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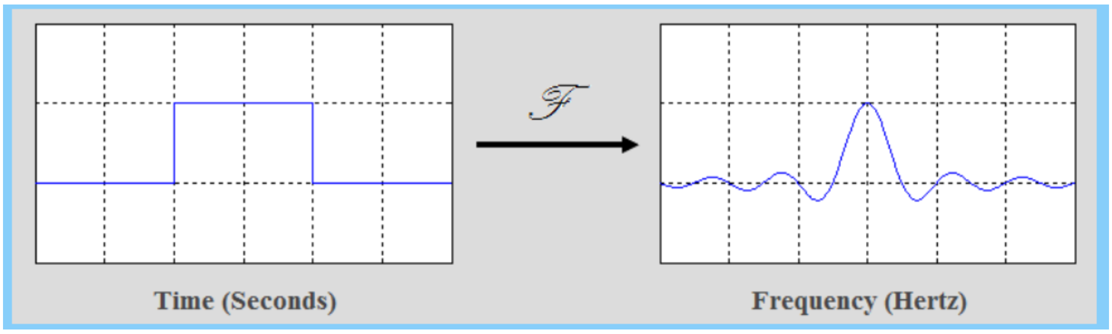
Contents

Introduction to Fourier Transform

The Fourier Transform is a tool that **breaks a waveform** (a function or signal) into an alternate representation, characterized by the sine and cosine functions of varying frequencies. The Fourier Transform shows that any waveform can be re-written as the sum of sinusoidals.

The Fourier Transform is the mathematical tool that shows us **how to deconstruc**t the waveform into its sinusoidal components. This has a multitude of applications, aides in the understanding of the universe, and just makes life much easier for the practicing engineer or scientist.



What is Fourier transform used for?

- The Fourier Transform is an important image processing tool which is used **to decompose an image into its sine and cosine components**. The output of the transformation represents the image in the Fourier or frequency domain, while the input image is the spatial domain equivalent.

- The Fourier transform can be used **to interpolate functions and to smooth signals**. For example, in the processing of pixelated images, the high spatial frequency edges of pixels can easily be removed with the aid of a two-dimensional Fourier transform.

- The Fourier transform **gives us insight into what sine wave frequencies make up a signal**. You can apply knowledge of the frequency domain from the Fourier transform in very useful ways, such as: Audio processing, detecting specific tones or frequencies and even altering them to produce a new signal.

- Fourier transforms is an extremely powerful mathematical tool that **allows you to view your signals in a different domain**, inside which several difficult problems become very simple to analyze

Conditions for Existence of Fourier Transform

Any function f(t) can be represented by using Fourier transform only when the function satisfies Dirichlet's conditions. i.e.

- The function f(t) has finite number of maxima and minima
- There must be finite number of discontinuities in the signal f(t),in the given interval of time.

- It must be absolutely integrable in the given interval of time i.e.

$$\int_{\infty}^{\infty} |f(t)|dt < \infty$$

Properties of Fourier Transform:

- **Linearity:**
  Addition of two functions corresponding to the addition of the two frequency spectrum is called the linearity. If we multiply a function by a constant, the Fourier transform of the resultant function is multiplied by the same constant.

The Fourier transform of sum of two or more functions is the sum of the Fourier transforms of the functions.

Case I.

If h(x) -> H(f) then ah(x) -> aH(f)

Case II.

If h(x) -> H(f) and g(x) -> G(f) then h(x)+g(x) -> H(f)+G(f)

- **Scaling:**
  Scaling is the method that is used to the change the range of the independent variables or features of data. If we stretch a function by the factor in the time domain then squeeze the Fourier transform by the same factor in the frequency domain.

If f(t) -> F(w) then f(at) -> (1/|a|)F(w/a)

- **Differentiation:**
  Differentiating function with respect to time yields to the constant multiple of the initial function.

If f(t) -> F(w) then f'(t) -> jwF(w)

- **Convolution:**
  It includes the multiplication of two functions. The Fourier transform of a convolution of two functions is the point-wise product of their respective Fourier transforms.

