

UNIVERSITÀ DEGLI STUDI DI SALERNO

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE ED ELETTRICA
E MATEMATICA APPLICATA

Corso:
DESIGN AND ANALYSIS OF ALGORITHMS



APPUNTI

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ANNO ACCADEMICO 2025/2026

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Capitolo 1

Array Stack

Listing 1.1: Implementazione Python dell'ADT Stack utilizzando una lista per la memorizzazione degli elementi.

```
1 class Empty(Exception):
2     pass
3
4 class ArrayStack:
5     """Implementazione di ADT Stack che utilizza un oggetto list di Python
6         per la memorizzazione."""
7
8     def __init__(self):
9         """Crea uno stack vuoto."""
10        self._data = []           # istanza di list non pubblica
11
12    def __len__(self):
13        """Restituisce il numero di elementi nello stack."""
14        return len(self._data)
15
16    def is_empty(self):
17        """Restituisce True se lo stack è vuoto."""
18        return len(self._data) == 0
19
20    def push(self, e):
21        """Aggiunge l'elemento e al top dello stack."""
22        self._data.append(e)     # il nuovo elemento è aggiunto in coda alla list
```

```

22
23 def top(self):
24     """Restituisce (ma non rimuove) l'elemento al top dello stack.
25     Raise Empty exception se lo stack è vuoto."""
26     if self.is_empty():
27         raise Empty('lo stack è vuoto')
28         # print("lo stack è vuoto")
29     return self._data[-1]          # legge l'ultimo elemento della list
30
31 def pop(self):
32     """Rimuove e restituisce l'elemento al top dello stack.
33     Raise Empty exception se lo stack è vuoto."""
34     if self.is_empty():
35         raise Empty('lo stack è vuoto')
36         # print("lo stack è vuoto")
37     return self._data.pop()        # rimuove l'ultimo elemento della list

```

Capitolo 2

Array Queue

Listing 2.1: Classe astratta che implementa l'ADT Queue.

```
1 class Queue:
2     """Classe astratta che implementa l'ADT Queue."""
3
4     def __len__(self):
5         """Restituisce il numero di elementi nella coda."""
6         raise NotImplementedError('deve essere implementato dalla sottoclasse.')
7
8     def is_empty(self):
9         """Restituisce True se la coda è vuota."""
10        raise NotImplementedError('deve essere implementato dalla sottoclasse.')
11
12    def first(self):
13        """Restituisce (ma non rimuove) l'elemento al front della coda.
14        Raise Empty exception se la coda è vuota."""
15        raise NotImplementedError('deve essere implementato dalla sottoclasse.')
16
17    def dequeue(self):
18        """Rimuove e restituisce l'elemento al front della coda.
19        Raise Empty exception se la coda è vuota."""
20        raise NotImplementedError('deve essere implementato dalla sottoclasse.')
21
22    def enqueue(self, e):
23        """Aggiunge un elemento al back della coda."""
24        raise NotImplementedError('deve essere implementato dalla sottoclasse.')
```

Listing 2.2: Implementazione Python dell'ADT Queue utilizzando una lista per la memorizzazione degli elementi.

```
1 from .queue import Queue
2
3 class Empty(Exception):
4     pass
5
6 class ArrayQueue(Queue):
7     """Implementazione di ADT Queue basata sul tipo list di Python usato come
8         array circolare."""
9
10    DEFAULT_CAPACITY = 10          # dimensione di default di nuove code
11
12    def __init__(self):
13        """Crea una coda vuota."""
14        self._data = [None] * ArrayQueue.DEFAULT_CAPACITY
15        self._size = 0
16        self._front = 0
17
18    def __len__(self):
19        """Restituisce il numero di elementi nella coda."""
20        return self._size
21
22    def is_empty(self):
23        """Restituisce True se la coda è vuota."""
24        return self._size == 0
25
26    def first(self):
27        """Restituisce (ma non rimuove) l'elemento al front della coda.
28            Raise Empty exception se la coda è vuota.
29        """
30        if self.is_empty():
31            raise Empty('Queue is empty')
32        return self._data[self._front]
33
34    def dequeue(self):
35        """Rimuove e restituisce l'elemento al front della coda.
36            Raise Empty exception se la coda è vuota.
37        """
38        if self.is_empty():
39            raise Empty('Queue is empty')
40        answer = self._data[self._front]
41        self._data[self._front] = None          # favorisce garbage collection
42        self._front = (self._front + 1) % len(self._data)
43        self._size -= 1
44        return answer
```

```

43
44 def enqueue(self, e):
45     """Aggiunge un elemento al back della coda."""
46     if self._size == len(self._data):
47         self._resize(2 * len(self._data))    # raddoppia la dimensione
48         dell'array se pieno
49     avail = (self._front + self._size) % len(self._data)
50     self._data[avail] = e
51     self._size += 1
52
53 def _resize(self, cap):    # we assume cap >= len(self)
54     """Ridimensiona l'array portandolo a lunghezza cap."""
55     old = self._data       # conserva la vecchia copia dell'array
56     self._data = [None] * cap    # alloca una nuova list di dimensione cap
57     j = self._front
58     for k in range(self._size):
59         self._data[k] = old[j]    # shifta gli indici per riallinearli
60         j = (j + 1) % len(old)    # usa la vecchia dimensione come modulo
61     self._front = 0            # front riallineato a 0

```


Capitolo 3

Alberi

Listing 3.1: Classe astratta che rappresenta una struttura ad albero.

```
1 from ..queue.array_queue import ArrayQueue
2 # import collections
3
4 class Tree:
5     """Abstract base class representing a tree structure."""
6
7     #----- start nested Position class -----
8     class Position:
9         """An abstraction representing the location of a single element within
10            a tree.
11
12            Note that two position instances may represent the same inherent
13            location in a tree.
14            Therefore, users should always rely on syntax 'p == q' rather than 'p
15            is q' when testing
16            equivalence of positions.
17        """
18
19        def element(self):
20            """Return the element stored at this Position."""
21            raise NotImplementedError('must be implemented by subclass')
22
23        def __eq__(self, other):
24            """Return True if other Position represents the same location."""
25            raise NotImplementedError('must be implemented by subclass')
26
27        def __ne__(self, other):
28            """Return True if other does not represent the same location."""
29            return not (self == other)          # opposite of __eq__
30
31     #----- end nested Position class -----
```

```

29
30 # ----- abstract methods that concrete subclass must support -----
31 def root(self):
32     """Return Position representing the tree's root (or None if empty)."""
33     raise NotImplementedError('must be implemented by subclass')
34
35 def parent(self, p):
36     """Return Position representing p's parent (or None if p is root)."""
37     raise NotImplementedError('must be implemented by subclass')
38
39 def num_children(self, p):
40     """Return the number of children that Position p has."""
41     raise NotImplementedError('must be implemented by subclass')
42
43 def children(self, p):
44     """Generate an iteration of Positions representing p's children."""
45     raise NotImplementedError('must be implemented by subclass')
46
47 def __len__(self):
48     """Return the total number of elements in the tree."""
49     raise NotImplementedError('must be implemented by subclass')
50
51 # ----- concrete methods implemented in this class -----
52 def is_root(self, p):
53     """Return True if Position p represents the root of the tree."""
54     return self.root() == p
55
56 def is_leaf(self, p):
57     """Return True if Position p does not have any children."""
58     return self.num_children(p) == 0
59
60 def is_empty(self):
61     """Return True if the tree is empty."""
62     return len(self) == 0
63
64 def depth(self, p):
65     """Return the number of levels separating Position p from the root."""
66     if self.is_root(p):
67         return 0
68     else:
69         return 1 + self.depth(self.parent(p))

```

```

70
71 def _height1(self):          # works, but  $O(n^2)$  worst-case time
72     """Return the height of the tree."""
73     return max(self.depth(p) for p in self.positions() if self.is_leaf(p))
74
75 def _height2(self, p):      # time is linear in size of subtree
76     """Return the height of the subtree rooted at Position p."""
77     if self.is_leaf(p):
78         return 0
79     else:
80         return 1 + max(self._height2(c) for c in self.children(p))
81
82 def height(self, p=None):
83     """Return the height of the subtree rooted at Position p.
84     If p is None, return the height of the entire tree.
85     """
86     if p is None:
87         p = self.root()
88     return self._height2(p)    # start _height2 recursion
89
90 def __iter__(self):
91     """Generate an iteration of the tree's elements."""
92     for p in self.positions(): # use same order as positions()
93         yield p.element()      # but yield each element
94
95 def positions(self):
96     """Generate an iteration of the tree's positions."""
97     return self.preorder()     # return entire preorder iteration

```

```

98
99 def preorder(self):
100     """Generate a preorder iteration of positions in the tree."""
101     if not self.is_empty():
102         for p in self._subtree_preorder(self.root()): # start recursion
103             yield p
104
105 def _subtree_preorder(self, p):
106     """Generate a preorder iteration of positions in subtree rooted at p."""
107     yield p # visit p before its subtrees
108     for c in self.children(p): # for each child c
109         for other in self._subtree_preorder(c): # do preorder of c's subtree
110             yield other # yielding each to our caller
111
112 def postorder(self):
113     """Generate a postorder iteration of positions in the tree."""
114     if not self.is_empty():
115         for p in self._subtree_postorder(self.root()): # start recursion
116             yield p
117
118 def _subtree_postorder(self, p):
119     """Generate a postorder iteration of positions in subtree rooted at
120     p."""
121     for c in self.children(p): # for each child c
122         for other in self._subtree_postorder(c): # do postorder of c's
123             subtree
124             yield other # yielding each to our caller
125     yield p # visit p after its subtrees
126
127 def breadthfirst(self):
128     """Generate a breadth-first iteration of the positions of the tree."""
129     if not self.is_empty():
130         fringe = ArrayQueue() # known positions not yet yielded
131         fringe.enqueue(self.root()) # starting with the root
132         while not fringe.is_empty():
133             p = fringe.dequeue() # remove from front of the queue
134             yield p # report this position
135             for c in self.children(p):
136                 fringe.enqueue(c) # add children to back of queue

