NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2024-2025

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

November / December 2024 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with six joints is shown in Figure 1.

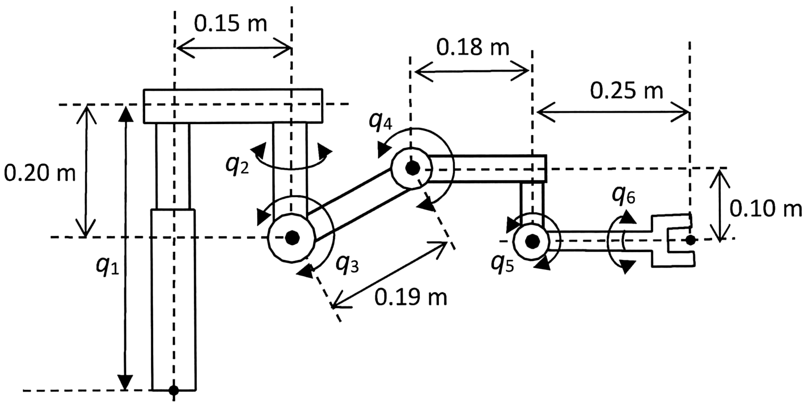

Figure 1

(a) Obtain the link coordinate diagram by using the Denavit-Hartenberg (D-H) algorithm.

(12 Marks)

Note: Question No. 1 continues on page 2.

(b) Derive the kinematic parameters of the robot based on the coordinate diagram obtained in part (a).

(8 Marks)

2. The dynamic equations of a robot when it is in contact with a workpiece are given as follows:

$$(5q_2^2+1)\,\ddot{q}_1+10q_2\dot{q}_1\dot{q}_2+3\dot{q}_1+47.5\,q_2\,cos(q_1)+d_1=u_1$$

$$5\ddot{q}_2-5q_2\dot{q}_1^2+5\dot{q}_2+47.5\,sin(q_1)+d_2=u_2$$

$$3\ddot{q}_3+2\dot{q}_3+29.4+d_3+f=u_3$$

where u_1, u_2, u_3 are the control inputs, q_1, q_2, q_3 are the joint variables, d_1, d_2, d_3 are the unknown disturbances and $f=22(q_3-0.2)$ is the contact force. The system possesses unmodelled resonances at 5 rad/s, 10 rad/s and 15 rad/s.

(a) If d_1, d_2, d_3 are zero, design a hybrid position and force controller for the robot to track the desired trajectories q_{1d}, q_{2d} and the desired force f_d. The motion control subspace should be overdamped with a damping ratio of 1.05, and the force control subspace should be critically damped. The control system should not excite all the unmodelled resonances.

(14 Marks)

(b) If d_1, d_2, d_3 are not zero, derive the error equations of the system based on the hybrid position and force controller designed in part (a).

(6 Marks)

3. (a) A mobile platform is shown in Figure 2 The platform has one steered standard wheel and two standard wheels. A local reference frame (x_r, y_r) is assigned as shown in the figure. The radius of each standard wheel is 10 cm. If the rotational velocities of the steered standard wheel and the two standard wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_{s1}$, $\dot{\varphi}_{s2}$ respectively, derive the rolling and sliding constraints of the mobile platform.

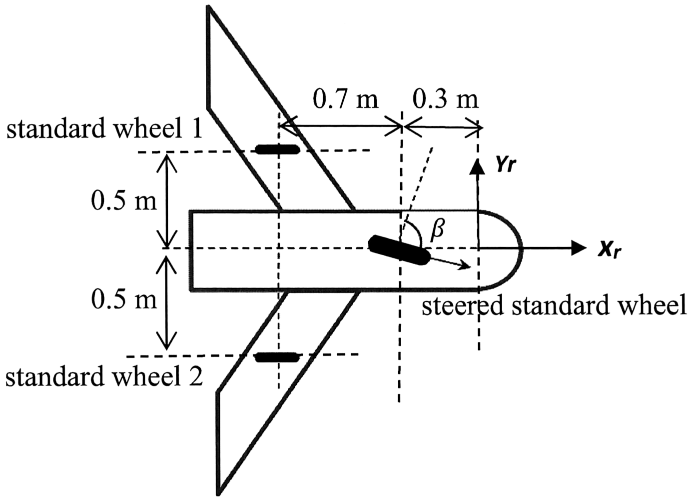

Figure 2

(8 Marks)

(b) A robot manipulator with four joint variables is mounted on a mobile platform. The transformation matrix from tool tip to base coordinate of the robot is given as:

$$T_{base}^{tool} = \begin{bmatrix} -S_1S_3C_4 - C_1S_4 & -S_1S_3S_4 - C_1C_4 & -S_1C_3 & S_1q_2 + 0.25\,S_1C_3 \\ C_1S_3C_4 - S_1S_4 & -C_1C_3S_4 - S_1C_4 & C_1C_3 & C_1q_2 + 0.25\,C_1C_3 \\ -C_3C_4 & -C_3S_4 & -S_3 & -0.25S_3 + 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where q_1, q_2, q_3 are the joint variables for the major axes, q_4 is the tool roll angle, $C_k = cos\, q_k$ and $S_k = sin\, q_k$.

(i) Derive the tool configuration Jacobian matrix of the manipulator.

(ii) Given that $x = 0.2$, $y = 0.2$, $z = 0$, solve the inverse kinematic problem to obtain q_1, q_2, q_3. (Note: orientation is not required).

