



## Operations Research

Publication details, including instructions for authors and subscription information:  
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To cite this article:

Alois Geyer, William T. Ziemba, (2008) The Innovest Austrian Pension Fund Financial Planning Model InnoALM. Operations Research 56(4):797-810. <https://doi.org/10.1287/opre.1080.0564>

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## OR PRACTICE

# The Innovest Austrian Pension Fund Financial Planning Model InnoALM

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This paper describes the financial planning model InnoALM we developed at Innovest for the Austrian pension fund of the electronics firm Siemens. The model uses a multiperiod stochastic linear programming framework with a flexible number of time periods of varying length. Uncertainty is modeled using multiperiod discrete probability scenarios for random return and other model parameters. The correlations across asset classes, of bonds, stocks, cash, and other financial instruments, are state dependent using multiple correlation matrices that correspond to differing market conditions. This feature allows InnoALM to anticipate and react to severe as well as normal market conditions. Austrian pension law and policy considerations can be modeled as constraints in the optimization. The concave risk-averse preference function is to maximize the expected present value of terminal wealth at the specified horizon net of expected discounted convex (piecewise-linear) penalty costs for wealth and benchmark targets in each decision period. InnoALM has a user interface that provides visualization of key model outputs, the effect of input changes, growing pension benefits from increased deterministic wealth target violations, stochastic benchmark targets, security reserves, policy changes, etc. The solution process using the IBM OSL stochastic programming code is fast enough to generate virtually online decisions and results and allows for easy interaction of the user with the model to improve pension fund performance. The model has been used since 2000 for Siemens Austria, Siemens worldwide, and to evaluate possible pension fund regulation changes in Austria.

*Subject classifications:* scenarios; correlation matrices; pension fund planning; stochastic linear programming.

*Area of review:* OR Practice.

*History:* Received July 2003; revisions received January 2006, August 2006, January 2007, March 2007; accepted March 2007.

## Introduction

The Siemens pension fund, established in 1998, is the largest corporate pension plan in Austria. More than 15,000 employees and 5,000 pensioners are members of the pension plan, with 510 million euro in assets under management as of December 1999. Innovest Finanzdienstleistungs AG, founded in 1998, is the investment manager of the Siemens pension plan and other institutional investors, with more than 7 billion euro in assets under management. The pension fund asset-liability management model InnoALM has been in use at Innovest since 2000. Meanwhile, it has become the only consistently implemented and fully integrated proprietary tool for assessing pension allocation issues within Siemens AG worldwide.

Several factors led Innovest to develop InnoALM, primarily the realization that changing demographics are creating a much higher ratio of retirees to workforce. Furthermore, the asset allocation constraints imposed by Austrian pension law are relaxed if a pension fund is

using advanced modeling tools and proves adequate risk management capability. This makes it paramount that the pension plan be managed using systematic asset-liability management models as a tool in the decision-making process. A promising way to approach this was a multiperiod stochastic linear programming model.

InnoALM is one of the first implemented models to fully exploit the power of the multiperiod stochastic programming optimization approach in a European pension fund. The mathematical aspects of such models applied to asset-liability management are documented in Ziemba and Mulvey (1998), Gondzio and Kouwenberg (2001), and Wallace and Ziemba (2005). While following to some extent the work on the Russell-Yasuda insurance company planning models (Cariño et al. 1994, 1998; Cariño and Ziemba 1998), the application to European pension funds is new, and this model has new features such as state-dependent correlation matrices, fat-tailed asset return distributions, and output not in previous models. Zenios

(1999) surveys large-scale asset-liability applications to bond and fixed-income portfolio management. Gondzio and Kouwenberg (2001) solved Dutch pension fund asset-liability management problems with millions of scenarios, constraints, and variables. Publicly available codes to solve large stochastic programming problems are detailed in Wallace and Ziemba (2005).

What is crucial are models that represent well the situation at hand, are user friendly, and provide the essential information quickly to those who need to make sound pension fund asset-liability decisions. The multiperiod stochastic programming approach includes more of the essential elements of the real problem faced by the pension plan than alternative approaches such as static mean-variance analysis (see, e.g., Sharpe and Tint 1990), continuous-time modeling (see, e.g., Campbell and Viceira 2002 and Rudolf and Ziemba 2004), shortfall risk minimization (see, e.g., Leibowitz and Henriksson 1988), and other approaches (see, e.g., Ziemba and Mulvey 1998). Key elements that make InnoALM superior to other models are the flexibility to formulate constraints and targets in combination with a broad and deep array of scenario-specific results. This allows Innovest to investigate path-dependent behavior of assets and liabilities as well as scenario-based risk assessment. Some of these aspects are illustrated in the application presented in §4.

InnoALM implements state-dependent correlation across asset classes, as asked for in discussions by Lo (1999) and Merton (2000). This feature allows the model to react to extreme events and to plan in advance to do so. Models that assume constant correlation matrices make a conceptual error that is one of the major factors appearing in most of the financial trading disasters of the 1990s and beyond, such as Orange County in 1994, Barrings in 1995, Niederhoffer in 1997 and 2006, Long Term Capital Management in 1998, the Tiger and Soros Hedge Funds in 2000, and Amarath in 2006. When funds are nondiversified and overleveraged, a plausible but low-probability extreme scenario can lead to a financial disaster. Consideration of the state-dependent correlations in advance should lead to portfolios that can react better to an extreme scenario and still produce good results when other, more probable scenarios occur. This feature is documented in the application presented in §4.

The paper briefly discusses the pension fund situation in Austria and Europe in §1. Section 2 develops the stochastic programming model formulation. Section 3 discusses the scenario generation and statistical inputs available for use in the model. Section 4 presents an illustrative application, using an example of a model formulation with five decision periods and four asset classes in various circumstances, including the difficult market conditions faced in 2000–2003. Section 5 provides conclusions and final remarks.

## 1. The Pension Fund Situation in Austria and Europe

The world's populations are aging rapidly. By 2030 there will be roughly a doubling from about 20% to about 40% of those 65 and older, the *retiree group*, compared to those 15–64, the *worker group*, in most countries of the world (see Bos 1994 or Roseveare et al. 1996). This demographic effect will have a major impact on public and private pension plans in Europe and across the world. European Union state pensions (usually labeled as Pillar 1) account for about 88% of total pension costs. While pay-as-you-go plans, where the contributions of current workers support current pensioners similar to U.S. social security, are the most threatened by aging populations, defined contribution plans are also at risk. Without changes, the pension payouts will grow from 10% of GDP in 1997 to over 15% of GDP in 2030 for many EU countries. Contribution rates must be raised significantly to enable the public social security system to cope. Reforms of the public pension systems will be necessary, together with an effective environment for Pillar 2 (company pensions) and Pillar 3 (private pension systems).

This paper describes a model for the effective operation Pillar 2 private pension funds in Austria. These funds usually work on a funded basis, where the pension benefits depend on an employment contract or the pursuit of a particular profession. Schemes are administered by private institutions, and benefits are not guaranteed by the state. Normally, contributions to such systems are made by the employer and, on an optional basis for additional benefits, by employees. Defined contribution plans (DCP), such as the Siemens pension plan for Austria, have fixed contributions, but the pensions are not fixed and the payout depends on the capital accumulation of the plan. Defined benefit plans (DBP) have payouts guaranteed by the company, and the contribution is variable, depending on the capital accumulation over time. For DCPs, which have become more popular, the employees and pensioners bear the risk of low asset returns. There is no direct financial risk for the employer, although with poor returns the employer would suffer negative image effects. For example, if there would be a headline “pensions for the Siemens’ pensioners must be reduced by 3% in the next year,” there would be reputation risk for Siemens.

The liability side of the Siemens pension plan consists of employees, for whom Siemens is contributing payments based on the DCP outline, and retired employees who receive pension payments. Contributions are computed on an individual level as a fixed fraction of salaries, which varies across employees. The set of retired employees is treated according to Austrian mortality and marital tables. Widows and widowers (a much smaller group) are entitled to 60% of the pension payments. Retired employees receive pension payments after reaching age 65 for men and 60 for women in accordance with the legal pension plan. Payments to retired employees are based on the individually

accumulated contribution and the fund performance during active employment.

