15

MULTIUSER COMMUNICATIONS

Our treatment of communication systems up to this point has been focused on a single communication link involving a transmitter and a receiver. In this chapter, the focus shifts to multiple users and multiple communication links. We explore the various ways in which the multiple users access a common channel to transmit information. The multiple access methods that are described in this chapter form the basis for current and future wireline and wireless communication networks, such as satellite networks, cellular and mobile communication networks, and underwater acoustic networks.

15-1 INTRODUCTION TO MULTIPLE ACCESS TECHNIQUES

It is instructive to distinguish among several types of multiuser communication systems. One type is a multiple access system in which a large number of users share a common communication channel to transmit information to a receiver. Such a system is depicted in Fig. 15-1-1. The common channel may be the up-link in a satellite communication system, or a cable to which are connected a set of terminals that access a central computer, or some frequency band in the radio spectrum that is used by multiple users to communicate with a radio receiver. For example, in a mobile cellular communication system, the users are the mobile transmitters in any particular cell of the system and the receiver resides in the base station of the particular cell.

A second type of multiuser communication system is a broadcast network in which a single transmitter sends information to multiple receivers as depicted

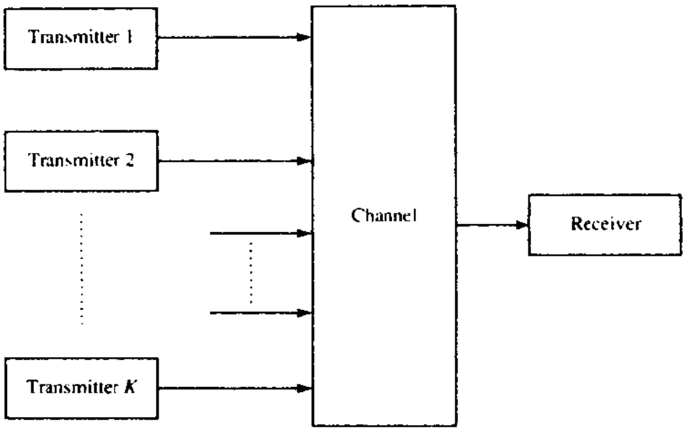


FIGURE 15-1-1 A multiple access system.

in Fig. 15-1-2. Examples of broadcast systems include the common radio and TV broadcast systems, as well as the down-links in a satellite system.

The multiple access and broadcast networks are probably the most common multiuser communication systems. A third type of multiuser system is a store-and-forward network, as depicted in Fig. 15-1-3. Yet a fourth type is the two-way communication system shown in Fig. 15-1-4.

In this chapter, we focus on multiple access methods for multiuser communications. In general, there are several different ways in which multiple users can send information through the communication channel to the receiver. One simple method is to subdivide the available channel bandwidth into a number, say N, of frequency nonoverlapping subchannels, as shown in Fig. 15-1-5, and to assign a subchannel to each user upon request by the users. This

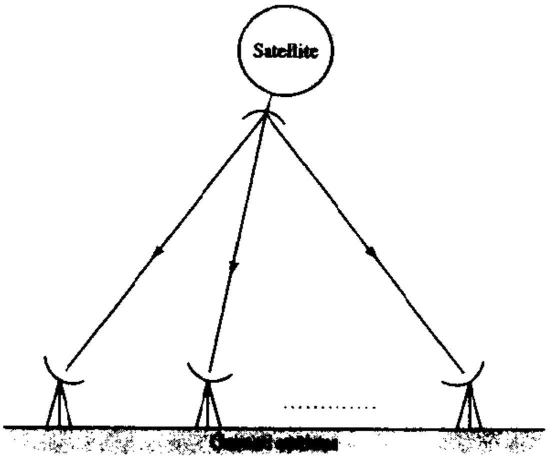


FIGURE 15-1-2 A broadcast network.

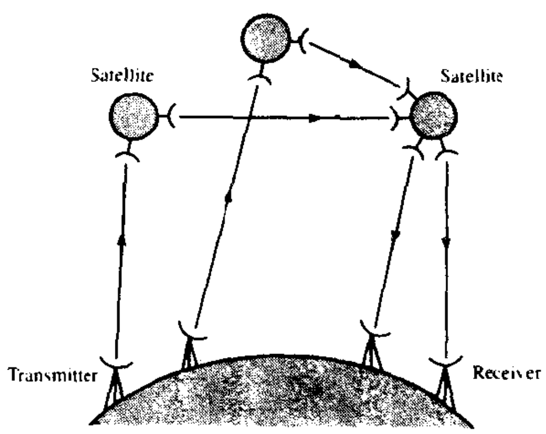


FIGURE 15-1-3 A store-and-forward communication network with satellite relays.

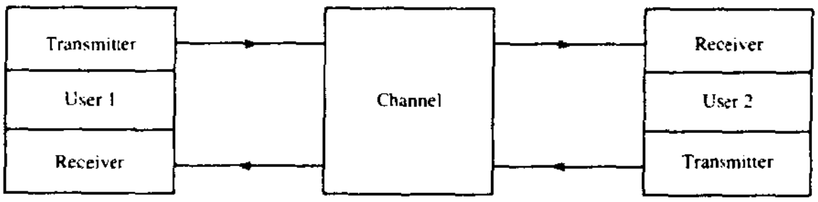


FIGURE 15-1-4 A two-way communication channel.

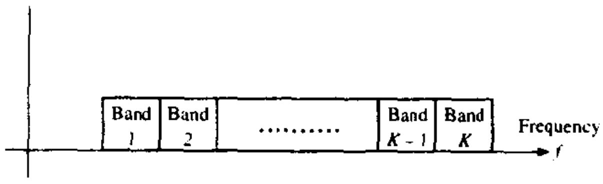


FIGURE 15-1-5 Subdivisions of the channel into nonoverlapping frequency bands.

method is generally called *frequency-division multiple access* (FDMA), and is commonly used in wireline channels to accommodate multiple users for voice and data transmission.

Another method for creating multiple subchannels for multiple access is to subdivide the duration T_f, called the *frame duration*, into, say, N nonoverlapping subintervals, each of duration T_f/N. Then each user who wishes to transmit information is assigned to a particular time slot within each frame. This multiple access method is called *time-division multiple access* (TDMA) and it is frequently used in data and digital voice transmission.

We observe that in FDMA and TDMA, the channel is basically partitioned into independent single-user subchannels. In this sense, the communication system design methods that we have described for single-user communication are directly applicable and no new problems are encountered in a multiple access environment, except for the additional task of assigning users to available channels.

The interesting problems arise when the data from the users accessing the network is bursty in nature. In other words, the information transmissions from a single user are separated by periods of no transmission, where these periods of silence may be greater than the periods of transmission. Such is the case generally with users at various terminals in a computer communications network that contains a central computer. To some extent, this is also the case in mobile cellular communication systems carrying digitized voice, since speech signals typically contain long pauses.

In such an environment where the transmission from the various users is bursty and low-duty-cycle, FDMA and TDMA tend to be inefficient because a certain percentage of the available frequency slots or time slots assigned to users do not carry information. Ultimately, an inefficiently designed multiple access system limits the number of simultaneous users of the channel.

An alternative to FDMA and TDMA is to allow more than one user to share a channel or subchannel by use of direct-sequence spread spectrum signals. In this method, each user is assigned a unique code sequence or *signature sequence* that allows the user to spread the information signal across the assigned frequency band. Thus signals from the various users are separated at the receiver by cross-correlation of the received signal with each of the possible user signature sequences. By designing these code sequences to have relatively small cross-correlations, the crosstalk inherent in the demodulation of the signals received from multiple transmitters is minimized. This multiple access method is called *code-division multiple access* (CDMA).

In CDMA, the users access the channel in a random manner. Hence, the signal transmissions among the multiple users completely overlap both in time and in frequency. The demodulation and separation of these signals at the receiver is facilitated by the fact that each signal is spread in frequency by the pseudo-random code sequence. CDMA is sometimes called *spread-spectrum multiple access* (SSMA).

An alternative to CDMA is nonspread random access. In such a case, when two users attempt to use the common channel simultaneously, their transmissions collide and interfere with each other. When that happens, the information is lost and must be retransmitted. To handle collisions, one must establish protocols for retransmission of messages that have collided. Protocols for scheduling the retransmission of collided messages are described below.

15-2 CAPACITY OF MULTIPLE ACCESS METHODS

It is interesting to compare FDMA, TDMA, and CDMA in terms of the information rate that each multiple access method achieves in an ideal AWGN channel of bandwidth W. Let us compare the capacity of K users, where each user has average power $P_i = P$, for all $1 \leq i \leq K$. Recall that in an ideal band-limited AWGN channel of bandwidth W, the capacity of a single user is

$$C = W \log_2\left(1 + \frac{P}{WN_0}\right) \tag{15-2-1}$$

where $\frac{1}{2}N_0$ is the power spectral density of the additive noise.

In FDMA, each user is allocated a bandwidth W/K. Hence, the capacity of each user is

$$C_K = \frac{W}{K}\log_2\left[1 + \frac{P}{(W/K)N_0}\right] \tag{15-2-2}$$

and the total capacity for the K users is

$$KC_K = W \log_2\left(1 + \frac{KP}{WN_0}\right) \tag{15-2-3}$$

Therefore, the total capacity is equivalent to that of a single user with average power $P_{av} = KP$.

It is interesting to note that for a fixed bandwidth W, the total capacity goes to infinity as the number of users increases linearly with K. On the other hand, as K increases, each user is allocated a smaller bandwidth (W/K) and, consequently, the capacity per user decreases. Figure 15-2-1 illustrates the capacity C_K per user normalized by the channel bandwidth W, as a function of $\mathscr{E}_b/N_0$, with K as a parameter. This expression is given as

$$\frac{C_K}{W} = \frac{1}{K}\log_2\left[1 + K\frac{C_K}{W}\left(\frac{\mathscr{E}_b}{N_0}\right)\right] \tag{15-2-4}$$

A more compact form of (15-2-4) is obtained by defining the normalized

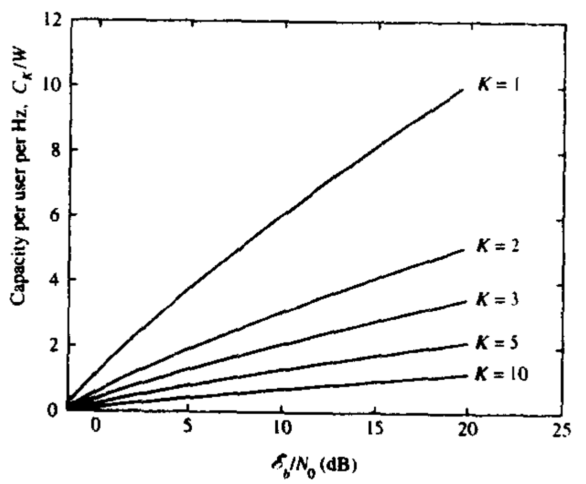


FIGURE 15-2-1 Normalized capacity as a function of $\mathscr{E}_b/N_0$ for FDMA.

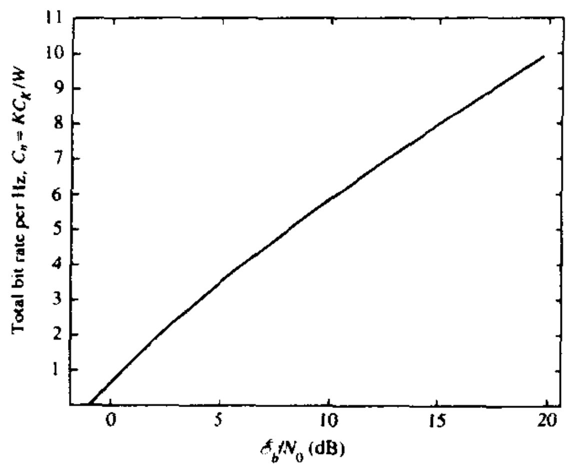


FIGURE 15-2-2 Total capacity per hertz as a function of $\mathscr{E}_b/N_0$ for FDMA.

total capacity $C_n = KC_K/W$, which is the total bit rate for all K users per unit of bandwidth. Thus, (15-2-4) may be expressed as

$$C_n = \log_2\left(1 + C_n\frac{\mathscr{E}_b}{N_0}\right) \tag{15-2-5}$$

or, equivalently,

$$\frac{\mathscr{E}_b}{N_0} = \frac{2^{C_n} - 1}{C_n} \tag{15-2-6}$$

The graph of C_n versus $\mathscr{E}_b/N_0$ is shown in Fig. 15-2-2. We observe that C_n increases as $\mathscr{E}_b/N_0$ increases above the minimum value of $\ln 2$.

