Quine and Boolos on Second-Order Logic:
An Examination of the Debate

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Abstract

The aim of this thesis is to examine the debate between Quine and Boolos over the logical status of higher-order logic—with Quine taking the position that higher-logic is more properly understood as set theory and Boolos arguing in opposition that higher-order logic is of a genuinely logical character. My purpose here then will be to stay as neutral as possible over the question of whether or not higher-order logic counts as logic and to instead focus on the exposition of the debate itself as exemplified in the work of Quine and Boolos.

Chapter I will be a detailed consideration of Quine's conception of logic and its place within the wider context of his philosophy. Only once this backdrop is in place will I then examine his views on higher-order logic. In Chapter II, I turn to Boolos's response to Quine—his attempt to examine the extent to which we may want to count higher-order logic as logic and the extent to which we may want to count it as set theory. With each point Boolos raises, I attempt to give what I think would have been Quine's reply. Finally, in Chapter III, I consider Boolos's attempt to show that monadic second-order logic (MSOL) should be understood as pure logic as it does not commit us to the existence of classes, as we may take the standard interpretation of MSOL to do. I discuss here some of the major reactions to Boolos's plural interpretation (Resnik, Parsons, and Linnebo), and conclude with more speculative remarks on what Quine's own response might have been. Throughout this thesis, my primary method has been one of close textual analysis.

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Table of Contents

Introduction

The aim of this thesis is to examine the debate between Quine and Boolos over the logical status of higher-order logic—with Quine taking the position that higher-logic is more properly understood as set theory and Boolos arguing in opposition that higher-order logic is of a genuinely logical character. Many philosophers, especially since Boolos's 1975 paper "On Second-Order Logic", have taken sides in the debate but very little, if any work has been done to explain what Quine's and Boolos's respective positions actually were. This has been particularly detrimental, I think, to our understanding of Quine where the exposition of his views on higher-order logic often gets little beyond alluding to his well-known aphorism that higher-order logic is set theory in sheep's clothing. My purpose here then will be to stay as neutral as possible over the question of whether or not higher-order logic counts as logic and to instead focus on the exposition of the debate itself as exemplified in the work of Quine and Boolos. Though, admittedly, through my work on this thesis, I have become increasingly convinced of the strength of Quine's position and that the burden of proof, so to speak, lies with those who want to claim that higher-order logic is logic in the same sense that first-order logic is.

Chapter I will focus on the exposition of Quine's conception of logic, particularly as he presents it in § 33 of *Word and Object*, "Aims and Claims of Regimentation". My aim will be to explicate how Quine conceives of logic from within his overall philosophical project of arriving at the clearest, simplest understanding of the world. I will then turn more directly to consider his criticisms of higher-order logic, again emphasizing how he develops these criticisms from within the broader framework of what he sees as the task of philosophy generally. Another related feature of Quine's conception of logic that I hope to bring out in this section is the way in which he relies on a fairly intuitive and traditional characteristic of logic, that it is self-evident or obvious, so as to carve out a body of theory that we can reasonably identify as logic. In many ways this tradition of logic is pre-Fregean though Frege himself also appealed to it when he offered a foundation for arithmetic grounded in logic. The aim will be to show that Quine's position is at least well-motivated and interesting even if we should ultimately decide to reject it. In fact, that we may decide not to accept Quine's characterization of logic will be consistent with what he claims to have shown in drawing the boundary of logic at first-order quantification theory. Ultimately, I think Quine is rejecting the view that we need to supply logic with a general philosophical account needs of it.

Chapter II will consider in detail the criticisms Boolos's 1975 criticisms of Quine's position and the kinds of responses that Quine could have offered. In many places it will seem that the two philosophers are talking past one another; that their respective positions in a sense rules out any common ground for debate. Yet, we will continue to see how Quine's position gives him a way to respond to nearly all of Boolos's criticisms. It seems that here Boolos is at a disadvantage. He appeals to Frege and the logicist tradition to suggest that higher-order logic is indeed well within the bounds of what may be traditionally thought of as logic. Though Boolos gives up the view that logic in this tradition has any claim to epistemic privilege; it suffers from the same epistemic debilities as set theory. Furthermore, he admits that higher-order logic is committed to the existence of sets, or at least subsets of the domain. It is hard to see how granting either of these features does not further Quine's own view of higher-order logic as a substantial mathematical theory, too substantial to be thought of as pure logic. A better strategy for Boolos might have been to reject this standard view of higher order logic as we will see him do in Chapter III.