The actuarial computation of liabilities is based on the assumption that active employees are in steady state; that is, staff is replaced by a new employee with the same qualification and sex, which gives rise to the constant number of employees. Newly employed staff starts with less salary than retired staff, which implies that total contributions grow less rapidly than individual salaries. There exist agreements with employees such that the annual pension payments are based on a discount rate of 6% and the remaining life expectancy at the time of retirement. It is also agreed that these annuities grow by 1.5% annually to compensate for inflation. Hence, the wealth of the pension fund must grow by 7.5% per year to match liability commitments; see the InnoALM wealth target described in §2.

Some EU member states rely on quantitative restrictions on asset allocations to ensure proper pension fund investments. Such rules are usually established to protect the pensioners but also lead to weakly diversified asset holdings. For example, in 2002 Austria's pension funds had a relatively low share of only 13.4% in equities and almost 75% in fixed income (OECD 2004). However, the European Commission (1997) stressed the importance of a relaxation of restrictive quantitative rules on pension fund investing. The diversification of investments is more important than rules on different investments. Recent changes in Austrian pension regulation respond to this point and have made the InnoALM approach even more relevant. The rigid asset-based limits (e.g., not more than 40% in equities; at least 40% in Eurobonds) are relaxed for institutions that prove sufficient risk management expertise for both assets and liabilities of the plan. Thus, the implementation of a scenario-based asset allocation model at Innovest leads to more flexibility that allows for more risk tolerance, and should ultimately result in better long-term investment performance. In fact, some results of InnoALM have been used by the Austrian regulatory authorities to assess the potential risk stemming from less-constrained pension plans. The European Commission recommends the use of such modern asset and liability management techniques for pension planning, although the problem of high costs of such models is of concern. InnoALM is a model that responds to that recommendation and demonstrates that a small team of researchers with a limited budget can quickly produce a valuable modeling system that can be operated by non-stochastic programming specialists on a single PC.

## 2. Formulating the InnoALM as a Multistage Stochastic Linear Programming Model

The model determines the optimal purchases and sales for each of  $N$  assets in each of  $T$  planning periods. Typical asset classes used at Innovest are U.S., Pacific, European, and emerging market equities and U.S., U.K., Japanese, and

European bonds. A concave risk-averse utility function is to maximize expected terminal wealth less convex penalty costs subject to linear constraints. The convex risk measure is approximated by a piecewise-linear function, so the model is a multiperiod stochastic linear program.

The nonnegative decision variables are wealth (after transactions)  $W_{it}$ , purchases  $P_{it}$ , and sales  $S_{it}$  for each asset ( $i = 1, \dots, N$ ). Purchases and sales are in periods  $t = 0, \dots, T - 1$ . Except for  $t = 0$ , purchases and sales are scenario dependent.

Wealth accumulates over time for a  $T$  period model according to

$$\begin{aligned} W_{i0} &= W_i^{init} + P_{i0} - S_{i0}, \quad t = 0, \\ \tilde{W}_{i1} &= \tilde{R}_{i1} W_{i0} + \tilde{P}_{i1} - \tilde{S}_{i1}, \quad t = 1, \\ \tilde{W}_{it} &= \tilde{R}_{it} \tilde{W}_{i,t-1} + \tilde{P}_{it} - \tilde{S}_{it}, \quad t = 2, \dots, T - 1, \quad \text{and} \\ \tilde{W}_{iT} &= \tilde{R}_{iT} \tilde{W}_{i,T-1}, \quad t = T. \end{aligned}$$

$W_i^{init}$  is the prespecified initial value of asset  $i$ . There is no uncertainty in the initialization period  $t = 0$ . Tildes denote scenario-dependent random parameters or decision variables. Returns are associated with time intervals.  $\tilde{R}_{it}$  ( $t = 1, \dots, T$ ) are the (random) gross returns for asset  $i$  between  $t - 1$  and  $t$ . The scenario generation and statistical properties of returns are discussed in §3.

The budget constraints are

$$\begin{aligned} \sum_{i=1}^N P_{i0}(1 + tcp_i) &= \sum_{i=1}^N S_{i0}(1 - tcs_i) + C_0, \quad t = 0 \quad \text{and} \\ \sum_{i=1}^N \tilde{P}_{it}(1 + tcp_i) &= \sum_{i=1}^N \tilde{S}_{it}(1 - tcs_i) + C_t, \quad t = 1, \dots, T - 1, \end{aligned}$$

where  $tcp_i$  and  $tcs_i$  denote asset-specific linear transaction costs for purchases and sales, and  $C_t$  is the fixed (nonrandom) net cashflow (inflow if positive).

Portfolio weights can be constrained over linear combinations (subsets) of assets or individual assets via

$$\begin{aligned} \sum_{i \in U} \tilde{W}_{it} - \theta_U \sum_{i=1}^N \tilde{W}_{it} &\leq 0 \quad \text{and} \\ -\sum_{i \in L} \tilde{W}_{it} + \theta_L \sum_{i=1}^N \tilde{W}_{it} &\leq 0, \quad t = 1, \dots, T - 1, \end{aligned}$$

where  $\theta_U$  is the maximum percentage and  $\theta_L$  is the minimum percentage of the subsets  $U$  and  $L$  of assets  $i = 1, \dots, N$  included in the restrictions. The  $\theta_U$ s,  $\theta_L$ s,  $U$ s, and  $L$ s might be time dependent.

Risk is measured as a weighted discounted convex function of target violation shortfalls of various types in various periods. In a typical application, the deterministic wealth target  $\bar{W}_t$  is assumed to grow by 7.5% in each year. The wealth targets are modeled via

$$\sum_{i=1}^N (\tilde{W}_{it} - \tilde{P}_{it} + \tilde{S}_{it}) + \tilde{M}_t^w \geq \bar{W}_t, \quad t = 1, \dots, T,$$

where  $\tilde{M}_t^W$  are nonnegative wealth target shortfall variables. The shortfall is penalized using a piecewise-linear convex risk measure using the variables and constraints

$$\tilde{M}_t^W = \sum_{j=1}^m \tilde{M}_{jt}^W, \quad t = 1, \dots, T,$$

$$\tilde{M}_{jt}^W \leq b_j - b_{j-1}, \quad t = 1, \dots, T; j = 1, \dots, m-1,$$

where  $\tilde{M}_{jt}^W$  is the wealth-target shortfall associated with segment  $j$  of the cost function,  $b_j$  is the  $j$ th breakpoint of the risk-measure function ( $b_0 = 0$ ), and  $m$  is the number of segments of the function. A piecewise-linear approximation to the convex quadratic risk measure is used so the model remains linear. The appropriateness of the quadratic function is discussed below. Convexity guarantees that if  $\tilde{M}_{jt}^W > 0$ , then  $\tilde{M}_{j-1,t}^W$  is at its maximum; and if  $\tilde{M}_{jt}^W$  is not at its maximum, then  $\tilde{M}_{j+1,t}^W = 0$ .

Stochastic benchmark goals can also be set by the user and are similarly penalized for underachievement. The benchmark target  $\tilde{B}_t$  is scenario dependent. It is based on stochastic asset returns and fixed asset weights  $\alpha_i$  defining the benchmark portfolio

$$\tilde{B}_t = W_0 \sum_{j=1}^t \sum_{i=1}^N \alpha_i \tilde{R}_{ij}.$$

The corresponding shortfall constraints are

$$\sum_{i=1}^N (\tilde{W}_{it} - \tilde{P}_{it} + \tilde{S}_{it}) + \tilde{M}_t^B \geq \tilde{B}_t, \quad t = 1, \dots, T,$$

where  $\tilde{M}_t^B$  is the benchmark target shortfall. These shortfalls are also penalized with a piecewise-linear convex risk measure.