```

Listing 3.2: Classe astratta che rappresenta una struttura ad albero binario.

```
1 from .tree import Tree
2
3 class BinaryTree(Tree):
4     """Abstract base class representing a binary tree structure."""
5
6     # ----- additional abstract methods -----
7     def left(self, p):
8         """Return a Position representing p's left child.
9         Return None if p does not have a left child.
10        """
11         raise NotImplementedError('must be implemented by subclass')
12
13     def right(self, p):
14         """Return a Position representing p's right child.
15         Return None if p does not have a right child.
16        """
17         raise NotImplementedError('must be implemented by subclass')
18
19     # ----- concrete methods implemented in this class -----
20     def sibling(self, p):
21         """Return a Position representing p's sibling (or None if no
22         sibling)."""
23         parent = self.parent(p)
24         if parent is None: # p must be the root
25             return None # root has no sibling
26         else:
27             if p == self.left(parent):
28                 return self.right(parent) # possibly None
29             else:
30                 return self.left(parent) # possibly None
31
32     def children(self, p):
33         """Generate an iteration of Positions representing p's children."""
34         if self.left(p) is not None:
35             yield self.left(p)
36         if self.right(p) is not None:
37             yield self.right(p)
```

```

37
38 def inorder(self):
39     """Generate an inorder iteration of positions in the tree."""
40     if not self.is_empty():
41         for p in self._subtree_inorder(self.root()):
42             yield p
43
44 def _subtree_inorder(self, p):
45     """Generate an inorder iteration of positions in subtree rooted at p."""
46     if self.left(p) is not None:          # if left child exists, traverse
47         its subtree
48         for other in self._subtree_inorder(self.left(p)):
49             yield other
50     yield p                                # visit p between its subtrees
51     if self.right(p) is not None:         # if right child exists, traverse
52         its subtree
53         for other in self._subtree_inorder(self.right(p)):
54             yield other
55
56 # override inherited version to make inorder the default
57 def positions(self):
58     """Generate an iteration of the tree's positions."""
59     return self.inorder()                # make inorder the default

```

Listing 3.3: Implementazione tramite lista linkata di una struttura ad albero binario.

```

1 from .binary_tree import BinaryTree
2
3 class LinkedBinaryTree(BinaryTree):
4     """Linked representation of a binary tree structure."""
5
6     #----- nested _Node class -----
7     class _Node:
8         """Lightweight, nonpublic class for storing a node."""
9         __slots__ = '_element', '_parent', '_left', '_right' # streamline
            memory usage
10
11     def __init__(self, element, parent=None, left=None, right=None):
12         self._element = element
13         self._parent = parent
14         self._left = left
15         self._right = right
16
17     #----- nested Position class -----
18     class Position(BinaryTree.Position):
19         """An abstraction representing the location of a single element."""
20
21     def __init__(self, container, node):
22         """Constructor should not be invoked by user."""
23         self._container = container
24         self._node = node
25
26     def element(self):
27         """Return the element stored at this Position."""
28         return self._node._element
29
30     def __eq__(self, other):
31         """Return True if other is a Position representing the same
            location."""
32         return type(other) is type(self) and other._node is self._node
33
34     #----- utility methods -----
35     def _validate(self, p):
36         """Return associated node, if position is valid."""
37         if not isinstance(p, self.Position):
38             raise TypeError('p must be proper Position type')
39         if p._container is not self:
40             raise ValueError('p does not belong to this container')
41         if p._node._parent is p._node: # convention for deprecated nodes
42             raise ValueError('p is no longer valid')
43         return p._node
44
45     def _make_position(self, node):
46         """Return Position instance for given node (or None if no node)."""
47         return self.Position(self, node) if node is not None else None

```

```

48
49 #----- binary tree constructor -----
50 def __init__(self):
51     """Create an initially empty binary tree."""
52     self._root = None
53     self._size = 0
54
55 #----- public accessors -----
56 def __len__(self):
57     """Return the total number of elements in the tree."""
58     return self._size
59
60 def root(self):
61     """Return the root Position of the tree (or None if tree is empty)."""
62     return self._make_position(self._root)
63
64 def parent(self, p):
65     """Return the Position of p's parent (or None if p is root)."""
66     node = self._validate(p)
67     return self._make_position(node._parent)
68
69 def left(self, p):
70     """Return the Position of p's left child (or None if no left child)."""
71     node = self._validate(p)
72     return self._make_position(node._left)
73
74 def right(self, p):
75     """Return the Position of p's right child (or None if no right
76         child)."""
77     node = self._validate(p)
78     return self._make_position(node._right)
79
80 def num_children(self, p):
81     """Return the number of children of Position p."""
82     node = self._validate(p)
83     count = 0
84     if node._left is not None:    # left child exists
85         count += 1
86     if node._right is not None:  # right child exists
87         count += 1
88     return count

```



```

88
89 #----- nonpublic mutators -----
90 def _add_root(self, e):
91     """Place element e at the root of an empty tree and return new Position.
92
93     Raise ValueError if tree nonempty.
94     """
95     if self._root is not None:
96         raise ValueError('Root exists')
97     self._size = 1
98     self._root = self._Node(e)
99     return self._make_position(self._root)
100
101 def _add_left(self, p, e):
102     """Create a new left child for Position p, storing element e.
103
104     Return the Position of new node.
105     Raise ValueError if Position p is invalid or p already has a left child.
106     """
107     node = self._validate(p)
108     if node._left is not None:
109         raise ValueError('Left child exists')
110     self._size += 1
111     node._left = self._Node(e, node)          # node is its parent
112     return self._make_position(node._left)
113
114 def _add_right(self, p, e):
115     """Create a new right child for Position p, storing element e.
116
117     Return the Position of new node.
118     Raise ValueError if Position p is invalid or p already has a right
119         child.
120     """
121     node = self._validate(p)
122     if node._right is not None:
123         raise ValueError('Right child exists')
124     self._size += 1
125     node._right = self._Node(e, node)         # node is its parent
126     return self._make_position(node._right)
127
128 def _replace(self, p, e):
129     """Replace the element at position p with e, and return old element."""
130     node = self._validate(p)
131     old = node._element
132     node._element = e
133     return old

```

```

133
134 def _delete(self, p):
135     """Delete the node at Position p, and replace it with its child, if any.
136
137     Return the element that had been stored at Position p.
138     Raise ValueError if Position p is invalid or p has two children.
139     """
140     node = self._validate(p)
141     if self.num_children(p) == 2:
142         raise ValueError('Position has two children')
143     child = node._left if node._left else node._right # might be None
144     if child is not None:
145         child._parent = node._parent # child's grandparent becomes parent
146     if node is self._root:
147         self._root = child # child becomes root
148     else:
149         parent = node._parent
150         if node is parent._left:
151             parent._left = child
152         else:
153             parent._right = child
154     self._size -= 1
155     node._parent = node # convention for deprecated node
156     return node._element
157
158 def _attach(self, p, t1, t2):
159     """Attach trees t1 and t2, respectively, as the left and right subtrees
160     of the external Position p.
161
162     As a side effect, set t1 and t2 to empty.
163     Raise TypeError if trees t1 and t2 do not match type of this tree.
164     Raise ValueError if Position p is invalid or not external.
165     """
166     node = self._validate(p)
167     if not self.is_leaf(p):
168         raise ValueError('position must be leaf')
169     if not type(self) is type(t1) is type(t2): # all 3 trees must be
170         same type
171         raise TypeError('Tree types must match')
172     self._size += len(t1) + len(t2)
173     if not t1.is_empty(): # attached t1 as left subtree of node
174         t1._root._parent = node
175         node._left = t1._root
176         t1._root = None # set t1 instance to empty
177         t1._size = 0
178     if not t2.is_empty(): # attached t2 as right subtree of node
179         t2._root._parent = node
180         node._right = t2._root
181         t2._root = None # set t2 instance to empty
182         t2._size = 0

```

Capitolo 4

Binary Search Tree (BST)

Listing 4.1: Classe astratta di base che include una classe non pubblica `_Item` per la memorizzazione di coppie chiave-valore.

```
1 from collections.abc import MutableMapping
2
3 class MapBase(MutableMapping):
4     """Our own abstract base class that includes a nonpublic _Item class."""
5
6     #----- nested _Item class -----
7     class _Item:
8         """Lightweight composite to store key-value pairs as map items."""
9         __slots__ = '_key', '_value'
10
11     def __init__(self, k, v):
12         self._key = k
13         self._value = v
14
15     def __eq__(self, other):
16         return self._key == other._key    # compare items based on their keys
17
18     def __ne__(self, other):
19         return not (self == other)        # opposite of __eq__
20
21     def __lt__(self, other):
22         return self._key < other._key     # compare items based on their keys
```