This final chapter will consider Boolos's plural interpretation of monadic second-order logic (MSOL). Here, he appears to confront Quine more directly in that this interpretation of MSOL apparently shows a way in which higher-order logic makes no additional ontological commitments to sets and can be understood in terms of our ordinary English plural locutions. Because Quine himself wrote almost nothing on Boolos's plural interpretation for MSOL, this chapter takes on a slightly different structure than the previous two. It in includes a discussion of the major reactions to Boolos's plural interpretation, such as those found in the work of Michael Resnik, Charles Parsons, and Øystein Linnebo and concludes with some more speculative remarks on what Quine's own thoughts on this matter might have been. Throughout this thesis, my primary method has been one of close textual analysis.

Before we begin I would like to highlight two topics that I have not discussed outright in this thesis but that I think are implicitly at play throughout much of what follows. One is the issue of what is to count as set theory, or a set theory. Much of the debate as it is presented here is roughly over whether higher-order logic is to be counted as logic or as set theory. I have focused mostly on Quine's and Boolos's differing views of what logic is. However, it seems to me that they are also at odds over what is to count as set theory. Thus, the debate between them could have been approached from this side as well. The other is the issue of the relationship between logic and model theory and what the relevance of the one to the other is, if any thing at all. At least in his 1975 paper, Boolos's arguments often rely heavily on the model theory for higher-order logic, a topic with which Quine often shows minimal concern. I highlight this topic not merely because of its relevance to the Quine-Boolos debate over higher-order logic, but also because it is a topic currently receiving a great deal of consideration throughout the literature on the development of analytic philosophy and its interaction with mathematical logic.[1]

Unfortunately, I leave both of these issues for some future work.

[1] For an overview of the debate and references see Juliet Floyd, "Frege, Semantics, and the Double-Definition Stroke," in *The Story of Analytic Philosophy*, Anat Biletzki and Anat Matar, eds. (London: Routledge, 1998), pp. 141-66.

Chapter I: Quine on [Higher-Order] Logic

In his contribution to the 1988 Washington University conference on Quine's philosophy, Burton Dreben remarked of Quine's 1932 Ph.D. dissertation "The Logic of Sequences: a Generalization of *Principia Mathematica*", that the generalization itself is unimpressive as a technical contribution to the development of mathematical logic (a view with which Quine agrees), but "[t]he true significance of the dissertation lies elsewhere: It shows in full the independence and force of mind, the special, if not unique, logical concerns, and the deepest philosophical impulses that have characterized and governed Quine to this very day."[1] The aim of the present chapter is to bring out what these "special, if not unique, logical concerns" are in the context of Quine's criticisms of higher-order logic. What we will see is that the issue for Quine is in a sense very much a terminological one. Quine's aim in logic is clarity; once we have made explicit what we are doing when we do logic, how we apply the label 'logic' itself is of little consequence. However, being a terminological issue does not make it a trivial issue for the pursuit of clarity runs deep throughout Quine's philosophy. Indeed it could be identified as the driving force behind his entire philosophical outlook.

The reason these two aims are not to be distinguished is because part of simplifying and clarifying our science, our understanding of the world, is the extent to which we are able to paraphrase our scientific theory into a canonical notation that makes explicit the ontological commitments of the theory and the logical relations among its sentences, in addition to removing ambiguities of ordinary language from the theory generally. In section **I**, I will attempt to show how Quine implements this strategy in his chief philosophical work, the 1960 *Word and Object*.[2]

[1] Burton S. Dreben, "Quine," in *Perspectives on Quine*, Robert B. Barrett and Roger F. Gibson, eds. (Oxford: Basil Blackwell, 19??), p. 81; Quine's dissertation was published years later as W.V. Quine, *The Logic of Sequences: A Generalization of <u>Principia Mathematica</u>*, (New York: Garland Publishers, 1990); see in particular Quine's preface.
[2] W.V. Quine, *Word and Object* (Cambridge: MIT Press, 1960).