In a TDMA system, each user transmits for $1/K$ of the time through the channel of bandwidth W, with average power KP. Therefore, the capacity per user is

$$C_K = \left(\frac{1}{K}\right)W \log_2\left(1 + \frac{KP}{WN_0}\right) \tag{15-2-7}$$

which is identical to the capacity of an FDMA system. However, from a practical standpoint, we should emphasize that, in TDMA, it may not be possible for the transmitters to sustain a transmitter power of KP when K is very large. Hence, there is a practical limit beyond which the transmitter power cannot be increased as K is increased.

In a CDMA system, each user transmits a pseudo-random signal of a bandwidth W and average power P. The capacity of the system depends on the level of cooperation among the K users. At one extreme is noncooperative CDMA, in which the receiver for each user signal does not know the spreading waveforms of the other users, or chooses to ignore them in the demodulation process. Hence, the other users signals appear as interference at the receiver of each user. In this case, the multiuser receiver consists of a bank of K

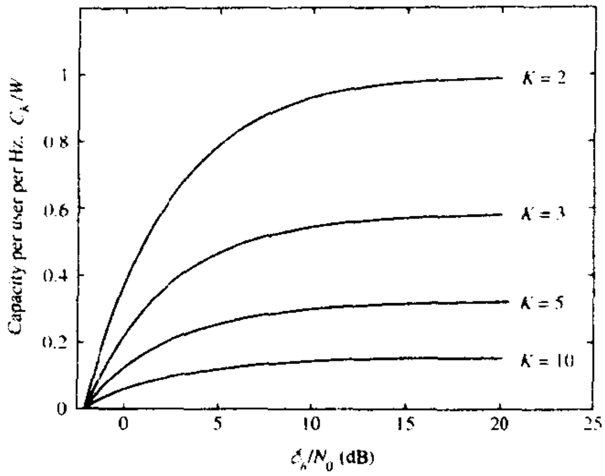


FIGURE 15-2-3 Normalized capacity as a function of $\mathscr{E}_b/N_0$ for noncooperative CDMA.

single-user receivers. If we assume that each user's pseudorandom signal waveform is gaussian then each user signal is corrupted by gaussian interference of power $(K-1)P$ and additive gaussian noise of power WN_0. Therefore, the capacity per user is

$$C_K = W \log_2 \left[1 + \frac{P}{WN_0 + (K-1)P}\right] \tag{15-2-8}$$

or, equivalently,

$$\frac{C_K}{W} = \log_2 \left[1 + \frac{C_K}{W} \frac{\mathscr{E}_b/N_0}{1 + (K-1)(C_K/W)\mathscr{E}_b/N_0}\right] \tag{15-2-9}$$

Figure 15-2-3 illustrates the graph of C_K/W versus $\mathscr{E}_b/N_0$, with K as a parameter.

For a large number of users, we may use the approximation $\ln(1+x) \le x$. Hence,

$$\frac{C_K}{W} \le \frac{C_K}{W} \frac{\mathscr{E}_b/N_0}{1 + K(C_K/W)(\mathscr{E}_b/N_0)} \log_2 e \tag{15-2-10}$$

or, equivalently,

$$\begin{aligned} C_n &\le \log_2 e - \frac{1}{\mathscr{E}_b/N_0} \\ &\le \frac{1}{\ln 2} - \frac{1}{\mathscr{E}_b/N_0} < \frac{1}{\ln 2} \end{aligned} \tag{15-2-11}$$

In this case, we observe that the total capacity does not increase with K as in TDMA and FDMA.

On the other hand, suppose that the K users cooperate by transmitting synchronously in time, and the multiuser receiver knows the spreading waveforms of all users and jointly demodulates and detects all the users' signals. Thus, each user is assigned a rate R_i, $1 \leq i \leq K$, and a codebook containing a set of 2^{nR_i} codewords of power P. In each signal interval, each user selects an arbitrary codeword, say $\mathbf{X}_i$, from its own codebook and all users transmit their codewords simultaneously. Thus, the decoder at the receiver observes

$$\mathbf{Y} = \sum_{i=1}^{K} \mathbf{X}_i + \mathbf{Z} \tag{15-2-12}$$

where $\mathbf{Z}$ is an additive noise vector. The optimum decoder looks for the K codewords, one from each codebook, that have a vector sum closest to the received vector $\mathbf{Y}$ in euclidean distance.

The achievable K-dimensional rate region for the K users in an AWGN channel, assuming equal power for each user, is given by the following equations:

$$R_i < W \log_2\left(1 + \frac{P}{WN_0}\right), \quad 1 \leq i \leq K \tag{15-2-13}$$

$$R_i + R_j < W \log_2\left(1 + \frac{2P}{WN_0}\right), \quad 1 \leq i, j \leq K \tag{15-2-14}$$

$$\vdots$$

$$\sum_{i=1}^{K} R_i < W \log_2\left(1 + \frac{KP}{WN_0}\right) \tag{15-2-15}$$

In the special case when all the rates are identical, the inequality (15-2-15) is dominant over the other $K-1$ inequalities. It follows that if the rates $\{R_i, 1 \leq i \leq K\}$ for the K cooperative synchronous users are selected to fall in the capacity region specified by the inequalities given above then the probabilities of error for the K users tend to zero as the code block length n tends to infinity.

From the above discussion, we conclude that the sum of the rates of the K users goes to infinity with K. Therefore, with cooperative synchronous users, the capacity of CDMA has a form similar to that of FDMA and TDMA. Note that if all the rates in the CDMA system are selected to be identical to R then (15-2-15) reduces to

$$R < \frac{W}{K} \log_2\left(1 + \frac{KP}{WN_0}\right) \tag{15-2-16}$$

which is identical to the rate constraint in FDMA and TDMA. In this case, CDMA does not yield a higher rate than TDMA and FDMA. However, if the rates of the K users are selected to be unequal such that the inequalities (15-2-13)–(15-2-15) are satisfied then it is possible to find the points in the achievable rate region such that the sum of the rates for the K users in CDMA exceeds the capacity of FDMA and TDMA.

Example 15-2-1

Consider the case of two users in a CDMA system that employs coded signals as described above. The rates of the two users must satisfy the inequalities

$$R_1 < W \log_2\left(1 + \frac{P}{WN_0}\right)$$

$$R_2 < W \log_2\left(1 + \frac{P}{WN_0}\right)$$

$$R_1 + R_2 < W \log_2\left(1 + \frac{2P}{WN_0}\right)$$

where P is the average transmitted power of each user and W is the signal bandwidth. Let us determine the capacity region for the two-user CDMA system.

The capacity region for the two-user CDMA system with coded signal waveforms has the form illustrated in Fig. 15-2-4, where

$$C_i = W \log_2\left(1 + \frac{P_i}{WN_0}\right), \qquad i = 1, 2$$

are the capacities corresponding to the two users with $P_1 = P_2 = P$. We note that if user 1 is transmitting at capacity C_1, user 2 can transmit up to a maximum rate

$$\begin{aligned} R_{2m} &= W \log_2\left(1 + \frac{2P}{WN_0}\right) - C_1 \\ &= W \log_2\left(1 + \frac{P}{P + WN_0}\right) \end{aligned} \tag{15-2-17}$$

which is illustrated in Fig. 15-2-4 as point A. This result has an interesting

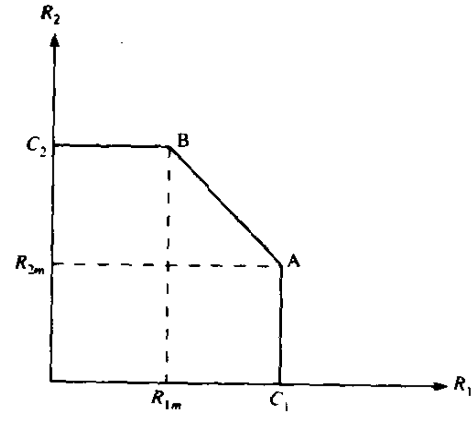


FIGURE 15-2-4 Capacity region of two-user CDMA multiple access gaussian channel.

interpretation. We note that rate R_{2m} corresponds to the case in which the signal from user 1 is considered as an equivalent additive noise in the detection of the signal of user 2. On the other hand, user 1 can transmit at capacity C_1, since the receiver knows the transmitted signal from user 2 and, hence, it can eliminate its effect in detecting the signal of user 1.

Due to symmetry, a similar situation exists if user 2 is transmitting at capacity C_2. Then, user 1 can transmit up to a maximum rate $R_{1m} = R_{2m}$, which is illustrated in Fig. 15.2.4 as point B. In this case, we have a similar interpretation as above, with an interchange in the roles of user 1 and user 2.

The points A and B are connected by a straight line. It is easily seen that this straight line is the boundary of the achievable rate region, since any point on the line corresponds to the maximum rate $W \log_2 (1 + 2P/WN_0)$, which can be obtained by simply time-sharing the channel between the two users.

In the next section, we consider the problem of signal detection for a multiuser CDMA system and assess the performance and the computational complexity of several receiver structures.

15-3 CODE-DIVISION MULTIPLE ACCESS

As we have observed, TDMA and FDMA are multiple access methods in which the channel is partitioned into independent, single-user subchannels, i.e., nonoverlapping time slots or frequency bands, respectively. In CDMA, each user is assigned a distinct signature sequence (or waveform), which the user employs to modulate and spread the information-bearing signal. The signature sequences also allow the receiver to demodulate the message transmitted by multiple users of the channel, who transmit simultaneously and, generally, asynchronously.

In this section, we treat the demodulation and detection of multiuser CDMA signals. We shall see that the optimum maximum-likelihood detector has a computational complexity that grows exponentially with the number of users. Such a high complexity serves as a motivation to devise suboptimum detectors having lower computational complexities. Finally, we consider the performance characteristics of the various detectors.

15-3-1 CDMA Signal and Channel Models

Let us consider a CDMA channel that is shared by K simultaneous users. Each user is assigned a signature waveform $g_k(t)$ of duration T, where T is the symbol interval. A signature waveform may be expressed as

$$g_k(t) = \sum_{n=0}^{L-1} a_k(n) p(t - nT_c), \qquad 0 \leq t \leq T \tag{15-3-1}$$

where $\{a_k(n), 0 \leq n \leq L-1\}$ is a pseudo-noise (PN) code sequence consisting of L chips that take values $\{\pm 1\}$, $p(t)$ is a pulse of duration T_c, and T_c is the chip interval. Thus, we have L chips per symbol and $T = LT_c$. Without loss of generality, we assume that all K signature waveforms have unit energy, i.e.,

$$\int_0^T g_k^2(t)\,dt = 1 \tag{15-3-2}$$

The cross-correlations between pairs of signature waveforms play an important role in the metrics for the signal detector and on its performance. We define the following cross-correlations:

$$\rho_{ij}(\tau) = \int_0^T g_i(t) g_j(t-\tau)\,dt, \qquad i \leq j \tag{15-3-3}$$

$$\rho_{ji}(\tau) = \int_0^T g_i(t) g_j(t+T-\tau)\,dt, \qquad i \leq j \tag{15-3-4}$$

For simplicity, we assume that binary antipodal signals are used to transmit the information from each user. Hence, let the information sequence of the kth user be denoted by $\{b_k(m)\}$, where the value of each information bit may be ± 1. It is convenient to consider the transmission of a block of bits of some arbitrary length, say N. Then, the data block from the kth user is

$$\mathbf{b}_k = [b_k(1) \quad \dots \quad b_k(N)]' \tag{15-3-5}$$

and the corresponding equivalent lowpass, transmitted waveform may be expressed as

$$s_k(t) = \sqrt{\mathscr{E}_k} \sum_{i=1}^{N} b_k(i) g_k(t - iT) \tag{15-3-6}$$

where $\mathscr{E}_k$ is the signal energy per bit. The composite transmitted signal for the K users may be expressed as

$$\begin{aligned} s(t) &= \sum_{k=1}^{K} s_k(t - \tau_k) \\ &= \sum_{k=1}^{K} \sqrt{\mathscr{E}_k} \sum_{i=1}^{N} b_k(i) g_k(t - iT - \tau_k) \end{aligned} \tag{15-3-7}$$

where $\{\tau_k\}$ are the transmission delays, which satisfy the condition $0 \leq \tau_k < T$ for $1 \leq k \leq K$. Without loss of generality, we assume that $0 \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_K < T$. This is the model for the multiuser transmitted signal in an asynchronous mode. In the special case of synchronous transmission, $\tau_k = 0$ for $1 \leq k \leq K$. The values of τ of interest in the cross-correlations given by (15-3-3) and (15-3-4) may also be restricted to $0 \leq \tau < T$, without loss of generality.

The transmitted signal is assumed to be corrupted by AWGN. Hence, the received signal may be expressed as

$$r(t) = s(t) + n(t) \tag{15-3-8}$$

where $s(t)$ is given by (15-3-7) and $n(t)$ is the noise, with power spectral density $\frac{1}{2}N_0$.