Listing 4.2: Implementazione di una sorted map utilizzando un albero di ricerca binario.

```
1 from ..tree.linked_binary_tree import LinkedBinaryTree
2 from .map_base import MapBase
3
4 class TreeMap(LinkedBinaryTree, MapBase):
5     """Sorted map implementation using a binary search tree."""
6
7     #----- override Position class -----
8     class Position(LinkedBinaryTree.Position):
9         def key(self):
10             """Return key of map's key-value pair."""
11             return self.element()._key
12
13         def value(self):
14             """Return value of map's key-value pair."""
15             return self.element()._value
16
17     #----- nonpublic utilities -----
18     def _subtree_search(self, p, k):
19         """Return Position of p's subtree having key k, or last node
20            searched."""
21         if k == p.key(): # found match
22             return p
23         elif k < p.key(): # search left subtree
24             if self.left(p) is not None:
25                 return self._subtree_search(self.left(p), k)
26             else: # search right subtree
27                 if self.right(p) is not None:
28                     return self._subtree_search(self.right(p), k)
29             return p # unsuccessful search
30
31     def _subtree_first_position(self, p):
32         """Return Position of first item in subtree rooted at p."""
33         walk = p
34         while self.left(walk) is not None: # keep walking left
35             walk = self.left(walk)
36         return walk
37
38     def _subtree_last_position(self, p):
39         """Return Position of last item in subtree rooted at p."""
40         walk = p
41         while self.right(walk) is not None: # keep walking right
42             walk = self.right(walk)
43         return walk
```

```

43
44 #----- public methods providing "positional" support -----
45 def first(self):
46     """Return the first Position in the tree (or None if empty)."""
47     return self._subtree_first_position(self.root()) if len(self) > 0 else
        None
48
49 def last(self):
50     """Return the last Position in the tree (or None if empty)."""
51     return self._subtree_last_position(self.root()) if len(self) > 0 else
        None
52
53 def before(self, p):
54     """Return the Position just before p in the natural order.
55
56     Return None if p is the first position.
57     """
58     self._validate(p)          # inherited from LinkedBinaryTree
59     if self.left(p):
60         return self._subtree_last_position(self.left(p))
61     else:
62         # walk upward
63         walk = p
64         above = self.parent(walk)
65         while above is not None and walk == self.left(above):
66             walk = above
67             above = self.parent(walk)
68         return above
69
70 def after(self, p):
71     """Return the Position just after p in the natural order.
72
73     Return None if p is the last position.
74     """
75     self._validate(p)          # inherited from LinkedBinaryTree
76     if self.right(p):
77         return self._subtree_first_position(self.right(p))
78     else:
79         walk = p
80         above = self.parent(walk)
81         while above is not None and walk == self.right(above):
82             walk = above
83             above = self.parent(walk)
84         return above

```

```

85
86 def find_position(self, k):
87     """Return position with key k, or else neighbor (or None if empty)."""
88     if self.is_empty():
89         return None
90     else:
91         p = self._subtree_search(self.root(), k)
92         self._rebalance_access(p)      # hook for balanced tree subclasses
93         return p
94
95 def delete(self, p):
96     """Remove the item at given Position."""
97     self._validate(p)                  # inherited from LinkedBinaryTree
98     if self.left(p) and self.right(p): # p has two children
99         replacement = self._subtree_last_position(self.left(p))
100         self._replace(p, replacement.element()) # from LinkedBinaryTree
101         p = replacement
102     # now p has at most one child
103     parent = self.parent(p)
104     self._delete(p)                    # inherited from LinkedBinaryTree
105     self._rebalance_delete(parent)     # if root deleted, parent is None
106
107 #----- public methods for (standard) map interface -----
108 def __getitem__(self, k):
109     """Return value associated with key k (raise KeyError if not found)."""
110     if self.is_empty():
111         raise KeyError('Key Error: ' + repr(k))
112     else:
113         p = self._subtree_search(self.root(), k)
114         self._rebalance_access(p)      # hook for balanced tree subclasses
115         if k != p.key():
116             raise KeyError('Key Error: ' + repr(k))
117         return p.value()

```

```

118
119 def __setitem__(self, k, v):
120     """Assign value v to key k, overwriting existing value if present."""
121     if self.is_empty():
122         leaf = self._add_root(self._Item(k,v))      # from LinkedBinaryTree
123     else:
124         p = self._subtree_search(self.root(), k)
125         if p.key() == k:
126             p.element()._value = v                # replace existing item's value
127             self._rebalance_access(p)              # hook for balanced tree subclasses
128             return
129         else:
130             item = self._Item(k,v)
131             if p.key() < k:
132                 leaf = self._add_right(p, item)    # inherited from
133                                                         LinkedBinaryTree
134             else:
135                 leaf = self._add_left(p, item)     # inherited from
136                                                         LinkedBinaryTree
137         self._rebalance_insert(leaf)                # hook for balanced tree
138                                                         subclasses
139
140 def __delitem__(self, k):
141     """Remove item associated with key k (raise KeyError if not found)."""
142     if not self.is_empty():
143         p = self._subtree_search(self.root(), k)
144         if k == p.key():
145             self.delete(p)                          # rely on positional version
146             return                                  # successful deletion complete
147         self._rebalance_access(p)                    # hook for balanced tree subclasses
148         raise KeyError('Key Error: ' + repr(k))
149
150 def __iter__(self):
151     """Generate an iteration of all keys in the map in order."""
152     p = self.first()
153     while p is not None:
154         yield p.key()
155         p = self.after(p)

```

```

153
154 #----- public methods for sorted map interface -----
155 def __reversed__(self):
156     """Generate an iteration of all keys in the map in reverse order."""
157     p = self.last()
158     while p is not None:
159         yield p.key()
160         p = self.before(p)
161
162 def find_min(self):
163     """Return (key,value) pair with minimum key (or None if empty)."""
164     if self.is_empty():
165         return None
166     else:
167         p = self.first()
168         return (p.key(), p.value())
169
170 def find_max(self):
171     """Return (key,value) pair with maximum key (or None if empty)."""
172     if self.is_empty():
173         return None
174     else:
175         p = self.last()
176         return (p.key(), p.value())
177
178 def find_le(self, k):
179     """Return (key,value) pair with greatest key less than or equal to k.
180
181     Return None if there does not exist such a key.
182     """
183     if self.is_empty():
184         return None
185     else:
186         p = self.find_position(k)
187         if k < p.key():
188             p = self.before(p)
189         return (p.key(), p.value()) if p is not None else None
190
191 def find_lt(self, k):
192     """Return (key,value) pair with greatest key strictly less than k.
193
194     Return None if there does not exist such a key.
195     """
196     if self.is_empty():
197         return None
198     else:
199         p = self.find_position(k)
200         if not p.key() < k:
201             p = self.before(p)
202         return (p.key(), p.value()) if p is not None else None

```



```

203
204 def find_ge(self, k):
205     """Return (key,value) pair with least key greater than or equal to k.
206
207     Return None if there does not exist such a key.
208     """
209     if self.is_empty():
210         return None
211     else:
212         p = self.find_position(k)           # may not find exact match
213         if p.key() < k:                     # p's key is too small
214             p = self.after(p)
215         return (p.key(), p.value()) if p is not None else None
216
217 def find_gt(self, k):
218     """Return (key,value) pair with least key strictly greater than k.
219
220     Return None if there does not exist such a key.
221     """
222     if self.is_empty():
223         return None
224     else:
225         p = self.find_position(k)
226         if not k < p.key():
227             p = self.after(p)
228         return (p.key(), p.value()) if p is not None else None
229
230 def find_range(self, start, stop):
231     """Iterate all (key,value) pairs such that start <= key < stop.
232
233     If start is None, iteration begins with minimum key of map.
234     If stop is None, iteration continues through the maximum key of map.
235     """
236     if not self.is_empty():
237         if start is None:
238             p = self.first()
239         else:
240             # we initialize p with logic similar to find_ge
241             p = self.find_position(start)
242             if p.key() < start:
243                 p = self.after(p)
244         while p is not None and (stop is None or p.key() < stop):
245             yield (p.key(), p.value())
246             p = self.after(p)

```

```

247
248 #----- hooks used by subclasses to balance a tree -----
249 def _rebalance_insert(self, p):
250     """Call to indicate that position p is newly added."""
251     pass
252
253 def _rebalance_delete(self, p):
254     """Call to indicate that a child of p has been removed."""
255     pass
256
257 def _rebalance_access(self, p):
258     """Call to indicate that position p was recently accessed."""
259     pass
260
261 #----- nonpublic methods to support tree balancing -----
262 def _relink(self, parent, child, make_left_child):
263     """Relink parent node with child node (we allow child to be None)."""
264     if make_left_child: # make it a left child
265         parent._left = child
266     else: # make it a right child
267         parent._right = child
268     if child is not None: # make child point to parent
269         child._parent = parent

```

```

270
271 def _rotate(self, p):
272     """Rotate Position p above its parent.
273
274     Switches between these configurations, depending on whether p==a or
275         p==b.
276
277         b                a
278         / \            /  \
279         a  t2          t0   b
280         / \            /  \
281         t0  t1          t1  t2
282
283     Caller should ensure that p is not the root.
284     """
285     """Rotate Position p above its parent."""
286     x = p._node
287     y = x._parent           # we assume this exists
288     z = y._parent           # grandparent (possibly None)
289     if z is None:
290         self._root = x      # x becomes root
291         x._parent = None
292     else:
293         self._relink(z, x, y == z._left)    # x becomes a direct child of z
294         # now rotate x and y, including transfer of middle subtree
295         if x == y._left:
296             self._relink(y, x._right, True)    # x._right becomes left child
297             # of y
298             self._relink(x, y, False)           # y becomes right child of x
299         else:
300             self._relink(y, x._left, False)    # x._left becomes right child
301             # of y
302             self._relink(x, y, True)           # y becomes left child of x

