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Information Theory and Network Coding

FINAL EXAM

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INFORMATION THEORY AND NETWORK CODING

FINAL EXAMINATION

1) The different goals of source coding, channel coding and network coding

⇒ Source coding

→ It represent information as accurately as possible using as few bits as possible.

→ The main aim is to convert information waveforms for example text, audio images and videos.

→ It deals with data compression, and error control.

⇒ Channel coding

→ Channel coding provides the GSM receiver with the ability to detect transmission errors.

→ It correct errors from a bit point of view.

→ The main aim for channel coding is to convert bits to signal waveform (encoder) and decoder converts waveform back to bits.

⇒ Network coding

→ Network coding is just a networking technique in which transmitted data is encoded ~~de~~ and decoded.

→ The main aim is to increase throughput and reduce delay.

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2) The Source Coding Theorem

- Source coding theorem is known as Shannon's source coding theorem
- It is also called noiseless coding theorem.
- Source coding theorem state that we can compress N independent and identically distributed (i.i.d) random ~~vd~~variables each with entropy (H) down to (NH) bits with negligible loss of information as $N \to \infty$.
- The theorem is:-

let X be an ensemble with $H(X) = H$ bits.

- given $Y > 0$ and $0 < E < 1$
- N is positive N_0 for $N > N_0$.

$$\left| \frac{1}{N} H_Y(X^N) - H \right| < E$$

The channel coding theorem

- Shannon's noisy channel
- It state that all rates below capacity C are achievable,
- For every rate $R \le C$, there exists a sequence of $(2^{nR}, n)$ codes with maximum probability of error $P_E \to 0$.

3)

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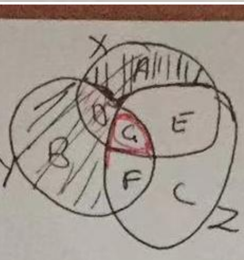

Information measures corresponding to:-

(i) Area $A = H(X|Y,Z)$

(ii) Area $D = I(X,Y|Z)$

(iii) Area $A+B+D = H(X,Y|Z)$

(iv) Area $E+F+G = I(X,Y;Z)$

(v) G is possible to take a negative value.

4) 4 binary codes:- M20216102g

C_1 =

code	L(x)
000	3
11	2
001	3
010	3
101	3

prefix

$\Rightarrow 4 \cdot \frac{1}{2^3} + \frac{1}{2^2} \Rightarrow 0,75$

$C_2 \Rightarrow$

code	L(x)
0	1
10	2
0010	4
0011	4

not prefix

$\Rightarrow \frac{1}{2^1} + \frac{1}{2^2} + 2 \cdot \frac{1}{2^4} \Rightarrow 0,875$

C_3

code	L(x)
0	1
10	2
11	2
0100	4

not prefix

$\Rightarrow \frac{1}{2^1} + 2 \cdot \frac{1}{2^2} + \frac{1}{2^4} \Rightarrow 1\frac{1}{16}$ ×

$C_4 \Rightarrow$

code	L(x)
01	2
00	2
011	3
001	3

not prefix

$\Rightarrow 2 \cdot \frac{1}{2^2} + 2 \cdot \frac{1}{2^3} \Rightarrow 0,75$

a) C_3 doesnot satisfy kraft's inequality

b) C_2 includes(0 , 10 and 0010, 0011) ; C_3 includes(0, 10 and 0100)

C_4 includes(01, 00 and 011, 001) are not prefix codes and are not uniquely decodable.

