Lecture 2
Wireless Channel

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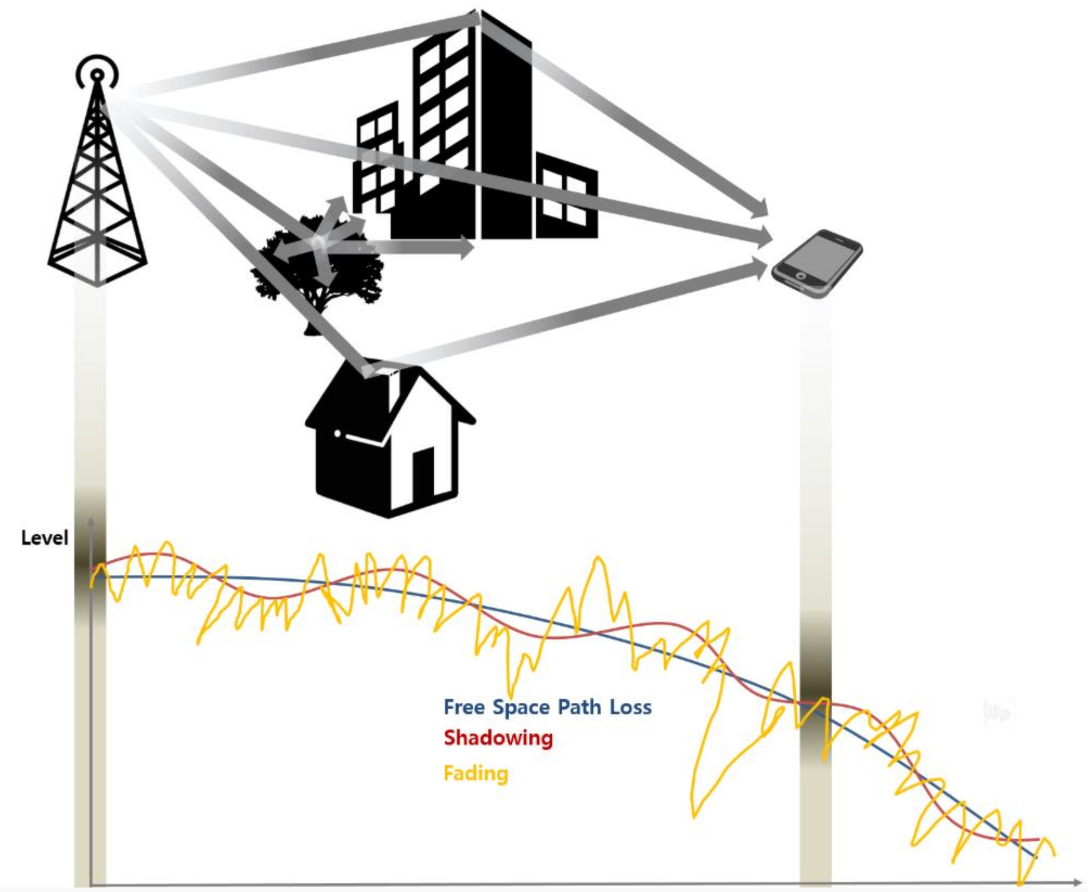
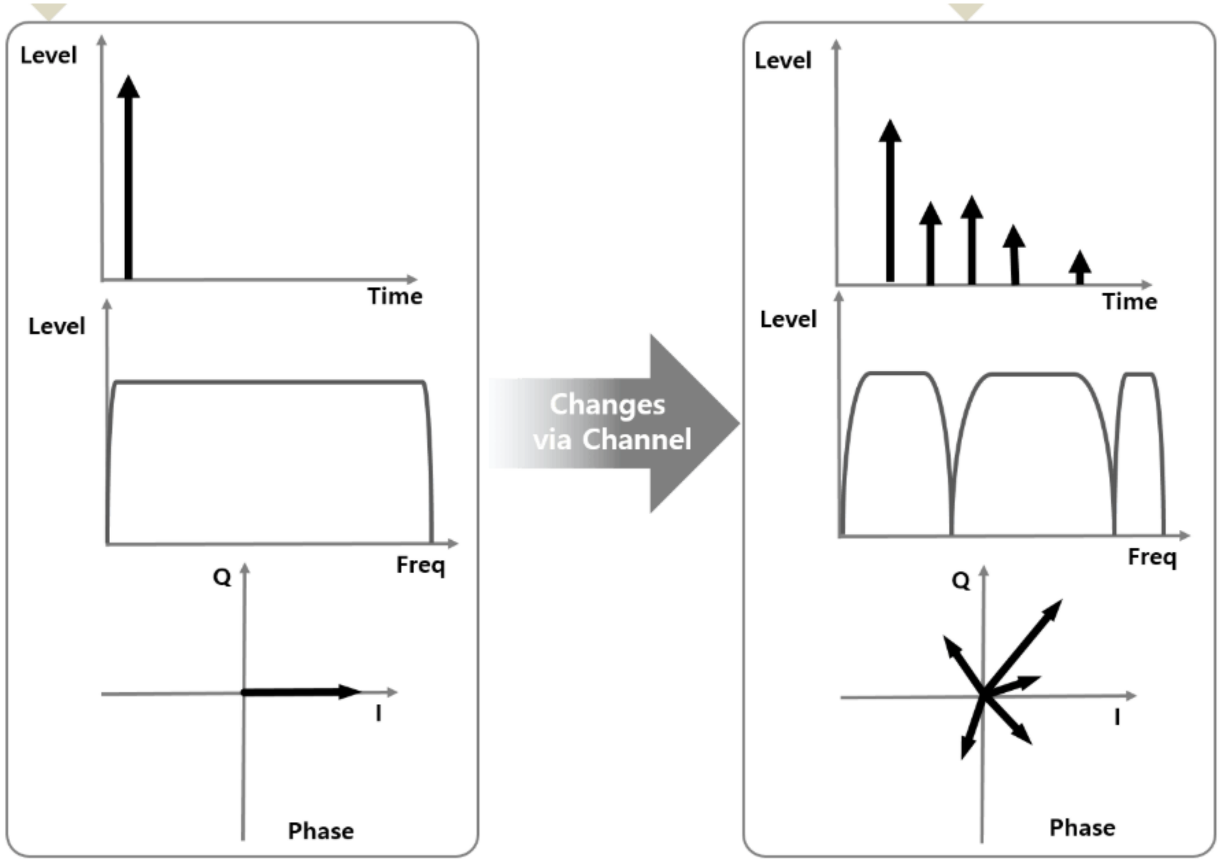
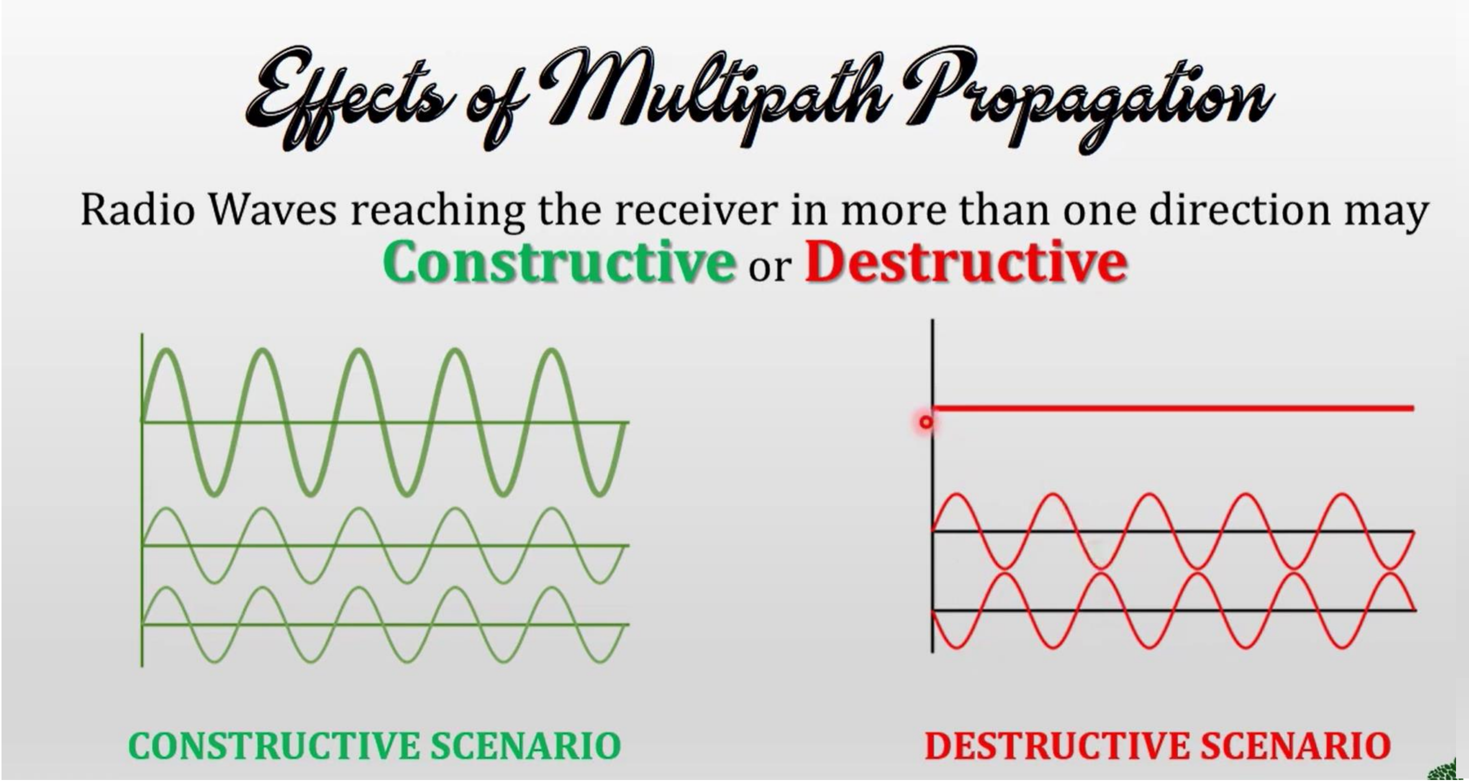
Major effect of the wireless channel