```

```

300
301 def _restructure(self, x):
302     """Perform a trinode restructure among Position x, its parent, and its
303         grandparent.
304
305     Return the Position that becomes root of the restructured subtree.
306
307     Assumes the nodes are in one of the following configurations:
308
309         z=a          z=c          z=a          z=c
310         / \          / \          / \          / \
311     t0  y=b        y=b  t3      t0  y=c        y=a  t3
312         / \          / \          / \          / \
313     t1  x=c        x=a  t2      x=b  t3      t0  x=b
314         / \          / \          / \          / \
315     t2  t3        t0  t1        t1  t2        t1  t2
316
317     The subtree will be restructured so that the node with key b becomes
318     its root.
319
320         b
321        / \
322       a   c
323      / \  / \
324     t0 t1 t2 t3
325
326     Caller should ensure that x has a grandparent.
327     """
328     """Perform trinode restructure of Position x with parent/grandparent."""
329     y = self.parent(x)
330     z = self.parent(y)
331     if (x == self.right(y)) == (y == self.right(z)): # matching alignments
332         self._rotate(y) # single rotation (of y)
333         return y # y is new subtree root
334     else: # opposite alignments
335         self._rotate(x) # double rotation (of x)
336         self._rotate(x)
337         return x # x is new subtree root

```

Capitolo 5

AVL Tree

Listing 5.1: Implementazione della classe AVLTreeMap e gestione del bilanciamento.

```
1 from .binary_search_tree import TreeMap
2
3 class AVLTreeMap(TreeMap):
4     """Sorted map implementation using an AVL tree."""
5
6     #----- nested _Node class -----
7     class _Node(TreeMap._Node):
8         """Node class for AVL maintains height value for balancing.
9
10        We use convention that a "None" child has height 0, thus a leaf has
11        height 1.
12        """
13        __slots__ = '_height'          # additional data member to store height
14
15        def __init__(self, element, parent=None, left=None, right=None):
16            super().__init__(element, parent, left, right)
17            self._height = 0           # will be recomputed during balancing
18
19        def left_height(self):
20            return self._left._height if self._left is not None else 0
21
22        def right_height(self):
23            return self._right._height if self._right is not None else 0
24
25        #----- positional-based utility methods -----
26
27        def _recompute_height(self, p):
28            p._node._height = 1 + max(p._node.left_height(), p._node.right_height())
29
30        def _isbalanced(self, p):
31            return abs(p._node.left_height() - p._node.right_height()) <= 1
```

```

30 def _tall_child(self, p, favorleft=False): # parameter controls tiebreaker
31     if p._node.left_height() + (1 if favorleft else 0) >
        p._node.right_height():
32         return self.left(p)
33     else:
34         return self.right(p)
35
36 def _tall_grandchild(self, p):
37     child = self._tall_child(p)
38     # if child is on left, favor left grandchild; else favor right
        grandchild
39     alignment = (child == self.left(p))
40     return self._tall_child(child, alignment)
41
42 def _rebalance(self, p):
43     while p is not None:
44         old_height = p._node._height # trivially 0
            if new node
45         if not self._isbalanced(p): # imbalance
            detected!
46         # perform trinode restructuring, setting p to resulting root,
47         # and recompute new local heights after the restructuring
48         p = self._restructure(self._tall_grandchild(p))
49         self._recompute_height(self.left(p))
50         self._recompute_height(self.right(p))
51         self._recompute_height(p) # adjust for
            recent changes
52         if p._node._height == old_height: # has height
            changed?
53         p = None # no further
            changes needed
54     else:
55         p = self.parent(p) # repeat with
            parent
56
57 #----- override balancing hooks
        -----
58 def _rebalance_insert(self, p):
59     self._rebalance(p)
60
61 def _rebalance_delete(self, p):
62     self._rebalance(p)

```

Capitolo 6

RB Tree

Listing 6.1: Implementazione della classe RedBlackTreeMap e gestione del colore dei nodi.

```
1 from .binary_search_tree import TreeMap
2
3 class RedBlackTreeMap(TreeMap):
4     """Sorted map implementation using a red-black tree."""
5
6     #----- nested _Node class -----
7     class _Node(TreeMap._Node):
8         """Node class for red-black tree maintains bit that denotes color."""
9         __slots__ = '_red'      # add additional data member to the Node class
10
11         def __init__(self, element, parent=None, left=None, right=None):
12             super().__init__(element, parent, left, right)
13             self._red = True    # new node red by default
14
15         #----- positional-based utility methods -----
16
17         # we consider a nonexistent child to be trivially black
18         def _set_red(self, p): p._node._red = True
19         def _set_black(self, p): p._node._red = False
20         def _set_color(self, p, make_red): p._node._red = make_red
21         def _is_red(self, p): return p is not None and p._node._red
22         def _is_red_leaf(self, p): return self._is_red(p) and self.is_leaf(p)
23
24         def _get_red_child(self, p):
25             """Return a red child of p (or None if no such child)."""
26             for child in (self.left(p), self.right(p)):
27                 if self._is_red(child):
28                     return child
29             return None
```

```

29
30 #----- support for insertions -----
31 def _rebalance_insert(self, p):
32     self._resolve_red(p) # new node is always red
33
34 def _resolve_red(self, p):
35     if self.is_root(p):
36         self._set_black(p) # make root black
37     else:
38         parent = self.parent(p)
39         if self._is_red(parent): # double red problem
40             uncle = self.sibling(parent)
41             if not self._is_red(uncle): # Case 1: misshapen 4-node
42                 middle = self._restructure(p) # do trinode restructuring
43                 self._set_black(middle) # and then fix colors
44                 self._set_red(self.left(middle))
45                 self._set_red(self.right(middle))
46             else: # Case 2: overfull 5-node
47                 grand = self.parent(parent)
48                 self._set_red(grand) # grandparent becomes red
49                 self._set_black(self.left(grand)) # its children become black
50                 self._set_black(self.right(grand))
51                 self._resolve_red(grand) # recur at red grandparent
52
53 #----- support for deletions -----
54 def _rebalance_delete(self, p):
55     if len(self) == 1:
56         self._set_black(self.root()) # special case: ensure that root is
57         black
58     elif p is not None:
59         n = self.num_children(p)
60         if n == 1: # deficit exists unless child is a red
61             leaf
62             c = next(self.children(p))
63             if not self._is_red_leaf(c):
64                 self._fix_deficit(p, c)
65         elif n == 2: # removed black node with red child
66             if self._is_red_leaf(self.left(p)):
67                 self._set_black(self.left(p))
68             else:
69                 self._set_black(self.right(p))

```



```

68
69 def _fix_deficit(self, z, y):
70     """Resolve black deficit at z, where y is the root of z's heavier
       subtree."""
71     if not self._is_red(y): # y is black; will apply Case 1 or 2
72         x = self._get_red_child(y)
73         if x is not None: # Case 1: y is black and has red child x; do
           "transfer"
74             old_color = self._is_red(z)
75             middle = self._restructure(x)
76             self._set_color(middle, old_color) # middle gets old color of z
77             self._set_black(self.left(middle)) # children become black
78             self._set_black(self.right(middle))
79         else: # Case 2: y is black, but no red children; recolor as "fusion"
80             self._set_red(y)
81             if self._is_red(z):
82                 self._set_black(z) # this resolves the problem
83             elif not self.is_root(z):
84                 self._fix_deficit(self.parent(z), self.sibling(z)) # recur upward
85         else: # Case 3: y is red; rotate misaligned 3-node and repeat
86             self._rotate(y)
87             self._set_black(y)
88             self._set_red(z)
89             if z == self.right(y):
90                 self._fix_deficit(z, self.left(z))
91             else:
92                 self._fix_deficit(z, self.right(z))

```


Capitolo 7

Hash

Listing 7.1: Classe base HashMapBase con funzione di hashing MAD.

```
1 from .map_base import MapBase
2 from random import randrange          # used to pick MAD parameters
3
4 class HashMapBase(MapBase):
5     """Abstract base class for map using hash-table with MAD compression.
6
7     Keys must be hashable and non-None.
8     """
9
10    def __init__(self, cap=11, p=109345121):
11        """Create an empty hash-table map.
12
13        cap        initial table size (default 11)
14        p          positive prime used for MAD (default 109345121)
15        """
16        self._table = cap * [ None ]
17        self._n = 0                                # number of entries in
18                                                    # the map
19        self._prime = p                            # prime for MAD
20                                                    # compression
21        self._scale = 1 + randrange(p-1)           # scale from 1 to p-1 for
22                                                    # MAD
23        self._shift = randrange(p)                 # shift from 0 to p-1 for
24                                                    # MAD
25
26    def _hash_function(self, k):
27        return (hash(k)*self._scale + self._shift) % self._prime %
28                len(self._table)
29
30    def __len__(self):
31        return self._n
```

```

27
28 def __getitem__(self, k):
29     j = self._hash_function(k)
30     return self._bucket_getitem(j, k)           # may raise KeyError
31
32 def __setitem__(self, k, v):
33     j = self._hash_function(k)
34     self._bucket_setitem(j, k, v)              # subroutine maintains
        self._n
35     if self._n > len(self._table) // 2:         # keep load factor <= 0.5
36         self._resize(2 * len(self._table) - 1) # number 2^x - 1 is often
        prime
37
38 def __delitem__(self, k):
39     j = self._hash_function(k)
40     self._bucket_delitem(j, k)                 # may raise KeyError
41     self._n -= 1
42
43 def _resize(self, c):
44     """Resize bucket array to capacity c and rehash all items."""
45     old = list(self.items())                    # use iteration to record existing items
46     self._table = c * [None]                  # then reset table to desired capacity
47     self._n = 0                                # n recomputed during subsequent adds
48     for (k,v) in old:
49         self[k] = v                            # reinsert old key-value pair