In sections **II.i** and **II.ii**, I will turn directly to Quine's criticisms of higher-order logic, particularly as they appear in his 1970 *Philosophy of Logic*[3] by addressing his attitude towards quantification over predicate letters, the completeness theorem, and his characterization of logical truths as obvious. Again we will see that Quine's aim is to dispel confusions and to make assumptions explicit. For if regimentation into canonical notation is to clarify and simplify our scientific theorizing, the canonical notation itself must aspire to this aim of clarity and simplicity. What we will see is that Quine's philosophy of logic gives expression to a certain Quinean view that philosophy is science gone self-reflective. In both science and philosophy (and so in logic), our aim is understanding, and the means for achieving this is to strive for the simplest and clearest systematization of the world we are capable of constructing.

I

Quine begins §33 of his *Word and Object* entitled "Aims and Claims of Regimentation" reflecting upon the useful purpose served by practical temporary departures from ordinary language.[4] These departures achieve many advantages but most important among them are understanding of the referential work of language, clarification of our conceptual scheme, and simplification of theory. Consider, for example, the use of parentheses. Quine remarks that to limit their value only to the resolution of ambiguities of grouping fails to recognize their far-reaching importance as they also allow for the iteration of identical constructions without requiring repeated variation of their expression so as to maintain grouping. In this way,

[3] W.V. Quine, *Philosophy of Logic*, 2nd ed. (Cambridge: Harvard University Press, 1986).

[4] W.V. Quine, *Word and Object*, pp. 157-61. In developing my understanding of Quine's conception of logic, I am much indebted to Thomas Ricketts's papers "Frege, Carnap, and Quine: Continuities and Discontinuities" in *Carnap Brought Home: The View from Jena*, Steve Awodey and Carsten Klein, eds. (Chicago and LaSalle: Open Court Publishing Company, 2004), pp. 181-202; and "Languages and Calculi" in *Logical Empiricism in North America*, Gary L. Hardcastle and Alan W. Richardson, eds. (Minneapolis: University of Minnesota Press, 2004), pp. 257-80.

parentheses allow us to minimize the number of basic constructions and the techniques required for their employment. They also allow for the possibility of subjecting both long and short expressions to a uniform algorithm and to argue by substitution of long expressions for short ones and vice versa without forcing a readjustment of context. Were it not "for parentheses or some alternative convention yielding the foregoing benefits," he remarks, "mathematics would not have come far."[5]

On Quine's view, the introduction of the logical notation for the truth-functional connectives and the quantifier-variable notation for generality not to differ in kind from other linguistic innovations used to simplify or clarify scientific theory. In fact, given these devices, augmented with classes and the predicate "∈" for class membership, Quine holds that his canonical notation is sufficient for the regimentation of the sentences of any scientific theory and the demonstration of the logical relationships between these sentences once so regimented.[6] Additionally, the quantifier-variable notation provides an objective standard by which to judge the ontological commitments of a particular theory, a significant advance over ordinary language with its tendency towards nominalization. The ontological commitments of a theory regimented into canonical notation are displayed by the range of the values of the bound variables.[7] For Quine, this simplification and clarification by means of a canonical logical notation is continuous with the aims of scientific theory generally:

> The same motives that impel scientists to seek ever simpler and clearer theories adequate to the subject matter of their special sciences are motives for simplification and clarification of the broader framework shared by all the sciences. Here the objective is called philosophical, because of the breadth of the framework concerned; but the motivation is the same. The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most

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[5] Ibid., p. 158; for a similar account see also "Logic as a Source of Syntactical Insights," in *The Way of Paradox and Other Essays*, rev. ed. (Cambridge: Harvard University Press, 1976), p. 44. An alternate convention is found in the logical notation used by the Polish logicians.
[6] W.V. Quine, "The Scope and Language of Science," in *Ways of Paradox*, p. 242-44.
[7] W.V. Quine, "On What There Is," in *From a Logical Point of View*, 2nd rev. ed. (Cambridge: Harvard University Press, 1980), pp. 12-19; "Scope and Language," pp. 242-44; *Word and Object*, pp. 242-43.

