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Numerical Analysis
Assignment 7

亚历克上 M202161029

Assignment-7 Ex: 5.2

1. Using Euler's method to approximate the solution for each following initial value problems :-

a) $y' = te^{3t} - 2y$ where $0 \le t \le 1$, $y(0) = 0$ while $h = 0{,}5$

$\therefore t_0 = t_{i-1} + h$

$w_i = w_{i-1} + h f'(t_i, w_i)$ $\quad t_0 = 0$ and $y_0 = 0$

$t_1 = t_0 + h = 0 + 0{,}5 \qquad w_1 = w_0 + h f(t_i, y_i) = 0 + 0{,}5 f(t_i, y_i)$

$t_1 = 0{,}5 \qquad w_1 = 0$

$t_2 = t_1 + 0{,}5 = 0{,}5 + 0{,}5 \qquad w_2 = w_1 + h f'(t_i, y_i) = 0 + 0{,}5 f(t_i, y_i)$

$t_2 = 1 \qquad w_2 = 1{,}1204223$

$\therefore$ The output is :-

n	t_i	w_i
0	0	0
1	0,5	0
2	1	1,1204

b) $y' = 1 + (t - y^2)$ where $2 \le t \le 3$, $y(2) = 1$ while $h = 0{,}5$

$\therefore t_0 = 2$, $y_0 = 1 \qquad w_0 = 1$

$t_1 = t_0 + h = 2 + 0{,}5 \qquad w_1 = w_0 + h f'(t_0, y_0)$

$t_1 = 2{,}5 \qquad = 2 + 0{,}5(1(2-1)^2)$

$\qquad = 2$

$t_2 = t_1 + h = 2{,}5 + 0{,}5 \qquad w_2 = w_1 + h f'(t_1, w_1)$

$t_2 = 3 \qquad w_2 = 2 + 0{,}5(1 + 2{,}5 - 2)^2)$

$\qquad = 2{,}6250$

The output is :-

n	t_i	w_i
0	2	1
1	2,5	2
2	3	2,6250

2a) The actual solutions to the initial value problem in Exercise 1 are given here and compare the actual error at each step to the error bound

a) $y(t) = \frac{1}{5} te^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$

$\therefore$ Actual value

$y(t) = y(0) = \frac{1}{5}(0)e^{3(0)} - \frac{1}{25}e^{3(0)} + \frac{1}{25}e^{-2(0)} = 0 + 1 - 1 = 0$

$y(t) = y(0,5) = \frac{1}{5}(0,5)e^{3(0,5)} - \frac{1}{25}e^{3(0,5)} + \frac{1}{25}e^{-2(0,5)} = \underline{\underline{0,2836}}$

$y(t) = y(1) = \frac{1}{2}(1)e^{3(1)} - \frac{1}{25}e^{3(1)} + \frac{1}{25}e^{-2(1)} = \underline{\underline{3,2191}}$

Actual error = (Actual value − Approximated value)

$t_0 \Rightarrow (0-0) = 0$

$t_1 \Rightarrow 0,2836165 - 0 \Rightarrow \underline{\underline{0,2836165}}$

$t_2 \Rightarrow 3,2190993 - 1,1204223 = \underline{\underline{2,098677}}$

<u>Error bound</u>

Error Bound $= |y_i - w_i| \leq \frac{hM}{2L}[e^{L(t_i - a)} - 1]$ in range $a \leq t \leq b$

$\left(\frac{\partial f}{\partial y}\right) \leq L \left|\frac{\partial f}{\partial y}(te^{3t} - 2y) = 2\right| = 2$

$\left|\frac{\partial f}{\partial y}\right| \leq 2 \quad \therefore L = 2$

$|y''(t)| \leq m;$

$y'(t) = \frac{1}{5} te^{3t} + \frac{2}{25}e^{3t} - \frac{2}{25}e^{-2t}$

$\Rightarrow \frac{9}{5}te^{3t} + \frac{21}{25}e^{3t} + \frac{4}{25}e^{-2t}$

at $t = 1$ (using mean value $(b) = 1$)

$y''(t) = y''(1) = \frac{9}{5}(1)e^{3(1)} + 21e^{3(1)} + \frac{4}{25}e^{-2(1)}$

$M = y''(1) = \underline{\underline{53,04747}}$

2b) $y(t) = t + \frac{1}{1-t}$

$\therefore$ Actual value

$y(t_0) = y(2) = 2 + \frac{1}{1-2} = 1$

$y(t_1) = y(2{,}5) = 2{,}5 + \frac{1}{1-2{,}5} = 1{,}8333$

$y(t_2) = y(3) = 3 + \frac{1}{1-3} = 2{,}5$

Actual Error

$y(t_1) = (1-1) = 0$

$(t_2) = 1{,}8333 - 2 = -0{,}16667 \therefore 0{,}16667$

$t_3 = 2{,}5 - 2{,} = 0{,}125$

Error bound $= |y(t) - y_i| \leq \frac{hM}{2L}\left[e^{L(t_i-a)} - 1\right]$

$\left|\frac{\partial f}{\partial y}\right| \leq L = \left|\frac{\partial}{\partial y}(1 + (t-y)^2\right| \Rightarrow |2(y-t)|$