```

Capitolo 8

Sorted Unsorted Table

Listing 8.1: Implementazione di una mappa tramite tabella ordinata (Sorted Table).

```
1 from .map_base import MapBase
2
3 class SortedTableMap(MapBase):
4     """Map implementation using a sorted table."""
5
6     #----- nonpublic behaviors -----
7
8     def _find_index(self, k, low, high):
9         """Return index of the leftmost item with key greater than or equal to
10            k.
11
12            Return high + 1 if no such item qualifies.
13
14            That is, j will be returned such that:
15                all items of slice table[low:j] have key < k
16                all items of slice table[j:high+1] have key >= k
17            """
18
19         if high < low:
20             return high + 1 # no element qualifies
21         else:
22             mid = (low + high) // 2
23             if k == self._table[mid]._key:
24                 return mid # found exact match
25             elif k < self._table[mid]._key:
26                 return self._find_index(k, low, mid - 1) # Note: may return mid
27             else:
28                 return self._find_index(k, mid + 1, high) # answer is right of mid
```

```

26
27 #----- public behaviors
28
29 -----
30
31 def __init__(self):
32     """Create an empty map."""
33     self._table = []
34
35
36 def __len__(self):
37     """Return number of items in the map."""
38     return len(self._table)
39
40
41 def __getitem__(self, k):
42     """Return value associated with key k (raise KeyError if not found)."""
43     j = self._find_index(k, 0, len(self._table) - 1)
44     if j == len(self._table) or self._table[j]._key != k:
45         raise KeyError('Key Error: ' + repr(k))
46     return self._table[j]._value
47
48
49 def __setitem__(self, k, v):
50     """Assign value v to key k, overwriting existing value if present."""
51     j = self._find_index(k, 0, len(self._table) - 1)
52     if j < len(self._table) and self._table[j]._key == k:
53         self._table[j]._value = v                # reassign value
54     else:
55         self._table.insert(j, self._Item(k,v))    # adds new item
56
57
58 def __delitem__(self, k):
59     """Remove item associated with key k (raise KeyError if not found)."""
60     j = self._find_index(k, 0, len(self._table) - 1)
61     if j == len(self._table) or self._table[j]._key != k:
62         raise KeyError('Key Error: ' + repr(k))
63     self._table.pop(j)                            # delete item
64
65
66 def __iter__(self):
67     """Generate keys of the map ordered from minimum to maximum."""
68     for item in self._table:
69         yield item._key
70
71
72 def __reversed__(self):
73     """Generate keys of the map ordered from maximum to minimum."""
74     for item in reversed(self._table):
75         yield item._key
76
77
78 def find_min(self):
79     """Return (key,value) pair with minimum key (or None if empty)."""
80     if len(self._table) > 0:
81         return (self._table[0]._key, self._table[0]._value)
82     else:
83         return None
84
85

```

```

75 def find_max(self):
76     """Return (key,value) pair with maximum key (or None if empty)."""
77     if len(self._table) > 0:
78         return (self._table[-1]._key, self._table[-1]._value)
79     else:
80         return None
81
82 def find_le(self, k):
83     """Return (key,value) pair with greatest key less than or equal to k.
84
85     Return None if there does not exist such a key.
86     """
87     j = self._find_index(k, 0, len(self._table) - 1)      # j's key >= k
88     if j < len(self._table) and self._table[j]._key == k:
89         return (self._table[j]._key, self._table[j]._value)    # exact match
90     elif j > 0:
91         return (self._table[j-1]._key, self._table[j-1]._value) # Note use
92                             of j-1
93     else:
94         return None
95
96 def find_ge(self, k):
97     """Return (key,value) pair with least key greater than or equal to k.
98
99     Return None if there does not exist such a key.
100    """
101    j = self._find_index(k, 0, len(self._table) - 1)      # j's key >= k
102    if j < len(self._table):
103        return (self._table[j]._key, self._table[j]._value)
104    else:
105        return None
106
107 def find_lt(self, k):
108     """Return (key,value) pair with greatest key strictly less than k.
109
110     Return None if there does not exist such a key.
111     """
112    j = self._find_index(k, 0, len(self._table) - 1)      # j's key >= k
113    if j > 0:
114        return (self._table[j-1]._key, self._table[j-1]._value) # Note use
115                             of j-1
116    else:
117        return None

```

```

116
117 def find_gt(self, k):
118     """Return (key,value) pair with least key strictly greater than k.
119
120     Return None if there does not exist such a key.
121     """
122     j = self._find_index(k, 0, len(self._table) - 1)      # j's key >= k
123     if j < len(self._table) and self._table[j]._key == k:
124         j += 1      # advanced past match
125     if j < len(self._table):
126         return (self._table[j]._key, self._table[j]._value)
127     else:
128         return None
129
130 def find_range(self, start, stop):
131     """Iterate all (key,value) pairs such that start <= key < stop.
132
133     If start is None, iteration begins with minimum key of map.
134     If stop is None, iteration continues through the maximum key of map.
135     """
136     if start is None:
137         j = 0
138     else:
139         j = self._find_index(start, 0, len(self._table)-1)  # find first
140         result
141     while j < len(self._table) and (stop is None or self._table[j]._key <
142         stop):
143         yield (self._table[j]._key, self._table[j]._value)
144         j += 1

```


Listing 8.2: Implementazione di una mappa tramite tabella non ordinata (Unsorted Table).

```
1 from .map_base import MapBase
2
3 class UnsortedTableMap(MapBase):
4     """Map implementation using an unordered list."""
5
6     def __init__(self):
7         """Create an empty map."""
8         self._table = []                # list of _Item's
9
10    def __getitem__(self, k):
11        """Return value associated with key k (raise KeyError if not found)."""
12        for item in self._table:
13            if k == item._key:
14                return item._value
15        raise KeyError('Key Error: ' + repr(k))
16
17    def __setitem__(self, k, v):
18        """Assign value v to key k, overwriting existing value if present."""
19        for item in self._table:
20            if k == item._key:                # Found a match:
21                item._value = v                # reassign value
22            return                            # and quit
23        # did not find match for key
24        self._table.append(self._Item(k,v))
25
26    def __delitem__(self, k):
27        """Remove item associated with key k (raise KeyError if not found)."""
28        for j in range(len(self._table)):
29            if k == self._table[j]._key:        # Found a match:
30                self._table.pop(j)            # remove item
31            return                            # and quit
32        raise KeyError('Key Error: ' + repr(k))
33
34    def __len__(self):
35        """Return number of items in the map."""
36        return len(self._table)
37
38    def __iter__(self):
39        """Generate iteration of the map's keys."""
40        for item in self._table:
41            yield item._key                # yield the KEY
```


Capitolo 9

Chain Hash

Listing 9.1: Gestione delle collisioni tramite Separate Chaining nella classe ChainHashMap.

```
1 from .hash_map_base import HashMapBase
2 from .unsorted_table_map import UnsortedTableMap
3
4 class ChainHashMap(HashMapBase):
5     """Hash map implemented with separate chaining for collision
6         resolution."""
7
8     def _bucket_getitem(self, j, k):
9         bucket = self._table[j]
10        if bucket is None:
11            raise KeyError('Key Error: ' + repr(k))    # no match found
12        return bucket[k]                                # may raise KeyError
13
14    def _bucket_setitem(self, j, k, v):
15        if self._table[j] is None:
16            self._table[j] = UnsortedTableMap()        # bucket is new to the table
17        oldsize = len(self._table[j])
18        self._table[j][k] = v
19        if len(self._table[j]) > oldsize:               # key was new to the table
20            self._n += 1                                # increase overall map size
21
22    def _bucket_delitem(self, j, k):
23        bucket = self._table[j]
24        if bucket is None:
25            raise KeyError('Key Error: ' + repr(k))    # no match found
26        del bucket[k]                                    # may raise KeyError
27
28    def __iter__(self):
29        for bucket in self._table:
30            if bucket is not None:                        # a nonempty slot
31                for key in bucket:
```


Capitolo 10

Probe Hash

Listing 10.1: Implementazione di una mappa tramite tabella hash con indirizzamento aperto.

```
1 from .hash_map_base import HashMapBase
2
3 class ProbeHashMap(HashMapBase):
4     """Hash map implemented with linear probing for collision resolution."""
5     _AVAIL = object()      # sentinel marks locations of previous deletions
6
7     def _is_available(self, j):
8         """Return True if index j is available in table."""
9         return self._table[j] is None or self._table[j] is ProbeHashMap._AVAIL
10
11    def _find_slot(self, j, k):
12        """Search for key k in bucket at index j.
13
14        Return (success, index) tuple, described as follows:
15        If match was found, success is True and index denotes its location.
16        If no match found, success is False and index denotes first available
17        slot.
18        """
19        firstAvail = None
20        while True:
21            if self._is_available(j):
22                if firstAvail is None:
23                    firstAvail = j                # mark this as first avail
24                if self._table[j] is None:
25                    return (False, firstAvail)    # search has failed
26            elif k == self._table[j]._key:
27                return (True, j)                  # found a match
28            j = (j + 1) % len(self._table)        # keep looking (cyclically)
```

```

28
29 def _bucket_getitem(self, j, k):
30     found, s = self._find_slot(j, k)
31     if not found:
32         raise KeyError('Key Error: ' + repr(k))           # no match found
33     return self._table[s]._value
34
35 def _bucket_setitem(self, j, k, v):
36     found, s = self._find_slot(j, k)
37     if not found:
38         self._table[s] = self._Item(k,v)                   # insert new item
39         self._n += 1                                         # size has increased
40     else:
41         self._table[s]._value = v                           # overwrite existing
42
43 def _bucket_delitem(self, j, k):
44     found, s = self._find_slot(j, k)
45     if not found:
46         raise KeyError('Key Error: ' + repr(k))           # no match found
47     self._table[s] = ProbeHashMap._AVAIL                     # mark as vacated
48
49 def __iter__(self):
50     for j in range(len(self._table)):                       # scan entire table
51         if not self._is_available(j):
52             yield self._table[j]._key