> general traits of reality. Nor let it be retorted that such constructions are conventional affairs not dictated by reality; for may not the same be said of a physical theory?[8]

Quine illustrates this project vividly in his review of Strawson's *Introduction to Logical Theory* by considering the situation of a formal logician who is also a scientist or mathematician.[9] The logician-scientist's interest in ordinary language lies in its utility for the pursuit of his scientific aims. If a departure from ordinary language will improve upon this utility, the logician-scientist has no qualms about doing so. He may, for example, introduce the notation '$\supset$' to replace 'if-then' of his ordinary language knowing full-well that '$\supset$' does not capture exactly what 'if-then' did, but such is not the purpose of the new notation. The scientist-logician is never under the illusion that his aim is synonymy between the ordinary and extraordinary language. Rather, '$\supset$' is meant to simplify the theory by increasing perspicuity and adding algorithmic facility. So long as '$\supset$' is fully adequate to fulfill the role that 'if-then' originally played in the scientific work at hand, the scientist-logician can get along without this more cumbersome piece of ordinary language.[10] Quine compares this technique to that of paraphrasing in ordinary language so as to remove ambiguities, the difference being that there the purpose is to facilitate communication and here it is the application of logical theory. However, for both purposes he sees synonymy not just as unnecessary, but as wholly misplaced:

> In neither case is synonymy to be claimed for the paraphrase. Synonymy, for sentences generally, is not a notion that we can readily make adequate sense of; and even if it were, it would be out of place in these cases. *If we paraphrase a sentence to resolve ambiguity, what we seek is not a synonymous sentence, but one that is more informative by dint of resisting some alternative interpretations.*[11]

Quine's comparison of his use of logical notation to that of paraphrasing in ordinary language should be emphasized as a distinctive philosophical departure from his predecessors in the

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[8] Quine, *Word and Object*, p. 161.
[9] P.F. Strawson, *Introduction to Logical Theory* (New York: Wiley, 1952); W.V. Quine, "Mr. Strawson on Logical Theory," in *Ways of Paradox*, p. 150.
[10] Ibid., p. 150.
[11] Quine, *Word and Object*, p. 159. (My emphasis)

analytic tradition as is brought out particularly well by contrasting his view of the analysis of the ordered pair with that of Russell.[12]

Quine explains that the ordered pair appears as a sort of defective noun for it has the peculiar feature of allowing two objects to be treated as one. In previous cases, he determined that defective nouns, such as attributes or propositions, proved undeserving of their claim to denote objects and so dismissed them as irreferential components of their containing phrases. Unlike these earlier cases though, the particular feature that makes the ordered pair seem defective is the very feature that gives it its utility; it is essential to its purpose that the ordered pair be treated as a single object. For example, this feature allows relations to be assimilated to classes by construing them as classes of ordered pairs. Without its objectual status, the ordered pair would be ineligible for class membership. The aim of philosophical analysis as Quine conceives of it is to make sense of how it is that the ordered pair can stand as a single object.

He explains that mathematicians have introduced the ordered pair by way of the condition

$$(1)\ \text{If } \langle x,y\rangle = \langle z,w\rangle \text{ then } x = z \text{ and } y = w$$

And hence, any already recognized object fulfilling this condition will fulfill the role of the ordered pair. Norbert Wiener offered the first such analysis in February 1914 defining the ordered pair as the class $\{\{x\}, \{y, \emptyset\}\}$.[13] Kazimierz Kuratowski followed in 1921 with the now more standard $\{\{x\}, \{x, y\}\}$. Both versions fulfill (1) while maintaining the important feature

[12] Ibid., § 53.

[13] Though arguably Felix Hausdorff might have produced $\{\{x, 1\}, \{y, 2\}\}$ first; see Akihiro Kanamori, "The Empty Set, the Singleton, and the Ordered Pair," *Bulletin of Symbolic Logic* 9(3) 2003, p. 291 n. 33.

that the apparent two objects of the pair are treated as a single object, a class.[14] Wiener's definition, Quine explains, captures what he sees as central to the task of philosophy:

> This construction is paradigmatic of what we are most typically up to when in a philosophical spirit we offer an "analysis" or "explication" of some hitherto inadequately formulated "idea" or expression. We do not claim synonymy. We do not claim to make clear and explicit what the users of the unclear expression had unconsciously in mind all along. We do not expose hidden meanings, as the words "analysis" and "explication" would suggest; we supply lacks. We fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fulfills those functions. Beyond those conditions of partial agreement, dictated by our interests and purposes, any traits of the explicans come under the head of "don't cares". Under this head we are free to allow the explicans all manner of novel connotations never associated with the explicandum.[15]