If total wealth exceeds the target, a fraction  $\gamma = 10\%$  of the exceeding amount is allocated to a reserve account and invested in the same way as other available funds. However, the wealth targets at future stages are adjusted. Additional nonnegative decision variables  $\tilde{D}_t$  are introduced and the wealth target constraints become

$$\begin{aligned} & \sum_{i=1}^N (\tilde{W}_{it} - \tilde{P}_{it} + \tilde{S}_{it}) - \tilde{D}_t + \tilde{M}_t^W \\ & = \bar{W}_t + \sum_{j=1}^{t-1} \gamma \tilde{D}_{t-j}, \quad t = 1, \dots, T-1, \text{ where } \tilde{D}_1 = 0. \end{aligned}$$

Because pension payments are based on wealth levels, increasing these levels increases pension payments. The reserves provide security for the pension plan's increase of pension payments at each future stage.

The pension plan's objective function is to maximize the expected discounted value of terminal wealth in period  $T$

net of the expected discounted penalty costs over the horizon from the convex risk measures  $c_k(\cdot)$  for the wealth and benchmark targets, respectively:

$$\text{Max } E \left[ d_T \sum_{i=1}^N \tilde{W}_{iT} - \lambda \sum_{t=1}^T d_t w_t \left( \sum_{k \in \{W, B\}} v_k c_k(\tilde{M}_t^k) \right) \right].$$

Expectation is over  $T$  period scenarios  $S_T$ . The discount factors  $d_t$  are related to the interest rate  $r$  by  $d_t = (1+r)^{-t}$ . Usually,  $r$  is taken to be the three- or six-month treasury bill rate. The  $v_k$  are weights for the wealth and benchmark shortfalls, and the  $w_t$  are weights for the weighted sum of shortfalls at each stage normalized via

$$\sum_{k \in \{W, B\}} v_k = 1 \quad \text{and} \quad \sum_{t=1}^T w_t = T.$$

Such concave objective functions with convex risk measures date to Kusu and Ziemba (1986), were used in the Russell-Yasuda model (Cariño and Ziemba 1998), and are justified in an axiomatic sense in Rockafellar and Ziemba (2000). Nontechnical decision makers find the increasing penalty for target violations a good approach and easy to understand.

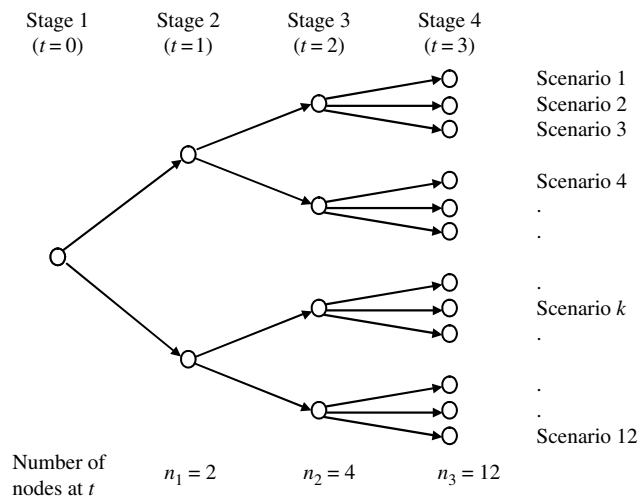
In the implementation of the model presented in §4, the penalty function  $c_k(M^k)$  corresponds to a quadratic utility function. Kallberg and Ziemba (1983) show, for normally distributed asset returns, that varying the average Arrow-Pratt absolute risk-aversion index  $R_A$  traces out the whole spectrum of risk attitudes of all concave utility functions. The most aggressive behavior is log utility, which has  $R_A = 1/\text{wealth}$ , which is essentially zero. Typical 60–40 stock-bond pension funds have  $R_A = 4$ . The Kallberg-Ziemba (1983) results indicate that for computational purposes, the quadratic utility function  $u(w) = w - R_A/2 w^2$  will suffice and is easier to use in the optimization. The error in this approximation is close to zero and well below the accuracy of the data.

The parameter  $\lambda$  in the objective corresponds to  $R_A/2$ , which in the quadratic utility function is the weight assigned to risk measured in terms of variance. The objective function of the InnoALM model penalizes only wealth and benchmark target shortfalls. If the target growth is roughly equal to the average return of the portfolio, shortfalls measure only negative deviations from the mean, whereas variance is based on positive and negative deviations. This implies that shortfalls account for only about half of the variance. Therefore, to obtain results in agreement with a quadratic utility function, we use  $\lambda = R_A$ , rather than  $R_A/2$ , in the objective function. To obtain a solution to the allocation problem for general levels of total initial wealth  $w_0$ , we use the rescaled parameter  $\lambda = R_A/w_0$  in the objective function.

Using a quadratic function, the penalty function  $c_k(M^k)$  is

$$c_k(M^k) = \sum_{j=1}^m \tilde{M}_{jt}^k (b_{j-1} + b_j), \quad \tilde{M}_{jt}^k \leq b_j - b_{j-1}, \text{ with } b_0 = 0.$$

**Figure 1.** Scenario tree with a 2-2-3 node structure (12 scenarios).



Uncertainty is modelled using multiperiod discrete probability scenarios using statistical properties of the assets' returns. A scenario tree is defined by the number of stages and the number of arcs leaving a particular node. Figure 1 shows a tree with a 2-2-3 node structure for a three-period problem with four stages and introduces some definitions and terminology. The tree always starts with a single node that corresponds to the present state ( $t = 0$ ). Decisions are made at each node of the tree and depend on the current state, which reflects previous decisions and uncertain future paths. A single scenario  $s_t$  is a trajectory that corresponds to a unique path leading from the single node at stage 1 ( $t = 0$ ) to a single node at  $t$ . Two scenarios  $s'_t$  and  $s''_t$  are identical until  $t - 1$  (i.e.,  $s'_{t-1} = s''_{t-1}$ ) and differ in subsequent periods  $t, \dots, T$ . The scenario assigns specific values to all uncertain parameters along the trajectory, i.e., asset returns and benchmark targets for all periods. Given all  $T$  period scenarios  $S_T$  and their respective probabilities, one has a complete description of the uncertainty of the model.

Allocations are based on optimizing the stochastic linear program with IBM's optimization solutions library using the stochastic extension library (OSLE version 3). IBM has ceased all sales of this product in 2004. While existing installations of OSLE can still be used, new implementations require alternative software such as the open source project COIN-OR (see <http://www.coin-or.org>). The library uses the Stochastic Mathematical Programming System (SMPS) input format for multistage stochastic programs (see King et al. 2005). The *core-file* contains information about the decisions variables, constraints, right-hand sides, and bounds. It contains all fixed coefficients and dummy entries for random elements. The *stoch-file* reflects the node structure of the scenario tree and contains all random elements—i.e., asset and benchmark returns—and probabilities. Nonanticipatory constraints are imposed to guarantee that a decision made at a specific node is identical for all scenarios leaving that node, so the future cannot be

anticipated. This is implemented by specifying an appropriate scenario structure in the stoch input file. The *time-file* assigns decision variables and constraints to stages. The required statements in the input files are automatically generated by the InnoALM system (see §4).

### 3. Scenario Generation and Statistical Inputs

The uncertainty of the random return and other parameters in InnoALM is modeled using discrete probability scenarios. These scenarios are approximations of the true underlying probability distributions. The accuracy of the set of scenarios chosen and the probabilities of these scenarios in relation to reality contribute greatly to model success. However, the scenario approach generally leads to superior investment performance even if there are errors in the estimations of both the actual scenario values and their probabilities. What the modeling effort attempts to do is to cover well the range of possible future evolution of the economic environment. Decisions take into account all these possible outcomes, weighted by their likelihood. This generally leads to superior performance of multiperiod stochastic programming models compared with other approaches, such as mean-variance analysis, fixed mix, stochastic control, stochastic programming with decision rules, etc. Studies showing this superiority, both in and out of sample, include Kusy and Ziemba (1986), Cariño and Turner (1998), Cariño et al. (1994, 1998), Cariño and Ziemba (1998), and Fleten et al. (2002).

Procedures for estimating the joint distribution of future bond and stock returns have been discussed by Chen et al. (1986), Keim and Stambaugh (1986), Ferson and Harvey (1993), Karolyi and Stultz (1996), and Bossaerts and Hillion (1999). Procedures for estimating discrete scenarios from joint multivariate bond and stock forecasting models have been discussed by Mulvey (1996), Jamshidian and Zhu (1997), Cariño et al. (1994, 1998), Cariño and Ziemba (1998), Zenios (1999), Høyland and Wallace (2001), Pflug (2001), and Roemisch and Heitsch (2003).

