

## Assignment



### ASSIGNMENT # 1

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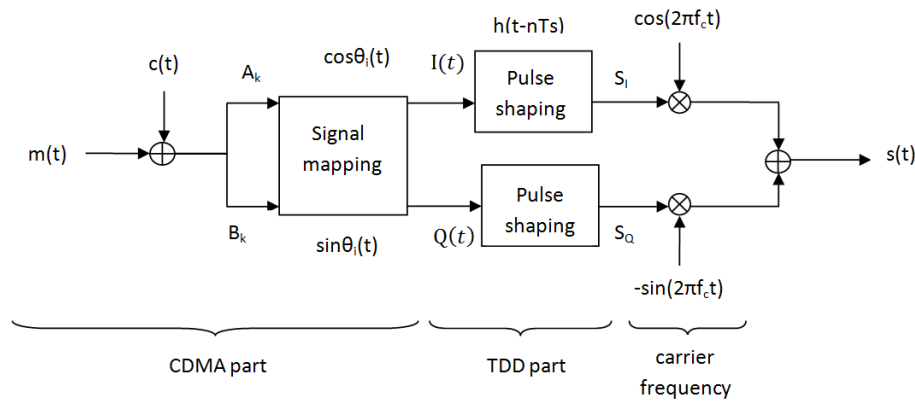
Submitted to: **Du Bing (杜冰)**

Subject: **Broadband Wireless Communication**

25th April 2022

## Modulation Process Math Model

TD-SCDMA adopts time division multiplex (TDD) operation instead of the frequency division multiplex (FDD) of W-CDMA, which may be considered a major difference between these two systems. In TDD mode, a 5ms frame is subdivided into 7 time slots, which can be flexibly assigned to either several users or to a single user who may require multiple time slots. Within one time slot, the system uses CDMA (Code Division Multiple Access) to further increase the capacity of the radio interface. Therefore, the TD-SCDMA system's mathematical model can be separated into TDMA and CDMA two processes.



Performance Comparison with LZ Algorithms

The mathematical model of the TD-SCDMA signal can be presented as

$$s(t) = s_I(t) \cos(2\pi f_0 t) - s_Q(t) \sin(2\pi f_0 t)$$

$$s_I(t) = \sum_{n=-\infty}^{\infty} I(n)h(t - nT_c)$$

$$s_Q(t) = \sum_{n=-\infty}^{\infty} Q(n)h(t - nT_c)$$

where  $S_I(t)$  and  $S_Q(t)$  denote pulse shaped spread spectrum signals in the in-phase (I) and quadrature (Q) channels respectively.  $f_0$  is the carrier frequency, and  $h(t)$  is the pulse shaping filter, which has linear phase and square root raise cosine frequency response. This is the time division part of the TD-SCDMA system.  $T_c$  is the chip period, equal to  $0.78125\mu s$  for the TD-SCDMA standard.

The pulse response of the filter is:

$$h(t) = \frac{\sin\left[\pi\frac{t}{T_c}(t - \alpha)\right] + 4\alpha\frac{t}{T_c}\cos\left[\pi\frac{t}{T_c}(t + \alpha)\right]}{\pi\frac{t}{T_c}\left[1 - \left(4\alpha\frac{t}{T_c}\right)^2\right]}.$$

The frequency response of the filter is:

$$H(f) = \begin{cases} 1 & , 0 \leq |f| \leq \frac{R_c}{2} - \beta \\ \cos \frac{\pi}{4\beta} \left( |f| - \frac{R_c}{2} + \beta \right) & , \frac{R_c}{2} - \beta \leq |f| \leq \frac{R_c}{2} + \beta \\ 0 & , |f| \geq \frac{R_c}{2} + \beta \end{cases}$$

where  $\alpha$  is the roll-off factor of the square root raise cosine filter, which determines the width of the transmitted band. The TD-SCDMA standard chooses a roll-off factor  $\alpha = 0.22$ ,  $\beta = \alpha/2T_c = 0.1408\text{MHz}$  is the excess bandwidth parameter.  $R_c = 1.28\text{MHz}$ .

$$B_N = \frac{R_c}{2} - \beta = 0.64\text{M} - 0.1408\text{M} = 0.4992\text{MHz} \quad , \quad B = \frac{R_c}{2} + \beta = 0.64\text{M} + 0.1408\text{M} = 0.7808\text{MHz};$$

$I(t)$  and  $Q(t)$  are the spread spectrum signals before the pulse shaping filter

$$I(t) = \sum_{i=1}^p m_i(t) c_i(t) \cos[\theta_i(t)]$$

$$Q(t) = \sum_{i=1}^p m_i(t) c_i(t) \sin[\theta_i(t)]$$

$m_i(t)$  = ith baseband quadrature phase-shift keying modulated signal,

$c_i(t)$  = ith pseudonoise binary code with a bandwidth of  $B$ ,

$\theta_i(t)$  = phase of the carrier associated with the ith SS signal,

$p$  = spreading factor of OVSF code.