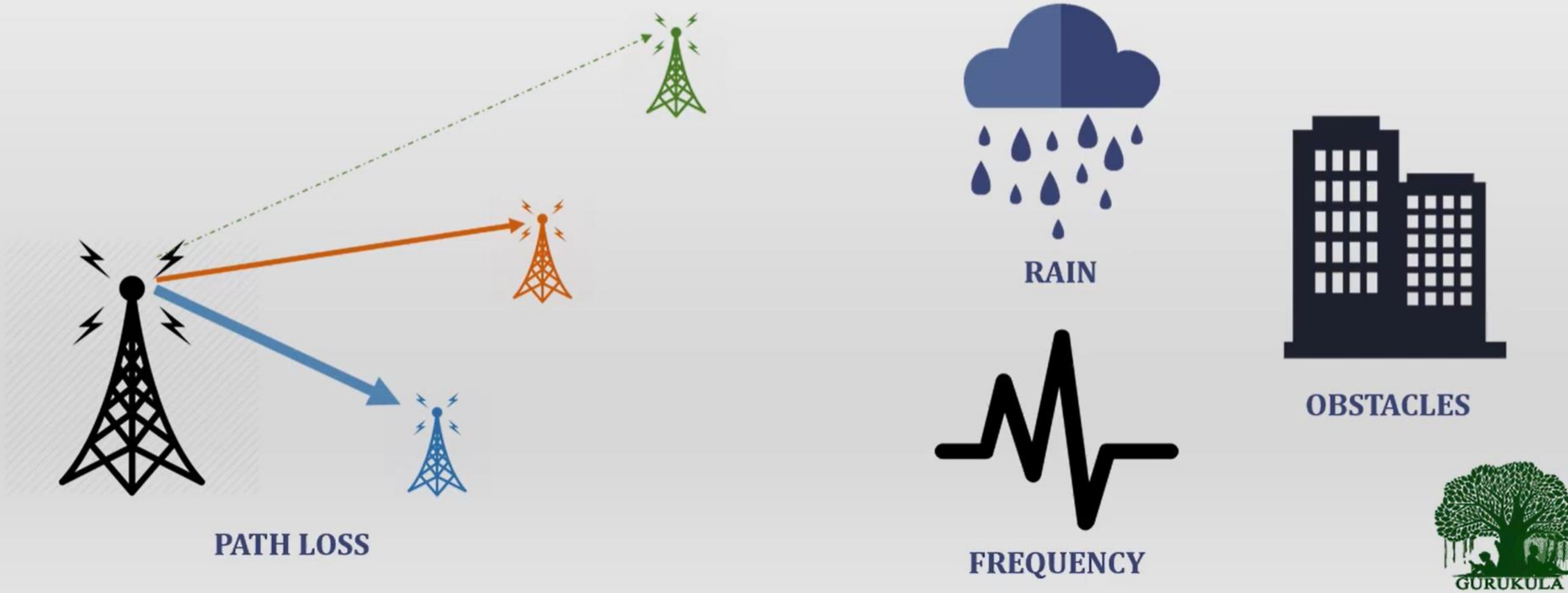
Signal in Wireless Channel



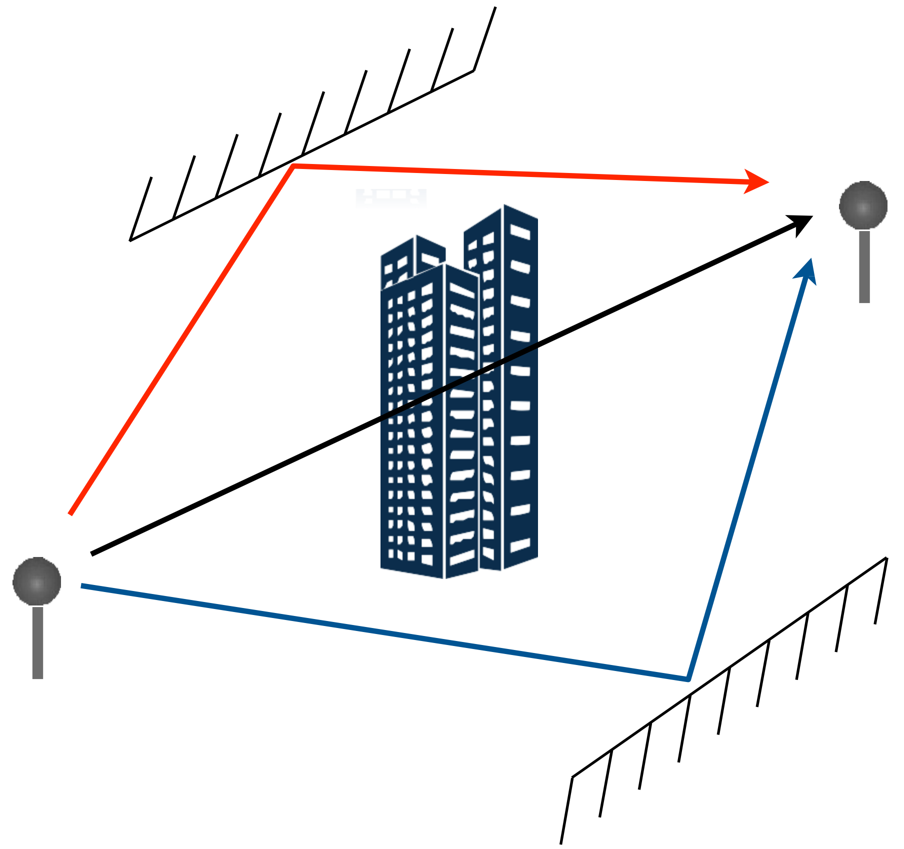
Fading

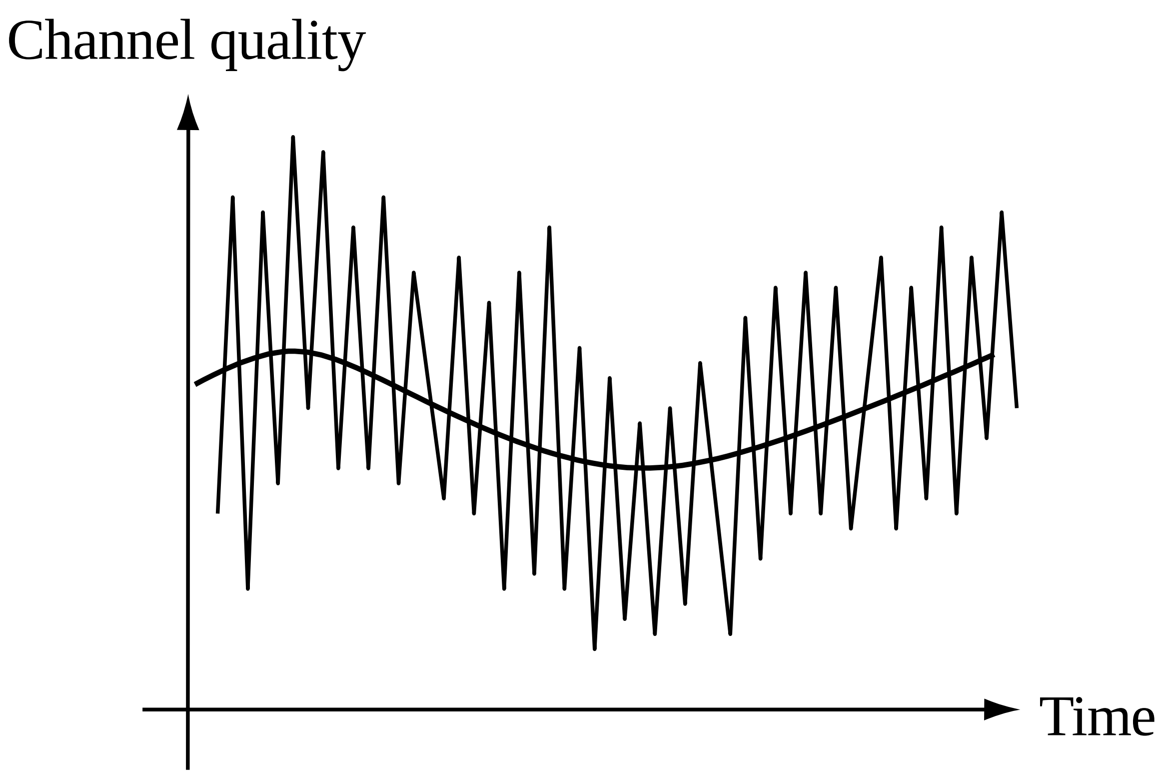
Fading

Fading is variation of the **attenuation of a signal** with various variables. These variables include **time**, **geographical position**, and **radio frequency** . In wireless systems, fading may either be due to multipath propagation, weather (particularly rain)



Wireless channels vary at two scales

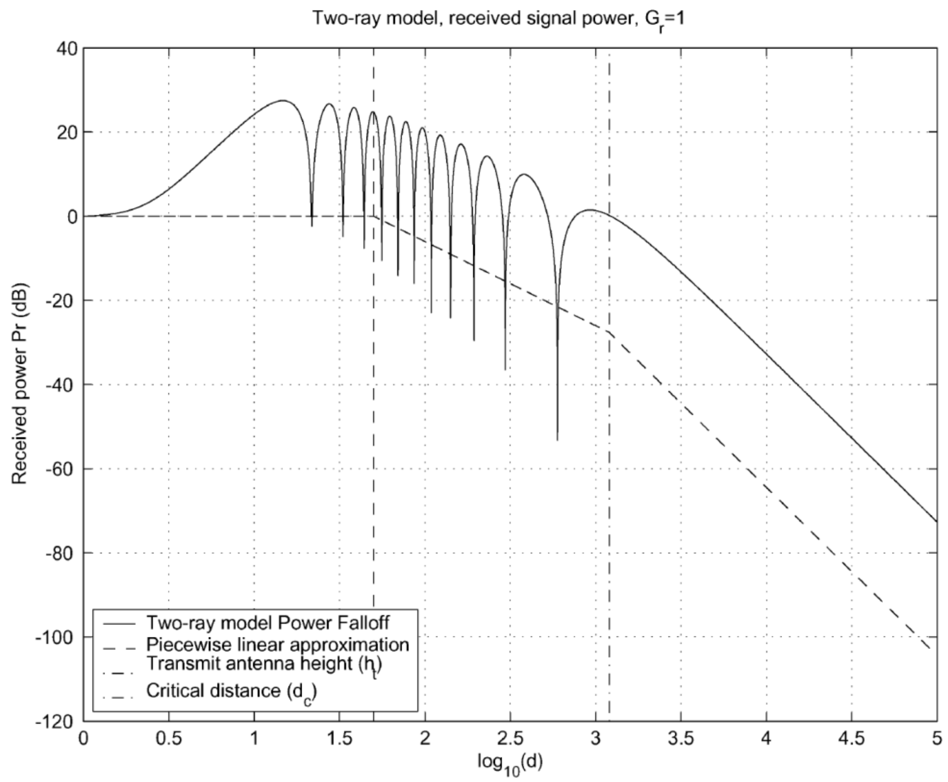




- Large-scale fading: path loss, shadowing, etc.
- Small-scale fading: constructive/destructive interference

Large - Scale Fading

- Path loss and Shadowing
  - In free space, received power $\propto \frac{1}{r^2}$
  - With reflections and obstacles, can attenuate faster than $\frac{1}{r^2}$
- Variation over time: very slow, order of seconds
- Critical for coverage and cell-cite planning



Small - Scale Fading

- Multipath fading: due to constructive and destructive interference of the waves

- Channel varies when the mobile moves a distance of the order of the carrier wavelength λ
  - Typical carrier frequency ~ 1GHz $\Rightarrow \quad \lambda = c/f = 0.3m$

- Variation over time: order of hundreds of microseconds

- Critical for design of communication systems

Plan

- Understand how *physical parameters* impact a wireless channel from the communication system point of view. Physical parameters such as
  - Carrier frequency
  - Mobile speed
  - Bandwidth
  - Delay spread
  - etc.

- Start with deterministic physical models

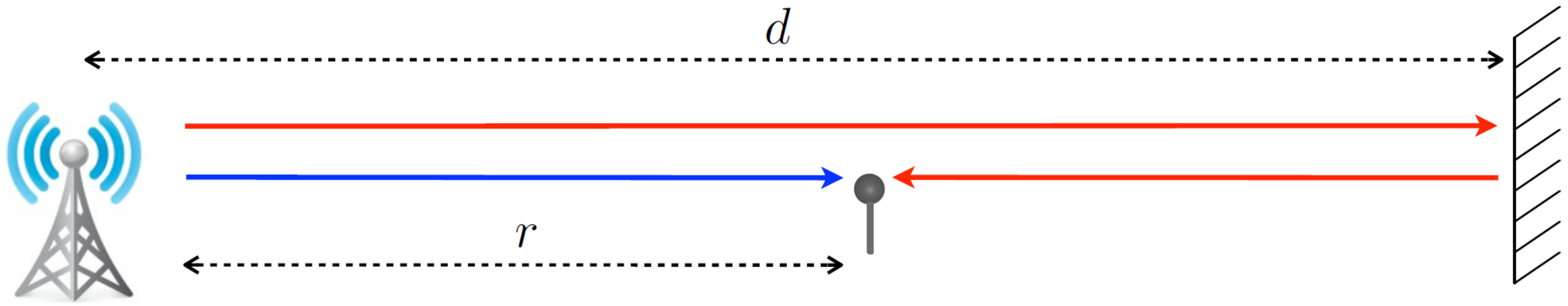
- Progress towards statistical models

Outline

- Physical modeling of wireless channels
- Deterministic Input-output model
- Time and frequency coherence
- Statistical models