The scenarios in InnoALM are defined in terms of the distribution of asset returns and their first- and second-order moments. The latter can be prespecified by the user or estimated from the built-in database of historical returns. James-Stein estimates, which have frequently been suggested as the preferred approach, can be used to estimate mean returns; see, e.g., Jorion (1985), Hensel and Turner (1998), and Grauer and Hakansson (1998). See also MacLean et al. (2007) for an alternative approach using truncated estimators. For each asset the user can choose from the normal, the  $t$ -, or the historical (empirical) distribution.

Empirical asset returns over short horizons (up to one month) typically are not normally distributed but have fat tails and are skewed. Jackwerth and Rubinstein (1997) show how much fatter the implied probability left tails of

**Table 1.** Statistical properties of asset returns.

	Stocks Eur 1/70-9/00	Stocks Eur 1/86-9/00	Stocks US 1/70-9/00	Stocks US 1/86-9/00	Bonds Eur 1/86-9/00	Bonds US 1/86-9/00
Monthly returns						
Mean (% p.a.)	10.6	13.3	10.7	14.8	6.5	7.2
Std. dev. (% p.a.)	16.1	17.4	19.0	20.2	3.7	11.3
Skewness	-0.90	-1.43	-0.72	-1.04	-0.50	0.52
Kurtosis	7.05	8.43	5.79	7.09	3.25	3.30
Jarque-Bera test	302.6	277.3	151.9	155.6	7.7	8.5
Annual returns						
Mean (%)	11.1	13.3	11.0	15.2	6.5	6.9
Std. dev. (%)	17.2	16.2	20.1	18.4	4.8	12.1
Skewness	-0.53	-0.10	-0.23	-0.28	-0.20	-0.42
Kurtosis	3.23	2.28	2.56	2.45	2.25	2.26
Jarque-Bera test	17.4	3.9	6.2	4.2	5.0	8.7

the S&P500 have become since the 1987 worldwide stock market crash because of investor fear of large declines.  $t$ -distributions model fat tails well (see Glasserman et al. 2000). The degrees-of-freedom parameter has to be set to a small value (e.g., five). However, both the normal and the  $t$ -distribution are symmetric distributions and might therefore underestimate the downside risk of an asset or portfolio. Skewed  $t$ -distributions are an alternative to account for skewness and fat tails but were not considered in this model.

Pension fund planning models typically use rebalancing intervals that are much longer than one month. InnoALM can accommodate, e.g., weekly, monthly, annual, or longer rebalancing intervals. In the example in §4, annual and biannual periods are considered. Annual returns have distinctly different distributional properties than monthly returns (see Table 1). Given the difficulty associated with choosing an appropriate parametric distribution, we also use a nonparametric approach to generate random samples reflecting the shape of the historical return distribution.

To simulate the historical distribution for a single asset, we compute standardized annual returns  $y_t$ . We use (overlapping) annual returns from monthly data rather than monthly returns because the planning intervals in the example presented in §4 are in years, and thus the distribution of annual returns is more appropriate than the distribution of monthly returns. A single element of the simulated historical return distribution is computed as follows. First, a random number  $u$  is drawn from a uniform distribution. This random number is treated as a probability and the corresponding percentile  $z$  is computed from the standardized returns. The percentile is a random draw from the historical, standardized distribution with the property  $P[y_t < z] = u$ . Multiplying  $z$  by a prespecified standard deviation and adding a prespecified mean yields the random return used at a particular node in the scenario tree (see below). Sampling from standardized rather than observed returns allows us to simulate historical distributions with means and standard deviations that might differ from the historically observed sample statistics.

This approach yields a random sample that matches the shape of the historical (fat tailed and/or skewed) distribution. The size of the random sample that can be generated by this approach is not limited by the number of available historical observations because any desired number of percentiles could be computed from historical returns. The approach cannot produce values that are more extreme than historically observed returns, however.

State-dependent correlation matrices of InnoALM are a new feature and have not yet been used in pension planning or asset allocation models. InnoALM uses three different correlation matrices and corresponding sets of standard deviations. The choice of a specific correlation matrix depends on the level of stock return volatility. We distinguish “extreme” (or “crash”) periods, “highly volatile” periods, and “normal” periods. Each of the three periods or regimes is assigned a probability of occurrence  $p^j$  ( $j = 1, 2, 3$ ). Harvey (1991), Karolyi and Stulz (1996), Solnik et al. (1996), and Das and Uppal (2004) study changing correlation structures over time. To estimate correlations and standard deviations for the three regimes, we use the regression approach suggested by Solnik et al. (1996). Using monthly time series, we compute moving average (window length 36 months) estimates of correlations among all assets and standard deviations of U.S. equity returns. Correlations are regressed on U.S. stock return volatilities. The estimated regression equations are used to predict correlations for the three regimes (more details on this are presented in §4.1).

Correlated random returns are simulated using the following procedure. For each asset  $i$ , we generate  $n_t$  standardized random numbers  $z_{it}$ , where  $n_t$  is the number of nodes in period  $t$  (see Figure 1).  $z_{it}$  could have a normal,  $t$ -, or historical distribution, depending on the asset and the choice of the user. The  $n_t$ -dimensional vectors  $z_{it}$  are used to compile the  $n_t \times N$  matrix  $\tilde{Z}$ . The correlation among assets is modeled by multiplying the matrix  $\tilde{Z}$  with the Cholesky decomposition of the correlation matrix  $C$ :  $\tilde{Y} = \tilde{Z} \cdot \text{chol}(C)$ . Thus, this procedure essentially uses a normal multivariate copula with normal or nonnormal marginals.

Simulated gross returns for each asset are obtained by multiplying each column of  $\tilde{Y}$  with the standard deviation  $\sigma_i$  of asset  $i$  and adding the asset's mean  $\mu_i$ , where both are adjusted for the length  $\tau_i$  of planning period  $t$  via

$$\tilde{R}_{it} = (1 + \mu_i)^{\tau_i} + \tilde{Y}_{it}\sigma_i\sqrt{\tau_i}.$$

Mixing of correlations is obtained by generating three subsets of simulated returns as described, but using a different correlation matrix  $C^j$  in each subset yielding three sets of returns  $\tilde{R}_{it}^j$  ( $j = 1, 2, 3$ ). At any particular period, the set of all nodes  $n_t$  is randomly partitioned into three subsets corresponding to the three volatility regimes. We use all available nodes of a period to define the subsets (i.e.,  $n_t = k_1 \cdot k_2 \cdots k_t$ , where  $k_i$  is the number of paths leaving from node  $i - 1$ ). The number of elements  $n_t^j$  in each set is determined by the prespecified probability  $p^j$  of the three regimes via  $n_t^j = n_t p^j$ , where  $n_t^j$  is rounded up to the nearest integer. All nodes  $n_t^j$  of a subset are used for moment-matching (e.g., if the node structure is 100-5-2 and  $p^j = 0.1$ , we have 10, 50, and 100 nodes available for regime  $j$ ). The simulated returns  $\tilde{R}_{it}^j$  are randomly distributed within each of the three subsets, and the subsets are randomly distributed across all nodes in that period. A tag is assigned to each return to identify the associated regime for later use as described in §4.3. A pseudo-code is included in the appendix to describe the procedure. It is well established empirically that short-term (daily or weekly) volatility is highly persistent. After extreme events, high levels of volatility are more likely than normal levels. However, estimating regime transition probabilities for annual or even longer intervals is not a trivial issue. Therefore, we assume that the probability for ending up in a particular regime is independent of the previous regime. If reliable regime transition probabilities were available, these could be included in applications of the model.

