

2021290010 - Nelay Mahadi Sajjad

1) $x^* = 0,542$ proximation to
 $x = 0,54225$

The relative error of x^* is $\frac{E_A}{X} = \frac{E_A}{V_A} \Rightarrow E_A \Rightarrow 0,54225 - 0,542 = \underline{0,00025}$

$$V_A \Rightarrow 0,542$$

$$\Rightarrow \frac{0,00025}{0,542} \Rightarrow \underline{0,00046125}$$

The Relative error of $x^* \Rightarrow \underline{0,00046125}$

The absolute error of x^* is $\underline{0,00025}$

$$2) \|x\|_1 \Rightarrow \begin{bmatrix} |0| + |2| \\ |3| + |1| \end{bmatrix} \Rightarrow \underline{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}$$

$$\|x\|_\infty \Rightarrow \begin{bmatrix} |0| & |2| \\ |3| & |1| \end{bmatrix} = \underline{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}$$

$$\|A\|_F \Rightarrow \begin{bmatrix} 1+4+0+2 \\ 3+6+3+1 \end{bmatrix} = \underline{\begin{bmatrix} 7 \\ 13 \end{bmatrix}}$$

$$\|x\|_2 \Rightarrow \begin{bmatrix} |0| & |2| \\ |3| & |1| \end{bmatrix} = \underline{\begin{bmatrix} 3 & 3 \end{bmatrix}}$$

3) 3 significant figures:

(1) $0.7285 \Rightarrow \underline{0.729} \Rightarrow 7.29 \times 10^{-1}$

(2) $2.23712 \Rightarrow \underline{2.24}$

(3) $0.4155 \Rightarrow \underline{4.16 \times 10^{-1}}$

(4) $0.5342 \Rightarrow \underline{5.34 \times 10^{-1}}$

II (1) Reasons for using simple partial pivoting

- It is used to avoid round off errors that could be caused when dividing every entry of a row by a pivot value that is relatively small in comparison to its remaining row entry.
- So that its easy to do row interchange if necessary.

(2) Method for getting dominant eigenvalue of a matrix

- The power method was introduced.
- To apply power method, one needs to begin with an initial guess for the eigenvector of the dominant eigenvalue.
- Power method is very useful but not applicable when a matrix is nondiagonalizable.
- The Inverse Power Method calculates the eigenvalue with smallest absolute value.

III PROVE THE THEOREM

- Maximum absolute error is an estimation given when the actual absolute error is unknown.
- The absolute error is the difference between the actual and measured value.
- If the actual value is known and the measured value is known then to calculate absolute error is simple we just subtract.

⇒ Proof

$$\text{Absolute error } (E_A) = \text{measure value } (V_m) - \text{Actual value } (V_A)$$

$$E_A = V_m - V_A$$

when V_m is known for example $\Rightarrow 5$
 $V_A = 3$

$$E_A = 5 - 3 \Rightarrow 2$$

Using Maximum absolute error :-

For example measuring the distance of the ~~building~~ building proportional to the foot.

The length = 30 feet.

we measure = 1 foot

Since a person has 2 foot and the measuring unit is 1 foot \therefore the maximum possible error is $\frac{1}{2}$

→ It might be $-\frac{1}{2}$ or $+\frac{1}{2}$ so denoted by $\pm \frac{1}{2}$

The final answer will be 30 ± 0.5 ft that will be the measurement

IV. JACOBI iterative

$$x_1 - 8x_2 + x_3 = 1$$

$$7x_1 + 2x_2 + 3x_3 = 3$$

$$x_1 + 2x_2 + 5x_3 = 2$$

$$x_1 = 1 + 8x_2 - x_3$$

$$x_2 = \frac{3}{2} - \frac{7}{2}x_1 - \frac{3}{2}x_3$$

$$x_3 = \frac{2}{5} - \frac{1}{5}x_1 + \frac{2}{5}x_2$$

initial values

$$x_1 = 0, x_2 = 0, x_3 = 0$$

$$x_1 = 1 + 0 - 0 = 1$$

$$x_2 = \frac{3}{2} - 0 - 0 = \frac{3}{2}$$

$$x_3 = \frac{2}{5} - 0 + 0 = \frac{2}{5}$$

second values

$$x_1 = 1, x_2 = \frac{3}{2}, x_3 = \frac{2}{5}$$

$$x_1 = 8$$

$$x_2 = -\frac{7}{2}$$

$$x_3 = \frac{1}{5}$$

Third values

$$x_1 = 2, x_2 = 2, x_3 = 2$$

$$x_1 = 15$$

$$x_2 = -8\frac{1}{2}$$

$$x_3 = \frac{4}{5}$$

fourth values

$$x_1 = 3, x_2 = 3, x_3 = 3$$

$$x_1 \Rightarrow 22$$

$$x_2 \Rightarrow 13\frac{1}{2}$$

$$x_3 \Rightarrow 1$$

fifth values

$$x_1 = 4, x_2 = 4, x_3 = 4$$

$$x_1 = 29$$

$$x_2 = -18\frac{1}{2}$$

$$x_3 = 1\frac{1}{5}$$

n	0	1	2	3	4	5
x_1	0,000	1	8	15	22	29
x_2	0,000	$\frac{3}{2}$	$-\frac{7}{2}$	$-8\frac{1}{2}$	$-13\frac{1}{2}$	$-18\frac{1}{2}$
x_3	0,000	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	1	$1\frac{1}{5}$

$$x_1 = 29$$

$$x_2 = -18\frac{1}{2}$$

$$x_3 = 1\frac{1}{5}$$

2. The missing term

x	2	3	4
$F(x)$	0.135	0.1175	0.100

→ Here we are given 2 value of $F(x)$. We assume that $F(x)$ is polynomial of degree 1

$$\text{Hence } \Delta^2 y_0 = 0$$

$$(E-1)^2 y_0 = 0$$

$$(E^2 - 2E + 1)y_0 = 0$$

$$y_2 - 2y_1 + y_0 = 0$$

$$\therefore 0.100 - 2(0.1175) + 0.135 = 0$$

$$-2(0.1175) + 0.135 = 0$$

$$0.1175 = \frac{0.135}{2}$$

$$\underline{0.1175}$$

V) The linear Lagrange interpolating polynomial.

$$P(0) = ?, P(1) = 4, P(2) = 1$$

$$L(x) = \frac{[(x-4)(x-1)]}{[(2-1)(4-1)]}$$

$$L(x) = \frac{(x-4)(x-1)}{3}$$

$$L(0) = -1$$

$$\underline{P = -1}$$

VI $\sqrt[3]{115}$

$$\text{let } f(x) = \sqrt[3]{115}$$

$$\frac{d}{dx} (\sqrt[3]{115}) = 0$$

$$\therefore f(x) = 0$$

Since we ~~can't~~ ^{get} find derivative $= 0$ that means ~~of~~ our third value is $\sqrt[3]{115}$ it ~~does~~ ^{does} not change

VII The relationship of computer science and numerical analysis →

- Numerical analysis is an area of mathematics and in computer science it creates, analyzes and implements algorithms for obtaining numerical solution to problems involving continuous variables.
- It works well on systems that predict things.
- Also works on simulations.
- The backbone of software algorithms.