Wireless Communications

Homework 1

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Summary

The communication on a wireless channel is inherently different from that on a wireline channel. The main difference is that unlike wireline channel, wireless is a shared medium. The entire spectrum is split into many licensed and unlicensed bands.

The transmission over a wireless channel is restricted to a range of frequencies ($f_c - W/2, f_c + W/2$) around the central carrier frequency f_c. The wire is a low pass filter and hence the carrier frequency for the wireline channel is $f_c = 0$.

It turns out that we can always work in with the baseband signal (i.e., the signal with $f_c = 0$) even for the wireless communication and then convert the baseband signal to the passband signal (a signal that is centered around some nonzero carrier fraquency) with the desired carrier frequency. This makes the design of the transmitter and receiver transparent to the carrier frequency. Thus, only the front end of of the sytem needs to be changed if we change f_c.

Also since the bandwidth of the signal W (typically in KHz) is much smaller than the carrier frequency f_c (typically in MHz), the design of DAC and ADC becomes much easier and modular.

The focus of this homework will be on the conversion of the baseband signal to the passband signal and vice-versa. Also, the actual wireless channel affects the passband signal. How do these effects translate in the baseband domain, i.e., is there a baseband equivalent of the wireless channel? We will also address this question.

Baseband Representation of the Passband Signals

The most of the processing such as coding/decoding, modulation/ demodulation etc. is done at the baseband. At the transmitter, the last stage of the operation is to “up-convert” or ”mix” the signal with the carrier frequency and transmit it via the antenna. Similarly, the first step at the receiver is to “down-convert” the RF signal to the baseband before processing. Therefore it is most important to have a baseband equivalent representation of signals.

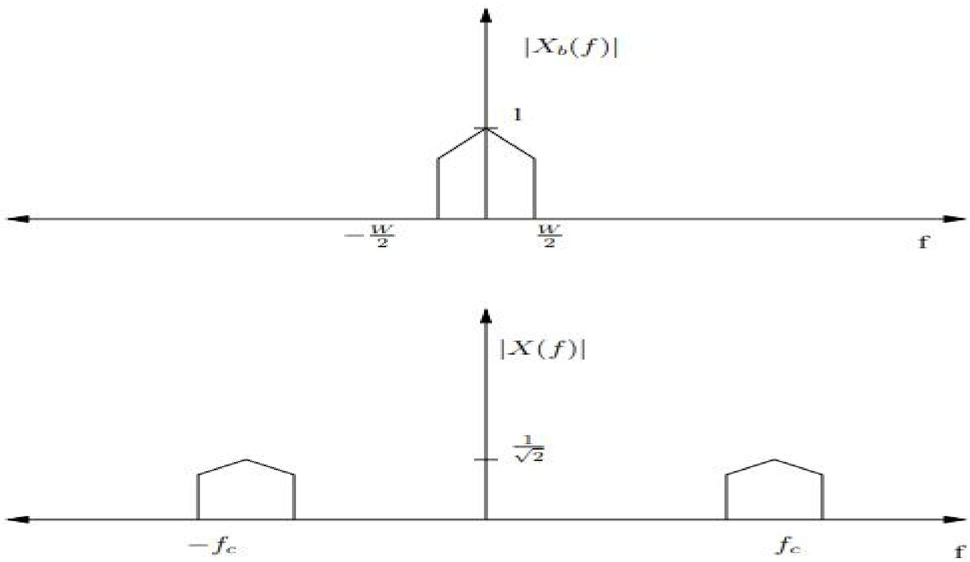

Figure 1: Magnitude spectrum of the real baseband signal and its passband signal

Let’s begin with the real baseband signal $x_b(t)$ (of double sided bandwidth W) that we want to transmit over the wireless channel in a band centered around f_c. In wireline channel, $x_b(t)$ would be the signal at the output of the DAC.

We know that we can up-convert this signal by multiplying it by $\cos 2\pi f_c t$.

$$x(t) = x_b(t) \cdot \sqrt{2} \cos 2\pi fct \quad (1)$$

The resulting signal x(t) has spectrum centered around fc and −fc. Figure 1 shows this transformation diagramatically. We scale the carrier by $\sqrt{2}$ as $\cos 2\pi f_c t$ has power 1/2. Thus, by scaling, we are keeping the power in $x_b(t)$ and x(t) same. Note that since $x_b(t)$ is real, the magnitude of its Fourier transform, $X_b(f)$ is symmetric in f and hence the magnitude of the spectrum of the RF signal, X(f) is symmetric around fc and $-f_c$. We note that to get real x(t), we need not have X(f) symmetric around fc and −fc. This is a consequence of $x_b(t)$ being real.