Physical Model: Warm–up Examples

Physical Model: Simple Example 1



Transmitted Waveform (electric field): $\cos 2\pi f t$

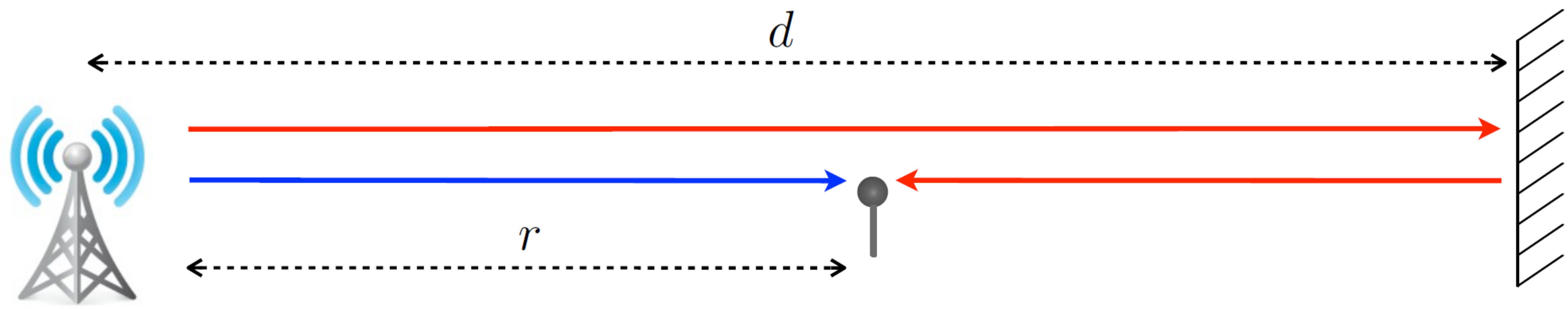
Received Waveform (path 1): $\frac{\alpha}{r} \cos 2\pi f \left(t - \frac{r}{c}\right)$

Received Waveform (path 2): $-\frac{\alpha}{2d - r} \cos 2\pi f \left(t - \frac{2d - r}{c}\right)$

$\Longrightarrow$ Received Waveform (aggregate):

$$\boxed{\frac{\alpha}{r} \cos 2\pi f \left(t - \frac{r}{c}\right) - \frac{\alpha}{2d - r} \cos 2\pi f \left(t - \frac{2d - r}{c}\right)}$$

Physical Model: Simple Example 1



Transmitted Waveform (electric field): $\cos 2\pi f t$

Received Waveform (aggregate):

$$\boxed{\frac{\alpha}{r}\cos 2\pi f\left(t-\frac{r}{c}\right)-\frac{\alpha}{2d-r}\cos 2\pi f\left(t-\frac{2d-r}{c}\right)}$$

Phase Difference between the two sinusoids:

$$\Delta\theta = \left\{\frac{2\pi f(2d-r)}{c}+\pi\right\}-\frac{2\pi f r}{c} = 2\pi\boxed{\frac{(2d-r)-r}{c}}f+\pi$$

T_d

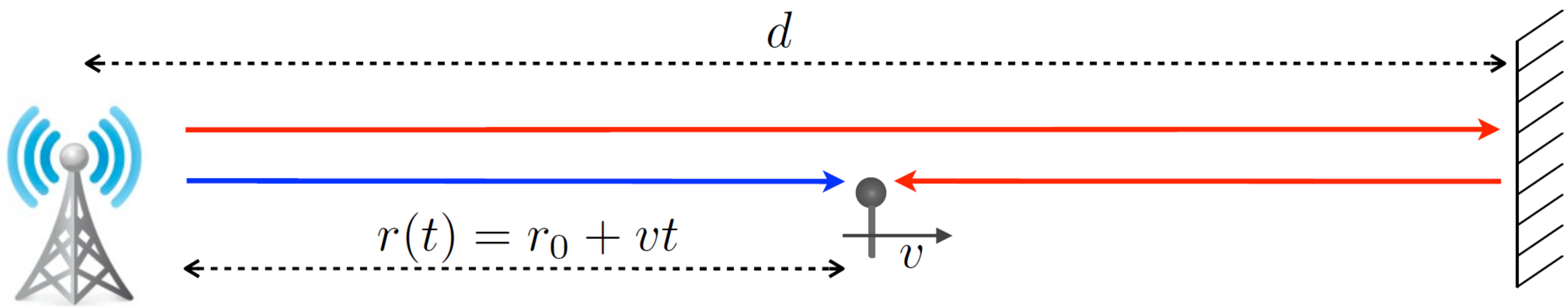
Delay Spread: difference between delays

$$= \begin{cases} 2n\pi, & \text{constructive interference} \\ (2n+1)\pi, & \text{destructive interference} \end{cases}$$

Delay Spread and Coherence Bandwidth

- Delay spread T_d : difference between delays of paths
- If frequency f change by $1/(2T_d)$, then the combined received sinusoid move from peak to valley
- Therefore, the frequency-variation scale is of the order of $\frac{1}{T_d}$
- Coherence bandwidth $W_c := \frac{1}{T_d}$

Physical Model: Simple Example 2



Transmitted Waveform (electric field): $\cos 2\pi f t$

Received Waveform (path 1): $\dfrac{\alpha}{r(t)} \cos 2\pi f \left(t - \dfrac{r(t)}{c} \right)$

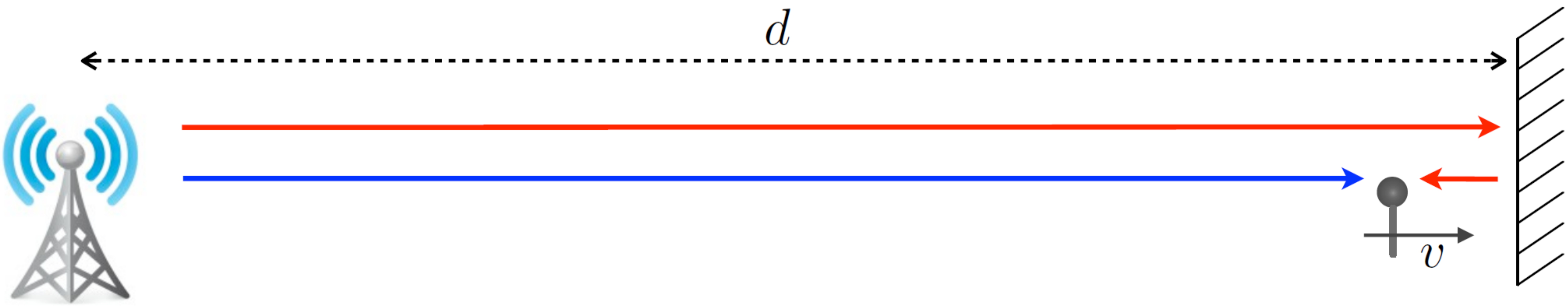
Received Waveform (path 2): $-\dfrac{\alpha}{2d - r(t)} \cos 2\pi f \left(t - \dfrac{2d - r(t)}{c} \right)$

$\Longrightarrow$ Received Waveform (aggregate):

$$\frac{\alpha}{r(t)} \cos 2\pi f \left(t - \frac{r(t)}{c} \right) - \frac{\alpha}{2d - r(t)} \cos 2\pi f \left(t - \frac{2d - r(t)}{c} \right)$$

$$= \boxed{\frac{\alpha}{r_0 + vt} \cos 2\pi f \left[\left(1 - \frac{v}{c} \right) t - \frac{r_0}{c} \right] - \frac{\alpha}{2d - r_0 - vt} \cos 2\pi f \left[\left(1 + \frac{v}{c} \right) t - \frac{2d - r_0}{c} \right]}$$

Physical Model: Simple Example 2



Approximation: distance to mobile Rx $\ll$ distance to Tx

$\Longrightarrow$ Received Waveform (aggregate):

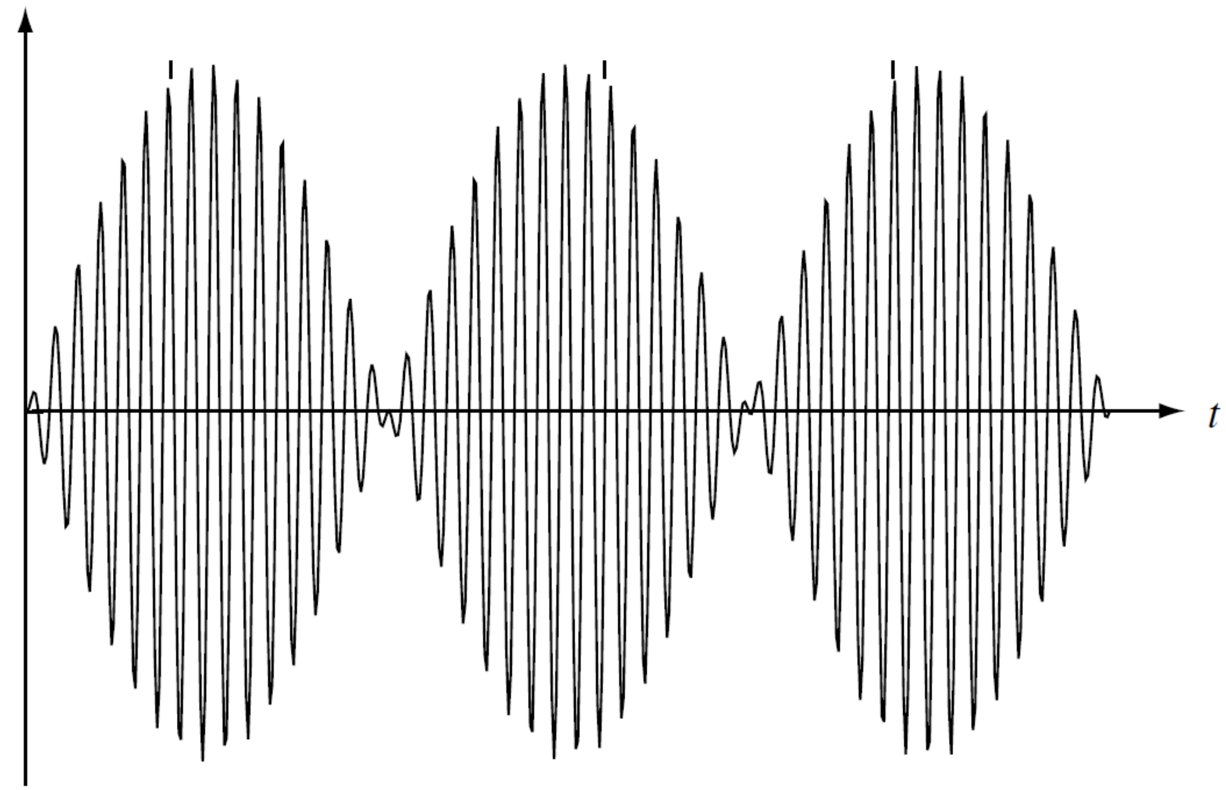
$$= \frac{\alpha}{r_0 + vt} \cos 2\pi f \left[\left(1 - \frac{v}{c}\right) t - \frac{r_0}{c} \right] - \frac{\alpha}{2d - r_0 - vt} \cos 2\pi f \left[\left(1 + \frac{v}{c}\right) t - \frac{2d - r_0}{c} \right]$$

$$\approx \boxed{\frac{2\alpha}{r_0 + vt} \sin 2\pi f \left(\frac{vt}{c} + \frac{r_0 - d}{c} \right)} \boxed{\sin 2\pi f \left(t - \frac{d}{c} \right)}$$

Time-varying amplitude

Time-invariant shift of the original input waveform

Physical Model: Simple Example 2



Time-varying envelope $$\boxed{\frac{2\alpha}{r_0 + vt}} \sin \boxed{2\pi f \left(\frac{vt}{c}\right.} + \left.\frac{r_0 - d}{c} \right)$$

Time-variation scale: r_0/v (seconds or minutes), much smaller than that of the second term

Doppler Spread $D_s = \dfrac{2fv}{c}$

Difference of the Doppler shifts of the two paths, cause this variation over time.