II $P(X=0, Y=a) = 1/4$
$P(X=0, Y=b) = 0$
$P(X=0, Y=c) = 1/4$
$P(Y=1, Y=a) = 0$
$P(Y=1, Y=b) = 1/4$
$P(Y=1, Y=c) = 1/4$

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$P_X(x) = \sum_y P(X,Y)$ ϕ $P_Y(y) = \sum_x P(Y,X)$
$P_X(a) = \sum_y P_{XY}(X=0,y) = 1/4 + 0 + 1/4 = 1/2$

$H(X,Y) = \sum_x \sum_y P_{XY}(x,y) \log P_{XY}(x,y)$
$\Rightarrow 1/4 \log_2 4 + 2\cdot 0 \cdot 1/4 \log_2 4 + 1/4 \log_2 4$
$\Rightarrow 2$

$H(X) \Rightarrow \sum P(x) \cdot \log_2 P(x)$
$\Rightarrow 1/2 \log_2 2 + 1/2 \log_2 2$
$\Rightarrow 1$

$H(Y|X) \Rightarrow \sum P(x) H(Y|X=x)$
$\Rightarrow H(X,Y) - H(X)$
$\Rightarrow 2 - 1$
$= 1$

$H(Y/X=0) \Rightarrow P_X(x) + P_X(a)$
$\Rightarrow 1/2 + 1/2$
$\Rightarrow 1$

$I(X;Y) \Rightarrow H(X) - H(X/Y)$
$\Rightarrow H(X) + H(Y) - H(X,Y)$
$\Rightarrow 1 + 1 - 2$
$\Rightarrow 0$

III. $X = \{x_1, x_2, x_3\} - \{0{,}6; 0{,}3; 0{,}1\}$

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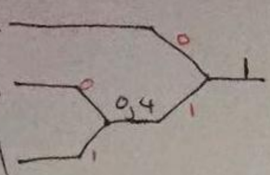
1) Huffman code X:

1	0	0,6
2	01	0,3
2	11	0,1



H code x $\Rightarrow \{0, 01, 11\}$

$\bar{L}_1 \Rightarrow (1 \cdot 0{,}6) + (2 \cdot 0{,}3) + (2 \cdot 0{,}1) \Rightarrow 1{,}4$

$n_1 = H(X)/\bar{L}_1 \Rightarrow H(0{,}6; 0{,}3; 0{,}1)/1{,}4$

$\Rightarrow \dfrac{\left(\frac{3}{5}\log_2\frac{5}{3}\right) + \left(\frac{3}{10}\log_2\frac{10}{3}\right) + \left(\frac{1}{10}\log_2 10\right)}{7/4}$

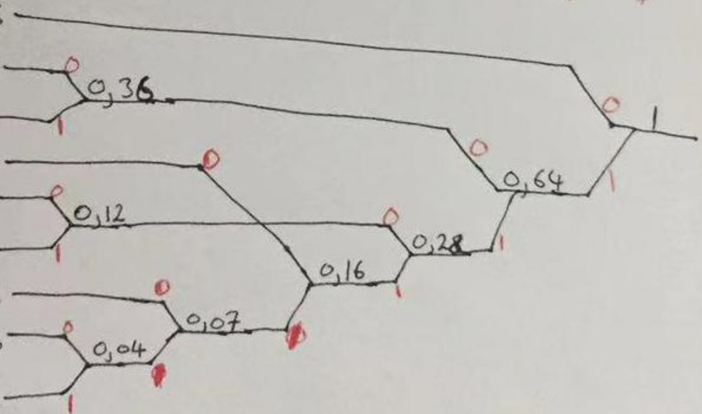
$\Rightarrow 0{,}92532988$

$\Rightarrow 0{,}925$

2) Huffman code for $X^2 = (0{,}6; 0{,}3; 0{,}1)^2 \Rightarrow (0{,}6; 0{,}3; 0{,}1)(0{,}6; 0{,}3; 0{,}1) \Rightarrow$

$\Rightarrow (\underset{1}{0{,}36}; \underset{2}{0{,}18}; \underset{5}{0{,}06}; \underset{3}{0{,}18}; \underset{4}{0{,}09}; \underset{7}{0{,}03}; \underset{6}{0{,}06}, \underset{8}{0{,}03}; \underset{9}{0{,}01})$

