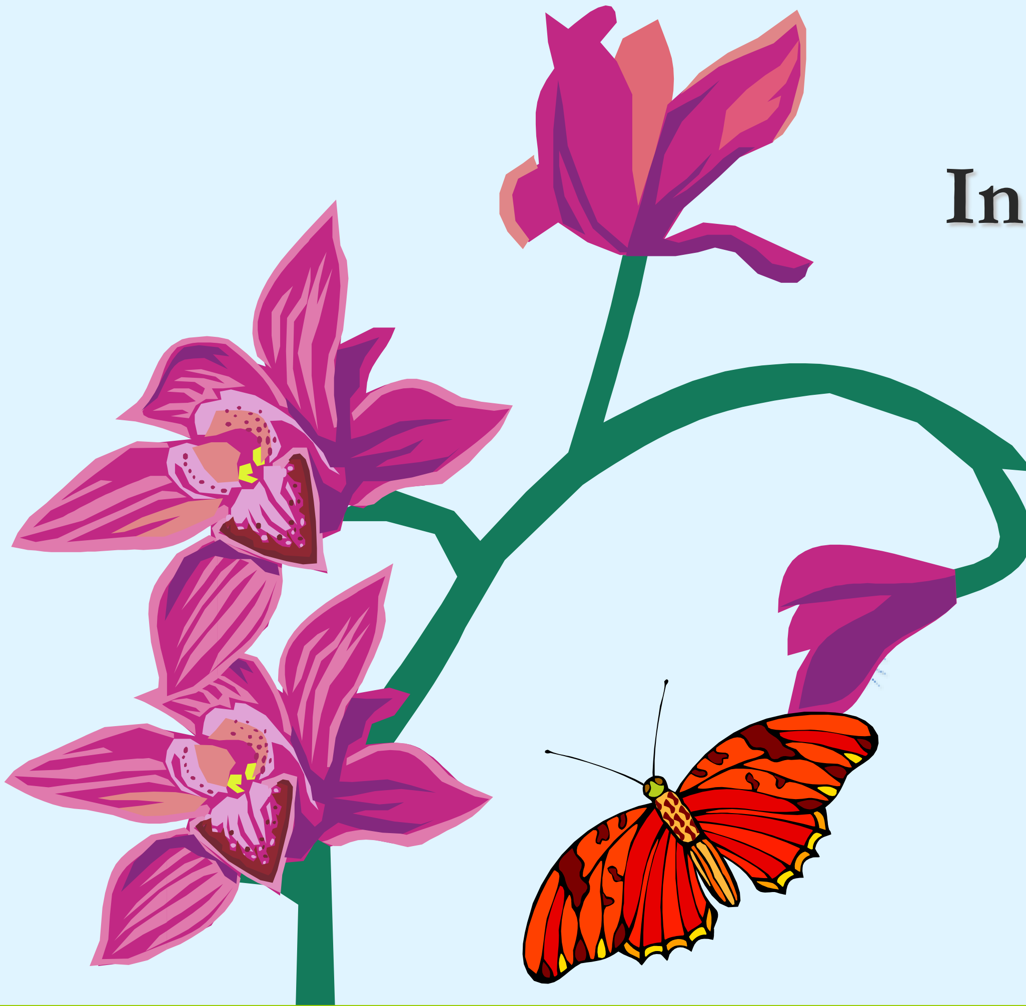
Information Theory and Network Coding

Instructor: Qifu T Sun

qfsun@ustb.edu.cn



Spring Term, 2021-22

***Information theory**, together with the **transistor** and **optical fiber** laid the foundation of the information age and has dramatically changed the daily life of every of us.*

Course information (1)

- **Goal of the course:**
  - (1st part, 2 weeks) Preliminaries on probability theory.
  - (2nd part, 4.5 weeks) Learn the ingredients of *information theory (IT)*, including ***information measures***, ***source coding theorem***, ***channel coding theorem***.
  - (3rd part, 0.75 week) Understand the most basic concept and theorems of *network coding*, a major new century breakthrough in IT society.
  - (4th part, 0.75 week) Project presentation and final exam.

Course information (2)

- Textbook: Raymond Yeung, *Information theory and network coding*, Springer, 2008.

Prof. Yeung is IEEE 2021 Hamming Medal Receipient!

"For fundamental contributions to **information theory** and pioneering **network coding** and its applications."

