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"Assignment No. 2nd"

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⇒ Passband Signal:

⇒ The signal in which the frequency of a signal (carrier signal) is modulated for the transmission of bits.

⇒ It has a frequency spectrum concentrated around a carrier frequency (f_c).

Mathematical Representation:

Consider any real-valued signal $s(t)$. In terms of positive frequencies, we can represent it as:

$$S_+(f) = 2u(f)\,S(f) \quad \text{(i)}$$

where $u(f)$ = Fourier Transform and $u(f)$ = step function.

In Time-Domain:

$$\text{eq. (i)} \Rightarrow S_+(t) = \int_{-\infty}^{\infty} S_+(f)\, e^{j2\pi f t}\, df \quad \text{(ii)}$$

$$\Rightarrow S_+(t) = F^{-1}[2u(f)] \star F^{-1}[S(f)] \quad \text{(iii)}$$

As we know that;

$$F^{-1}[S(f)] = s(t) \quad \text{(a)}$$

and

$$F^{-1}[2u(f)] = \delta(t) + \frac{j}{\pi t} \quad \text{(b)}$$

Using (a) and (b):

$$\text{eq. (iii)} \Rightarrow S_+(t) = [\delta(t) + j/\pi t] \star s(t)$$

$$\Rightarrow S_+(t) = s(t) + j\,\hat{s}(t).$$

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⇒ Lowpass Representation:

We will perform the frequency translation of $S_+(f)$ for obtaining equivalent lowpass representation.

$$\Rightarrow \quad S_l(f) = S_+(f+f_c)$$

In Time-Domain Relation:

$$S_l(t) = S_+(t)\, e^{-j2\pi f_c t}$$

Replacing $S_+(t)$ with $s(t) + j\hat{s}(t)$

$$\Rightarrow \quad S_l(t) = [s(t) + j\hat{s}(t)]\, e^{-j2\pi f_c t}$$

⇒ We can also represent lowpass signal using Real and imaginary part in combination as given below:

$$S_l(t) = x(t) + jy(t).$$

It can be also be represented as;

$$s(t) = x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t$$

which is the desired form of bandpass signal.

where $x(t)$ and $y(t)$ are the quadrature components.

Another Form of $s(t)$:

$$s(t) = Re\{[x(t) + jy(t)]\, e^{j2\pi f_c t}\}$$

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⇒ Fourier Series Representation of Signal $x(t)$:

⇒ Consider two signals as given in (i) and (ii):

$$x(t) = \cos w_0 t \quad \text{(i)}$$

$$x(t) = e^{jw_0 t} \quad \text{(ii)}$$

Both these signals are periodic with:

Fundamental Frequency $= F = w_0 = 2\pi F$

Fundamental period $= T = \frac{2\pi}{w_0}$

As (ii) is a contineous periodic signal, So, there will be a set of complex exponentials, associated with it, ranging from $-\infty$ to $+\infty$.

$$\Rightarrow \quad \phi_k = \sum_{k=-\infty}^{\infty} e^{jkw_0 t}$$

By using vector anology, we can write;

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$$

which is the required Fourier Series for representing any signal in the space.

Note:

a_k = Fourier Series co-efficient.

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⇒ Fourier Series Co-efficient :

As we know that ;

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

multiplying both sides by $e^{-jn\omega_0 t}$

$$\Rightarrow x(t).\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k .\, e^{jk\omega_0 t} .\, e^{-jn\omega_0 t}$$

Taking $j\omega_0 t$ as common

$$\Rightarrow x(t) \cdot e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k\, e^{j(k-n)\omega_0 t}$$

Taking $\int_0^T$ on both sides

$$\Rightarrow \int_0^T x(t).\, e^{-jn\omega_0 t}\, dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k\, e^{j(k-n)\omega_0 t}\, dt$$

Taking constants out of integral.

$$\Rightarrow \int_0^T x(t).\, e^{-jn\omega_0 t}\, dt = \sum_{k=-\infty}^{\infty} a_k \left[\int_0^T e^{j(k-n)\omega_0 t}\, dt \right] \rightarrow (i)$$

After applying Euler's Formula on $\int_0^T e^{j(k-n)\omega_0 t}\, dt$.

