# New Frontiers in Benchmarking and Liability-Driven Investing

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# Abstract

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Meeting the challenges of modern investment practice involves the design of novel forms of investment solutions, as opposed to investment products, customised to meet investors' long-term objectives while respecting the short-term (regulatory or otherwise) constraints they have to face. We argue in this paper that such new forms of investment solutions should rely on the use of improved performance-seeking and liability-hedging building-block portfolios, as well as on the use of improved dynamic allocation strategies. Although each of the ingredients discussed in this paper may already be found separately in existing investment products, we suggest that it is only by putting the pieces of the puzzle together, and by combining the underlying sources of expertise and added value that the asset management industry will satisfactorily address investors' needs.

# Introduction

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Asset management is justified as an industry by the capacity of adding value through the design of investment solutions that match investors' needs. For more than fifty years, the industry has in fact focused mostly on security selection as a single source of added value. This focus has somewhat distracted the industry from another key source of added value, namely, portfolio construction and asset allocation decisions. In the face of recent crises, and given the intrinsic difficulty of delivering added value through security selection decisions alone, the relevance of the old paradigm has been questioned with heightened intensity, and a new paradigm is starting to emerge.

In a nutshell, the new paradigm recognises that the art and science of portfolio management consists of constructing dedicated portfolio *solutions*, as opposed to one-size-fits-all investment *products*, so as to reach the return objectives defined by the investor, while respecting the investor's constraints expressed in terms of (absolute or relative) risk budgets. In this broader context, asset allocation and portfolio construction decisions appear as the main source of added value by the investment industry, with security selection being a third-order problem. As argued throughout this paper, asset allocation and portfolio construction decisions are intimately related to risk management. In the end, the quintessence of investment management is essentially about finding optimal ways to spend risk budgets that investors are reluctantly willing to set, with a focus on allowing the greatest possible access to performance potential while respecting such risk budgets. Risk diversification, risk hedging, and risk insurance will be shown to be three useful

approaches to optimal spending of investors' risk budgets.

Academic research has provided very useful guidance to the ways asset allocation and portfolio construction decisions should be analysed so as to best improve investors' welfare. In brief, the "fund separation theorems" that lie at the core of modern portfolio theory advocate separate management of performance and risk-control objectives. In the context of asset allocation decisions with consumption/liability objectives, it can be shown that the suitable expression of the fund separation theorem provides rational support for *liability-driven investment* (LDI) techniques that have recently been promoted by a number of investment banks and asset management firms. These solutions involve, on the one hand, the design of a *customised liability-hedging portfolio* (LHP), the sole purpose of which is to hedge away as effectively as possible the impact of unexpected changes in risk factors affecting liability values (most notably interest rate and inflation risks), and, on the other hand, the design of a *performance-seeking portfolio* (PSP), whose *raison d'être* is to provide investors an optimal risk/return trade-off.[1]

One of the implications of this LDI paradigm is that one should distinguish two different levels of asset allocation decisions: allocation decisions involved in the design of the performance-seeking or the liability-hedging portfolio (design of better building blocks, or BBBs), and asset allocation decisions involved in the optimal split between the PSP and the LHP (design of advanced asset allocation decisions, or AAAs). Each level of analysis involves its

1 - More generally, there are other forms of hedging demand that allow investors to neutralise the impact of unexpected changes in risk factors affecting the opportunity set or the wealth process. This is discussed in more detail later in this paper, where we cover life-cycle investment strategies.

# Introduction

own challenges and difficulties, and while the LDI paradigm is now widely adopted in the institutional world, very few market participants adopt an implementation approach of the paradigm that is fully consistent with the state-of-the-art of academic research.

Our ambition in this paper is to describe the most advanced forms of LDI strategies. Our focus is to provide not so much a thorough and rigorous treatment of all technical questions related to asset allocation and portfolio construction as a holistic overview of the key conceptual challenges involved. We address both questions (BBB and AAA) in this paper. More specifically, we first focus here on how to construct efficient performance-seeking and liability-hedging portfolios, and then move on to provide information on how to allocate optimally to these two building blocks once they have been designed.

In the next section, we present the challenges related to asset allocation and portfolio construction decisions within the PSP. We then discuss the challenges related to asset allocation and portfolio construction decisions within the LHP. The last section provides an introduction to optimal allocation to the PSP and the LHP for a long-term investor facing short-term constraints, once these two key building blocks have been properly designed. A few concluding thoughts can be found in a final section. Further technical details are relegated to an appendix.

# Introduction

# 1. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Performance-Seeking Portfolio

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Modern portfolio theory again provides some useful guidance to the optimal design of a PSP that would best suit investors' needs. More precisely, the prescription is that the PSP should be obtained as the result of a portfolio optimisation procedure aiming at generating the highest risk-reward ratio.

Portfolio optimisation is a straightforward procedure, at least in principle. In a mean-variance setting, for example, the prescription consists of generating a maximum Sharpe ratio (MSR) portfolio based on expected return, volatility, and pairwise correlation parameters for all assets to be included in the portfolio, a procedure which can even be handled analytically in the absence of portfolio constraints.

More precisely, consider a simple mean-variance problem:

$$\max_{w} \mu_p - \frac{1}{2}\gamma\sigma_p^2$$

Here, the control variable is a vector $w$ of optimal weight allocated to various risky assets, $\mu_p$ is the portfolio expected return, and $\sigma_p$ the portfolio volatility. We further assume that the investor is facing the following investment opportunity set: a riskless bond paying the risk-free rate $r$, and a set of $N$ risky assets with expected return vector $\mu$ (of size $N$) and covariance matrix $\Sigma$ (of size $N$x$N$), all assumed constant so far.

With these notations, the portfolio expected return and volatility respectively are given by:

$$\mu_p = w'(\mu - re) + r$$
$$\sigma_p^2 = w'\Sigma w$$

In this context, it is straightforward to show by standard arguments that the only efficient portfolio composed with risky assets is the maximum Sharpe ratio portfolio, also known as the tangency portfolio. Appendix A.1 provides more details.

Finally, the Sharpe ratio reads (where we further let $e$ be a vector of ones of size $N$):

$$SR = \frac{w'(\mu - re)}{(w'\Sigma w)^{1/2}}$$

And the optimal portfolio is given by:

$$\max_{w}\left(\mu_p - \frac{1}{2}\gamma\sigma_p^2\right)$$
$$\Rightarrow w_0^* = \frac{1}{\gamma}\Sigma^{-1}(\mu - re) = \frac{e'\Sigma^{-1}(\mu - re)}{\gamma}\underbrace{\frac{\Sigma^{-1}(\mu - re)}{e'\Sigma^{-1}(\mu - re)}}_{PSP}$$

(1)

This is a two-fund separation theorem, which gives the allocation to the MSR performance-seeking portfolio (PSP), with the rest invested in cash, as well as the composition of the MSR performance-seeking portfolio.

In practice, investors end up holding more or less imperfect proxies for the truly optimal performance-seeking portfolio, if only because of the presence of parameter uncertainty, which makes it impossible to obtain a perfect estimate for the maximum Sharpe ratio portfolio. If we let $\lambda$ be the Sharpe ratio of the (generally inefficient) PSP actually held by the investor, and $\sigma$ be its volatility, we obtain the following optimal allocation strategy:

$$w_0^* = \frac{\lambda}{\gamma\sigma}PSP \qquad (2)$$

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Hence, the allocation to the performance-seeking portfolio is a function of two objective parameters, the PSP volatility and the PSP Sharpe ratio, and one subjective parameter, the investor's risk aversion. The optimal allocation to the PSP is inversely proportional to the investor's risk-aversion. If risk aversion rises to infinity, the investor holds only the risk-free asset, as should be expected. For finite risk aversion, the allocation to the PSP is inversely proportional to the PSP volatility, and it is proportional to the PSP Sharpe ratio. As a result, if the Sharpe ratio of the PSP is increased, one can invest more in risky assets. Hence, risk management is not only about risk reduction; it is also about performance enhancement through a better expenditure of investors' risk budgets. We revisit this point later in the paper.

The expression (1) is useful because, in principle, it provides a straightforward expression for the optimal portfolio starting from a set of $N$ risky assets. In the presence of a realistically large number $N$ of securities, the curse of dimensionality, however, makes it practically impossible for investors to implement such direct one-step portfolio optimisation decisions involving all individual components of the asset mix. The standard alternative approach widely adopted in investment practice consists instead of first grouping individual securities in various asset classes by various dimensions, e.g., country, sector, and/or style within the equity universe, or country, maturity, and credit rating within the bond universe, and subsequently generating the optimal portfolio through a two-stage process. On the one hand, investable proxies are generated for maximum Sharpe ratio (MSR) portfolios within each asset class in the investment universe. We call this step,

which is typically delegated to professional money managers, the *portfolio construction step*. On the other hand, when the MSR proxies are obtained for each asset class, an optimal allocation to the various asset classes is eventually generated so as to generate the maximum Sharpe ratio at the global portfolio level. This step is called the *asset allocation step*, and it is typically handled by a centralised decision maker (*e.g.*, a pension fund CIO) with or without the help of specialised consultants, as opposed to being delegated to decentralised asset managers. We discuss both of these steps in what follows.

## 1.1. Portfolio Construction Step: Designing Efficient Benchmarks

In the absence of active views, the default option consists of using market-cap-weighted indices as proxies for the asset class MSR portfolio. Academic research, however, has found that such indices were likely to be severely inefficient portfolios (Haugen and Baker 1991; Grinold 1992; Amenc, Goltz, and Le Sourd 2006). In sum, market-cap-weighted indices are not good choices as investment benchmarks because they are poorly diversified portfolios. In fact, capitalisation weighting tends to lead to exceedingly high concentration in relatively few stocks. As a result of their lack of diversification, cap-weighted indices have empirically been found to be highly inefficient portfolios that do not provide investors fair rewards for the risks they take. For the same reason, they have been found to be dominated by equally-weighted benchmarks (De Miguel, Garlappi, and Uppal 2009); equally-weighted benchmarks use a naive weighting scheme that ensures that they are well diversified, but they are optimal in the mean-variance sense

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if and only if all securities have identical expected returns and volatilities and all pairs of correlation are identical.

In what follows, we analyse in some detail a number of alternatives based on practical implementation of modern portfolio theory that have been suggested to generate more efficient proxies for the MSR portfolio in the equity or fixed-income investment universes (see exhibit 1).

Modern portfolio theory was born with the efficient frontier analysis of Markowitz in 1952. Unfortunately, early applications of the technique, based on naïve estimates of the input parameters, have been of little use because they lead to unreasonable portfolio allocations.

We explain below first how to help bridge the gap between portfolio theory and portfolio construction by showing how to generate enhanced parameter estimates and to improve the quality of the portfolio optimisation outputs (optimal portfolio weights). We begin by focusing on enhanced covariance parameter estimates and explain how to meet the main challenge of sample risk reduction.[2] Against this backdrop, we present the state-of-the art methods of reducing the problem of dimensionality and estimating the covariance matrix with multi-factor models. We then turn to expected return estimation. We argue that statistical methods are not likely to generate any robust expected return estimates, which suggests that economic models such as the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT) should instead be used to estimate expected returns. Finally, we present evidence that proxies for

expected return estimates should include systematic risk measures, idiosyncratic risk measures, and downside risk measures.

### 1.1.1. Robust Estimators for Covariance Parameters

In practice, successful implementation of a theoretical model relies not only on its conceptual grounds but also on the reliability of the input to the model. In mean-variance (MV) optimisation the results will depend greatly on the quality of the parameter estimates: the covariance matrix and the expected returns of assets.

Several improved estimates for the covariance matrix have been proposed, including most notably the factor-based approach (Sharpe 1963), the constant correlation approach (Elton and Gruber 1973), and the statistical shrinkage approach (Ledoit and Wolf 2004). In addition, Jagannathan and Ma (2003) find that imposing (non-short selling) constraints on the weights in the optimisation programme improves the risk-adjusted out-of-sample performance in a manner that is similar to some of the aforementioned improved covariance matrix estimators.

In these papers, the focus was on testing the out-of-sample performance of *global minimum variance* (GMV) portfolios, as opposed to the MSR portfolios (also known as *tangency portfolios*), given that there is a consensus that purely statistical estimates of expected returns are not robust enough to be used, a point we return to later in this paper when we look at expected return estimation.