Therefore, from equations 15, 16, 19, and 20, a general model of the TD-SCDMA signal is given by

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} \sum_{i=1}^p m_i(n) c_i(n) \cos[\theta_i(n)] h(t - nT_c) \cos(2\pi f_0 t) \\ &\quad - \sum_{n=-\infty}^{\infty} \sum_{i=1}^p m_i(n) c_i(n) \sin[\theta_i(n)] h(t - nT_c) \sin(2\pi f_0 t) \\ &= \text{Re} \left\{ \left[ \sum_{n=-\infty}^{\infty} \sum_{i=1}^p m_i(n) c_i(n) h(t - nT_c) e^{j\theta_i(n)} \right] e^{j2\pi f_0 t} \right\} \\ &= \text{Re} \left\{ \left[ \sum_{n=-\infty}^{\infty} f(n) h(t - nT_c) \right] e^{j2\pi f_0 t} \right\} \\ &= \text{Re} \left\{ [g(t)] e^{j2\pi f_0 t} \right\} \end{aligned}$$

With

$$f(t) = \sum_{i=1}^p m_i(t) c_i(t) e^{j\theta_i(t)}$$

$$g(t) = f(t) * h(t) = \sum_{nT_s=-\infty}^{\infty} f(nT_s) h(t - nT_s).$$

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We could further express  $s(t)$  as

$$s(t) = \tilde{s}(t)\cos[2\pi f_0 t + \theta(t)]$$

where

$$\tilde{s}(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$$

$$\theta(t) = \tan^{-1} \frac{s_Q(t)}{s_I(t)}$$

where  $\text{Re}\{\}$  denotes the real part of  $\{\}$ , and the spread spectrum signal part is separated from  $s(t)$  as  $f(t)$ ,  $g(t)$  is the pulse shaped spread spectrum, and  $s(t)$  is the envelope of  $s(t)$ .

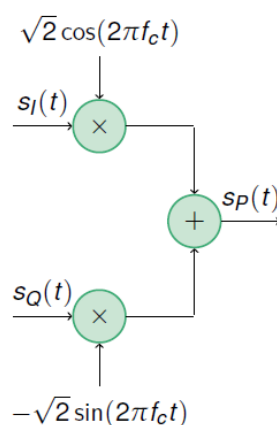
## Passband Signals

We have seen that many signal sets include both  $\sin(2\pi f_c t)$  and  $\cos(2\pi f_c t)$ . Examples include PSK and QAM signal sets. Such signals are referred to as passband signals. Passband signals have frequency spectra concentrated around a carrier frequency  $f_c$ . This is in contrast to baseband signals with spectrum centered at zero frequency. Baseband signals can be converted to passband signals through up-conversion. Passband signals can be converted to baseband signals through down-conversion.

## Baseband – Passband Conversion

The passband signal  $S_p(t)$  is constructed from two (digitally modulated) baseband signals,  $S_I(t)$  and  $S_Q(t)$ . Note that two signals can be carried simultaneously!

$S_I(t)$  and  $S_Q(t)$  are the in-phase (I) and quadrature (Q) components of  $S_p(t)$ . This is a consequence of  $S_I(t) \cos(2\pi f_c t)$  and  $S_Q(t) \sin(2\pi f_c t)$  being orthogonal. When the carrier frequency  $f_c$  is much greater than the bandwidth of  $S_I(t)$  and  $S_Q(t)$ .



### Exercise: Orthogonality of In-phase and Quadrature Signals

Show that  $S_I(t) \cos(2\pi f_c t)$  and  $S_Q(t) \sin(2\pi f_c t)$  are orthogonal when  $f_c \gg B$ , where  $B$  is the bandwidth of  $S_I(t)$  and  $S_Q(t)$ . You can make your argument either in the time-domain or the frequency domain.

## Baseband Equivalent Signals

The passband signal  $S_p(t)$  can be written as

$$s_P(t) = \sqrt{2}s_I(t) \cdot \cos(2\pi f_c t) - \sqrt{2}s_Q(t) \cdot \sin(2\pi f_c t).$$

If we define  $S(t) = S_I(t) + j \cdot S_Q(t)$ , then  $S_P(t)$  can also be expressed as

$$\begin{aligned} s_P(t) &= \sqrt{2} \cdot \Re\{s(t)\} \cdot \cos(2\pi f_c t) - \sqrt{2} \cdot \Im\{s(t)\} \cdot \sin(2\pi f_c t) \\ &= \sqrt{2} \cdot \Re\{s(t) \cdot \exp(j2\pi f_c t)\}. \end{aligned}$$

The signal  $s(t)$  is called the baseband equivalent, or the complex envelope of the passband signal  $S_P(t)$ . It contains the same information as  $S_P(t)$ . Note that  $S(t)$  is complex-valued.

