北京科技大学 2019—2020 年 第 一 学期

Computational Methods B(计算方法 B) Time:2 Hours

Name: Student ID

Part A: (50)

1. Explain the advantages and disadvantages of the Bisection method and Newton Method for finding the roots of $f(x)=0$.
2. Why are polynomials often used in interpolation? Write nth Lagrange interpolating polynomial for n+1 points $(x_0, f(x_0))$, $(x_1, f(x_1))$,…, $(x_n, f(x_n))$.and explain its advantages and disadvantages.
4. Explain the advantages and disadvantages of Gauss elimination method for solving a system of linear equations.
5. Explain the idea of numerical integration for Composite Trapezoidal rule and Composite Simpson's rule.

Part B: (50 points)

1.(10points) Use Newtonian Method to find a root accurate to within 10^{-3} between 1 and 2 for the problem $f(x)=e^x+2^{-x}+2\cos x-6=0$.

2.(10points) The table 1 lists values of a function $f(x)$ at various points, to find approximation value $f(3.0)$ by using Lagrange interpolating polynomial of degree 1 and 2.

Table 1

x	2.0	2.5	4.0
$f(x)$	0.5	0.4	0.25

3.(10points) Use the composite Trapezoidal rule with the value of 4 to approximate the integration $\int_1^2 x\ln x \,\mathrm{d}x$, $n=4$

4. (10points) Use Euler's method to approximate the solution for the initial value problem

$$\begin{cases} y' = y^2 - 2ty + t^2 + 1, \quad 2 \le t \le 3, \\ y(2) = 1, \quad h = 0.5 \end{cases}$$

5.(10points) Find the iterations of Jacobi and Gauss-Seidel iterative for the following linear systems

$$\begin{cases} 3x_1 + 2x_2 + x_3 = -10 \\ -x_1 + 4x_2 + 2x_3 = 10 \\ 2x_1 - 3x_2 + 15x_3 = 8 \end{cases}$$

write the iterative method in matrix form. For Jacobi iterative methods, to find $X^{(2)}$ using $X^{(0)} = (0,0,0)^T$ ·

Numerical Analysis (2019-2020, 2 Hours)Name: Student

Part A: (50)

1. Explain the advantages and disadvantages of the Bisection method and Newtonian Method for finding the roots of $f(x)=0$.
2. Explain the method of Fixed-Point Iteration to determine a root of an equation of the form $f(x)=0$ and specify when the method fails. Describe Newton method for finding the roots of a function.

3. Why are polynomials often used in interpolation? Why are Taylor polynomials not useful for interpolation? What is meant by Lagrangian interpolation?
3. Explain Gauss elimination method for solving a set of simultaneous linear equations.
4. Explain the method of divided differences, why it is more advantageous than Lagrangian polynomial method for interpolation?

Part B: ($6\times 7=42$)

1. Using Newtonian Method to find a root accurate to within 10^{-3} between 1 and 2 for the problem $f(x)=e^{x}+2^{-x}+2\cos x-6=0$.(page 75, 6)(a) For p0=1, we have p8=1.829384
2. (10points) The table 1 lists values of a function $f(x)$ at various points, to find approximation value $f(3.0)$ by using Lagrange interpolating polynomial of degree 1 and 2.

Table 1

x	2.0	2.5	4.0
$f(x)$	0.5	0.4	0.25

EXAMPLE 1 Using the numbers (or *nodes*) $x_0 = 2$, $x_1 = 2.5$, and $x_2 = 4$ to find the second interpolating polynomial for $f(x) = 1/x$ requires that we determine the coefficient polynomials $L_0(x)$, $L_1(x)$, and $L_2(x)$. In nested form they are

$$L_0(x) = \frac{(x-2.5)(x-4)}{(2-2.5)(2-4)} = (x-6.5)x + 10,$$

$$L_1(x) = \frac{(x-2)(x-4)}{(2.5-2)(2.5-4)} = \frac{(-4x+24)x-32}{3},$$

and

$$L_2(x) = \frac{(x-2)(x-2.5)}{(4-2)(4-2.5)} = \frac{(x-4.5)x+5}{3}.$$

Since $f(x_0) = f(2) = 0.5$, $f(x_1) = f(2.5) = 0.4$, and $f(x_2) = f(4) = 0.25$, we have

$$\begin{aligned} P(x) &= \sum_{k=0}^{2} f(x_k)L_k(x) \\ &= 0.5((x-6.5)x+10) + 0.4\frac{(-4x+24)x-32}{3} + 0.25\frac{(x-4.5)x+5}{3} \\ &= (0.05x - 0.425)x + 1.15. \end{aligned}$$