2 - Another key challenge is the presence of non-stationary risk parameters, which can be accounted for with conditional factor models capturing time-dependencies (e.g., GARCH-type models) and state-dependencies (e.g., Markov regime-switching models) in risk parameter estimates.

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Exhibit 1: Inefficiency of cap-weighted benchmarks and the quest for an efficient proxy for the true tangency portfolio.
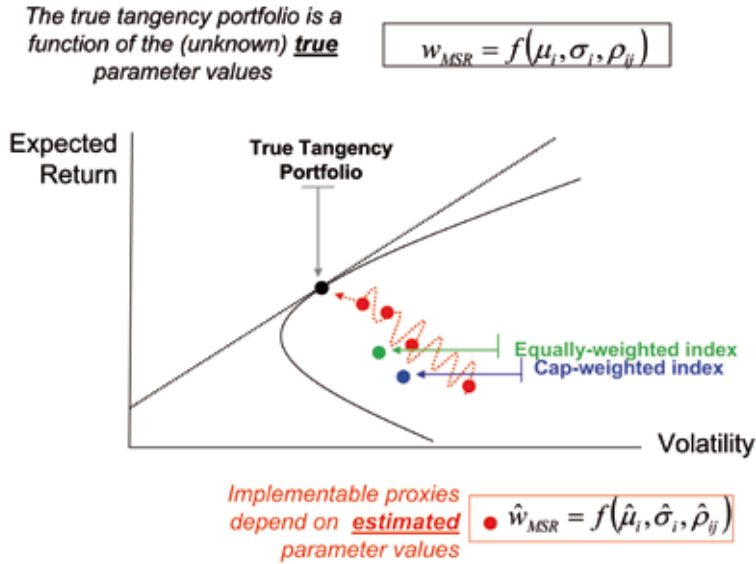


The key problem in covariance matrix estimation is the curse of dimensionality; when a large number of stocks are considered, the number of parameters to estimate grows exponentially, where the majority of them are pairwise correlations.

Therefore, at the estimation stage, the challenge is to reduce the number of factors that come into play. In general, a multifactor model decomposes the (excess) return (in excess of the risk-free asset) of an asset into its expected rewards for exposure to the "true" risk factors as follows:

$$r_{i\,t} = \alpha_{i\,t} + \sum_{j=1}^{K} \beta_{i,j\,t} \cdot F_{j\,t} + \varepsilon_{i\,t}$$

or in matrix form for all $N$ assets:

$$r_t = \alpha_t + \beta_t F_t + \varepsilon_t$$

where $\beta_t$ is a $NxK$ matrix containing the sensitivities of each asset $i$ to the corresponding $j$-th factor movements; $r_t$ is the vector of the $N$ assets' (excess) returns, $F_t$ a vector containing the $K$ risk factors'

(excess) returns, and $\varepsilon_t$ the $Nx1$ vector containing the zero mean uncorrelated residuals $\varepsilon_{i\,t}$. The covariance matrix for the asset returns, implied by a factor model, is given by:

$$\Omega = \beta \cdot \Sigma_F \cdot \beta^T + \Sigma_\varepsilon$$

where $\Sigma_F$ is the $KxK$ covariance matrix of the risk factors and $\Sigma_\varepsilon$ an $NxN$ covariance matrix of the residuals corresponding to each asset.

Although the factor-based estimator is expected to allow for a reasonable trade-off between sample risk and model risk, there is still the problem of choosing the "right" factor model. One popular approach attempts to rely as little as possible on strong theoretical assumptions by using *principal component analysis* (PCA) to determine the underlying risk factors from the data. The PCA method is based on a spectral decomposition of the sample covariance matrix and its goal is to use only a few linear combinations of the original

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stochastic variables—combinations that will constitute the set of (unobservable) factors—to explain covariance structures.

Bengtsson and Holst (2002) and Fujiwara *et al.* (2006) motivate the use of PCA in a similar way, extracting principal components to estimate expected correlation within MV portfolio optimisation. The latter find that the realised risk-return of portfolios based on the PCA method outperforms the portfolio based on a single index and that the optimisation gives a practically reasonable asset allocation. On the whole, the main strength of the PCA approach at this point is that it leads to "letting the data talk" and having them tell us what the underlying risk factors that govern most of the variability of the assets at each point in time are. This strongly contrasts with having to rely on the assumption that a particular factor model is the true pricing model and reduces the specification risk embedded in the factor-based approach, all while keeping the sample risk reduction.

The question of determining the appropriate number of factors to structure the correlation matrix is critical for the risk estimation when PCA is used as a factor model. Several options, some on greater theoretical grounds than others, have been proposed to answer this question.

As a final note, we need to recognise that the discussion is so far cast in a mean-variance setting, which can, in principle, be rationalised only for normally distributed asset returns. In the presence of non-normally distributed asset returns, optimal portfolio selection techniques require estimates for variance-covariance parameters, along with estimates for

higher-order moments and co-moments of the return distribution. This is a formidable challenge that greatly exacerbates the dimensionality problem already present with mean-variance analysis. In a recent paper, Martellini and Ziemann (2010) extend the existing literature, which has focused mostly on the covariance matrix, by introducing improved estimators for the co-skewness and co-kurtosis parameters. On the one hand, they find that the use of these enhanced estimates generates a significant improvement in investor welfare. On the other hand, they find that when the number of constituents in the portfolios is large (more than twenty, for example), the increase in sample risk arising from the need to estimate higher-order co-moments far outweighs the benefits of considering a more general portfolio optimisation procedure. When portfolios with large numbers of assets are optimised, maximising the Sharpe ratio leads to better out-of-sample results than does maximising a return-to-VaR ratio. It does so even when portfolio performance is assessed with measures that rely on VaR rather than on volatility to adjust for risk. Similar arguments hold for other extreme risk measures such as CVaR. In the end, using extreme risk measures in portfolios with large numbers of assets leads to a formidable estimation problem, and empirical results suggest that it is sensible to stay with the mean-variance approach, in which reliable input estimates can be derived.

## 1.1.2. Robust Estimators for Expected Returns

Although it appears that risk parameters can be estimated with a fair degree of accuracy, expected returns are difficult to obtain with a reasonable estimation error

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(Merton 1980). What makes the problem worse is that optimisation techniques are very sensitive to differences in expected returns, so portfolio optimisers usually allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (Britten-Jones 1999; Michaud 1998).

In view of the difficulty of using sample-based expected return estimates in a portfolio optimisation context, a reasonable alternative is to use some risk estimate as a proxy for excess expected returns.[3] This approach is based on the most basic principle in finance, i.e., the natural relationship between risk and reward. In fact, standard asset pricing theories such as the APT imply that expected returns should be positively related to systematic volatility, as measured through a factor model that summarises individual stock return exposure to a number of rewarded risk factors.

More recently, several papers have focused on the explanatory power of idiosyncratic rather than systematic risk for the cross-section of expected returns. In particular, Malkiel and Xu (2006), extending an insight from Merton (1987), show that an inability to hold the market portfolio, whatever the cause, will force investors to deal, to some degree, with total risk in addition to market risk, so firms with larger firm-specific variances require higher average returns to compensate investors for holding imperfectly diversified portfolios.[4] That stocks with high idiosyncratic risk earn higher returns has also been confirmed in a number of recent empirical studies.

Taken together, these findings suggest that *total* risk, a model-free quantity given by the sum of systematic and specific risk, should be positively related to expected return. Most commonly, total risk is the volatility of a stock's returns. Martellini (2008) has investigated the portfolio implications of these findings and found that tangency portfolios constructed on the assumption that the cross-section of excess expected returns could be approximated by the cross-section of volatility posted better out-of-sample risk-adjusted performance than their market-cap-weighted counterparts.

More generally, recent research suggests that the cross-section of expected returns might best be explained by risk indicators taking into account higher-order moments. Theoretical models have shown that, in exchange for higher skewness and lower kurtosis of returns, investors are willing to accept expected returns lower (and volatility higher) than those of the mean-variance benchmark (Rubinstein 1973; Krauz and Litzenberger 1976). More specifically, skewness and kurtosis in individual stock returns (as opposed to the skewness and kurtosis of aggregate portfolios) have been shown to matter in several papers. High skewness is associated with lower expected returns in several studies (Barberis and Huang 2004; Brunnermeier, Gollier, and Parker 2007; Mitton and Vorkink 2007). The intuition behind this result is that investors like to hold positively skewed portfolios. The highest skewness is achieved by concentrating portfolios in a small number of stocks that themselves have positively skewed returns. Thus investors tend to be underdiversified and drive up the price of stocks with high positive skewness, which in

3 - This discussion focuses on estimating the fair neutral reward for holding risky assets. If one has access to active views on expected returns, one may use a disciplined approach (*e.g.*, the Black-Litterman model) to combine the active views and the neutral estimates.
4 - For a similar conclusion from a behavioural perspective, see Barberis and Huang (2001).

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turn reduces their future expected returns. Stocks with negative skewness are relatively unattractive and thus have low prices and high returns. The preference for kurtosis is in the sense that investors like low kurtosis and thus expected returns should be positively related to kurtosis. Two studies provide empirical evidence that individual stocks' skewness and kurtosis are indeed related to future returns (Boyer, Mitton, and Vorkink 2010; Conrad, Dittmar, and Ghysels 2008). An alternative to direct consideration of the higher moments of returns is to use a risk measure that aggregates the dimensions of risk. In this respect, Bali and Cakici (2004) show that future returns on stocks are positively related to their Value-at-Risk and Estrada (2000) and Chen, Chen, and Chen (2009) show that there is a relationship between downside risk and expected returns.

### 1.1.3. Implications for Benchmark Portfolio Construction

Once careful estimates for risk and return parameters have been obtained, one may then design efficient proxies for asset class benchmarks with attractive risk/return profiles. For example Amenc *et al.* (2010) find

that efficient equity benchmarks designed on the basis of robust estimates for risk and expected return parameters substantially outperform in terms of risk-adjusted performance the market-cap-weighted indices often used as default options for investment benchmarks in spite of their well-documented lack of efficiency (Haugen and Baker 1991; Grinold 1992).

Exhibit 2, borrowed from Amenc *et al.* (2010), shows summary performance statistics for an efficient index constructed in keeping with the aforementioned principles. For the average return, volatility, and Sharpe ratio, we report differences with respect to cap-weighting and assess whether this difference is statistically significant.

Exhibit 2 shows that the efficient weighting of index constituents leads to higher average returns, lower volatility, and higher Sharpe ratios. All these differences are statistically significant at the 10% level, whereas the difference in Sharpe ratios is significant even at the 0.1% level. Given the data, it is highly unlikely that the unobservable true performance of efficient weighting was not different from that of capitalisation

*Exhibit 2: Risk and return characteristics for the efficient index*

| Index | Ann. average return (compounded) | Ann. standard deviation | Sharpe ratio (compounded) | Information ratio | Tracking error |
|---|---|---|---|---|---|
| Efficient index | 11.63% | 14.65% | 0.41 | 0.52 | 4.65% |
| Cap-weighted | 9.23% | 15.20% | 0.24 | 0.00 | 0.00% |
| Difference (efficient minus cap-weighted) | 2.40% | -0.55% | 0.17 | - | - |
| p-value for difference | 0.14% | 6.04% | 0.04% | - | - |

*The table shows risk and return statistics portfolios constructed with the same set of constituents as the cap-weighted index. Rebalancing is quarterly subject to an optimal control of portfolio turnover (by setting the reoptimisation threshold to 50%). Portfolios are constructed by maximising the Sharpe ratio given an expected return estimate and a covariance estimate. The expected return estimate is set to the median total risk of stocks in the same decile when sorting by total risk. An implicit factor model for stock returns is used to estimate the covariance matrix. Weight constraints are set so that each stock's weight is between 1/2N and 2/N, where N is the number of index constituents. P-values for differences are computed using the paired t-test for the average, the F-test for volatility, and a Jobson-Korkie test for the Sharpe ratio. The results are based on weekly return data from 01/1959 to 12/2008.*

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weighting. Economically, the performance difference is pronounced, as the Sharpe ratio increases by about 70%.

## 1.2. Asset Allocation Step: Putting the Efficient Benchmarks Together

After efficient benchmarks have been designed for various asset classes, these building blocks can be assembled in a second step, the asset allocation step, to build a well-designed multiclass performance-seeking portfolio. Although the methods we have discussed so far can, in principle, be applied in both contexts, a number of key differences should be emphasised.

