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Numerical Analysis
Assignment 8

亚历克上 M202161029

Ex 6.2 Q 1b, 2, Q3

1)
$$x_1 + x_2 - x_3 = 1$$
$$x_1 + x_2 + 4x_3 = 2$$
$$2x_1 - x_2 + 2x_3 = 3$$

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

We have three rules:-

1) Interchanging two rows
$R_1 \Leftrightarrow R_2$

2) Multiplying a row by a non zero constant
$2R_1 \rightarrow R_1$

3) Adding a multiple of one row to another row
$R_1 + 2R_2 \rightarrow R_1$

∴ Row interchange is necessary:-

⇒ **Using Gaussian Elimination with backward substitution algorithm**

⇒ Interchange row 2 and 3
$R_2 \Leftrightarrow R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 1 \\ 2 & -1 & 2 & 3 \\ 1 & 1 & 4 & 2 \end{array}\right] \begin{array}{l} \\ 2R_1 - R_2 \rightarrow R_2 \\ +1R_1 - R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 1 \\ 0 & 3 & -4 & 1 \\ 0 & 0 & -5 & -1 \end{array}\right] = \left[\begin{array}{r} x_1 + x_2 - x_3 = 1 \\ 3x_2 - 4x_3 = 1 \\ -5x_3 = -1 \end{array}\right]$$

$x_3 = -1/5$
$\Rightarrow \underline{\underline{1/5}}$

$x_2 = 1 + 4x_3$
$\Rightarrow 1 + 4(1/5)$
$\Rightarrow \underline{\underline{14/5}}$

$x_1 = 1 + x_3 - x_2$
$\Rightarrow 1 + 1/5 - 14/5$
$\Rightarrow 6/5 - 4/5 \Rightarrow \underline{\underline{-3/5}}$

2) b) Using Gaussian Elimination with partial Pivoting algorithm

$x_1 + x_2 - x_3 = 1$

$x_1 + x_2 + 4x_3 = 2$

$2x_1 - x_2 + 2x_3 = 3$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A^{(1)} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 1 & -1 \\ 2 & -1 & 2 \end{pmatrix} \quad L^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad b^{(1)} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$A^{(1)}x = b^{(1)}$ we have the relationship $L^{(1)}A^{(1)} = P^{(1)}A$

$\therefore$ we multiply $L^{(2)}A^{(2)} = P^{(2)}A$ by P_{23} $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ to get $P_{23}L^{(2)}A^{(2)}A$.

$P_{23}P^{(2)} = P^{(3)}$ so $P_{23}L^{(2)}A^{(2)} = P^{(3)}A$

we have $P_{23}L^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 6 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 6 & 0 & 0 \end{pmatrix}$

$\therefore$ row 2 and 3 are not interchanged.

3) b) Gausian Elimination with scaled Partial Pivoting and Gausian Elimination with Complete Pivoting algorithm.

$$x_1 + x_2 - x_3 = 1$$
$$x_1 + x_2 + 4x_3 = 2$$
$$2x_1 - x_2 + 2x_3 = 3$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A^{(-1)}x \Rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & -1 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & 4 & 2 & 3 \end{array}\right]$$

$\Rightarrow$ Interchange row 2 and 3

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ -1 & 4 & +2 & 3 \\ 1 & 1 & -1 & 2 \end{array}\right]$$

using $\cancel{2}4R_2 \to R_2$ & $R_1 + 2R_3 \to R_3$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ -1/4 & 1 & 1/2 & 3/4 \\ 2 & 2 & 1 & 3 \end{array}\right]$$