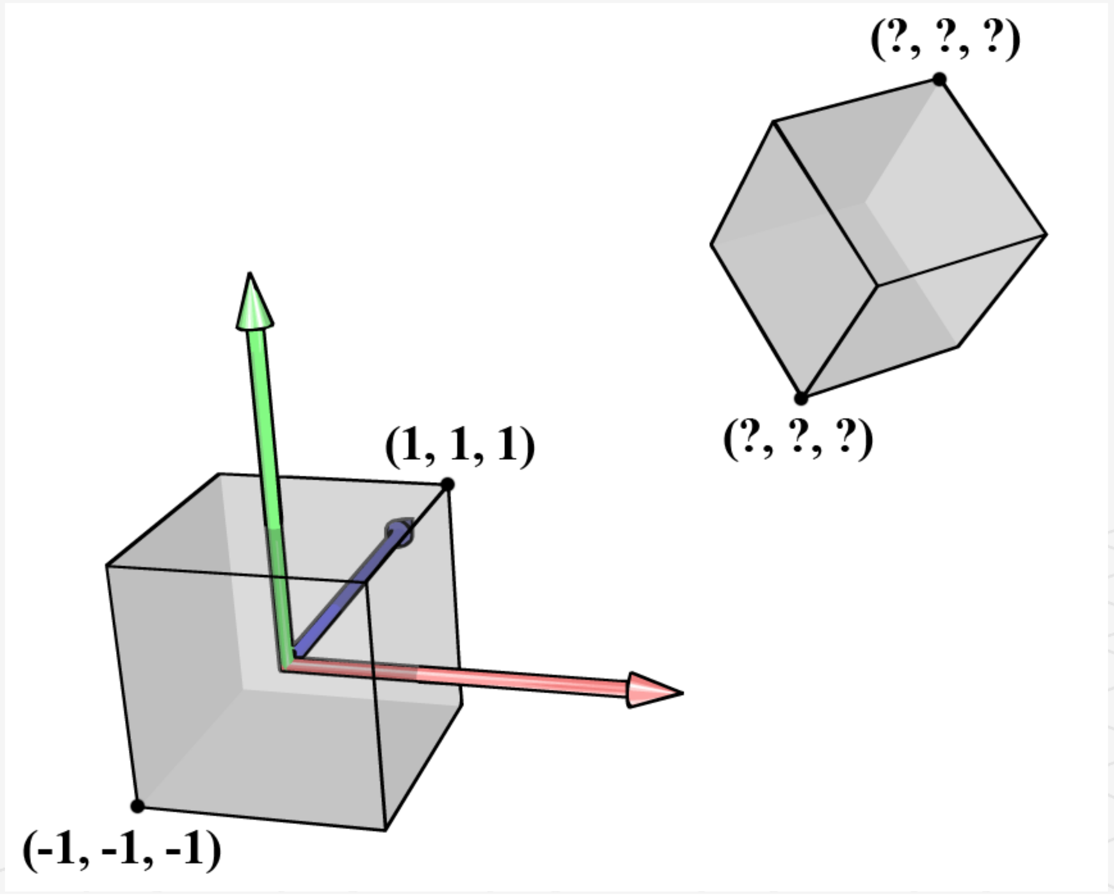
Vertex Shader Stage

- After the primitives have been assembled, the vertices are fed into the vertex shader stage
- The vertex shader can be thought of as a function that inputs a vertex and outputs a vertex
- Many special effects can be done in the vertex shader such as transformations, lighting, and displacement mapping
- We have access to the input vertex data and we also can access textures and other data stored in GPU memory such as transformation matrices, and scene lights
- Once the vertices of the 3D model have been defined in local space, it is placed in the global scene
- We specify where we want the origin and axes of the local space coordinate system relative to the global scene coordinate system, then execute a change of coordinate transformation
- This process of changing coordinates relative to a local coordinate system into the global scene coordinate system is called the *world transform*, and the corresponding matrix is called the *world matrix*
- If you want to define an object directly in the world space, then you can supply an identity world matrix.



World Matrix

Change of Coordinate from local space to world space (AKA. World Matrix). A more common approach is to define **W** as a sequence of transformations, say **W** = **SRT**, the product of a scaling matrix **S** to scale the object into the world, followed by a rotation matrix **R** to define the orientation of the local space relative to the world space, followed by a translation matrix **T** to define the origin of the local space relative to the world space.



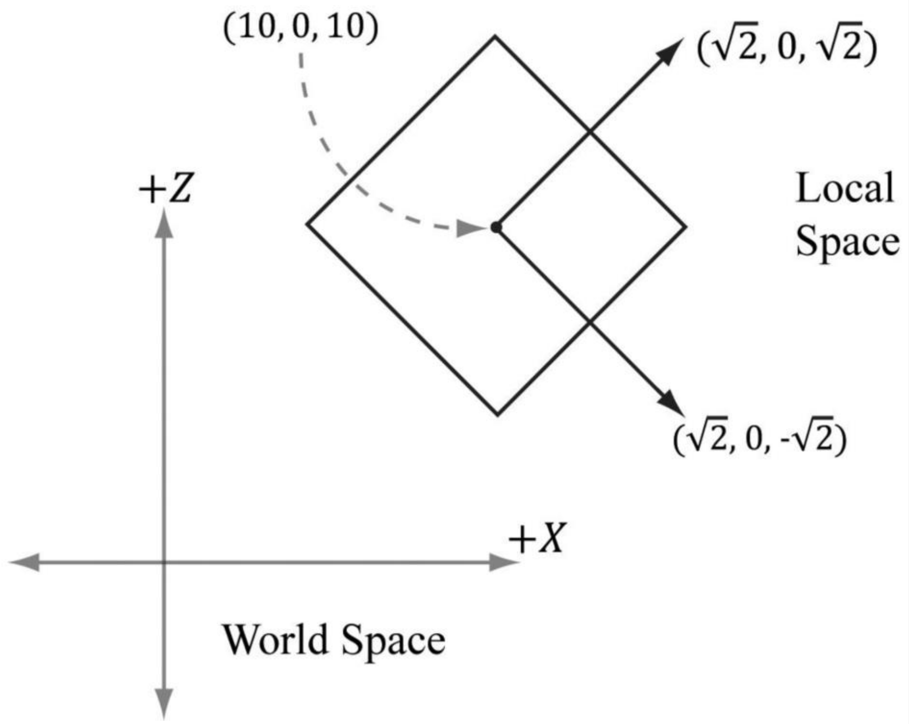
$$\mathbf{W} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ Q_x & Q_y & Q_z & 1 \end{bmatrix}$$

Example

Suppose we have a unit square defined relative to some local space with minimum and maximum points (−0.5, 0, −0.5) and (0.5, 0, 0.5), respectively. Find the world matrix such that the square has a length of 2 in world space, the square is rotated 45° clockwise in the *xz*-plane of the world space, and the square is positioned at (10, 0, 10) in world space. We construct **S**, **R**, **T**, and **W** as follows:

$$\mathbf{S} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 10 & 0 & 10 & 1 \end{bmatrix}$$

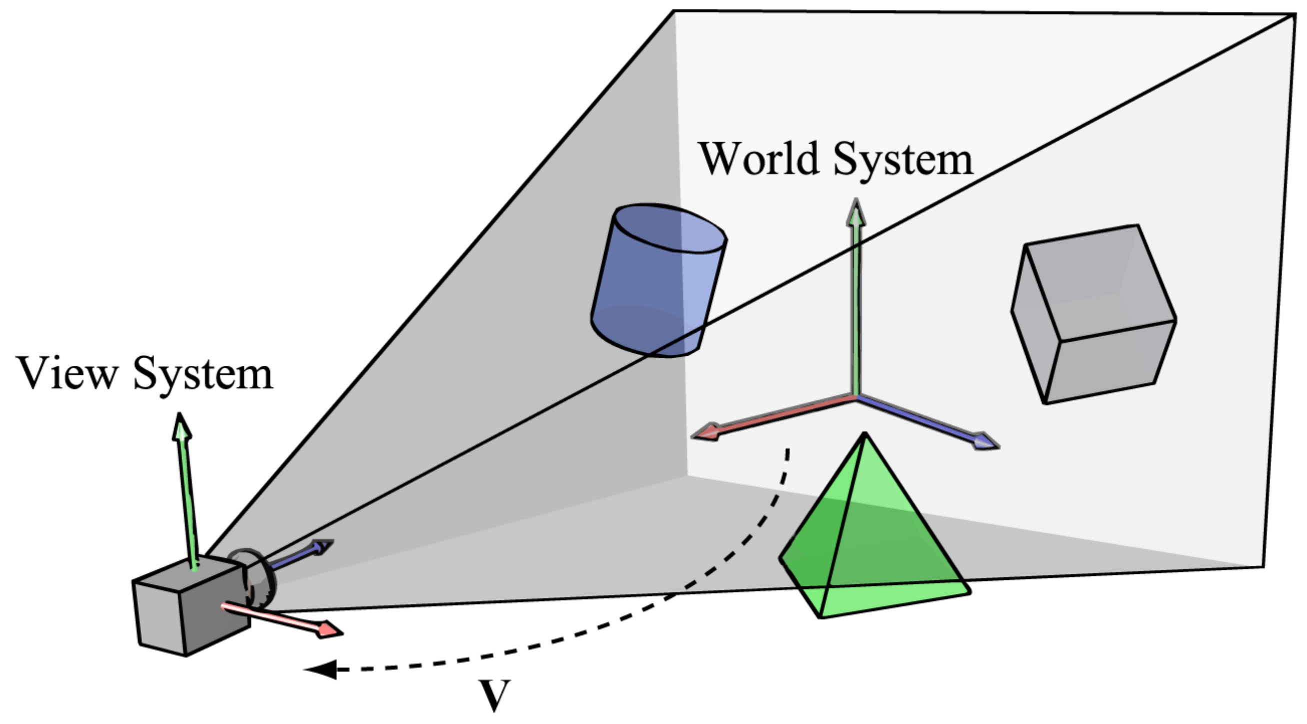
$$\mathbf{W} = \mathbf{SRT} = \begin{bmatrix} \sqrt{2} & 0 & -\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{2} & 0 & \sqrt{2} & 0 \\ 10 & 0 & 10 & 1 \end{bmatrix}$$



View Space

In order to form a 2D image of the scene, we must place a virtual camera in the scene. The camera specifies what volume of the world the viewer can see and thus what volume of the world we need to generate a 2D image of.

The camera sits at the origin looking down the positive *z*-axis, the *x*-axis aims to the right of the camera, and the *y*-axis aims above the camera. Instead of describing our scene vertices relative to the world space, it is convenient for later stages of the rendering pipeline to describe them relative to the camera coordinate system. The change of coordinate transformation from world space to view space is called the *view transform*, and the corresponding matrix is called the *view matrix*.