(iii) Determine the approach vector of the robot at this joint configuration.

(12 Marks)

4. As shown in Figure 3, a moving camera takes three images of the same object at three poses. Three coordinate frames represented by F_p, F_q, and F_d are attached to the projection centre of the camera at the three poses, respectively. Twelve feature points O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10}, O_{11}, and O_{12} are on the same plane, with 7 groups of points on the same line, respectively, as shown in the figure. Let G, H, and J denote the Euclidean homography matrix from F_p to F_q, F_q to F_d, and F_p to F_d, respectively. Their scaled homography matrices are given by G_n, H_n, and J_n, respectively, where scaled homography matrix is defined as the corresponding Euclidean homography matrix divided by its third row third column element.

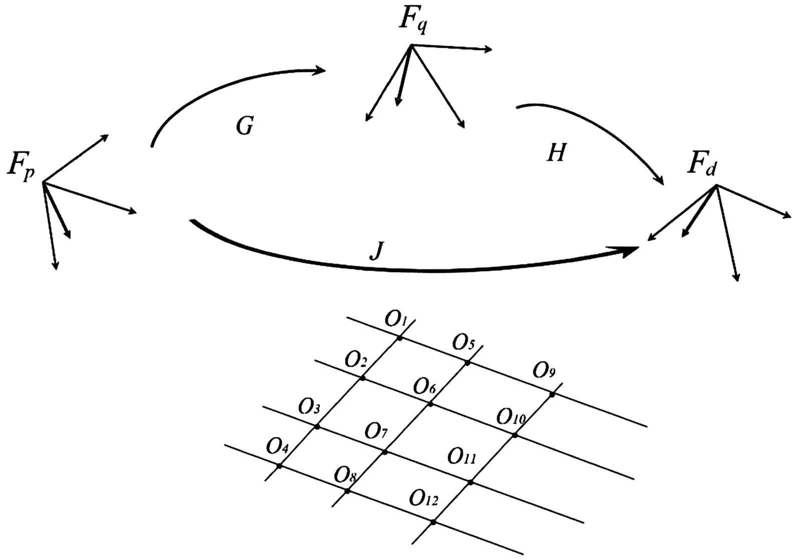

Figure 3

Eight feature points O_1, O_2, O_3, O_4, O_5, O_6, O_7, and O_8 can be detected in the image taken at the pose attached to F_p. Their corresponding normalized coordinates in F_p are given by $m_{ip} = \left[p_{ix}, p_{iy}, 1\right]^T$ with $i = 1,2,3,4,5,6,7,8$, respectively.

All the 12 feature points can be detected in the image taken at the pose attached to F_q. Their corresponding normalized coordinates in F_q are given by $m_{iq} = \left[q_{ix}, q_{iy}, 1\right]^T$ with $i = 1,2,3,4,5,6,7,8,9,10,11,12$, respectively.

Eight feature points O_5, O_6, O_7, O_8, O_9, O_{10}, O_{11}, and O_{12} can be detected in the image taken at the pose attached to F_d. Their corresponding normalized coordinates in F_d are given by $m_{id} = \left[d_{ix}, d_{iy}, 1\right]^T$ with $i = 5,6,7,8,9,10,11,12$, respectively.

Note: Question No. 4 continues on page 5.

(a) Choose one group of feature points and derive G_n.

(7 Marks)

(b) How many groups of four feature points can be used to derive H_n ?

(6 Marks)

(c) Derive J_n.

(7 Marks)

5. Three sensors are used to measure a state variable x_k that can be modelled by $x_{k+1} = x_k$. The measurements of the three sensors are given by y_{1k}, y_{2k}, and y_{3k}, respectively, which are governed by the models $y_{1k} = x_k + v_{1k}$, $y_{2k} = x_k + v_{2k}$, and $y_{3k} = x_k + v_{3k}$ where v_{1k}, v_{2k}, and v_{3k} are zero mean Gaussian sensor noises with variance given by a^2, $4a^2$, and $9a^2$, respectively.

Let $\hat{x}_k$ represent the estimate of x_k and $\hat{x}_{k+1}$ represent the estimate of x_{k+1}. Let the estimation errors be $\tilde{x}_k = x_k - \hat{x}_k$ and $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error $\tilde{x}_k$ and the noise terms v_{1k}, v_{2k}, and v_{3k} are uncorrelated. Assume that $E\left[\tilde{x}_k\right] = 0$ and $E\left[\tilde{x}_{k+1}\right] = 0$. Let the estimation error variances be $p_k = E\left[\tilde{x}_k^2\right]$ and $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

(a) Design an estimation strategy as follows:

$$\hat{x}_{k+1} = \hat{x}_k + L_{1k}\left(y_{1k} - \hat{x}_k\right) + L_{2k}\left(y_{2k} - \hat{x}_k\right) + L_{3k}\left(y_{3k} - \hat{x}_k\right)$$

where L_{1k}, L_{2k}, and L_{3k} represent Kalman gains. Design the update laws for the Kalman gains to minimize the estimation error variance.

(10 Marks)

(b) Design an estimation strategy using one Kalman gain for the three sensors. Compare this estimation strategy to the estimation strategy in part (a) and discuss the advantages and disadvantages of each strategy. Justify your answer.

(10 Marks)

END OF PAPER

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1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.