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Numerical Analysis
Assignment 9

亚历克上 M202161029

Ex: 6.3 Q 1a, 2

1a) $\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$ Determing either its ~~single~~ nonsingular and computing the inverse:-

⇒ The matrix is invertible (nonsingular) if matrix is an $n \times n$ matrix such that $AB = BA = I_n$

⇒ I_n is the identity matrix and B is the inverse of A. $B = A^{-1}$

⇒ The matrix is invertible (nonsingular) if $\det(A) \neq 0$

* We have 2 main methods to find deteminant $\det(A)$:-

(1) The diagonal method

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix} - \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix}$$

$$\Rightarrow (aei + bfg + cgh) - (ceg + afh + bdi)$$

2) Method of cofactors

Using method (1) ⇒ $\det(A)$ ⇒

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix} \Rightarrow \begin{matrix} 4 & 2 & 6 & 4 & 2 \\ 3 & 0 & 7 & 3 & 0 \\ -2 & -1 & -3 & -2 & -1 \end{matrix} - \begin{matrix} 4 & 2 & 6 & 4 & 2 \\ 3 & 0 & 7 & 3 & 0 \\ -2 & -1 & -3 & -2 & -1 \end{matrix}$$

$$\Rightarrow ((4)(0)(-3) + (2)(7)(-2) + (6)(3)(-1)) - ((6)(0)(-2) + (4)(7)(-1) + (2)(3)(-3))$$

$$\Rightarrow (0 + (-28) - 18) - (0 - 28 - 18)$$

~~Det(A) = -46~~ : $(-46) - (-46)$ $\det(A) = 0$

∴ The matrix is singular.

2) 4 : 3x3 linear systems having the same coefficient matrix :-

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 2 \\ x_1 + x_2 - x_3 &= -1 \\ -x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 6 \\ x_1 + x_2 - x_3 &= 4 \\ -x_1 + x_2 - 3x_3 &= 5 \end{aligned}$$

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + x_2 - x_3 &= 1 \\ -x_1 + x_2 - 3x_3 &= -3 \end{aligned}$$

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= -1 \\ x_1 + x_2 - x_3 &= 0 \\ -x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

$$\therefore \left[\begin{array}{ccc:c} 2 & -3 & 1 & 2 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & -3 & 0 \end{array}\right] \quad \left[\begin{array}{ccc:c} 2 & -3 & 1 & 6 \\ 1 & 1 & -1 & 4 \\ -1 & 1 & -3 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 2 & -3 & 1 & 0 \\ 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \end{array}\right] \quad \left[\begin{array}{ccc:c} 2 & -3 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & -3 & 0 \end{array}\right]$$

Row Operations

$$\left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 1 & 1 & -1 & -1 & 4 & 1 & 0 \\ -1 & 1 & -3 & 0 & 5 & -3 & 0 \end{array}\right]$$

$$Ax = b$$

$$\therefore \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 0 & -1 \\ -1 & 4 & 1 & 0 \\ 0 & 5 & -3 & 0 \end{bmatrix}$$

$Ax = b$

$A^{-1}Ax = A^{-1}B$

$I_n x = A^{-1}B$

$x = A^{-1}B$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \frac{1}{(-14)} \begin{bmatrix} -2 & -5 & 2 \\ 8 & -5 & -1 \\ 2 & -1 & 5 \end{bmatrix} \times$$

$$\begin{bmatrix} 2 & 6 & 0 & -1 \\ -1 & 4 & 1 & 0 \\ 0 & 5 & -3 & 0 \end{bmatrix}$$

$\Rightarrow \frac{1}{14}$ [~~$(-2 \times 2) + (-5 \times -1) + (2 \times 0)$~~

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↓

$A^{-1} \Rightarrow |\vec{A}| \Rightarrow a_{11}\alpha_{11} + a_{12}\alpha_{12} + a_{13}\alpha_{13}$

$A^{-1} = \frac{1}{|A|} \, ad_j A$

$\alpha_{11} = (2)\begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix} = -2$

$\alpha_{12} = (-3)\begin{vmatrix} 1 & -1 \\ -1 & -3 \end{vmatrix} = -5$

$\alpha_{13} = (1)\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$

$\alpha_{21} = (1)\begin{vmatrix} -3 & 1 \\ 1 & -3 \end{vmatrix} = 8$

$\alpha_{22} = (1)\begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix} = -5$

$\alpha_{23} = (-1)\begin{vmatrix} 2 & -3 \\ -1 & +1 \end{vmatrix} = -1$

$\alpha_{31} = (1)\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$

$\alpha_{32} = (-1)\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} = -1$

$\alpha_{33} = (-3)\begin{vmatrix} 2 & -3 \\ +1 & 1 \end{vmatrix} = 5$

$$A^{-1} \Rightarrow \frac{1}{(-14)} \begin{bmatrix} -2 & -5 & 2 \\ 8 & -5 & -1 \\ 2 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \frac{1}{(-14)}\begin{bmatrix}-2&-5&2\\8&-5&-1\\2&-1&5\end{bmatrix}\begin{bmatrix}2&6&0&-1\\-1&4&1&0\\0&5&-3&0\end{bmatrix}$$

$3\times 3 \cdot 3\times 4$

multiplication applicable

3×4

the output

$$\Rightarrow \frac{1}{-14}\begin{bmatrix}(-2\times 2)+(-5\times -1)+(2\times 0) & (-2\times 6+-5\times 4+2\times 5) & (-2\times 0+-5\times 1+2\times -3) & (-2\times -1+-5\times 0\ 0)\\ (8\times 2)+(-5\times -1)+(-1\times 0) & (8\times 6+(-5\times 4)+(-1\times 5)) & \cdots & \cdots\\ \vdots & \vdots & \vdots & \vdots\\ (2\times 2)+(-1\times -1)+(5\times 0) & \vdots & \vdots & (2\times -1)+(-1\times 0)+5\times 0)\end{bmatrix}$$

$$\Rightarrow \frac{1}{-14}\begin{bmatrix}1&-22&-11&2\\21&23&-2&-8\\5&33&-16&-2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-2&-5&2\\8&-5&-1\\2&-1&5\end{bmatrix}$$

$\therefore$ The coeffient matrix $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-2&-5&2\\8&-5&-1\\2&-1&5\end{bmatrix}$