15-3-2 The Optimum Receiver

The optimum receiver is defined as the receiver that selects the most probable sequence of bits $\{b_k(n), 1 \le n \le N, 1 \le k \le K\}$ given the received signal $r(t)$ observed over the time interval $0 \le t \le NT + 2T$. First, let us consider the case of synchronous transmission; later, we shall consider asynchronous transmission.

Synchronous Transmission In synchronous transmission, each (user) interferer produces exactly one symbol which interferes with the desired symbol. In additive white gaussian noise, it is sufficient to consider the signal received in one signal interval, say $0 \le t \le T$, and determine the optimum receiver. Hence, $r(t)$ may be expressed as

$$r(t) = \sum_{k=1}^{K} \sqrt{\mathscr{E}_k}\, b_k(1) g_k(t) + n(t), \qquad 0 \le t \le T \tag{15-3-9}$$

The optimum maximum-likelihood receiver computes the log-likelihood function

$$\Lambda(\mathbf{b}) = \int_0^T \left[r(t) - \sum_{k=1}^{K} \sqrt{\mathscr{E}_k}\, b_k(1) g_k(t) \right]^2 dt \tag{15-3-10}$$

and selects the information sequence $\{b_k(1), 1 \le k \le K\}$ that minimizes $\Lambda(\mathbf{b})$. If we expand the integral in (15-3-10), we obtain

$$\begin{aligned} \Lambda(\mathbf{b}) = \int_0^T r^2(t)\, dt &- 2 \sum_{k=1}^{K} \sqrt{\mathscr{E}_k}\, b_k(1) \int_0^T r(t) g_k(t)\, dt \\ &+ \sum_{j=1}^{K} \sum_{k=1}^{K} \sqrt{\mathscr{E}_j \mathscr{E}_k}\, b_k(1) b_j(1) \int_0^T g_k(t) g_j(t)\, dt \end{aligned} \tag{15-3-11}$$

We observe that the integral involving $r^2(t)$ is common to all possible sequences $\{b_k(1)\}$ and is of no relevance in determining which sequence was transmitted. Hence, it may be neglected. The term

$$r_k = \int_0^T r(t) g_k(t)\, dt, \qquad 1 \le k \le K \tag{15-3-12}$$

represents the cross-correlation of the received signal with each of the K signature sequences. Instead of cross-correlators, we may employ matched filters. Finally, the integral involving $g_k(t)$ and $g_j(t)$ is simply

$$\rho_{jk}(0) = \int_0^T g_j(t) g_k(t)\, dt \tag{15-3-13}$$

Therefore, (15-3-11) may be expressed in the form of correlation metrics

$$C(\mathbf{r}_K, \mathbf{b}_K) = 2 \sum_{k=1}^{K} \sqrt{\mathscr{E}_k}\, b_k(1) r_k - \sum_{j=1}^{K} \sum_{k=1}^{K} \sqrt{\mathscr{E}_j \mathscr{E}_k}\, b_k(1) b_j(1) \rho_{jk}(0) \tag{15-3-14}$$

These correlation metrics may also be expressed in vector inner product form as

$$C(\mathbf{r}_K, \mathbf{b}_K) = 2\mathbf{b}_K^t \mathbf{r}_K - \mathbf{b}_K^t \mathbf{R}_s \mathbf{b}_K \tag{15-3-15}$$

where

$$\mathbf{r}_K = [r_1 \quad r_2 \quad \dots \quad r_K]^t, \qquad \mathbf{b}_K = [\sqrt{\mathscr{E}_1}\, b_1(1) \quad \dots \quad \sqrt{\mathscr{E}_K}\, b_K(1)]$$

and $\mathbf{R}_s$ is the correlation matrix, with elements $\rho_{jk}(0)$. It is observed that the optimum detector must have knowledge of the received signal energies in order to compute the correlation metrics.

There are 2^K possible choices of the bits in the information sequence of the K users. The optimum detector computes the correlation metrics for each sequence and selects the sequence that yields the largest correlation metric. We observe that the optimum detector has a complexity that grows exponentially with the number of users, K.

In summary, the optimum receiver for symbol-synchronous transmission consists of a bank of K correlators or matched filters followed by a detector that computes the 2^K correlation metrics given by (15-3-15) corresponding to the 2^K possible transmitted information sequences. Then, the detector selects the sequence corresponding to the largest correlation metric.

Asynchronous Transmission In this case, there are exactly two consecutive symbols from each interferer that overlap a desired symbol. We assume that the receiver knows the received signal energies $\{\mathscr{E}_k\}$ for the K users and the transmission delays $\{\tau_k\}$. Clearly, these parameters must be measured at the receiver or provided to the receiver as side information by the users via some control channel.

The optimum maximum-likelihood receiver computes the log-likelihood function

$$\begin{aligned}
\Lambda(\mathbf{b}) &= \int_0^{NT+2T} \left[r(t) - \sum_{k=1}^{K} \sqrt{\mathscr{E}_k} \sum_{i=1}^{N} b_k(i) g_k(t - iT - \tau_k) \right]^2 dt \\
&= \int_0^{NT+2T} r^2(t)\, dt - 2 \sum_{k=1}^{K} \sqrt{\mathscr{E}_k} \sum_{i=1}^{N} b_k(i) \int_0^{NT+2T} r(t) g_k(t - iT - \tau_k)\, dt \\
&\quad + \sum_{k=1}^{K} \sum_{l=1}^{K} \sqrt{\mathscr{E}_k \mathscr{E}_l} \sum_{i=1}^{N} \sum_{j=1}^{N} b_k(i) b_l(j) \int_0^{NT+2T} g_k(t - iT - \tau_k) g_l(t - jT - \tau_l)\, dt
\end{aligned} \tag{15-3-16}$$

where $\mathbf{b}$ represents the data sequences from the K users. The integral involving $r^2(t)$ may be ignored, since it is common to all possible information sequences. The integral

$$r_k(i) \equiv \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t) g_k(t - iT - \tau_k)\, dt, \qquad 1 \le i \le N \tag{15-3-17}$$

represents the outputs of the correlator or matched filter for the kth user in each of the signal intervals. Finally, the integral

$$\int_0^{NT+2T} g_k(t - iT - \tau_k) g_l(t - jT - \tau_l)\, dt$$
$$= \int_{-iT-\tau_k}^{NT+2T-iT-\tau_k} g_k(t) g_l(t + iT - jT + \tau_k - \tau_l)\, dt \quad (15\text{-}3\text{-}18)$$

may be easily decomposed into terms involving the cross-correlation $\rho_{kl}(\tau) = \rho_{kl}(\tau_k - \tau_l)$ for $k \le l$ and $\rho_{lk}(\tau)$ for $k > l$. Therefore, we observe that the log-likelihood function may be expressed in terms of a correlation metric that involves the outputs $\{r_k(i), 1 \le k \le K, 1 \le i \le N\}$ of K correlators or matched filters—one for each of the K signature sequences. Using vector notation, it can be shown that the NK correlator or matched filter outputs $\{r_k(i)\}$ can be expressed in the form

$$\mathbf{r} = \mathbf{R}_N \mathbf{b} + \mathbf{n} \quad (15\text{-}3\text{-}19)$$

where, by definition

$$\begin{aligned} \mathbf{r} &= [\mathbf{r}'(1) \quad \mathbf{r}'(2) \quad \dots \quad \mathbf{r}'(N)]' \\ \mathbf{r}(i) &= [r_1(i) \quad r_2(i) \quad \dots \quad r_K(i)]' \end{aligned} \quad (15\text{-}3\text{-}20)$$

$$\begin{aligned} \mathbf{b} &= [\mathbf{b}'(1) \quad \mathbf{b}'(2) \quad \dots \quad \mathbf{b}'(N)]' \\ \mathbf{b}(i) &= [\sqrt{\mathscr{E}_1} b_1(i) \quad \sqrt{\mathscr{E}_2} b_2(i) \quad \dots \quad \sqrt{\mathscr{E}_K} b_K(i)]' \end{aligned} \quad (15\text{-}3\text{-}21)$$

$$\begin{aligned} \mathbf{n} &= [\mathbf{n}'(1) \quad \mathbf{n}'(2) \quad \dots \quad \mathbf{n}'(N)]' \\ \mathbf{n}(i) &= [n_1(i) \quad n_2(i) \quad \dots \quad n_K(i)]' \end{aligned} \quad (15\text{-}3\text{-}22)$$

$$\mathbf{R}_N = \begin{bmatrix} \mathbf{R}_a(0) & \mathbf{R}'_a(1) & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{R}_a(1) & \mathbf{R}_a(0) & \mathbf{R}'_a(1) & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_a(1) & \mathbf{R}_a(0) & \mathbf{R}'_a(1) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_a(1) & \mathbf{R}_a(0) \end{bmatrix} \quad (15\text{-}3\text{-}23)$$

and $\mathbf{R}_a(m)$ is a $K \times K$ matrix with elements

$$R_{kl}(m) = \int_{-\infty}^{\infty} g_k(t - \tau_k) g_l(t + mT - \tau_l)\, dt \quad (15\text{-}3\text{-}24)$$

The gaussian noise vectors $\mathbf{n}(i)$ have zero mean and autocorrelation matrix

$$E[\mathbf{n}(k)\mathbf{n}'(j)] = \tfrac{1}{2} N_0 \mathbf{R}_a(k - j) \quad (15\text{-}3\text{-}25)$$

Note that the vector $\mathbf{r}$ given by (15-3-19) constitutes a set of sufficient statistics for estimating the transmitted bits $b_k(i)$.

If we adopt a block processing approach, the optimum ML detector must compute 2^{NK} correlation metrics and select the K sequences of length N that correspond to the largest correlation metric. Clearly, such an approach is much too complex computationally to be implemented in practice, especially when K and N are large. An alternative approach is ML sequence estimation employing the Viterbi algorithm. In order to construct a sequential-type detector, we make use of the fact that each transmitted symbol overlaps at most with $2K-2$ symbols. Thus, a significant reduction in computational complexity is obtained with respect to the block size parameter N, but the exponential dependence on K cannot be reduced.

It is apparent that the optimum ML receiver employing the Viterbi algorithm involves such a high computational complexity that its use in practice is limited to communication systems where the number of users is extremely small, e.g., $K < 10$. For larger values of K, one should consider a sequential-type detector that is akin to either the sequential decoding or the stack algorithms described in Chapter 8. Below, we consider a number of sub-optimums detectors whose complexity grows linearly with K.

15-3-3 Suboptimum Detectors

In the above discussion, we observed that the optimum detector for the K CDMA users has a computational complexity, measured in the number of arithmetic operations (additions and multiplications/divisions) per modulated symbol, that grows exponentially with K. In this subsection we describe suboptimum detectors with computational complexities that grow linearly with the number of users, K. We begin with the simplest suboptimum detector, which we call the conventional (single-user) detector.

Conventional Single-User Detector In conventional single-user detection, the receiver for each user consists of a demodulator that correlates (or match-filters) the received signal with the signature sequence of the user and passes the correlator output to the detector, which makes a decision based on the single correlator output. Thus, the conventional detector neglects the presence of the other users of the channel or, equivalently, assumes that the aggregate noise plus interference is white and gaussian.

Let us consider synchronous transmission. Then, the output of the correlator for the kth user for the signal in the interval $0 \leq t \leq T$ is

$$r_k = \int_0^T r(t) g_k(t)\, dt \tag{15-3-26}$$

$$= \sqrt{\mathscr{E}_k}\, b_k(1) + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sqrt{\mathscr{E}_j}\, b_j(1) \rho_{jk}(0) + n_k(1) \tag{15-3-27}$$

where the noise component $n_k(1)$ is given as

$$n_k(1) = \int_0^T n(t) g_k(t)\, dt \tag{15-3-28}$$

Since $n(t)$ is white gaussian noise with power spectral density $\frac{1}{2}N_0$, the variance of $n_k(1)$ is

$$E[n_k^2(1)] = \tfrac{1}{2}N_0 \int_0^T g_k^2(t)\, dt = \tfrac{1}{2}N_0 \tag{15-3-29}$$

Clearly, if the signature sequences are orthogonal, the interference from the other users given by the middle term in (15-3-27) vanishes and the conventional single-user detector is optimum. On the other hand, if one or more of the other signature sequences are not orthogonal to the user signature sequence, the interference from the other users can become excessive if the power levels of the signals (or the received signal energies) of one or more of the other users is sufficiently larger than the power level of the kth user. This situation is generally called the *near–far problem* in multiuser communications, and necessitates some type of power control for conventional detection.

In asynchronous transmission, the conventional detector is more vulnerable to interference from other users. This is because it is not possible to design signature sequences for any pair of users that are orthogonal for all time offsets. Consequently, interference from other users is unavoidable in asynchronous transmission with the conventional single-user detection. In such a case, the near–far problem resulting from unequal power in the signals transmitted by the various users is particularly serious. The practical solution generally requires a power adjustment method that is controlled by the receiver via a separate communication channel that all users are continuously monitoring. Another option is to employ one of the multiuser detectors described below.