- The *information theory* part in the course partially follows the MOOCs open course video.
- http://www.inc.cuhk.edu.hk/InformationTheory/index.html

Course information (3) — Grading system

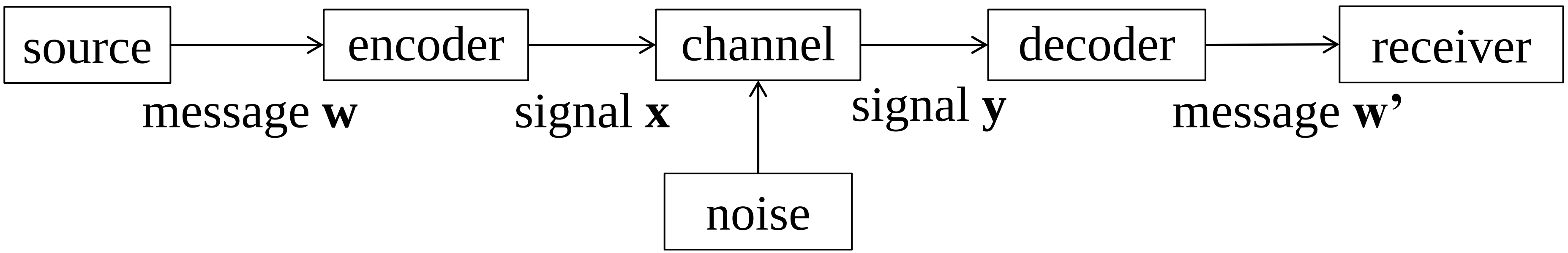
- **Class participation** (40%)
  - Attendance & In-class exercise & Quiz
  - Assignment
  - Project Presentation
- **Open-book take-home final exam** (60%)
  - On Week 8, 13 Apr, (Tentative)

Teaching Format

- **Tencent meeting for online teaching**

- **Wechat group to disseminate lecture notes/assignments/notification**

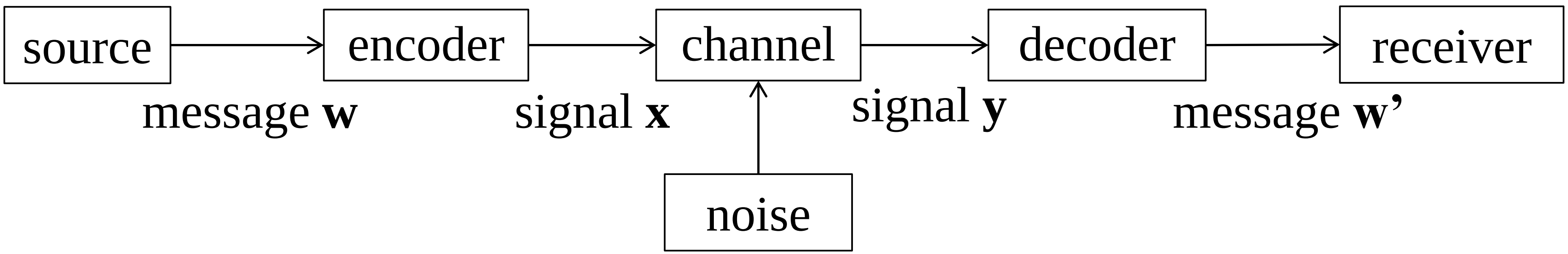
A glance at information theory and coding theory



The end-to-end data transmission model

The main goal of digital communication is to study how to **reliably** and **efficiently** transmit data as much as possible from one end to the other.

A glance at information theory and coding theory



The end-to-end data transmission model

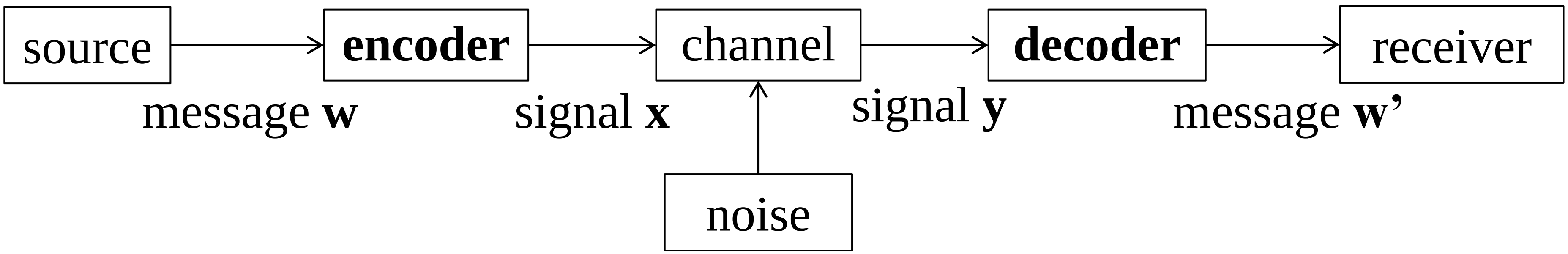
- **Information theory:** study the ***fundamental limits*** on digital signal processing operations such as data compression and reliable data transmission.

  Information theory is considered as **the root of information science**.

- **Coding theory**: How to design *practical coding schemes* with performance approaching to the information theoretical limits.