## 4. Implementation and Sample Results

Out of a large number of calculations and tests, we present examples that show interesting features of InnoALM but do not disclose any important proprietary aspects of InnoALM. The purpose of the sample application is to highlight the importance of considering mixing correlations, the impact of various assumptions about return distributions, and the effects of rebalancing as implied by the optimal solution. We also show the impact of constraints on the asset allocation as prescribed by Austria's pension law. Compared to the actual applications at Innovest the example is simplified, but the results highlight key lessons to be learned from inspecting the model's wealth and risk implications across time and across scenarios.

InnoALM has a user interface to select assets, and define the number of stages and the scenario node structure. The user can specify the wealth targets, cash inflows and outflows, and other parameters (e.g., risk aversion, constraints

on asset weights, weights that define the benchmark target). Historical data on the asset classes considered are embedded into the model. This includes a monthly data set ranging from 1970 to January 2002 for equities (MSCI index) and from 1986 to January 2002 for bonds (JP Morgan index). The period October 2000 to January 2002 was reserved for out-of-sample tests (see §4.3). Statistical properties of returns can be computed from the historical database or specified by the user. Calculations are easy to make using various assumptions, such as those from the past 101 years in Dimson et al. (2002, 2006). Different parameters can be specified for each stage of the planning period.

Statistical analysis and simulation uses the GAUSS programming language (Aptech Systems, Inc., [www.aptech.com](http://www.aptech.com)). This language is also used to automatically generate the SMPS input files. This greatly facilitates experimenting with the model because there is no need to do any recoding or manipulating SMPS files if different assets are considered, a different node structure is assumed, or other modifications are made. The problem is solved with IBM's optimization solutions library using the stochastic extension library (OSLE version 3). The solution is written to an output file that is used to generate summary tables and graphs.

A typical application as described below, with 10,000 scenarios, takes about 7–8 minutes for simulation, generating SMPS files, solving and producing output on a 1.2 GHz Pentium III notebook with 376 MB RAM, although for some problems execution times can be 15–20 minutes.

### 4.1. Sample Application—Assumptions

To illustrate some of the model's features, we present results for an application with four asset classes (Stocks Europe, Stocks US, Bonds Europe, and Bonds US), and five periods (six stages). Periods 1 and 2 are one year in length, Periods 3 and 4 are two years in length, and Period 5 is four years long (10 years in total), which reflects Innovest's rebalancing policy. We assume discrete compounding, which implies that the mean return for asset  $i$  ( $\mu_i$ ) used in simulations is  $\mu_i = \exp(\bar{y} + 0.5\sigma^2) - 1$ , where  $\bar{y}$  and  $\sigma^2$  are mean and variance of log-returns. We generate 10,000 scenarios using a 100-5-5-2-2 node structure, which should provide sufficient detail about the distribution of assets and still allow for reasonable solution times. Using only a few branches and fitting distributions across all nodes may induce (unintended) serial correlation between stages. This might bias the results because of the implied return predictability. This shortcoming can be avoided with very large sample sizes (i.e., many branches), which was not feasible in this application. Initial wealth is 100 units, and the wealth target grows at an annual rate of 7.5%. No benchmark target and no cash inflows and outflows are considered in this sample application, and the interest rate is fixed at 5%. We use  $R_A = 4$ , which corresponds



roughly with a simple static mean-variance model to a standard 60–40 stock-bond pension fund mix; see Kallberg and Ziemba (1983). Hence, it is appropriate for this application.

Assumptions about the statistical properties of log-returns (including dividends) are based on a sample of monthly data from January 1970 for stocks and from 1986 for bonds to September 2000. All asset returns are measured in nominal Euros. Summary statistics for monthly and annual log-returns are in Table 1. The U.S. and European equity means for the longer period 1970–2000 are much lower than for 1986–2000 and slightly less volatile. Table 1 shows that monthly stock returns are non-normal and negatively skewed. Monthly stock returns are fat-tailed, whereas monthly bond returns are close to normal (the critical value of the Jarque-Bera test for  $\alpha = 0.01$  is 9.2).

However, for long-term planning models such as InnoALM with its one-year review period, properties of monthly returns are less relevant. The second panel of Table 1 contains statistics for annual returns. While average returns and volatilities remain about the same (we lose one year of data when we compute annual returns), the distributional properties change dramatically. While we still find negative skewness, there is no evidence for fat tails in annual returns except for European stocks (1970–2000).

The mean returns from this sample are comparable to the 1900–2000 mean returns estimated by Dimson et al. (2002, 2006). Their estimate of the nominal mean equity return for the United States is 12.0%, and that for Germany and the United Kingdom is 13.6% (the simple average of the two countries' means). The mean of bond returns is 5.1% for the United States and 5.4% for Germany and the United Kingdom.

Assumptions about means, standard deviations, and correlations for the applications of InnoALM appear in Table 2 and are based on the sample statistics presented in Table 1. Projecting future rates of returns from past data is difficult. We use the equity means from the period 1970–2000 because the period 1986–2000 had high stock returns that are not assumed to prevail in the long run. Thus, the results might be considered to be based on conservative assumptions. The asset classes Innovest uses are well diversified and therefore do not have an excessive amount of own-company (Siemens) stock. See Douglass et al. (2004) for an analysis of the problems that can arise in U.S. pension plans with high allocations to own-company stock that can be justified only by very low risk aversion or high expected own-company stock returns.

The correlation matrices in Table 2 for the three different regimes are based on the regression approach described above. Results for the estimated regression equations appear in Table 3. We consider three different regimes and assume that 10% of the time equity markets are extremely volatile, 20% of the time markets are characterized by high volatility, and 70% of the time markets are normal. The 35th percentile of U.S. equity return volatility located at the

**Table 2.** Means, standard deviations, and correlations assumptions.

		Stocks Europe	Stocks US	Bonds Europe	Bonds US
Normal periods (70% of the time)	Stocks US	0.755			
	Bonds Europe	0.334	0.286		
	Bonds US	0.514	0.780	0.333	
	Standard deviation	14.6	17.3	3.3	10.9
High volatility (20% of the time)	Stocks US	0.786			
	Bonds Europe	0.171	0.100		
	Bonds US	0.435	0.715	0.159	
	Standard deviation	19.2	21.1	4.1	12.4
Extreme periods (10% of the time)	Stocks US	0.832			
	Bonds Europe	−0.075	−0.182		
	Bonds US	0.315	0.618	−0.104	
	Standard deviation	21.7	27.1	4.4	12.9
Average period	Stocks US	0.769			
	Bonds Europe	0.261	0.202		
	Bonds US	0.478	0.751	0.255	
	Standard deviation	16.4	19.3	3.6	11.4
All periods	Mean	10.6	10.7	6.5	7.2

center of the 70% “normal” range defines “normal” periods. “Highly volatile” periods are based on the 80th volatility percentile and “extreme” periods on the 95th percentile. The associated correlations are computed using the results from Table 3 and reflect the return relationships that typically prevailed during those market conditions. For example, if the 35th percentile of volatility is 0.173 (p.a.) the expected correlation between U.S. and European stocks is  $0.62 + 2.7 \cdot 0.173 / \sqrt{12} = 0.755$ . The correlations in Table 2 show a distinct pattern across the three regimes. Correlations among stocks tend to increase as stock return volatility rises, whereas the correlations between stocks and bonds tend to decrease. European bonds may serve as a hedge for equities during extremely volatile periods because bond and stock returns, which are usually positively correlated, are then negatively correlated.

A crucial aspect for Innovest is to choose appropriate return distributions. To facilitate this choice, we calculate and compare optimal portfolios for seven cases. We distinguish cases with and without mixing of correlations and consider normal,  $t$ -, and historical distributions. Cases NM, HM, and TM use mixing correlations. Case NM assumes normal distributions for all assets. Case HM uses the historical distributions of each asset, whereby we capture the skewness found for all annual asset returns (see Table 1). Case TM assumes  $t$ -distributions with five degrees of freedom for stock returns to account for their fat tails, whereas bond returns are assumed to have normal distributions. While  $t$ -distributions are not empirically justified for annual returns (see Table 1), we have made this assumption to investigate how the model can deal with severe (worse than

**Table 3.** Regression equations relating asset correlations and U.S. stock return volatility (monthly returns; Jan. 1989–Sept. 2000; 141 observations).