To get back the baseband signal, we multiply x(t) again by √2cos2πfct and then pass the signal through a low pass filter with bandwidth W.

$$x(t)\ \sqrt{2}\cos 2\pi f_c t = 2\cos^2(2\pi f_c t).x_b(t) \quad (2)$$

$$= (1 + \cos 4\pi f_c t)x_b(t) \quad (3)$$

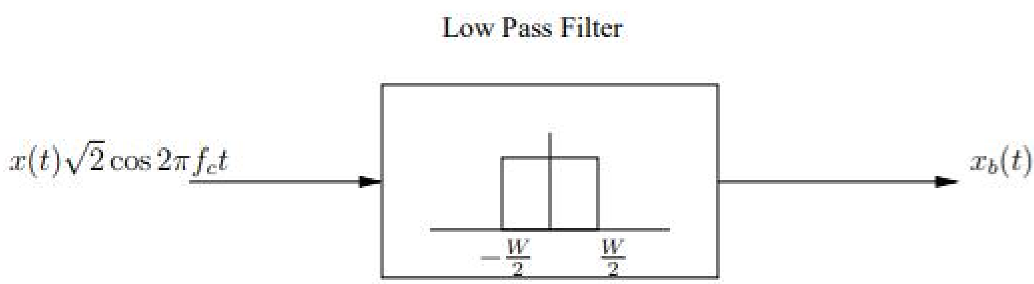

Figure 2: Down-conversion at the receiver

The low pass filter will discard the signal $x_b(t)\cos 4\pi f_c t$ as it is the bandpass signal centered around $2f_c$. Figure 2 shows this transformation diagramatically.

One can see that if we multiply x(t) by $\sqrt{2}\sin 2\pi f_c t$ instead of $\sqrt{2}\cos 2\pi f_c t$, we get $x_b(t)$sin4πfct and low pass filter will discard this signal completely. There will be a similar outcome had we modulated the baseband signal on √2sin2πfct and try to recover it by using √2cos2πfct. Thus,

1. Since the only difference in $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ is the phase lag of π 2, synchronization of carrier phase is crucial in up-conversion and down-conversion.

2. We also note that the signals modulated on √2cos2πfct and $\sqrt{2}\sin 2\pi f_c t$ never get mixed up in the process of down-conversion. Though both the signals share same frequency band, they are orthogonal to each other. Thus, we could have transmitted two real baseband signals in the same frequency band and doubled the data rate. This is possible as now we are using total double sided bandwidth of 2W instead of W as in wireline channel. The resulting RF signal is still real. However, the magnitude of the spectrum of the RF signal need not be symmetric around fc and $-f_c$.

Thus, we can now have the RF signal x(t) which is

$$x(t) = x_{b1}(t)\ \sqrt{2}\cos 2\pi f_c t - x_b 2(t)\sqrt{2}\sin 2\pi f_c t \quad (4)$$

The baseband signals $x_{b1}(t)$ and $x_{b2}(t)$ are obtained at the receiver by multiplying x(t) by $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ separately and then passing both the outputs through the low pass filters. Here we are modulating the amplitude of the carrier by the baseband data. Such a scheme is called amplitude modulation. When we modulate both sin and cos parts of the carrier by two undependent baseband signals, the scheme is called Quadrature Amplitude Modulation (QAM).[1]

The baseband signal xb(t) is now defined in terms of the pair (x_{b1} (t), $x_{v2}(t)$). In literature, this pair is denoted as ($x^{I/B}(t)$, $x^{Q/b}(t)$), where I stands for “in phase” signal and Q stands for “quadrature phase” signal. To make the notation compact we can think of $x_b(t)$ as a complex signal defined as follows:

$$X_b(t) \overset{def}{=} x^{I/b}(t) + jx^{Q/b}(t) \quad (5)$$

We will follow this notation hereafter.

If the wireless channel is just the AWGN channel, then we know how to recover the baseband signal from the RF signal at the receiver and we are done. However, wireless channel is not AWGN channel. If h(t) denote the impulse response of the (time-invarient) wireless channel, the received RF signal is

$$y(t) = h(t) * x(t) + w(t) \quad (6)$$

where w(t) is the RF noise. We will ignore the noise for the time being. Then, $y(t) = h(t) * x(t)$. x(t) is obtained by up-converting the baseband signal $x_b(t)$. We obtain the baseband signal $y_b(t)$ at the receiver by down-converting the received RF signal y(t). The question we want to address now is

How does the channel impulse response manifests itself in baseband? How are the the baseband signals $y_b(t)$ and $x_b(t)$ related?