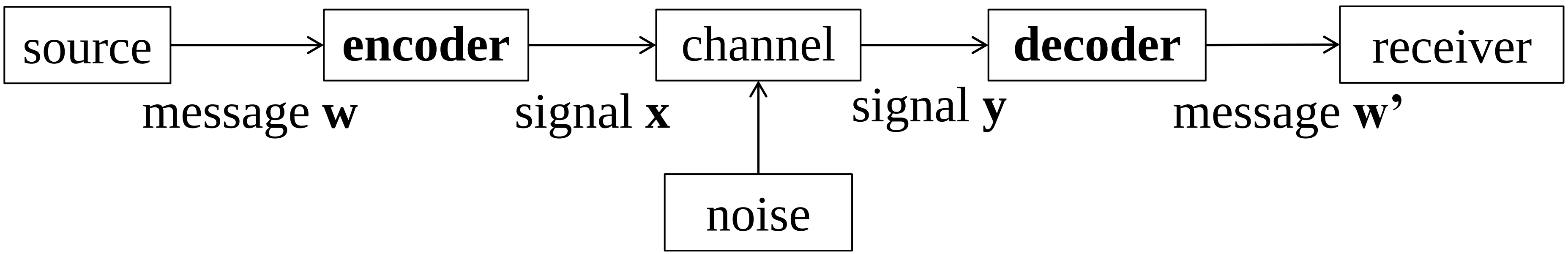
Information theory and coding theory can be regarded as *dual parts*.

A glance at information theory and coding theory



- Corresponding to information theoretical limits on data compression and reliable data communication, there are two most basic aspects in coding theory:
  - **Source coding** (For *data compression*, reduce redundancy among data)
  - **Channel coding** (For *error correction*, increase redundancy to combat with transmission error due to channel noise)
- **Network coding** is a new technique in coding theory, which enhances *multicast* throughput.

Examples for source coding

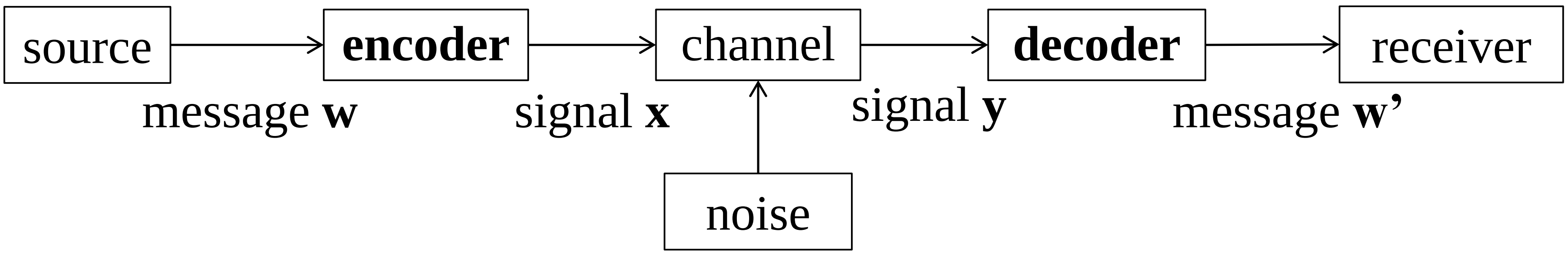


Example. W = {A, B, C, D}. p(A) = 1/2, p(B) = 1/4, p(C) = p(D) = 1/8. How to encode messages in W into binary bits for data transmission?

- Method 1: A → 00, B → 01, C → 10, D → 11 √
- Method 2: A → 0, B → 1, C → 10, D → 11 ×
- Method 3: A → 0, B → 10, C → 110, D → 111 √ Huffman coding

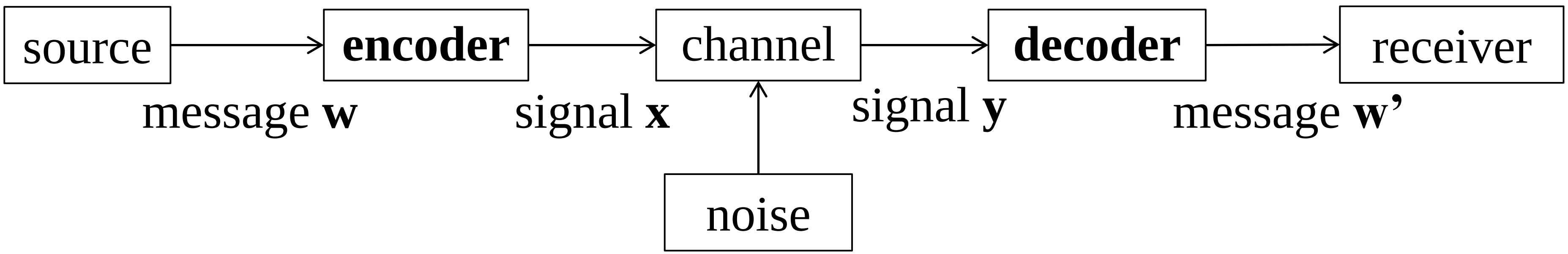
Method 3 is better than method 1, and is the **best** we can do for W.