Correlation between	Constant	Slope w.r.t. U.S. stock volatility	<i>t</i> -statistic of slope	<i>R</i> <sup>2</sup>
Stocks Europe–Stocks US	0.62	2.7	6.5	0.23
Stocks Europe–Bonds Europe	1.05	−14.4	−16.9	0.67
Stocks Europe–Bonds US	0.86	−7.0	−9.7	0.40
Stocks US–Bonds Europe	1.11	−16.5	−25.2	0.82
Stocks US–Bonds US	1.07	−5.7	−11.2	0.48
Bonds Europe–Bonds US	1.10	−15.4	−12.8	0.54

“normal”) market conditions, an aspect of key concern to Innovest and regulatory authorities. The cases NA, HA, and TA are based on the same distribution assumptions with no mixing of correlations matrices. Instead, the correlations and standard deviations used in these cases correspond to an “average” period where 10%, 20%, and 70% weights are used to compute averages of correlations and standard deviations used in the three different regimes. Comparisons of the average (A) cases and mixing (M) cases are mainly intended to investigate the effect of mixing correlations. Finally, in the case TMC, we maintain all assumptions of case TM but constrain asset weights according to Austria’s pension law. Eurobonds must be at least 40% and equity at most 40%, and these constraints are binding. As mentioned above, those constraints can be relaxed if a pension fund is managed according to the standards of the regulating authority (mainly if certain risk measures are in an acceptable range). Comparisons of constrained and unconstrained results, as presented below, have been used by Austrian regulatory authorities to assess the potential effects of such constraints. We consider the unconstrained cases also because the associated effects of mixing correlations and varying distributions are more pronounced if constraints are omitted.

## 4.2. Sample Application—Results

Table 4 shows the optimal initial asset weights at stage 1 for the various cases. Table 5 shows results for the final stage. These tables show a distinct pattern: the mixing correlation cases initially assign a much lower weight to European bonds than the “average” period cases. Single-period, mean-variance optimization and the “average” period cases (NA, HA, and TA) suggest an approximate 45–55 mix between equities and bonds. The mixing correlation cases (NM, HM, and TM) imply a 65–35 mix. Investing in U.S. bonds is not optimal at stage 1, mainly due to the relatively high volatility of U.S. bonds. Chopra and Ziemba

**Table 4.** Optimal initial asset weights at stage 1 by case (percentage).

	Stocks Europe	Stocks US	Bonds Europe	Bonds US
Single-period, mean-variance optimal weights (average periods)	34.8	9.6	55.6	0.0
Case NA: no mixing (average periods) normal distributions	27.2	10.5	62.3	0.0
Case HA: no mixing (average periods) historical distributions	40.0	4.1	55.9	0.0
Case TA: no mixing (average periods) <i>t</i> -distributions for stocks	44.2	1.1	54.7	0.0
Case NM: mixing correlations normal distributions	47.0	27.6	25.4	0.0
Case HM: mixing correlations historical distributions	37.9	25.2	36.8	0.0
Case TM: mixing correlations <i>t</i> -distributions for stocks	53.4	11.1	35.5	0.0
Case TMC: mixing correlations historical distributions; constraints on asset weights	35.1	4.9	60.0	0.0

(1993) point out that the asset allocation is very sensitive to the accuracy of the estimated mean return. For example, assuming a mean return for U.S. stocks equal to the long-run mean of 12% as estimated by Dimson et al. (2002, 2006), the model invests 100% into U.S. and European equities. Conversely, a mean return for U.S. stocks of 9% implies a 30%–70% mix of equities and bonds.

Table 5 shows that the distinction between “A” and “M” cases becomes less pronounced over time. However, European equities still have a consistently higher weight in the mixing cases than in no-mixing cases. This higher weight is mainly at the expense of Eurobonds. In general, the proportion of equities at the final stage is much higher than in the first stage because the expected portfolio wealth at later stages is far above the target wealth level (206.1 at stage 6), and the higher risk associated with stocks is less important (see §4.3). This can also be derived from Figure 2, which shows the wealth distribution across all stages for case TM. Extreme poorly performing scenarios (i.e., large shortfalls) do not seem to be a problem for the mixing correlation cases, as also indicated by the final column in Table 5. The constraints in case TMC lead to lower expected portfolio wealth throughout the horizon and to a higher shortfall probability than any other case. Calculations show that initial wealth would have to be 35% higher to compensate for the loss in terminal expected wealth due to those constraints. In all cases, the optimal weight of equities is much higher than the 13.4% in 2002 in Austria (OECD 2004).

The expected terminal wealth levels and the shortfall probabilities at the final stage make the difference between mixing and no-mixing cases even clearer (see Table 5).

**Table 5.** Expected portfolio weights at the final stage by case (percentage), expected terminal wealth, expected reserves, and the probability for wealth-target shortfalls (percentage) at the final stage.

	Stocks Europe	Stocks US	Bonds Europe	Bonds US	Expected terminal wealth	Expected reserves at stage 6	Probability of target shortfall	Probability shortfall >10%
NA	34.3	49.6	11.7	4.4	328.9	202.8	11.2	2.7
HA	33.5	48.1	13.6	4.8	328.9	205.2	13.7	3.7
TA	35.5	50.2	11.4	2.9	327.9	202.2	10.9	2.8
NM	38.0	49.7	8.3	4.0	349.8	240.1	9.3	2.2
HM	39.3	46.9	10.1	3.7	349.1	235.2	10.0	2.0
TM	38.1	51.5	7.4	2.9	342.8	226.6	8.3	1.9
TMC	20.4	20.8	46.3	12.4	253.1	86.9	16.1	2.9

Mixing correlations implies higher levels of terminal wealth and lower shortfall probabilities.

If the level of portfolio wealth exceeds the target, the surplus  $\tilde{D}_j$  is allocated to a reserve account; see §2. The reserves in  $t$  are computed from  $\sum_{j=1}^t \tilde{D}_j$  and are shown in Table 5 for the final stage. These values are in monetary units given an initial wealth level of 100. They can be put into context by comparing them to the wealth target (206.1 at stage 6). Expected reserves exceed the target level at the final stage by up to 16%. Depending on the scenario, the reserves can be as high as 1,800. Their standard deviation (across scenarios) ranges from five at the first stage to 200 at the final stage. The constraints in case TMC lead to a much lower level of reserves compared to the other cases, which implies, in fact, less security against future increases of pension payments.

Summarizing, the main lesson to be learned from this application is that scenario-dependent correlations imply higher levels of wealth, less risk, and more reserves than constant alternatives. We also find that optimal allocations, expected wealth, and shortfall probabilities are more affected by considering mixing correlations while the type of distribution (i.e., accounting for skewness or fat tails) has a smaller impact. The results illustrate the insights pension fund planners and regulating authorities can obtain from

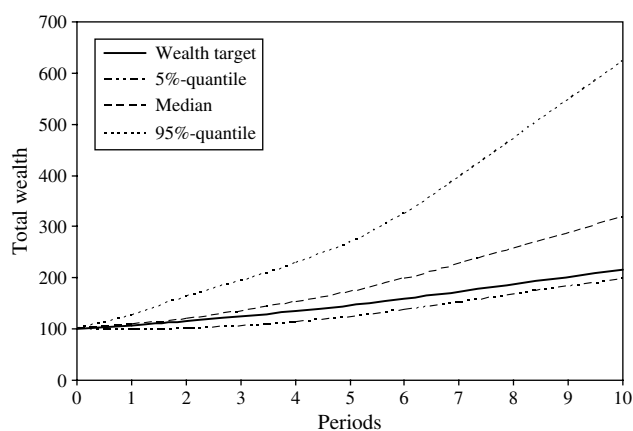
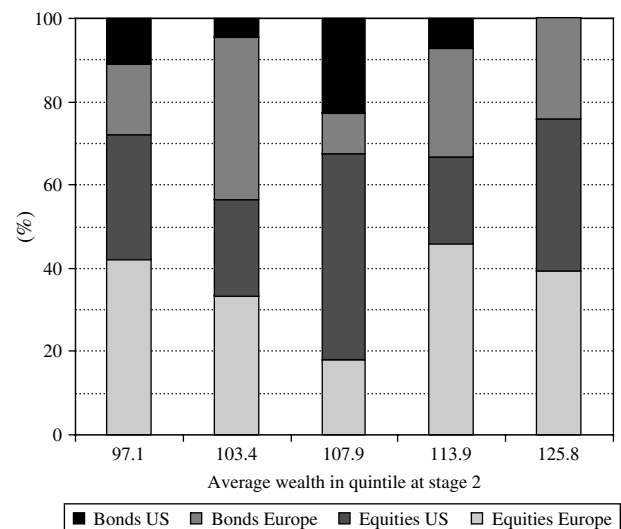
inspecting the model's wealth and risk implications across time and across scenarios.

