1 Probability and Distributions

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

$$\binom{-x}{r} = (-1)^r \binom{x+r-1}{r}$$

$$\binom{n}{r_1, r_2, \dots, r_k} = \binom{n}{r_1} \dots \binom{n - r_1 - \dots - r_{k-1}}{r_k}$$

$$\binom{x}{r} = \frac{x(x-1)\dots(x-r+1)}{r!}$$

$$\binom{a}{0}\binom{b}{n} + \binom{a}{1}\binom{b}{n-1} + \dots + \binom{a}{n}\binom{b}{0} = \binom{a+b}{n}$$

$$\text{Bernoulli}(p) = p^x(1-p)^{1-x}$$

$$p \mid p(1-p) \mid 1-p+pe^t$$

$$\text{Binomial}(n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$np \mid np(1-p) \mid (1-p+pe^t)^n$$

$$\text{Poisson}(\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \mid \lambda \mid \lambda \mid e^{\lambda(e^t-1)}$$

$$\text{Geometric}(p) = (1-p)^{x-1}p$$

$$\frac{1}{p} \mid \frac{1-p}{p^2} \mid \frac{pe^t}{1-(1-p)e^t}$$

$$\text{Uniform}(a,b) = \frac{1}{b-a}$$

$$\frac{a+b}{2} \mid \frac{(b-a)^2}{12} \mid \frac{e^{tb}-e^{ta}}{t(b-a)}$$

$$\text{Normal}(\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\text{E}(Z^{2r+1}) = 0 \mid \text{E}(Z^{2r}) = \frac{(2r)!}{(2^r r!)}$$

$$\mu \mid \sigma^2 \mid e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$\text{Gamma}(\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$\frac{\alpha}{\beta} \mid \frac{\alpha}{\beta^2} \mid \left(\frac{\beta}{\beta-t}\right)^\alpha$$

$$\text{E}(\ln(X)) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} - \ln(\beta)$$

$$\theta = \frac{1}{\beta} \mid \sum \Gamma(\alpha_i, \beta) = \Gamma(\sum \alpha_i, \beta)$$

$$\Gamma(1,\beta) = \text{Exponential}(\beta) \mid \Gamma(\frac{n}{2}, \frac{1}{2}) = \chi^2_n$$

$$Y \sim \text{Gamma}(\alpha,\beta), cY \sim \text{Gamma}(\alpha, \frac{\beta}{c})$$

$$\text{Beta}(\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

$$\frac{\alpha}{\alpha+\beta} \mid \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \mid \varnothing$$

$$\text{B}(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$\text{Beta}(1,1) = \text{U}[0,1] \mid \mu_r' = \text{E}(X^r)$$

$$\text{ZIP}(\phi,\lambda) := ZX | (1-\phi)\lambda | (1-\phi)\lambda(1+\phi\lambda)$$

$$Z = \text{Bernoulli}(1-\phi), X = \text{Poisson}(\lambda)$$

$$\text{M}_{\text{ZIP}}(t) = \phi + (1-\phi)e^{\lambda(e^t-1)}$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$$

$$X_1 | X_2 \mid \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(x_2 - \mu_2) \mid \sigma_1^2(1-\rho^2)$$

$$\text{Skewness} = \frac{\mu_3}{\sigma^3} \mid \text{Kurtosis} = \frac{\mu_4}{\sigma^4}$$

$$\text{P}(g(X) \ge c) \le \frac{\text{E}(g(x))}{C}$$

$$\text{P}(|X-\mu| \ge c\sigma) \le \frac{1}{c^2}$$

$$\text{P}(|X-\mu| \ge c) \le \frac{\text{E}(|X|^r)}{c^r}$$

$$\text{E}(XY)^2 \le \text{E}(X^2)\text{E}(Y^2)$$

$$\text{E}(g(X)) \ge g(\text{E}(X)), \forall \text{凸函数 } g(x^2)$$

$$\text{E}(X) = \text{E}[\text{E}(X|Y)]$$

$$\text{Var}(X) = \text{E}[\text{Var}(X|Y)] + \text{Var}[\text{E}(X|Y)]$$

$$\text{q 分位数}\xi_q := \text{F}(\xi_q) = \text{P}(X \le \xi_q) \ge q$$

$$\text{pgf} := \text{G}_X(z) = \text{E}(z^X) = \sum_{x \in \mathbb{S}} z^x \text{P}_X(X = x)$$

Additivity： N, Poisson, NB, Binomial

$$IBF := f_X(x) \propto \frac{f_{X|Y}(x|y_0)}{f_{Y|X}(y_0|x)}$$

2 Sampling Distributions

$$g(y_1,y_2) = f(x_1,x_2) \times |\mathbf{J}|$$

$$\mathbf{J} = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}$$

$$\overline{X} = \frac{1}{n}\sum X_i \mid S^2 = \frac{1}{n-1}\sum (X_i - \overline{X})^2$$

$$\overline{X} \sim \text{N}(\mu, \frac{\sigma^2}{n}) \mid \overline{X} \perp S^2$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \mid \sum Z_i^2 \sim \chi^2(n)$$

$$T = \frac{Z}{\sqrt{\frac{Y}{n}}} \sim \text{t}(n), Y \sim \chi^2(n)$$

$$T = \frac{\sqrt{n}(\overline{X}-\mu)}{S} \sim \text{t}(n-1)$$

$$W = \frac{\frac{U}{M}}{\frac{V}{n}} \sim \text{F}(m,n), U \sim \chi^2(m), V \sim \chi^2(n)$$

$$\text{F}(X_{(n)}) = [F(x)]^n$$

$$f_{X_{(r)}} = \frac{n!}{(r-1)!(n-r)!} f(x)[F(x)]^{r-1}[1-F(x)]^{n-r}$$

$$\frac{\sqrt{n}(\overline{X}_n - \mu)}{\sigma} \xrightarrow{L} Z \sim N(0,1), \forall \text{随机变量} X$$

3 Point Estimation

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$$

$$\ell(\theta) = \ln L(\theta) = \sum_{i=1}^{n} \ln f(x_i;\theta)$$

$$h(\theta) = h(\hat{\theta}), \forall \text{MLE } \hat{\theta}$$

$$\frac{1}{n}\sum X_i^2 - \overline{X}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

$$\hat{\mu}_k = \text{E}(X^k)$$

$$p(\theta|x) = \frac{f(x|\theta)\pi(\theta)}{f(x)} \propto f(x|\theta) \times \pi(\theta)$$

$$\text{Unbiasedness: E}(\hat{\theta}) = \theta \;/\; \text{Bias}(\hat{\theta}) = \text{E}(\hat{\theta}) - \theta$$

$$\text{MSE}(\hat{\theta}) = \text{E}[(\hat{\theta}-\theta)^2] = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$$

$$\text{Fisher Information: } I_n(\theta) = E\left[-\frac{\partial^2 \log L(\theta)}{\partial\theta^2}\right]$$

$$\text{CRLB: } Var(\hat{\theta}) \ge \frac{1}{I_n(\theta)}$$

$$\text{Sufficiency: } f(x;\theta) = g(T(x),\theta) \times h(x), \forall \text{Statistics T}$$

$$\hat{\theta}_n \xrightarrow{P} \theta, n \xrightarrow{\infty}$$

$$\sqrt{nI(\theta)}(\hat{\theta}_n - \theta) \xrightarrow{d} N(0,1) \text{ or } \hat{\theta}_n \dot{\sim} N\left(\theta, \frac{1}{I_n(\theta)}\right)$$