In the asset allocation context, the number of constituents is small, and using time- and state-dependent covariance matrix estimates is reasonable; nonetheless, these estimates do not necessarily improve the situation in portfolio construction contexts, in which the number of constituents is large. Similarly, although it is not, in general, feasible to optimise the portfolio with higher-order moments in a portfolio construction context, in which the number of constituents is typically large, it is reasonable to go beyond mean-variance analysis in an asset allocation context, in which the number of constituents is limited.

Furthermore, in asset allocation, the universe is not homogeneous, which has implications for expected returns and covariance estimation. In terms of covariance matrix, it will not prove easy to obtain a universal factor model for the entire investment universe. In this context, it is arguably better to use statistical shrinkage towards, say, the constant correlation model, than to take

a factor model approach (Ledoit and Wolf 2003; Ledoit and Wolf 2004).

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# 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability-Hedging Portfolio

# 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

Risk diversification is only one possible form of risk management, focusing merely on achieving the best risk/return trade-off regardless of investment objectives and constraints. On the other hand, one should recognise that diversification is simply not the appropriate tool when it comes to protecting long-term liability needs.

One key academic insight from the pioneering work of Robert Merton in the nineteen-seventies is that the presence of state variables impacting the asset return and/or wealth process will lead to the introduction of dedicated *hedging demands*, in addition to cash and optimally diversified PSP (which is still needed).

In particular, it is clear that the risk factors impacting pension liability values should be hedged rather than diversified away. Two of these factors, interest rate risk and inflation risk, stand out. Although constructing interest rate and inflation hedging benchmarks might seem straightforward compared to constructing performance-seeking benchmarks, some challenges remain, which we discuss now.

## 2.1. Towards the Design of Improved Interest Rate Risk Benchmarks

A first approach to the design of the LHP, called *cash-flow matching*, involves ensuring a perfect *static* match between the cash flows from the portfolio of assets and the commitments in the liabilities. Let us assume, for example, that a pension fund has a commitment to pay out a monthly pension to a retired person. Leaving aside the complexity relating to the uncertain life expectancy of the retiree, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the real value of which is known today, but for which the nominal value is typically matched with an inflation index. It is possible, in theory, to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments. Doing so would involve purchasing—assuming that securities of that kind exist on the market—inflation-linked zero-coupon bonds with a maturity corresponding to the dates on which the monthly pension instalments are paid out, with amounts that are proportional to the amount of real commitments.

Although this technique has the advantage of simplicity and, in theory, allows perfect risk management, it has a number of limitations and implementation poses several challenges. In particular, finding bond portfolios with the proper duration is hardly feasible, especially in the corporate bond segment.

The conflicts of interest between issuers and investors about the duration of corporate bonds is known as the duration problem. Each bond investor has in mind a specific time horizon, and there is no reason to expect that these needs correspond to the optimal financing plan of the issuers. In fact, the duration structure of outstanding bonds reflects the preferences of the issuers in their aim to minimise the cost of capital. This minimisation is fundamentally opposed to the interest of the investors, who usually try to maximise their returns. Although as such a part of the suitability problem mentioned above, the duration mismatch in the corporate bond market is of primordial importance to investors. Pension funds have some fixed nominal liabilities originating from their defined-benefit plans. Given this

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long-term perspective, long-term bonds are a much better hedge than short-term debt. Issuers of such bonds therefore have to pay only a small yield premium—even though they are more volatile. In contrast, for short-term investors with no fixed time horizon in mind such investments are far less attractive. The duration of the indices is nonetheless a result of the sell-side of corporate bonds—so no investor should hold just this benchmark duration. Hence, many corporate bond indices are not well suited to serving as benchmarks for corporate bond investors.

More worrisome perhaps is that the characteristics of corporate bond indices can change over time. So, efforts to design stable corporate bond indices optimised in an attempt not only to maximise their risk-adjusted performance but also to display a (quasi-) constant duration and allocation by rating class over time are required.

## 2.2. Towards the Design of Improved Inflation-Hedging Benchmarks

A recent surge in worldwide inflation has increased the need for investors to hedge against unexpected changes in prices. Inflation hedging has in fact become a concern of critical importance for private investors, who consider inflation a direct threat to their purchasing power, as well as for pension funds, which must make pension payments often indexed to consumer prices or wages.

In this context, novel forms of institutional investment solutions have been promoted by asset managers and investment banks, focusing on the design of customised

liability-matching portfolios, the sole purpose of which is to hedge away as effectively as possible the impact of unexpected changes in the risk factors—most notably inflation—affecting liability values. A variety of cash instruments (Treasury inflation protected securities, or TIPS) as well as dedicated OTC derivatives (such as inflation swaps) are used to achieve a customised exposure to consumer price inflation. One outstanding problem, however, is that such solutions generate very modest performance given that real returns on inflation-protected securities, negatively impacted by the presence of a significant inflation risk premium, are usually very low. In this context, it has been argued that some other asset classes, such as stocks, real estate, or commodities, could provide useful inflation protection, especially when long-term horizons are considered, at a cost lower than that of investing in TIPS.

Empirical evidence suggests that there is in fact a negative relationship between expected stock returns and expected inflation, which is consistent with the intuition that higher inflation depresses economic activity and thus depresses stock returns. On the other hand, higher future inflation leads to higher dividends and thus higher returns on stocks, so equity investments should offer significant inflation protection over long horizons (as it happens, several recent empirical studies [Boudoukh and Richardson 1993; Schotman and Schweizer 2000] have confirmed that equities provide a good hedge against inflation over the long term). This property is particularly appealing for long-term investors such as pension funds, which need to match price increases at the horizon but

## 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

not on a monthly basis. Obviously, different kinds of stocks offer different inflation-hedging benefits, and it is in fact possible to select stocks or sectors on the basis of their ability to hedge against inflation. For example, utilities and infrastructure typically have revenues highly correlated with inflation, and as a result they tend to provide better-than-average inflation protection. So it seems possible to select stocks or sectors on the basis of their ability to hedge against inflation (hedging demand), as opposed to selecting them as a function of their outperformance potential (speculative demand). In this context, one can envision selecting stocks or sectors in an attempt to maximise the inflation-hedging property of equity-based inflation-hedging solutions. The analysis typically involves two separate phases, selection and optimisation. The goal of the selection phase is to select the set of stocks likely to exhibit the most attractive inflation-hedging properties. In the second phase, a portfolio of selected stocks will be formed in such a way as to optimise the expected inflation-hedging benefits.

Going beyond the equity universe, similar inflation-hedging properties are expected for bond returns. Indeed, bond yields may be decomposed into a real yield and an expected inflation component. Since expected and realised inflation move together over the long term, a positive long-term correlation between bond returns and changes in inflation is expected. In the short-term, however, expected inflation may deviate from actual realised inflation, leading to low or negative correlations in the short term. It has also been recently argued that alternative forms of investment offer attractive inflation-hedging benefits.

Commodity prices, in particular, are believed to be leading indicators of inflation in that they are quick to respond to economy-wide shocks to demand. Commodity prices are usually set in highly competitive auction markets and consequently tend to be more flexible than prices in general. In addition, recent inflation is fuelled heavily by increases in commodity prices, in particular in agriculture, minerals, and energy. In the same vein, commercial and residential real estate provide at least a partial hedge against inflation, and portfolios that include real estate realise an increase in inflation hedgeability, especially over longer horizons.

Exhibit 3 (taken from Amenc, Martellini, and Ziemann [2009], a paper to which we refer for further details on the calibration of the VAR and VECM models), which displays a set of estimated term structure of correlation coefficients between asset returns and inflation-linked liability returns. It confirms that various asset classes have different inflation-hedging properties over various horizons; inflation-hedging capacity increases in tandem with the horizon for stocks, bonds, and real estate.

As a consequence of the aforementioned findings, it is tempting to investigate whether novel liability-hedging investments can be designed to decrease the cost to the investor of inflation insurance. In particular, it is possible to construct different versions of the inflation-hedging portfolio to assess the impact of introducing investment classes such as equities, commodities, and real estate in addition to inflation-linked bonds. Amenc, Martellini, and Ziemann (2009) have shown that the increased expected return generated by adding asset classes

# 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

Exhibit 3: Term structure of correlation coefficients between different asset returns and inflation-linked liability returns for various horizons
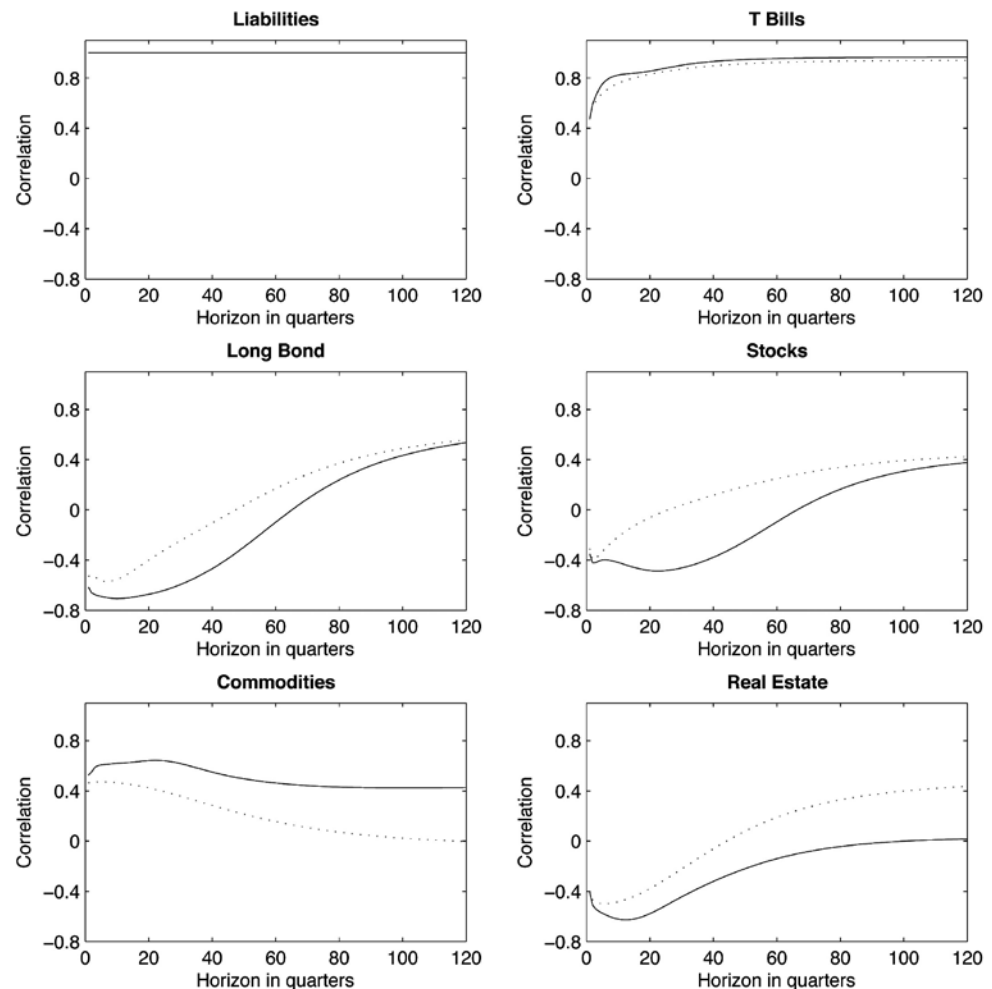


Dotted lines correspond to implied correlations estimated from a vector auto regressive (VAR) model, whereas solid lines correspond to implied correlations estimated from a vector error correction model (VECM).
Source: Amenc, Martellini, and Ziemann (2009).

with good long-term inflation-hedging properties allows pension fund sponsors to maintain contributions and lower exposure to downside risk.

Other advanced solutions may involve hedging a particular segment of inflation distribution, with an expected focus on hedging large, as opposed to moderate, inflation shocks, in an attempt to reduce yet again the costs of inflation hedging

and hence enhance the performance of the inflation-hedging portfolio.

As a final note, we analyse in the next section situations in which the separation of the performance-seeking and liability-hedging portfolios does not apply in a strict sense. More specifically, we consider situations in which there are multiple and equally attractive candidates for the PSP and LHP.