View Matrix

If $\boldsymbol{Q}_W = (Q_x, Q_y, Q_z, 1), \boldsymbol{u}_W = (u_x, u_y, u_z, 0), \boldsymbol{v}_W = (v_x, v_y, v_z, 0), and\ \boldsymbol{w}_W = (w_x, w_y, w_z, 0)$ describe, respectively, the origin, *x*-, *y*-, and *z*-axes of view space with homogeneous coordinates relative to world space. Then the change of coordinate matrix from view space to world space is:

$$\mathbf{W} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ Q_x & Q_y & Q_z & 1 \end{bmatrix}$$

We want the reverse transformation from world space to view space. That reverse transformation is just given by the inverse.

$\mathbf{W}^{-1}$ transforms from world space to view space.

The world coordinate system and view coordinate system generally differ by position and orientation only, so it makes intuitive sense that **W** = **RT** (i.e., the world matrix can be decomposed into a rotation followed by a translation). This form makes the inverse easier to compute:

$$\mathbf{V} = \mathbf{W}^{-1} = (\mathbf{RT})^{-1} = \mathbf{T}^{-1}\mathbf{R}^{-1} = \mathbf{T}^{-1}\mathbf{R}^{T}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Q_x & -Q_y & -Q_z & 1 \end{bmatrix} \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ -\mathbf{Q}\cdot\mathbf{u} & -\mathbf{Q}\cdot\mathbf{v} & -\mathbf{Q}\cdot\mathbf{w} & 1 \end{bmatrix}$$

Camera Local Coordinates

Given the position of the camera, the target point, and the world "up" direction, how to compute the local coordinate system of the camera?

Let **Q** be the position of the camera and let **T** be the target point the camera is aimed at. Furthermore, let **j** be the unit vector that describes the "up" direction of the world space. We use the world *xz*-plane as our world "ground plane" . The direction the camera is looking is given by:

$$\mathbf{w} = \frac{\mathbf{T}-\mathbf{Q}}{||\mathbf{T}-\mathbf{Q}||}$$

This vector describes the local *z*-axis of the camera.

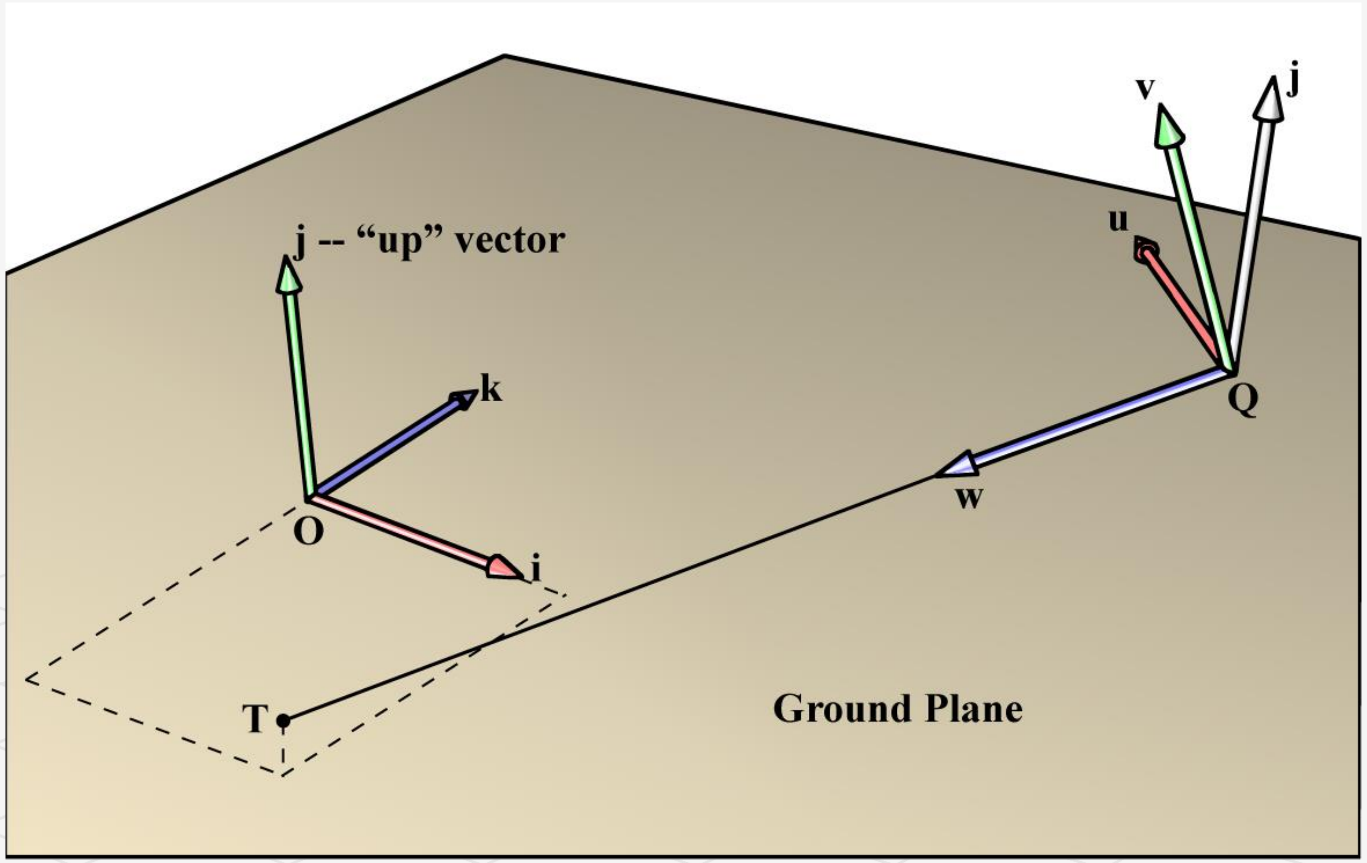
A unit vector that aims to the "right" of **w** is given by:

$$\mathbf{u} = \frac{\mathbf{j}\times\mathbf{w}}{||\mathbf{j}\times\mathbf{w}||}$$

This vector describes the local *x*-axis of the camera.

A vector that describes the local *y*-axis of the camera is given by:

$$\mathbf{v} = \mathbf{w}\times\mathbf{u}$$



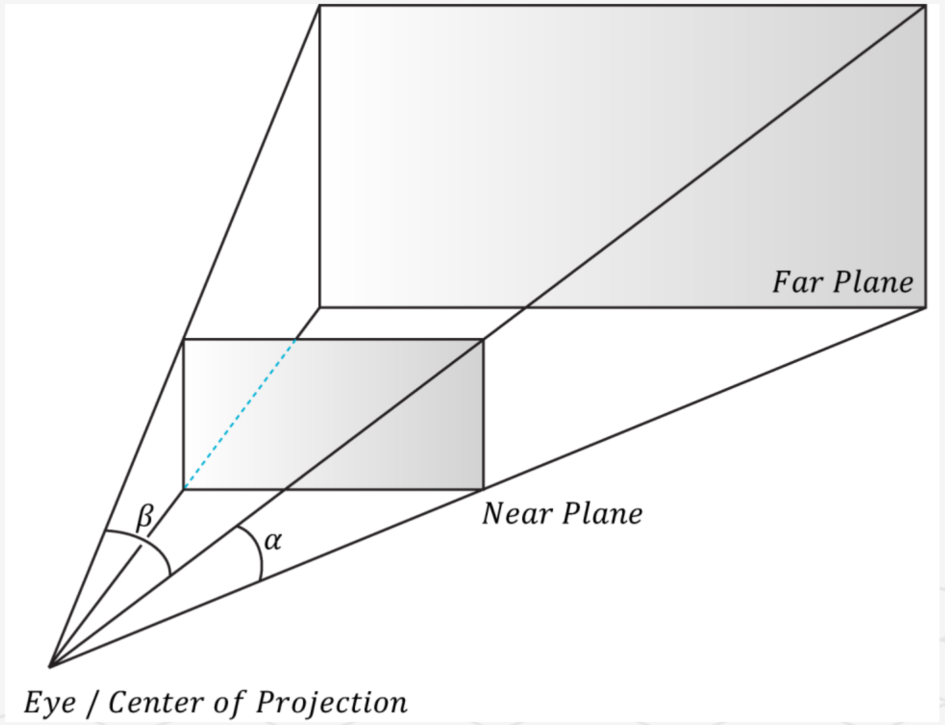
XMMatrixLookAtLH

The DirectXMath library provides the following function for computing the view matrix based on the just described process:

```
XMMATRIX XM_CALLCONV XMMatrixLookAtLH( // Outputs view matrix V

FXMVECTOR EyePosition, // Input camera position Q

FXMVECTOR FocusPosition, // Input target point T

FXMVECTOR UpDirection); // Input world up direction j

// Build the view matrix. Look at: void BoxApp::Update(const GameTimer& gt)

    XMVECTOR pos = XMVectorSet(x, y, z, 1.0f); // camera position

    XMVECTOR target = XMVectorZero(); // target position (world centre)

    XMVECTOR up = XMVectorSet(0.0f, 1.0f, 0.0f, 0.0f); //(y-axis)

    XMMATRIX view = XMMatrixLookAtLH(pos, target, up);
```

Projection and Homogeneous Clip Space

There is another component to a camera, which is the volume of space the camera sees. This volume is described by a frustum:



Perspective projection matrix

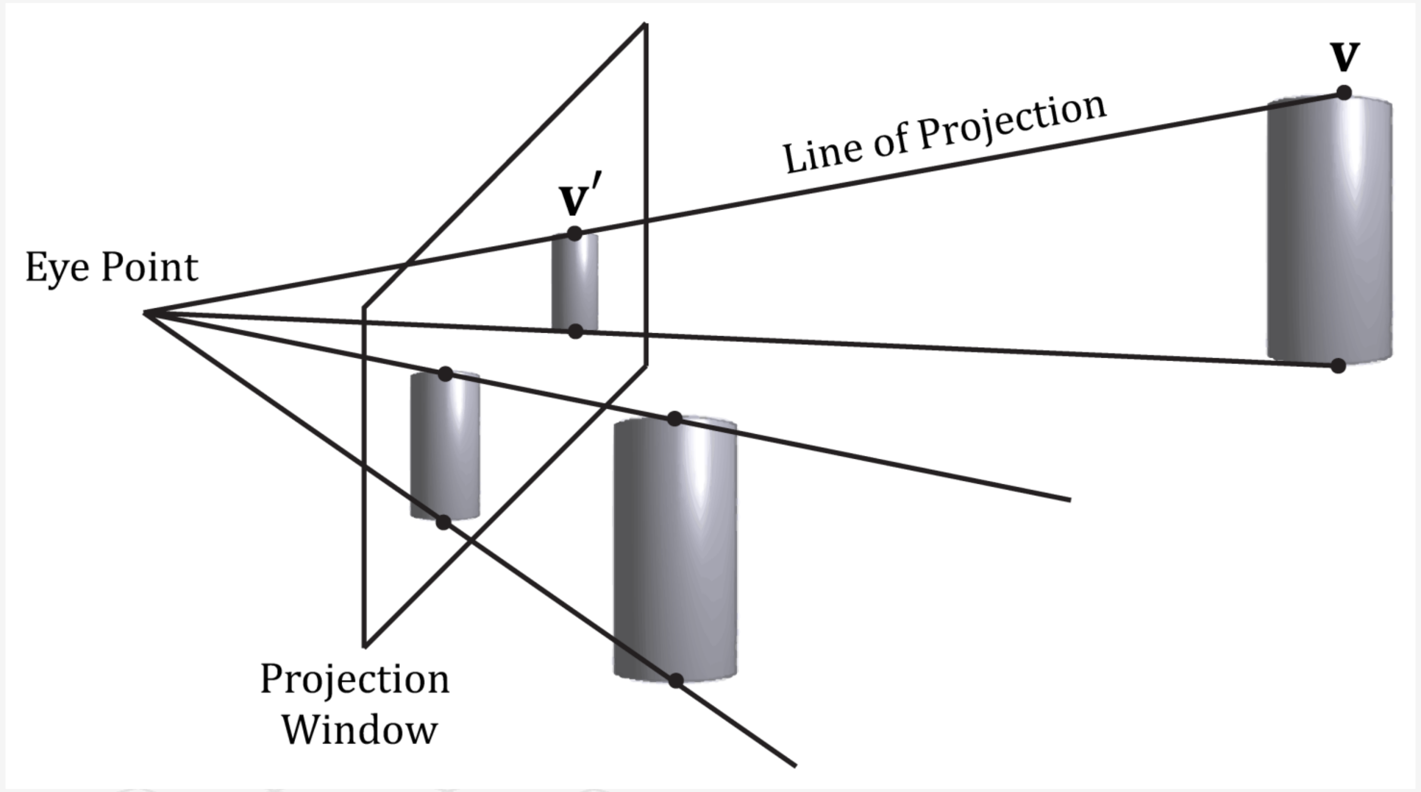
How to project the 3D geometry inside the frustum onto a 2D projection window?

The projection must be done in such a way that parallel lines converge to a vanishing point as the 3D depth of an object increases, and the size of its projection diminishes; a perspective projection does this.

We call the line from a vertex to the eye point the *vertex's line of projection*.

Then we define the *perspective projection transformation* as the transformation that transforms a 3D vertex **v** to the point **v**′ where its line of projection intersects the 2D projection plane;

We say that **v**′ is the projection of **v**. The projection of a 3D object refers to the projection of all the vertices that make up the object.



Both cylinders in 3D space are the same size but are placed at different depths. The projection of the cylinder closer to the eye is bigger than the projection of the farther cylinder. Geometry inside the frustum is projected onto a projection window; geometry outside the frustum, gets projected onto the projection plane, but will lie outside the projection window.

Defining a Frustum

We can define a frustum in view space, with center of projection at the origin and looking down the positive *z*-axis, by the following four quantities: a near plane *n*, far plane *f*, vertical field of view angle α, and aspect ratio *r*.

In view space, the near plane and far plane are parallel to the *xy*-plane; thus we simply specify their distance from the origin along the *z*-axis.

The aspect ratio is defined by *r* = *w*/*h* where *w* is the width of the projection window and *h* is the height of the projection window.

The projection window is essentially the 2D image of

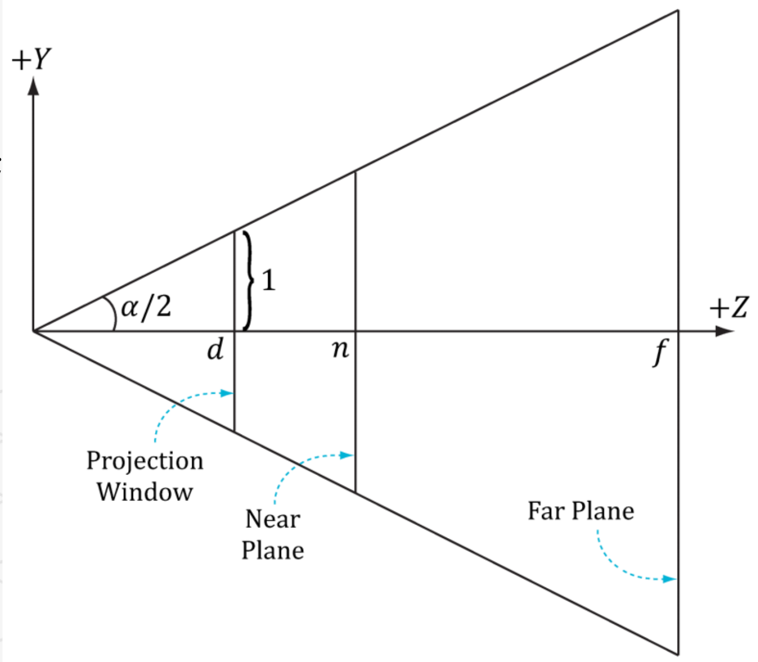
the scene in view space.

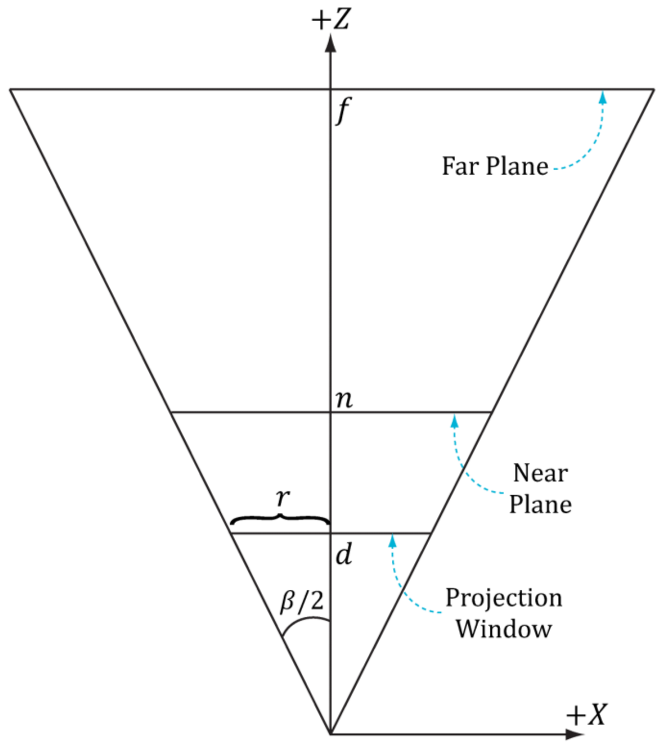
The image here will eventually be mapped to the back buffer;
back buffer dimensions (aspect ratio)

We label the horizontal field of view angle β

β is determined by the vertical field of view angle α and

aspect ratio *r (next slide)*.





Defining a Frustum

In order to have the specified vertical field of view α, the projection window must be placed a distance *d* from the origin:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{d} \Rightarrow d = \cot\left(\frac{\alpha}{2}\right)$$

We have now fixed the distance *d* of the projection window along the *z*-axis to have a vertical field of view α when the height of the projection window is 2.

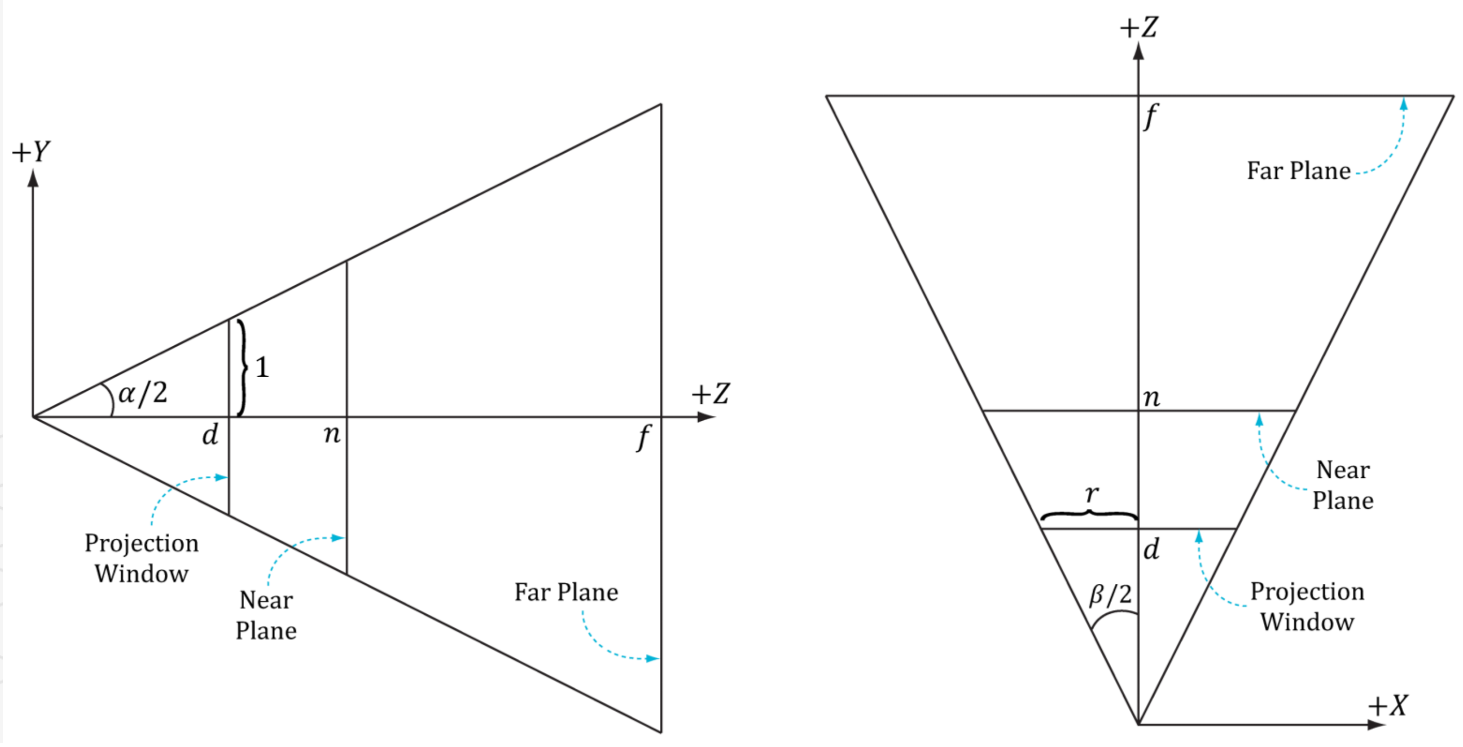
Now we can solve for β.

$$\tan\left(\frac{\beta}{2}\right) = \frac{r}{d} = \frac{r}{\cot\left(\frac{\alpha}{2}\right)} = r.\tan\left(\frac{\alpha}{2}\right)$$

So given the vertical field of view angle α and the aspect ratio *r*,

we can always get the horizontal field of view angle β:

$$\beta = 2tan^{-1}\left(r.\tan\left(\frac{\alpha}{2}\right)\right)$$



Projecting Vertices

Given a point (*x*, *y*, *z*), we wish to find its projection (*x*′, *y*′, *d*), on the projection plane *z* = *d*.

