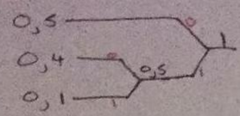
Alec Mabhiza Chirawu - 亚历克上 - M202161029

Assignment 3

1) a) $X = \{x_1; x_2; x_3\}$

$\{0{,}5; 0{,}4; 0{,}1\}$



0	1
10	2
11	2

$(0{,}5 \times 1) + (0{,}4 \times 2) + (0{,}1 \times 2) \Rightarrow 1{,}5$

$\underline{L = 1{,}5}$

$H(x) = 0{,}5 \log 2 + 0{,}4 \log \frac{5}{2} + 0{,}1 \log 10 \Rightarrow \underline{1{,}36096404}$

$n_1 \Rightarrow \frac{H(x)}{L_1} \Rightarrow \frac{1{,}36096404}{1{,}5} \Rightarrow \underline{0{,}90730936}$

b) $L \Rightarrow x^2 \to \{(0{,}5)^2; (0{,}4)^2; (0{,}1)^2)$

$\Rightarrow (0{,}5^2 \times 1) + (0{,}4^2 \times 2) + (0{,}1^2 \times 2)$

$\Rightarrow \underline{0{,}59}$

$H(x^2) \Rightarrow \cancel{0{,}5^2 \log} \{0{,}25; 0{,}16; 0{,}01\}$

$\Rightarrow 0{,}25 \log \frac{4}{1} + 0{,}16 \log \frac{25}{4} + 0{,}01 \log 100$

$\Rightarrow \underline{0{,}98945555}$

$n_2 \Rightarrow \frac{H(x^2)}{L_2} \Rightarrow \frac{0{,}98945555}{0{,}59} \Rightarrow \underline{1{,}6770433}$

c) comparison

$n_1 \Rightarrow L_1 \Rightarrow 1{,}5$ $\qquad$ $n_2 \Rightarrow L_2 \Rightarrow 0{,}59$

$H(x) \Rightarrow 1{,}361$ $\qquad$ $H(x^2) \Rightarrow 0{,}989$

$\underline{n_1 \Rightarrow 0{,}91}$ $\qquad$ $\underline{n_2 = 1{,}677}$, $\quad$ $\underline{n_3 = 4{,}84093115}$

$\Rightarrow$ As the value of X^n increases to ∞ the value of n_n also increases.

$\Rightarrow$ The value of n start at 0,907 and keep expanding as n progresses.

2) A a)

C_x	$L(x)$
000	3
10	2
00	2
11	2

To satisfy kraft inequality $\sum_x \frac{1}{b^{L(x)}} \leq 1$

b=2 for binary numbers.

$= \frac{1}{2^3} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} \Rightarrow 7/8$

{000, 10, 00, 11} ~~it~~ satisfy kraft ~~th~~ inequality.

b)

C_x	$L(x)$
100	3
101	3
0	1
11	2

$\Rightarrow \left(2 \cdot \frac{1}{2^3}\right) + \left(\frac{1}{2^1}\right) + \frac{1}{2^2} \Rightarrow 1$

{100, 101, 0, 11} satisfy kraft inequality.

c)

$C(x)$	$L(x)$
01	2
100	3
011	3
00	2
111	3
1010	4
1011	4
1101	4

$\Rightarrow \left(2 \cdot \frac{1}{2^2}\right) + \left(3 \cdot \frac{1}{2^3}\right) + \left(3 \cdot \frac{1}{2^4}\right) \Rightarrow 1\frac{1}{16}$

{01, 100, 011, 00, 111, 1010, 1011, 1101} ~~sati~~ does not satisfy kraft inequality.

d)

C_x	$L(x)$
01	2
111	3
011	3
00	2
010	3
110	3

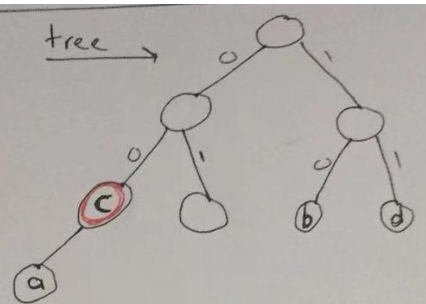
$= \left(2 \cdot \frac{1}{2^2}\right) + \left(4 \cdot \frac{1}{2^3}\right) \Rightarrow 1$

{01, 111, 011, 00, 010, 110} satisfy kraft inequality.

