Information Theory and Network Coding

Final Examination

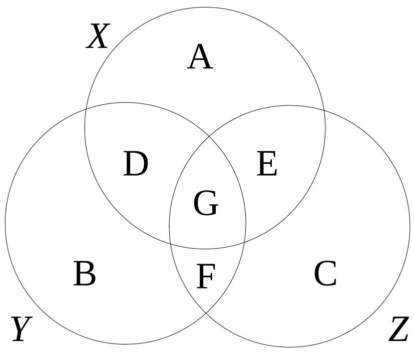
Due date: **2pm, Sun., 17 April, 2021 (Beijing Time)**

Notice: Do not forget to write your name and SID on your answer sheet.
Merge your answer sheets into a SINGLE FILE, with your Student ID as the file name;
Email your single answer file to me qfsun@ustb.edu.cn before the deadline.

Name: ____________ Student ID: _____________

I. Short Answer Questions

1. (10pts) Briefly describe the different goals of source coding, channel coding, and network coding.
2. (10pts) Briefly describe what the Source Coding Theorem and the Channel Coding Theorem respectively state.
3. (10pts) In the Venn diagram representation below, H(X, Y, Z) can be divided into 7 areas, labeled with A, B, C, D, E, F, G respectively. Write down the respective information measure corresponding to *area A, area D, area A+B+D*, and *area E+F+G*. Which one among the 7 areas (A to G) is possible to take a negative value?
   // e.g. H(X | Z) corresponds to area A+D.)



4. (10pts) There are 4 binary codes C_1 = {000, 11, 001, 010, 101}, C_2 = {0, 10, 0010, 0011}, C_3 = {0, 10, 11, 0100}, C_4 = {01, 00, 011, 001}. Which ones do not satisfy Kraft's inequality? Which are not uniquely decodable? Which are prefix codes?

II. (15pts) The *joint distribution* of two binary random variables X, Y is given by the table

$X \backslash Y$	a	b	C
0	1/4	0	1/4
1	0	1/4	1/4

Calculate H(X, Y), H(X), H(Y|X), H(Y | X = 0), I(X; Y).

III. (15pts) Let X = $\{x_1, x_2, x_3\}$ be a ternary random variable with probability distribution {0.6, 0.3, 0.1}.

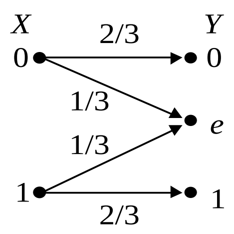
1. Construct a binary Huffman code for X. Calculate the code's expected length $\overline{L}_1$ and code efficiency $\eta_1 = H(X)/\overline{L}_1$?
2. Construct a binary Huffman code for two i.i.d. copies X^2 of X, calculate the code's expected length $\overline{L}_2$ and code efficiency $\eta_2 = H(X^2)/\overline{L}_2 = 2H(X)/\overline{L}_2$?

IV. (15pts) Consider a stationary (1st order) Markov source $X_1, X_2, \ldots X_j, \ldots$. Every random variable X_j in the stationary source has an identical probability distribution (1/4, 1/4, 1/4, 1/4). The conditional probability distribution of X_{j+1} given X_j is known via the transition matrix

$$P_{X_{j+1}|X_j} = \begin{bmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 & 0 \\ 1/4 & 0 & 1/4 & 1/2 \end{bmatrix}$$

1. Calculate $H(X_j)$, the entropy rate H_X of this source, and $H(X_j, X_{j+1})/2$. Rank $H(X_j)$, $H(X_j, X_{j+1})/2$, and H_X in a descending order.
2. What value is $I(X_{j-1}; X_{j+1} \mid X_j)$ equal to?

V. (15pts) The figure on the right depicts a channel with input random variable X $\in$ {0, 1} and output random variable Y $\in$ {0, *e*, 1}.



1. Write down corresponding transition matrix of this channel.
2. When p(X = 1) = 0, p(X = 1) = 1, calculate H(Y | X), H(Y), and I(X; Y).
3. When p(X = 0) = 1/2, p(X = 1) = 1/2, calculate H(Y | X), H(Y), and I(X; Y);
4. What is the capacity of this channel? Explain it.