1 Can we up-convert one more baseband signal and still be able to recover it at the receiver? The answer is negative. This is because, we can express $\cos(2\pi fct+\theta)$ as $\cos\theta \cos 2\pi fct - \sin\theta \sin 2\pi fct$. Thus any phase change θ is uniquely determined by the amplitudes of $\cos 2\pi fct$ and $\sin 2\pi fct$.

It turns out that there is an baseband equivalent filter hb(t) of the channel filter h(t). The transmitted baseband signal $x_b(t)$ is filtered through the baseband channel filter $h_b(t)$ to give the received baseband signal *$y_b(t)$.*[2]

$$yb(t) = hb(t) * xb(t) \quad (7)$$

To understand the relation between h(t) and $h_B(t)$, let's consider a few examples.

1. Let's take the simple case when *h(t) = δ(t). Then, y(t) = x(t) and hence yb(t) = xb(t). Hence, hb(t) = δ(t).*

2. Let's consider *$h(t) = \delta(t - t_0)$.* In this case,

$$y(t) = x(t - t_0) = xI/b(t - t_0)\ \sqrt{2}\cos 2\pi fc(t - t_0) - x\ Q\ b\ (t - t_0)\ \sqrt{}\ \sin 2\pi fc(t - t_0) \quad (8)$$

We obtain the baseband signal $y_b(t)$ as

$$Y^{I/b}(t) = LPF\ (\ y(t)\ \sqrt{}\cos 2\pi f_c t\ ) \quad (9)$$

$$= LPF(\ (2\cos 2\pi fc(t - t0) \cos 2\pi fct)\ x^{I/b}\ (t - t_0)$$

$$-(2\sin 2\pi f_c(t - t_0) \cos 2\pi f_c t) x^{Q/b}\ (t - t_0)\ ) \quad (10)$$

$$= LPF\ (\cos 2\pi f_c(2t - t_0) + \cos 2\pi f_c t_0) x^{I/b}\ (t - t_0)\ )$$

$$-(\sin 2\pi f_c(2t - t_0) - \sin 2\pi f_c t_0) x^{Q/b}\ (t - t_0) \quad (11)$$

$$= x^{I/b}\ (t - t_0) \cos 2\pi f_c t_0 + x^{Q/b}\ (t - t_0) \sin 2\pi f_c t_0 \quad (12)$$

$$= R\ \{\ x_b(t - t_0)\ e^{-j2\pi fct0}\} \quad (13)$$

[2]Similarly, we obtain $y_b^Q\ (t)$ as

$$y^{Q/b}\ (t) = LPF\ (\ -y_b(t)\ \sqrt{}\ 2\sin 2\pi f_c t\ ) \quad (14)$$

$$= I\ \{x_b(t - t_0)\ ^{-j2\pi fct0}\} \quad (15)$$

Thus,

$$y_b(t) = x_b(t - t_0)\ )\ e^{-j2\pi fct0} \quad (16)$$

Hence,

$$h_b(t) = e^{-j2\pi fct0}\ \delta(t - t_0) \quad (17)$$

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[2] *The intuition behind this relation can be obtained by representing the signals in frequency domain. We know that in frequency domain, Y (f) = H(f)X(f). We have also seen from Figure 1 that the X(f) and Xb(f) are related by translation in frequency domain and so are Y (f) and Yb(f). Thus, if we translate H(f) appropriately, we will have Hb(f) so that Yb(f) = Hb(f)Xb(f), i.e., in time domain, yb(t) = hb(t) ∗ xb(t).*

Thus, the baseband signal also gets delayed by the same amount as the passband signal. However, its phase also changes. This phase lag depends on the delay t_0 as well as on the carrier frequency f_c.

We can generalize the second example to obtain the baseband equivalent representation of a generalized channel. Suppose the wireless channel is given by

$$h(t) = \sum_{t=0}^{L-1} a_l \, \delta(t - t_l) \quad (18)$$

Then, the baseband equivalent of the channel will be

$$h_b(t) = \sum_{t=0}^{L-1} a_l \, e^{-j2\pi fctl} \, \delta(t - t_l) \quad (19)$$