Problem Set 3

Submission date: Monday, 11 Apr. 2022

1. Let X = $\{x_1, x_2, x_3\}$ be a ternary random variable with probability distribution {0.5, 0.4, 0.1}.
   a) Construct a binary Huffman code for X. Calculate the code's expected length $\bar{L}_1$ and code efficiency $\eta_1 = H(X)/\bar{L}_1$.
   b) Construct a binary Huffman code for two i.i.d. copies X^2 of X, calculate the code's expected length $\bar{L}_2$ and code efficiency $\eta_2 = H(X^2)/\bar{L}_2 = 2H(X)/\bar{L}_2$.
   c) Make a comparison between η_1 and η_2. If a binary Huffman code is used for n i.i.d. copies X^n, what is the asymptotic value of $\eta_n = H(X^n)/\bar{L}_n = nH(X)/\bar{L}_n$ when $n\to\infty$?
2. Consider 4 different codes:

   {000, 10, 00, 11};

   {100, 101, 0, 11};

   {01, 100, 011, 00, 111, 1010, 1011, 1101};

   {01, 111, 011, 00, 010, 110}

   a) Which ones do not satisfy Kraft inequality?
   b) Is each of these codes qualified as a prefix code? If not, please explain.

3. For the following three codes, which ones cannot be constructed by Huffman coding?
   a) {0, 10, 11};
   b) {00, 01, 10, 110};
   c) {01, 10};
4. Consider a binary erasure channel (BEC) with X = {0, 1}, Y = {0, e, 1}, and $p_{Y|X} = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$. Assume $P(X = 0) = p_0$, $P(X = 1) = 1 - p_0$.
   a) Calculate H(Y|X);
   b) Find the distribution of Y;
   c) What is the value of p_0 that maximizes H(Y)?
   d) What is the capacity C of this channel?