By considering the *x*- and *y*-coordinates separately and using similar triangles, we find:

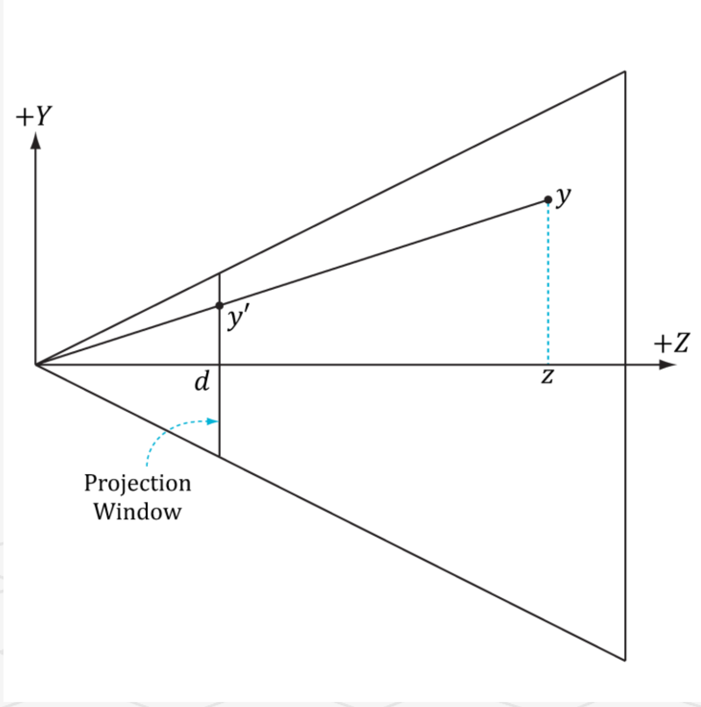
$$\frac{x'}{d} = \frac{x}{z} \Rightarrow x' = \frac{xd}{z} = \frac{x\cot(\alpha/2)}{z} = \frac{x}{z\tan(\alpha/2)}$$

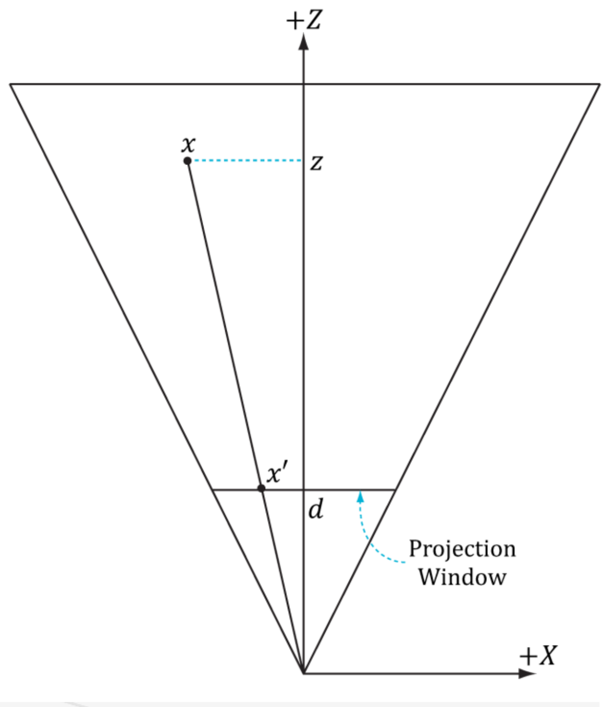
and

$$\frac{y'}{d} = \frac{y}{z} \Rightarrow y' = \frac{yd}{z} = \frac{y\cot(\alpha/2)}{z} = \frac{y}{z\tan(\alpha/2)}$$

Observe that a point (x, y, z) is inside the frustum if and only if

$$-r \le x' \le r$$
$$-1 \le y' \le 1$$
$$n \le z \le f$$





Normalized Device Coordinates (NDC)

In view space, the projection window has a height of 2 and a width of 2*r*, where *r* is the aspect ratio.

$$r = \frac{w}{h} = \frac{w}{2} \Rightarrow w = 2r$$

It would be more convenient if we could remove this dependency on the aspect ratio. The solution is to scale the projected *x*-coordinate from the interval [−*r*, *r*] to [−1, 1] like so:

$$-r \le x' \le r \Rightarrow -1 \le \frac{x'}{r} \le 1$$

After this mapping, the *x*- and *y*-coordinates are said to be *normalized device coordinates* (NDC) (the *z*-coordinate has not yet been normalized), and a point (*x*, *y*, *z*) is inside the frustum if and only if

$$-1 \le \frac{x'}{r} \le 1, -1 \le \mathrm{y}' \le 1, \mathrm{n} \le \mathrm{z} \le \mathrm{f}$$

The transformation from view space to NDC space can be viewed as a unit conversion. We have the relationship that one NDC unit equals *r* units in view space (i.e., 1ndc = *r* vs) on the *x*-axis. So given *x* view space units, we can use this relationship to convert units:

$$x\ vs. \frac{(1\ ndc)}{r\ vs} = \frac{x}{r}\ ndc$$

We can modify our projection formulas to give us the projected *x*- and *y*-coordinates directly in NDC coordinates:

$$x' = \frac{x}{rz\tan\left(\frac{\alpha}{2}\right)}\ and\ y' = \frac{y}{xz\tan\left(\frac{\alpha}{2}\right)}$$

Writing the Projection Equations with a Matrix

Equation $x' = \frac{x}{rztan\left(\frac{\alpha}{2}\right)}$ $and\ y' = \frac{y}{xztan\left(\frac{\alpha}{2}\right)}$ is nonlinear, so it does not have a matrix representation.

The "trick" is to separate it into two parts: a linear part and a nonlinear part.

The nonlinear part is the divide by *z*.

To do this, we take advantage of homogeneous coordinates and copy the input *z*-coordinate to the output *w*-coordinate.

In terms of matrix multiplication, this is done by setting entry [2][3] = 1 and entry [3][3] = 0 (zero-based indices). Our projection matrix looks like this:

A, B constants will be used to transform the input *z*-coordinate into the normalized range.

$$\mathbf{P} = \begin{bmatrix} \dfrac{1}{r\tan(\alpha/2)} & 0 & 0 & 0 \\ 0 & \dfrac{1}{\tan(\alpha/2)} & 0 & 0 \\ 0 & 0 & A & 1 \\ 0 & 0 & B & 0 \end{bmatrix}$$

Perspective Projection Matrix

Multiplying an arbitrary point (*x*, *y*, *z*, 1) by projection matrix gives:

After multiplying by the projection matrix (the linear part),

we complete the transformation by dividing each coordinate

by *w* = *z* (the nonlinear part).

you may wonder about a possible divide by zero;

however, the near plane should be greater than zero;

so such a point would be clipped.

$$[x, y, z, 1]\begin{bmatrix} \frac{1}{r\tan(\alpha/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\alpha/2)} & 0 & 0 \\ 0 & 0 & A & 1 \\ 0 & 0 & B & 0 \end{bmatrix}$$

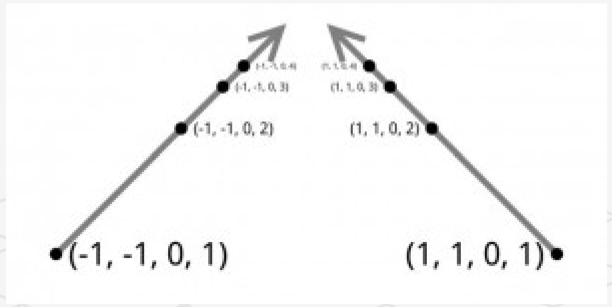
$$= \left[\frac{x}{r\tan(\alpha/2)}, \frac{y}{\tan(\alpha/2)}, Az + B, z\right]$$

The divide by *w* is sometimes called the *perspective divide* or *homogeneous divide*.

$$\left[\frac{x}{r\tan(\alpha/2)}, \frac{y}{\tan(\alpha/2)}, Az + B, z\right] \xrightarrow{\text{divide by } w} \left[\frac{x}{rz\tan(\alpha/2)}, \frac{y}{z\tan(\alpha/2)}, A + \frac{B}{z}, 1\right]$$

Perspective Divide

- The perspective projection doesn’t actually create the 3D effect; for that, we need to do something called the *perspective divide*.
- Each coordinate actually has four components, X, Y, Z, and W.
- The projection matrix sets things up so that after multiplying with the projection matrix, each coordinate’s W will increase the further away the object is.
- DirectX will then divide by w: X, Y, Z will be divided by W.
- The further away something is, the more it will be pulled towards the center of the screen.



Normalized Depth Value

We still need 3D depth information around for the depth buffering algorithm.

Direct3D wants the depth coordinates in the normalized range [0, 1].

We must construct an order preserving function $g(z)$ that maps the interval $[n, f]$ onto [0, 1].

Because the function is order preserving, if $z1, z2 \in [n, f]$ and $z1 < z2$, then $g(z1) < g(z2)$; so we can still correctly compare depths in the normalized interval.