1	0	0,36
3	100	0,18
3	101	0,18
4	1110	0,09
4	1100	0,06
4	1101	0,06
5	11110	0,03
6	111110	0,03
6	111111	0,01



$\bar{L}_2 \Rightarrow (1 \cdot 0{,}36) + (3 \cdot 18) + (3 \cdot (18)) + (4 \cdot 0{,}09) + (4 \cdot 0{,}06) + (4 \cdot 0{,}06) + (5 \cdot 0{,}03) + (6 \cdot 0{,}03) + (6 \cdot 0{,}01)$

$\Rightarrow \cancel{1{,}6788} \Rightarrow 2{,}67$

$n_2 = H(X^2)/\bar{L}_2 \Rightarrow 2H(X)/\bar{L}_2$

$\Rightarrow \frac{36}{100}\log_2\frac{100}{36} + 2 \cdot \frac{18}{100}\log_2\frac{100}{18} + \frac{9}{100}\log_2\frac{100}{9} + 2 \cdot \frac{6}{100}\log_2\frac{100}{6} + 2 \cdot \frac{3}{100}\log_2\frac{100}{3} + \frac{1}{100}\log_2 100$

$\Rightarrow 2{,}59092368 / \cancel{1{,}6788}\ 2{,}67$

$\cancel{\Rightarrow 1{,}60052117}$

$\cancel{\Rightarrow 1{,}601}$ $\Rightarrow 0{,}9703834$

$\Rightarrow 0{,}970$

IV)

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1. $\cancel{\Rightarrow} H(X_j) \Rightarrow H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) = 4 \cdot \frac{1}{4} \log_2 4 \Rightarrow 2$

$$H(X_{j+1}|X_j) = \underbrace{p(X_j = a)}_{1/4} H(X_2|X_j = a) + \underbrace{p(X_2 = b)}_{1/4} H(X_2|H_j = b) +$$
$$\underbrace{p(X_1 = c)}_{1/4} H(X_2|X_j = c) + \underbrace{p(X_j = d)}_{1/4} H(X_2|X_j = d)$$

$\cancel{H(X_{j+1}|X_j)}$

$H(X_2|X_j = a) = \left(\frac{1}{2}, \frac{1}{4}, 0, \frac{1}{4}\right) = \frac{1}{2}\log_2 2 + 2 \cdot \frac{1}{4}\log_2 4 + 0\log_2 0 \Rightarrow 1,5$

$H(X_2|X_j = b) \Rightarrow \left(0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right) \Rightarrow 0\log_2 0 + \frac{1}{2}\log 2 + 2\frac{1}{4}\log_2 4 \rightarrow 1,5$

$H(X_2|X_j = c) \Rightarrow \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right) \Rightarrow \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 + 0 \Rightarrow 1,5$

$H(X_2|X_j = d) \Rightarrow \left(\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}\right) \Rightarrow 2 \cdot \frac{1}{4}\log_2 4 + 0 + \frac{1}{2}\log_2 2 \Rightarrow 1,5$

$\therefore H(X_j|X_{j+1}) \Rightarrow \frac{1}{4}(1,5) + \frac{1}{4}(1,5) + \frac{1}{4}(1,5) + \frac{1}{4}(1,5) \Rightarrow 1,5$

$$\Rightarrow H(X_j, X_{j+1})/2 \Rightarrow \frac{H(X_j) + H(X_{j+1}|X_j)}{2} = \frac{2 + 1,5}{2} \Rightarrow 1,75$$

$$\Rightarrow H_x \Rightarrow \cancel{\lim_{j\to\infty}} \lim_{j\to\infty} H(X_{j+1}|X_j, X_{j-1}, \dots X_1) = H(X_{j+1}|X_j)$$