We will get :

$$\int_0^T e^{j(k-n)\omega_0 t}\, dt = \int_0^T \cos(k-n)\omega_0 t\, dt + j\int_0^T \sin(k-n)\omega_0 t\, dt$$

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There are two possibilities :

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \begin{cases} T, & k = n \\ 0, & k \neq n \end{cases}$$

So, we can write eq.(i) as:

$$a_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt.$$

which is the value of co-efficient of Fourier series.

Fourier Transform from Fourier Series :

$\Rightarrow$ The Fourier series equation for a signal $x(t)$ can be expressed as :

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega t}$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} X[k] . e^{jk\frac{2\pi}{T}t} \quad (i) \quad \because \omega = \frac{2\pi}{T}$$

We can write: change in frequency $= \Delta f = \frac{1}{T}$

$$\text{eq. (i)} \Rightarrow x(t) = \sum_{k=-\infty}^{\infty} X[k] . e^{jk2\pi\Delta f t}$$

$$\because X[k] = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) . e^{-jk\omega t} dt$$

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$$\Rightarrow X[K] = \Delta f \int_{t_0}^{t_0+T} x(t).\, e^{-jK2\pi \Delta f t} dt \quad (ii) \qquad \because \Delta f = \frac{1}{T}$$

put eq.(ii) in eq. (i)

$$eq.(i) \Rightarrow x(t) = \sum_{K=-\infty}^{\infty} \left[\Delta f \int_{t_0}^{t_0+T} x(t).\, e^{-jK2\pi \Delta f t} dt \right] . e^{jK2\pi \Delta f t}$$

let; $t_0 = -T/2$

$$\Rightarrow x(t) = \sum_{K=-\infty}^{\infty} \left[\Delta f \int_{-T/2}^{T/2} x(t).\, e^{-j2\pi K \Delta f t} . dt \right] . e^{j2\pi K \Delta f t} \quad (iii)$$

By taking limit : $-\infty$ to ∞ i.e $T \to \infty$.
we will make the following changes:

"Σ" becomes "$\int$".
"Δf" becomes "df"
"$K\Delta f$" becomes "f"

$$eq(iii) \Rightarrow x(t) = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} x(t).\, e^{-j2\pi f t} . dt .\, e^{j2\pi f t}$$

$$\Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X[\omega].\, e^{j\omega t} . d\omega \quad (iv)$$

from eq. (iv) : we can write:

$$X[\omega] = \int_{-\infty}^{\infty} x(t).\, e^{-j\omega t} dt \quad (V)$$

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Hence;

eq. (iv) is The Inverse Fourier Transform while eq. (v) is The Fourier Tranform of any signal.

⇒ Hilbert Transform :

It is The transformation in which phase angle of all components of a signal is shifted by $\pm \pi/2$.

Mathematically;

Let $x(t)$ be any signal. Then, its Hilbert transform can be written as;

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(k)}{t-k} \, dk$$

Note :

⇒ $x(t)$ and $\hat{x}(t)$ have same amplitude spectrum.

⇒ Both have same energy spectral density.

⇒ Energy of a signal :

The energy of a signal $s(t)$ can be given by;

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 \, dt$$

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⇒ Energy association of $x_d(f)$, $x(f)$ and $x_+(f)$:

$$\Rightarrow \quad E_{x_d} = 2E_x = 4E_{x_+}.$$

⇒ Rayleigh's Energy Theorem:

It states that,

"The integral of the square of magnitude of a function is equal to the integral of the square of magnitude of its Fourier transform."

⇒ It means that the energy of a signal in Time-Domain will be equal to its energy in Frequency-Domain.

Mathematical Representation:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |x(f)|^2\,df.$$

⇒ Product of two signals:

The product of two continuous-time signals can be obtained by multiplying their values at every instant of time.

Mathematically;

For two signals, $x_1(t)$ and $x_2(t)$, the inner product can be represented as:

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$$\langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{\infty} x_1(t) \cdot x_2^*(t)\, dt.$$

⇒ The signals are said to be orthogonal, if their inner product is zero.