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Assignment 1

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1) a) $P(z=1) \Rightarrow P(x=1, y=1) + P(x=-1, y=-1)$

$\Rightarrow P(x=1)\,P(y=1) + P(x=-1)\,P(y=-1)$

$\Rightarrow 2(\frac{1}{2})(\frac{1}{2})$

$\Rightarrow \frac{1}{2}$

Probability distribution of z

$P(z=-1) = 1 - P(z=1) = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore P_z = \{\frac{1}{2}, \frac{1}{2}\}$

b) $P(x=1, y=1, z=1) = \frac{1}{4}$

$P(x=1)\,P(y=1)\,P(z=1) = \frac{1}{8}$

We conclude that X, Y and z are not independent.

⇒ For X independent of Z

$P(x=1, z=1) = \frac{1}{4} \Leftrightarrow P(x=1)\,P(z=1) = \frac{1}{4}$

$P(x=1, z=-1) = \frac{1}{4} \Leftrightarrow P(x=1)\,P(z=-1) = \frac{1}{4}$

$P(x=-1, z=1) = \frac{1}{4} \Leftrightarrow P(x=-1)\,P(z=1) = \frac{1}{4}$

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} We conclude that X is independent of Z

⇒ For Y independent of Z

$P(y=1, z=1) = \frac{1}{4} \Leftrightarrow P(y=1)\,P(z=1) = \frac{1}{4}$

$P(y=1, z=-1) = \frac{1}{4} \Leftrightarrow P(y=1)\,P(z=-1) = \frac{1}{4}$

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} We conclude that Y is independent of Z

2) $E[X]^2 \Rightarrow$ Formular $\left[\left(\sum_{i=1}^{n} P(X=x_i)\cdot x_i\right)^2\right]$

$\Rightarrow ((-1)(0,2)+0(0,5)+1(0,3))^2 = 0,01$

$E[X^2] \Rightarrow$ Formular $\left[\sum_{i=1}^{n} P(X=x_i)\cdot x_i^2\right]$

$\Rightarrow (-1)^2(0,2)+0^2(0,5)+1^2(0,3)$

$\Rightarrow 0,2+0,3 \Rightarrow 0,5$

3) Is X independent of Y?

$P(X=1,Y=1) = p(X=1)\,p(Y=1)$

$\Rightarrow P(X=1,Y=1)=0 \neq P(X=1)P(Y=1)=1/9$ } We conclude that X is not independent of Y

$E[X^2] =$ Formular $\left[\sum_{i=1}^{n} P(X=x)\cdot x_i^2\right]$

$\Rightarrow (1/3)(1^2)+(2/3)(2^2)$

$\Rightarrow 1/3+8/3$

$\Rightarrow 3$

$E[Y^2] =$ Formular $\left[\sum_{i=1}^{n} P(Y=y_i)\cdot y_i^2\right]$

$\Rightarrow (1/3)(1^2)+(2/3)(2^2)$

$\Rightarrow 1/3+8/3$

$\Rightarrow 3$

$E[X^2+Y^2] \Rightarrow E[X^2]+E[Y^2] = 3+3 \Rightarrow 6$

$E[X^2Y^2] \Rightarrow$ Formular $\left[\sum_{i=1}^{\infty}\sum_{j=1}^{n} P(x_i,y_j)\cdot x_i^2 y_j^2\right]$

$\Rightarrow 0(1^2)(2^2)+1/3((1^2)(2^2)(2)+(2^2)(2^2)) \Rightarrow 8$

4. a) The distribution of Z, i.e., $P(Z=0)$ & $P(Z=1)$

$P(Z=0) = P(X=0,Y=0,Z=0) + P(X=1,Y=1,Z=0) + P(X=1,Y=0,Z=0) +$
$P(X=0,Y=1,Z=0)$

$\Rightarrow 2\,(0,2+0,05)$

$\Rightarrow 0,5$

$P(Z=1) = 1 - P(Z=0)$

$= 0,5$

b) $P(X=0,Y=0 \mid Z=1)$, $P(X=1,Y=0 \mid Z=1)$

$P(X=0,Y=0 \mid Z=1) = P(X=0,Y=0,Z=1) / P(Z=1)$

$= 0,05/0,5 = 0,1$

$P(X=1,Y=0,Z=1) = P(X=1,Y=0,Z=1) / P(Z=1)$

$= 0,2/0,5 = 0,4$

5) a) $P(Z=0 \mid X=0,Y=0) = 1$

$P(Z=1 \mid X=0,Y=0) = 0$

b) $P(X=Z) = 1/2 + 1/4 + 1/2 = 5/6$

$P(Y=Z) = 1/12 + 1/6 + 1/2 = 3/4$

c) $P(x,y,z) : x \in X, y \in Y, z \in Z$

with the aid of a diagram

| (x,y) \ z | 0 | 1 |
|---|---|---|
| (0,0) | 1/12 | 0 |
| (0,1) | 1/4 | 0 |
| (1,0) | 1/6 | 0 |
| (1,1) | 0 | 1/2 |

d) $(Z=0) = P(g(X,Y)=0) = \sum_{(x,y):\, g(x,y)=0} f_{XY}(x,y)$

$P(Z=0) \Rightarrow 1/12 + 1/4 + 1/3 \Rightarrow 1/2$

$P(Z=1) \Rightarrow 1 - 1/2 = 1/2$

e) $E[Z] = 1/2\,(0) + (1/2)(1) = 1/2$

6) $P(X=1, Y=1) = P(Y=1 \mid X=1) P(X=1) = (0,7)(0,5) = \underline{0,35}$

$P(X=1) \Rightarrow$ formular $\left[ \sum_{i=1}^{2} P(X=1 \mid X=x_i) p(X=x_i) \right] = (0,5)(0,7+0,1) = \underline{0,40}$

$$P(X=1 \mid Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{0,35}{0,40} = \underline{0,87532}$$

$$P(X \neq Y) = P(X=0, Y \neq 0) + P(X=1, Y \neq 1) = 0,15 + 0,15 = \underline{0,3}$$

7) $P(X=1) = P(X=2) = P(X=3) = 1/3$

$P(Y=R \mid X=1) = P(Y=B \mid X=2) = 1$, $P(Y=B \mid X=1) = P(Y=R \mid X=2) = 0$,

$P(Y=B \mid X=3) = P(Y=R \mid X=3) = 1/3$

$$P(X=3 \mid Y=R) = \frac{P(Y=R \mid X=3) P(X=3)}{\sum_{i=1}^{3} P(Y=R \mid X=x_i) P(X=x_i)}$$

$$\Rightarrow \frac{(1/2)(1/3)}{(1)(1/3) + (0)(1/3) + (1/2)(1/3)}$$

$$\Rightarrow \underline{1/3}$$