EE6221 Final

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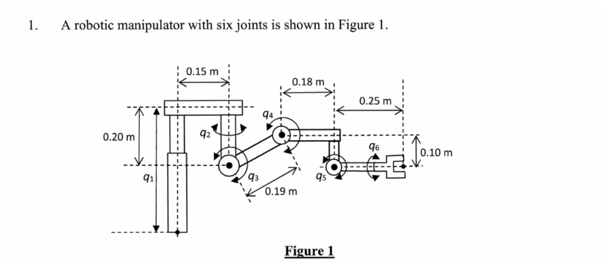
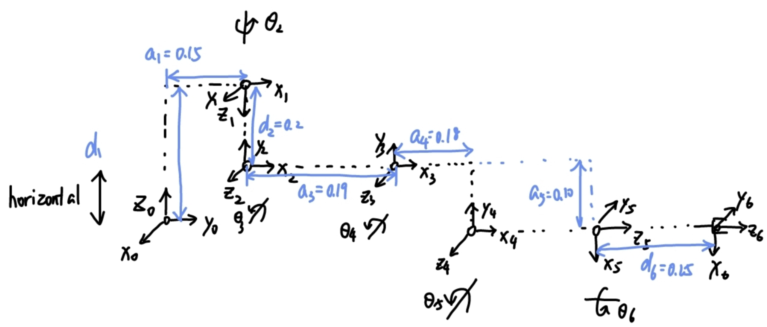
1 2024-2025 Term 1 Problem 1



1. A robotic manipulator with six joints is shown in Figure 1.



	θ	d	a	α
1	$\pi/2$	q_1	0.15	π
2	q_2	0.2	0	0
3	q_3	0	0.19	0
4	q_4	0	0.18	0
5	q_5	0	0.10	$-\pi/2$
6	q_6	0.15	0	0

Figure 1: 2024-2025 Problem 1 (a)

2 2024-2025 Term 1 Problem 2

The whole controller:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 5q_2^2+1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{M(\theta)\ddot{\theta}} + \underbrace{\begin{bmatrix} 10q_2\dot{q}_1\dot{q}_2 + 3\dot{q}_1 \\ -51_2\dot{q}_1^2 + 5\dot{q}_2 \\ 2\dot{q}_3 \end{bmatrix}}_{C(\theta,\dot{\theta})\dot{\theta}} + \underbrace{\begin{bmatrix} 47.5q_2\cos(q_1) \\ 47.5\sin(q_1) \\ 0 \end{bmatrix}}_{g(\theta)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 22(q_3-0.2) \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{d}$$

For motion control,

$$\xi_m = 1.05, \quad w_n = 0.5 \times \min w_{res} = 0.5 \times 5 = 2.5\text{rad/s}$$

$$k_v = 2\xi_m w_n = 5.25, \quad k_p = w_n^2 = 6.25$$
$$v_1 = \ddot{q}_1 + 5.25\dot{e}_1 + 6.25e_1$$
$$v_2 = \ddot{q}_2 + 5.25\dot{e}_2 + 6.25e_2$$

For force control,

$$\xi_f = 1, \quad w_n = 0.5 \times \min w_{res} = 0.5 \times 5 = 2.5\text{rad/s}$$
$$k_v = 2\xi_m w_n = 5, \quad k_p = w_n^2 = 6.25$$
$$v_3 = \frac{1}{22}(\ddot{q}_3 + 5\dot{e}_3 + 6.25e_3)$$

To be more detailed, take u_1 as an example:

$$u_1 = \alpha_1 v_1 + \beta_1$$

where

$$\alpha_1 = 5_{q_2^2} + 1$$
$$\beta_1 = 10q_2\dot{q}_1\dot{q}_2 + 3\dot{q}_1 + 47.5q_2\cos(q_1) + d_1$$

Let $e_1 = q_{1d} - q_1$, we have $\ddot{e_1} = \ddot{q}_{1d} - \ddot{q}_1$.