Decorrelating Detector We observe that the conventional detector has a complexity that grows linearly with the number of users, but its vulnerability to the near–far problem requires some type of power control. We shall now devise another type of detector that also has a linear computational complexity but does not exhibit the vulnerability to other-user interference.

Let us first consider the case of symbol-synchronous transmission. In this case, the received signal vector $\mathbf{r}_K$ that represents the output of the K matched filters is

$$\mathbf{r}_K = \mathbf{R}_s \mathbf{b}_K + \mathbf{n}_K \tag{15-3-30}$$

where $\mathbf{b}_K = [\sqrt{\mathscr{E}_1}b_1(1) \ \sqrt{\mathscr{E}_2}b_2(1) \ \ldots \ \sqrt{\mathscr{E}_K}b_K(1)]'$ and the noise vector with elements $\mathbf{n}_K = [n_1(1) \ n_2(1) \ \ldots \ n_K(1)]'$ has a covariance

$$E(\mathbf{n}_K \mathbf{n}_K') = \mathbf{R}_s \tag{15-3-31}$$

Since the noise is gaussian, $\mathbf{r}_K$ is described by a K-dimensional gaussian pdf with mean $\mathbf{R}_s\mathbf{b}_K$ and covariance $\mathbf{R}_s$. That is,

$$p(\mathbf{r}_K \mid \mathbf{b}_K) = \frac{1}{\sqrt{(2\pi)^K \det \mathbf{R}_s}} \exp\left[-\tfrac{1}{2}(\mathbf{r}_K - \mathbf{R}_s\mathbf{b}_K)'\mathbf{R}_s^{-1}(\mathbf{r}_K - \mathbf{R}_s\mathbf{b}_K)\right] \tag{15-3-32}$$

The best linear estimate of $\mathbf{b}_K$ is the value of $\mathbf{b}_K$ that minimizes the likelihood function

$$\Lambda(\mathbf{b}_K) = (\mathbf{r}_K - \mathbf{R}_s\mathbf{b}_K)'\mathbf{R}_s^{-1}(\mathbf{r}_K - \mathbf{R}_s\mathbf{b}_K) \tag{15-3-33}$$

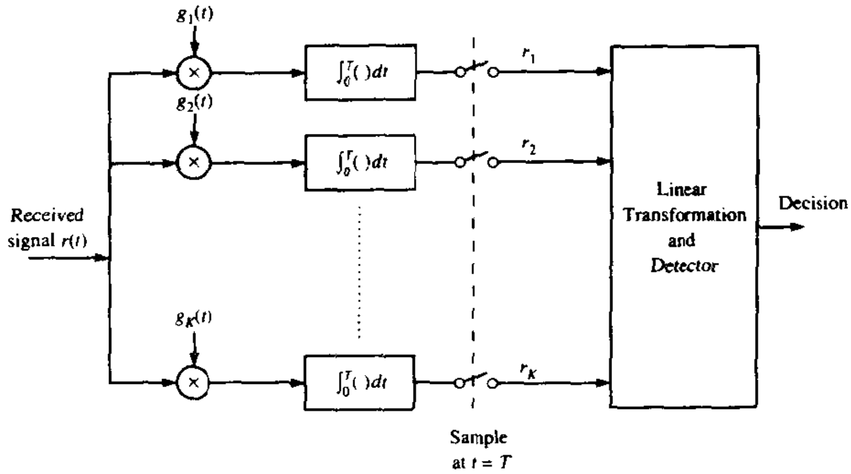


FIGURE 15-3-1 Receiver structure for decorrelation receiver.

The result of this minimization yields

$$\mathbf{b}_K^0 = \mathbf{R}_s^{-1}\mathbf{r}_K \tag{15-3-34}$$

Then, the detected symbols are obtained by taking the sign of each element of $\mathbf{b}_K^0$, i.e.

$$\hat{\mathbf{b}}_K = \operatorname{sgn}(\mathbf{b}_K^0) \tag{15-3-35}$$

Figure 15-3-1 illustrates the receiver structure. Note from (15-3-34) and (15-3-35) that the decorrelator requires knowledge of the relative delays, in general, to form $\mathbf{R}_s$; no knowledge of the signal amplitudes is required.

Since the estimate $\mathbf{b}_K^0$ is obtained by performing a linear transformation on the vector of correlator outputs, the computational complexity is linear in K.

The reader should observe that the best (maximum-likelihood) linear estimate of $\mathbf{b}_K$ given by (15-3-34) is different from the optimum nonlinear ML sequence detector that finds the best discrete-valued $\{\pm 1\}$ sequence that maximizes the likelihood function. It is also interesting to note that the estimate $\mathbf{b}_K^0$ is the best linear estimate that maximizes the correlation metric given by (15-3-15).

An interesting interpretation of the detector that computes $\mathbf{b}_K^0$ as in (15-3-34) and makes decisions according to (15-3-35) is obtained by considering the case of $K = 2$ users. In this case,

$$\mathbf{R}_s = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{15-3-36}$$

$$\mathbf{R}_s^{-1} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix} \tag{15-3-37}$$

where

$$\rho = \int_0^T g_1(t)g_2(t)\,dt \tag{15-3-38}$$

Then, if we correlate the received signal

$$r(t) = \sqrt{\mathscr{E}_1}b_1g_1(t) + \sqrt{\mathscr{E}_2}b_2g_2(t) + n(t) \tag{15-3-39}$$

with $g_1(t)$ and $g_2(t)$, we obtain

$$\mathbf{r}_2 = \begin{bmatrix} \sqrt{\mathscr{E}_1}b_1 + \rho\sqrt{\mathscr{E}_2}b_2 + n_1 \\ \rho\sqrt{\mathscr{E}_1}b_1 + \sqrt{\mathscr{E}_2}b_2 + n_2 \end{bmatrix} \tag{15-3-40}$$

where n_1 and n_2 are the noise components at the output of the correlators. Therefore,

$$\begin{aligned}\mathbf{b}_2^0 &= \mathbf{R}_s^{-1}\mathbf{r}_2 \\ &= \begin{bmatrix} \sqrt{\mathscr{E}_1}b_1 + (n_1 - \rho n_2)/(1-\rho^2) \\ \sqrt{\mathscr{E}_2}b_2 + (n_2 - \rho n_1)/(1-\rho^2) \end{bmatrix}\end{aligned} \tag{15-3-41}$$

This is a very interesting result, because the transformation $\mathbf{R}_s^{-1}$ has eliminated the interference components between the two users. Consequently, the near–far problem is eliminated and there is no need for power control.

It is interesting to note that a result similar to (15-3-41) is obtained if we correlate $r(t)$ given by (15-3-39) with the two modified signature waveforms

$$g_1'(t) = g_1(t) - \rho g_2(t) \tag{15-3-42}$$

$$g_2'(t) = g_2(t) - \rho g_1(t) \tag{15-3-43}$$

This means that, by correlating the received signal with the modified signature waveforms, we have tuned out or *decorrelated* the multiuser interference. Hence, the detector based on (15-3-34) is called a *decorrelating detector.*

In asynchronous transmission, the received signal at the output of the correlators is given by (15-3-19). Hence, the log-likelihood function is given as

$$\Lambda(\mathbf{b}) = (\mathbf{r} - \mathbf{R}_N\mathbf{b})^t\mathbf{R}_N^{-1}(\mathbf{r} - \mathbf{R}_N\mathbf{b}) \tag{15-3-44}$$

where $\mathbf{R}_N$ is defined by (15-3-23) and $\mathbf{b}$ is given by (15-3-21). It is relatively easy to show that the vector $\mathbf{b}$ that minimizes $\Lambda(\mathbf{b})$ is

$$\mathbf{b}^0 = \mathbf{R}_N^{-1}\mathbf{r} \tag{15-3-45}$$

This is the ML estimate of $\mathbf{b}$ and it is again obtained by performing a linear transformation of the outputs from the bank of correlators of matched filters.

Since $\mathbf{r} = \mathbf{R}_N\mathbf{b} + \mathbf{n}$, it follows from (15-3-45) that

$$\mathbf{b}^0 = \mathbf{b} + \mathbf{R}_N^{-1}\mathbf{n} \tag{15-3-46}$$

Therefore, $\mathbf{b}^0$ is an unbiased estimate of $\mathbf{b}$. This means that the multiuser interference has been eliminated, as in the case of symbol-synchronous transmission. Hence, this detector for asynchronous transmission is also called a *decorrelating detector*.

A computationally efficient method for obtaining the solution given by (15-3-45) is the square-root factorization method described in Appendix D. Of course, there are many other methods that may be used to invert the matrix $\mathbf{R}_N$. Iterative methods to decorrelate the signals have also been explored.

Minimum Mean-Square-Error Detector In the above discussion, we showed that the linear ML estimate of $\mathbf{b}$ is obtained by minimizing the quadratic log-likelihood function in (15-3-44). Thus, we obtained the result given by (15-3-45), which is an estimate derived by performing a linear transformation on the outputs of the bank of correlators or matched filters.

Another, somewhat different, solution is obtained if we seek the linear transformation $\mathbf{b}^0 = \mathbf{Ar}$, where the matrix $\mathbf{A}$ is to be determined so as to minimize the mean square error (MSE)

$$\begin{aligned} J(\mathbf{b}) &= E[(\mathbf{b}-\mathbf{b}^0)'(\mathbf{b}-\mathbf{b}^0)] \\ &= E[(\mathbf{b}-\mathbf{Ar})'(\mathbf{b}-\mathbf{Ar})] \end{aligned} \tag{15-3-47}$$

It is easily shown that the optimum choice of $\mathbf{A}$ that minimizes $J(\mathbf{b})$ is

$$\mathbf{A}^0 = (\mathbf{R}_N + \tfrac{1}{2}N_0\mathbf{I})^{-1} \tag{15-3-48}$$

and, hence,

$$\mathbf{b}^0 = (\mathbf{R}_N + \tfrac{1}{2}N_0\mathbf{I})^{-1}\mathbf{r} \tag{15-3-49}$$

The output of the detector is then $\hat{\mathbf{b}} = \operatorname{sgn}(\mathbf{b}^0)$.

The estimate given by (15-3-49) is called the *minimum MSE* (MMSE) estimate of $\mathbf{b}$. Note that when $\frac{1}{2}N_0$ is small compared with the diagonal elements of $\mathbf{R}_N$, the MMSE solution approaches the ML solution given by (15-3-45). On the other hand, when the noise level is large compared with the signal level in the diagonal elements of $\mathbf{R}_N$, $\mathbf{A}^0$ approaches the identity matrix (scaled by $\frac{1}{2}N_0$). In this low-SNR case, the detector basically ignores the interference from other users, because the additive noise is the dominant term. It should also be noted that the MMSE criterion produces a biased estimate of $\mathbf{b}$. Hence, there is some residual multiuser interference.

To perform the computations that lead to the values of $\mathbf{b}$, we solve the set of linear equations

$$(\mathbf{R}_N + \tfrac{1}{2}N_0\mathbf{I})\mathbf{b} = \mathbf{r} \tag{15-3-50}$$

This solution may be computed efficiently using a square-root factorization of the matrix $\mathbf{R}_N + \frac{1}{2}N_0\mathbf{I}$ as indicated above. Thus, to detect NK bits requires $3NK^2$ multiplications. Therefore, the computational complexity is $3K$ multiplications per bit, which is independent of the block length N and is linear in K.

Other Types of Detectors The decorrelating detector and the MMSE detector described above involve performing linear transformations on a block of data from a bank of K correlators or matched filters. The MMSE detector is akin to the linear MSE equalizer described in Chapter 10. Consequently, MMSE multiuser detection can be implemented by employing a tapped-delay-line filter with adjustable coefficients for each user and selecting the filter coefficients to minimize the MSE for each user signal. Thus, the received information bits are estimated sequentially with finite delay, instead of as a block.

The estimate $\mathbf{b}^0$ given by (15-3-46), which is obtained by processing a block of N bits by a decorrelating detector, can also be computed sequentially. Xie *et al.* (1990) have demonstrated that the transmitted bits may be recovered sequentially from the received signal, by employing a form of a decision-feedback equalizer with finite delay. Thus, there is a similarity between the detection of signals corrupted by ISI in a single-user communication system and the detection of signals in a multiuser system with asynchronous transmission.

15-3-4 Performance Characteristics of Detectors

The bit error probability is generally the desirable performance measure in multiuser communications. In evaluating the effect of multiuser interference on the performance of the detector for a single user, we may use as a benchmark the probability of a bit error for a single-user receiver in the absence of other users of the channel, which is

$$P_k(\gamma_k) = Q(\sqrt{2\gamma_k}) \tag{15-3-51}$$

where $\gamma_k = \mathscr{E}_k/N_0$, $\mathscr{E}_k$ is the signal energy per bit and $\frac{1}{2}N_0$ is the power spectral density of the AWGN.

