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Numerical Analysis
Assignment 2-2

Ex 2.3 , 1, 4, 5(a), 6(a)

1) let $f(x) = x^2 - 6$ and $p_0 = 1$. Using Newton's method to find p_1

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)}$$

$\Rightarrow$ let $p = f(p)$

$$f(p) = p^2 - 6$$

$$\therefore p_1 = 1 - \frac{-5}{2}$$

$$= 1 - (-2,5)$$

$$p_1 = 3,5$$

4) let $f(x) = -x^3 - \cos x$. With $P_0 = -1$ and $P_1 = 0$ find P_3.

a) Using the secant method

→ Definition

$$\boxed{P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}}$$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 0 - \frac{f(0)(0+1)}{f(0) - f(-1)} = -\frac{f(0)}{f(0) - f(-1)}$$

$$= -\frac{-\cos 0}{-\cos 0 - (-(-1)^3 - \cos(-1))} = -\frac{-1}{-1 - (1 - \cos(-1))}$$

$$= \frac{1}{-2 + \cos(-1)} = \frac{1}{-2 + 0{,}5403} = -0{,}6851$$

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_1)}{f(P_2) - f(P_1)} = -0{,}6851 - \frac{f(-0{,}6851)(-0{,}6851 - 0)}{f(-0{,}6851) - f(0)}$$

$$= -0.6851 - \frac{-0{,}6851(-(-0{,}6851)^3 - \cos(-0{,}6851))}{(0.6851)^3 - \cos(-0{,}6851) + \cos(0)} \Rightarrow -1.252$$

$\therefore$ $f(-1{,}252) = 1{,}649$, which is far from 0, we might need three more iteration.

4b) Using the method of False Position

Using the formular

$$x_n = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

I	P_1	P_0	$f(P_0)$	$f(P_1)$	P_n	x_n
1	0	-1	0,4597	-1	-1,85081	-6,06362
2	-6,0636	-1	0,4597	-222,94515	1,01404	-1,57126
3	-1,57126	-1	0,4597	3,8796919	-0,8413677	0,05232

$P_3 = -0,84136$

5a) Using Newton's method to find solutions accurate to 10^{-4}

$x^3 - 2x^2 - 5$, $[1, 4]$

Formular : $X_{n+1} = X_n - \dfrac{f(x_n)}{f'(x_n)}$

First X_0 is my starting point in the range $[1, 4]$

$f(x) = x^3 - 2x^2 - 5$

$f'(x) = 3x^2 - 4x$

$X_0 = 2 \rightarrow$ I am starting at 2.

$n = 0$

The answer is $2,6906$

$$X_1 = X_{0+1} = 2 - \frac{2^3 - 2(2^2) - 5}{3 \cdot 2^2 - 4(2)} = 2 - \frac{-5}{4} = 3,25$$

$$X_2 = 3,25 - \frac{(3,25)^3 - 2(3,25)^2 - 5}{3(3,25)^2 - 4(3,25)} \Rightarrow \cancel{3,25} - \frac{\cancel{8,203125}}{\cancel{69,984375}} = \overset{2,8110367893}{\cancel{3,1588803399}}$$

$$X_3 = \overset{2,8110367}{\cancel{3,15883}} - \frac{(\overset{2,811036}{\cancel{3,1588}})^3 - 2(\overset{2,8110}{\cancel{3,1588}})^2 - 5}{3(\overset{2,8110}{\cancel{3,1588}})^2 - 4(\overset{2,8110}{\cancel{3,1588}})} = \cancel{3,1588} - \frac{\cancel{6,34318}}{\cancel{17,423774}} = 2,69798950247 \Rightarrow \cancel{3,0787249722}$$

$$X_4 = \overset{2,697989}{\cancel{3,0787}} - \frac{(\overset{2,6979}{\cancel{3,0787}})^3 - 2(\overset{2,69798}{\cancel{3,0787}})^2 - 5}{\cancel{3(3,0787)^2} \; 3(2,697989)^2 - 4(2,697989)} \Rightarrow 3,6627051178$$

$$X_5 = 3,662705 - \frac{(3,6627)^3 - 2(3,6627)^2 - 5}{3(3,6627)^2 - 4(3,6627)} = 2,98657379$$

$$X_6 = 2,98657 - \frac{(2,98657)^3 - 2(2,98657)^2 - 5}{3(2,98657)^2 - 4(2,98657)} = 2,73004388947$$

$$X_7 = 2,730043 - \frac{(2,730043)^3 - 2(2,730043)^2 - 5}{3(2,730043)^2 - 4(2,730043)} = 2,69148198143$$

$$X_8 = 2,691489 - \frac{(2,6914819)^3 - 2(2,6914819)^2 - 5}{3(2,6914819)^2 - 4(2,6914819)} \Rightarrow 2,690647375$$

6a) Using Newton's method within 10^{-5} for $1 \leq x \leq 2$

$e^{x} + 2^{-x} + 2\cos x - 6 = 0$

Formular : $X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

$f(x) = e^{x} + 2^{-x} + 2\cos x - 6$

$f'x = e^{x} - 2^{-x}\ln(2) - 2\sin x$

$X_n = 1$ first guess

$n = 0$

$X_1 = 1 - \frac{e^{1} + 2^{-1} + 2\cos(1) - 6}{e^{1} - 2^{(-1)}\ln(2) - 2\sin(1)} \Rightarrow 1,7625 2476$

$X_2 = 1,76252476 - \frac{e^{1,766252476} + 2^{-1,7625} + 2\cos(1,7625) - 6}{e^{1,7625} - 2^{(1,7625)}\ln(2) - 2\sin(1,7625)} \Rightarrow 1,78129599$

$X_3 = 1,78129599 - \frac{e^{1,78129} + 2^{-1,78129} + 2\cos(1,78129) - 6}{e^{1,78129} - 2^{(1,78129)}\ln(2) - 2\sin(1,78129)} \Rightarrow 1,77 13207$

$X_4 = 1,7713207 - \frac{e^{1,7713207} + 2^{-1,7713207} + 2\cos(1,7713207) - 6}{e^{1,7713207} + 2^{-1,7713207}\ln(2) - 2\sin(1,7713207)} \Rightarrow 1,77745761$

$X_5 = 1,77745761 - \frac{e^{1,77745761} + 2^{-1,77745761} + 2\cos(1,77743761) - 6}{e^{1,77745761} + 2^{-1,77745}\ln(2) - 2\sin(1,77745)} \Rightarrow 1,77711057$

$X_6 = 1,77711 - \frac{e^{1,7711} + 2^{1,7711} + 2\cos(1,77711) - 6}{e^{1,77711} + 2^{-1,77711}\ln(2) - 2\sin(1,77711)} \Rightarrow 1,77720047$

The solution ~~exact~~ accurate to $10^{-5} \Rightarrow \underline{\underline{1,77711}}$