Control Law:

$$v_1 = \ddot{q}_{1d} + k_{v1}\dot{e}_1 + k_{p1}e_1$$

Substitute $\ddot{e_1} = \ddot{q}_{1d} - \ddot{q}_1$ into

$$\ddot{q}_1 = v_1 = \ddot{q}_{1d} + k_{v1}\dot{e}_1 + k_{p1}e_1$$

We get error equation

$$\ddot{e}_1 + k_{v1}\dot{e}_1 + k_{p1}e_1 = 0$$

Use Laplace Transformation to error equation, we can get the closed-loop characteristics equation

$$s^2 + k_{v1}\dot{e}_1 + k_{p1}e_1 = 0$$

The characteristic equation of a second order system can be expressed in the standard form:

$$s^2 + 2\xi w_n \dot{e} + w_n^2 e = 0$$

Hence,

$$k_{v1} = 2\xi w_n, \quad k_{p1} = w_n^2$$

So, for u_1, the controller is

$$u_1 = (5_{q2}^{\ 2} + 1)v_1 + (10q_2\dot{q}_1\dot{q}_2 + 3\dot{q}_1 + 47.5q_2\cos(q_1) + d_1)$$

the control law is

$$v_1 = \ddot{q}_{1d} + k_{v1}\dot{e}_1 + k_{p1}e_1$$

where $k_{v1} = 2\xi w_n, k_{p1} = w_n^2$.

3 2024-2025 Term 1 Problem 3(a)

$$J(\beta) = [\sin(\alpha+\beta), -\cos(\alpha+\beta), -l\cos(\beta)]$$
$$C(\beta) = [\cos(\alpha+\beta), \sin(\alpha+\beta), l\sin(\beta)]$$

Standard Wheel & Steered Standard Wheel:

Rolling Constraint:

$$J(\beta)R(\theta)\dot{\xi}_I - r_{ss}\dot{\phi}_{ss} = 0$$

Sliding Constraint:

$$C(\beta)R(\theta)\dot{\xi}_I = 0$$

Caster Wheel:

Rolling Constraint:

$$J(\beta)R(\theta)\dot{\xi}_I - r_{ss}\dot{\phi}_{ss} = 0$$

Sliding Constraint:

$$C(\beta)R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

4 2024-2025 Term 1 Problem 3(b)

(i) Derive the tool configuration Jacobian matrix of the manipulator.

$$w = \begin{bmatrix} S_1q_2 + 0.25S_1C_3 \\ C_1q_2 + 0.25C_1C_3 \\ -0.25S_3 + 0.25 \\ \exp(q_4/\pi)(-S_1C_3) \\ \exp(q_4/\pi)(C_1C_3) \\ \exp(q_4/\pi)(-S_3) \end{bmatrix}$$

The column i of tool configuration Jacobian matrix of the manipulator is

$$v^i(q) = \frac{\partial w}{\partial q_i}$$

Then we have

$$v^1(q) = \begin{bmatrix} C_1 q_2 + 0.25 C_1 C_3 \\ -S_1 q_2 - 0.25 S_1 C_3 \\ 0 \\ \exp(q_4/\pi)(-C_1 C_3) \\ \exp(q_4/\pi)(-S_1 C_3) \\ 0 \end{bmatrix} \quad v^2(q) = \begin{bmatrix} S_1 \\ C_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

$$v^3(q) = \begin{bmatrix} -0.25 S_1 S_3 \\ -0.25 C_1 S_3 \\ -0.25 C_3 \\ \exp(q_4/\pi)(S_1 S_3) \\ \exp(q_4/\pi)(-C_1 S_3) \\ \exp(q_4/\pi)(-C_3) \end{bmatrix} \quad v^4(q) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \exp(q_4/\pi)(-S_1 C_3)/\pi \\ \exp(q_4/\pi)(C_1 C_3)/\pi \\ \exp(q_4/\pi)(-S_3)/\pi \end{bmatrix} \tag{2}$$

(ii) Obtain q_1, q_2, q_3.

$$\begin{cases} x = 0.2 = S_1 q_2 + 0.25 S_1 C_3 \\ y = 0.2 - C_1 q_2 + 0.25 C_1 C_3 \\ z = 0 = -0.25 S_3 + 0.25 \end{cases} \Longrightarrow \begin{cases} q_1 = \pi/4 \\ q_2 = \sqrt{2}/5 \\ q_3 = \pi/2 \end{cases}$$

(iii) Determine the approach vector of the robot at this joint configuration.

$$r^3 = \begin{bmatrix} -S_1 C_3 \\ C_1 C_3 \\ -S_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

5 2024-2025 Term 1 Problem 4

(a) Choose one group of feature points to derive G_n

To obtain homography matrix, we need to choose at least 4 coplane and noncolinear common feature points which can be detected by both F_p and F_q.