We see from last equation (perspective divide – previous slide), that the z-coordinate undergoes the transformation: $g(z) = A + \frac{B}{z}$

We now need to choose A and B subject to the constraints:

Condition 1: $g(n) = A + B/n = 0$ (the near plane gets mapped to zero)

Condition 2: $g(f) = A + B/f = 1$ (the far plane gets mapped to one)

Solving condition 1 for B yields: $B = -An$. Substituting this into condition 2 and

solving for A gives:

$$A + \frac{-An}{f} = 1$$

$$\frac{Af - An}{f} = 1$$

$$Af - An = f$$

$$A = \frac{f}{f - n}$$

$$g(z) = \frac{f}{f-n} - \frac{nf}{(f-n)z}$$

Graph of *g*(*z*) for different near planes

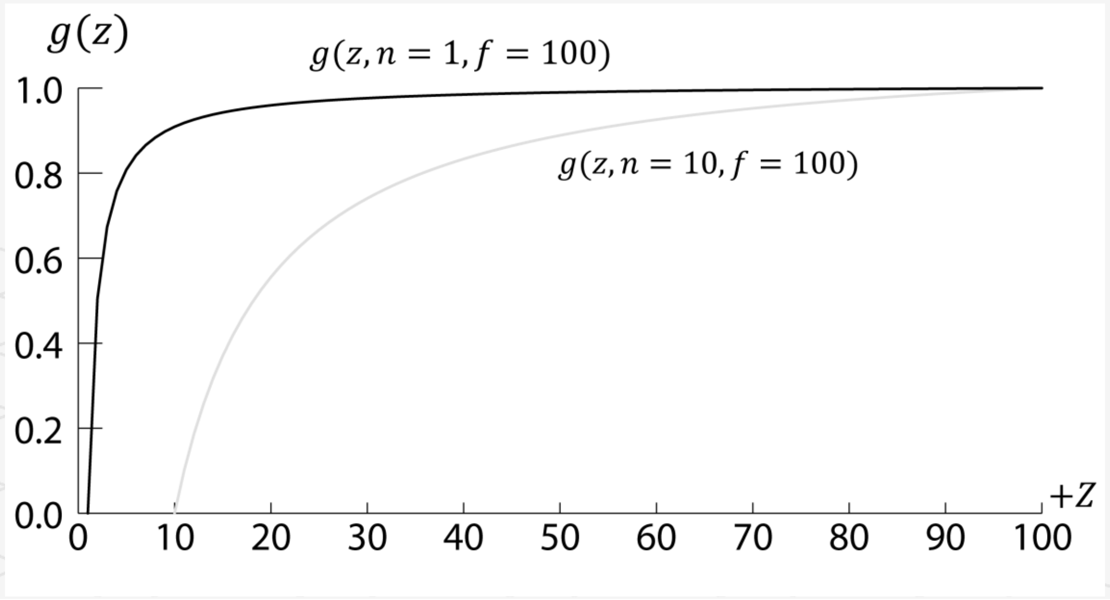
A graph of *g* shows it is strictly increasing (order preserving) and nonlinear. It also shows that most of the range is "used up" by depth values close to the near plane. Consequently, the majority of the depth values get mapped to a small subset of the range.

This can lead to depth buffer precision problems (the computer can no longer distinguish between slightly different transformed depth values due to finite numerical representation).

The general advice is to make the near and far planes as close as possible to minimize depth precision problems.

Now that we have solved for *A* and *B*, we can state the full *perspective projection matrix*:

$$\mathbf{P} = \begin{bmatrix} \frac{1}{r\tan(\alpha/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\alpha/2)} & 0 & 0 \\ 0 & 0 & \frac{f}{f-n} & 1 \\ 0 & 0 & \frac{-nf}{f-n} & 0 \end{bmatrix}$$



XMMatrixPerspectiveFovLH

A perspective projection matrix can be built with the following DirectX Math function:

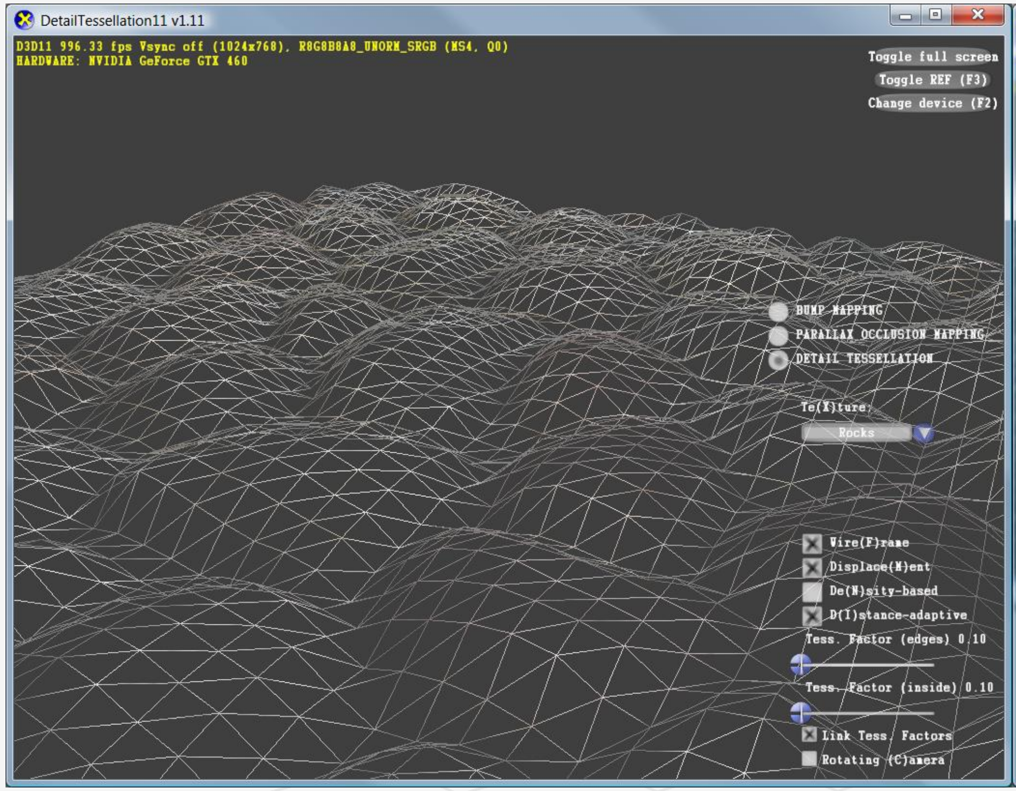
```
inline XMMATRIX XM_CALLCONV XMMatrixPerspectiveFovLH
(
    float FovAngleY,
    float AspectRatio,
    float NearZ,
    float FarZ
)
```

Field of view: 45º , Near plane at: 1 , far plane at: 1000 (these numbers are in view space)

```
void BoxApp::OnResize()
{
D3DApp::OnResize();

    // The window resized, so update the aspect ratio and recompute the projection matrix.
    XMMATRIX P = XMMatrixPerspectiveFovLH(0.25f*MathHelper::Pi, AspectRatio(), 1.0f, 1000.0f);
    XMStoreFloat4x4(&mProj, P);
}
```

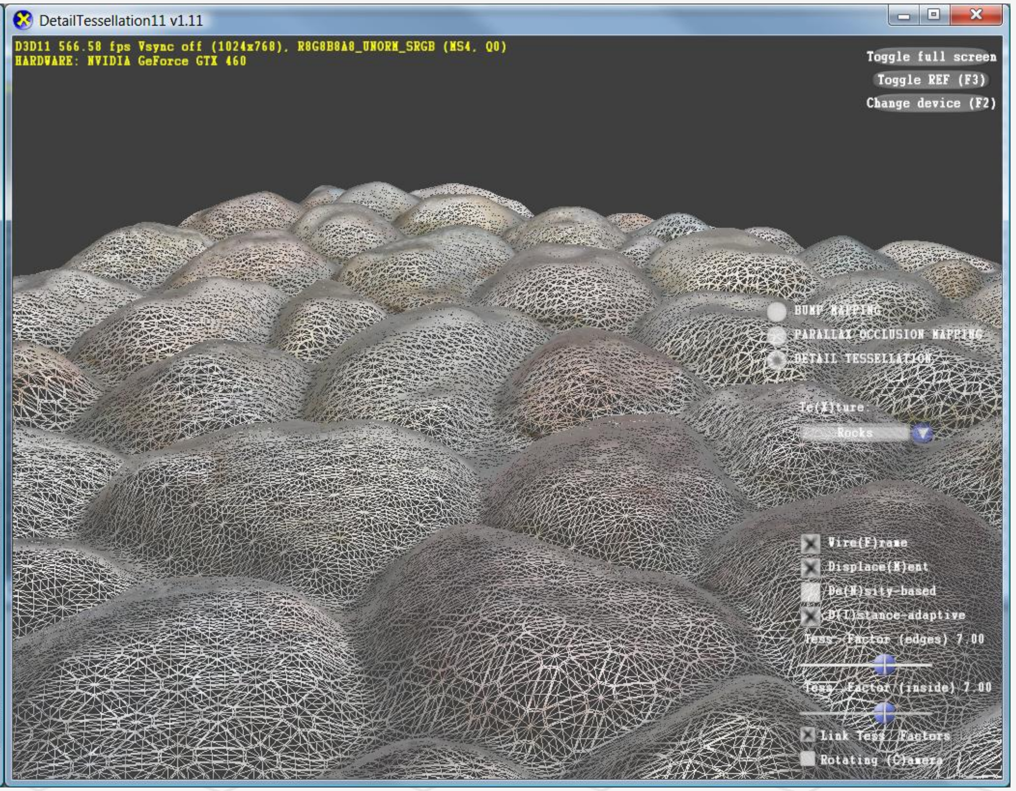
THE TESSELLATION STAGES

Tessellation refers to subdividing the triangles of a mesh to add new triangles. These new triangles can then be offset into new positions to create finer mesh detail.



```
D3D11 996.33 fps Vsync off (1024x768), R8G8B8A8_UNORM_SRGB (MS4, Q0)
HARDWARE: NVIDIA GeForce GTX 460
```

[✓] Wire(F)rame
[✓] Displace(M)ent
[ ] De(N)sity-based
[✓] D(I)stance-adaptive
Tess. Factor (edges) 0.10
Tess. Factor (inside) 0.10
[✓] Link Tess. Factors
[ ] Rotating (C)amera



```
D3D11 566.58 fps Vsync off (1024x768), R8G8B8A8_UNORM_SRGB (MS4, Q0)
HARDWARE: NVIDIA GeForce GTX 460
```

[✓] Wire(F)rame
[✓] Displace(M)ent
[ ] De(N)sity-based
[✓] D(I)stance-adaptive
Tess. Factor (edges) 7.00
Tess. Factor (inside) 7.00
[✓] Link Tess. Factors
[ ] Rotating (C)amera

Tessellation Benefits

- We can implement a level-of-detail (LOD) mechanism, where triangles near the camera are tessellated and triangles far away from the camera are not tessellated

- We keep a simpler low-poly mesh (low triangle count) in memory, and add the extra triangles on the fly, thus saving memory

- We do operations like animation and physics on a simpler low-poly mesh, and only use the tessellated high-poly mesh for rendering

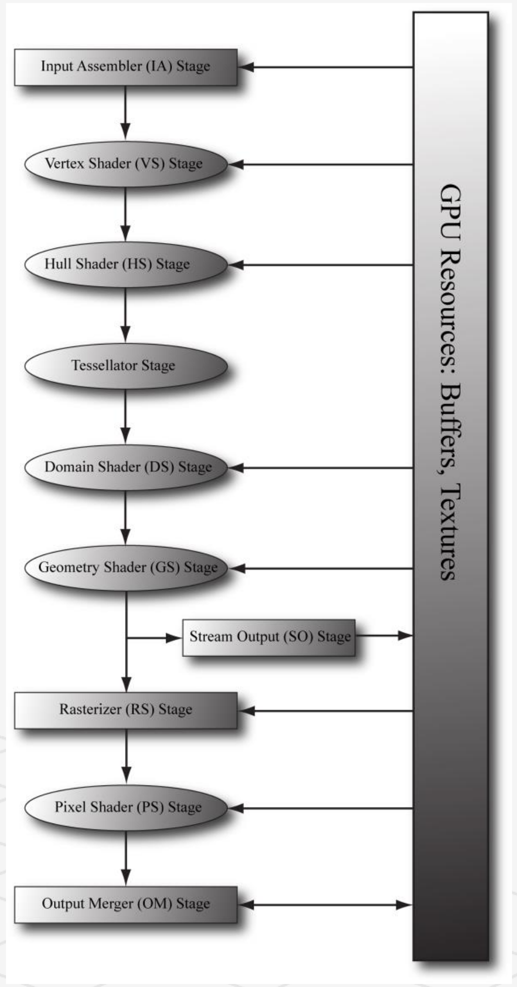
The tessellation stages are new to Direct3D 11 and 12, and they provide a way to tessellate geometry on the GPU. Before Direct3D 11, if you wanted to implement a form of tessellation, it would have to be done on the CPU, and then the new tessellated geometry would have to be uploaded back to the GPU for rendering.