Wiener's construction of the ordered pair may be what Quine is most typically up to when he offers a philosophical analysis, but it is clearly not what the tradition had in mind. Russell, in a way, conceived of the aim of analysis to be precisely what Quine says it is not: to expose hidden meanings.[16] In his 1903 *Principles of Mathematics*, Russell states that the purpose of the logical analysis of mathematics is the discovery of the logical indefinables, or logical constants, "in order that the mind may have that kind of acquaintance with them which it has with redness or the taste of a pineapple."[17] The success of this reduction would then allow philosophy to provide an answer to the question, what does mathematics mean? Russell explains,

> Mathematics in the past was unable to answer, and Philosophy answered by introducing the totally irrelevant notion of mind. But now Mathematics is able to answer, so far at least as to reduce the whole of its propositions to certain fundamental notions of logic. At this point, the discussion must be resumed by Philosophy. I shall endeavour to indicate what are the fundamental notions involved, to prove at length that no others occur in mathematics, and to point out briefly the philosophical difficulties involved in the analysis of these notions.[18]

Later in life, reflecting back on his mathematical work, Russell wrote specifically of the Wiener-Kuratowski analysis,

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[14] The proof is relatively straight forward. See, for example, Herbert B. Enderton, *Elements of Set Theory* (London: Academic Press, 1977), p. 36.
[15] Quine, *Word and Object*, pp. 258-59.
[16] For a detailed account of Russell's various views of philosophical analysis see Peter Hylton, "Beginning with Analysis," in *Bertrand Russell and the Origins of Analytic Philosophy*, Ray Monk and Anthony Palmer, eds. (Bristol: Thoemmes Press, 1997), pp. 183-215.
[17] Bertrand Russell, *The Principles of Mathematics*, 1903, 2nd ed., (London: George Allen and Unwin Ltd., 1937), p. xv.
[18] Ibid., p. 4.

> I thought of relations in those days [circa 1900], almost exclusively as *intensions*. I thought of sentences such as, '*x* precedes *y*', '*x* is greater than *y*', '*x* is north of *y*'. It seemed to me—as, indeed, it still seems—that, *although from the point of view of a formal calculus one can regard a relation as a set of ordered couples, it is the intension alone which gives unity to the set.*[19]

Elsewhere Russell is reported to have called the analysis "a trick".[20] This contrast between Russell and Quine over the significance of the Wiener-Kuratowski definition of the ordered pair helps to illustrate the extent to which Quine is willing to do away with traditional philosophical concerns so as to obtain clarity about a troublesome but useful notion of our current ongoing science. That Wiener's and Kuratowski's definitions differ is of no matter for Quine. They both fulfill condition (1) and where their differences lie has no effect on this; there is no hidden meaning, no essence, for the analysis to discover. What distinguishes Quine's attitude towards philosophical analysis is that he does not judge the success of the analysis by some preconceived philosophical notion of what the analysis should be, a guiding metaphysical assumption concerning the intensionality of relations as in Russell's case. Rather his guiding concern is whether the proposed analysis fulfills the condition that makes the ordered pair worthwhile for science. So long as the analysis does this, any other features of it may be consigned to the realm of don't cares.[21] For Quine

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[19] Bertrand Russell, *My Philosophical Development* (London: George Allen and Unwin Ltd., 1959), p. 87 (My emphasis); and of Quine, in particular, he wrote, "Professor Quine, for example, has produced systems which I admire greatly on account of their skill, but which I cannot feel to be satisfactory because they seem to be created *ad hoc* and not to be such as even the cleverest logician would have thought of if he had not known of the contradictions" (p. 80).

[20] W.D. Hart, "Clarity," in *The Analytic Tradition*, David Bell and Neil Cooper, eds., (Oxford: Basil Blackwell Ltd., 1990), p. 207.

[21] This of course is also the view Quine holds with regard to numbers. As he remarks in "Ontological Relativity,"

> So, though Russell was wrong in suggesting that numbers need more than their arithmetical properties, he was right in objecting to the definition of numbers as any things fulfilling arithmetic. The subtle point is that any progression will serve as a version of number so long and only so long as we stick to one and the same progression. Arithmetic is, in this sense, all there is to number: there is no saying absolutely what the numbers are; there is only arithmetic. (p. 45)

See "Ontological Relativity," in *Ontological Relativity and Other Essays*, (New York: Columbia University Press, 1969), pp. 26-68, and especially pp. 43-5; and "Reply to Charles Parsons," in *The Philosophy of W.V. Quine*, expanded ed., Lewis Edwin Hahn and Paul Arthur Schilpp, eds. (Chicago and LaSalle: Open Court Publishing Company, 1998), pp. 400-1. The classic development of this view of numbers is Paul Benacerraf, "What Numbers Could not Be," *The Philosophical Review* 74:1 (1965), pp. 47-73.