Time-variation scale: c/fv (ms)

Doppler Spread and Coherence Time

- Mobility causes time-varying delays (Doppler shift)
- Doppler spread D_s: difference between Doppler shifts of multiple signal paths
- If time t change by $1/(2D_s)$, then the combined received sinusoidal envelope move from peak to valley
- Therefore, the time-variation scale is of the order of $\frac{1}{D_s}$
- Coherence time $T_c := \frac{1}{D_s}$

What we learned from the examples

- Delay spread/coherence bandwidth and Doppler spread/ coherence time seem fundamental

- However, it is difficult to derive the explicit received waveform mathematically.
  - Out of scope – EM wave theory

- Instead, we construct useful input/output models, and take measurements to determine the parameters in the models

Physical Model:
Input/Output Relations

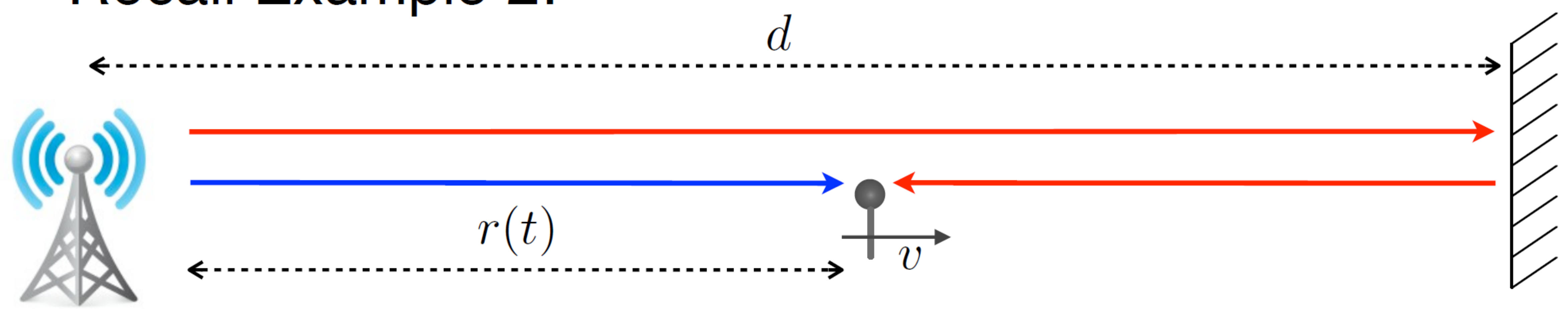
Physical Input/Output Model

- Wireless channels as linear time-varying systems:

$$y(t) = \sum_i a_i(t) x\left(t - \tau_i(t)\right)$$

$$a_i(t)\text{: gain of path } i \qquad \tau_i(t)\text{: delay of path } i$$

- Recall Example 2:



$$x(t) = \cos 2\pi f t$$

$$a_1(t) = \frac{|\alpha|}{r_0 + vt} \qquad \tau_1(t) = \frac{r_0 + vt}{c}$$

$$a_2(t) = \frac{|\alpha|}{2d - r_0 - vt} \qquad \tau_2(t) = \frac{2d - r_0 - vt}{c} - \frac{\pi}{2\pi f}$$

Physical Input/Output Model

- Wireless channels as linear time-varying systems:

$$y(t) = \sum_i a_i(t) x\left(t - \tau_i(t)\right)$$

$$a_i(t)\text{: gain of path } i \qquad \tau_i(t)\text{: delay of path } i$$

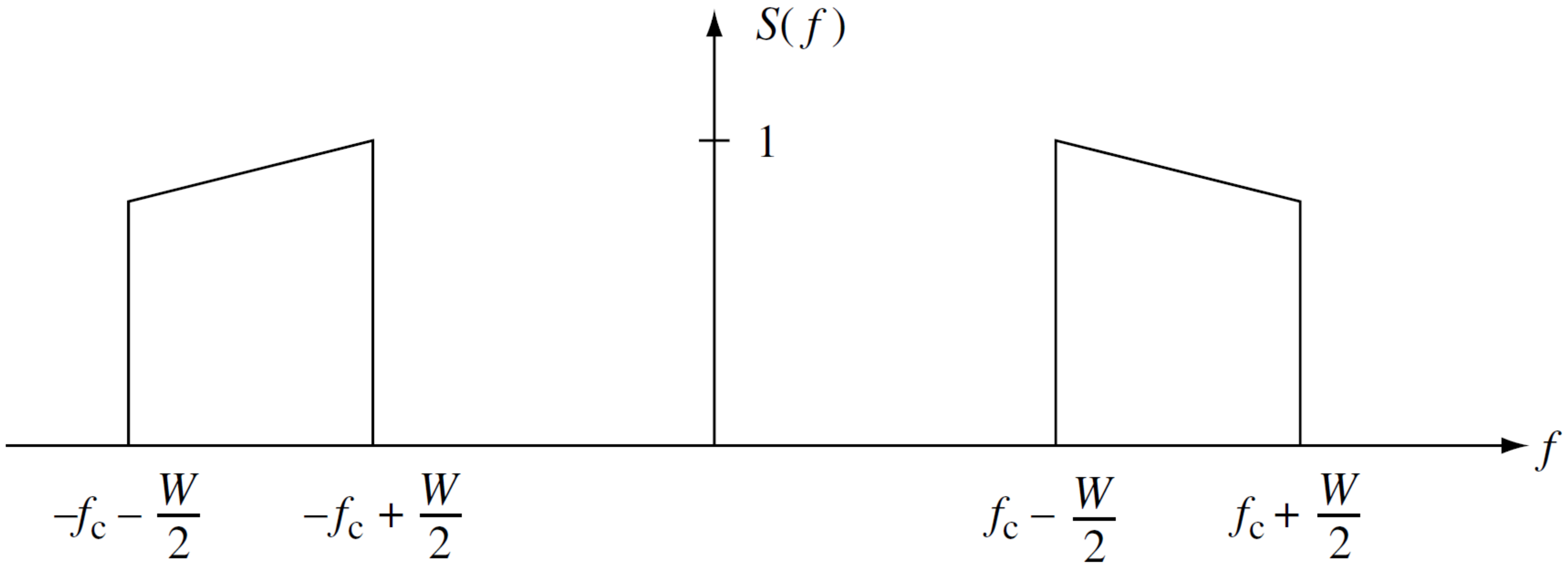
- Impulse response:

$$x(t) \longrightarrow \boxed{h\left(\tau, t\right)} \longrightarrow y(t) = \sum_i a_i(t) x\left(t - \tau_i(t)\right)$$

$$h(\tau, t) = \sum_i a_i(t) \delta\left(\tau - \tau_i(t)\right)$$

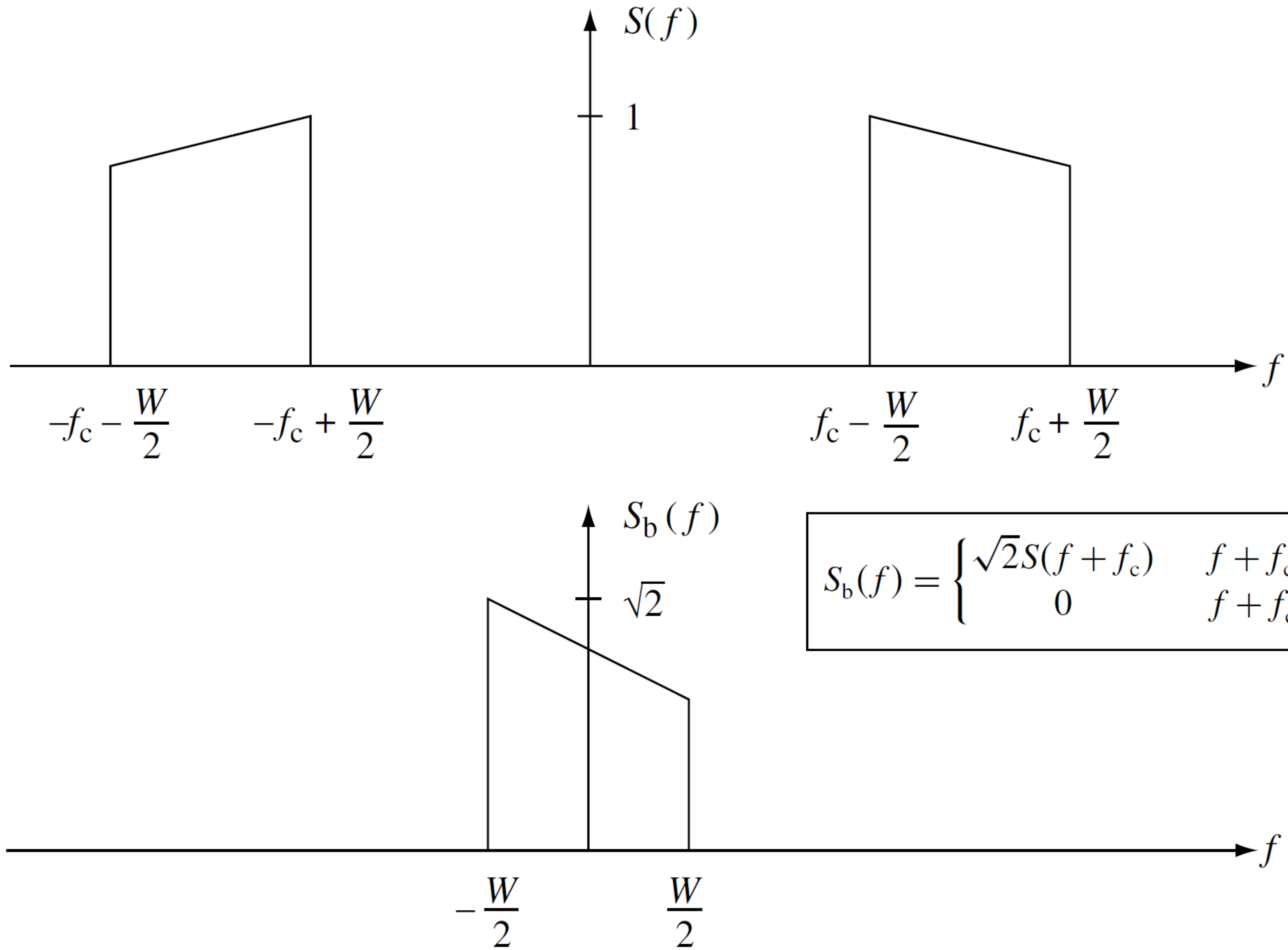
- Frequency response: $H(f;t) = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}$

Passband–Baseband Conversion



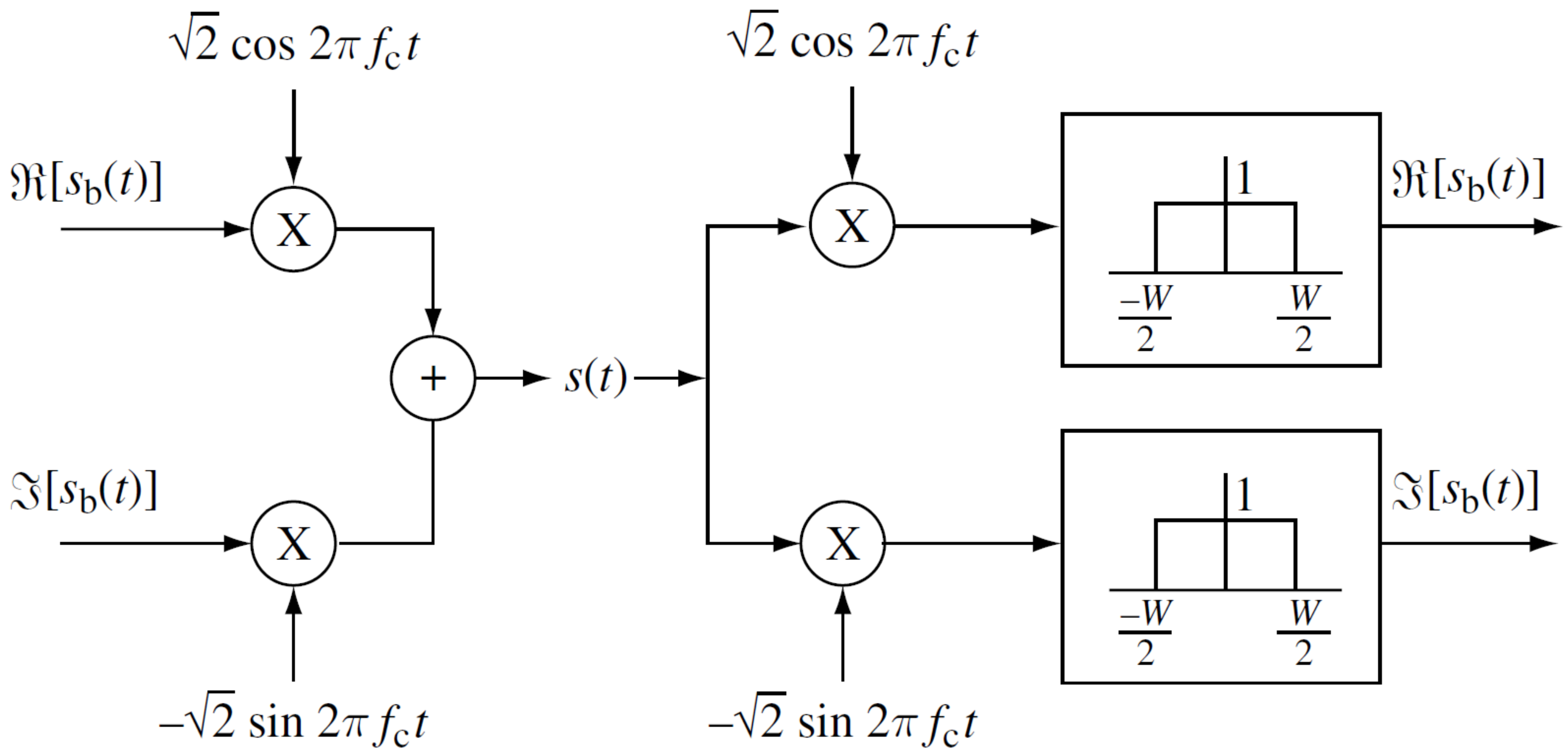
- Communications takes place in a passband
  - Carrier frequency f_c
  - Bandwidth $W < 2f_c$
  - Real signal $s(t)$