In the case of the optimum detector for either synchronous or asynchronous transmission, the probability of error is extremely difficult and tedious to evaluate. In this case, we may use (15-3-51) as a lower bound and the performance of a suboptimum detector as an upper bound.

Let us consider, first, the suboptimum, conventional single-user detector. For synchronous transmission, the output of the correlator for the kth user is given by (15-3-27). Therefore, the probability of error for the kth user, conditional on a sequence $\mathbf{b}_i$ of bits from other users, is

$$P_k(\mathbf{b}_i) = Q\left(\sqrt{2\left[\sqrt{\mathscr{E}_k} + \sum_{\substack{j=1 \\ j\neq k}}^{K} \sqrt{\mathscr{E}_j}\, b_j(1)\rho_{jk}(0)\right]^2 \Big/ N_0}\right) \tag{15-3-52}$$

Then, the average probability of error is simply

$$P_k = (\tfrac{1}{2})^{K-1} \sum_{\substack{i=1 \\ i\neq k}}^{K} P_k(\mathbf{b}_i) \tag{15-3-53}$$

The probability in (15-3-53) will be dominated by the term that has the smallest argument in the Q function. The smallest argument will result in an SNR of

$$(SNR)_{\min} = \frac{1}{N_0}\left[\sqrt{\mathscr{E}_k} - \sum_{\substack{j=1 \\ j\neq k}}^{K} \sqrt{\mathscr{E}_j}\,|\rho_{jk}(0)|\right]^2 \tag{15-3-54}$$

Therefore,

$$(\tfrac{1}{2})^{K-1}Q(\sqrt{2(SNR)_{\min}}) < P_k < (\tfrac{1}{2})^{K-1}(K-1)Q(\sqrt{2(SNR)_{\min}}) \tag{15-3-55}$$

A similar development can be used to obtain bounds on the performance for asynchronous transmission.

In the case of a decorrelating detector, the other-user interference is completely eliminated. Hence, the probability of error may be expressed as

$$P_k = Q(\mathscr{E}_k/\sigma_k^2) \tag{15-3-56}$$

where σ_k^2 is the variance of the noise in the kth element of the estimate $\mathbf{b}^0$.

Example 15-3-1

Consider the case of synchronous, two-user transmission, where $\mathbf{b}_2^0$ is given by (15-3-41). Let us determine the probability of error.

The signal component for the first term in (15-3-41) is $\sqrt{\mathscr{E}_1}$. The noise component is

$$n = \frac{n_1 - \rho n_2}{1-\rho^2}$$

where ρ is the correlation between the two signature signals. The variance of this noise is

$$\begin{aligned}\sigma_1^2 &= \frac{E[(n_1-\rho n_2)]^2}{(1-\rho^2)^2} \\ &= \frac{1}{1-\rho^2}\frac{N_0}{2}\end{aligned} \tag{15-3-57}$$

and

$$P_1 = Q\left(\sqrt{\frac{2\mathscr{E}_1}{N_0}(1-\rho^2)}\right) \tag{15-3-58}$$

A similar result is obtained for the performance of the second user. Therefore, the noise variance has increased by the factor $(1-\rho^2)^{-1}$. This noise enhancement is the price paid for the elimination of the multiuser interference by the decorrelation detector.

The error rate performance of the MMSE detector is similar to that for the decorrelation detector when the noise level is low. For example, from

(15-3-49), we observe that when N_0 is small relative to the diagonal elements of the signal correlation matrix $\mathbf{R}_N$,

$$\mathbf{b}^0 \approx \mathbf{R}_N^{-1}\mathbf{r} \tag{15-3-59}$$

which is the solution for the decorrelation detector. For low multiuser interference, the MMSE detector results in a smaller noise enhancement compared with the decorrelation detector, but has some residual bias resulting from the other users. Thus, the MMSE detector attempts to strike a balance between the residual interference and the noise enhancement.

An alternative to the error probability as a figure of merit that has been used to characterize the performance of a multiuser communication system is the ratio of SNRs with and without the presence of interference. In particular, (15-3-51) gives the error probability of the kth user in the absence of other-user interference. In this case, the SNR is $\gamma_k = \mathscr{E}_k/N_0$. In the presence of multiuser interference, the user that transmits a signal with energy $\mathscr{E}_k$ will have an error probability P_k that exceeds $P_k(\gamma_k)$. The *effective SNR* γ_{ke} is defined as the SNR required to achieve the error probability

$$P_k = P_k(\gamma_{ke}) = Q(\sqrt{2\gamma_{ke}}) \tag{15-3-60}$$

The *efficiency* is defined as the ratio γ_{ke}/γ_k and represents the performance loss due to the multiuser interference. The desirable figure of merit is the *asymptotic efficiency*, defined as

$$\eta_k = \lim_{N_0 \to 0} \frac{\gamma_{ke}}{\gamma_k} \tag{15-3-61}$$

This figure of merit is often simpler to compute than the probability of error.

Example 15-3-2

Consider the case of two symbol-synchronous users with signal energies $\mathscr{E}_1$ and $\mathscr{E}_2$. Let us determine the asymptotic efficiency of the conventional detector.

In this case, the probability of error is easily obtained from (15-3-52) and (15-3-53) as

$$P_1 = \tfrac{1}{2}Q\left(\sqrt{2(\sqrt{\mathscr{E}_1} + \rho\sqrt{\mathscr{E}_2})^2/N_0}\right) + \tfrac{1}{2}Q\left(\sqrt{2(\sqrt{\mathscr{E}_1} - \rho\sqrt{\mathscr{E}_2})^2/N_0}\right)$$

However, the asymptotic efficiency is much easier to compute. It follows from the definition (15-3-61) and from (15-3-52) that

$$\eta_1 = \left[\max\left(0, 1 - \sqrt{\frac{\mathscr{E}_2}{\mathscr{E}_1}}|\rho|\right)\right]^2$$

A similar expression is obtained for η_2.

The asymptotic efficiency of the optimum and suboptimum detectors that we have described has been evaluated by Verdu (1986), Lupas and Verdu

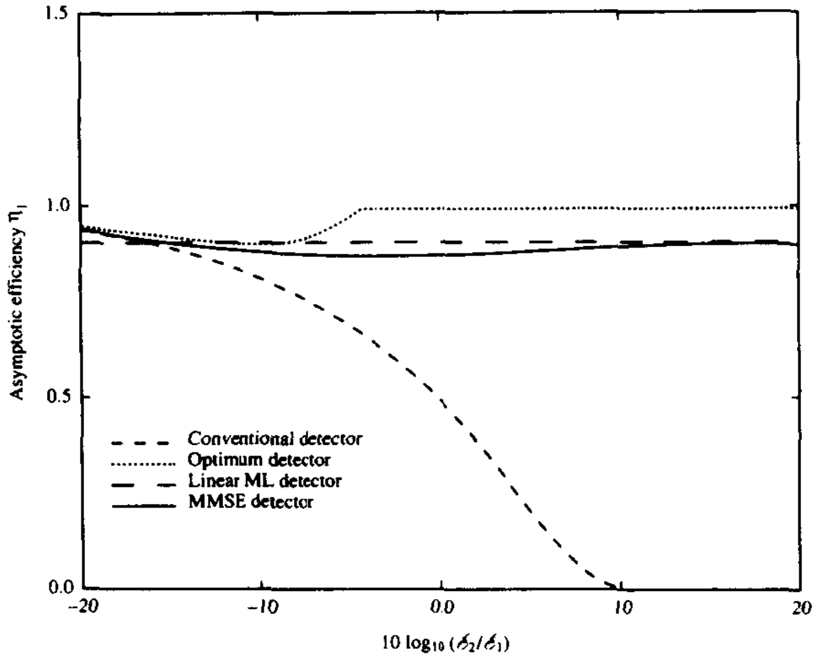


FIGURE 15-3-2 Asymptotic efficiencies of optimum (Viterbi) detector, conventional detector, MMSE detector, and linear ML detector in a two-user synchronous DS/SSMA system. [*From Xie* et al. *(1990), ©IEEE.*]

(1989), and Xie *et al.* (1990). Figure 15-3-2 illustrates the asymptotic efficiencies of these detectors when $K = 2$ users are transmitting synchronously. These graphs show that when the interference is small ($\mathscr{E}_2 \rightarrow 0$), the asymptotic efficiencies of these detectors are relatively large (near unity) and comparable. As $\mathscr{E}_2$ increases, the asymptotic efficiency of the conventional detector deteriorates rapidly. However, the other linear detectors perform relatively well compared with the optimum detector. Similar conclusions are reached by computing the error probabilities, but these computations are often more tedious.

15-4 RANDOM ACCESS METHODS

In this section, we consider a multiuser communication system in which users transmit information in packets over a common channel. In contrast to the CDMA method described in Section 15-3, the information signals of the users are not spread in frequency. As a consequence, simultaneous transmission of signals from multiple users cannot be separated at the receiver. The access methods described below are basically random, because packets are generated according to some statistical model. Users access the channel when they have one or more packets to transmit. When more than one user attempts to transmit packets simultaneously, the packets overlap in time, i.e., they collide,

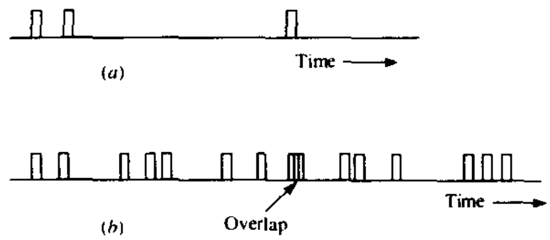


FIGURE 15-4-1 Random access packet transmission: (a) packets from a typical user; (b) packets from several users.

and, hence, a conflict results, which must be resolved by devising some channel protocol for retransmission of the packets. Below, we describe several random access channel protocols that resolve conflicts in packet transmission.

15-4-1 ALOHA Systems and Protocols

Suppose that a random access scheme is employed where each user transmits a packet as soon as it is generated. When a packet is transmitted by a user and no other user transmits a packet for the duration of the time interval then the packet is considered successfully transmitted. However, if one or more of the other users transmits a packet that overlaps in time with the packet from the first user, a collision occurs and the transmission is unsuccessful. Figure 15-4-1 illustrates this scenario. If the users know when their packets are transmitted successfully and when they have collided with other packets, it is possible to devise a scheme, which we may call a *channel access protocol*, for retransmission of collided packets.

Feedback to the users regarding the successful or unsuccessful transmission of packets is necessary and can be provided in a number of ways. In a radio broadcast system, such as one that employs a satellite relay as depicted in Fig. 15-4-2, the packets are broadcast to all the users on the down-link. Hence, all

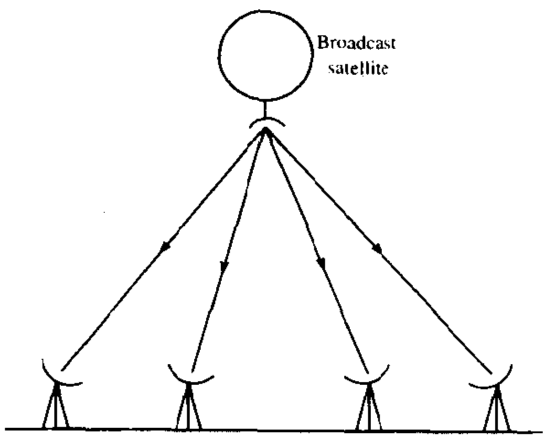


FIGURE 15-4-2 Broadcast system.

the transmitters can monitor their transmissions and, thus, obtain the following ternary information: no packet was transmitted, or a packet was transmitted successfully, or a collision occurred. This type of feedback to the transmitters is generally denoted as $(0, 1, c)$ feedback. In systems that employ wireline or filter-optic channels, the receiver may transmit the feedback signal on a separate channel.

The ALOHA system devised by Abramson (1973, 1977) and others at the University of Hawaii employs a satellite repeater that broadcasts the packets received from the various users who access the satellite. In this case, all the users can monitor the satellite transmissions and, thus, establish whether or not their packets have been transmitted successfully.

There are basically two types of ALOHA systems: *synchronized or slotted* and *unsynchronized or unslotted.* In an unslotted ALOHA system, a user may begin transmitting a packet at any arbitrary time. In a slotted ALOHA, the packets are transmitted in time slots that have specified beginning and ending times.

We assume that the start time of packets that are transmitted is a Poisson point process having an average rate of λ packets/s. Let T_p denote the time duration of a packet. Then, the normalized channel traffic G, also called the *offered channel traffic*, is defined as

$$G = \lambda T_p \tag{15-4-1}$$

There are many channel access protocols that can be used to handle collisions. Let us consider the one due to Abramson (1973). In Abramson's protocol, packets that have collided are retransmitted with some delay τ, where τ is randomly selected according to the pdf

$$p(\tau) = \alpha e^{-\alpha\tau} \tag{15-4-2}$$

where α is a design parameter. The random delay τ is added to the time of the initial transmission and the packet is retransmitted at the new time. If a collision occurs again, a new value of τ is randomly selected and the packet is retransmitted with a new delay from the time of the second transmission. This process is continued until the packet is transmitted successfully. The design parameter α determines the average delay between retransmissions. The smaller the value of α, the longer the delay between retransmissions.