Examples for source coding

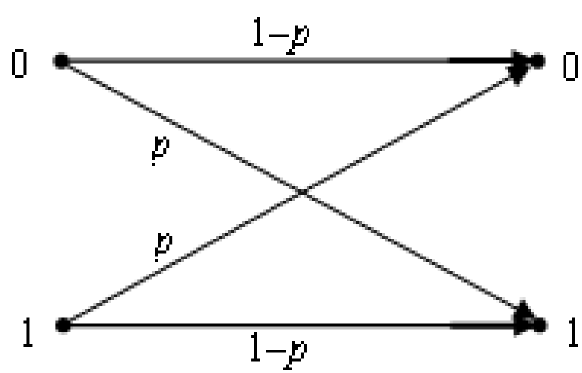


- **Source coding** (For *data compression*, reduce redundancy among data)
  - .zip, .rar, .png files (*lossless* data compression)
  - .jpg, .mp3, .mp4 files (*lossy* data compression)

Examples for channel coding



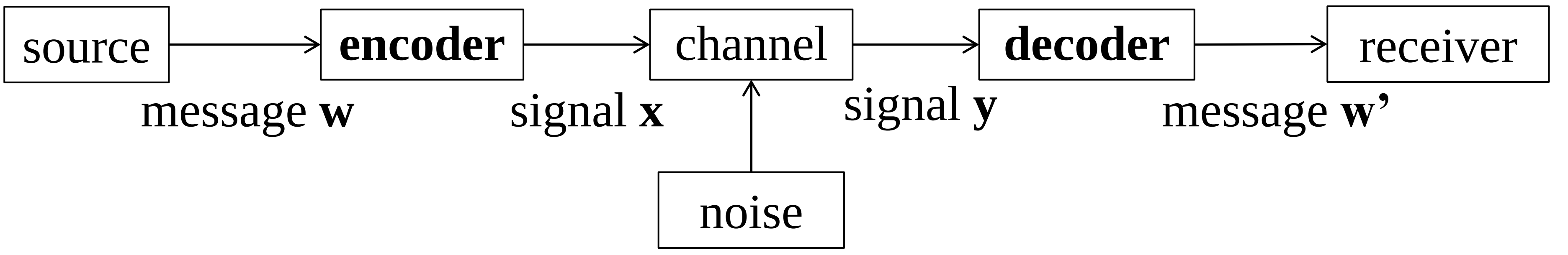
Example. Consider the binary symmetric channel (BSC), $p < 1/2$.



- Method 1: directly transmit 0 or 1. $P_e = p$.
- Method 2: transmit '000' or '111'.

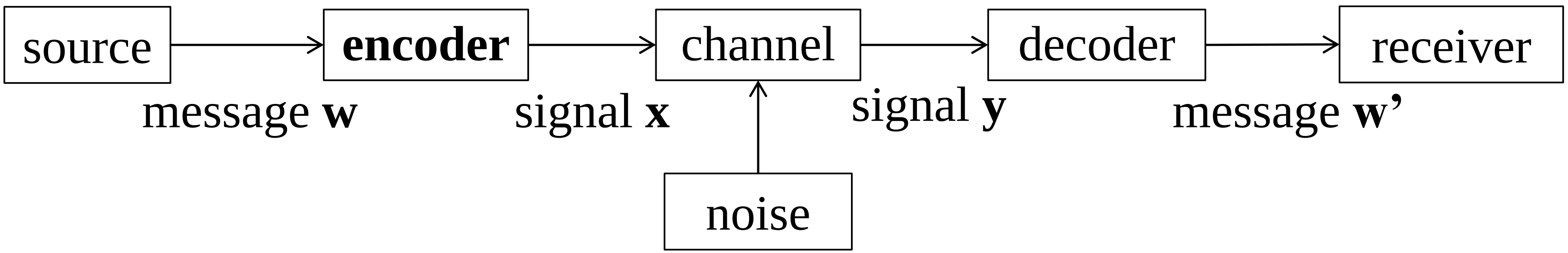
$$P_e = p\{2 \text{ bit errors}\} + p\{3 \text{ bit errors}\} = 3p^2 - 2p^3 < p$$

Examples for channel coding



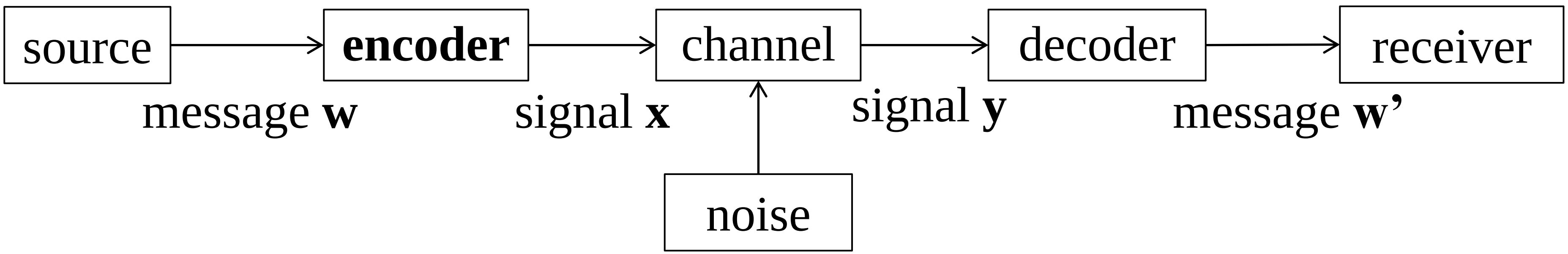
- **Channel coding** (For *error correction*, increase redundancy to combat with transmission error due to channel noise)
  - Reed-Solomon codes (algebraic block codes, used in CD, Blu-ray, DSL, deep-space communication)
  - Fountain codes (capacity-approaching codes, used in WiFi, WiMax, digital video broadcasting)
  - LDPC codes, polar codes (adopted in 5G standards)