# 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

## 2.3.Performance-Seeking Portfolios with Attractive Liability- /Inflation-Hedging Properties

As previously mentioned, asset pricing theory is grounded on *separation theorems* that state that risk and performance are two conflicting objectives best managed separately. According to this paradigm, performance generation is obtained first through optimal exposure to rewarded risk factors to alleviate the burden on contributions, while hedging against unexpected shocks that impact current value of (assets and) liabilities is accounted for by a separate dedicated portfolio.

This clear separation of performance and hedging portfolios is very useful and has a number of important implications not only for portfolio construction and asset allocation techniques but also for the organisational structure of the institutional investor.

If an investor were given a choice of two (or more) performance-seeking portfolios with identical risk/reward ratios but distinct liability-hedging properties he would obviously favour the performance portfolio with the most attractive liability-hedging properties.

In fact, this hypothetical question would not arise if the efficient frontier is strictly concave, which would ensure the existence and uniqueness of the maximum risk/ reward ratio portfolio, as is the case in the standard mean-variance paradigm with perfect information.

It has been shown, however, that when a general, not strictly convex risk measure is used the efficient frontier may not be strictly concave, and as a result the max reward/ risk portfolio may not be unique (Stoyanov, Rachev, and Fabozzi 2007). A similar result would hold for a mean-variance objective in the absence of perfect information regarding the risk/return parameters. See exhibit 4 for an illustration. As a result, in

*Exhibit 4: Multiple true or estimated tangency portfolios*
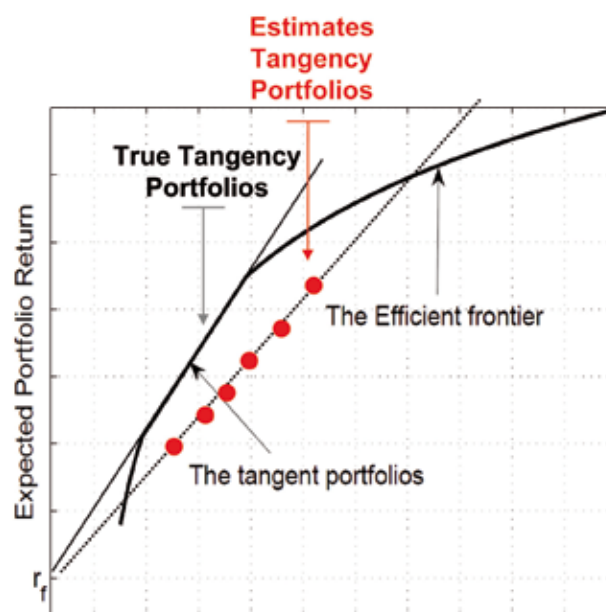
# 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

most realistic situations, if given a choice of seemingly attractive portfolio-seeking portfolios, it would be rational for an investor to select the performance portfolio with the most attractive liability-hedging properties, and this would not conflict with the fund separation theorem. Conversely, if an investor had a choice of liability-hedging portfolios with equally attractive inflation-hedging benefits, the investor would tend to favour the hedging portfolio with the most attractive risk/reward ratio.

## 2. Asset Allocation and Portfolio Construction Decisions in the Optimal Design of the Liability–Hedging Portfolio

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

Assuming that reasonable proxies for performance-seeking and liability-hedging portfolios have been designed using some of the aforementioned methods, we must still determine the optimal strategy to allocate assets to those two building blocks. Several novel paradigms, which we describe next, are reshaping our approaches to long-term investment decisions for long-term investors facing liability commitments and short-term performance constraints.

## 3.1. Accounting for the Presence of Investors' Liability Commitments: The Liability-Driven Investment Paradigm

As explained earlier in this paper, investors with consumption/liability objectives need to invest in two distinct portfolios, in addition to cash: one performance-seeking portfolio and one liability-hedging portfolio, construction methods for which have been discussed in previous sections.

Formally, under the assumption of a constant opportunity set,[5] we obtain the following expression of the fund separation theorem in the intertemporal context when trading is possible between current date and investment horizon:

$$w_t^* = \frac{\lambda}{\gamma\sigma} PSP + \left(1 - \frac{1}{\gamma}\right)\beta \ LHP \qquad (3)$$

This expression is similar to that in equation (2), extended to the asset/liability management setting. As appears from equation (3), the allocation to the "risky" building block is still an increasing function of the PSP Sharpe ratio $\lambda$ and a decreasing function of the investor's risk aversion $\gamma$ and PSP volatility $\sigma$, as was

already the case in equation (2).[6] The allocation to the "safe" building block is an increasing function of the beta $\beta$ of liability portfolio with respect to the liability-hedging portfolio. If there is an asset portfolio that perfectly matches the liability value, then the beta is 1, and an infinitely risk-averse investor fully allocates to the LHP. This is consistent with the intuition that for an investor facing liability commitments the LHP, as opposed to cash, is the true risk-free asset.

Equation (2) is the solution to a static optimisation problem, and the corresponding strategy is (by design) of the buy-and-hold kind. Equation (3) is the solution to a dynamic optimisation problem, as shall be evidenced by the presence of an explicit time dependency in the expression for the optimal allocation strategy. The corresponding strategy is a fixed-mix strategy, where, in principle, constant trading occurs to rebalance the portfolio allocation back to the constant target. The fund separation theorem is expressed here under the assumption of a constant opportunity set. In later sections, we shall relax this assumption and analyse how the allocation is impacted by the introduction of time variation to the expected return and volatility of the PSP.

Exhibit 5 shows the distribution of the funding ratio, defined as asset value divided by liability value, at horizon. The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund.[7] On the left-hand side the Sharpe ratio of the PSP is assumed to be 0.24, whereas it is assumed to have improved by 50% to

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

0.36 on the right-hand side. The idea here is to capture possible improvements to the performance-seeking Sharpe ratio that would result from the use of efficient rather than cap-weighted benchmarks, as discussed in previous sections.

The expected funding ratio has increased significantly. Volatility increases as well, but this is mostly the result of higher dispersion on the upside. Regarding downside risk, the shortfall probability (formally defined as the probability that the funding ratio ends up below 100%) decreases from 19.23% to 11.97% when the Sharpe ratio of the PSP portfolio rises from 0.24 to 0.36. On the whole, the substantial improvement in the distribution of the funding ratio at horizon is the result of two effects. On the one hand, if the Sharpe ratio of the PSP rises, the expected value of the funding ratio improves, with associated benefits from an ALM perspective. On the other hand, the allocation to the PSP has also increased. Overall, we obtain that improving the risk/reward ratio is a key component in meeting investors' long-term objectives.

## 3.2. Accounting for the Presence of Investors' Long-Term Objectives: The Life-Cycle Investment Paradigm

Although it may be acceptable to assume a constant opportunity set when investors have a short-term horizon, a long horizon, typical of most investors' problems, makes it necessary to go beyond Markowitz static portfolio selection analysis. The next important step after Markowitz (1952) is Merton (1969, 1971), who takes portfolio construction techniques beyond the static setting and shows how to use dynamic programming to solve dynamic portfolio optimisation problems. In terms of industry implications, the development of dynamic asset pricing theory has led to the emergence of improved investment solutions that take into account the changing nature of investment opportunities. These novel forms of investment are broadly referred to as *life-cycle investing* strategies.

Current forms of life-cycle investing are sometimes grossly sub-optimal. For example, a popular asset allocation strategy for managing equity risk on

*Exhibit 5: Distribution of funding ratio with inefficient versus efficient PSP*
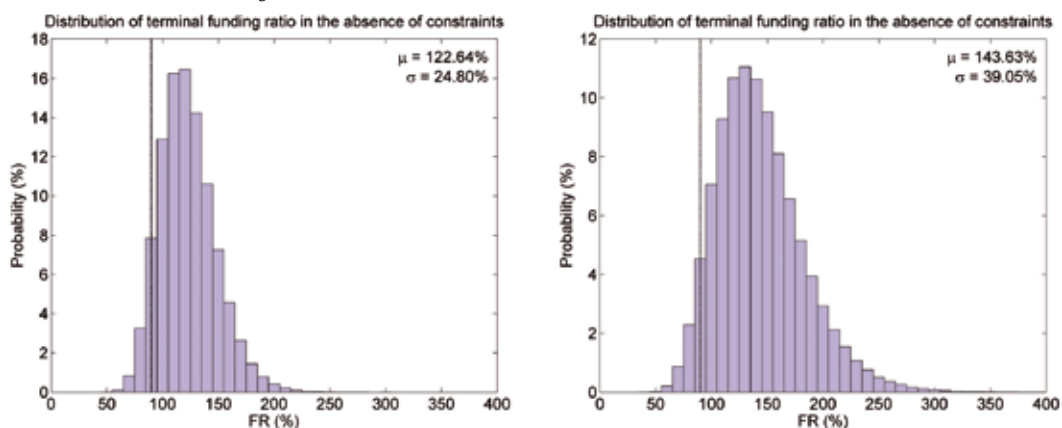


*The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund). On the left-hand side the Sharpe ratio of the PSP is assumed to be 0.24, whereas it is assumed to have improved by 50% to 0.36 on the right-hand side.*

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

behalf of a private investor in the context of a defined-contribution pension plan is known as deterministic life-cycle investing. In the early stages, when the retirement date is far away, the contributions are invested entirely in equities. Then, beginning on a predetermined date (ten years, say) before retirement, the assets are switched gradually to bonds at some pre-defined rate (10% a year, say). By the date of retirement, all the assets are held in bonds.[8] This is somewhat reminiscent of the rule of thumb put forward by Shiller (2005), advocating a percentage allocation to equity given by 100 minus the investor's age in years.

While deterministic life-cycle investing is a simple strategy popular with investment managers and consultants, and it is widely used by defined-contribution pension providers, there is no evidence that it is an optimal strategy in a rational sense, and we argue below that these strategies are very imperfect proxies for truly optimal stochastic life-cycle investing strategies. In general, the presence of risk factors can impact the opportunity set (risk and return parameters), and they can have a direct impact on the wealth process for non-self-financed portfolios when inflows and outflows of cash are taking place.

### 3.2.1. Accounting for Risk Factors Impacting the Investment Opportunity Set

A large body of empirical research has shown that interest and inflation rates, as well as expected return, volatility, and correlation parameters are stochastically time-varying, as a function of key state variables that describe the state of the business cycle. Unexpected changes

in these variables have an impact on portfolio risk and performance (through changes in interest rates and risk premium process parameters), which should be managed optimally (Detemple and Rindisbacher 2009). In fact, one can show that the dynamic asset allocation problem involves an optimal hedging of the state variables impacting the risk-free rate and the risk-premium processes. Except for a myopic investor, the optimal dynamic solution does not consist of merely taking the static solution and updating it with time-varying parameters; one should also introduce dedicated *hedging* portfolios.

For example, when interest rates are stochastic, cash is no longer a risk-free asset; the risk-free asset is instead a bond with a term to maturity matching the investor's horizon. It is hardly surprising, then, that the optimal allocation decision in the presence of interest rate risk involves an additional building block, the bond with a term to maturity matching the investor's horizon, in addition to cash and the highest risk/reward performance-seeking portfolio (a three-fund rather than two-fund separation theorem). As another illustration, let us assume that the expected return on most risky assets is, for example, negatively impacted by increases in oil prices. To compensate for the deterioration of the investment opportunity set in the event of a sharp increase in oil prices, the investor will benefit from holding a long position in a portfolio optimised to exhibit the highest possible correlation with oil prices.

Kim and Omberg (1996), for example, have analysed a model including a stochastic equity risk premium with a mean-

# 3. Dynamic Allocation Decisions to the Performance–Seeking and Liability–Hedging Portfolios

reverting component. In this model, one can show that the optimal allocation involves not only a deterministic decrease of the allocation to equity as the investor gets closer to the time horizon, which is consistent with standard target date fund practice, but also a state-dependent component, suggesting that the allocation to equity should be increased (respectively, decreased) when, as measured by such proxies as dividend yields or price-earnings ratios, equity has become cheap and decreased when it has become expensive.

We obtain the following expression for the optimal allocation strategy, assuming for simplicity that an equity benchmark is the only risky asset so that the PSP is 100% invested in that benchmark (see appendix A.3 for details):

$$w_t^* = \frac{\lambda_t^S}{\gamma \sigma_S} PSP - \underbrace{\left(1 - \frac{1}{\gamma}\right)\left[A(T-t) + B(T-t)\lambda_t^S\right]\frac{\rho_{S\lambda}\sigma_\lambda}{\sigma_S}}_{\text{hedging demand } HD_{t,T}^{\lambda}}$$

Here we let $\rho_{\lambda S}$ be the correlation of the Sharpe ratio of the equity index, denoted by $\lambda^S$, and the equity index return; a negative correlation means that high realised return periods tend to be followed by low expected return periods, which is supported by empirical evidence. Moreover, $\sigma_\lambda$ is the volatility of the equity Sharpe ratio process and $\sigma_S$ is the volatility on the stock index.