B

L	code
a	000
b	10
c	00
d	11

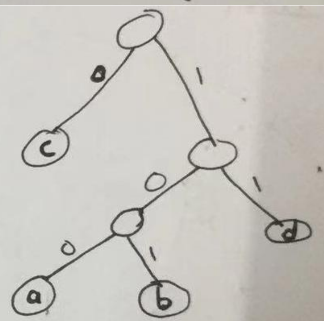
tree →



$\{000, 10, 00, 11\}$ ⇒ is not a prefix code because 00 is a prefix to 000.
⇒ no unique decodability.

L	code
a	100
b	101
c	0
d	11

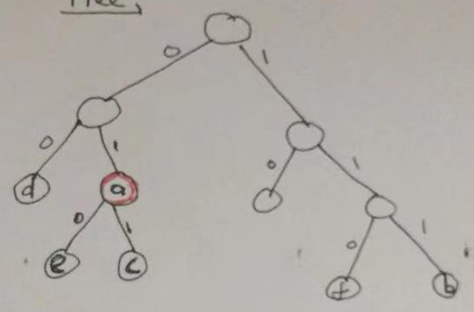
tree →



$\{100, 101, 0, 11\}$ is a prefix code
⇒ all codes are unique.

L	code
a	01
b	111
c	011
d	00
e	010
f	110

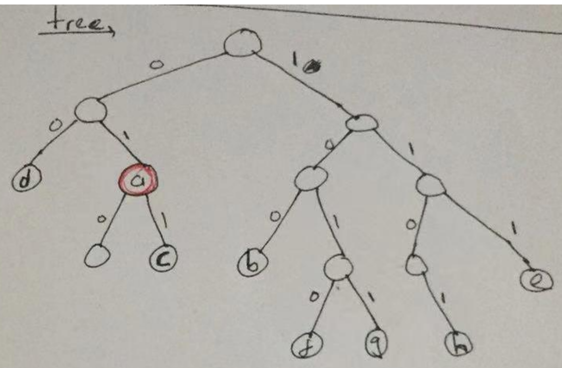
tree,



$\{01, 111, 011, 00, 010, 110\}$ is not a prefix code because 01 is also a prefix code for 011 and 010,
⇒ no unique decodability.

L	code
a	01
b	100
c	011
d	00
e	111
f	1010
g	1011
h	1101

tree,

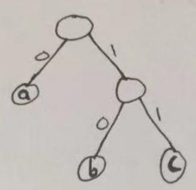


{01, 100, 011, 00, 111, 1010, 1011, 1101} is not a prefix code because 01 is a prefix code for 011.

$\Rightarrow$ no unique decodability.

3) a) {0, 10, 11}

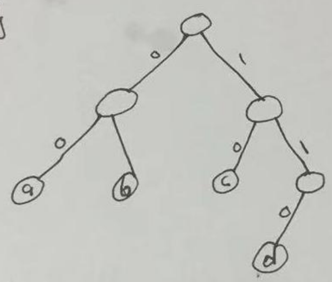
L	code
a	0
b	10
c	11



It can be constructed by Huffman coding.

b) {00, 01, 10, 110}

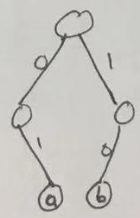
L	code
a	00
b	01
c	10
d	110



It can be constructed by Huffman coding.

c) {01, 10}

L	code
a	01
b	10



It can be constructed by Huffman coding.

4a) $H(Y/X) = p_0 H(Y/X=0) + (1-p_0) H(Y/X=1)$

$\Rightarrow H(1/2, 1/4, 1/4) = 1/2 \log 2 + 2 \cdot 1/4 \log 4 = \underline{1,5}$

b) $Y = P(Y=0) = 1/2 p_0 + 1/4(1-p_0)$, $P(Y=e) = 1/4$, $P(Y=1) = 1/4(1-p_0) + 1/4$

$\Rightarrow \underline{1/4 - 1/4 p_0 , 1/4 , 1/2 - 1/4 p_0}$

c) maximizes $H(Y)$: $p_0 = 1/2$, $P(Y=0) = P(Y=1) \Rightarrow \underline{3/8}$

d) $C \Rightarrow H(Y) - H(Y/X) = H(3/8, 1/4, 3/8) - 1,5$

$\Rightarrow 3/8 \log 8/3 + 1/4 \log 4 + 3/8 \log 8/3 - 1,5$

$\cancel{\Rightarrow \approx 0,04565599}$

$\Rightarrow \underline{0,06127812}$