北京科技大学 2021-2022 年 第 一 学期

Exam for Computational Method B(计算方法 B) Time:2 Hours

Name: Student ID

Part A: (50points)

1. Explain the advantages and disadvantages of Bisection, Fixed-Point Iteration, and Newton's methods for finding the roots of equation $f(x)=0$.

2. Explain the idea of Interpolation and Polynomial Approximation, the advantages and disadvantages of Lagrange interpolating polynomial and Newtonian interpolating polynomial.

3. Explain the idea of direct methods (Gauss elimination method, Pivoting Strategies) and Iterative Techniques for solving linear systems of equations.

4. Explain the idea of numerical differentiation and integration, the idea ofComposite Trapezoidal rule and Composite Simpson's rule.

5. Let $\boldsymbol{A}=\begin{bmatrix} 1 & -1 & 2 \\ 1 & -2 & -3 \\ -2 & 4 & 5 \end{bmatrix}$, to compute $\|A\|_1=$, $\|A\|_\infty=$.

Part B: (50 points)

1. (a)Use the Bisection method to find P3 for $f(x)=\sqrt{x}-\cos x=0$ on [0, 1].

(b) Let $f(x)=x^2-6=0$, Use Newton's method to find P2 with $p_0=-1$.

2. Table 1 lists values of a function $f(x)$ at various points, to find approximation value $f(1.5)$ by using Lagrange interpolating polynomial of degree 1 and of degree 2 by $P_{2,3,4}(1.5)$.

Table 1

x	1.0	1.3	1.6	1.9
$f(x)$	0.7651977	0.6200860	0.4554022	0.2818186

3. Use the composite Trapzoidal rule with the value of 4 to approximate the integral $\int_{-2}^{2} e^x x^2 \mathrm{d}x$, $n = 4$

4. Find the Jacobi iterations and Gauss-Seidel iterations for the following linear systems, write the iterative method in matrix form. For Jacobi iterative methods, to find $x^{(2)}$ by using initial value $x^{(0)} = (0,0,0)^t$

$$\begin{bmatrix} 10 & -1 & -1 \\ -1 & 10 & -2 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 8.5 \\ 3.2 \end{bmatrix}$$

5. Use Euler's method to approximate the solution for the following initial-value problem with $h = 0.1$.

$$\begin{cases} y' = -5y + 5t^2 + 2t, & 0 \le t \le 1 \\ y(0) = \dfrac{1}{3} \end{cases},$$