Here, we choose O_1, O_2, O_5, O_6.

$$m_{ip} = [p_{ix}, p_{iy}, 1]^T, m_{iq} = [q_{ix}, q_{iy}, 1]^T, i = 1, 2, 5, 6$$

$$G = \begin{pmatrix} g_1 & g_2 & g_3 \\ g_4 & g_5 & g_6 \\ g_7 & g_8 & g_9 \end{pmatrix} \implies g = [g_1, g_2, ..., g_9]^T$$

We can obtain G by solving $M_p G = m_q$, where

$$M_{pj} = \begin{pmatrix} p_{jx} & p_{jy} & 1 & 0 & 0 & 0 & -p_{jx}q_{jx} & -p_{jy}q_{jx} \\ 0 & 0 & 0 & p_{jx} & p_{jy} & 1 & -p_{jx}q_{jy} & -p_{jy}q_{jy} \end{pmatrix}$$

and M_p is obtained by concatenating M_{pj}.

(b) How many groups of four feature points can be used to derive H_n?

$$(\frac{4 \times 3}{2})^2 + 2 \times 4 \times 4 = 68$$

$(\frac{4\times3}{2})^2$: Choose 2 points from O_5, O_6, O_7, O_8 and 2 points from $O_9, O_{10}, O_{11}, O_{12}$.

$2 \times 4 \times 4$: Choose 3 points from one column and 1 point from the other for column O_5, O_6, O_7, O_8 and $O_9, O_{10}, O_{11}, O_{12}$.

(c) Derive J_n

Not sure.

Answer 1: cannot solve because all the common feature points of F_p and F_d are colinear.

Answer 2: $J = H * G$

6 2024-2025 Term 1 Problem 5

State-space model:

$$x_{k+1} = x_k$$

Measurement model:

$$y_{1k} = x_k + v_{1k}, v_{1k} \sim N(0, a^2)$$

$$y_{2k} = x_k + v_{2k}, v_{2k} \sim N(0, 4a^2)$$

$$y_{3k} = x_k + v_{3k}, v_{3k} \sim N(0, 9a^2)$$

$$R_k = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & 4a^2 & 0 \\ 0 & 0 & 9a^2 \end{pmatrix}$$

Measurement model can be written as

$$z_k = H_k x_k + v_k$$

where

$$z_k = [y_{1k}, y_{2k}, y_{3k}]^T$$
$$H_k = [1, 1, 1]^T$$
$$v_k = [v_{1k}, v_{2k}, v_{3k}]^T$$

Given:

$$\tilde{x}_k = x_k - \hat{x}_k, \tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$$
$$\mathbb{E}[\tilde{x}_k] = 0, \mathbb{E}[\tilde{x}_{k+1}] = 0$$
$$p_k = \mathbb{E}[\tilde{x}_k^2], p_{k+1} = \mathbb{E}[\tilde{x}_{k+1}^2]$$

(a) Design an estimation strategy as follows:

$$\hat{x}_{k+1} = \hat{x}_k + L_{1k}(y_{1k} - \hat{x}_k) + L_{2k}(y_{2k} - \hat{x}_k) + L_{3k}(y_{3k} - \hat{x}_k)$$

where L_{1k}, L_{2k}, L_{3k} represents Kalman gains.

Since $y_{ik} = x_k + v_{ik}$, we have $y_{ik} - \hat{x}_k = x_k - \hat{x}_k + v_{ik} = \tilde{x}_k + v_{ik}$.