Passband–Baseband Conversion



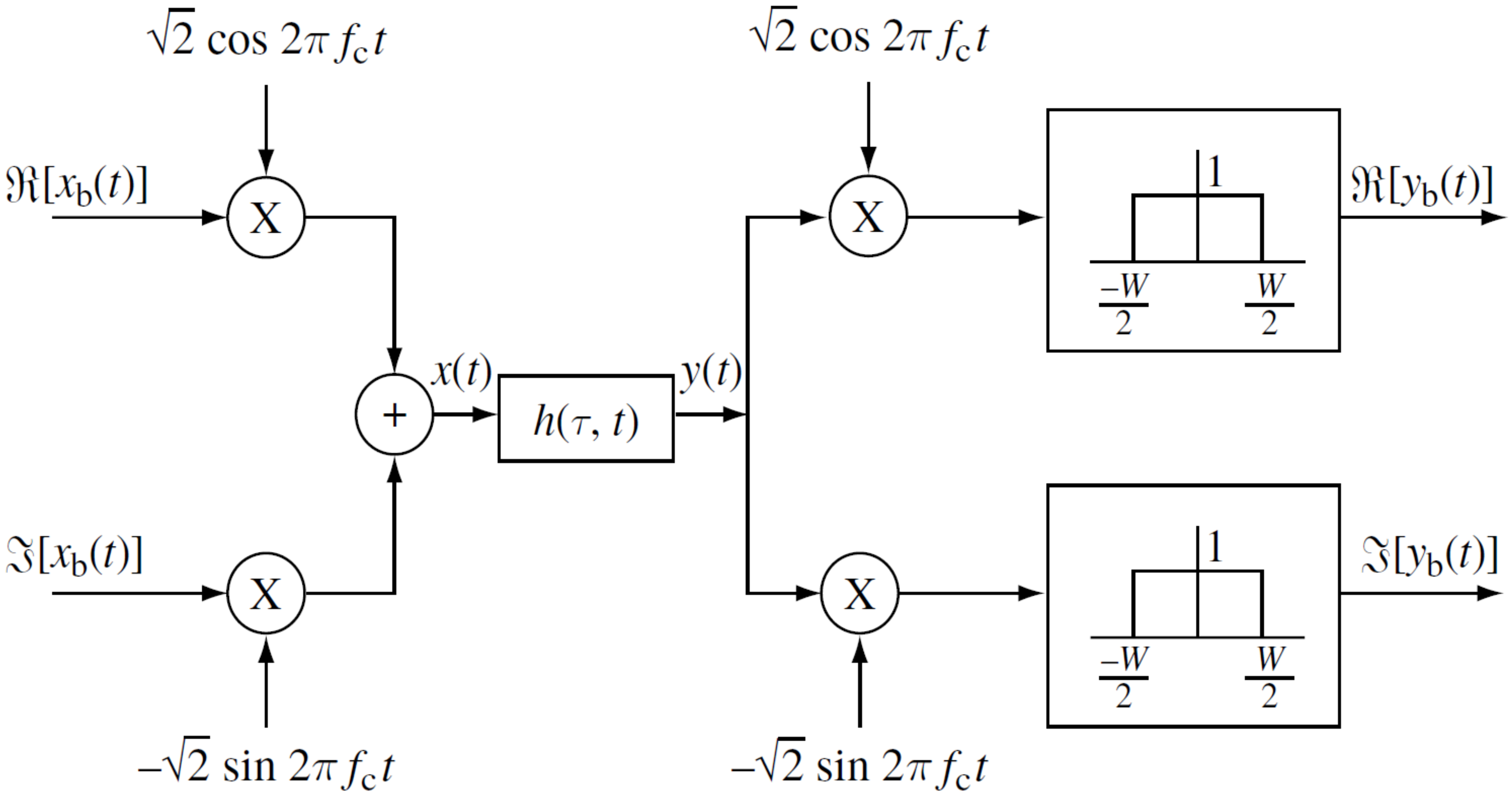
$$S_b(f) = \begin{cases} \sqrt{2}S(f+f_c) & f+f_c > 0, \\ 0 & f+f_c \le 0. \end{cases}$$

Passband–Baseband Conversion



$$\boxed{s(t) = \frac{1}{\sqrt{2}}\left[s_{\mathrm{b}}(t)\mathrm{e}^{\mathrm{j}2\pi f_c t} + s_{\mathrm{b}}^*(t)\mathrm{e}^{-\mathrm{j}2\pi f_c t}\right] = \sqrt{2}\Re\left[s_{\mathrm{b}}(t)\mathrm{e}^{\mathrm{j}2\pi f_c t}\right]}$$

Baseband System Architecture



$$y_b(t) = \sum_i a_i^b(t) x_b\left(t - \tau_i(t)\right),$$

where $a_i^b(t) := a_i(t) e^{-j2\pi f_c \tau_i(t)}$

Continuous - time Baseband Model

- Complex baseband equivalent channel:

$$x_b(t) \longrightarrow \boxed{h_b(\tau, t)} \longrightarrow y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t))$$

$$h_b(\tau, t) = \sum_i a_i^b(t) \delta(\tau - \tau_i(t)),$$

$$\text{where } a_i^b(t) := a_i(t) e^{-j2\pi f_c \tau_i(t)}$$

- Frequency response: shifted from passband to baseband

$$H_b(f; t) = H(f + f_c; t)$$

- Each path is associated with a delay and a complex gain

Modulation and Sampling

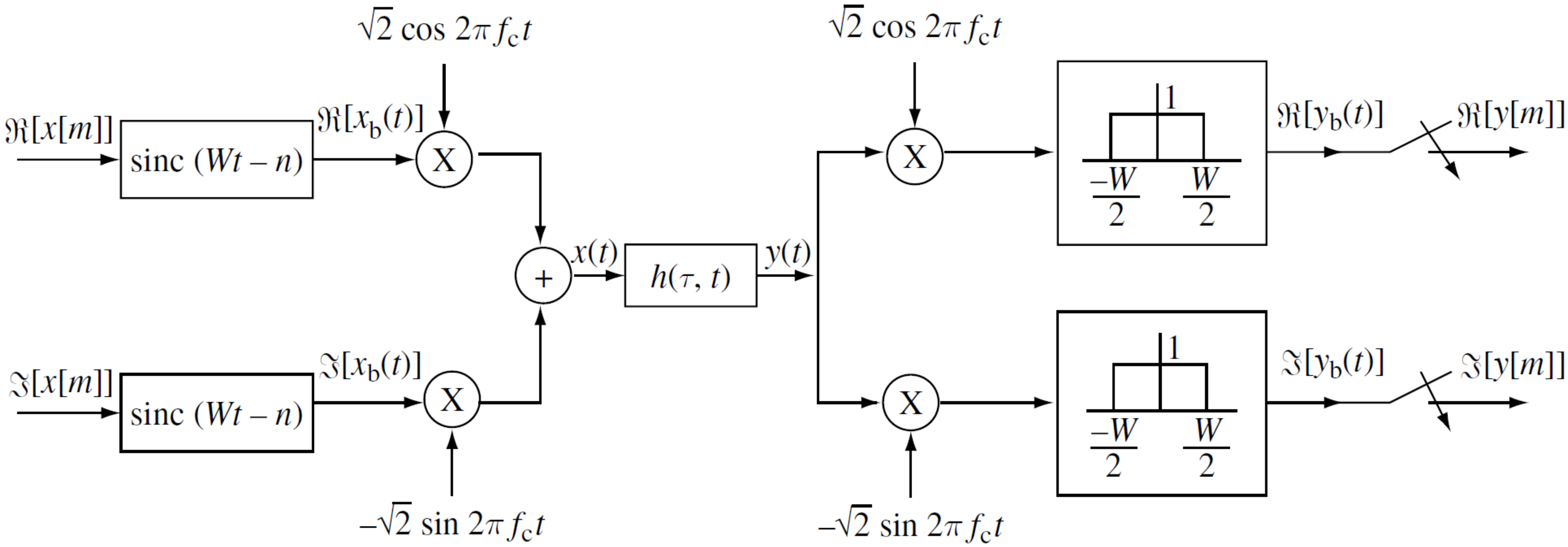
- Modern communication systems are digitized, (partially) thanks to sampling theorem

- Our baseband signal can be represented as follows:

$$x_b(t) = \sum_n x[n]\mathrm{sinc}(Wt - n),$$

$$x[n] := x_n(n/W), \quad \mathrm{sinc}(t) := \frac{\sin \pi t}{\pi t}$$

Modulation and Sampling



$$y[m] = \sum_l h_l[m]x[m-l],$$

$$\text{where } h_l[m] := \sum_i a_i^b(m/W)\text{sinc}\left[l - \tau_i(m/W)W\right]$$

Discrete - Time Baseband Model

- Discrete-time channel model

$$x[m] \longrightarrow \boxed{h_l[m]} \longrightarrow y[m] = \sum_l h_l[m]x[m-l]$$

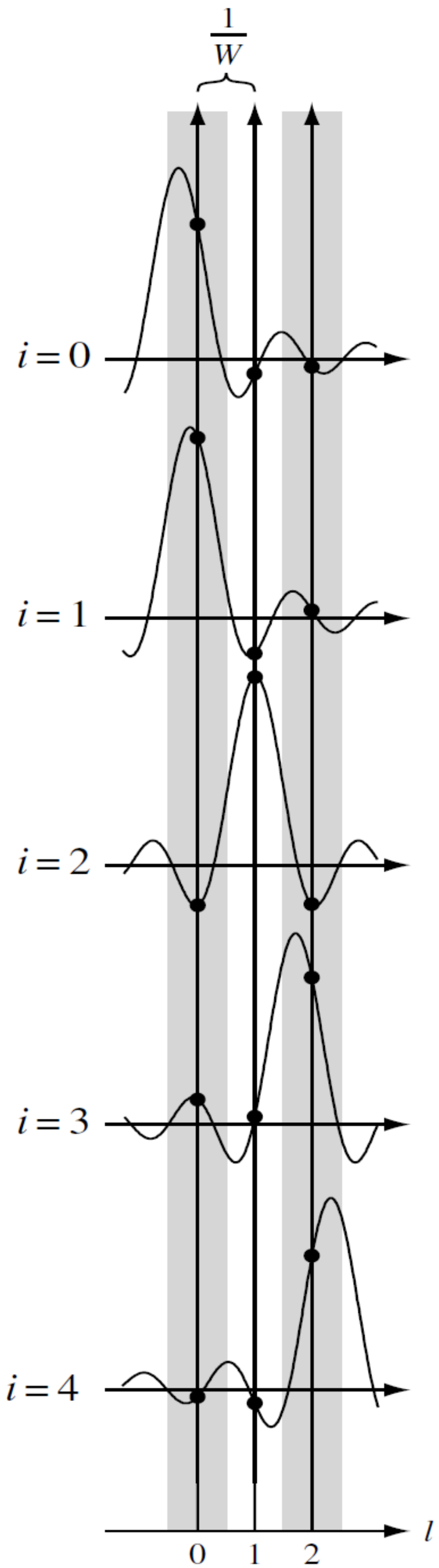
$$h_l[m] := \sum_i a_i^b(m/W)\text{sinc}\left[l - \tau_i(m/W)W\right]$$

