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COURSE: DIGITAL COMMUNICATION

Fourier Transform

The Fourier Transform decomposes a waveform (a function or signal) into sinusoidal or an alternate representation, characterized by the sine and cosine functions of varying frequencies. The Fourier Transform shows that any waveform can be re-written as the sum of sinusoidal. In other word, it is a mathematical technique that transforms a function of time, x(t), to a function of frequency, X(ω). The Fourier Transform is extensively used in the field of Signal Processing. In fact, the Fourier Transform is probably the most important tool for analysing signals in that entire field.

The result produced by the Fourier transform is a complex valued function of frequency. The Fourier transform is also called a generalization of the Fourier series. This term can also be applied to both the frequency domain representation and the mathematical function used. The Fourier transform helps in extending the Fourier series to non-periodic functions, which allows viewing any function as a sum of simple sinusoids.

$$f(x) = \int_{-\infty}^{\infty} F(k)\, e^{2\pi i k x}\, dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i k x}\, dx$$

Where F(k) can be obtained using inverse Fourier transform.

Some of the properties of Fourier transform include:

- **It is a linear transform** – If g(t) and h(t) are two Fourier transforms given by G(f) and H(f) respectively, then the Fourier transform of the linear combination of g and t can be easily calculated.
- **Time shift property** – The Fourier transform of g(t–a) where a is a real number that shifts the original function has the same amount of shift in the magnitude of the spectrum.
- **Modulation property** – A function is modulated by another function when it is multiplied in time.

- **Parseval's theorem** – Fourier transform is unitary, i.e., the sum of square of a function g(t) equals the sum of the square of its Fourier transform, G(f).
- **Duality** – If g(t) has the Fourier transform G(f), then the Fourier transform of G(t) is g(-f).

Fourier Series

The Fourier Series breaks down a periodic function into the sum of sinusoidal functions. It is the Fourier Transform for periodic functions. It could potentially possess an infinite number of harmonics. A function is periodic, with fundamental period T, if the following is true for all t:

f(t+T)=f(t)

A Fourier Series, with period T, is an infinite sum of sinusoidal functions (cosine and sine), each with a frequency that is an integer multiple of 1/T (the inverse of the fundamental period). The Fourier Series also includes a constant, and hence can be written as:

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$