Now, let λ', where $\lambda' < \lambda$, be the rate at which packets are transmitted successfully. Then, the normalized channel throughput is

$$S = \lambda' T_p \tag{15-4-3}$$

We can relate the channel throughput S to the offered channel traffic G by making use of the assumed start time distribution. The probability that a packet will not overlap a given packet is simply the probability that no packet

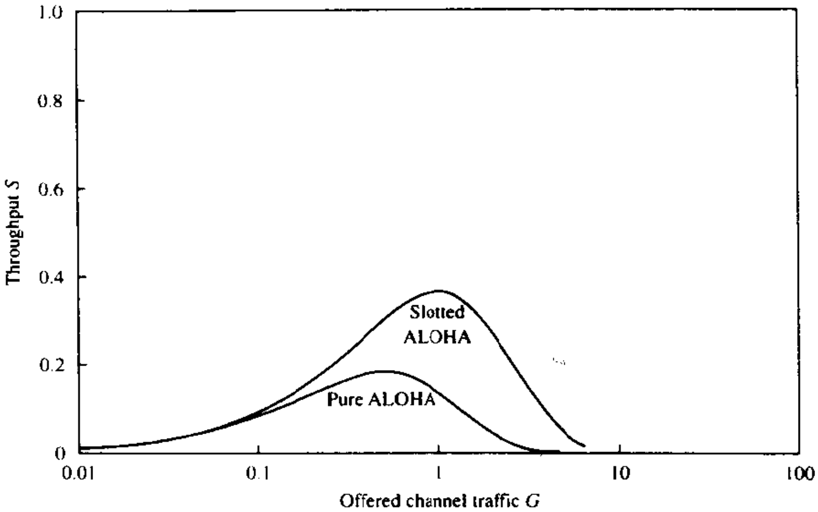


FIGURE 15-4-3 Throughput in ALOHA systems.

begins T_p s before or T_p s after the start time of the transmitted packet. Since the start time of all packets is Poisson-distributed, the probability that a packet will not overlap is $\exp(-2\lambda T_p) = \exp(-2G)$. Therefore,

$$S = Ge^{-2G} \tag{15-4-4}$$

This relationship is plotted in Fig. 15-4-3. We observe that the maximum throughput is $S_{\max} = 1/2e = 0.184$ packets per slot, which occurs at $G = \frac{1}{2}$. When $G > \frac{1}{2}$, the throughput S decreases. The above development illustrates that an unsynchronized or unslotted random access method has a relatively small throughput and is inefficient.

Throughput for slotted ALOHA To determine the throughput in a slotted ALOHA system, let G_i be the probability that the ith user will transmit a packet in some slot. If all the K users operate independently and there is no statistical dependence between the transmission of the user's packet in the current slot and the transmission of the user's packet in previous time slots, the total (normalized) offered channel traffic is

$$G = \sum_{i=1}^{K} G_i \tag{15-4-5}$$

Note that, in this case, G may be greater than unity.

Now, let $S_i \leq G_i$ be the probability that a packet transmitted in a time slot is received without a collision. Then, the normalized channel throughput is

$$S = \sum_{i=1}^{K} S_i \tag{15-4-6}$$

The probability that a packet from the ith user will not have a collision with another packet is

$$Q_i = \prod_{\substack{j=1 \\ j \neq i}}^{K} (1 - G_j) \tag{15-4-7}$$

Therefore,

$$S_i = G_i Q_i \tag{15-4-8}$$

A simple expression for the channel throughput is obtained by considering K identical users. Then,

$$S_i = \frac{S}{K}, \qquad G_i = \frac{G}{K}$$

and

$$S = G\left(1 - \frac{G}{K}\right)^{K-1} \tag{15-4-9}$$

Then, if we let $K \to \infty$, we obtain the throughput

$$S = Ge^{-G} \tag{15-4-10}$$

This result is also plotted in Fig. 15-4-3. We observe that S reaches a maximum throughput of $S_{max} = 1/e = 0.368$ packets per slot at $G = 1$, which is twice the throughput of the unslotted ALOHA system.

The performance of the slotted ALOHA system given above is based on Abramson's protocol for handling collisions. A higher throughput is possible by devising a better protocol.

A basic weakness in Abramson's protocol is that it does not take into account the information on the amount of traffic on the channel that is available from observation of the collisions that occur. An improvement in throughput of the slotted ALOHA system can be obtained by using a tree-type protocol devised by Capetanakis (1979). In this algorithm, users are not allowed to transmit new packets that are generated until all earlier collisions are resolved. A user can transmit a new packet in a time slot immediately following its generation, provided that all previous packets that have collided have been transmitted successfully. If a new packet is generated while the channel is clearing the previous collisions, the packet is stored in a buffer. When a new packet collides with another, each user assigns its respective packet to one of two sets, say A or B, with equal probability (by flipping a coin). Then, if a packet is put in set A, the user transmits it in the next time slot. If it collides again, the user will again randomly assign the packet to one of two sets and the process of transmission is repeated. This process continues until all packets contained in set A are transmitted successfully. Then, all packets in set B are transmitted following the same procedure. All the users monitor the state of the channel, and, hence, they know when all the collisions have been serviced.

When the channel becomes available for transmission of new packets, the earliest generated packets are transmitted first. To establish a queue, the time scale is subdivided into subintervals of sufficiently short duration such that, on average, approximately one packet is generated by a user in a subinterval. Thus, each packet has a "time tag" that is associated with the subinterval in which it was generated. Then, a new packet belonging to the first subinterval is transmitted in the first available time slot. If there is no collision then a packet from the second subinterval is transmitted, and so on. This procedure continues as new packets are generated and as long as any backlog of packets for transmission exists. Capetanakis has demonstrated that this channel access protocol achieves a maximum throughput of 0.43 packets per slot.

In addition to throughput, another important performance measure in a random access system is the average transmission delay in transmitting a packet. In an ALOHA system, the average number of transmissions per packet is G/S. To this number we may add the average waiting time between transmissions and, thus, obtain an average delay for a successful transmission. We recall from the above discussion that in the Abramson protocol, the parameter α determines the average delay between retransmissions. If we select α small, we obtain the desirable effect of smoothing out the channel load at times of peak loading, but the result is a long retransmission delay. This is the trade-off in the selection of α in (15-4-2). On the other hand, the Capetanakis protocol has been shown to have a smaller average delay in the transmission of packets. Hence, it outperforms Abramson's protocol in both average delay and throughput.

Another important issue in the design of random access protocols is the stability of the protocol. In our treatment of ALOHA-type channel access protocols, we implicitly assumed that for a given offered load, an equilibrium point is reached where the average number of packets entering the channel is equal to the average number of packets transmitted successfully. In fact, it can be demonstrated that any channel access protocol, such as the Abramson protocol, that does not take into account the number of previous unsuccessful transmissions in establishing a retransmission policy is inherently unstable. On the other hand, the Capetanakis algorithm differs from the Abramson protocol in this respect and has been proved to be stable. A thorough discussion of the stability issues of random access protocols is found in the paper by Massey (1988).

15-4-2 Carrier Sense Systems and Protocols

As we have observed, ALOHA-type (slotted and unslotted) random-access protocols yield relatively low throughput. Furthermore, a slotted ALOHA system requires that users transmit at synchronized time slots. In channels where transmission delays are relatively small, it is possible to design random

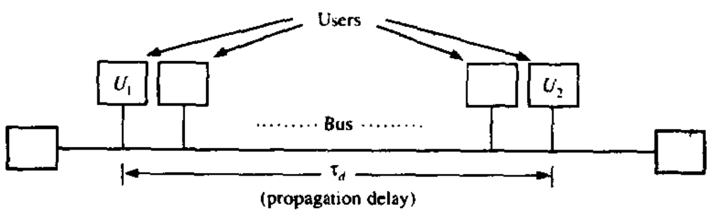


FIGURE 15-4-4 Local area network with bus architecture.

access protocols that yield higher throughput. An example of such a protocol is *carrier sensing* with collision detection, which is used as a standard Ethernet protocol in local area networks. This protocol is generally known as *carrier sense multiple access with collision detection* (CSMA/CD).

The CSMA/CD protocol is simple. All users listen for transmissions on the channel. A user who wishes to transmit a packet seizes the channel when it senses that the channel is idle. Collisions may occur when two or more users sense an idle channel and begin transmission. When the users that are transmitting simultaneously sense a collision, they transmit a special signal, called a *jam signal,* that serves to notify all users of the collision and abort their transmissions. Both the carrier sensing feature and the abortion of transmission when a collision occurs result in minimizing the channel down-time and, hence, yield a higher throughput.

To elaborate on the efficiency of CSMA/CD, let us consider a local area network having a bus architecture, as shown in Fig. 15-4-4. Consider two users U_1 and U_2 at the maximum separation, i.e., at the two ends of the bus, and let τ_d be the propagation delay for a signal to travel the length of the bus. Then, the (maximum) time required to sense an idle channel is τ_d. Suppose that U_1 transmits a packet of duration T_p. User U_2 may seize the channel τ_d s later by using carrier sensing, and begins to transmit. However, user U_1 would not know of this transmission until τ_d s after U_2 begins transmission. Hence, we may define the time interval $2\tau_d$ as the (maximum) time interval to detect a collision. If we assume that the time required to transmit the jam signal is negligible, the CSMA/CD protocol yields a high throughput when $2\tau_d \ll T_p$.

There are several possible protocols that may be used to reschedule transmissions when a collision occurs. One protocol is called *nonpersistent CSMA,* a second is called *1-persistent CSMA,* and a generalization of the latter is called p-persistant CSMA.

Nonpersistent CSMA In this protocol, a user that has a packet to transmit senses the channel and operates according to the following rule.

(a) If the channel is idle, the user transmits a packet.
(b) If the channel is sensed busy, the user schedules the packet

transmission at a later time according to some delay distribution. At the end of the delay interval, the user again senses the channel and repeats steps (a) and (b).

1-Persistent CSMA This protocol is designed to achieve high throughput by not allowing the channel to go idle if some user has a packet to transmit. Hence, the user senses the channel and operates according to the following rule.

(a) If the channel is sensed idle, the user transmits the packet with probability 1.

(b) If the channel is sensed busy, the user waits until the channel becomes idle and transmits a packet with probability one. Note that in this protocol, a collision will always occur when more than one user has a packet to transmit.

p**-Persistent CSMA** To reduce the rate of collisions in 1-persistent CSMA and increase the throughput, we should randomize the starting time for transmission of packets. In particular, upon sensing that the channel is idle, a user with a packet to transmit sends it with probability p and delays it by τ with probability $1-p$. The probability p is chosen in a way that reduces the probability of collisions while the idle periods between consecutive (nonoverlapping) transmissions is kept small. This is accomplished by subdividing the time axis into minislots of duration τ and selecting the packet transmission at the beginning of a minislot. In summary, in the p-persistent protocol, a user with a packet to transmit proceeds as follows.

(a) If the channel is sensed idle, the packet is transmitted with probability p, and with probability $1-p$ the transmission is delayed by τ s.

(b) If at $t=\tau$, the channel is still sensed to be idle, step (a) is repeated. If a collision occurs, the users schedule retransmission of the packets according to some preselected transmission delay distribution.

(c) If at $t=\tau$, the channel is sensed busy, the user waits until it becomes idle, and then operates as in (a) and (b) above.

Slotted versions of the above protocol can also be constructed.

The throughput analysis for the nonpersistent and the p-persistent CSMA/CD protocols has been performed by Kleinrock and Tobagi (1975), based on the following assumptions:

1 the average retransmission delay is large compared with the packet duration T_p;

2 the interarrival times of the point process defined by the start times of all the packets plus retransmissions are independent and exponentially distributed.