- Note: the l-th tap h_l contains contributions mostly for the paths that have delays that lie inside the bin (roughly)

$$\left[\frac{l}{W} - \frac{1}{2W}, \frac{l}{W} + \frac{1}{2W}\right]$$

- System resolves the multipaths up to delays of $\frac{1}{W}$

Multipath Resolution



$$\boxed{h_l[m] := \sum_i a_i^b(m/W)\mathrm{sinc}\left[l - \tau_i(m/W)W\right]}$$

- $\mathrm{sinc}(t)$ vanish quickly outside of the interval $[-0.5, 0.5]$ (roughly)
- The peak of the i-th translated sinc lies at τ_i
- To contribute significantly to h_l, the delay must fall inside

$$\left[\frac{l}{W} - \frac{1}{2W}, \frac{l}{W} + \frac{1}{2W}\right]$$

Time and Frequency Coherence

Varying Channel Tap

- The discrete-time baseband channel model is the equivalent one in designing communication systems
- It only matters how the taps $h_l[m]$ vary over time m and carrier frequency f_c
- l-th tap of the discrete-time baseband channel model

$$
\begin{aligned}
h_l[m] &:= \sum_i a_i^b(m/W)\mathrm{sinc}\left[l - \tau_i(m/W)W\right] \\
&= \sum_i a_i\left(t_m\right) e^{-j2\pi f_c \tau_i(t_m)} \mathrm{sinc}\left[l - \tau_i(t_m)W\right] \qquad t_m := \frac{m}{W} \\
&\approx \sum_{i \in l\text{-th delay bin}} a_i\left(t_m\right) e^{-j\boxed{2\pi f_c \tau_i(t_m)}}
\end{aligned}
$$

Difference in phases (over the paths that contribute significantly to the tap), causes variation of the tap gain

Frequency Variation

$$h_l[m] \approx \sum_{i \in l\text{-th delay bin}} a_i(t_m)\, e^{-j2\pi f_c \tau_i(t_m)}$$

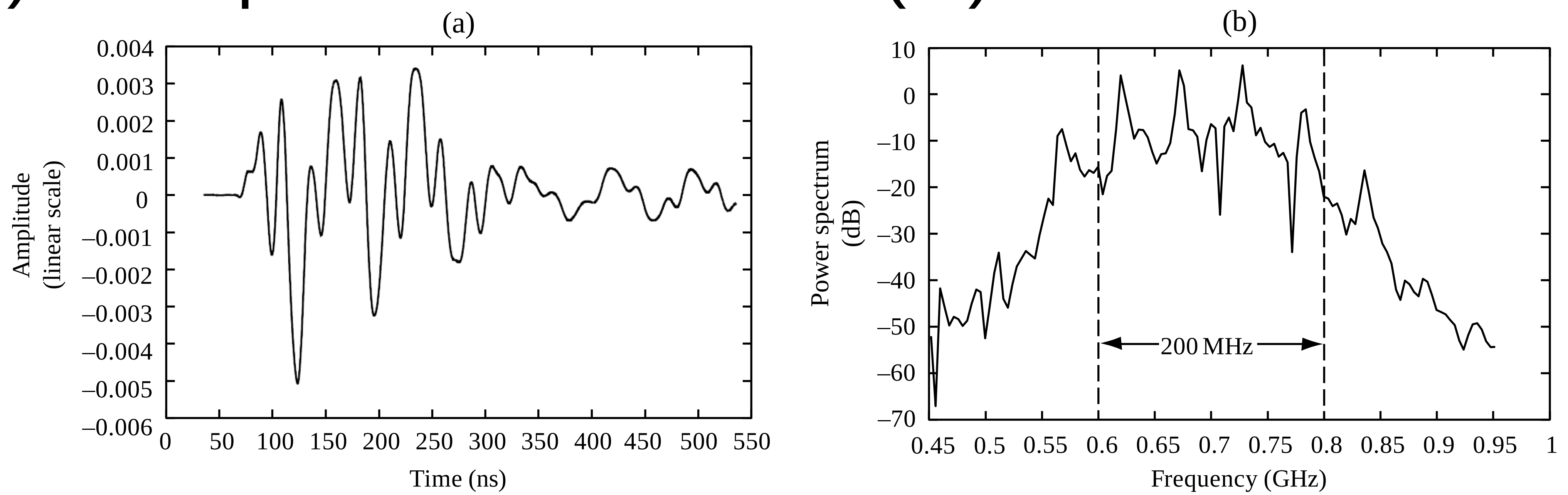
- Delay Spread $T_d := \max_{i,j} |\tau_i(t) - \tau_j(t)|$
- Coherence Bandwidth $W_c := \dfrac{1}{T_d}$
- For a system with bandwidth W

$$W_c \gg W \implies \text{single tap, flat fading}$$

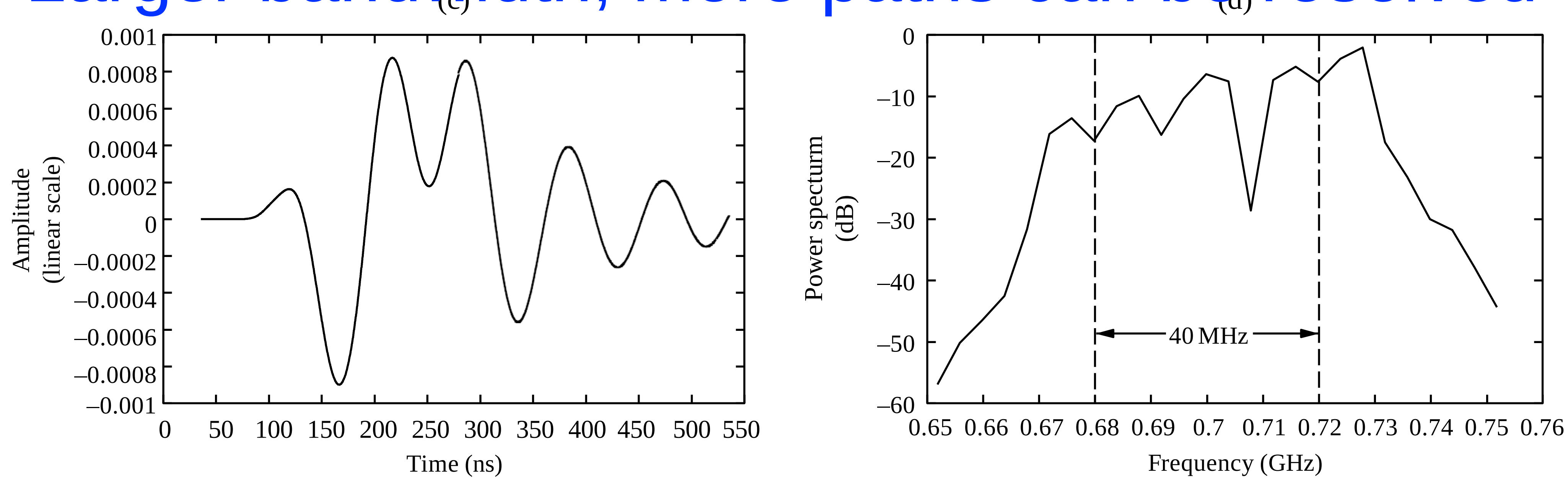
$$W_c < W \implies \text{multiple taps, frequency-selective fading}$$

Flat and Frequency - Selective Fading

- Effective channel depends on both physical environment (W_c) and operation bandwidth (W)



Larger bandwidth, more paths can be resolved



Time Variation

$$h_l[m] \approx \sum_{i \in l\text{-th delay bin}} a_i(t_m)\, e^{-j2\pi f_c \tau_i(t_m)}$$

- Doppler Spread $D_s := \max_{i,j} f_c |\tau_i{}'(t) - \tau_j{}'(t)|$
- Coherence Time $T_c := \frac{1}{D_s}$
- For a system with delay requirement (application dependent) T

$$T_c \gg T \implies \text{slow fading}$$
$$T_c < T \implies \text{fast fading}$$

Representitive Numbers

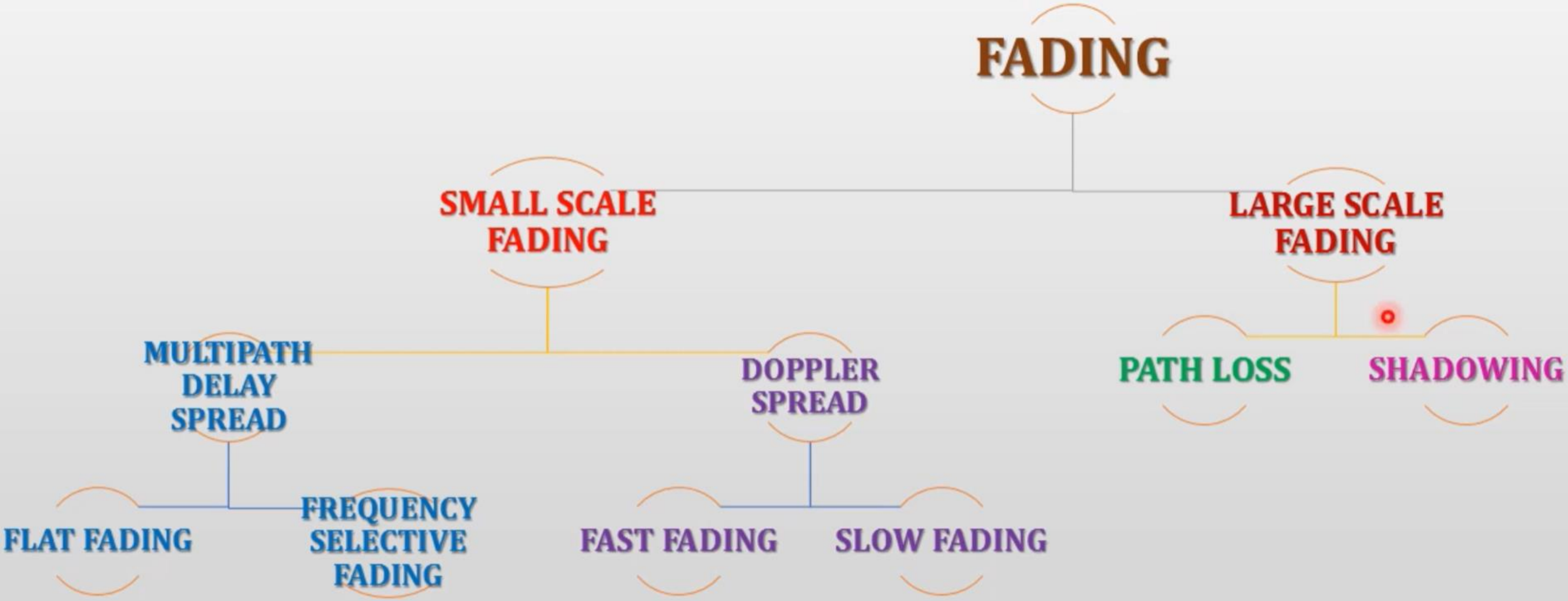
Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$D = f_c v/c$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 μs
Coherence bandwidth	$W_c = 1/(2T_d)$	500 kHz

Types of Channels

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Underspread	$T_d \ll T_c$

- Typical channels are underspread
- Coherence time T_c depends on carrier frequency and mobile speed, of the order of ms or more
- Delay spread T_d depends on distance to scatters and cell size, of the order of ns (indoor) to µs (outdoor)

Types of Fading



Types of Small Scale Fading

Based on Time Delay Spread

Leads to **TIME** Dispersion

FLAT FADING

- BW of signal < BW of Channel $B_s << B_c$
- Delay Spread < Symbol Period $\sigma_\tau \ll T_s$

FREQUENCY SELECTIVE FADING

- BW of signal > BW of Channel $B_s >> B_c$
- Delay Spread > Symbol Period $\sigma_\tau \gg T_s$

Based on Doppler Spread

Leads to **FREQUENCY** Dispersion

FAST FADING

- High Doppler Spread
- Coherence Time < Symbol Period $T_c << T_s$
- BW of Channel Varies FASTER than BW of signal $B_s << B_D$

SLOW FADING

- Low Doppler Spread
- Coherence Time > Symbol Period $T_c >> T_s$
- BW of Channel Varies SLOWER than the BW of Signal $B_s >> B_D$

