

# Answer Set Solving in Practice

Exercise 1 (Answer Set Programming by Examples)

You can check your answers running `clingo` with the program files in the directory `exercise01`. For example, if the file is named `example.lp`, the command `clingo example.lp 0` will compute all stable models.

## Exercise 1.1 (Positive Programs)

Follow some programs and their stable models:

a.	a.	b :- a.	b :- a, c.	b :- a, c.
b.	b :- a.	a.	a.	a.
-----	-----	-----	-----	-----
{ a, b }	{ a, b }	{ a, b }	{ a }	{ a, b, c }

a :- b.	a :- b.	a :- b.	a :- b.	a :- b.
b :- a.	b :- a.	b :- a.	b :- a.	b :- a, b.
-----	a.	a :- a.	a :- c.	a :- c.
{ }	-----	-----	c.	c.
	{ a, b }	{ }	-----	-----
			{ a, b, c }	{ a, c }

Find the stable models of the next positive programs:

a.	a.	b :- a, c.	b :- a, c.	b :- a, c.
b.	b :- a, c.	a.	a.	a.
c.	c.	c.	d :- b.	c :- a.
-----	-----	-----	-----	d :- a, b, c.
				-----

a :- b.	a :- b.	a :- b.	a :- b.	a :- b.
b :- a, c.	b :- a, c.	b :- a.	b :- a, c.	b :- a, b.
c.	a.	a :- a, c.	a :- c, d.	a :- c.
-----	c.	c.	c.	c.
	-----	-----	d :- c.	d :- a, c.
			-----	b :- d.
				-----

**Exercise 1.2** (Negation)

Follow some programs and their stable models:

a :- not b. ----- { a }	a :- not b. b. ----- { b }	a :- not b. b :- not c. ----- { b }	a :- not b. b :- not c. c :- d. d. ----- { a, c, d }
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a :- not b. b :- not c. c :- d. d :- c. ----- { b }	a :- not b. b :- not c. d :- a. e :- b. ----- { b, e }	a :- not b. b :- not c. a. b :- a. ----- { a, b }	a :- not b. b :- not c. c :- not d. d. ----- { b, d }
--	---	--	--

Find the stable models of the next programs:

a :- not b, c. c. -----	a :- not b, c. b. c. -----	a :- not b, d. b :- not c. d. -----	a :- not b, c. b :- not c. c :- d. d :- not e. -----
-------------------------------	-------------------------------------	--	--

a :- not b. b :- not d. c :- d, not e. d :- c, not e. -----	a :- not b. b :- not c. d :- a. e :- not c. -----	a :- not b. b :- not c. a :- not d. b :- e. e. -----	a :- d. a :- not b. b :- not c, d. c :- not d, b. d. -----
---	---	---	---

**Exercise 1.3** (Choice Rules)

Follow some programs and their stable models:

$\{ a \}.$ ----- $\{ \}$ $\{ a \}$	$0 \{ a \} 1.$ ----- $\{ \}$ $\{ a \}$	$1 \{ a \}.$ ----- $\{ a \}$	$2 \{ a \}.$ -----
$\{ a \}.$ $b :- \text{not } a.$ $c :- a.$ ----- $\{ b \}$ $\{ a, c \}$	$\{ a \}.$ $b :- \text{not } a.$ $b :- a.$ ----- $\{ b \}$ $\{ a, b \}$	$a.$ $\{ b \} :- a.$ ----- $\{ a \}$ $\{ a, b \}$	$\{ a \}.$ $\{ b \} :- a.$ ----- $\{ \}$ $\{ a \}$ $\{ a, b \}$

Find the stable models of the next programs:

$\{ a \}.$ $b :- \text{not } a.$ $c :- a, \text{not } d.$ $e :- b, c.$ -----	$\{ a \}.$ $b :- \text{not } a, c.$ $b :- a, d.$ $c.$ -----	$a.$ $b :- a.$ $\{c\} :- b, \text{not } d.$ -----	$\{ a \}.$ $b :- \text{not } a.$ $\{ c \} :- b.$ -----
--	---	--	---