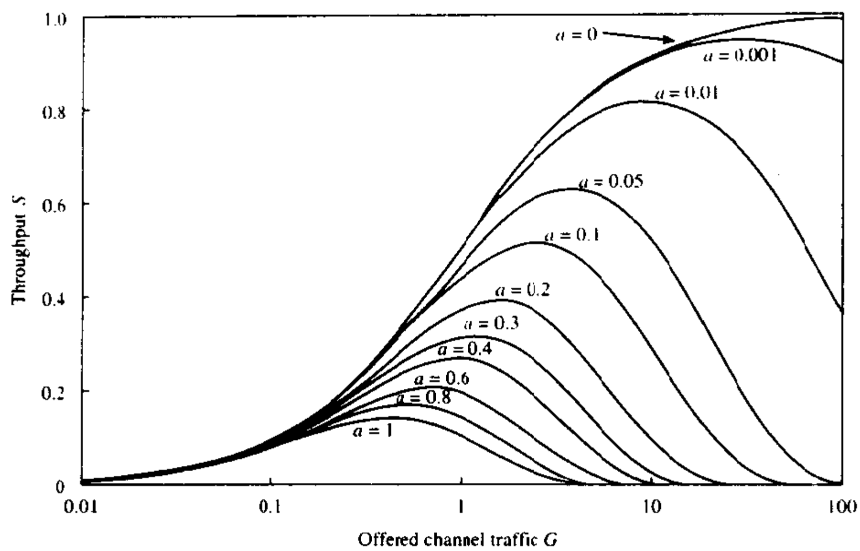


FIGURE 15-4-5 Throughput in nonpersistent CSMA. [*From Kleinrock and Tobagi (1975),* © *IEEE.*]

For the nonpersistent CSMA, the throughput is

$$S = \frac{Ge^{-aG}}{G(1+2a)+e^{-aG}} \tag{15-4-11}$$

where the parameter $a = \tau_d/T_p$. Note that as $a \to 0$, $S \to G/(1+G)$. Figure 15-4-5 illustrates the throughput versus the offered traffic G, with a as a parameter. We observe that $S \to 1$ as $G \to \infty$ for $a = 0$. For $a > 0$, the value of S_{max} decreases.

For the 1-persistent protocol, the throughput obtained by Kleinrock and Tobagi (1975) is

$$S = \frac{G[1+G+aG(1+G+\frac{1}{2}aG)]e^{-G(1+2a)}}{G(1+2a)-(1-e^{-aG})+(1+aG)e^{-G(1+a)}} \tag{15-4-12}$$

In this case,

$$\lim_{a\to 0} S = \frac{G(1+G)e^{-G}}{G+e^{-G}} \tag{15-4-13}$$

which has a smaller peak value than the nonpersistent protocol.

By adopting the p-persistent protocol, it is possible to increase the throughput relative to the 1-persistent scheme. For example, Fig. 15-4-6 illustrates the throughput versus the offered traffic with $a = \tau_d/T_p$ fixed and with p as a parameter. We observe that as p increases toward unity, the maximum throughput decreases.

The transmission delay was also evaluated by Kleinrock and Tobagi (1975). Figure 15-4-7 illustrates the graphs of the delay (normalized by T_p) versus the

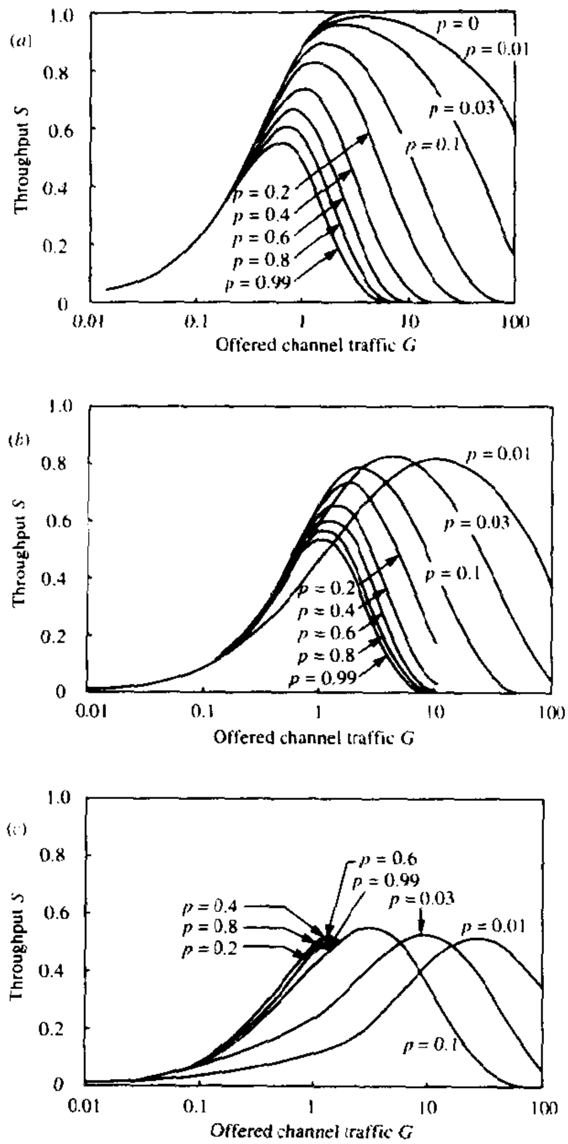


FIGURE 15-4-6 Channel throughput in p-persistent CSMA: (a) $a = 0$; (b) $a = 0.01$; (c) $a = 0.1$ [*From Kleinrock and Tobagi (1975).* ©*IEEE.*]

throughput S for the slotted nonpersistent and p-persistent CSMA protocols. Also shown for comparison is the delay versus throughput characteristic of the ALOHA slotted and unslotted protocols. In this simulation, only the newly generated packets are derived independently from a Poisson distribution. Collisions and uniformly distributed random retransmissions are handled without further assumptions. These simulation results illustrate the superior performance of the p-persistent and the nonpersistent protocols relative to the ALOHA protocols. Note that the graph labeled "optimum p-persistent" is

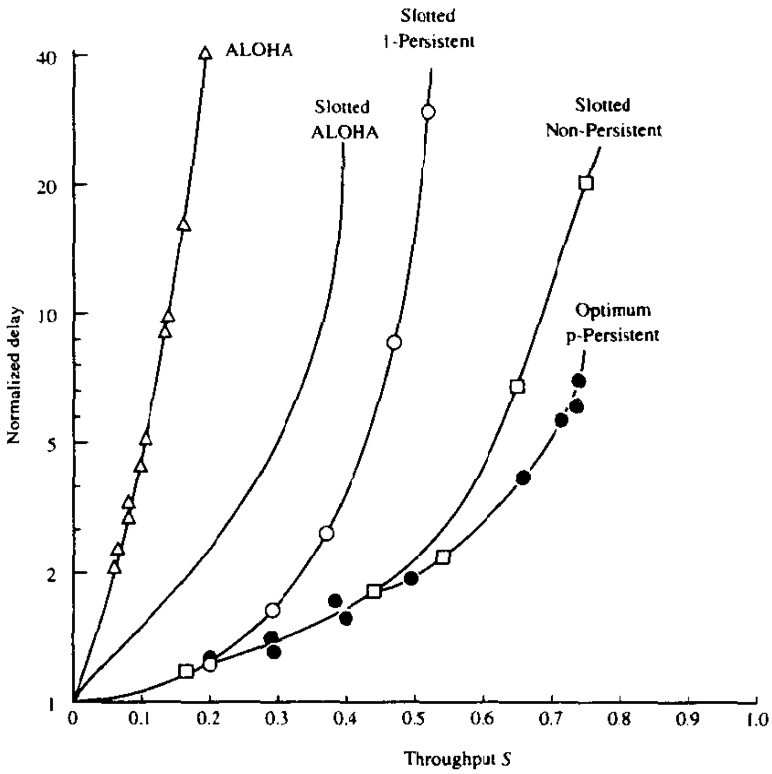


FIGURE 15-4-7 Throughput versus delay from simulation ($a = 0.01$). [*From Kleinrock and Tobagi (1975),* © *IEEE.*]

obtained by finding the optimum value of p for each value of the throughput. We observe that for small values of the throughput, the 1-persistent ($p = 1$) protocol is optimal.

15-5 BIBLIOGRAPHICAL NOTES AND REFERENCES

FDMA was the dominant multiple access scheme that has been used for decades in telephone communication systems for analog voice transmission. With the advent of digital speech transmission using PCM, DPCM, and other speech coding methods, TDMA has replaced FDMA as the dominant multiple access scheme in telecommunications. CDMA and random access methods, in general, have been developed over the past three decades, primarily for use in wireless signal transmission and in local area wireline networks.

Multiuser information theory deals with basic information-theoretic limits in source coding for multiple sources, and channel coding and modulation for multiple access channels. A large amount of literature exists on these topics. In the context of our treatment of multiple access methods, the reader will find the papers by Cover (1972), El Gamal and Cover (1980) Bergmans and Cover (1974), and Hui (1984) particularly relevant. The capacity of a cellular CDMA system has been considered in the paper by Gilhousen *et al.* (1991).

Signal demodulation and detection for multiuser communications has received considerable attention in recent years. The reader is referred to the papers by Verdu (1986a–c, 1989), Lupas and Verdu (1990), Xie *et al.* (1990a, b), Poor and Verdu (1988), Zhang and Brady (1993), and Zvonar and Brady (1995). Earlier work on signal design and demodulation for multiuser communications is found in the papers by Van Etten (1975, 1976), Horwood and Gagliardi (1975), and Kaye and George (1970).

The ALOHA system, which was one of the earliest random access systems, is treated in the papers by Abramson (1970, 1977) and Roberts (1975). These papers contain the throughput analysis for unslotted and slotted systems. Stability issues regarding the ALOHA protocols may be found in the papers by Carleial and Hellman (1975), Ghez *et al.* (1988), and Massey (1988). Stable protocols based on tree algorithms for random access channels were first given by Capetanakis (1977). The carrier sense multiple access protocols that we described are due to Kleinrock and Tobagi (1975). Finally, we mention the IEEE Press book edited by Abramson (1993), which contains a collection of papers dealing with multiple access communications.

PROBLEMS

15-1 In the formulation of the CDMA signal and channel models described in Section 15-3-1, we assumed that the received signals are real. For $K > 1$, this assumption implies phase synchronism at all transmitters, which is not very realistic in a practical system. To accommodate the case where the carrier phases are not synchronous, we may simply alter the signature waveforms for the K users, given by (15-3-1), to be complex-valued, of the form

$$g_k(t) = e^{j\theta_k} \sum_{n=0}^{L-1} a_k(n) p(t - nT_c), \qquad 1 \le k \le K$$

where θ_k represents the constant phase offset of the kth transmitter as seen by the common receiver.

a Given this complex-valued form for the signature waveforms, determine the form of the optimum ML receiver that computes the correlation metrics analogous to (15-3-15).

b Repeat the derivation for the optimum ML detector for asynchronous transmission that is analogous to (15-3-19).

15-2 Consider a TDMA system where each user is limited to a transmitted power P, independent of the number of users. Determine the capacity per user, C_K, and the total capacity KC_K. Plot C_K and KC_K as functions of $\mathscr{E}_b/N_0$ and comment on the results as $K \to \infty$.

15-3 Consider an FDMA system with $K = 2$ users, in an AWGN channel, where user 1 is assigned a bandwidth $W_1 = \alpha W$ and user 2 is assigned a bandwidth $W_2 = (1 - \alpha)W$, where $0 \le \alpha \le 1$. Let P_1 and P_2 be the average powers of the two users.

a Determine the capacities C_1 and C_2 of the two users and their sum $C = C_1 + C_2$ as a function of α. On a two-dimensional graph of the rates R_2 versus R_1, plot the graph of the points (C_2, C_1) as α varies in the range $0 \leq \alpha \leq 1$.

b Recall that the rates of the two users must satisfy the conditions

$$R_1 < W_1 \log_2\left(1 + \frac{P_1}{W_1 N_0}\right)$$

$$R_2 < W_2 \log_2\left(1 + \frac{P_2}{W_2 N_0}\right)$$

$$R_1 + R_2 < W \log_2\left(1 + \frac{P_1 + P_2}{W N_0}\right)$$

Determine the total capacity C when $P_1/\alpha = P_2/(1-\alpha) = P_1 + P_2$, and, thus, show that the maximum rate is achieved when $\alpha/(1-\alpha) = P_1/P_2 = W_1/W_2$.

15-4 Consider a TDMA system with $K = 2$ users in an AWGN channel. Suppose that the two transmitters are peak-power-limited to P_1 and P_2, and let user 1 transmit for $100\alpha\%$ of the available time and user 2 transmit $100(1-\alpha)\%$ of the time. The available bandwidth is W.

a Determine the capacities C_1, C_2, and $C = C_1 + C_2$ as functions of α.

b Plot the graph of the points (C_2, C_1) as α varies in the range $0 \leq \alpha \leq 1$.

15-5 Consider a TDMA system with $K = 2$ users in an AWGN channel. Suppose that the two transmitters are average-power-limited, with powers P_1 and P_2. User 1 transmits $100\alpha\%$ of the time and user 2 transmits $100(1-\alpha)\%$ of the time. The channel bandwidth is W.

a Determine the capacities C_1, C_2, and $C = C_1 + C_2$ as functions of α.

