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Numerical Analysis
Assignment 10

亚历克上 M202161029

Chapter 6 Ex: 6.5 Question 1a), 2a,

1a) <u>Solving linear systems:</u>

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow z_1 = 2$$

$$z_2 = 2z_1 + z_2 = -1 \Rightarrow -5$$

$$2z_1 + (-5) + z_3 = 1 \Rightarrow 3$$

$$\begin{pmatrix} z_1 \Rightarrow 2 \\ z_2 \Rightarrow -5 \\ z_3 \Rightarrow 3 \end{pmatrix}$$

For :-

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$$

$$x_3 \Rightarrow 3x_3 = 3 \Rightarrow 1$$

$$x_2 \Rightarrow -2x_2 + 3 \Rightarrow -5 \Rightarrow -3$$

$$x_1 = 2x_1 + 3(-3) + (-1) = 3 \Rightarrow 6$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$$

2) The permutation matrix P so that PA can be factored into the product LU :-

a) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(i) Changing A in upper matrix U.

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix} \Rightarrow R_3 \to R_3 - R_1 \Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

(ii) Lower matrix L using the concept of factorisation

$$U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix} \quad \& \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

now consider the permutation matrix P corresponding to $P = R_2$ so that $PA = LU$, we option the P by factor row method :-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_1 \longleftrightarrow R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$PA = LU$

$$PA \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 1 & -1 & 4 \end{bmatrix} \quad \therefore \underline{\underline{PA = LU}}$$