The hedging demand $HD_{t,T}^{\lambda}$ against unexpected changes in the PSP Sharpe ratio has the following properties (for $\rho_{\lambda S} < 0$ and $\gamma > 1$):
• The investor with $\gamma > 1$ holds more stocks when equity Sharpe ratio is mean-reverting

than when it is constant ($\sigma_\lambda = 0$);
• The hedging demand disappears if there is no equity risk premium risk ($\sigma_\lambda = 0$), or if the risk exists but cannot be hedged away ($\rho_{\lambda S} = 0$).
• The investment in stock decreases when approaching horizon $T$; this is consistent with the prescriptions of target date funds.

We also confirm that there is one additional state-dependent factor: if/when equity prices are low (high), and therefore expected return is high (low), one should allocate more (less) to stocks, regardless of horizon.

### 3.2.2. Accounting for Risk Factors Impacting the Wealth Process
As noted earlier, the presence of risk factors can also have a direct impact on the wealth process, even in the case of a constant opportunity set. In fact, most portfolios are not self-financed portfolios, because of the presence of outflows of cash (consumption, liability payments) and/or inflows of cash (endowment, contribution, income, and so on). For example, labour income risk will have an impact on optimal portfolio decisions and will legitimise the introduction of a dedicated hedging demand. More generally, the present value of liability and endowment flows are impacted by state variables (interest rate risk, inflation risk, income risk, etc.), the impact of which must be hedged away.

A pension fund, for example, should hold a long position in a liability-hedging portfolio to hedge away the implicit short position in liability flows. Conversely, a sovereign wealth fund from an oil-rich

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

country should hold a short position in oil prices to hedge away implicit long positions in endowment flows. The intuition is rather straightforward: the sovereign wealth fund is implicitly holding a long position in an asset whose value is given as the present value of future sovereign contributions. If the fund implements the standard asset allocation decision optimal in the absence of contributions, it will invest in the highest risk/reward portfolio and cash as a function of risk aversion. This will result in exposure to oil prices greater than that of the optimal composition of the highest risk/reward portfolio. This position should be offset by holding a short position in securities positively correlated with oil prices, or a long position in securities negatively correlated with oil prices.

Generally speaking, a sound investment solution involves a dynamic asset allocation strategy that takes into account (i) the stochastic features of the investor's lifetime income progression (where is the money coming from), (ii) the stochastic features of the investor's expected pension value (what the money is going to be used for), and (iii) the stochastic features of the assets held in his portfolio. These advances in dynamic asset allocation have potential applications for the design of stochastic, state-dependent, asset allocation policies, which stand in contrast to the deterministically time-dependent allocation strategies embedded in existing life-cycle investment products.

If we focus on a retirement product, the optimal asset allocation strategy will involve a state-dependent allocation to three building blocks: (a) a performance-

seeking portfolio (heavily invested in equities as well as in bonds and real estate); (b) an income-hedging portfolio (heavily invested in cash but also invested in equities, which have appealing wage-inflation-hedging properties, especially over long horizons), and; (c) a pension-hedging portfolio (heavily invested in bonds to hedge against interest rate changes, and in real estate for inflation-hedging motives). In the early stages of the plan, the income-hedging fund is the dominant low-risk component of the investment strategy, but as the retirement date approaches, there is a gradual, albeit non-deterministic, switch from the income-hedging building block to the pension-hedging building block. This switching resembles deterministic life-cycle investing only superficially; instead of switching from high-risk assets to low-risk assets, as in deterministic life-cycle investing, the optimal stochastic lifestyle strategy switches between types of hedging demands. Moreover, the relative weights of the hedging portfolios are no longer deterministic; they take into account the current level of all variables of interest.

In other words, stochastic life-cycle investing has at least two advantages over deterministic life-cycle investing: for one, in keeping with the tenets of the fund separation theorem, it focuses not on asset classes but on fundamental building blocks; for another, it involves a stochastic allocation to these three building blocks, which evolves as a function of the current values of state variables of interest (above all, the interest rate, inflation rate, and income rate). On the whole, state-dependent asset allocation strategies can

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be shown to strongly dominate various static and deterministic strategies, and the utility costs associated with such inefficient strategies have been found substantial (Cairns, Blake, and Dowd 2006). A similar result is obtained by Martellini and Milhau (2010a), who confirm that the opportunity cost involved in purely deterministic life-cycle strategies such as those implemented by target date funds is substantial for reasonable parameter values. Surprising, perhaps, they also find that even very reasonably fine partitions of the set of investors and market conditions, perfectly consistent with implementation in retail money management and only marginally more complex than current partitions based solely on time horizon, allow substantial welfare gains compared to deterministic life-cycle strategies. These results have major potential implications for the design of improved forms of target date funds, since they suggest that more financial innovation can be used to design improved forms of target date funds based on stochastic rather than deterministic life-cycle investing.

## 3.3. Accounting for the Presence of Investors' Short-Term Constraints: The Risk-Controlled Investment Paradigm

One key element missing from the analysis presented so far is the incorporation of short-term constraints to the design of the optimal allocation strategy. In fact, it may be that most investors, even those (such as pension funds or sovereign wealth funds) with the longest possible horizons inevitably face a number of short-term performance constraints, imposed by accounting or regulatory pressure (or both), political pressure, peer pressure, and so on. In private wealth management, there is also strong evidence that investors face (mostly self-imposed) short-term constraints, *e.g.*, maximum drawdown constraints. These constraints are managed not through diversification strategies (which are dedicated to the design of the PSP) or hedging strategies but through insurance strategies. From a technical standpoint, the introduction of short-term constraints can be formalised in a portfolio selection problem based on a key insight into the profound correspondence between pricing and portfolio problems. On the one hand, asset pricing problems are equivalent to dynamic asset allocation problems: Merton's (1973) interpretation of the Black and Scholes (1973) option pricing formula. On the other hand, dynamic asset allocation problems (Merton 1973) are equivalent to asset pricing problems: the *martingale* or *convex duality* approach to dynamic asset allocation problems (Karatzas, Lehoczky, and Shreve 1997; Cox and Huang 1989).

The practical implication of the introduction of short-term constraints is that optimal investment in a performance-seeking satellite portfolio (PSP) is a function not only of risk aversion but also of risk budgets (margin for error), as well as of the likelihood of the risk budget's being spent before the horizon. In a nutshell, a pre-commitment to risk management allows one to adjust risk exposure in an optimal state-dependent manner and therefore to generate the highest exposure to PSP upside potential and respect risk constraints.

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

### 3.3.1. Dynamic Liability-Driven Investing

Constant proportion portfolio insurance techniques, originally designed to ensure the respect of absolute performance, can be extended to a relative return context. Martellini and Milhau (2010a) show that an approach similar to standard constant proportion portfolio insurance (CPPI) can be taken to offer the investor a relative performance guarantee, with a cap on underperformance of the liability-driven benchmark (see Black and Jones [1987] and Black and Perold [1992] for an introduction to the basic CPPI technique). The techniques of traditional CPPI still apply, provided that the risky asset is re-interpreted as the performance-seeking portfolio, which contains risk relative to the liability benchmark, and the risk-free asset is re-interpreted as the liability-hedging portfolio, which contains no risk relative to the liability benchmark.

Investors with consumption/liability objectives must still invest in two distinct portfolios: the "risky" and the "safe" building blocks; their allocation to the risky block must still be increasing in the PSP Sharpe ratio $\lambda$ and decreasing in the investor's risk aversion $\gamma$ and PSP volatility $\sigma$. The novelty is that the amounts allocated to the PSP and to the LHP respectively are also a function of the risk budget, a quantity defined as the difference between the asset value $A_t$ and a (probability-weighted) floor $p_{t,T}F_t$. When implemented in continuous-time, this portfolio strategy makes it possible to truncate the final distribution of the funding ratio to the minimum funding ratio denoted by $F_t$.

In what follows, we present the optimal allocation strategy.[9]

$$w_t^{*c} = \frac{1}{\gamma}\frac{\lambda}{\sigma}\left(1 - p_{t,T}\frac{F_t}{A_t}\right)PSP$$
$$+ \left(1 - \frac{1}{\gamma}\left(1 - p_{t,T}\frac{F_t}{A_t}\right)\right)\beta\ LHP \quad (4)$$

A simplified version consists of the following expression:

$$w_t^{*c} = \frac{1}{\gamma}\frac{\lambda}{\sigma}\left(1 - \frac{F_t}{A_t}\right)PSP$$
$$+ \left(1 - \frac{1}{\gamma}\left(1 - \frac{F_t}{A_t}\right)\right)\beta\ LHP \quad (5)$$

The allocation strategy presented in equation (5) has a constant proportion portfolio insurance (CPPI) flavour: the dollar allocation to the risky PSP is not a function of risk aversion, PSP volatility, and the Sharpe ratio alone; it is also a function of the risk budget (or margin for error) defined as the distance between the asset value $A_t$ and the short-term floor $F_t$ (typically known as the "cushion" in CPPI terminology):

$$A_t w_t^{*c} = \frac{1}{\gamma}\frac{\lambda}{\sigma}\left(A_t - F_t\right)PSP$$
$$+ \left(A_t - \frac{1}{\gamma}\left(A_t - F_t\right)\right)\beta\ LHP$$

When the margin for error disappears, *i.e.*, when the investor's short-term risk budget is spent, the allocation to the PSP becomes 0.

Equation (4) defines a more general allocation strategy, with more aggressive expenditures of the risk budget. Here, the risk budget is $A_t - p_{t,T}F_t$ rather than $A_t - F_t$, where the number $p_{t,T}$ is like a probability, hence between 0 and 1, and can be

# 3. Dynamic Allocation Decisions to the Performance–Seeking and Liability–Hedging Portfolios

interpreted as the estimated probability that the risk budget will be violated by the corresponding unconstrained strategy (see exhibit 6).

Although the risk budget $A_t - p_{t,T}F_t > A_t - F_t$ looks higher than the investor would like, we still have that $A_t \geq F_t$ for all $t$ because $p_{t,T}$ adjusts in an optimal manner in the sense that $p_{t,T} \to 1$ when and if the satellite portfolio does so poorly that the risk budget is almost entirely spent. Conversely, when the margin for error increases, we have that $p_{t,T} \to 0$, hence allowing fuller access to the upside potential of the satellite. Intuitively, it is straightforward to understand that state-dependent expenditure of the risk budget is better than a deterministic (constant) spending scheme. Having constant spending of the risk budget is in general sub-optimal and has an opportunity cost for the investor with a finite time horizon.

In fact, equation (4) describes an asset allocation strategy that is somewhat reminiscent of (a dynamic replication of) option-based portfolio insurance (OBPI) strategies, which it extends in a number of dimensions. First, the underlying asset is not a risky asset but the underlying optimal unconstrained strategy. Moreover, the risk-free asset is no longer cash, but the investor's liability benchmark. These modifications to the standard OBPI strategy allow one to transport its structure to relative risk management.

Formally, the terminal wealth $A_T^{*c}$ obtained by using the optimal constrained strategy can be written as:

$$A_T^{*c} = F_T + \max\left(\xi A_T^{*u} - F_T, 0\right)$$

Here, $F_t$ is the floor and $A_T^{*u}$ is the terminal wealth generated by the optimal unconstrained strategy. This

*Exhibit 6: Violations of risk budgets with unconstrained strategies*
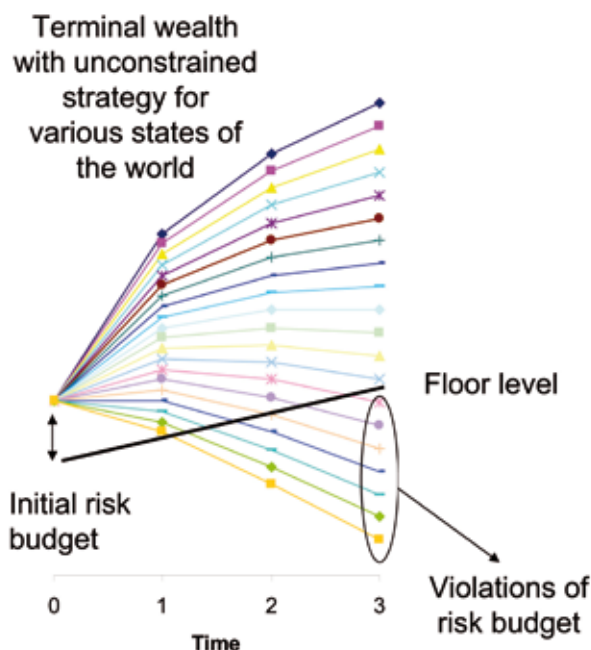
# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

unconstrained strategy is fixed-mix when the opportunity set is constant, and in general has a life-cycle component in the presence of a stochastic opportunity set.