If f(t) -> F(w) and g(t) -> G(w)

then f(t)*g(t) -> F(w)*G(w)

- **Frequency Shift:**Frequency is shifted according to the co-ordinates. There is a duality between the time and frequency domains and frequency shift affects the time shift.

If f(t) -> F(w) then f(t)exp[jw't] -> F(w-w')

- **Time Shift:**
  The time variable shift also effects the frequency function. The time shifting property concludes that a linear displacement in time corresponds to a linear phase factor in the frequency domain.

If f(t) -> F(w) then f(t-t') -> F(w)exp[-jwt']

Fourier Series

The Fourier Series breaks down a periodic function into the sum of sinusoidal functions. It is the Fourier Transform for periodic functions. To start the analysis of Fourier Series, let's define periodic functions.

A function is periodic, with fundamental period T, if the following is true for all t:

f(t+T)=f(t)

Limitations of Fourier series

Methods based on the Fourier transform are almost synonymous with frequency domain processing of signals. There is no doubt about how incredibly powerful Fourier analysis can be. However, its popularity and effectiveness have a downside. It has led to **a very specific** and **limited view of frequency** in the context of signal processing. Simply put, frequencies, in the context of Fourier methods, are just a collection of the individual frequencies of periodic signals that a given signal is composed of.

Why we need Fourier series and Fourier transform?

The Fourier series is used to represent a periodic function by a discrete sum of complex exponentials, while the Fourier transform is then used to represent a general, nonperiodic function by a continuous superposition or integral of complex exponentials. The Fourier transform can be viewed as the limit of the Fourier series of a function with the period approaches to infinity, so the limits of integration change from one period to $(-\infty,\infty)(-\infty,\infty)$.

In a classical approach it would not be possible to use the Fourier transform for a periodic function which cannot be in $L1(-\infty,\infty)L1(-\infty,\infty)$. The use of generalized functions, however, frees us of that restriction and makes it possible to look at the Fourier transform of a periodic function. It can be shown that the Fourier series coefficients of a periodic function are sampled values of the Fourier transform of one period of the function.

The significance of fourier transform in communication engineering?

Communication engineering mainly deal with **signals** and hence signals are of various type like continues, discrete,periodic,non-periodic and many of many types.Now Fourier transform helps us to converted time domain signal in frequency domain.

Now let us assumed we want to know syestem output

$$Y(t)=x(t)*h(t)$$

Where x(t) is input and h(t) is impulse of that syestem

Now outout in time domain will be convolute of x(t) and h(t). But in frequency domain out will be just multiplication of x(f) and h(f). Since doing convolution mathematically tough and multiplication is easy hence we apply Fourier transform on input signal and impulse response of signal to know output. Another thing is it become easy to write algorithm for multiplication of frequency than convolution.

Fourier transform is used to transform **periodic** and n**on-periodic signals** from **time** domain to **frequency** domain. It can also transform Fourier series into the frequency domain, as Fourier series is nothing but a simplified form of time domain periodic function.

Fourier transform vs Fourier series

Fourier series

1. Periodic function => converts into a *discrete* exponential or sine and cosine function.
2. Non-periodic function => not applicable

Fourier transform

1. Periodic function => converts its Fourier series in the frequency domain.
2. non-Periodic function => converts it into *continuous* frequency domain.

Applications

A Fourier Series has many applications in mathematical analysis as it is defined as the sum of multiple sines and cosines. Thus, it can be easily differentiated and integrated, which usually analyses the functions such as saw waves which are periodic signals in experimentation. It also provides an analytical approach to solve the discontinuity problem. In calculus, this helps in solving complex differential equations.

The Fourier series has many such applications in **electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, shell theory**, etc.

The Fourier transform can be used to interpolate functions and to smooth signals. For example, in the processing of pixelated images, the high spatial frequency edges of pixels can easily be removed with the aid of a two-dimensional Fourier transform.