Summary of encoder components



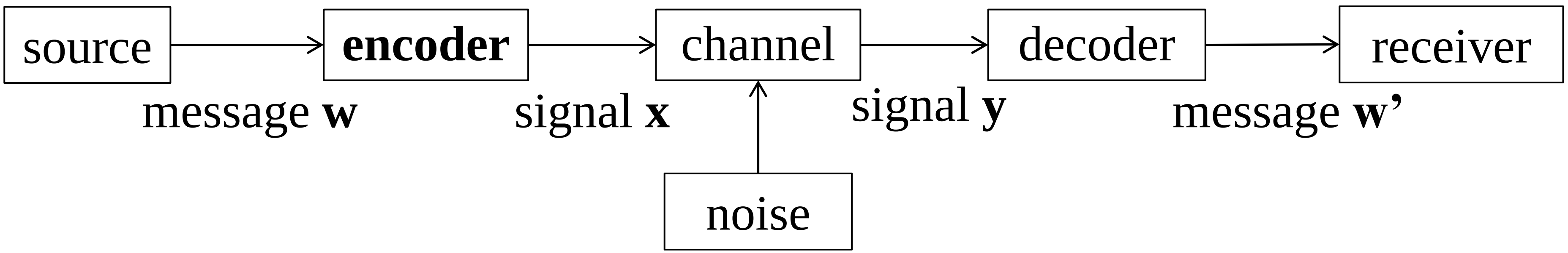
- The major components in data encoder:
  - **Source coding** (For *data compression*, reduce redundancy among data)
  - **Channel coding** (For *error correction*, increase redundancy to combat with transmission error due to channel noise)
- *Q: What is the difference between source and channel coding?*

Summary of encoder components



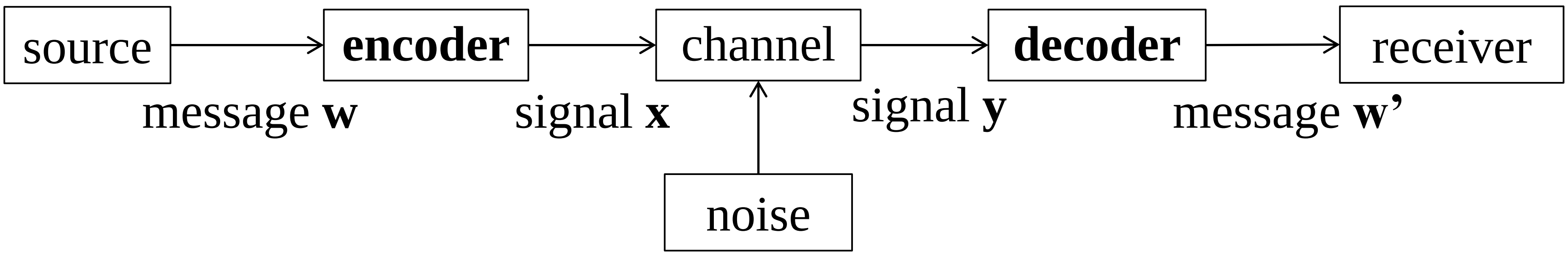
- The major components in data encoder:
  - **Source coding** (For *data compression*, reduce redundancy among data)
  - **Channel coding** (For *error correction*, increase redundancy to combat with transmission error due to channel noise)
  - **Modulation** (map the encoded data to different signals suitable to different channels)

Summary of decoder components



- The major components in signal decoder:
  - Demodulation
  - Channel decoding
  - Source decoding

Example for network coding



- Classical information theory and coding theory are developed for *point-to-point* communication systems.
- When we extend our scope to networks, much less is known on the network capacity (theoretical limit of the reliable transmission rate).
  - *Network information theory*
  - ***Network coding theory***