### 4.3. Model Tests

Because state-dependent correlations have a significant impact on allocation decisions, we further investigate their nature and their implications to test the model. While the focus of the previous section was to compare various stochastic assumptions and to highlight the model's benefits to Innovest, the purpose of the present section is to test the advantages of using mixing correlations in an out-of-sample context and a controlled experiment.

Positive effects on the pension fund performance and its risk profile induced by the stochastic, multiperiod planning approach will be realized only if the portfolio is dynamically rebalanced, as implied by the optimal scenario tree. We first illustrate the decision rule implied by the model. We form quintiles of wealth and compute the average optimal weights assigned to each quintile.

Figure 3 shows the distribution of weights for each of the five average levels of wealth for case TM at stage 2, which

**Figure 2.** Total wealth distribution over time for case TM.**Figure 3.** Optimal TM weights conditional on quintiles of portfolio wealth at stage 2.

depends on the wealth level in a specific way. For example, if average attained wealth is 103.4, which is slightly below the target, a very cautious strategy is chosen. Bonds have the highest weight in this case (almost 50%). In this situation, the model implies that the risk of even stronger underachievement of the target is to be minimized. If attained wealth is far below the target (97.1), the model implies more than 70% equities and a high share (10.9%) of relatively risky U.S. bonds. With such strong underachievement, there is no room for a cautious strategy to attain the target level again. If average attained wealth is close to the target 107.5 putting more money into U.S. assets (72.4%), which are more risky than the corresponding European assets, is acceptable because risk does not play a big role in that situation. For wealth levels above the target, most of the portfolio is switched to European assets, which are safer than U.S. assets, thereby preserving the high levels of attained wealth. These results follow the objective of the fund, although individual pension beneficiaries might have different preferences.

The decision rules implied by the optimal solution can be used to perform a test of the model using the following rebalancing strategy. Consider the 10-year period from January 1992 to January 2002. In the first month of this period, we assume that wealth is allocated according to the optimal solution for stage 1 (see Table 4). In each of the subsequent months, the portfolio is rebalanced as follows: we identify the current volatility regime (normal, highly volatile, or extreme) by comparing the U.S. stock return volatility estimated from the past 36 months of U.S. equity returns (see §3, p. 13) to the 35th, 80th, and 95th percentiles of volatility as defined in §4.1. The regime might have been identified incorrectly. Then, we search the scenario tree across all stages to find a node that corresponds to the current volatility regime (using the tags assigned to simulated return) and has the same or a similar level of wealth. Given the high short-run persistence of return covariance, it is plausible to assume that the current volatility and correlation regime prevails in the subsequent month and we use the optimal weights from that node to determine the rebalancing decision. For the no-mixing cases NA, TA, and HA, the information about the current volatility regime cannot be used to identify optimal weights. In those cases, we use the weights from a node with a level of wealth as close as possible to the current level of wealth.

Table 6 presents summary statistics for the complete sample time period and the out-of-sample time period October 2000 to January 2002. The mixing correlation solutions assuming normal and *t*-distributions (cases NM and TM) provide a higher average return with lower standard deviation than the corresponding nonmixing cases (NA and TA). The advantage might be substantial, as indicated by the 14.9% average return of TM compared to 10.0% for TA. The *t*-statistic for this difference is 1.7 and is significant at the 5% level (one-sided test). Using the historical distribution and mixing correlations (HM) yields a lower average

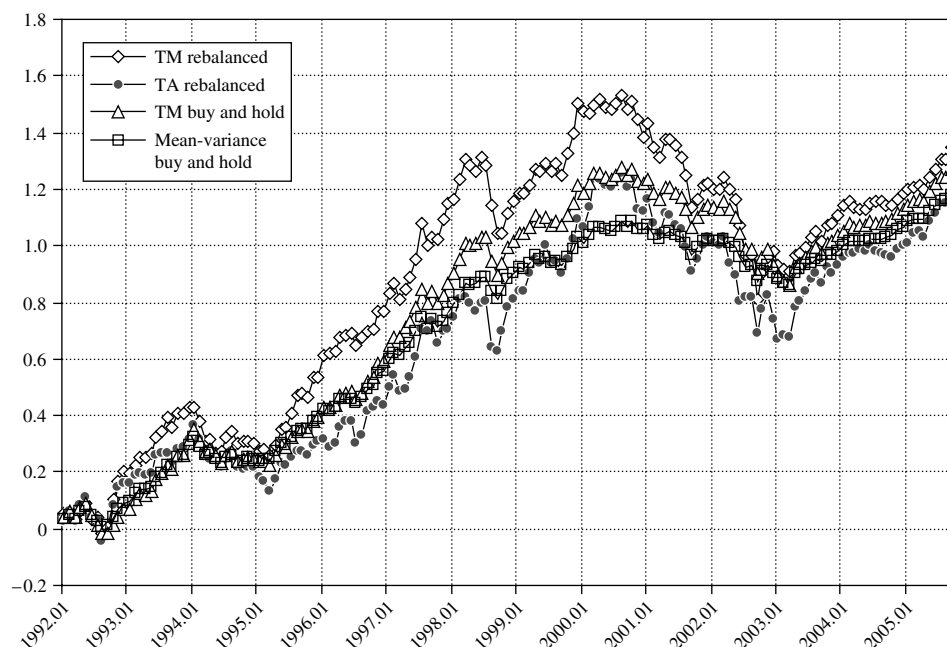
return than no-mixing (HA). This result can be explained by a strong asymmetry in the pattern of conditional weights of HA (as shown in Figure 3 for TM) in favor of stocks, which contributes to the superior performance of HA compared to HM. In the constrained case TMC, the average return for the complete sample is in the same range as for the unconstrained cases. This is mainly due to relatively high weights assigned to U.S. bonds, which had high returns during the test period. The standard deviation of returns is much lower because the constraints imply a lower degree of rebalancing.

To emphasize the difference between the cases TM and TA, Figure 4 compares the cumulated monthly returns obtained from the rebalancing strategy for the two cases as well as the results of two buy-and-hold strategies. One of the buy-and-hold strategies assumes that the portfolio weights on January 1992 are fixed at the optimal TM weights throughout the test period, and the other uses the weights from single-period mean-variance analysis (see Table 4). Rebalancing on the basis of the optimal TM scenario tree provides a substantial gain when compared to the buy-and-hold strategies or the performance using TA results, where rebalancing does not account for different correlation and volatility regimes.

The results in Table 6 suggest that the TMC is the most attractive strategy after adjusting for risk (e.g., using a Sharpe ratio). However, such in- and out-of-sample comparisons depend on the observed asset returns and the test period. To isolate the potential benefits from considering state-dependent correlations, we performed the following controlled simulation experiment. Consider 10,000 10-year periods where simulated annual returns of the four assets are assumed to have the statistical properties summarized in Table 2. The means and standard deviations of simulated returns across all 100,000 years are equal to those given in the last two rows of Table 2. One of the 10 years is assumed to be an “extreme” year, two years correspond to “highly volatile” markets, and seven years are “normal” years using the corresponding correlation matrix and standard deviations from Table 2. Returns have normal, historical, or

**Table 6.** Results of asset allocation strategies using the decision rule implied by the optimal scenario tree.

	Complete sample 01/92–01/02		Out-of-sample 10/00–01/02	
	Mean	Std. dev.	Mean	Std. dev.
NA	11.6	16.1	−17.1	18.6
NM	13.1	15.5	−9.6	16.9
HA	12.6	16.5	−15.7	21.1
HM	11.8	16.5	−15.8	19.3
TA	10.0	16.0	−14.6	18.9
TM	14.9	15.9	−10.8	17.6
TMC	12.4	8.5	0.6	9.9

**Figure 4.** Cumulative monthly returns for different strategies.

$t$ -distributions. We compare rebalancing strategies that use the implied decision rules of the optimal scenario tree, as explained in the in- and out-of-sample tests above. Only the rebalancing strategies of the cases NM, HM, and TM are appropriate in this simulated environment because the strategies are based on the optimal solution derived from a stochastic tree with mixing correlations. The strategies for cases NA, HA, and TA are based on the optimal solution assuming an average correlation matrix. For simplicity, we assume that the current volatility regime is known in the mixing strategies.