However, uploading new geometry from CPU memory to GPU memory is slow, and it also burdens the CPU with computing the tessellation.

Direct3D 12 provides an API to do tessellation completely in hardware with a Direct3D 12 capable video card.

THE GEOMETRY SHADER STAGE

- The geometry shader inputs entire primitives
- The main advantage of the geometry shader is that it can create or destroy geometry
- For example, the input primitive can be expanded into one or more other primitives, or the geometry shader can choose not to output a primitive based on some condition. This is in contrast to a vertex shader, which cannot create vertices: it inputs one vertex and outputs one vertex.
- A common example of the geometry shader is to expand a point into a quad or to expand a line into a quad
- The geometry shader can stream-out vertex data into a buffer in memory, which can later be drawn. This is an advanced technique, and will be discussed later



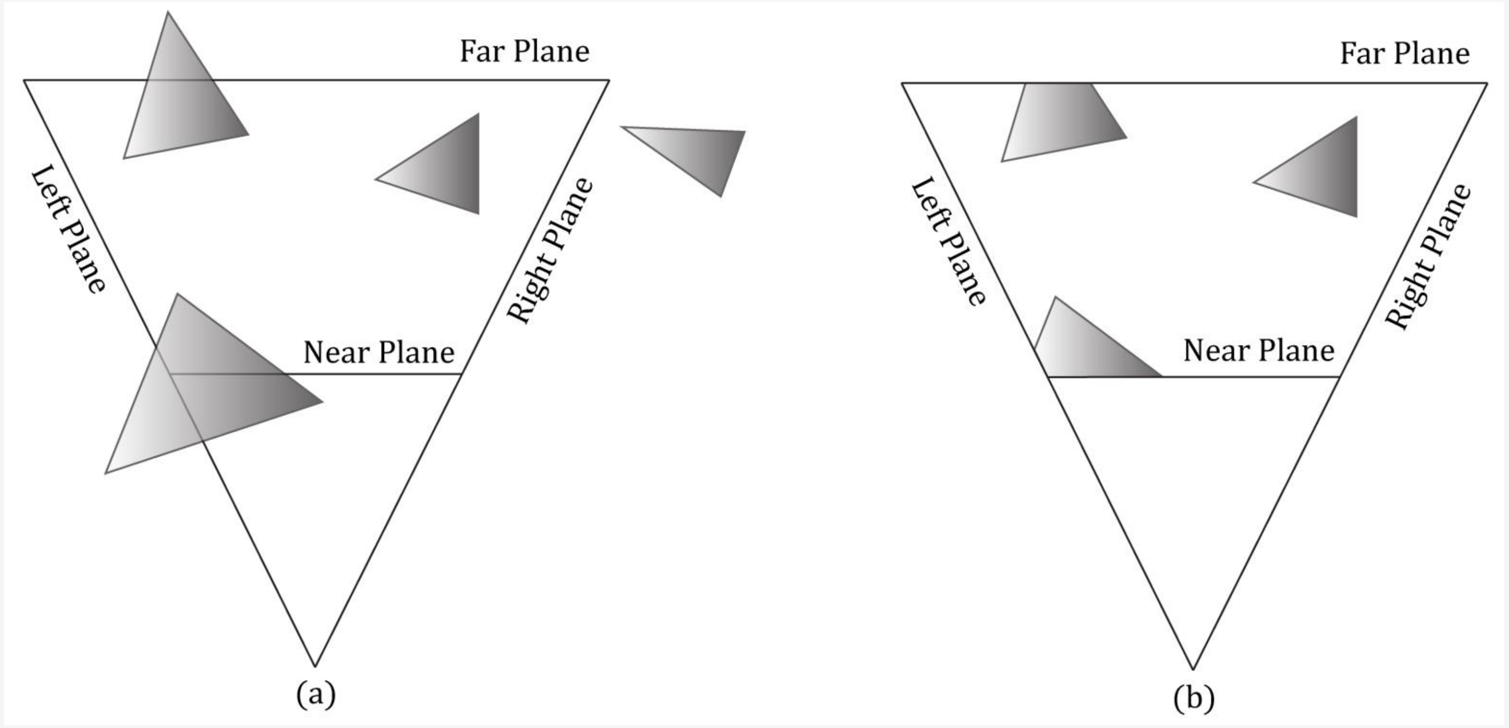
Clipping

- *Vertex positions leaving the geometry shader must be transformed to homogeneous clip space.*

- Geometry completely outside the viewing frustum needs to be discarded, and geometry that intersects the boundary of the frustum must be clipped, so that only the interior part remains.

Think of the frustum as being the region bounded by six planes: the top, bottom, left, right, near, and far planes.

To clip a polygon against the frustum, we clip it against each frustum plane one-by-one



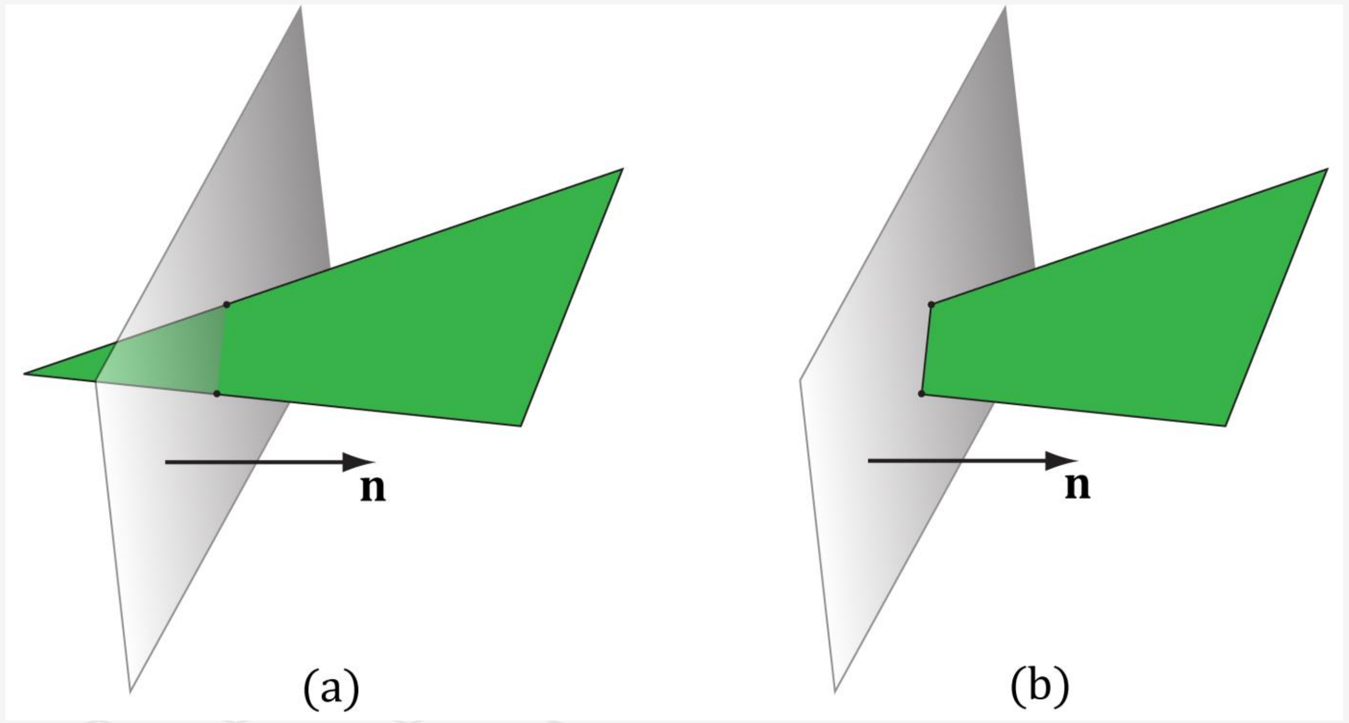
Clipping

▪ When clipping a polygon against a plane the part in the positive half-space of the plane is kept, and the part in the negative half space is discarded.

Because the hardware does clipping for us, we will not cover the details here; instead, we refer you to the popular Sutherland-Hodgeman clipping algorithm.

It basically amounts to finding the intersection points between the plane and polygon edges, and then ordering the vertices to form the new clipped polygon.

Note that the clipped triangle is not a triangle, but a quad. Thus the hardware will need to triangulate the resulting quad, which is straightforward to do for convex polygons.



Blinn

Blinn describes how clipping can be done in 4D homogeneous space. After the perspective divide, points inside the view frustum are in normalized device coordinates and bounded as follows:

$$-1 \le \frac{x}{w} \le 1,\ \ -1 \le \frac{y}{w} \le 1,\ \ 0 \le \frac{z}{w} \le 1$$

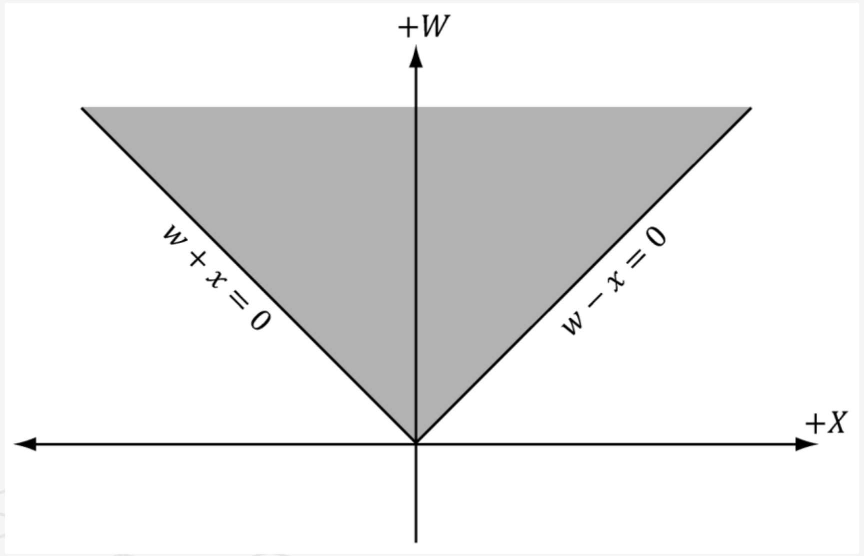
So in homogeneous clip space, before the divide, 4D points (*x*, *y*, *z*, *w*) inside the frustum are bounded as follows:

$$-w \le x \le w, -w \le y \le w, 0 \le z \le w$$

That is, the points are bounded by the simple 4D planes:

Left: *w* = −*x*, Right: *w* = *x*, Bottom: *w* = −*y*, Top: *w* = *y*, Near: *z* = 0, Far: *z* = *w*

The figure shows the frustum boundaries in the *xw*-plane in homogeneous clip space:



Rasterization

The main job of the rasterization stage is to compute pixel colors from the projected 3D triangles

- This stage can be broken down into viewport transformation, backface culling and vertex attribute interpolation.