> *explication is elimination.* We have, to begin with, an expression or form of expression that is somehow troublesome. It behaves partly like a term but not enough so, or it is vague in ways that bother us, or it puts kinks in a theory or encourages one or another confusion. But also it serves certain purposes that are not to be abandoned. Then we find a way of accomplishing those same purposes through other channels, using other and less troublesome forms of expression. The old perplexities are resolved.[22]

It is in this vein that Quine considers the ordered pair to be a paradigm of philosophical analysis and philosophy of science to be philosophy enough.[23]

Though the simplification and clarification of scientific theory, alone, is of sufficient gain to recommend regimentation, the regimentation of scientific theory into canonical notation also aims at the clarification of the science of logic itself. Having regimented a theory, we can more easily identify the logical relationships between its sentences by applying logical theory to them, what Quine describes as "the systematic study of logical truths." Chief among such logical interdependencies among sentences is logical implication as Quine explains, "Logical implication is the central business of logic. Logical truth would be of little concern to us on its own account, but it is important as an avenue to implication. It is simpler to theorize about truth than implication because it is attributable to single sentences whereas implication relates sentences in pairs."[24]

The importance of logical implication to scientific theorizing is not to be understated. For Quine this logical relationship is "the lifeblood of theories" as it is what links a theory to its empirical checkpoints.[25] Having established a hypothesis, implication allows for its testing. One side of the implication, the theoretical, is made up of our backlog of accepted theory plus the hypothesis; this side does the implying. On the other side, the observational, is an implied

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[22] Quine, *Word and Object*, p. 260 (Quine's italics); For my account of the significance of the ordered pair in Quine's philosophy, particularly as it indicates a decisive contrast between the philosophical outlooks of Quine and Russell, I am indebted to Kanamori, "Empty Set," pp. 288-93 and to Hylton, "Beginning with Analysis," pp. 213-15.
[23] Quine, "Strawson," p. 151.
[24] W.V. Quine, "Grammar, Truth, and Logic," in *Philosophy and Grammar*, Stig Kanger and Sven Öhman, eds., (Dordrecht, Holland: D. Reidel Publishing Company, 1981), pp. 17; see also *Methods of Logic*, 1950, 4th ed. (Cambridge: Harvard University Press, 1982), p. 4; and *Philosophy of Logic* [1986], pp. vii, 48-9.
[25] W.V. Quine, *From Stimulus to Science* (Cambridge: Harvard University Press, 1995), p. 51.

generality of the form "Whenever this, that", Quine's observation categorical, available to the experimenter for direct testing. So long as our canonical notation includes the resources to represent the relation of implication between a body of scientific theory and an observation categorical, this notation will be sufficient for the needs of science.[26] And determining whether one sentence implies another does in fact require nothing beyond the resources of Quine's canonical notation, the truth functional connectives and quantification. Implication holds simply when the conditional formed from the two sentences is valid, when it is a logical truth.[27] This raises an important and illuminating question for Quine's conception of logic: how is he to characterize logical truth? The definition of logical truth, or validity, offered by most contemporary philosophers and logicians goes by way of model theory: a sentence is *logically true*, or *valid*, if every model in a model-theoretic semantics *S* is a model of the sentence.[28] However, in his *Philosophy of Logic*, Quine first defines logical truth in terms of sentence substitution: a logical truth is a sentence that yields only truths when we substitute sentences for its simple sentences.[29] He then notes that his substitutional definition can also be given by a two step method employing the notion of a *valid logical schema.*

He describes a schema as a sort of "dummy sentence" that depicts the logical structure of what could be an actual sentence, a sentence of the fully interpreted object language. The logical structure of a sentence is its composition in terms of truth functions, quantifiers, and variables. Sentences then, by Quine's account, are composed only of logical structure and predicates. A schema depicts the logical structure of a sentence by replacing the predicates with schematic