Example for network coding

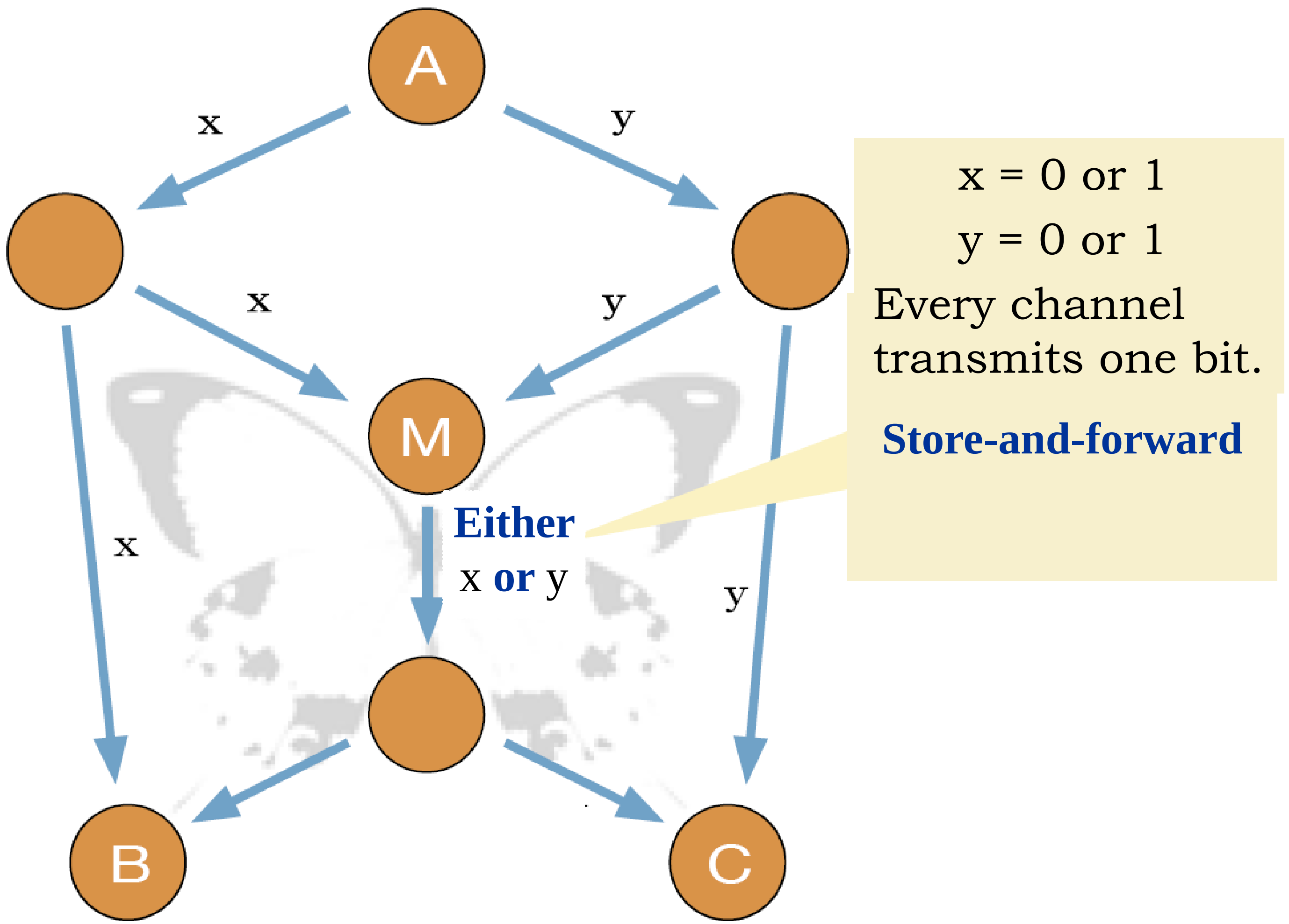


Figure adapted from *Scientific American*, Chinese 7/2007 edition

Store-and-forward

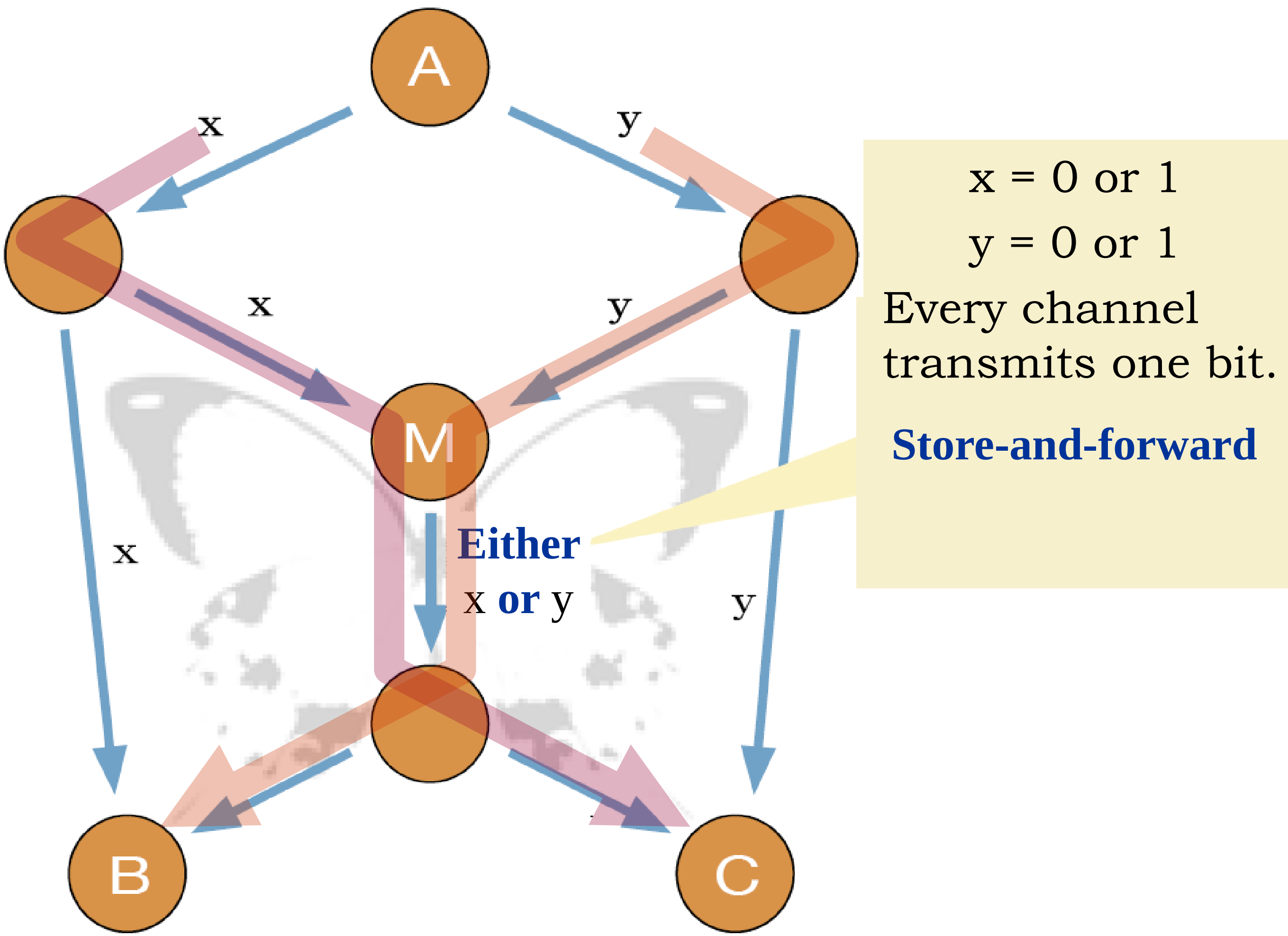


Figure adapted from *Scientific American*, Chinese 7/2007 edition

Store-and-forward

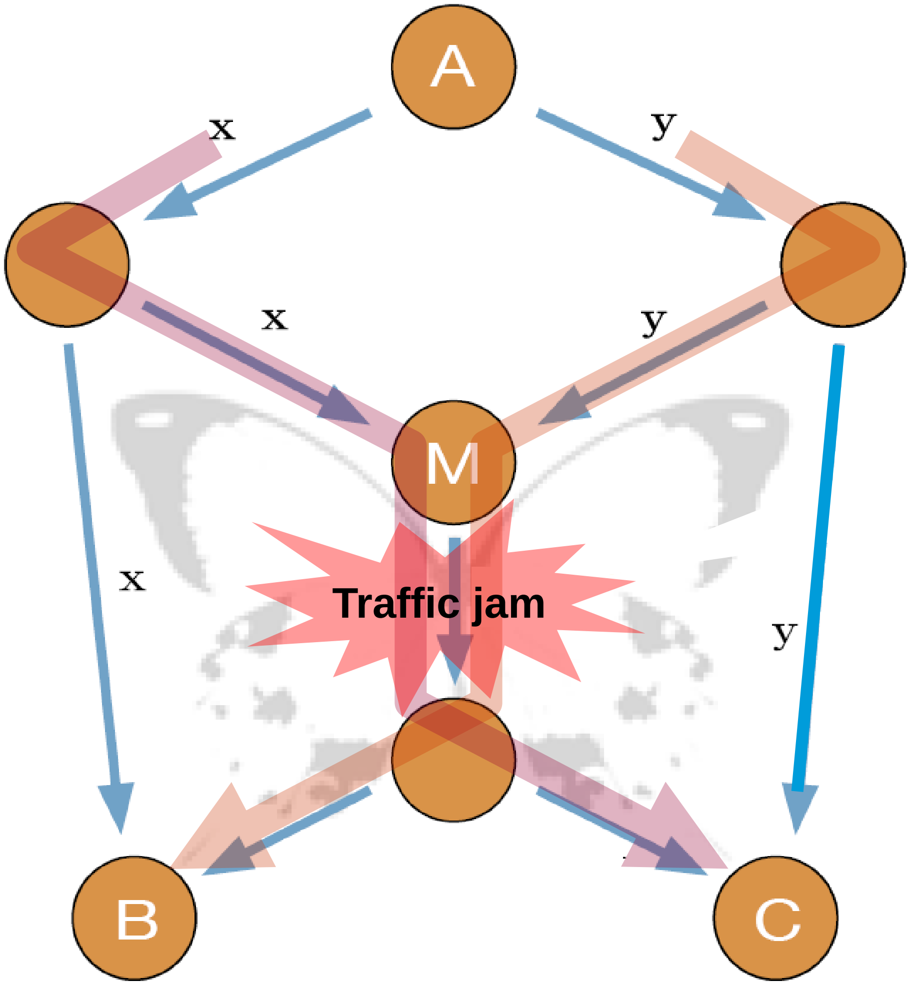