After clipping, the hardware can do the perspective divide to transform from homogeneous clip space to normalized device coordinates (NDC).

Once vertices are in NDC space, the 2D *x*- and *y*- coordinates forming the 2D image are transformed to a rectangle on the back buffer called the viewport.

After this transform, the *x* and *y*-coordinates are in units of pixels.

Usually the viewport transformation does not modify the *z*-coordinate, as it is used for depth buffering, but it can by modifying the MinDepth and MaxDepth values of the D3D12_VIEWPORT structure.

The MinDepth and MaxDepth values must be between 0 and 1.

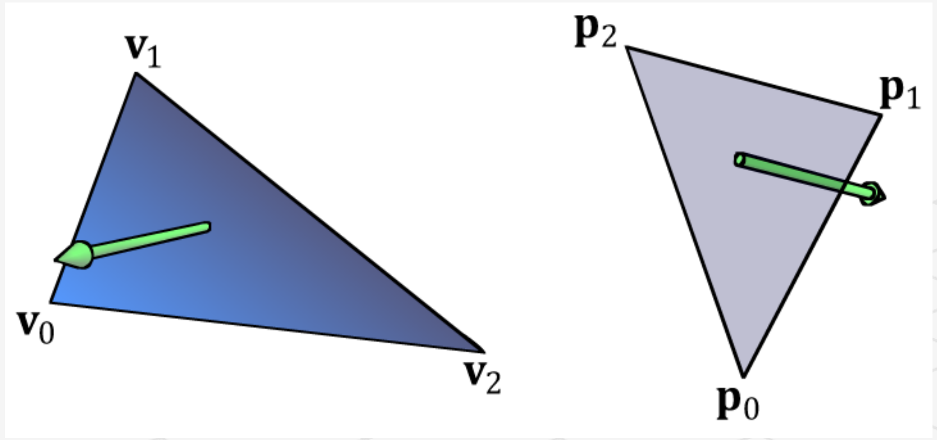
Backface culling

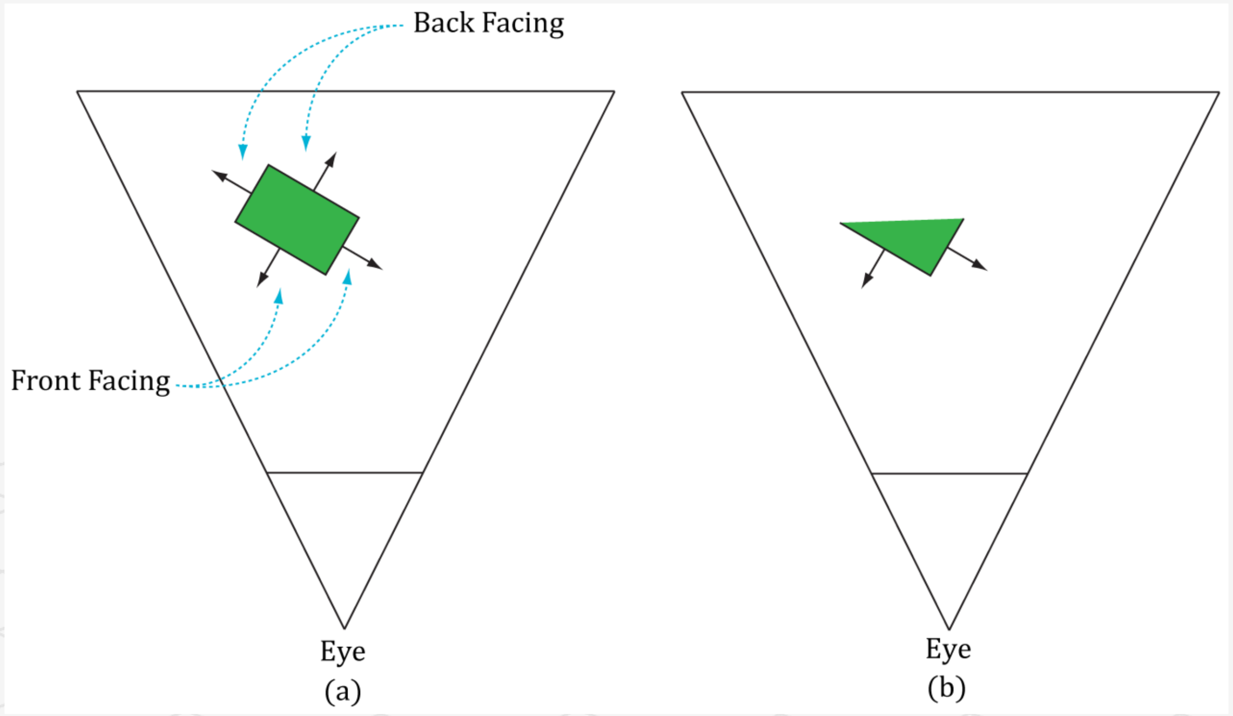
A triangle has two sides. To distinguish between the two sides we use the following convention. If the triangle vertices are ordered **v**0, **v**1, **v**2 then we compute the triangle normal **n.** The side the normal vector emanates from is the *front side* and the other side is the *back side*. The left triangle is front-facing from our viewpoint, and the right triangle is back-facing from our viewpoint. *Backface culling* refers to the process of discarding backfacing triangles from the pipeline. This can potentially reduce the amount of triangles that need to be processed by half.

$$\mathbf{e}_0 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_1 = \mathbf{v}_2 - \mathbf{v}_0$$

$$\mathbf{n} = \frac{\mathbf{e}_0 \times \mathbf{e}_1}{\mathbf{e}_0 \times \mathbf{e}_1}$$





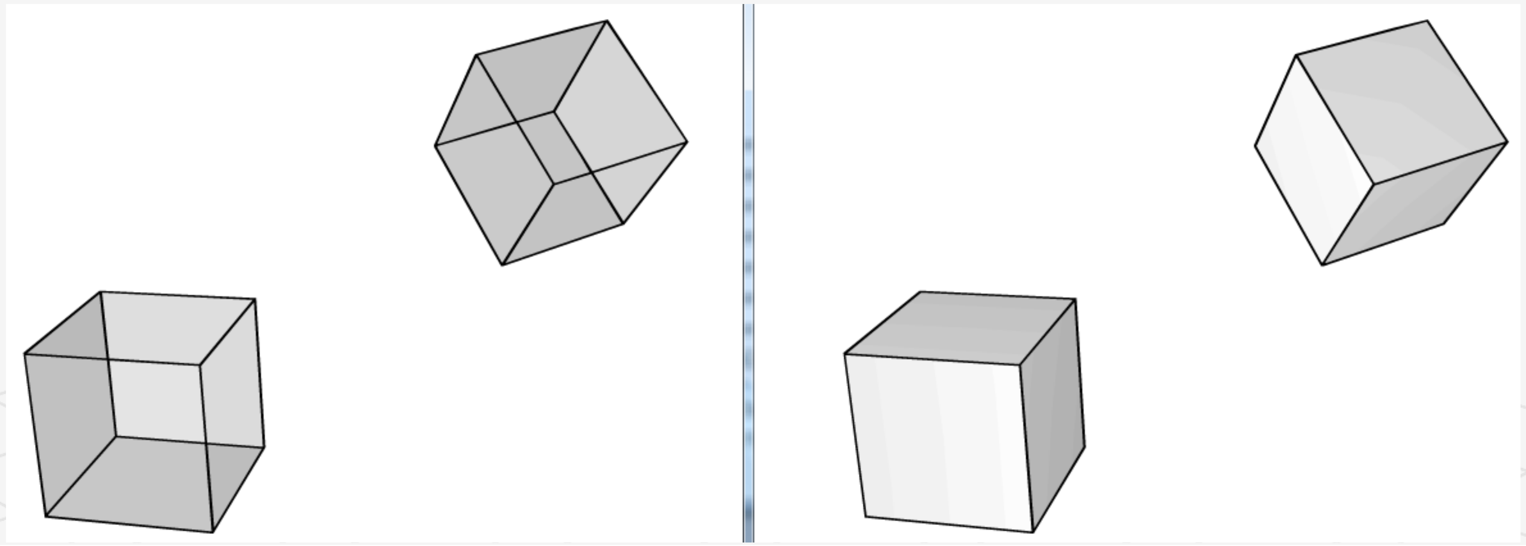
Winding Order

By default, Direct3D treats triangles with a clockwise winding order (with respect to the viewer) as front-facing, and triangles with a counterclockwise winding order (with respect to the viewer) as back-facing.

However, this convention can be reversed with a Direct3D render state setting.

(Left) We draw the cubes with transparency so that you can see all six sides.

(Right) We draw the cubes as solid blocks. Note that we do not see the three back-facing sides since the three front-facing sides occlude them—therefor the back-facing triangles can actually be discarded from further processing and no one will notice.