An approximation to $f(3) = \frac{1}{3}$ (see Figure 3.6) is

$$f(3) \approx P(3) = 0.325.$$

and $x_1 = 1.6$. The value of the interpolating polynomial at 1.5 is

$$P_1(1.5) = \frac{(1.5-1.6)}{(1.3-1.6)}(0.6200860) + \frac{(1.5-1.3)}{(1.6-1.3)}(0.4554022) = 0.5102968.$$

Two polynomials of degree 2 can reasonably be used, one by letting $x_0 = 1.3$, $x_1 = 1.6$, and $x_2 = 1.9$, which gives

$$\begin{aligned} P_2(1.5) &= \frac{(1.5-1.6)(1.5-1.9)}{(1.3-1.6)(1.3-1.9)}(0.6200860) + \frac{(1.5-1.3)(1.5-1.9)}{(1.6-1.3)(1.6-1.9)}(0.4554022) \\ &\quad + \frac{(1.5-1.3)(1.5-1.6)}{(1.9-1.3)(1.9-1.6)}(0.2818186) \\ &= 0.5112857, \end{aligned}$$

3. Use the composite Trapzoidal rule with the value of 4 to approximate the integral

$\int_1^2 x\ln x \, dx$, $n=4$ (page 203)(a) 0.639900

4. Use Euler's method to approximate the solution for the initial value problem

$$\begin{cases} y' = 1 + (t-y)^2, & 2 \le t \le 3, \\ y(2) = 1, & h = 0.5 \end{cases}$$ (page 263,1(b))

b. i	t_i	w_i	$y(t_i)$
1	2.500	2.0000000	1.8333333
2	3.000	2.6250000	2.5000000

5. 设方程组为

$$\begin{cases} 3x_1 + 2x_2 + x_3 = -10 \\ -x_1 + 4x_2 + 2x_3 = 10 \\ 2x_1 - 3x_2 + 15x_3 = 8 \end{cases}$$

试分别写出其 Jacobi 迭代格式和 Gauss-Seidel 迭代格式以及相应的迭代矩阵.

解 （1）Jacobi 迭代格式为 $\begin{cases} x_1^{(k+1)} = \frac{1}{3}\left(-10 - 2x_2^{(k)} - x_3^{(k)}\right) \\ x_2^{(k+1)} = \frac{1}{4}\left(10 + x_1^{(k)} - 2x_3^{(k)}\right) \\ x_3^{(k+1)} = \frac{1}{15}\left(8 - 2x_1^{(k)} + 3x_2^{(k)}\right) \end{cases}$ 整理后即得：

$$\begin{cases} x_1^{(k+1)} = -\frac{2}{3}x_2^{(k)} - \frac{1}{3}x_3^{(k)} - \frac{10}{3} \\ x_2^{(k+1)} = \frac{1}{4}x_1^{(k)} - \frac{1}{2}x_3^{(k)} + \frac{5}{2} \\ x_3^{(k+1)} = -\frac{2}{15}x_1^{(k)} + \frac{1}{5}x_2^{(k)} + \frac{8}{15} \end{cases}$$

故 Jacobi 迭代矩阵为

$$\boldsymbol{B}_J = \begin{bmatrix} 0 & -\frac{2}{3} & -\frac{1}{3} \\ \frac{1}{4} & 0 & -\frac{1}{2} \\ -\frac{2}{15} & \frac{1}{5} & 0 \end{bmatrix}$$

（2）Gauss-Seidel 迭代格式为 $\begin{cases} x_1^{(k+1)} = -\dfrac{2}{3}x_2^{(k)} - \dfrac{1}{3}x_3^{(k)} - \dfrac{10}{3} \\ x_2^{(k+1)} = \dfrac{1}{4}x_1^{(k+1)} - \dfrac{1}{2}x_3^{(k)} + \dfrac{5}{2} \\ x_3^{(k+1)} = -\dfrac{2}{15}x_1^{(k+1)} + \dfrac{1}{5}x_2^{(k+1)} + \dfrac{8}{15} \end{cases}$

从式中解出 $x_i^{(k+1)}, i=1,2,3$，得 $\begin{cases} x_1^{(k+1)} = -\dfrac{2}{3}x_2^{(k)} - \dfrac{1}{3}x_3^{(k)} - \dfrac{10}{3} \\ x_2^{(k+1)} = -\dfrac{1}{6}x_2^{(k)} - \dfrac{7}{12}x_3^{(k)} + \dfrac{5}{3} \\ x_3^{(k+1)} = \dfrac{5}{90}x_2^{(k)} - \dfrac{13}{180}x_3^{(k)} + \dfrac{59}{45} \end{cases}$

故可得 Gauss-Seidel 迭代矩阵为

$$\boldsymbol{B}_{GS} = \begin{bmatrix} 0 & -\dfrac{2}{3} & -\dfrac{1}{3} \\ 0 & -\dfrac{1}{6} & -\dfrac{7}{12} \\ 0 & \dfrac{5}{90} & -\dfrac{13}{180} \end{bmatrix}$$