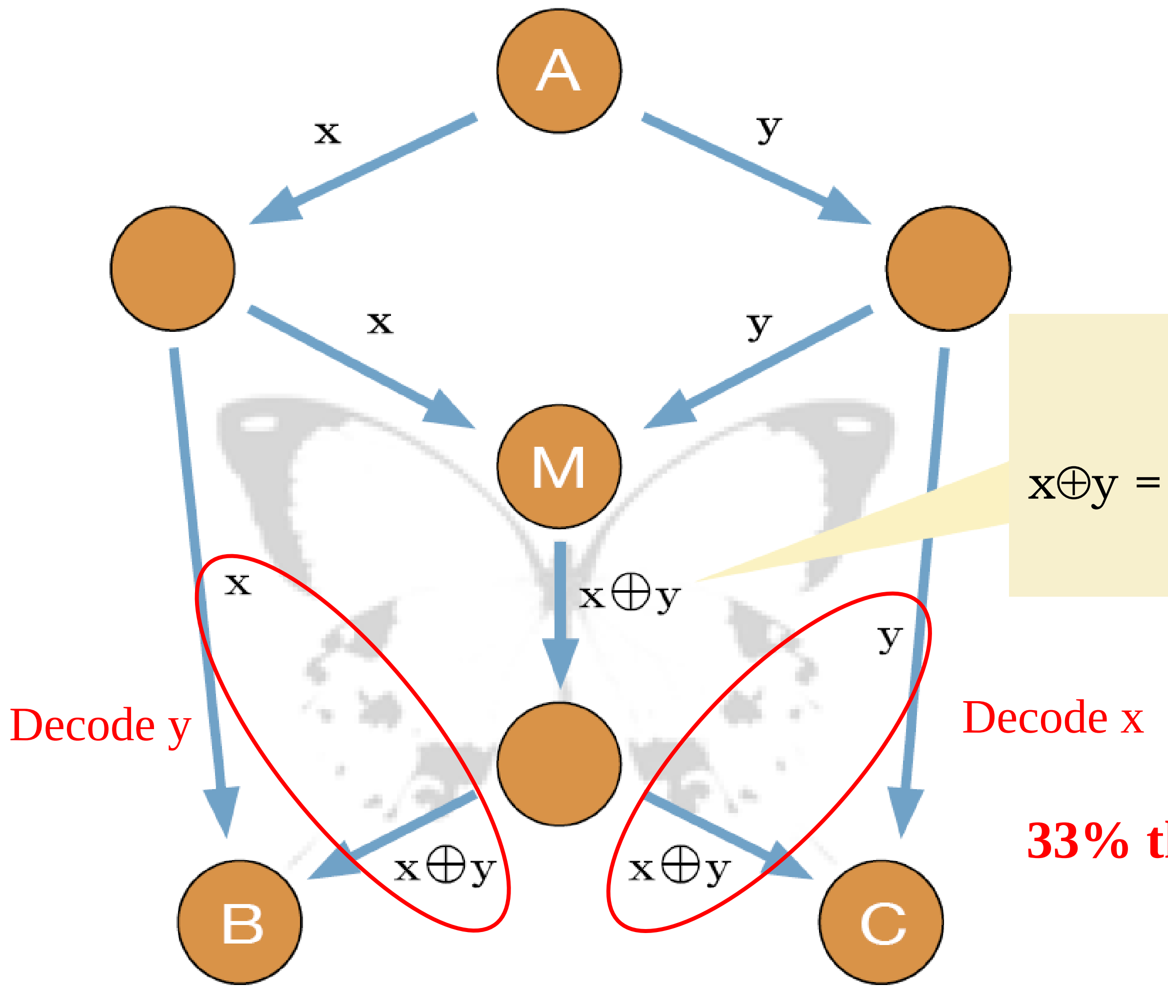


Figure adapted from *Scientific American*, Chinese 7/2007 edition

Network coding (NC)



$$x \oplus y = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

33% throughput gain!

Figure adapted from *Scientific American*, Chinese 7/2007 edition

Syllubus on Information Theory

Chapter 1 The Science of Information
Chapter 2 Information Measures
Chapter 3 The I-Measure
Chapter 4 Zero-Error Data Compression
Chapter 5 Weak Typicality
Chapter 6 Strong Typicality
Chapter 7 Discrete Memoryless Channels
Chapter 8 Rate-Distortion Theory
Chapter 9 The Blahut-Arimoto Algorithms
Chapter 10 Differential Entropy
Chapter 11 Continuous-Valued Channels