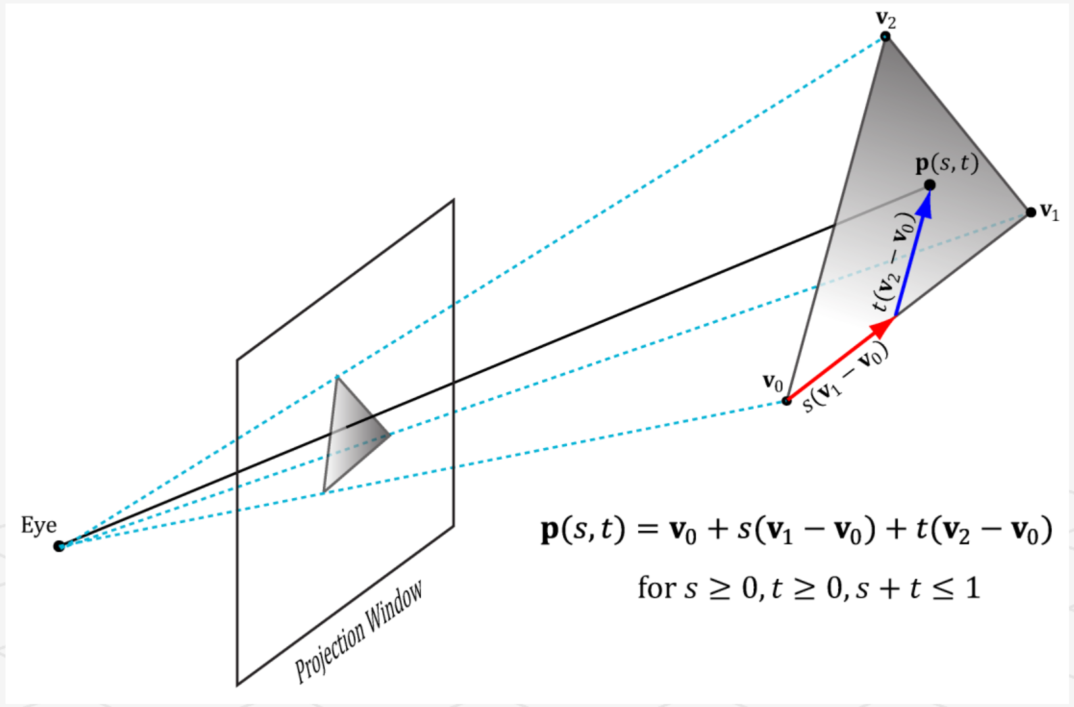


Vertex Attribute Interpolation

In addition to position, we can attach attributes to vertices such as colors, normal vectors, and texture coordinates.

After the viewport transform, these attributes need to be interpolated for each pixel covering the triangle. In addition to vertex attributes, vertex depth values need to get interpolated so that each pixel has a depth value for the depth buffering algorithm.

The vertex attributes are interpolated in screen space in such a way that the attributes are interpolated linearly across the triangle in 3D space. This requires the *perspective correct interpolation*. Essentially, interpolation allows us to use the vertex values to compute values for the interior pixels.



$$\mathbf{p}(s,t) = \mathbf{v}_0 + s(\mathbf{v}_1 - \mathbf{v}_0) + t(\mathbf{v}_2 - \mathbf{v}_0)$$

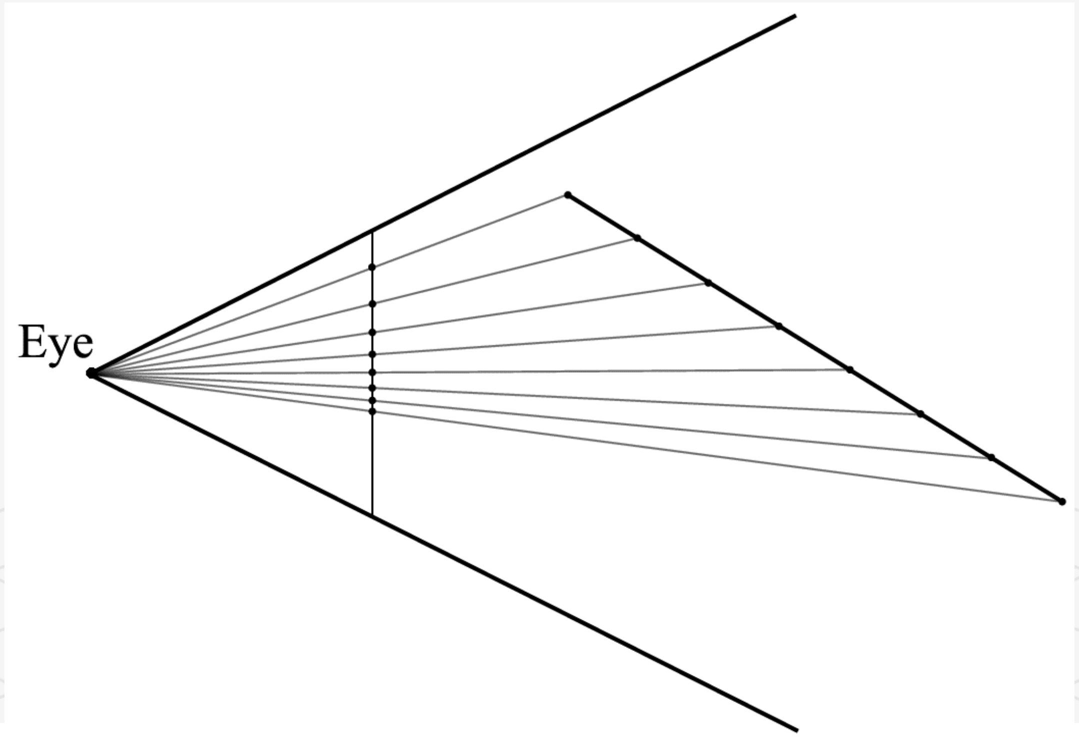
$$\text{for } s \geq 0, t \geq 0, s + t \leq 1$$

The perspective correct attribute interpolation

The mathematical details of perspective correct attribute interpolation are not something we need to worry about since the hardware does it.

A 3D line is being projected onto the projection window (the projection is a 2D line in screen space). We see that taking uniform step sizes along the 3D line corresponds to taking non-uniform step sizes in 2D screen space.

Therefore to do linear interpolation in 3D space, we need to do nonlinear interpolation in screen space.



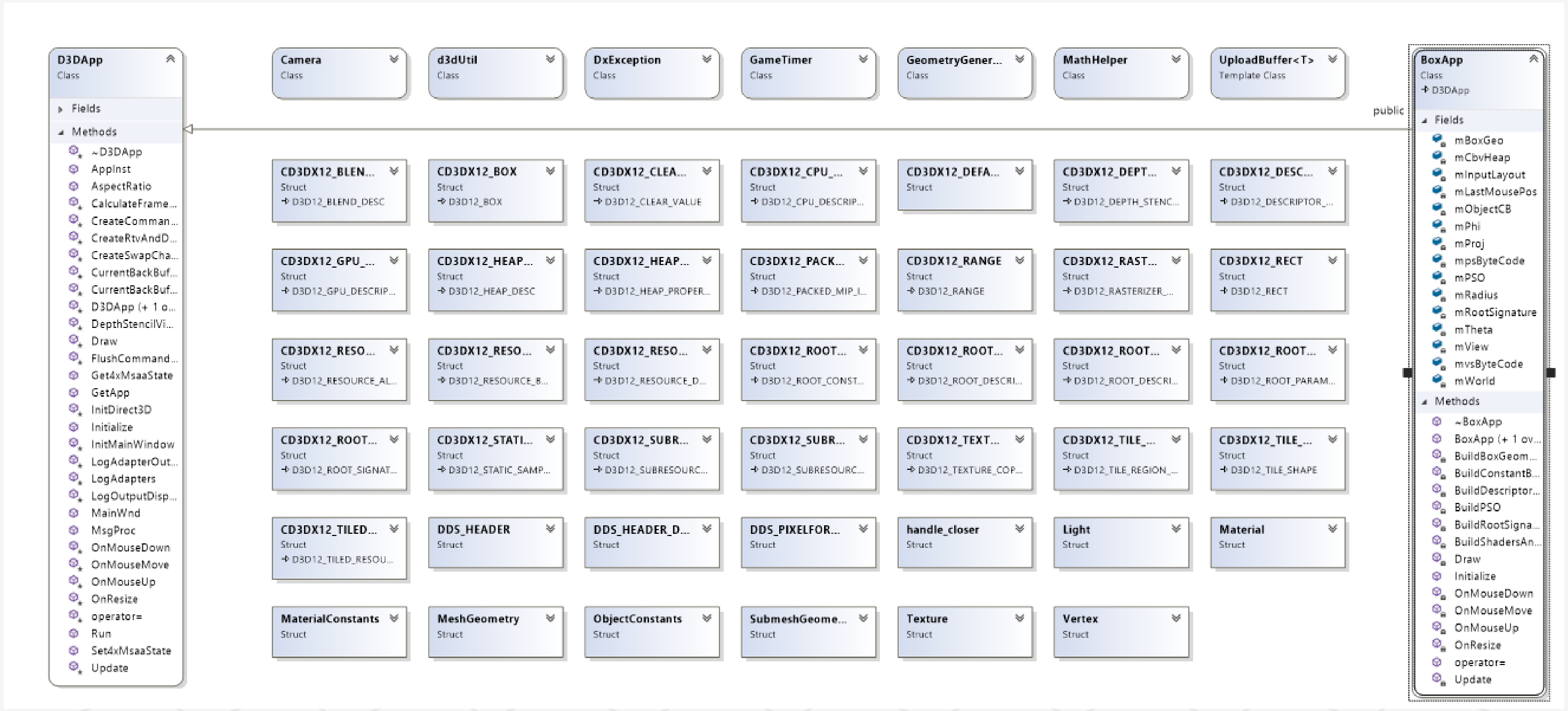
THE PIXEL SHADER STAGE

- Pixel shaders are programs we write that are executed on the GPU
- A pixel shader is executed for each pixel fragment and uses the interpolated vertex attributes as input to compute a color.
- A pixel shader can be as simple as returning a constant color, to doing more complicated things like per-pixel lighting, reflections and shadowing effects.

Output Merger Stage

- After pixel fragments have been generated by the pixel shader, they move onto the output merger (OM) stage.
- In this stage, some pixel fragments may be rejected (e.g., from the depth or stencil buffer tests).
- Pixel fragments that are not rejected are written to the back buffer.
- Blending is also done in this stage, where a pixel may be blended with the pixel currently on the back buffer instead of overriding it.
- Some special effects like transparency are implemented with blending.

Class Diagram


D3DApp
Class
Fields
Methods
~D3DApp
AppInst
AspectRatio
CalculateFrame...
CreateComman...
CreateRtvAndD...
CreateSwapCha...
CurrentBackBuf...
CurrentBackBuf...
D3DApp (+ 1 o...
DepthStencilVi...
Draw
FlushCommand...
Get4xMsaaState
GetApp
InitDirect3D
Initialize
InitMainWindow
LogAdapterOut...
LogAdapters
LogOutputDisp...
MainWnd
MsgProc
OnMouseDown
OnMouseMove
OnMouseUp
OnResize
operator=
Run
Set4xMsaaState
Update
Camera
Class
d3dUtil
Class
DxException
Class
GameTimer
Class
GeometryGener...
Class
MathHelper
Class
UploadBuffer<T>
Template Class
CD3DX12_BLEN...
Struct
D3D12_BLEND_DESC
CD3DX12_BOX
Struct
D3D12_BOX
CD3DX12_CLEA...
Struct
D3D12_CLEAR_VALUE
CD3DX12_CPU_...
Struct
D3D12_CPU_DESCRIP...
CD3DX12_DEFA...
Struct
CD3DX12_DEPT...
Struct
D3D12_DEPTH_STENC...
CD3DX12_DESC...
Struct
D3D12_DESCRIPTOR_...
CD3DX12_GPU_...
Struct
D3D12_GPU_DESCRIP...
CD3DX12_HEAP...
Struct
D3D12_HEAP_DESC
CD3DX12_HEAP...
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