Using this experiment, we assess the value of scenario-dependent correlation matrices by comparing NM to NA, HM to HA, and TM to TA (see Table 7). Case TMC is included to test the impact of constraints on asset weights. The average annual return over 10,000 repetitions using TM weights is 10.0%, compared with 9.51% using the

TA weights. The mean difference has a  $t$ -statistic of 11.2, which indicates a highly significant advantage of using state-dependent correlations. The rebalancing strategy in case TM applies appropriate decision rules, while the strategy in case TA cannot respond adequately to different regimes. The expected wealth at the end of year 10 is 314.5 for TM, compared to only 292.7 for TA. The shortfall probability in case TM is also smaller. Similar differences can be observed by comparing the pairs NA-NM and HA-HM, although the advantage of the mixing cases is not uniform across distributions. For the constrained case TMC, we obtain an average return of only 8.06%, which indicates that the constraints imply insufficient rebalancing capacity. Thus, the relatively good performance of the TMC rebalancing strategy in the sample period 1992–2002 might have been positively biased by the favorable conditions during that time. Rebalancing strategies always outperform buy-and-hold strategies (e.g., 10% compared to 9.2% in case TM).

Summarizing, the main implications derived from the model tests for practical applications are significant benefits of using scenario-dependent correlation matrices. The controlled experiments indicate that the advantages measured in terms of wealth and shortfall probabilities are not sample dependent.

**Table 7.** Average annual returns, expected terminal wealth, and the probability for wealth-target shortfalls (percentage) for different rebalancing strategies.

	Average annual return (%)	Expected terminal wealth	Probability of target shortfall
NA	9.87	307.1	31.0
NM	9.98	313.3	30.7
HA	9.28	283.2	33.7
HM	9.53	294.3	32.8
TA	9.51	292.7	32.1
TM	10.0	314.5	30.1
TMC	8.06	231.9	38.0

## 5. Conclusions and Final Remarks

The model InnoALM provides an easy-to-use tool to help Austrian pension funds' investment allocation committees evaluate the effect of various policy choices in light of changing economic conditions and various goals, constraints, and liability commitments. The model includes

features that reflect real investment practices. These include multiple scenarios, nonnormal distributions, and different volatility and correlation regimes. The model provides a systematic way to estimate in advance the likely results of particular policy changes and asset return realizations. This provides more confidence and justification to policy changes that might be controversial, such as a higher weight in equity and less in bonds than has traditionally been the case in Austria.

The model is an advance on previous models and includes new features such as state-dependent correlation matrices. Crucial to the success of the results are the scenario inputs and especially the mean return assumptions. The model has a number of ways to estimate such scenarios. Given good inputs, the policy recommendations can improve current investment practice and provide greater confidence to the asset allocation process. The following quote by Konrad Kontriner (member of the board) and Wolfgang Herold (senior risk strategist) of Innovest emphasizes the practical importance of InnoALM:

“The InnoALM model has been in use by Innovest, an Austrian Siemens subsidiary, since its first draft versions in 2000. Meanwhile it has become the only consistently implemented and fully integrated proprietary tool for assessing pension allocation issues within Siemens AG worldwide. Apart from this, consulting projects for various European corporations and pensions funds outside of Siemens have been performed on the basis of the concepts of InnoALM.

The key elements that make InnoALM superior to other consulting models are the flexibility to adopt individual constraints and target functions in combination with the broad and deep array of results, which allows to investigate individual, path-dependent behavior of assets and liabilities as well as scenario based and Monte-Carlo like risk assessment of both sides.

In light of recent changes in Austrian pension regulation the latter even gained additional importance, as the rather rigid asset-based limits were relaxed for institutions that could prove sufficient risk management expertise for both assets and liabilities of the plan. Thus, the implementation of a scenario-based asset allocation model will lead to more flexible allocation restraints that will allow for more risk tolerance and will ultimately result in better long-term investment performance.

Furthermore, some results of the model have been used by the Austrian regulatory authorities to assess the potential risk stemming from less-constraint pension plans.”

## Appendix. Pseudo-Code for Scenario Generation

```
SET N % number of assets
SET T % number of periods
SET cp, hp, np % probabilities for crash, high
and normal regimes
SET m % T × N matrix of mean gross returns adjusted
for period length
```

```
SET cs, hs, ns % T × N matrices of volatilities
adjusted for period length
```

```
SET cc, hc, nc % N × N correlations for crash, high
and normal regimes
```

```
FOR period = 1 to T
    nz = n_nodes[period] % total number of nodes in
    the current period
```

```
COMPUTE nz × N matrix Z % antithetic random
numbers with mean 0 and std.dev 1; Z can have normal,
t- or historical distribution
```

```
DEFINE ic, ih, in % arrays with random pointers to
rows of Z; ic, ih, and in have dimension CEIL(cp*nz),
CEIL(hp*nz), and nz-CEIL(cp*nz)-CEIL(hp*nz); the
joint index set is nonoverlapping; in each period a
different set of random pointers is used
```

```
COMPUTE mean 0 returns with required correlation
and volatility in each regime:
```

```
Y[ic,.] = (Z[ic,.]*CHOL(cc)).*cs[period,.]
Y[ih,.] = (Z[ih,.]*CHOL(hc)).*hs[period,.]
Y[in,.] = (Z[in,.]*CHOL(nc)).*ns[period,.]
```

```
COMPUTE R_period = m[period,.] + Y % gross
returns for current period
```

```
ENDFOR
```

## Acknowledgments

This research was supported by Innovest and the Natural Sciences and Engineering Research Council of Canada. Thanks to Melania Paunescu for computer help; Wolfgang Herold, Konrad Kontriner, and Johann Maurer for encouragement and support; Alan King and Alex Weissensteiner; the former area editor, Rick Rosenthal; the current area editor, Steve Graves; an anonymous associate editor, and the referees for helpful comments on earlier versions of this paper.

This paper was presented at the Hermes Centre of Excellence in Computational Finance and Economics, the Conference on Asset and Liability Management: From Institutions to Households, Nicosia, Cyprus, May 2001; the International Conference on Financial Engineering, E-Commerce and Supply Chain, Athens, Greece, May 2001; the Chicago Fed/DePaul University Finance Seminar, June 2001; the 9th International Conference on Stochastic Programming, Berlin, August 2001; the IFIP/IIASA/GAMM Workshop on Dynamic Stochastic Optimization, IIASA, Laxenburg, Austria, March 2002; Southampton University and Imperial College, September 2002; the Euro Working Group on Financial Modeling, Cyprus and University of Bergamo, November 2002; the SAID Business School, Oxford University, March 2003; the University of Zurich and the Mathematical Institute, Humboldt University, June 2003; the Operations Research Department of the ETH Zurich, July 2003; the Work-

shop on Asset Liability Management in Insurance in Vienna, September 2004; the University of Washington Statistics and Industrial Engineering Departments, June and November 2005; the Control Engineering Department, ETH, Zurich, October 2005; the 19th International Conference on Mathematical Programming, Rio de Janeiro, August 2006; the International Workshop on Forecasting and Risk Control, Chinese Academy of Sciences, Beijing, December 2006; the Mathematical Institute, Oxford University and the Department of Mathematics, University of Edinburgh, March 2007; Universities of Bergamo and Venice, May 2008; and Universities of Florence, Zurich, and Reading, June 2008.

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