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1) $x^* = 90,142$

Relative error of $x^* \Rightarrow \frac{E_A}{V_A} \Rightarrow \frac{0,0005}{90,1415} \Rightarrow 0,00000554$

Absolute error of $x^* \Rightarrow |X - x|$

$\Rightarrow |90,1415 - 90.142|$

$\Rightarrow 0,0005$

2) $(1,-4,3,2)^T$, $A = \begin{bmatrix} -5 & 2 \\ 3 & 4 \end{bmatrix}$

$\|x\|_1 \Rightarrow \begin{matrix} |-5| + |2| \\ |3| + |4| \end{matrix} \Rightarrow \begin{bmatrix} 7 \\ 7 \end{bmatrix}$

$\|x\|_8 \Rightarrow \begin{matrix} |3|+|1| + |-5| + |3| \\ |-4| + |2| + |2| + |4| \end{matrix} \Rightarrow \begin{bmatrix} 12 \\ 12 \end{bmatrix}$

$\|x\|_2 \Rightarrow 2\begin{bmatrix} 7 \\ 7 \end{bmatrix}$

$\|A\|_8 \Rightarrow \begin{matrix} |1| + |4| + |-5| + |2| \\ |3| + |2| + |3| + |4| \end{matrix} \Rightarrow \begin{bmatrix} 12 \\ 12 \end{bmatrix} \Rightarrow 12$

3) (1) 0,75885 ⇒ 0,759

(2) 11,03712 ⇒ 11,04

(3) 50,415 ⇒ 50,42

(4) 0,73429 ⇒ 0,734

II

1) Reason for simple partial pivoting.

- It helps reduce rounding errors
- It makes an element above or below a leading one into a zero.

→ Its Gaussian with partial pivoting.

→ the formular is $$a_{ik}^{(k)} = \max_{k \leq i \leq n} | a_{ik}^{(k)}$$

→ Its more efficient

→ Fast convergence

2) Methods for solving system of linear equations

Gauss Eliminiation

→ An augumented matrix is form from the linear systems.

→ Operation are a) $\lambda E_i \to E_i$

$$E_i + \lambda E_i \to E_i$$

→ It can solve more than 2 linear equations simultaneously

→ Useful for solving large problems.

Pivot strategies

→ Its Gaussian with partial pivoting

→ The formular is $a_{ik} = \max_{k \le i \le n} |a_{ik}^{(k)}|$

Iterative strategies

III Prove the theorem

Absolute error → so the magnitude of the difference between the true value of the quantity and individual measurement value is called the absolute error.

The theorm says the maximum ~~absta~~ absolute error does not exceed the sum of absolute error that is, if x_i $(i = 1, 2, 3, \dots, n)$ be the n approximate numbers and u is their difference, then $\Delta_u \le \Delta_{x_1} + \Delta_{x_2} + \dots + \Delta_{x_n}$.

we prove by :-

Let n_1, n_2 be approximate number to N_1, N_2 with with errors E_1, E_2 so $N_1 = n_1 + E_2$, $N_2 = n_2 + E_2$ then $N_1 + N_2 = (n_1 + E_1) + (n_1 + E_2) \Rightarrow (n_1 + n_2) + (E_1 + E_2)$.

Let $N_1 + N_2 = N$, $n_1 + n_2 = n$ and $N = n + E$ then.

$N = n + (E_1 + E_2)$

$N - n = E_1 + E_2$

$E = E_1 + E_2$

IV. $x_1 - 5x_2 + x_3 = 3$

$9x_1 + 2x + 3x_3 = 1$

$x_1 + 2x_2 + 4x_3 = 2$

Formular

$$x_i^{(k)} = \frac{1}{a_{ii}}\left[\sum_{\substack{j=1 \\ j \neq i}}^{n} (-a_{ij} x_j^{(k-1)}) + b_i\right]$$

for $i = 1, 2, \cdots, n$

$x_1 = \frac{1}{1}(3 + 5x_2 - 1x_3)$ ~~$x_1 = 3 + 5x_2 - x_3$~~

$x_2 \Rightarrow \frac{1}{2}(1 - 9x_1 - 3x_3) \Rightarrow$ ~~$x_2 = \frac{1}{4} - \frac{2}{4}x_2 - \frac{1}{3}x_3$~~

$x_3 \Rightarrow \frac{1}{4}(2 - 2x_2 - 1x_1) \Rightarrow$ ~~$x_3 = 2 - 2x - 4x_3$~~

~~when $x_2 = 0$~~ $\Rightarrow x_1 = 3 + 5(0) - x_3$

$x_1 \Rightarrow 3 + 5x_2 - x_3$

$x_2 = \frac{1}{2} - \frac{9}{2}x_1 - \frac{3}{2}x_3$

$x_3 = \frac{1}{2} - \frac{1}{4}x_1 - \frac{1}{2}x_2$

$\Rightarrow$ when $x_2 = 0$ then :-

$x_1 = 3 - x_3$

$x_2 = \frac{1}{2} - \frac{9}{2}x_1 - \frac{3}{2}x_3$

$x_3 = \frac{1}{2} - \frac{1}{4}x_1$

when $x_1 = 1$

$x_1 = 3 - x_3$

$x_2 = \frac{1}{2} - \frac{9}{2}(1) - \frac{3}{2}x_3$

$x_3 = \frac{1}{2} - \frac{1}{4}$

$\Rightarrow$ $x_1 = 2{,}75$

$x_2 = -4\frac{3}{8}$

$x_3 \Rightarrow 0{,}25$

V Crout's method

$$x_1 - 5x_2 + x_3 = 3$$

$$9x_1 + 2x_2 + 3x_3 = 1$$

$$x_1 + 2x_2 + 4x_3 = 2$$

$$\begin{bmatrix} 1 & -5 & 1 \\ 9 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

Let, $A = LU$ that

$$\begin{bmatrix} 1 & -5 & 1 \\ 9 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} \quad \text{i.e } LUX = B$$

$$\begin{bmatrix} 1 & -5 & 1 \\ 9 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} a & ag & ah \\ b & bg+c & bh+ci \\ d & dg+e & dh+ei+f \end{bmatrix}$$

$\therefore a = 1, b = 9, d = 1$

$ag = -5 \therefore g = \frac{-5}{1} = 5$

$bg + c = 2 \therefore c = 2 - (-3)(-5) \Rightarrow 13$

$dg + e = 2 \therefore e = -4 - (-1)(-1) \Rightarrow -5$

$ah = 1 \therefore h = 1$

$bh + ci = +3 \therefore i = (3)(2) + 1 \Rightarrow 7$

$dh + ei + f = 4 \therefore f = 3 + 4(-2) + 1 = -4$

$y = 4 \quad = (3)(2) +$

$y = 3 +$

$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 9 & 13 & 0 \\ 1 & -5 & -4 \end{bmatrix}$ and $U \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $LY = B$ Where $Y = UX$

$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 9 & 13 & 0 \\ 1 & -5 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$y_1 = 1$

$y_2 = 1 \quad \therefore \quad x = y = z = 1 \Longleftrightarrow$ proved!!!

$y_3 = 1$