Obviously, no such long-maturity options written on customised dynamic liability-driven investing (LDI)/life-cycle investing (LCI) strategies can be found, even as OTC contracts, and investors will have to implement some form of dynamic allocation strategy that will allow replication of the optimal payoff.

This approach, known as *risk-controlled investing*, allows an investor to truncate the relative return distribution so as to allocate the probability weights away from severe under-performance relative to the liabilities and to greater potential outperformance. The right panel of exhibit 7 shows the distribution of the funding ratio at horizon under the same assumption as in exhibit 5, but under a risk-controlled strategy with a minimum funding ratio at 90% (for comparison, the left panel presents the base-case unconstrained strategy analysed in exhibit 5). Here, the Sharpe ratio of the

PSP is assumed to be taken at the base-case value 0.24.

We find that, as expected, the introduction of the risk-controlled strategy makes it possible to truncate the left-side of the distribution of the final funding ratio at 90%. Downside risk protection has a cost, however, as shown by the expected terminal funding ratio (117.43% on the right panel), lower for the risk-controlled strategy than for the unconstrained strategy (122.64% on the left panel).

Exhibit 8 shows the distribution of the funding ratio at horizon under the same assumption as in exhibit 7, but under the assumption of a Sharpe ratio of a PSP improved by 50% (taken to be 0.36 as opposed to 0.24).

We find again that, as expected, the introduction of the risk-controlled strategy makes it possible to truncate the left-side of the distribution of the final funding ratio at 90%. The implicit cost of downside risk protection can be seen from the fact that the expected terminal funding ratio is lower for the
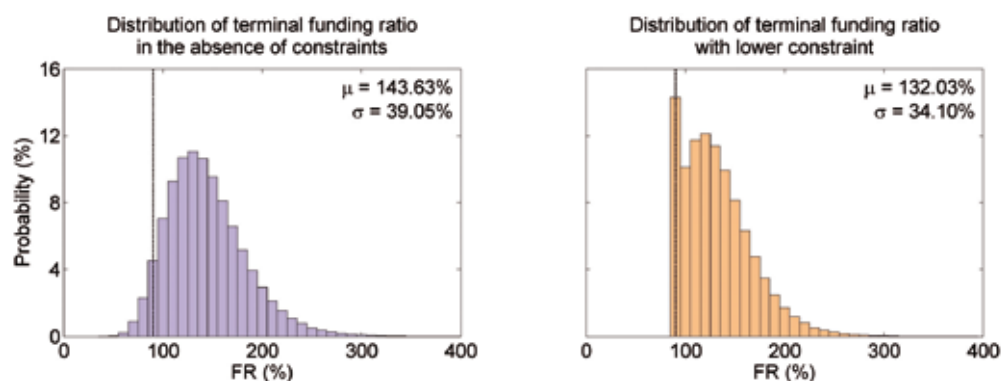
*Exhibit 7: Distribution of terminal funding ratio under a risk-controlled strategy with an inefficient PSP*
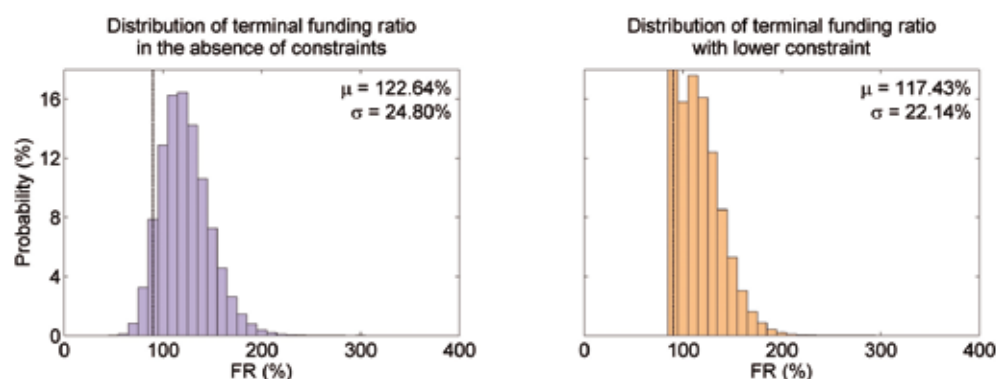


*The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund. The distribution in the right panel is that generated by a risk-controlled strategy with a minimum funding ratio at 90%. For more details, see Martellini and Milhau (2010a).*

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*Exhibit 8: Distribution of terminal funding ratio under a risk-controlled strategy with an improved PSP*



*The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund. For more details, see Martellini and Milhau (2010a).*

risk-controlled strategy (132.03% on the right panel) than for the unconstrained strategy (143.63% on the left panel). Interestingly, we find that the improved PSP has a very substantial impact on the final distribution of the constrained funding ratio. In particular, the mean of the distribution of the funding ratio is higher with the risk-controlled strategy and the efficient PSP than it is with the base-case strategy and the inefficient PSP, and this while allowing for downside protection below the 90% minimum.

In practice, the starting point consists of generating stochastic scenarios for the return on risky asset classes, as well as the return on liabilities. These scenarios will be used to analyse the distribution of the final surplus, expected return, volatility, max drawdown, and so on, generated by risk-controlled strategies. Most notably, they allow one to provide a formal analysis of the costs and benefits of increasing or decreasing the multiplier and protection, as well as introducing various floors and goals as discussed below. A standard Monte-Carlo analysis or some form of historical simulation (bootstrapping) can be used to generate these scenarios.

Beyond the calibration stage, there are a number of key improvements that can be used in implementation. The first of these improvements has to do with the introduction of a time-varying multiplier, allowing one to benefit from the clustering effect in volatility dynamics, with increases in volatility typically associated with bear market environments. The second of these improvements has to do with strategies based on a trading frequency that takes place at regular space rather than time intervals.

Although the original approach was developed in a simple framework, it can be extended in a number of important directions, allowing the introduction of more complex floors. A great variety of floors can in fact be introduced (simultaneously if necessary) to accommodate the needs of different investors. For their relevance to investors, the following possible floors stand out: capital guarantee floors enabling protection of a fraction of the initial capital, benchmark protection floors enabling protection of a fraction of the value of any given stochastic benchmark (with the liability portfolio being the most

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natural benchmark for investors bearing liabilities), max drawdown floors limiting maximum consecutive losses, trailing performance floors enabling protection of a fraction of the earlier value of the portfolio on a rolling basis, and so on.

In addition to accounting for the presence of floors, dynamic risk-controlled strategies can accommodate *goals* or *ceilings*. Goal-directed strategies recognise that the investor has no utility over a ceiling of wealth $G_t$, the investor's goal (actually a cap), which can be a constant, deterministic, or stochastic function of time. Goal-directed strategies involve an optimal switch at some suitably defined threshold that defines the point at which hopeful behaviour becomes fearful (Chen and Liao 2006; Browne 2000). From a conceptual standpoint, it is not clear, *a priori*, why any investor should want to impose a strict limit on upside potential. The intuition is that by forgoing performance beyond a threshold at which greater wealth affords relatively lower utility, investors benefit from a decrease

in the cost of the downside protection (a short position in a convex payoff in addition to the long position, reminiscent of a collar). In other words, without the performance cap, when their wealth is already very high, investors are more likely to fail to reach a goal they have nearly reached.

The right panel of exhibit 9 shows the distribution of the funding ratio at horizon under the same assumption as in exhibit 5, but under a risk-controlled strategy with a minimum funding ratio at 90% and a maximum funding ratio of 150% (for comparison, the left panel presents the base-case unconstrained strategy analysed in exhibit 5, and the panel in the middle presents the strategy with minimum funding ratio constraint analysed in exhibit 7). Here, the Sharpe ratio of the PSP is assumed to be taken at the base-case value 0.24.

By giving up part of the upside potential beyond a point at which the marginal utility of wealth (relative to liabilities)

*Exhibit 9: Distribution of terminal funding ratio under a risk-controlled strategy with upper and lower bounds on the funding ratio and an inefficient PSP*
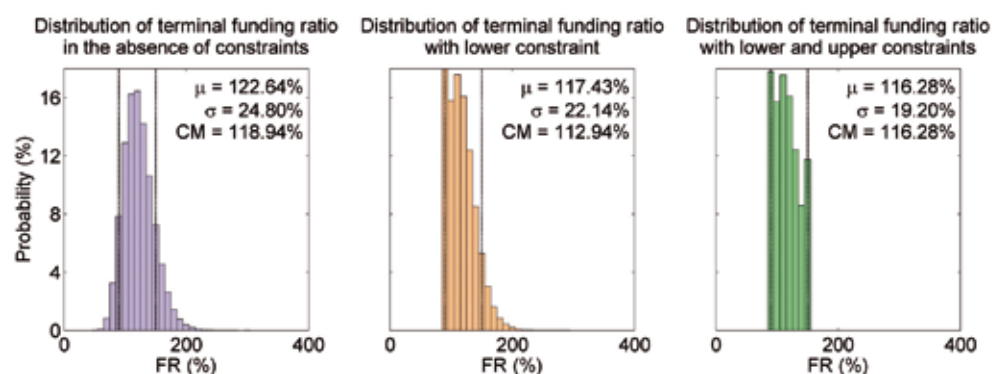


*The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund. The distribution in the middle panel is that generated by a risk-controlled strategy with a minimum funding ratio at 90% and the distribution in the right panel is that generated by a risk-controlled strategy with a minimum funding ratio at 90% and a maximum at 150%. For more details, see Martellini and Milhau (2010a).*

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is low or almost zero, the investor can decrease the cost of downside protection, as can be seen from the fact that the conditional mean of the funding ratio CM for values between the minimum 90% and the maximum 150% is higher on the left panel (116.28%) than in the middle panel (112.94%).

Again, using an improved PSP would lead to substantially better results (see exhibit 10).

### 3.3.2. Putting the Pieces Together: Stochastic Life-Cycle Investing with Risk Budgets

In closing, we briefly discuss the extension to a setting with a stochastic opportunity set, and we argue that the two motivations behind dynamic asset allocation decisions— the risk management and the revision of strategic asset allocation motivations— are often perceived as inconsistent and mutually exclusive.

In practice, dynamic risk-controlled strategies, which usually imply a reduction

to equity allocation when a drop of equity prices has led to a substantial fall in the risk budget, have often been blamed for their pro-cyclical nature. Long-term investors are often reluctant to sell equity holdings after a fall in equity prices because they know that the phenomenon of mean reversion in equity risk premia makes equity markets particularly attractive after falls.

In other words, it is widely perceived that short-term constraints lead to tension between hedging long-term risk and insurance. In fact, recent research suggests that long-term objectives and short-term constraints need not be mutually exclusive and can be integrated in a comprehensive asset allocation framework (Martellini and Milhau 2010b).