```

Capitolo 11

Priority Queue

Listing 11.1: Classe base `PriorityQueueBase`: definizione della struttura `_Item` e dell'interfaccia per le code di priorità.

```
1 class PriorityQueueBase:
2     """Abstract base class for a priority queue."""
3
4     #----- nested _Item class
5     class _Item:
6         """Lightweight composite to store priority queue items."""
7         __slots__ = '_key', '_value'
8
9         def __init__(self, k, v):
10             self._key = k
11             self._value = v
12
13         def __lt__(self, other):
14             return self._key < other._key    # compare items based on their keys
15
16         def __repr__(self):
17             return '({0},{1})'.format(self._key, self._value)
18
19     #----- public behaviors
20     def is_empty(self):
21         """Return True if the priority queue is empty."""
22         return len(self) == 0
23
24     def __len__(self):
25         """Return the number of items in the priority queue."""
26         raise NotImplementedError('deve essere implementato dalla sottoclasse.')
27
28     def add(self, key, value):
29         """Add a key-value pair."""
30         raise NotImplementedError('deve essere implementato dalla sottoclasse.')
```

```
31
32 def min(self):
33     """Return but do not remove (k,v) tuple with minimum key.
34
35     Raise Empty exception if empty.
36     """
37     raise NotImplementedError('deve essere implementato dalla sottoclasse.')
38
39 def remove_min(self):
40     """Remove and return (k,v) tuple with minimum key.
41
42     Raise Empty exception if empty.
43     """
44     raise NotImplementedError('deve essere implementato dalla sottoclasse.')
```


Capitolo 12

Sorted Unsorted Priority Queue

Listing 12.1: Implementazione di SortedPriorityQueue.

```
1 from .priority_queue_base import PriorityQueueBase
2 from ..list.positional_list import PositionalList
3
4 class Empty(Exception):
5     pass
6
7 class SortedPriorityQueue(PriorityQueueBase): # base class defines _Item
8     """A min-oriented priority queue implemented with a sorted list."""
9
10    #----- public behaviors -----
11
12    def __init__(self):
13        """Create a new empty Priority Queue."""
14        self._data = PositionalList()
15
16    def __len__(self):
17        """Return the number of items in the priority queue."""
18        return len(self._data)
19
20    def add(self, key, value):
21        """Add a key-value pair."""
22        newest = self._Item(key, value) # make new item instance
23        walk = self._data.last() # walk backward looking for smaller key
24        while walk is not None and newest < walk.element():
25            walk = self._data.before(walk)
26        if walk is None:
27            self._data.add_first(newest) # new key is smallest
28        else:
29            self._data.add_after(walk, newest) # newest goes after walk
```

```

29
30 def min(self):
31     """Return but do not remove (k,v) tuple with minimum key.
32
33     Raise Empty exception if empty.
34     """
35     if self.is_empty():
36         raise Empty('Priority queue is empty.')
37     p = self._data.first()
38     item = p.element()
39     return (item._key, item._value)
40
41 def remove_min(self):
42     """Remove and return (k,v) tuple with minimum key.
43
44     Raise Empty exception if empty.
45     """
46     if self.is_empty():
47         raise Empty('Priority queue is empty.')
48     item = self._data.delete(self._data.first())
49     return (item._key, item._value)

```

Listing 12.2: Implementazione di UnsortedPriorityQueue.

```

1 from .priority_queue_base import PriorityQueueBase
2 from ..list.positional_list import PositionalList
3
4 class Empty(Exception):
5     pass
6
7
8 class UnsortedPriorityQueue(PriorityQueueBase): # base class defines _Item
9     """A min-oriented priority queue implemented with an unsorted list."""
10
11     #----- nonpublic behavior -----
12
13     def _find_min(self):
14         """Return Position of item with minimum key."""
15         if self.is_empty(): # is_empty inherited from base class
16             raise Empty('Priority queue is empty')
17         small = self._data.first()
18         walk = self._data.after(small)
19         while walk is not None:
20             if walk.element() < small.element():
21                 small = walk
22             walk = self._data.after(walk)
23         return small

```

```

23
24 #----- public behaviors
25     -----
26 def __init__(self):
27     """Create a new empty Priority Queue."""
28     self._data = PositionalList()
29
30 def __len__(self):
31     """Return the number of items in the priority queue."""
32     return len(self._data)
33
34 def add(self, key, value):
35     """Add a key-value pair."""
36     self._data.add_last(self._Item(key, value))
37
38 def min(self):
39     """Return but do not remove (k,v) tuple with minimum key.
40
41     Raise Empty exception if empty.
42     """
43     p = self._find_min()
44     item = p.element()
45     return (item._key, item._value)
46
47 def remove_min(self):
48     """Remove and return (k,v) tuple with minimum key.
49
50     Raise Empty exception if empty.
51     """
52     p = self._find_min()
53     item = self._data.delete(p)
54     return (item._key, item._value)

```


Capitolo 13

Heap Priority Queue

Listing 13.1: Implementazione di HeapPriorityQueue: gestione di un heap binario tramite array con algoritmi di upheap e downheap per il bilanciamento.

```
1 from .priority_queue_base import PriorityQueueBase
2
3 class Empty(Exception):
4     pass
5
6
7 class HeapPriorityQueue(PriorityQueueBase): # base class defines _Item
8     """A min-oriented priority queue implemented with a binary heap."""
9
10    #----- nonpublic behaviors -----
11
12    def _parent(self, j):
13        return (j-1) // 2
14
15    def _left(self, j):
16        return 2*j + 1
17
18    def _right(self, j):
19        return 2*j + 2
20
21    def _has_left(self, j):
22        return self._left(j) < len(self._data) # index beyond end of list?
23
24    def _has_right(self, j):
25        return self._right(j) < len(self._data) # index beyond end of list?
26
27    def _swap(self, i, j):
28        """Swap the elements at indices i and j of array."""
29        self._data[i], self._data[j] = self._data[j], self._data[i]
```

```

29
30 def _upheap(self, j):
31     parent = self._parent(j)
32     if j > 0 and self._data[j] < self._data[parent]:
33         self._swap(j, parent)
34         self._upheap(parent)           # recur at position of parent
35
36 def _downheap(self, j):
37     if self._has_left(j):
38         left = self._left(j)
39         small_child = left             # although right may be smaller
40         if self._has_right(j):
41             right = self._right(j)
42             if self._data[right] < self._data[left]:
43                 small_child = right
44         if self._data[small_child] < self._data[j]:
45             self._swap(j, small_child)
46             self._downheap(small_child) # recur at position of small child
47
48 def _heapify(self):
49     """Bottom-up construction of a heap in O(n) time."""
50     # start at the parent of the last element
51     start = self._parent(len(self._data) - 1)
52     for j in range(start, -1, -1): # go backward from start to 0
53         self._downheap(j)
54
55 #----- public behaviors -----
56
57 def __init__(self, contents=()):
58     """Create a new empty Priority Queue.
59
60     If contents is provided, it should be an iterable of (k,v) tuples.
61     """
62     self._data = [self._Item(k, v) for k, v in contents]
63     if len(self._data) > 1:
64         self._heapify()
65
66     """ without Heapify
67     def __init__(self):
68         Create a new empty Priority Queue.
69         self._data = []
70     """
71
72 def __len__(self):
73     """Return the number of items in the priority queue."""
74     return len(self._data)

```

```

74
75 def add(self, key, value):
76     """Add a key-value pair to the priority queue."""
77     self._data.append(self._Item(key, value))
78     self._upheap(len(self._data) - 1)           # upheap newly added
79                                                 position
80
81 def min(self):
82     """Return but do not remove (k,v) tuple with minimum key.
83
84     Raise Empty exception if empty.
85     """
86     if self.is_empty():
87         raise Empty('Priority queue is empty.')
88     item = self._data[0]
89     return (item._key, item._value)
90
91 def remove_min(self):
92     """Remove and return (k,v) tuple with minimum key.
93
94     Raise Empty exception if empty.
95     """
96     if self.is_empty():
97         raise Empty('Priority queue is empty.')
98     self._swap(0, len(self._data) - 1)           # put minimum item at the
99                                                 end
100    item = self._data.pop()                       # and remove it from the
101                                                 list;
102    self._downheap(0)                             # then fix new root
103    return (item._key, item._value)

```


Capitolo 14

Adaptable Priority Queue

Listing 14.1: Implementazione di AdaptableHeapPriorityQueue: estensione dello heap con oggetti Locator per la modifica (update) e rimozione (remove) efficiente di elementi.

```
1 from .heap_priority_queue import HeapPriorityQueue
2
3 class AdaptableHeapPriorityQueue(HeapPriorityQueue):
4     """A locator-based priority queue implemented with a binary heap."""
5
6     #----- nested Locator class
7     #-----
8     class Locator(HeapPriorityQueue._Item):
9         """Token for locating an entry of the priority queue."""
10         __slots__ = '_index'          # add index as additional field
11
12         def __init__(self, k, v, j):
13             super().__init__(k,v)
14             self._index = j
15
16         #----- nonpublic behaviors
17         #-----
18
19         # override swap to record new indices
20         def _swap(self, i, j):
21             super()._swap(i,j)          # perform the swap
22             self._data[i]._index = i    # reset locator index (post-swap)
23             self._data[j]._index = j    # reset locator index (post-swap)
24
25         def _bubble(self, j):
26             if j > 0 and self._data[j] < self._data[self._parent(j)]:
27                 self._upheap(j)
28             else:
29                 self._downheap(j)
```

```

27
28 #----- public behaviors
29
30 -----
29 def add(self, key, value):
30     """Add a key-value pair."""
31     token = self.Locator(key, value, len(self._data)) # initiaize locator
32     index
33     self._data.append(token)
34     self._upheap(len(self._data) - 1)
35     return token
36
37 def update(self, loc, newkey, newval):
38     """Update the key and value for the entry identified by Locator loc."""
39     j = loc._index
40     if not (0 <= j < len(self) and self._data[j] is loc):
41         raise ValueError('Invalid locator')
42     loc._key = newkey
43     loc._value = newval
44     self._bubble(j)
45
46 def remove(self, loc):
47     """Remove and return the (k,v) pair identified by Locator loc."""
48     j = loc._index
49     if not (0 <= j < len(self) and self._data[j] is loc):
50         raise ValueError('Invalid locator')
51     if j == len(self) - 1:
52         # item at last position
53         self._data.pop()
54         # just remove it
55     else:
56         self._swap(j, len(self)-1)
57         # swap item to the last position
58         self._data.pop()
59         # remove it from the list
60         self._bubble(j)
61         # fix item displaced by the swap
62     return (loc._key, loc._value)