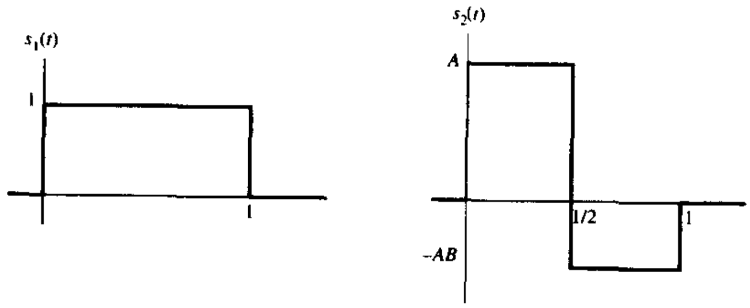
b Plot the graph of the points (C_2, C_1) as α varies in the range $0 \leq \alpha \leq 1$.

c What is the similarity between this solution and the FDMA system in Problem 15-3.

15-6 Consider the two-user, *synchronous*, multiple-access channel and the signature sequences shown in Fig. P15-6. The parameter $A \geq 0$ describes the relative strength between the two users, and $0 \leq B \leq 1$ describes the degree of correlation between the waveforms. Let

$$r(t) = \sum_{k=1}^{2} \sum_{i=-\infty}^{\infty} b_k(i) s_k(t-i) + n(t)$$

FIGURE P15-6



denote the received waveform at time t, where $n(t)$ is white gaussian noise with power spectral density σ^2, and $b_k(i) \in \{-1, +1\}$. In the following problems, you will compare the structure of the conventional multiuser detector to optimimum receiver structures for various values of A, $0 \leq B \leq 1$, and σ^2.

a Show that, given the observation $\{r(t), -\infty < t \leq 1\}$, a sufficient statistic for the data $b_1(0)$ and $b_2(0)$ is the observation during $t \in [0, 1]$.

b Conventional (suboptimum) multiuser detection chooses the data $b_k(0)$ according to the following rule:

$$b_k(0) = \operatorname{sgn}(y_k)$$

where

$$y_k = \int_0^1 r(t)s_k(t)\,dt$$

Determine an expression for the probability of bit error for user 1, using the notation

$$w_k = \int_0^1 s_k^2(t)\,dt$$

$$\rho_{12} = \int_0^1 s_1(t)s_2(t)\,dt.$$

c What is the form of this expression for $A \to 0$, $B < 1$, and arbitrary σ^2?

d What is the form of this expression for arbitrarily large A, $B < 1$, and arbitrary σ^2? What does this say about conventional detection?

e What is the form of this expression for $B = 1$, and arbitrary σ^2 and A? Why does this differ from the result in (d)?

f Determine the form of this expression for arbitrarily large σ^2, arbitrary A, and $B < 1$.

g *Determine the form of this expression for $\sigma^2 \to 0$, arbitrary A, and $B < 1$.*

15-7 Refer to Problem 15-6. The maximum-likelihood sequence receiver for this channel selects the data $b_1(0)$ and $b_2(0)$ transmitted during the interval $[0, 1]$ according to the rule

$$(\widehat{(b_1(0)}, \widehat{b_2(0))}) = \operatorname*{argmax}_{b_1, b_2} \Lambda[\{r(t), 0 < t < 1\} \mid b_1, b_2]$$

where $\Lambda[\{r(t), 0 < t < 1\} \mid b_1, b_2]$ is the likelihood function of b_1 and b_2 given an observation of $\{r(t), 0 < t < 1\}$. It will be helpful to write this maximization as

$$(\widehat{(b_1(0)}, \widehat{b_2(0))}) = \operatorname*{argmax}_{b_1} \operatorname*{argmax}_{b_2} \Lambda[\{r(t), 0 < t < 1\} \mid b_1, b_2]$$

where the value b_2^* that satisfies the inner maximization may depend on b_1. Note that the need for "sequence detection" is obviated.

a Express this maximization in the *simplest* possible terms, using the same notation as in Problem 15-6(b). Reduce this maximization to simplest form, using facts like

$$\operatorname*{argmax}_{x} Ke^{f_1(x)} = \operatorname*{argmax}_{x} f_1(x)$$

if, say, K is independent of x.

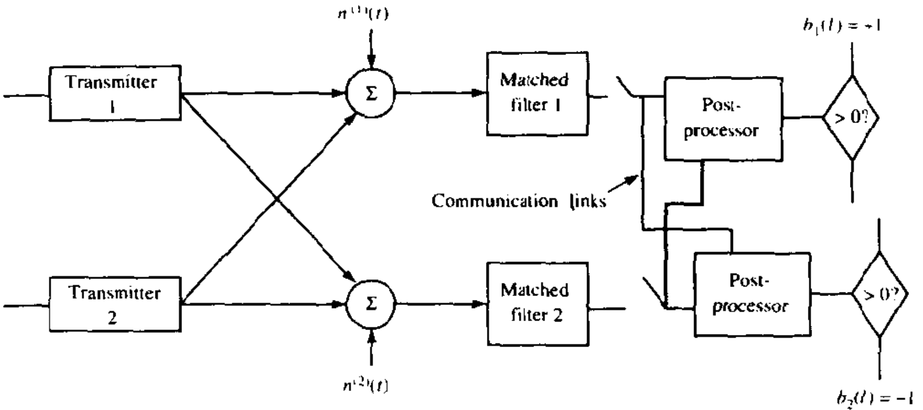


FIGURE P15-8

b What is the simplest structure of the MLS receiver as the relative strength of the interferer vanishes, $A \to 0$? How does it compare with conventional detection?

c What is the simplest structure of the MLS receiver for $B = 1$ and arbitrary A and σ^2? How does it compare with conventional detection? Why?

d What is the simplest structure of the MLS receiver for arbitrarily large σ^2 and arbitrary A and B? How does it compare with conventional detection? Determine the error rate for user 1 in this case. [*Hint:* Use the fact that $\operatorname{sgn}(y_2) = \operatorname{sgn}(y_2 \pm \rho_{12})$ with high probability in this case.]

e Determine the error probability of user 1 of the MLS receiver for $\sigma^2 \to 0$, and arbitrarily large A and $B < 1$? How does it compare with conventional detection?

f What is the structure of the MLS receiver for arbitrarily large A, and $B < 1$, and arbitrary σ_2? How does it compare with conventional detection? What does this say about conventional detection in this case? [*Hint:* Use the fact that $E|y_2|$ is roughly A times greater than $E|y_1|$.]

15-8 Consider the asynchronous communication system shown in Fig. P15-8. The two receivers are not colocated, and the white noise processes $n^{(1)}(t)$ and $n^{(2)}(t)$ may be considered to be independent. The noise processes are identically distributed, with power spectral density σ^2 and zero mean. Since the receivers are not colocated, the relative delays between the users are not the same—denote the relative delay of user k at receiver i by $\tau_k^{(i)}$. All other signal parameters coincide for the receivers, and the received signal at receiver i is

$$r^{(i)}(t) = \sum_{k=1}^{2} \sum_{l=-\infty}^{\infty} b_k(l) s_k(t - lT - \tau_k^{(i)}) + n^{(i)}(t)$$

where s_k has support on $[0, T]$. You may assume that the receiver i has full knowledge of the waveforms, energies, and relative delays $\tau_1^{(i)}$ and $\tau_2^{(i)}$. Although receiver i is eventually interested only in the data from transmitter i, note that there is a free communication link between the sampler of one receiver, and the postprocessing circuitry of the other. Following each postprocessor, the decision is attained by threshold detection. In this problem, you will consider options for postprocessing and for the communication link in order to improve performance.

a What is the bit error probability for users 1 and 2 of a receiver pair that does not utilize the communication link, and does not perform postprocessing. Use the following notation:

$$y_k(l) = \int s_k(t - lT - \tau_k^{(k)})r^{(k)}(t)\,dt$$

$$\rho_{12}^{(i)} = \int s_1(t - \tau_1^{(i)})s_2(t - \tau_2^{(i)})\,dt$$

$$\rho_{21}^{(i)} = \int s_1(t - \tau_1^{(i)})s_2(t + T - \tau_2^{(i)})\,dt$$

$$w_k = \int s_k^2(t - \tau_k^{(1)})\,dt = \int s_k^2(t - \tau_k^{(2)})\,dt$$

b Consider a postprocessor for receiver 1 that accepts $y_2(l-1)$ and $y_2(l)$ from the communication link, and implements the following postprocessing on $y_1(l)$

$$z_l(l) = y_1(l) - \rho_{21}^{(1)}\,\text{sgn}\,[y_2(l-1)] - \rho_{12}^{(1)}\,\text{sgn}\,[y_2(l)].$$

Determine an exact expression for the bit error rate for user 1.

c Determine the asymptotic multiuser efficiency of the receiver proposed in (b), and compare with that in (a). Does this receiver always perform better than that proposed in (a)?

15-9 The baseband waveforms shown in Fig. P15-6 are assigned to two users who share the same *asynchronous*, narrowband channel. Assume that $B = 1$ and $A = 4$. We should like to compare the performance of several receivers, with a criterion of $\mathcal{P}_1(0)$. Since this expression is too complicated in some cases, we shall also be interested in comparing the asymptotic multiuser efficiency η_1 of each receiver. Assume that $\tau_1 = 0$ but that $0 < \tau_2 < T$ is fixed and known at the receiver, and assume that we have infinite horizon transmission, $2M + 1 \to \infty$.

a For the conventional, multiuser detector:

(i) Find the exact bit probability of error for user 1. Express this result in terms of w_i, ρ_{12}, ρ_{21}, and σ^2. [*Hint:* Conditioning on $b_2(-1)$ and $b_2(0)$ will help.]

(ii) Plot the asymptotic multiuser efficiency η_1 as a function of τ_2. Indicate and explain the maximum and minimum values of η_1 in this plot.

b For the MLS receiver:

(i) Plot η_1 as a function of τ_2. Explain maximum and minimum values, and compare with (a)(ii).

(ii) Which error sequences are most likely for each value of τ_2?

c For the limiting decorrelating detector:

(i) Find an exact expression for the probability of error for user 1, with similar parameters as in (a)(i) [*Hint:* Don't forget to normalize ρ_{12} and ρ_{21}.]

(ii) Plot η_1 as a function of τ_2. Explain the minimum value of η_1 in this case, and compare with (a)(ii).

15-10 The symbol-by-symbol detector that minimizes the probability of a symbol error differs from the maximum-likelihood sequence detector. The former is more completely described as the detector that selects each $b_k(0)$ according to the rule

$$\widehat{b_k(0)} = \underset{b_k(0)}{\text{argmax}}\, \Lambda[\{r(t), 0 < t < 1\} \mid b_k(0)]$$

a Show that this decision rule minimizes $\Lambda[b_k(0) \neq \widehat{b_k(0)}]$ among all decision rules with observation $\{r(t), 0 < t < 1\}$. Subject to this criteria, it is superior to the MLS receiver.

b Show that the simplest structure of the minimum-probability-of-error receiver for user 1 is given by

$$\widehat{b_1(0)} = \underset{b_1}{\operatorname{argmax}} \left[\exp\left(\frac{b_1 y_1}{\sigma^2}\right) \cosh\left(\frac{y_2 - b_1 \rho_{12}}{\sigma^2}\right) \right] .$$

c Find the simplest form of the minimum-probability-of-error receiver for $B = 1$ and arbitrary A and σ^2. How does this compare with the above receivers?

d Find the limiting form of the minimum-probability-of-error receiver for arbitrarily large σ^2 and arbitrary A and B. Compare with the above receivers.

e Find the limiting form of the minimum-probability-of-error receiver for $A \gg 1$ and arbitrary σ^2 and B. Compare with the above receivers.

f Find the limiting form of the minimum-probability-of-error receiver for $A \gg 1$ $\sigma^2 \to 0$ and arbitrary B. Compare with the above receivers.

15-11 In a pure ALOHA system, the channel bit rate is 2400 bits/s. Suppose that each terminal transmits a 100 bit message every minute on the average.

a Determine the maximum number of terminals that can use the channel.

b Repeat (a) if slotted ALOHA is used.

15-12 Determine the maximum input traffic for the pure ALOHA and slotted ALOHA protocols.

15-13 For a Poisson process, the probability of k arrivals in a time interval T is

$$P(k) = \frac{e^{-\lambda T}(\lambda T)^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

a Determine the average number or arrivals in the interval T.

b Determine the variance σ^2 in the number of arrivals in the interval T.

c What is the probability of at least one arrival in the interval T?

d What is the probability of exactly one arrival in the interval T?

15-14 Refer to Problem 15-13. The average arrival rate is $\lambda = 10$ packets/s. Determine

a the average time between arrivals;

b the probability that another packet will arrive within 1 s; within 100 ms.

15-15 Consider a pure ALOHA system that is operating with a throughput $G = 0.1$ and packets are generated with a Poisson arrival rate λ. Determine

a the value of λ;

b the average number of attempted transmissions to send a packet.

15-16 Consider a CSMA/CD system in which the transmission rate on the bus is 10 Mbits/s. The bus is 2 km and the propagation delay is 5 μs/km. Packets are 1000 bits long. Determine

a the end-to-end delay τ_d;

b the packet duration T_p;

c the ratio τ_d / T_p;

d the maximum utilization of the bus and the maximum bit rate.