$\Rightarrow 1,5$

$\Rightarrow H_x$ in descending order:

$$H(X_j) > \frac{H(X_j|X_{j+1})}{2} > H_x$$

$$2 > 1,75 > 1,5$$

2) $I(X_{j-1}, X_{j+1}|X_j) \Rightarrow 1,5$

V) $X \in = \{0,1\}$ M202161029

$Y \in = \{0, e, 1\}$

1) transition matrix :-

$$P(Y|X) \Rightarrow \begin{array}{c|ccc} & 0 & e & 1 \\ 0 & P(Y=0/X=0) & P(Y=e/X=0) & P(Y=1/X=0) \\ 1 & P(Y=0/X=1) & P(Y=e/X=1) & P(Y=1/X=1) \end{array}$$

$$\Rightarrow \begin{bmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

$H(Y/X) \Rightarrow H(2/3, 1/3, 0) = 2/3 \log_2 3/2 + 1/3 \log_2 3 \Rightarrow 0,91829583 \Rightarrow 0,918$

$P(X=0)=0, P(X=1)=1$

$H(Y) = P(Y) = $ ~~$1/3 \log 3 + 2/3 \log 3/2 \Rightarrow 0,918$~~ $1/0 \log 1 + 1 \log 1 + 1 \log 0 = 0$

$P(X) = (0,1)$

$P(Y|X) = \frac{P(X.Y)}{P(X)}$

$P(X.Y) = P(Y/X) \cdot P(X)$

$$P(X.Y) \Rightarrow \begin{array}{c} 0 \times \\ 1 \times \end{array} \begin{bmatrix} 2/3, & 1/3, & 0 \\ 0, & 1/3, & 2/3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0, & 0 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

$P(X/Y) = \frac{P(X.Y)}{P(Y)}$

$P(Y) \Rightarrow [0, 1/3, 2/3]$

$$P(X/Y) = \frac{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}}{[0 \quad 1/3 \quad 2/3]}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$I(X;Y) = H(X) - H(X|Y)$

$\Rightarrow H(X) - P(H(X)$

$\Rightarrow 2/3 \, H(X)$

$\Rightarrow 2/3 \, P(X)$

$\Rightarrow 2/3 \cdot 0 + 2/3 \cdot 1$

$\Rightarrow 2/3$ bits/symbol.

$$\boxed{\begin{array}{l} \textbf{2)} \; H(Y/X) = 0,918 \\ H(Y) \Rightarrow 0 \\ I(X;Y) \Rightarrow 2/3 \text{ bits/symbol} \end{array}}$$

3) $P(X) = (1/2, 1/2)$

$$P(X.Y) \Rightarrow \begin{array}{c} 1/2 \cdot \\ 1/2 \cdot \end{array} \begin{bmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/3 & 1/6 & 0 \\ 0 & 1/6 & 1/3 \end{bmatrix}$$

$P(X/Y) = \frac{P(X.Y)}{P(Y)} \; \therefore P(Y) \Rightarrow [1/3 \quad 1/3 \quad 2/3]$

$$\Rightarrow \frac{\begin{bmatrix} 1/3 & 1/6 & 0 \\ 0 & 1/6 & 1/3 \end{bmatrix}}{[1/3 \quad 1/3 \quad 2/3]} \Rightarrow \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$H(Y) = [1 \quad 1 \quad 1/2]$

$= 1 \cdot 1 \log_2 1 + 1/2 \log 2$

$\Rightarrow 1$

$I(X;Y) \Rightarrow 2/3 \, H(X)$

$\Rightarrow (2/3 \cdot 1/2) + (2/3 \cdot 1/2)$

$\Rightarrow 2/3$ bits/symbol.

$$\boxed{\begin{array}{l} \textbf{3)} \; H(Y/X) \Rightarrow 0,918 \\ H(Y) \Rightarrow 1 \\ I(X;Y) \Rightarrow 2/3 \text{ bits/symbol} \end{array}}$$

4) $C = \max [I(X,Y)]$

$\Rightarrow \text{Max} [2/3 \, H(X)]$

$\therefore \text{Max } H(X) \Rightarrow 2/3$ bits/symbol

$\Rightarrow$ It becomes maximum only if the transitted symbol is 1.