```

Capitolo 15

Graph

Listing 15.1: Implementazione della struttura dati Grafo tramite mappe di adiacenza, con classi interne Vertex/Edge e supporto a grafi orientati e non orientati.

```
1 #----- nested Vertex class -----
2 class Vertex:
3     """Lightweight vertex structure for a graph."""
4     __slots__ = '_element'
5
6     def __init__(self, x):
7         """Do not call constructor directly. Use Graph's
8             insert_vertex(x)."""
9         self._element = x
10
11     def element(self):
12         """Return element associated with this vertex."""
13         return self._element
14
15     def __hash__(self):    # will allow vertex to be a map/set key
16         return hash(id(self))
17
18     def __str__(self):
19         return str(self._element)
20
21 #----- nested Edge class -----
22 class Edge:
23     """Lightweight edge structure for a graph."""
24     __slots__ = '_origin', '_destination', '_element'
25
26     def __init__(self, u, v, x):
27         """Do not call constructor directly. Use Graph's
28             insert_edge(u,v,x)."""
29         self._origin = u
30         self._destination = v
31         self._element = x
```

```

30
31 def endpoints(self):
32     """Return (u,v) tuple for vertices u and v."""
33     return (self._origin, self._destination)
34
35 def opposite(self, v):
36     """Return the vertex that is opposite v on this edge."""
37     if not isinstance(v, Graph.Vertex):
38         raise TypeError('v must be a Vertex')
39     if v is self._origin:
40         return self._destination
41     elif v is self._destination:
42         return self._origin
43     raise ValueError('v not incident to edge')
44
45 def element(self):
46     """Return element associated with this edge."""
47     return self._element
48
49 def __hash__(self):          # will allow edge to be a map/set key
50     return hash( (self._origin, self._destination) )
51
52 def __str__(self):
53     return
54         '({0},{1},{2})'.format(self._origin,self._destination,self._element)
55
56 class Graph:
57     """Representation of a simple graph using an adjacency map."""
58
59     def __init__(self, directed=False):
60         """Create an empty graph (undirected, by default).
61
62         Graph is directed if optional parameter is set to True.
63         """
64         self._outgoing = {}
65         # only create second map for directed graph; use alias for
66         # undirected
67         self._incoming = {} if directed else self._outgoing
68
69     def is_directed(self):
70         """Return True if this is a directed graph; False if undirected.
71
72         Property is based on the original declaration of the graph, not its
73         contents.
74         """
75         return self._incoming is not self._outgoing # directed if maps are
76             distinct

```

```

73
74 def vertex_count(self):
75     """Return the number of vertices in the graph."""
76     return len(self._outgoing)
77
78 def vertices(self):
79     """Return an iteration of all vertices of the graph."""
80     return self._outgoing.keys()
81
82 def edge_count(self):
83     """Return the number of edges in the graph."""
84     total = sum(len(self._outgoing[v]) for v in self._outgoing)
85     # for undirected graphs, make sure not to double-count edges
86     return total if self.is_directed() else total // 2
87
88 def edges(self):
89     """Return a set of all edges of the graph."""
90     result = set() # avoid double-reporting edges of undirected
91                     graph
92     for secondary_map in self._outgoing.values():
93         result.update(secondary_map.values()) # add edges to resulting
94                                             set
95     return result
96
97 def _validate_vertex(self, v):
98     """Verify that v is a Vertex of this graph."""
99     if not isinstance(v, self.Vertex):
100         raise TypeError('Vertex expected')
101     if v not in self._outgoing:
102         raise ValueError('Vertex does not belong to this graph.')
103
104 def get_edge(self, u, v):
105     """Return the edge from u to v, or None if not adjacent."""
106     return self._outgoing[u].get(v) # returns None if v not adjacent
107
108 def degree(self, v, outgoing=True):
109     """Return number of (outgoing) edges incident to vertex v in the
110     graph.
111
112     If graph is directed, optional parameter used to count incoming
113     edges.
114     """
115     adj = self._outgoing if outgoing else self._incoming
116     return len(adj[v])

```

```

114
115     def incident_edges(self, v, outgoing=True):
116         """Return all (outgoing) edges incident to vertex v in the graph.
117
118         If graph is directed, optional parameter used to request incoming
119         edges.
120         """
121         adj = self._outgoing if outgoing else self._incoming
122         for edge in adj[v].values():
123             yield edge
124
125     def insert_vertex(self, x=None):
126         """Insert and return a new Vertex with element x."""
127         v = self.Vertex(x)
128         self._outgoing[v] = {}
129         if self.is_directed():
130             self._incoming[v] = {} # need distinct map for incoming edges
131         return v
132
133     def insert_edge(self, u, v, x=None):
134         """Insert and return a new Edge from u to v with auxiliary element
135         x."""
136         e = self.Edge(u, v, x)
137         self._outgoing[u][v] = e
138         self._incoming[v][u] = e

```

```

1 def remove_vertex(self, v):
2     """Remove vertex v and all its incident edges from the graph."""
3     # Phase 1: Remove references to v from the adjacency maps of its
4         neighbors.
5
6     # For every neighbor w connected by an outgoing edge (v -> w),
7     # remove the link back to v from w's incoming map.
8     for w in list(self._outgoing[v]):
9         del self._incoming[w][v]
10
11     # If the graph is directed, we also need to handle incoming edges (u ->
12         v).
13     # We must remove the link to v from u's outgoing map.
14     # (In an undirected graph, _incoming is _outgoing, so the loop above
15         covered this).
16     if self.is_directed():
17         for u in list(self._incoming[v]):
18             del self._outgoing[u][v]
19
20     # Phase 2: Remove v from the graph's internal dictionaries.
21     del self._outgoing[v]
22     if self.is_directed():
23         del self._incoming[v]
24
25 def remove_edge(self, e):
26     """Remove edge e from the graph."""
27     u, v = e.endpoints()
28
29     # Remove the edge from u's outgoing map
30     del self._outgoing[u][v]
31
32     # Remove the edge from v's incoming map
33     del self._incoming[v][u]

```

Listing 15.2: Metodi di cancellazione per la classe Graph.

Capitolo 16

DFS

Listing 16.1: Algoritmo di ricerca in profondità (DFS): esplorazione ricorsiva del grafo, generazione della foresta di attraversamento e ricostruzione dei cammini.

```
1 def DFS(g, u, discovered):
2     """Perform DFS of the undiscovered portion of Graph g starting at Vertex
3         u.
4
5     discovered is a dictionary mapping each vertex to the edge that was used
6     to
7     discover it during the DFS. (u should be "discovered" prior to the call.)
8     Newly discovered vertices will be added to the dictionary as a result.
9     """
10    for e in g.incident_edges(u):    # for every outgoing edge from u
11        v = e.opposite(u)
12        if v not in discovered:    # v is an unvisited vertex
13            discovered[v] = e      # e is the tree edge that discovered v
14            DFS(g, v, discovered)  # recursively explore from v
15
16 def DFS_complete(g):
17     """Perform DFS for entire graph and return forest as a dictionary.
18
19     Result maps each vertex v to the edge that was used to discover it.
20     (Vertices that are roots of a DFS tree are mapped to None.)
21     """
22     forest = {}
23     for u in g.vertices():
24         if u not in forest:
25             forest[u] = None      # u will be the root of a tree
26             DFS(g, u, forest)
27     return forest
```

```

26
27 def construct_path(u, v, discovered):
28     """
29     Return a list of vertices comprising the directed path from u to v,
30     or an empty list if v is not reachable from u.
31
32     discovered is a dictionary resulting from a previous call to DFS started
33     at u.
34     """
35     path = [] # empty path by default
36     if v in discovered:
37         # we build list from v to u and then reverse it at the end
38         path.append(v)
39         walk = v
40         while walk is not u:
41             e = discovered[walk] # find edge leading to walk
42             parent = e.opposite(walk)
43             path.append(parent)
44             walk = parent
45         path.reverse() # reorient path from u to v
46     return path

```

Capitolo 17

BFS

Listing 17.1: Algoritmo di ricerca in ampiezza (BFS): esplorazione iterativa a livelli del grafo e generazione della foresta di attraversamento.

```
1 def BFS(g, s, discovered):
2     """Perform BFS of the undiscovered portion of Graph g starting at Vertex
3         s.
4         discovered is a dictionary mapping each vertex to the edge that was used
5         to discover it during the BFS (s should be mapped to None prior to the call).
6         Newly discovered vertices will be added to the dictionary as a result.
7     """
8     level = [s] # first level includes only s
9     while len(level) > 0:
10         next_level = [] # prepare to gather newly found
11                         vertices
12         for u in level:
13             for e in g.incident_edges(u): # for every outgoing edge from u
14                 v = e.opposite(u)
15                 if v not in discovered: # v is an unvisited vertex
16                     discovered[v] = e # e is the tree edge that discovered v
17                     next_level.append(v) # v will be further considered in next
18                                         pass
19         level = next_level # relabel 'next' level to become
20                             current
21
22 def BFS_complete(g):
23     """Perform BFS for entire graph and return forest as a dictionary.
24     Result maps each vertex v to the edge that was used to discover it.
25     (vertices that are roots of a BFS tree are mapped to None).
26     """
27     forest = {}
28     for u in g.vertices():
29         if u not in forest:
30             forest[u] = None # u will be a root of a tree
31             BFS(g, u, forest)
32     return forest
```