More formally, an explicit analytical representation of the relationship between optimal strategies in the presence and absence of short-term constraints can be derived, which allows us to disentangle the impact of short-term constraints from

*Exhibit 10: Distribution of terminal funding ratio under a risk-controlled strategy with upper and lower bounds on the funding ratio and an improved PSP*
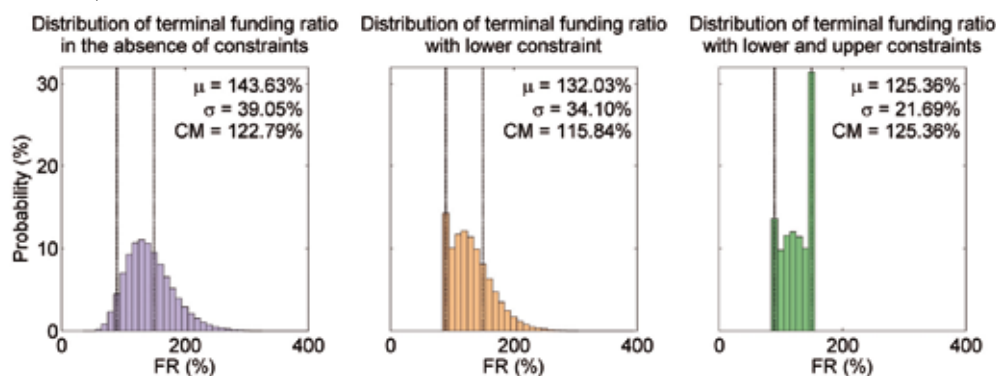


*The initial funding ratio is assumed to be 100%; the horizon is 11.32 years (taken to be the duration of liabilities of a Dutch defined-benefit pension fund). For more details, see Martellini and Milhau (2010a).*

# 3. Dynamic Allocation Decisions to the Performance-Seeking and Liability-Hedging Portfolios

the impact of return predictability on the optimal allocation decision.

For example, in the presence of a stochastic Sharpe ratio for the PSP, the optimal dollar allocation to the PSP can be defined as:

$$A_t \, w_t^{*PSP} = \frac{1}{\gamma} \frac{\lambda_t}{\sigma} \left( A_t - p_{t,T} F_t \right) PSP$$

Depending on market conditions and parameter values, the pro-cyclical risk-controlled motivation $A_t - p_{t,T} F_t$ may outweigh the revision of strategic asset allocation motivation $\lambda_t / \gamma \sigma$, or vice versa, with risk management always prevailing in the final instance. In other words, the risk-control method can be made entirely consistent with internal or external processes whose aim is to generate active asset allocation views. In fact, making this process part of a formal risk control strategy may be the only way to implement active asset allocation decisions and, at the same time, ensure that risk limits are not exceeded.

# Conclusion

# Conclusion

Rising to the challenges of modern investment practice involves designing new forms of investment—rather than mere products—customised to meet investors' expectations. These new forms rely on improved, more efficient PSP and LHP building blocks, as well as on improved dynamic allocation strategies. Although each of these ingredients can be found in current investment products, it is only by putting the pieces of the puzzle together and by combining all these sources of expertise that the asset management industry will address investors' needs satisfactorily.

From the technical perspective, these advanced investment solutions also rely on a sophisticated exploitation of the benefits of the three competing approaches to risk management: risk diversification (a key ingredient in the design of better benchmarks for performance-seeking portfolios), risk hedging (a key ingredient in the design of better benchmarks for hedging portfolios), and risk insurance (a key ingredient in the design of better dynamic asset allocation benchmarks for long-term investors facing short-term constraints), each of which represents a hitherto largely unexplored potential source of added value for the asset management industry.

Risk *management* is often mistaken for risk *measurement*. This is a problem since the ability to measure risk properly is at best a necessary but not sufficient condition to ensure proper risk management. Another misconception is that risk management is about risk reduction. In fact, it is at least as much about return enhancement as it is about risk reduction. Indeed, risk *management* is about maximising the probability of achieving investors' long-term objectives while respecting the short-term constraints they face.

In the end, the traditional (AM or ALM) static strategies without dynamic risk-controlled ingredient inevitably lead to under-spending investors' risk budgets in normal market conditions (with a high opportunity cost), and over-spending their risk budgets in extreme market conditions. This idea was intuitively discussed in Bernstein (1996): "The word risk derives from the Latin *risicare*, which means *to dare*. In this sense, risk is a choice rather than a fate. The actions we dare to take, which depend on how free we are to make choices, are what the story of risk is all about."

# Appendix

# Appendix

## A1. Static Asset Allocation Problem

First find the MSR portfolio:

$$SR_w = 0 \Leftrightarrow (w'\Sigma w)e'\Sigma^{-1}(\mu - re) = w'(\mu - re)e'w$$
$$\Leftrightarrow w'(\mu - re) = (w'\Sigma w)e'\Sigma^{-1}(\mu - re)$$
$$\Leftrightarrow (\mu - re) = \Sigma we'\Sigma^{-1}(\mu - re)$$
$$\Leftrightarrow \Sigma^{-1}(\mu - re) = we'\Sigma^{-1}(\mu - re)$$

We multiply by $e'$ on the left-hand side to obtain:

$$SR_w = (\mu - re)(w'\Sigma w)^{-\frac{1}{2}} - w'(\mu - re)(w'\Sigma w)^{-\frac{3}{2}}\Sigma w$$
$$SR_w = 0 \Leftrightarrow (\mu - re) = w'(\mu - re)(w'\Sigma w)^{-1}\Sigma w$$
$$SR_w = 0 \Leftrightarrow (w'\Sigma w)\Sigma^{-1}(\mu - re) = w'(\mu - re)w$$

Finally (note that weights sum up to 1):

$$w^*_{MSR} = \frac{\Sigma^{-1}(\mu - re)}{e'\Sigma^{-1}(\mu - re)}$$

The next step consists of finding the right allocation as a function of the investor's risk aversion:

$$\underset{w}{Max}\ \mu_p - \frac{\gamma}{2}\sigma_p^2 = \underset{w}{Max}\ w'(\mu - re) - \frac{\gamma}{2}w'\Sigma w$$

We obtain the solution by writing the first-order condition ($L$ is the Lagrangian for the problem, and $L_w$ its first derivative with respect to portfolio weights):

$$L_w = (\mu - re) - \gamma\Sigma w = 0 \Rightarrow w^*_0 = \frac{1}{\gamma}\Sigma^{-1}(\mu - re)$$
$$= \frac{e'\Sigma^{-1}(\mu - re)}{\gamma}\frac{\Sigma^{-1}(\mu - re)}{e'\Sigma^{-1}(\mu - re)}$$

## A2. Dynamic Asset-Liability Allocation Problem with Constant Opportunity Set

We now consider a dynamic asset allocation problem, with an investor allowed to rebalance portfolio between dates $0$ and $T$.

In this intertemporal context, information about asset return distribution over the horizon is not sufficient, and one needs to know the distribution of asset return at all points in time.

In what follows, we assume that the investor has access to $N$ locally risky assets and one risk-free asset paying the constant interest rate, with the following dynamics (where $W$ is a standard $N$-dimensional Brownian motion process):

$$\begin{cases} dS_t = \text{diag}(S_t)[(re + \sigma'_S\lambda_S)dt + \sigma'_S dW_t] \\ \dfrac{dB_t}{B_t} = rdt \Leftrightarrow B_t = B_0 e^{rt} \end{cases}$$

Here $\lambda_s$ is the price of risk vector for the $N$ assets, assumed to be constant, and $\sigma_s$ is the volatility matrix, which is not necessarily constant. The investor needs to finance the payment of a liability portfolio whose value is modelled here as an exogenous geometric Brownian motion process:[10]

$$dL_t = L_t\left[\mu_L dt + \sigma'_L dW_t\right]$$

The asset value process for a given (self-financed) portfolio strategy is given by:

$$dA_t = A_t\left[w'_t(\text{diag}\ S_t)^{-1}dS_t + (1 - w'_t e)\frac{dB_t}{B_t}\right]$$
$$= A_t\left[(r_t + w'_t\sigma'_S\lambda_t)dt + w'_t\sigma'_S dW_t\right]$$

The optimal dynamic asset allocation decisions with a constant opportunity set, with CRRA preferences $u(x) = x^{1-\gamma}/1-\gamma$, is given by equation (2), that is:

$$\underset{(w_t)}{\max}\ E\left[u\left(\frac{A_T}{L_T}\right)\right] \Rightarrow w^*_t = \frac{\lambda}{\gamma\sigma}PSP + \left(1 - \frac{1}{\gamma}\right)\beta\ LHP$$

Here, the PSP is again the portfolio that achieves the highest Sharpe ratio, and we let

# Appendix

$\lambda$ be its Sharpe ratio and $\sigma$ its volatility. *LHP* is the portfolio that achieves the highest correlation with the liability process; $\beta$ is the beta of changes in the liability portfolio value with respect to changes in the LHP value.

The expressions for the PSP, the LHP, and the weights are as follows:

$$PSP = \frac{\sigma_S^{-1}\lambda_S}{e'\sigma_S^{-1}\lambda_S}, \quad LHP = \frac{\sigma_S^{-1}\sigma_L}{e'\sigma_S^{-1}\sigma_L},$$

$$\frac{\lambda}{\sigma} = e'\sigma_S^{-1}\lambda_S, \quad \beta = e'\sigma_S^{-1}\sigma_L$$

The expression for the PSP in this dynamic model is actually the same as the expression for the MSR PSP in the static case. Formally, the relationship can be established as follows. Consider the vector of log-returns over the period $[0,T]$, $\ln S_T - \ln S_0$. Under the assumption of constant parameters, its first two moments are given by:

$$E[\ln S_T - \ln S_0] = \left[re + \sigma_S'\lambda_S - \frac{1}{2}\text{diag}(\sigma_S'\sigma_S)\right]T,$$

$$V[\ln S_T - \ln S_0] = \sigma_S'\sigma_S T$$

Then define $\mu$ and $\Sigma$ as:

$$\mu = E[\ln S_T - \ln S_0] + \frac{1}{2}\text{diag}(V[\ln S_T - \ln S_0]),$$

$$\Sigma = V[\ln S_T - \ln S_0]$$

where we observe that diag $(V[\ln S_T - \ln S_0])$ is the vector of variances.

Then we have:

$$PSP = \frac{\Sigma^{-1}(\mu - re)}{e'\Sigma^{-1}(\mu - re)}$$

which is the same expression as in the static case. The quantities $\mu$ and $\Sigma$ can be obtained using a proper econometric model for asset returns (see, for example, Amenc, Martellini, and Ziemann [2009] and the references therein).

The proof of the result in equation (2) can be obtained by applying the martingale approach of Cox and Huang (1989).[11] First, the dynamic portfolio problem is mapped into a static problem where the control variable is the terminal wealth:

$$\max_{A_T} E\left[\left(\frac{A_T}{L_T}\right)\right], \quad \text{s.t. } E[M_T A_T] = A_0$$

where $M_T$ is the pricing kernel:

$$M_T = \exp\left[-\left(r + \frac{\|\lambda_S\|^2}{2}\right)T - \lambda_S'W_T\right]$$

The optimal terminal wealth reads:

$$A_T^* = \frac{A_0}{E\left[(M_TL_T)^{1-\frac{1}{\gamma}}\right]} M_T^{-\frac{1}{\gamma}}L_T^{1-\frac{1}{\gamma}}$$

and the optimal wealth process is:

$$A_t^* = \frac{A_0}{E\left[(M_TL_T)^{1-\frac{1}{\gamma}}\right]} E_t\left[\left(\frac{M_TL_T}{M_tL_t}\right)^{1-\frac{1}{\gamma}}\right] M_t^{-\frac{1}{\gamma}}L_t^{1-\frac{1}{\gamma}}$$

Applying Ito's lemma to both sides of this equality and matching the diffusion terms, one obtains the optimal portfolio.

## A3. Dynamic Asset Allocation Problem with a Time-Varying Opportunity Set (Focus on Time-Varying Sharpe Ratio)

We now assume that the equity risk premium is time-varying with the business cycle, with a mean-reverting component. We let $\rho_{\lambda S}$ be the correlation between $\lambda^S$ and $S$; a negative correlation means that high realised return periods tend to be followed by low expected return periods, which is supported by empirical evidence.

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$$d\lambda_t^s = a_\lambda\left(b_\lambda - \lambda_t^s\right)dt + \sigma_\lambda dW_t^\lambda$$
$$dS_t = S_t\left[r + \sigma_S\lambda_t^s\right]dt + S_t\sigma_S dW_t^S$$

The interest rate $r$ is assumed to be constant for simplicity, and the stock index is assumed to be the only risky asset; this is an incomplete market setting except for a perfect negative correlation $\rho_{\lambda S} = -1$.