Follow some programs and their stable models:

$\{ a; b \}.$ ----- $\{ \}$ $\{ a \}$ $\{ b \}$ $\{ a, b \}$	$1 \{ a; b \}.$ ----- $\{ a \}$ $\{ b \}$ $\{ a, b \}$	$\{ a; b \} 1.$ ----- $\{ \}$ $\{ a \}$ $\{ b \}$	$1 \{ a; b \} 1.$ ----- $\{ a \}$ $\{ b \}$
$2 \{ a; b \}.$ ----- $\{ a, b \}$	$0 \{ a; b \} 2.$ ----- $\{ \}$ $\{ a \}$ $\{ b \}$ $\{ a, b \}$	$3 \{ a; b \}.$ -----	

{ a; b }.	1 { a; b } 1.	1 { a; b } 1.	{ a }.
c :- not a, not b.	c :- a.	c :- a.	1 { b; c } 1 :- a.
d :- not a, b.	d :- b.	d :- b.	d :- a, c.
e :- a, not b.	-----	c :- d.	-----
f :- a, b.	{ a, c }	d :- c.	{ }
-----	{ b, d }	-----	{ a, b }
{ c }		{ a, c, d }	{ a, c, d }
{ b, d }		{ b, c, d }	
{ a, e }			
{ a, b, f }			

Find the stable models of the next programs:

{ a; b }.	1 { a; b } 1.	1 { a; b } 1.	{ a }.
c :- not a.	c :- not b.	c :- a.	b :- a.
d :- b.	d :- not a.	d :- b.	1 { c; d } 1 :- b.
-----	e :- c, not d.	c :- d, not a.	e :- a, c.
	-----	d :- c, not b.	-----
		-----	

**Exercise 1.4** (Constraints)

Follow some programs and their stable models:

<pre>:- not a. -----</pre>	<pre>:- b. ----- { }</pre>	<pre>b :- not c. :- b. -----</pre>	<pre>b :- not c. c. :- b. ----- { c }</pre>
<pre>b. :- b. :- c. -----</pre>	<pre>b :- not d. d. :- b. :- c. ----- { d }</pre>	<pre>c :- d. d :- c. :- c. :- not d. -----</pre>	<pre>a :- c. b :- not c. c. :- not a, not b. ----- { a, c }</pre>

Find the stable models of the next programs:

<pre>d. :- not a, d. -----</pre>	<pre>b :- c. :- b. -----</pre>	<pre>b :- not c, d. d. :- b. -----</pre>	<pre>b :- not c, d. d. :- b. -----</pre>
<pre>c :- d. d. :- b. :- c. -----</pre>	<pre>b :- d. c :- d. :- b. :- c. -----</pre>	<pre>c :- d. d :- c. :- not c. :- d. -----</pre>	<pre>a :- not b. b :- not c. :- not a, not c. -----</pre>

**Exercise 1.5** (Programming with choice rules and constraints)

Follow some programs and their stable models:

<pre>{ a }. :- not a. ----- { a }</pre>	<pre>{ a }. :- not b, not a. :- not b, a. -----</pre>	<pre>{ a }. c :- a. :- not c. ----- { a, c }</pre>	<pre>{ a }. c :- a. d :- not a. :- c. :- d. -----</pre>
<pre>1 { a; b }. c :- a. d :- b. :- c, not e. ----- { b, d }</pre>	<pre>{ a; b } 1. c :- not a, not b. d :- b. :- not c, not d. ----- { c } { b, d }</pre>	<pre>1 { a; b } 1. c :- a, not b. d :- not a, b. :- c, d. ----- { a, c } { b, d }</pre>	<pre>{ a }. 1 { b; c } 1. :- not a, b. :- a, c. ----- { c } { a, b }</pre>