[26] W.V Quine, *Pursuit of Truth*, 1990, rev. ed. (Cambridge: Harvard University Press, 1992), pp. 9-10; I am indebted in this paragraph to Ricketts, "Frege, Carnap, and Quine," p. 198.
[27] Quine, *Methods* [1982], p. 46.
[28] For example see Stewart Shapiro, *Foundations without Foundationalism: A Case for Second-order Logic*, paperback ed., (Oxford: Clarendon Press, 2002), p. 6.
[29] Quine, *Philosophy of Logic* [1986], p. 50.

predicate letters 'F', 'G', etc. Unlike actual predicates, these schematic predicate letters are not part of the object language but serve instead only to diagrammatically mark positions where object language predicates could appear. For example, the sentence 'There is something that walks' can be rendered into canonical notation as '$(\exists x)$(x walks)'. Replacing the predicate '① walks' with a schematic predicate letter 'F' we have depicted its logical structure thus '$(\exists x)Fx$'. Quine then defines a logical schema as *valid* if every sentence obtainable from it by substitution of sentences for simple sentence schemata is true. Now, a *logical truth* is a truth obtainable by this substitution method from a valid logical schema.[30] He does not include separate substitution clauses for term letters, such as names or functions. Given identity as a logical primitive, name and function letters are superfluous; both can be paraphrased away by Russell's method of descriptions. This austere language of truth functions, quantifiers, variables, and predicates is enough though eliminable term letters may always be introduced for mere convenience.[31]

We should pause here to observe that Quine's substitutional definition will not work if he takes identity as a primitive logical predicate for then '$(\exists x)(\exists y)\neg x = y$' would count as a logical truth, and logic as traditionally conceived, and as Quine conceives it, does not pronounce on the number of objects in the world. Yet, if he does not take identity as a primitive logical predicate, truths of identity theory such as '$x = x$' or '$(\exists y)x = y$' would not count as logical truths as they could be falsified by substituting some other predicate for '='. This, too, seems an undesirable result as such truths are often considered logical truths. This tension, though, does not result from Quine's substitutional definition but from the identity predicate itself he explains, in that once identity is allowed as part of our genuinely logical vocabulary, some logical generalities

[30] Ibid., pp. 24, 49-50.
[31] Ibid., pp. 25-6; Quine, *Methods* [1982], pp. 274-77.

become directly expressible in the object language. Logic's concern extends in this way from talk of forms of sentences to the expression of genuine sentences.

Still, Quine thinks there are also reasons for wanting to include identity as part of the logical vocabulary. One is that, like quantification theory, there are complete proof procedures available for identity theory. Another is that identity theory, again like quantification theory, does not discriminate amongst objects in its application. Neither of these features holds for many branches of higher mathematics and, in particular, not for set theory.[32] He then offers a further consideration for favoring the inclusion of identity theory as part of logic:

> [F]or identity is marginal in a curious way. Namely, any theory with any finite number of other primitive predicates gets identity too as a bonus. For identity can be defined, or something to the same formal purpose, by exhaustion of those primitive predicates. For example, if the primitive predicates are '*P*', '*Q*', and a dyadic '*R*', we can define '$x = y$' as
>
> $\forall z(Px \equiv Py \,.\, Qx \equiv Qy \,.\, Rxz \equiv Ryz \,.\, Rzx \equiv Rzy)$.[33]

By defining identity this way, truths of identity theory gain the same schematic status as other logical generalizations. Ultimately, these considerations lead Quine to include identity theory as more appropriately part of logic than of some other higher branch of mathematics. What should be stressed here is that his initial considerations against counting identity as a primitive logical predicate are not to be understood merely as an ad hoc maneuver for securing his substitutional definition of logical truth. Identity presents difficulties for any view that sees logic as primarily concerned with form, and specifically for Quine, forms of sentences.

Though his two-step definition of logical truth comes to the same thing as the definition given in terms of sentence substitution the notion of a valid schema serves a further purpose as he explains, "Because of their freedom from subject matter, schemata are the natural medium for logical laws and proofs."[34] It is only after he gives his substitutional definition that Quine then

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[32] Quine, *Philosophy of Logic* [1986], pp. 61-4; Quine, *From Stimulus to Science*, p. 52.
[33] Quine, *From Stimulus to Science*, p. 52.
[34] Quine, *Philosophy of Logic* [1986], p. 51