### Exercise: Complex Envelope

Find the complex envelope representation of the signal

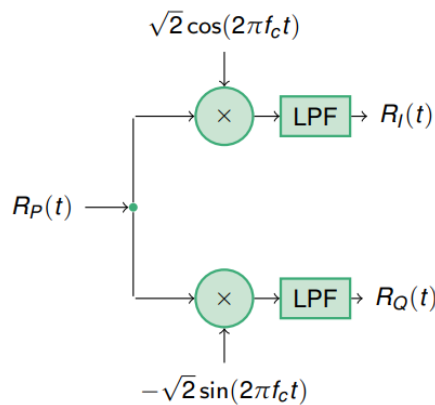
$$s_P(t) = \text{sinc}(t/T) \cos(2\pi f_c t + \frac{\pi}{4})$$

**Solution:**

$$\begin{aligned} s(t) &= \frac{e^{j\pi/4}}{\sqrt{2}} \text{sinc}(t/T) \\ &= \frac{1}{2}(\text{sinc}(t/T) + j\text{sinc}(t/T)) \end{aligned}$$

### Passband – Baseband Conversion

The down-conversion system is the mirror image of the up-conversion system. The top-branch recovers the in-phase signal  $S_I(t)$ . The bottom branch recovers the quadrature signal  $S_Q(t)$ .



Let the the passband signal  $S_P(t)$  be input to down-converter:

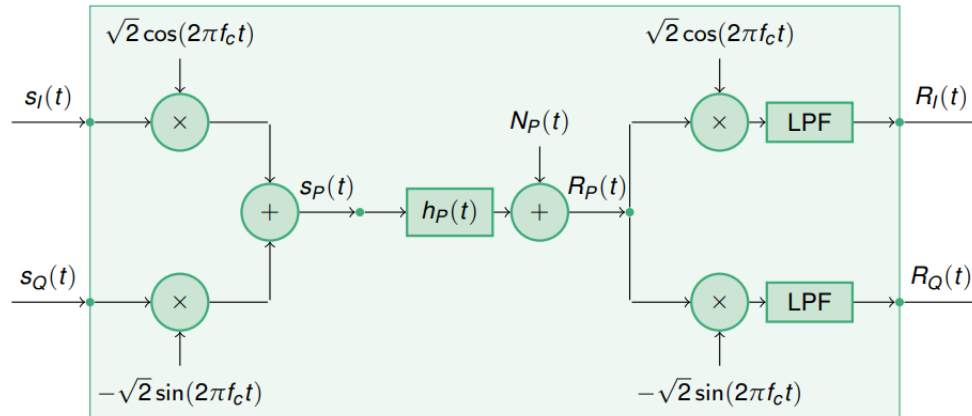
$$s_P(t) = \sqrt{2}(s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t))$$

Multiplying  $S_P(t)$  by  $\sqrt{2} \cos(2\pi f_c t)$  on the top branch yields  $S_P(t) \cdot$

$$\begin{aligned} &\sqrt{2} \cos(2\pi f_c t) \\ &= 2s_I(t) \cos^2(2\pi f_c t) - 2s_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t) \\ &= s_I(t) + s_I(t) \cos(4\pi f_c t) - s_Q(t) \sin(4\pi f_c t). \end{aligned}$$

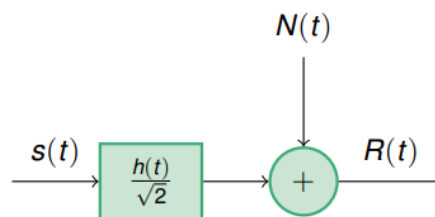
The low-pass filter rejects the components at  $\pm 2f_c$  and retains  $S_i(t)$ . A similar argument shows that the bottom branch yields  $S_q(t)$ .

### Complete Passband System



Complete pass-band system with channel (filter) and passband noise

### Baseband Equivalent System



The passband system can be interpreted as follows to yield an equivalent system that employs only baseband signals:

Baseband equivalent transmitted signal:  $S(t) = S_i(t) + j \cdot S_q(t)$  baseband equivalent channel with complex-valued impulse response:  $h(t)$ .

Baseband equivalent received signal:  $R(t) = R_i(t) + j \cdot R_q(t)$ . Complex valued, additive Gaussian noise:  $N(t)$  with spectral height  $N_0$ .