Capitolo 18

Transitive Closure

Listing 18.1: Calcolo della chiusura transitiva tramite l'algoritmo di Floyd-Warshall: approccio di programmazione dinamica per la raggiungibilità tra tutte le coppie di vertici.

```
1 from copy import deepcopy
2
3 def floyd_warshall(g):
4     """Return a new graph that is the transitive closure of g."""
5     closure = deepcopy(g)           # imported from copy module
6     verts = list(closure.vertices()) # make indexable list
7     n = len(verts)
8     for k in range(n):
9         for i in range(n):
10            # verify that edge (i,k) exists in the partial closure
11            if i != k and closure.get_edge(verts[i],verts[k]) is not None:
12                for j in range(n):
13                    # verify that edge (k,j) exists in the partial closure
14                    if i != j != k and closure.get_edge(verts[k],verts[j]) is not
15                        None:
16                        # if (i,j) not yet included, add it to the closure
17                        if closure.get_edge(verts[i],verts[j]) is None:
18                            closure.insert_edge(verts[i],verts[j])
19
20 return closure
```


Capitolo 19

Topological Sort

Listing 19.1: Ordinamento topologico di un grafo diretto aciclico (DAG): risoluzione sequenziale dei vincoli di precedenza basata sul grado entrante.

```
1 def topological_sort(g):
2     """Return a list of vertices of directed acyclic graph g in topological
3         order.
4
5     If graph g has a cycle, the result will be incomplete.
6     """
7     topo = []           # a list of vertices placed in topological order
8     ready = []          # list of vertices that have no remaining
9                           constraints
10    incount = {}         # keep track of in-degree for each vertex
11    for u in g.vertices():
12        incount[u] = g.degree(u, False) # parameter requests incoming degree
13        if incount[u] == 0:             # if u has no incoming edges,
14            ready.append(u)             # it is free of constraints
15    while len(ready) > 0:
16        u = ready.pop()                 # u is free of constraints
17        topo.append(u)                 # add u to the topological order
18        for e in g.incident_edges(u):  # consider all outgoing neighbors of u
19            v = e.opposite(u)
20            incount[v] -= 1             # v has one less constraint without u
21            if incount[v] == 0:
22                ready.append(v)
23    return topo
```


Capitolo 20

Shortest Path

Listing 20.1: Algoritmo di Dijkstra per i cammini minimi: calcolo delle distanze tramite rilassamento degli archi e ricostruzione dell'albero dei cammini minimi.

```
1 from ..priority_queue.adaptable_heap_priority_queue import
   AdaptableHeapPriorityQueue
2
3 def shortest_path_lengths(g, src):
4     """Compute shortest-path distances from src to reachable vertices of g.
5
6     Graph g can be undirected or directed, but must be weighted such that
7     e.element() returns a numeric weight for each edge e.
8
9     Return dictionary mapping each reachable vertex to its distance from src.
10    """
11    d = {}                                # d[v] is upper bound from
        s to v
12    cloud = {}                            # map reachable v to its
        d[v] value
13    pq = AdaptableHeapPriorityQueue()      # vertex v will have key
        d[v]
14    pqlocator = {}                        # map from vertex to its pq
        locator
15
16    # for each vertex v of the graph, add an entry to the priority queue, with
17    # the source having distance 0 and all others having infinite distance
18    for v in g.vertices():
19        if v is src:
20            d[v] = 0
21        else:
22            d[v] = float('inf')           # syntax for positive
        infinity
23    pqlocator[v] = pq.add(d[v], v)         # save locator for future
        updates
```

```

24
25 while not pq.is_empty():
26     key, u = pq.remove_min()
27     cloud[u] = key                                # its correct d[u] value
28     del pqlocator[u]                             # u is no longer in pq
29     for e in g.incident_edges(u):                 # outgoing edges (u,v)
30         v = e.opposite(u)
31         if v not in cloud:
32             # perform relaxation step on edge (u,v)
33             wgt = e.element()
34             if d[u] + wgt < d[v]:                 # better path to v?
35                 d[v] = d[u] + wgt                # update the distance
36                 pq.update(pqlocator[v], d[v], v) # update the pq entry
37
38 return cloud                                     # only includes reachable
    vertices
39
40 def shortest_path_tree(g, s, d):
41     """Reconstruct shortest-path tree rooted at vertex s, given distance map
42         d.
43
44     Return tree as a map from each reachable vertex v (other than s) to the
45     edge e=(u,v) that is used to reach v from its parent u in the tree.
46     """
47     tree = {}
48     for v in d:
49         if v is not s:
50             for e in g.incident_edges(v, False): # consider INCOMING edges
51                 u = e.opposite(v)
52                 wgt = e.element()
53                 if d[v] == d[u] + wgt:
54                     tree[v] = e                 # edge e is used to reach v
55     return tree

```

Capitolo 21

Minimum Spanning Tree

Listing 21.1: Struttura dati Partition (Union-Find): gestione di insiemi disgiunti con compressione del cammino e unione per dimensione.

```
1
2 class Partition:
3     """Union-find structure for maintaining disjoint sets."""
4
5     #----- nested Position class -----
6     class Position:
7         __slots__ = '_container', '_element', '_size', '_parent'
8
9         def __init__(self, container, e):
10             """Create a new position that is the leader of its own group."""
11             self._container = container          # reference to Partition instance
12             self._element = e
13             self._size = 1
14             self._parent = self                  # convention for a group leader
15
16         def element(self):
17             """Return element stored at this position."""
18             return self._element
19
20     #----- nonpublic utility -----
21     def _validate(self, p):
22         if not isinstance(p, self.Position):
23             raise TypeError('p must be proper Position type')
24         if p._container is not self:
25             raise ValueError('p does not belong to this container')
```

```

26
27 #----- public Partition methods
    -----
28 def make_group(self, e):
29     """Makes a new group containing element e, and returns its Position."""
30     return self.Position(self, e)
31
32 def find(self, p):
33     """Finds the group containing p and return the position of its
        leader."""
34     self._validate(p)
35     if p._parent != p:
36         p._parent = self.find(p._parent)    # overwrite p._parent after
            recursion
37     return p._parent
38
39 def union(self, p, q):
40     """Merges the groups containing elements p and q (if distinct)."""
41     a = self.find(p)
42     b = self.find(q)
43     if a is not b:                        # only merge if different groups
44         if a._size > b._size:
45             b._parent = a
46             a._size += b._size
47         else:
48             a._parent = b
49             b._size += a._size

```

Listing 21.2: Algoritmo di Prim: espansione del cluster tramite coda di priorità adattabile.

```

0
1
2 from ..priority_queue.heap_priority_queue import HeapPriorityQueue
3 from ..priority_queue.adaptable_heap_priority_queue import
  AdaptableHeapPriorityQueue
4 from .partition import Partition
5
6 def MST_Prim(g):
7     """Compute a minimum spanning tree of weighted graph g.
8
9     Return a list of edges that comprise the MST (in arbitrary order).
10    """
11    d = {}                # d[v] is bound on distance to tree
12    tree = []             # list of edges in spanning tree
13    pq = AdaptableHeapPriorityQueue() # d[v] maps to value (v, e=(u,v))
14    pqlocator = {}        # map from vertex to its pq locator
15
16    # for each vertex v of the graph, add an entry to the priority queue, with
17    # the source having distance 0 and all others having infinite distance
18    for v in g.vertices():
19        if len(d) == 0:    # this is the first node
20            d[v] = 0      # make it the root
21        else:
22            d[v] = float('inf') # positive infinity
23        pqlocator[v] = pq.add(d[v], (v, None))
24
25    while not pq.is_empty():
26        key, value = pq.remove_min()
27        u, edge = value    # unpack tuple from pq
28        del pqlocator[u]   # u is no longer in pq
29        if edge is not None:
30            tree.append(edge) # add edge to tree
31        for link in g.incident_edges(u):
32            v = link.opposite(u)
33            if v in pqlocator: # thus v not yet in tree
34                # see if edge (u,v) better connects v to the growing tree
35                wgt = link.element()
36                if wgt < d[v]: # better edge to v?
37                    d[v] = wgt # update the distance
38                    pq.update(pqlocator[v], d[v], (v, link)) # update the pq entry
39
40    return tree

```

Listing 21.3: Algoritmo di Kruskal: approccio greedy basato sull'ordinamento degli archi e gestione dei cicli tramite Partition.

```
41
42 def MST_Kruskal(g):
43     """Compute a minimum spanning tree of a graph using Kruskal's algorithm.
44
45     Return a list of edges that comprise the MST.
46
47     The elements of the graph's edges are assumed to be weights.
48     """
49     tree = []                # list of edges in spanning tree
50     pq = HeapPriorityQueue() # entries are edges in G, with weights as key
51     forest = Partition()     # keeps track of forest clusters
52     position = {}           # map each node to its Partition entry
53
54     for v in g.vertices():
55         position[v] = forest.make_group(v)
56
57     for e in g.edges():
58         pq.add(e.element(), e) # edge's element is assumed to be its weight
59
60     size = g.vertex_count()
61     while len(tree) != size - 1 and not pq.is_empty():
62         # tree not spanning and unprocessed edges remain
63         weight, edge = pq.remove_min()
64         u, v = edge.endpoints()
65         a = forest.find(position[u])
66         b = forest.find(position[v])
67         if a != b:
68             tree.append(edge)
69             forest.union(a, b)
70
71     return tree
```