GURUKULA

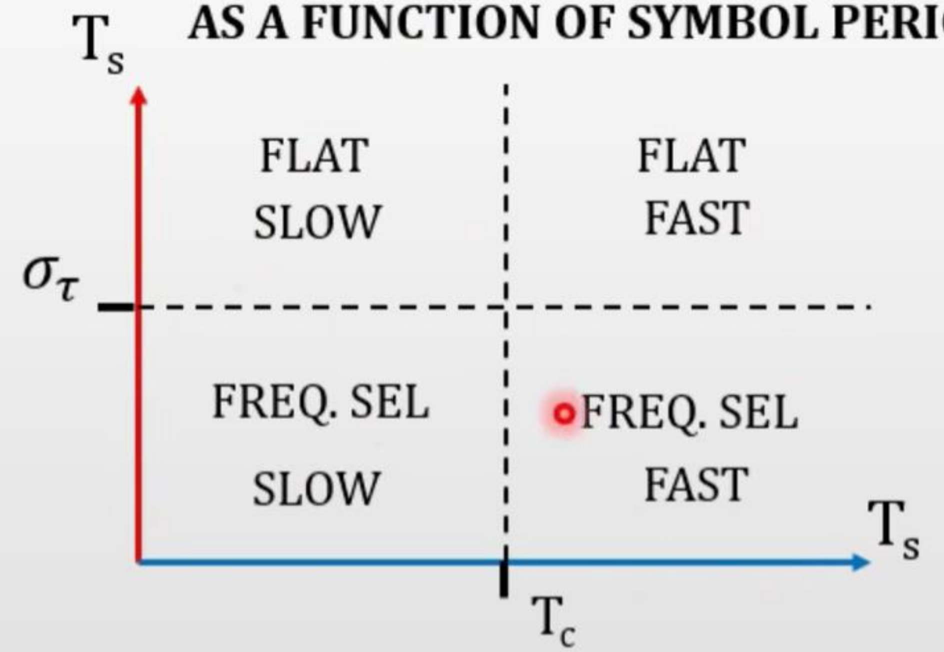
Based on Time Delay Spread

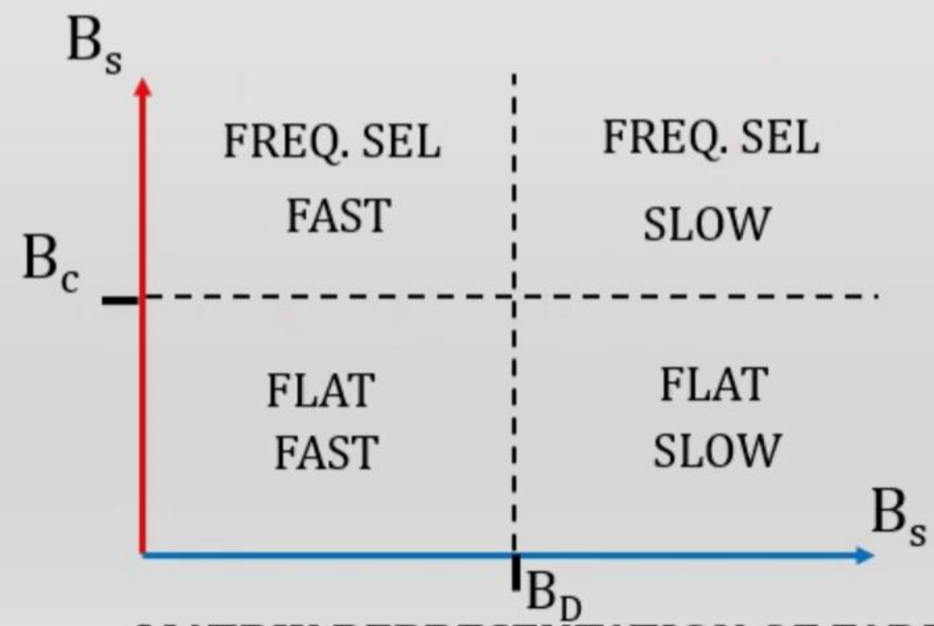
- **FLAT FADING**: $B_s << B_c$, $\sigma_\tau \ll T_s$
- **FREQUENCY SELECCTIVE FADING**: $B_s >> B_c$, $\sigma_\tau \gg T_s$

Based on Doppler Spread

- **FAST FADING**: $T_c << T_s$, $\boxed{B_s << B_D}$
- **SLOW FADING**: $T_c >> T_s$, $\boxed{B_s >> B_D}$

MATRIX REPRESENTATION OF FADING AS A FUNCTION OF SYMBOL PERIOD



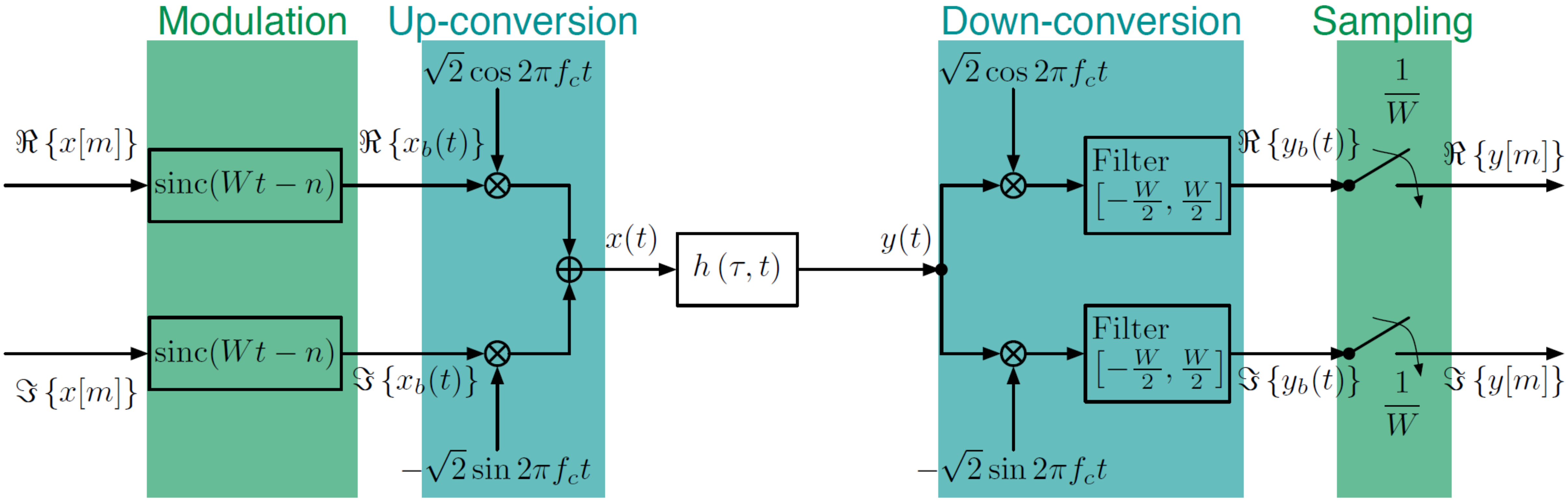


MATRIX REPRESENTATION OF FADING AS A FUNCTION OF SIGNAL BW

Stochastic Models

Recap: Deterministic Modeling

- Continuous-time Passband ⟶ Discrete-time Baseband:



- Continuous-time Passband (*Real*)

$$y(t)=\sum_i a_i(t)x\left(t-\tau_i(t)\right)\quad a_i(t)\text{: gain of path } i\quad \tau_i(t)\text{: delay of path } i$$

- Continuous-time Baseband (*Complex*)

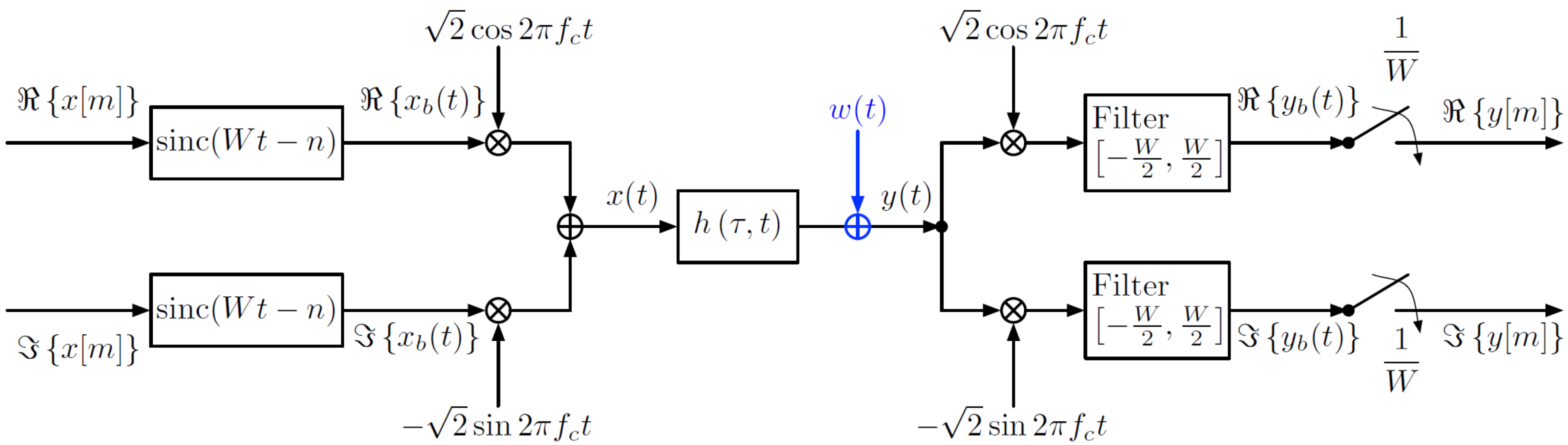
$$y_b(t)=\sum_i a_i^b(t)x_b\left(t-\tau_i(t)\right),\quad a_i^b(t):=a_i(t)e^{-j2\pi f_c\tau_i(t)}$$

- Discrete-time Baseband (*Complex*)

$$y[m]=\sum_l h_l[m]x[m-l],\quad h_l[m]:=\sum_i a_i^b(m/W)\mathrm{sinc}\left[l-\tau_i(m/W)W\right]$$

Noise

- Thermal noise at the receiver:



- Continuous-time Passband (*Real*)

$$y(t) = \sum_i a_i(t)x\left(t-\tau_i(t)\right) + w(t) \quad a_i(t)\text{: gain of path } i \quad \tau_i(t)\text{: delay of path } i$$

- Continuous-time Baseband (*Complex*)

$$y_b(t) = \sum_i a_i^b(t)x_b\left(t-\tau_i(t)\right) + w_b(t), \quad a_i^b(t) := a_i(t)e^{-j2\pi f_c\tau_i(t)}$$

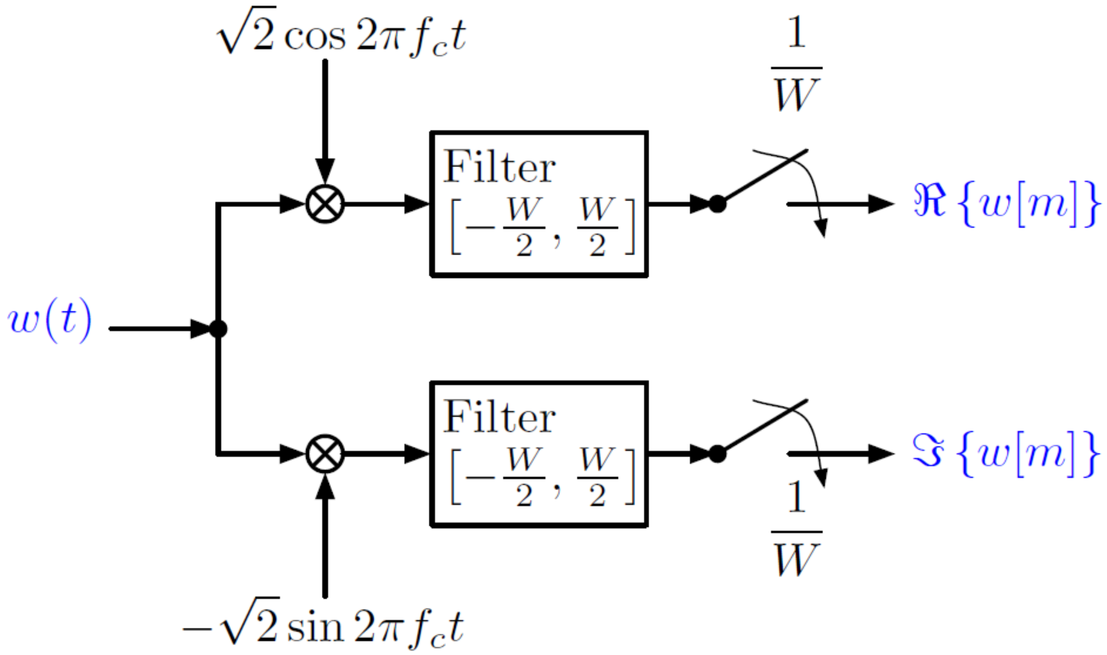
- Discrete-time Baseband (*Complex*)

$$y[m] = \sum_l h_l[m]x[m-l] + w[m], \quad h_l[m] := \sum_i a_i^b(m/W)\text{sinc}\left[l-\tau_i(m/W)W\right]$$