Find the stable models of the next programs:

<pre>{ a }. :- b. :- not a. -----</pre>	<pre>{ a }. b. :- b, not a. :- b, a. -----</pre>	<pre>{ a }. d :- c. c :- not a. :- not d. -----</pre>	<pre>{ a }. d :- not a. c :- a. e :- d. :- not e. -----</pre>
<pre>1 { a; b }. c :- not a. d :- not b. e :- a, b. :- not c, not e. -----</pre>	<pre>{ a; b } 1. c :- not a. d :- not b. :- c, d. -----</pre>	<pre>1 { a; b } 1. c :- a. d :- b. :- c. :- d. -----</pre>	<pre>{ a }. 1 { b; c } 1 :- a. d :- a, b. :- not d. -----</pre>

**Exercise 1.6** (Cardinality rules)

Follow some programs and their stable models:

<pre>{ a; b }. :- 1 { a;b } 1. ----- { } { a, b }</pre>	<pre>1 { a; b }. c :- 1 { a;b } 1. :- not c. ----- { a, c } { b, c }</pre>	<pre>1 { a; b }. c :- a. d :- b. :- { c;d } 1. ----- { a, b, c, d }</pre>	<pre>1 { a; b }. { c } :- { a;b } 1. ----- { a } { a, c } { b } { b, c } { a, b }</pre>
---	--	---	---

<pre>1 { a; b }. a :- 1{ a; b }. b :- 1{ a; b }. ----- { a, b }</pre>	<pre>1{ a; b }1. 1{ c; d } :- a. :- 3 { a;b;c;d }. ----- { a, c } { a, d } { b }</pre>	<pre>{ a; b } 1. c :- a. :- 2 { a;b;c }. ----- { } { b }</pre>	<pre>1 { a; b }. c :- a. d :- b. :- 2 { a;b;c;d } 2. ----- { a, b, c, d }</pre>
---	--	--	---

Find the stable models of the next programs:

<pre>{ a; b }. c. :- 2 { a;b;c } 2. -----</pre>	<pre>1 { a; b }. c :- 1 {a;b} 1. :- c. -----</pre>	<pre>1 { a; b }. a :- a. d :- b. :- 2 { c; d }. -----</pre>	<pre>1 { a; b }. { c } :- 1 { a;b } 1. -----</pre>
---	--	---	--

<pre>1 { a;b;c }. a :- 1 { a;b;c }. b :- 1 { a;b;c }. c :- 1 { a;b;c }. -----</pre>	<pre>1 { a;b } 1. 1 { c;d } 1 :- a. :- 2 { a;b;c;d }. -----</pre>	<pre>{ a; b } 1. c :- a. :- { a;b;c } 1. -----</pre>	<pre>1 { a; b }. c :- a. c :- b. :- 2 { a;b;c }. -----</pre>
---	---	--	--

**Exercise 1.7** (Aggregates)

Follow some programs and their stable models:

a. b. c.

```
a(V) :- V = #sum{ 1: a;      2: b;      5: c      }.
b(V) :- V = #sum{ 1: a;      2: b;      5: not c }.
c(V) :- V = #sum{ 1: a;      2: not b; 5: not c }.
d(V) :- V = #sum{ 1: not a; 2: not b; 5: not c }.
```

```
e(V) :- V = #sum{ 1: a;      1: b;      1: c      }.
f(V) :- V = #sum{ 1: a;      1: b;      1: not c }.
g(V) :- V = #sum{ 1: a;      1: not b; 1: not c }.
h(V) :- V = #sum{ 1: not a; 1: not b; 1: not c }.
```

```
i(V) :- V = #sum{ 1: a;      1: b;      5: c      }.
j(V) :- V = #sum{ 1: a;      5: b;      5: c      }.
k(V) :- V = #sum{ 5: a;      5: b;      5: c      }.
```

```
l(V) :- V = #sum{ 1,x: a;      1,y: b;      1,z: c      }.
m(V) :- V = #sum{ 1,x: a;      1,y: b;      1,z: not c }.
n(V) :- V = #sum{ 1,x: a;      1,y: not b; 1,z: not c }.
o(V) :- V = #sum{ 1,x: not a; 1,y: not b; 1,z: not c }.
```

```
p(V) :- V = #sum{ 1,x: a;      1,y: b;      5,z: c      }.
q(V) :- V = #sum{ 1,x: a;      5,y: b;      5,z: c      }.
r(V) :- V = #sum{ 5,x: a;      5,y: b;      5,z: c      }.
```

```
s(V) :- V = #sum{ 1,x: a;      1,x: b;      1,x: c      }.
t(V) :- V = #sum{ 1,x: a;      1,x: b;      1,x: not c }.
u(V) :- V = #sum{ 1,x: a;      1,x: not b; 1,x: not c }.
v(V) :- V = #sum{ 1,x: not a; 1,x: not b; 1,x: not c }.
```

```
w(V) :- V = #sum{ 1,x: a;      1,x: b;      5,x: c      }.
x(V) :- V = #sum{ 1,x: a;      5,x: b;      5,x: c      }.
y(V) :- V = #sum{ 5,x: a;      5,x: b;      5,x: c      }.
```

```
-----
{ a, b, c,
  a(8), b(3), c(1), d(0),
  e(1), f(1), g(1), h(0),
  i(6), j(6), k(5),
  l(3), m(2), n(1), o(0),
  p(7), q(11), r(15),
  s(1), t(1), u(1), v(0),
  w(6), x(6), y(5) }
```



```
{ a; b }.
:- 1 #sum{ 1,x: a; 1,y: b }.
-----
{ }
```

```
{ a; b }.
:- 2 #sum{ 1,x: a; 1,y: b }.
-----
{ }
{ a }
{ b }
```

```
{ a; b }.
:- #sum{ 1,x: a; 1,x: b } 1.
-----
```

```
{ a; b }.
:- #sum{ 1: a; 1: b } 1.
-----
```

```
{ a; b }.
:- -1 #sum{ -1,x: a; -1,y: b }.
-----
{ a, b }
```

```
{ a; b }.
:- 1 #sum{ 1,x: not a; 1,y: not b }.
-----
{ a, b }
```

```
{ a; b }.
c :- 1 #sum{ 1,x: a; 1,y: b }.
:- 2 #sum{ 1,x: a; 1,y: c }.
-----
{ }
{ b, c }
```

```
{ a; b }.
c :- 2 #sum{ 1: a; 2: b }.
:- 3 #sum{ 1: a; 2: c }.
-----
{ }
{ a }
{ b, c }
```

Find the stable models of the next programs:

```
{ a; b }.
c :- a.
:- 1 #sum{ 1,x: c; 1,y: b }.
-----
```

```
{ a; b }.
:- 2 #sum{ 1,x: a; 10,y: b }.
-----
```

```
{ a; b }.
c :- a.
d :- b.
:- #sum{ 1,x: c; 1,x: d } 1.
-----
```

```
{ a; b }.
c :- a.
d :- b.
:- #sum{ 1: c; 1: d } 1.
-----
```

```
{ a; b }.
:- 0 #sum{ -1,x: a; -1,y: b }.
-----
```

```
{ a; b; c}.
:- 1 #sum{ 1,x: not a;
           1,y: not b;
           1,z: not c}.
-----
```

```
{ a; b }.
c :- 2 #sum{ 1,x: a; 1,y: b }.
:- 2 #sum{ 2,x: a; 1,y: c }.
-----
```

```
{ a; b }.
c :- 3 #sum{ 1: a; 2: b }.
:- 2 #sum{ 1: c; 2: b }.
-----
```