Since $x_{k+1} = x_k$, we have $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1} = x_k - \hat{x}_{k+1}$.

Therefore,

$$\begin{aligned}
\hat{x}_{k+1} &= \hat{x}_k + L_{1k}(y_{1k} - \hat{x}_k) + L_{2k}(y_{2k} - \hat{x}_k) + L_{3k}(y_{3k} - \hat{x}_k) \\
&= \hat{x}_k + (L_{1k} + L_{2k} + L_{3k})\tilde{x}_k + L_{1k}v_{1k} + L_{2k}v_{2k} + L_{3k}v_{3k} \\
\tilde{x}_{k+1} &= x_k - \hat{x}_{k+1} \\
&= (1 - L_{1k} - L_{2k} - L_{3k})\tilde{x}_k - (L_{1k}v_{1k} + L_{2k}v_{2k} + L_{3k}v_{3k})
\end{aligned}$$

$$\begin{aligned}
p_{k+1} &= \mathbb{E}[\tilde{x}_{k+1}^2] \\
&= (1 - L_{1k} - L_{2k} - L_{3k})^2\mathbb{E}[\tilde{x}_k^2] + L_{1k}^2\mathbb{E}[v_{1k}^2] + L_{2k}^2\mathbb{E}[v_{2k}^2] + L_{3k}^2\mathbb{E}[v_{3k}^2] \\
&= (1 - L_{1k} - L_{2k} - L_{3k})^2 p_k + L_{1k}^2 \cdot a^2 + L_{2k}^2 \cdot 4a^2 + L_{3k}^2 \cdot 9a^2
\end{aligned}$$

(Use $\mathbb{E}[\tilde{x}_k] = 0, \mathbb{E}[v_{ik}] = 0$ and uncorrelated)

Set the partial derivatives to be 0, we can get the update law:

$$\begin{cases} \dfrac{\partial p_{k+1}}{\partial L_{1k}} = 0 \\ \dfrac{\partial p_{k+1}}{\partial L_{2k}} = 0 \\ \dfrac{\partial p_{k+1}}{\partial L_{3k}} = 0 \end{cases} \implies \begin{cases} L_{1k} = \dfrac{36p_k}{36a^2 + 49p_k} \\ L_{2k} = \dfrac{9p_k}{36a^2 + 49p_k} \\ L_{3k} = \dfrac{4p_k}{36a^2 + 49p_k} \end{cases}, p_{k+1} = \dfrac{36a^2 p_k}{36a^2 + 49p_k}$$

Update law:

$$L_{1k} = \frac{36p_k}{36a^2 + 49p_k}$$
$$L_{2k} = \frac{9p_k}{36a^2 + 49p_k}$$
$$L_{3k} = \frac{4p_k}{36a^2 + 49p_k}$$
$$p_{k+1} = \frac{36a^2 p_k}{36a^2 + 49p_k}$$

(b) Design an estimation strategy using one Kalman gain for the three sensors.

Estimation Strategy:

$$\hat{x}_{k+1} = \hat{x}_k + L_{ck}(y_{1k} + y_{2k} + y_{3k} - 3\hat{x}_k)$$

$$\begin{aligned}\tilde{x}_{k+1} &= \tilde{x}_k - L_{ck}(3\tilde{x}_k + v_{1k} + +v_{2k} + v_{3k}) \\ &= (1 - 3L_{ck})\tilde{x}_k - L_{ck}(v_{1k} + +v_{2k} + v_{3k})\end{aligned}$$

$$p_{k+1} = \mathbb{E}[\tilde{x}_{k+1}^2] = (1 - 3L_{ck})p_k + L_{ck}^2 \cdot 14a^2 \tag{3}$$

$$\frac{\partial p_{k+1}}{\partial L_{ck}} = 0 \implies L_{ck} = \frac{3p_k}{9p_k + 14a^2}, p_{k+1} = \frac{14a^2 p_k}{9p_k + 14a^2} \tag{4}$$

Comparison:

L_{1k}, L_{2k}, L_{3k}: Flexible. Computationally complex.

L_{ck}: Easy to compute. Less flexible.