1 Probability and Distributions

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

$$\binom{-x}{r} = (-1)^r \binom{x+r-1}{r}$$

$$\binom{n}{r_1, r_2, \ldots, r_k} = \binom{n}{r_1} \ldots \binom{n - r_1 - \cdots - r_{k-1}}{r_k}$$

$$\binom{x}{r} = \frac{x(x-1)\ldots(x-r+1)}{r!}$$

$$\binom{a}{0}\binom{b}{n} + \binom{a}{1}\binom{b}{n-1} + \cdots + \binom{a}{n}\binom{b}{0} = \binom{a+b}{n}$$

$$\mathrm{Bernoulli}(p) = p^x (1-p)^{1-x}$$

$$p \mid p(1-p) \mid 1 - p + pe^t$$

$$\mathrm{Binomial}(n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$np \mid np(1-p) \mid (1 - p + pe^t)^n$$

$$\mathrm{Poisson}(\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \mid \lambda \mid \lambda \mid e^{\lambda(e^t - 1)}$$

$$\mathrm{Geometric}(p) = (1-p)^{x-1} p$$

$$\frac{1}{p} \mid \frac{1-p}{p^2} \mid \frac{pe^t}{1 - (1-p)e^t}$$

$$\mathrm{Uniform}(a, b) = \frac{1}{b-a}$$

$$\frac{a+b}{2} \mid \frac{(b-a)^2}{12} \mid \frac{e^{tb} - e^{ta}}{t(b-a)}$$

$$\mathrm{Normal}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathrm{E}(Z^{2r+1}) = 0 \mid \mathrm{E}(Z^{2r}) = \frac{(2r)!}{(2^r r!)}$$

$$\mu \mid \sigma^2 \mid e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$\mathrm{Gamma}(\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$\frac{\alpha}{\beta} \mid \frac{\alpha}{\beta^2} \mid \left(\frac{\beta}{\beta - t}\right)^\alpha$$

$$\mathrm{E}(\ln(X)) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} - \ln(\beta)$$

$$\theta = \frac{1}{\beta} \mid \sum \Gamma(\alpha_i, \beta) = \Gamma(\sum \alpha_i, \beta)$$

$$\Gamma(1, \beta) = \mathrm{Exponential}(\beta) \mid \Gamma(\frac{n}{2}, \frac{1}{2}) = \chi_n^2$$

$$\mathrm{Beta}(\alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$$

$$\frac{\alpha}{\alpha + \beta} \mid \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \mid \varnothing$$

$$\mathrm{B}(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$\mathrm{Beta}(1, 1) = \mathrm{U}[0, 1] \mid \mu_r' = \mathrm{E}(X^r)$$

$$\mathrm{ZIP}(\phi, \lambda) := ZX | (1-\phi)\lambda | (1-\phi)\lambda(1+\phi\lambda)$$

$$Z = \mathrm{Bernoulli}(1-\phi), X = \mathrm{Poisson}(\lambda)$$

$$\mathrm{M}_{\mathrm{ZIP}}(t) = \phi + (1-\phi)e^{\lambda(e^t - 1)}$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$$

$$X_1 | X_2 \mid \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(x_2 - \mu_2) \mid \sigma_1^2(1-\rho^2)$$

$$\mathrm{Skewness} = \frac{\mu_3}{\sigma^3} \mid \mathrm{Kurtosis} = \frac{\mu_4}{\sigma^4}$$

$$\mathrm{P}(g(X) \geq c) \leq \frac{\mathrm{E}(g(x))}{C}$$

$$\mathrm{P}(|X - \mu| \geq c\sigma) \leq \frac{1}{c^2}$$

$$\mathrm{P}(|X - \mu| \geq c) \leq \frac{\mathrm{E}(|X|^r)}{c^r}$$

$$\mathrm{E}(XY)^2 \leq \mathrm{E}(X^2)\mathrm{E}(Y^2)$$

$$\mathrm{E}(g(X)) \geq g(\mathrm{E}(X)), \forall \text{凸函数 } g(x^2)$$

$$\mathrm{E}(X) = \mathrm{E}[\mathrm{E}(X|Y)]$$

$$\mathrm{Var}(X) = \mathrm{E}[\mathrm{Var}(X|Y)] + \mathrm{Var}[\mathrm{E}(X|Y)]$$

$$\text{q 分位数} \xi_q := \mathrm{F}(\xi_q) = \mathrm{P}(X \leq \xi_q) \geq q$$

$$\mathrm{pgf} := \mathrm{G}_X(z) = \mathrm{E}(z^X) = \sum_{x \in \mathbb{S}} z^x \mathrm{P}_X(X = x)$$

$$\text{Additivity}: \text{ N, Poisson, NB, Binomial}$$

$$IBF := f_X(x) \propto \frac{f_{X|Y}(x|y_0)}{f_{Y|X}(y_0|x)}$$

2 Sampling Distributions

$$g(y_1, y_2) = f(x_1, x_2) \times |\mathbf{J}|$$

$$\mathbf{J} = \begin{matrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{matrix}$$

$$\overline{X} = \frac{1}{n}\sum X_i \mid S^2 = \frac{1}{n-1}\sum (X_i - \overline{X})^2$$

$$\overline{X} \sim \mathrm{N}(\mu, \frac{\sigma^2}{n}) \mid \overline{X} \perp S^2$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

$$T = \frac{Z}{\sqrt{\frac{Y}{n}}} \sim \mathrm{t}(n), Y \sim \mathrm{X}^2(n)$$

$$T = \frac{\sqrt{n}(\overline{X} - \mu)}{S} \sim \mathrm{t}(n-1)$$

$$W = \frac{\frac{U}{M}}{\frac{V}{n}} \sim \mathrm{F}(m, n), U \sim \mathrm{X}^2(m), V \sim \mathrm{X}^2(n)$$