Additive White Noise Model

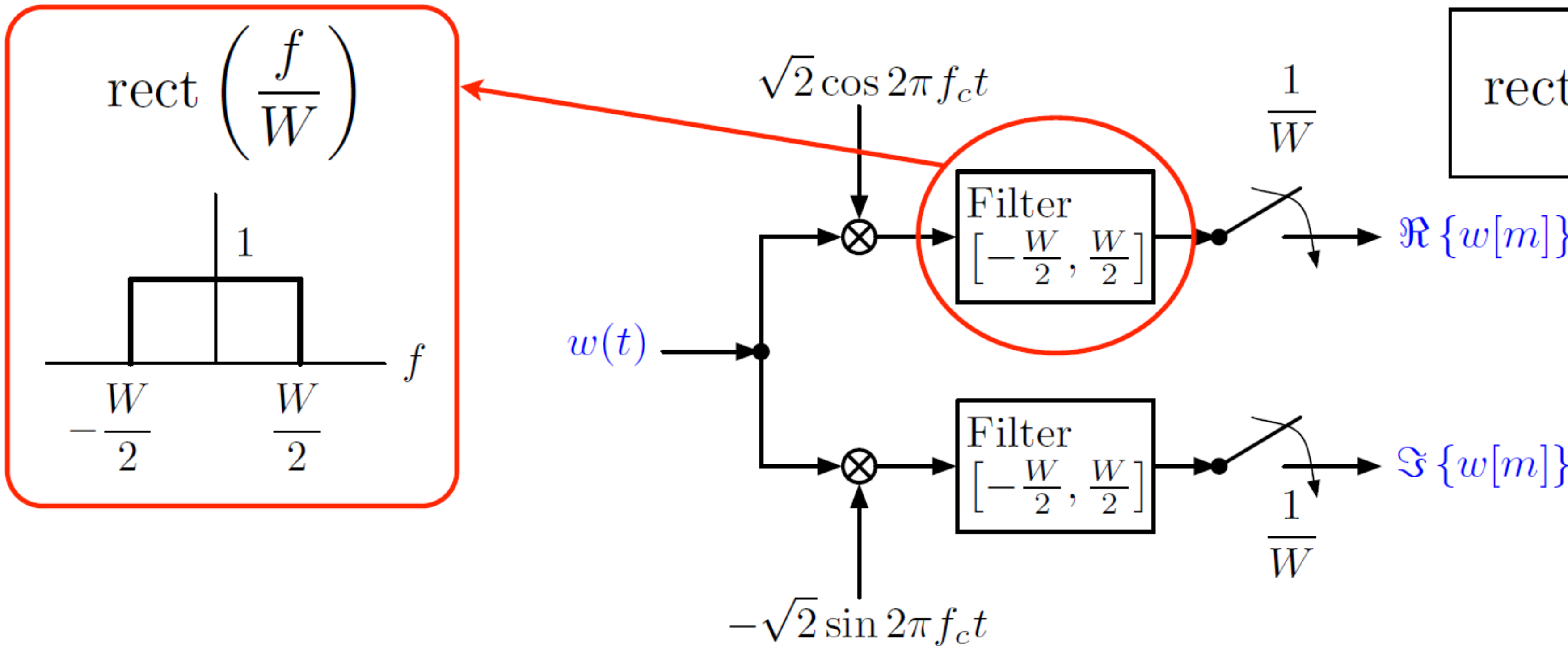
- Additive White Gaussian Noise (AWGN)
  - Standard modeling for thermal noise
  - $\{w(t)\}$: zero-mean (real) white Gaussian process with spectral density $N_0/2$
  - In other words, $\mathbb{E}\left[w(0)w(t)\right] = \frac{N_0}{2}\delta(t)$

- Discrete-time baseband equivalent noise:



- System is linear $\longrightarrow$ can separate the noise out
- Rectangle filter in frequency $\Leftrightarrow$ sinc in time

Equivalent Discrete-Time Baseband Noise



$$\boxed{\mathrm{rect}\left(\frac{f}{W}\right) \xrightarrow{\mathscr{F}^{-1}} W\,\mathrm{sinc}\,(Wt)}$$

$$\Re\{w[m]\} = \int_{-\infty}^{\infty} w(t) \underbrace{\left[\sqrt{2}W\cos(2\pi f_c t)\,\mathrm{sinc}\,(Wt-m)\right]}_{\psi_{m,1}(t)} dt$$

$$= \boxed{\langle w(t), \psi_{m,1}(t)\rangle}$$

$$\Im\{w[m]\} = \int_{-\infty}^{\infty} w(t) \underbrace{\left[-\sqrt{2}W\sin(2\pi f_c t)\,\mathrm{sinc}\,(Wt-m)\right]}_{\psi_{m,2}(t)} dt$$

$$= \boxed{\langle w(t), \psi_{m,2}(t)\rangle}$$

Equivalent Discrete-Time Baseband Noise

$$\Re\{w[m]\} = \int_{-\infty}^{\infty} w(t) \underbrace{\left[\sqrt{2}W\cos(2\pi f_c t)\,\mathrm{sinc}(Wt-m)\right]}_{\psi_{m,1}(t)} dt$$

$$= \boxed{\langle w(t), \psi_{m,1}(t)\rangle}$$

$$\Im\{w[m]\} = \int_{-\infty}^{\infty} w(t) \underbrace{\left[-\sqrt{2}W\sin(2\pi f_c t)\,\mathrm{sinc}(Wt-m)\right]}_{\psi_{m,2}(t)} dt$$

$$= \boxed{\langle w(t), \psi_{m,2}(t)\rangle}$$

Fact.

$\{\psi_{m,1}(t), \psi_{m,2}(t) \mid m \in \mathbb{Z}\}$ forms an orthogonal set of waveforms.

- The real and the imaginary parts of $w[m]$ are
  - Both Gaussian with zero-mean and variance $WN_0/2$
  - Independent and identically distributed (i.i.d.) over time (m)
  - "White" discrete-time processes

Circular Symmetric Complex Gaussian

$$w[m] = \Re\{w[m]\} + j\Im\{w[m]\}$$

$$\Re\{w[m]\} \sim \mathcal{N}\left(0, \frac{WN_0}{2}\right), \quad \Im\{w[m]\} \sim \mathcal{N}\left(0, \frac{WN_0}{2}\right)$$

$$(\Re\{w[m]\}, \Im\{w[m]\}): \text{ independent}$$

$$\iff w[m] \sim \mathcal{CN}(0, WN_0): \text{ circular symmetric complex Gaussian}$$

Fact.

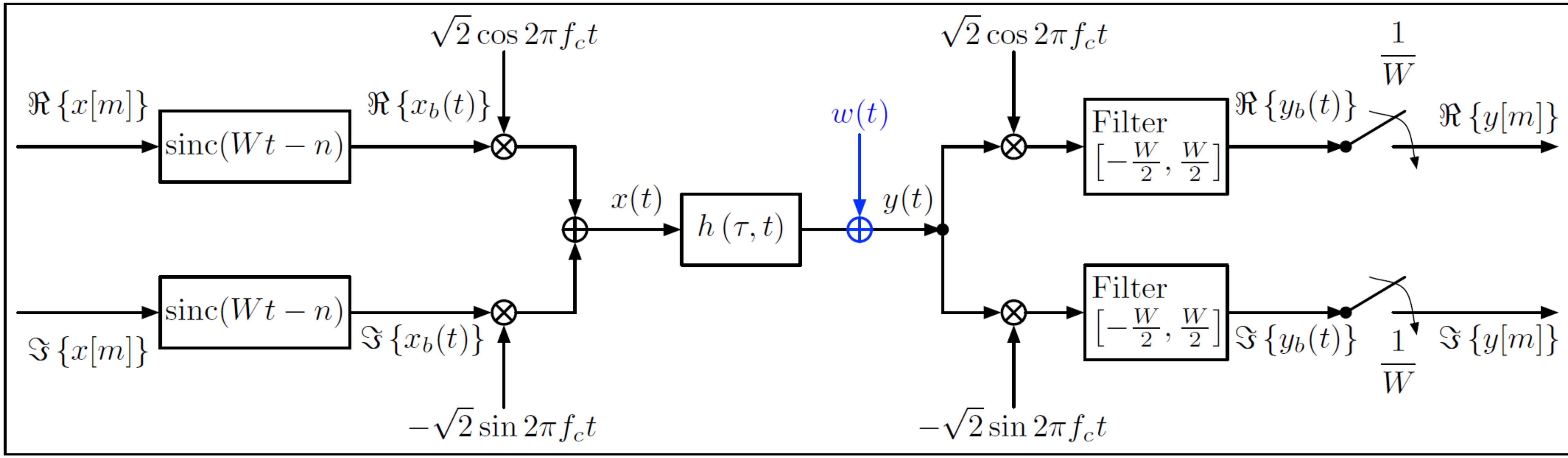
X is circular symmetric if $e^{i\theta}X \overset{\mathrm{d}}{=} X$ for all θ the same distribution

Fact.

$\{w[m] \mid m \in \mathbb{Z}\}$ is a zero-mean white circular symmetric complex Gaussian process with auto-correlation function

$$R[m] := \mathbb{E}\left[w[n+m]w[n]^*\right] = WN_0\delta[m]$$

Discrete Baseband Model with Noise



$$\equiv$$

$$x[m] \longrightarrow \boxed{h_l[m]} \longrightarrow y[m] = \sum_l h_l[m]x[m-l] + w[m]$$

$$w[m] \sim \mathcal{CN}(0, WN_0),$$

where $\frac{N_0}{2}$ is the spectral density of white Gaussian process $\{w(t)\}$

Fading

$$y[m] = \sum_l h_l[m]x[m-l] + w[m],$$

$$\text{where } h_l[m] := \sum_i a_i^b(m/W)\text{sinc}\left[l - \tau_i(m/W)W\right]$$

$$w[m] \sim \mathcal{CN}\left(0, WN_0\right)$$

- Additive noise $w[m]$
  - Essentially completely random, no correlation over time
  - Largely depends on nature
  - Can be dealt with using digital (wired) communication techniques
- Filter taps $h_l[m]$
  - Varying over time and frequency
  - Largely depends on nature
  - Why not use stochastic models for taps as well?

Rayleigh Fading

- Many small scattered paths for each tap:
  - Phase for each path is uniformly distributed over $[0, 2\pi]$

$$h_l[m] \approx \sum_i a_i\left(t_m\right) e^{\boxed{-j2\pi f_c \tau_i\left(t_m\right)}}$$

  - For each path it is a circular symmetric random variable

- Each tap: sum of many small independent *circular symmetric* random variables
  - By Central Limit Theorem (CLT), we can model

$$h_l[m] \sim \mathcal{CN}\left(0, \sigma_l^2\right)$$

  - Zero-mean because of rich scattering

Rician Fading

- If there is a strong line-of-sight path, then model it as

$$h_l[m] = \underbrace{\boxed{\sqrt{\frac{\kappa}{\kappa+1}}\sigma_l e^{j\theta}}}_{\text{Line-of-sight}} + \underbrace{\boxed{\sqrt{\frac{1}{\kappa+1}}\mathcal{CN}\left(0, \sigma_l^2\right)}}_{\text{Scattered Multipath}}$$

- K-Factor κ :
  - The larger it is, the more deterministic the channel will be.