Because of market incompleteness, we use the dynamic programming approach, with a value function defined as (assuming again CRRA preferences):

$$J\left(t, \lambda_t^S, A_t\right) = \sup_{(\omega_s)_{s \geq t}} E_t\left[\frac{A_T^{1-\gamma}}{1-\gamma}\right]$$

The Hamilton-Jacobi-Bellman (HJB) equation for $J$ reads:

$$0 = J_t + \sup_{w_S}\left[\frac{1}{2}w_S^2\sigma_S^2 A_t^2 J_{AA} + w_S\left(\mu_S - r\right)A_t J_A + w_S\rho_{\lambda S}\sigma_\lambda\sigma_S A_t J_{A\lambda^S}\right]$$
$$+ rA_t J_A + \frac{1}{2}\sigma_\lambda^2 J_{\lambda^S\lambda^S} + a_\lambda\left(b_\lambda - \lambda_t^S\right)J_{\lambda^S}$$

and the optimal portfolio strategy is:

$$w_t^S = -\frac{J_A}{A_t J_{AA}}\frac{\mu_S - r}{\sigma_S} - \frac{J_{A\lambda^S}}{A_t J_{AA}}\frac{\rho_{\lambda S}\sigma_\lambda}{\sigma_S}$$
$$= -\frac{J_A}{A_t J_{AA}}\frac{\lambda_t^S}{\sigma_S} - \frac{J_{A\lambda^S}}{A_t J_{AA}}\frac{\rho_{\lambda S}\sigma_\lambda}{\sigma_S}$$

Plugging the optimal portfolio strategy back into the HJB equation, we finally obtain the following partial differential equation (PDE):

$$0 = J_t + rA_t J_A + \frac{1}{2}\sigma_\lambda^2 J_{\lambda^S\lambda^S} + a_\lambda\left(b_\lambda - \lambda_t^S\right)J_{\lambda^S} - \frac{J_A^2}{2J_{AA}}\left(\lambda_t^S\right)^2$$
$$- \frac{J_{A\lambda^S}^2}{2J_{AA}}\rho_{\lambda S}^2\sigma_\lambda^2 - \frac{J_A J_{A\lambda^S}}{J_{AA}}\sigma_\lambda\rho_{\lambda S}\lambda_t^S$$

Given the affine structure of the model, we guess a solution to the HJB equation of the form:

$$J\left(t, A_t, \lambda_t^S\right) = \frac{A_t^{1-\gamma}}{1-\gamma}\exp\left[\frac{1-\gamma}{\gamma}\left[A(T-t)\lambda_t^S + \frac{1}{2}B(T-t)\left(\lambda_t^S\right)^2 + C(T-t)\right]\right]$$

Plugging the relevant derivatives of J back into the HJB equation, we obtain a system of three coupled ordinary differential equations (ODEs) for A, B, and C that can be solved to yield:

$$A(s) = -\frac{1}{\gamma}\frac{a_\lambda b_\lambda\left(1-e^{-\sqrt{q}s}\right)^2}{\sqrt{q}\left[2\sqrt{q} - \left(\sqrt{q}-\bar{b}_\lambda\right)\left(1-e^{-2\sqrt{q}s}\right)\right]}$$

$$B(s) = -\frac{1}{\gamma}\frac{1-e^{-2\sqrt{q}s}}{2\sqrt{q} - \left(\sqrt{q}-\bar{b}_\lambda\right)\left(1-e^{-2\sqrt{q}s}\right)}$$

$$\bar{b}_\lambda = b_\lambda - \frac{1-\gamma}{\gamma}\rho_{\lambda S}\sigma_\lambda; q = \bar{b}_\lambda^2 - \sigma_\lambda^2\left(\rho_{\lambda S}^2 + \gamma\left(1-\rho_{\lambda S}^2\right)\right)\frac{1-\gamma}{\gamma^2}$$

The optimal portfolio strategy is then obtained by taking the derivatives of the J function in:

$$w_t^* = -\frac{J_A}{A_t J_{AA}}\frac{\lambda_t^S}{\sigma_S} - \frac{J_{A\lambda^S}}{A_t J_{AA}}\frac{\rho_{\lambda S}\sigma_\lambda}{\sigma_S}$$

## A4. Dynamic Asset–Liability Allocation Problem with Short-Term Performance Constraints

We consider the case of an investor maximising expected utility from terminal funding ratio, subject to a short-term performance constraint, here a minimum funding ratio constraint formally defined as $A_t \geq F_t = kL_t$ for all $t \leq T$:

$$\max_{(w_t)} E\left[\left(\frac{A_T}{L_T}\right)\right], \quad \text{s.t. } A_t \geq kL_t \text{ for all } t \leq T$$

In order to avoid technical issues, we assume that the liability $L$ can be fully replicated by the LHP. In this context, the static form of the dynamic portfolio problem is a programme in which the series of short-term constraints can been replaced by a single long-term performance constraint:

$$\max_{A_T} E\left[\left(\frac{A_T}{L_T}\right)\right], \quad \text{s.t. } A_T \geq kL_T \text{ and } E\left[M_T A_T\right] = A_0$$

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The optimal payoff is then given by:

$$A_T^* = kL_T + \left(\xi A_T^{*u} - kL_T\right)^+$$

where $A_T^{*u}$ is the terminal wealth that would be optimal in the absence of performance constraints and $\xi$ is some constant whose value is adjusted so as to make the budget constraint hold $E[M_T A_{T^*}] = A_0$ . Under the assumption of a constant opportunity set, the exchange option between the unconstrained payoff and the floor can be valued using Margrabe's (1978) formula:

$$A_t^* = kL_t + \xi A_t^{*u} N(d_{1,t}) - kL_t N(d_{2,t})$$

$$d_{1,t} = \frac{1}{\Sigma_{t,T}}\left[\ln\frac{\xi A_t^{*u}}{kL_t} + \frac{1}{2}\Sigma_{t,T}^2\right], \quad d_{2,t} = d_{1,t} - \Sigma_{t,T}$$

12 - Under the assumption of a perfect liability matching portfolio, the parameter $\beta$ would be equal to 1.

where $\Sigma_{t,T}$ is the cumulated volatility of the unconstrained funding ratio $A^{*u}/L$ over the time span $[t, T]$. Applying Ito's lemma and identifying diffusion terms, we obtain the optimal strategy:

$$w_t^{*c} = \frac{\lambda}{\gamma\sigma}\left(1 - \frac{kN(-d_{2,t})L_t}{A_t^*}\right)PSP + \left[1 - \frac{1}{\gamma}\left(1 - \frac{kN(-d_{2,t})L_t}{A_t^*}\right)\right]\beta\,LHP$$

where $N(-d_{2t})$ can be interpreted as the risk-neutral probability that the exchange option ends out-of-the-money.[12]

# Appendix

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# About EDHEC-Risk Institute

# About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost French business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy for international excellence that led it to set up EDHEC-Risk in 2001. With 47 professors, research engineers and research associates, EDHEC-Risk has the largest asset management research team in Europe.

## The Choice of Asset Allocation and Risk Management

EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset/liability management context. This strategic choice is applied to all of the centre's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset/liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds.



- 40% Strategic Asset Allocation
- 45.5% Tactical Asset Allocation
- 11% Stock Picking
- 3.5% Fees
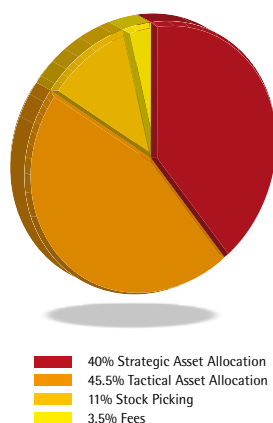
*Source EDHEC (2002) and Ibbotson, Kaplan (2000)*

## An Applied Research Approach

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by EDHEC-Risk's advisory board. This board is made up of internationally recognised researchers, the centre's business partners and representatives of major international institutional investors. The management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been undertaken :
- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the three EDHEC-Risk locations in London, Nice, and Singapore.

In addition, EDHEC-Risk has developed close partnerships with a small number of sponsors within the framework of research chairs. These research chairs involve a three-year commitment by EDHEC-Risk and the sponsor to research themes on which the parties to the chair have agreed.

# About EDHEC-Risk Institute

The following research chairs have been endowed :
- **Regulation and Institutional Investment,** *in partnership with AXA Investment Managers (AXA IM)*
- **Asset/Liability Management and Institutional Investment Management,** *in partnership with BNP Paribas Investment Partners*
- **Risk and Regulation in the European Fund Management Industry,** *in partnership with CACEIS*
- **Structured Products and Derivative Instruments,** *sponsored by the French Banking Federation (FBF)*
- **Private Asset/Liability Management,** *in partnership with ORTEC Finance*
- **Dynamic Allocation Models and New Forms of Target-Date Funds,** *in partnership with UFG*
- **Advanced Modelling for Alternative Investments,** *in partnership with Newedge Prime Brokerage*
- **Asset/Liability Management Techniques for Sovereign Wealth Fund Management,** *in partnership with Deutsche Bank*
- **Core-Satellite and ETF Investment,** *in partnership with Amundi ETF*
- **The Case for Inflation-Linked Bonds: Issuers' and Investors' Perspectives,** *in partnership with Rothschild & Cie*

The philosophy of the institute is to validate its work by publication in international journals, but also to make it available to the sector through its position papers, published studies and conferences.

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC-Risk Institutional Days.

EDHEC also provides professionals with access to its website, **www.edhec-risk.com**, which is entirely devoted to international asset management research. The website, which has more than 35,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC's analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 400,000 readers.

### EDHEC-Risk Institute: Key Figures, 2008-2009

| | |
|---|---|
| Number of permanent staff | 47 |
| Number of research associates | 17 |
| Number of affiliate professors | 5 |
| Overall budget | €8,700,000 |
| External financing | €5,900,000 |
| Number of conference delegates | 1,950 |
| Number of participants at EDHEC-Risk Executive Education seminars | 371 |

### Research for Business
EDHEC-Risk's activities have also given rise to executive education and research service offshoots.

EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset managementtraining across traditional and alternative classes.

# About EDHEC-Risk Institute

### The EDHEC-Risk Institute PhD in Finance

The EDHEC-Risk Institute PhD in Finance at EDHEC Business School is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC Business School, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world's best universities and enjoying the support of the research centre with the greatest impact on the European financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

### The EDHEC-Risk Institute MSc in Risk and Investment Management

The EDHEC-Risk Institute Executive MSc in Risk and Investment Management is designed for professionals in the investment management industry who wish to progress, or maintain leadership in their field, and for other finance practitioners who are contemplating lateral moves. It appeals to senior executives, investment and risk managers or advisors, and analysts. This postgraduate programme is designed to be completed in seventeen months of part-time study and is formatted to be compatible with professional schedules.

The programme has two tracks: an executive track for practitioners with significant investment management experience and an apprenticeship track for selected high-potential graduate students who have recently joined the industry. The programme is offered in Asia—from Singapore—and in Europe—from London and Nice.

### FTSE EDHEC-Risk Efficient Indices

FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the first set of FTSE EDHEC Risk Efficient Indices at the beginning of 2010. Initially offered for the UK, the Eurobloc, the USA, Developed Asia-Pacific, ex. Japan, and Japan, the index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk).

### EDHEC-Risk Alternative Indexes

The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a "true and fair" view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC-Risk was the first to launch composite hedge fund strategy indexes as early as 2003.

The thirteen EDHEC-Risk Alternative Indexes are published monthly on **www. edhec-risk.com** and are freely available to managers and investors.

# EDHEC-Risk Institute Publications and Position Papers (2007–2010)

# EDHEC-Risk Institute Publications (2007–2010)

## 2010

• Martellini, L., and V. Milhau. Capital structure choices, pension fund allocation decisions and the rational pricing of liability streams (July).

• Sender, S. EDHEC survey of the asset and liability management practices of European pension funds (June).

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• Amenc, N., F. Goltz, and A. Grigoriu. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation (January).

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• Goltz, F., and V. Le Sourd. Does finance theory make the case for capitalisation-weighted indexing? (January)

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• Le Sourd, V. Hedge fund performance in 2007 (February).

• Amenc, N., F. Goltz, V. Le Sourd, and L. Martellini. The EDHEC European investment practices survey 2008 (January).

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• Ducoulombier, F. EDHEC European real estate investment and risk management survey (November).

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• Le Sourd, V. Performance measurement for traditional investment (literature survey) (January).

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### 2010
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• Amenc, N., F. Ducoulombier, and P. Foulquier. Réactions après l'étude. Juste valeur ou non : un débat mal posé (December). With the EDHEC Financial Analysis and Accounting Research Centre.

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• Christory, C., S. Daul, and J.-R. Giraud. Quantification of hedge fund default risk (January).

# Notes

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