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Numerical Analysis
Assignment 2-1

Homework 2 / 2021/09/14 亚历克上 202161024

Ex. 2.2, 1, 5, 6 and Ex. 2.3, 1, 4, 5(a), 6(a)

Showing that the function has a ~~po~~ fixed point at p when $p=0$

$\Rightarrow f(x) = x^4 + 2x^2 - x - 3$

a) $g_1(x) = (3 + x - 2x^2)^{1/4}$

let $p = g_1(p)$

$\therefore \quad p = (3 + p - 2p^2)^{1/4}$

or $p^4 = 3 + p - 2p^3$

or $0 = p^4 + 2p^2 - p - 3$

$\therefore f(p) = 0$

b) $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

let $p = g_2(p)$

then $p = \left(\dfrac{p + 3 - p^4}{2}\right)^{1/2}$

$p^2 = \left(\dfrac{p + 3 - p^4}{2}\right)$

$2p^2 = p + 3 - p^4$

$0 = p^4 + 2p^2 - p - 3$

$\therefore f(p) = 0$

c) $g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$

let $p = g_3(p)$

$\therefore p = \left(\frac{p+3}{p^2+2}\right)^{1/2}$

$p^2 = \frac{p+3}{p^2+2}$

$p^2(p^2+2) = p+3$

$p^4+2p^2 = p+3$

$p^4+2p^2-p-3 = 0$

$f(p) = 0$

d) $g_4(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}$

let $p = g_4(p)$

$\therefore p = \frac{3p^4+2p^2+3}{4p^3+4p-1}$

$4p^4+4p^2-p = 3p^4+2p^2+3$

$0 = p^4+2p^2-p-3$

$f(p) = 0$

5) Using a fixed point iteration method to determine a solution accurate to within 10^{-2} for $x^4-3x^2-3=0$. on $[1,2]$, Using $P_0=1$.

$\therefore x^4-3x^2-3=0$

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$$x^4-3x^2-3=0$$

$$x^4=3x^2+3$$

$$x=\pm\left[3x^2+3\right]^{1/4}$$

$$\therefore g(x)=(3x^2+3)^{1/4}$$

Iteration	a_n	b_n	$f(a_n)$	$f(b_n)$	$P_n=\frac{a_n+b_n}{2}$	$f(P_n)$
1	1	2	1.5650845	1.9679598	1.56508500732	2,340347319
2	2,234034	2	2,099437	1,9679	1.79357287786	2,0495537782
3	2,09955	2	3,01103	1,9679	1.88594374301	1,384321
4	1,3843	2	2,84324	1,9679	1.92284784339	2,349521
5	2,34954	2	2,45236	1,9679	1.937507539	1,745634
6	1,7456	2	2,67734	1,9679	1.9433169899	

At 10^{-2} the solution is 1.9433169899 at 6 iterati

5) Using a fixed-point iteration method, to determine a solution accurate to within 10^{-2} for ~~$x^4 - 3x^2 - 3 = 0$~~ $x^3 - x - 1 = 0$ on $[1,2]$. Using $P_0 = 1$.

$\therefore x^3 - x - 1 = 0$ on the given interval, $x \neq 0$ so we can divide by x

$x^2 = 1 + 1/x$

$x = \sqrt{1 + 1/x} = g(x)$ $\therefore$ satisfying theorm 2.4

1) $g(x) \in [1,2]$

2) $g'(x) = \frac{1}{2\sqrt{1 + 1/x}} \cdot -x^{-2}$ $\therefore$ absolutely less than 1 for all x in the interval $[1,2]$

within 10^{-2} for $x^3 - x - 1 = 0$ on $[1,2]$. Using $P_0 = 1$.

P_i	$g(x) = \sqrt{1 + 1/x}$ from 1	difference
0	1.414213562	0.41421
1	1.306562965	0.10765
2	1.32867104	0.02211
3	1.323869976	0.0048
4		

$\therefore$ The solution accurate is

1.323869976