1 Probability and Distributions

$$\binom{n}{r}=\binom{n-1}{r}+\binom{n-1}{r-1}$$

$$\binom{-x}{r}=(-1)^r\binom{x+r-1}{r}$$

$$\binom{n}{r_1,r_2,\dots,r_k}=\binom{n}{r_1}\dots\binom{n-r_1-\cdots-r_{k-1}}{r_k}$$

$$\binom{x}{r}=\frac{x(x-1)\dots(x-r+1)}{r!}$$

$$\binom{a}{0}\binom{b}{n}+\binom{a}{1}\binom{b}{n-1}+\cdots+\binom{a}{n}\binom{b}{0}=\binom{a+b}{n}$$

$$\mathrm{Bernoulli}(p)=p^x(1-p)^{1-x}$$

$$p \mid p(1-p) \mid 1-p+pe^t$$

$$\mathrm{Binomial}(n,p)=\binom{n}{x}p^x(1-p)^{n-x}$$

$$np \mid np(1-p) \mid (1-p+pe^t)^n$$

$$\mathrm{Poisson}(\lambda)=\frac{\lambda^x e^{-\lambda}}{x!} \mid \lambda \mid \lambda \mid e^{\lambda(e^t-1)}$$

$$\mathrm{Geometric}(p)=(1-p)^{x-1}p$$

$$\frac{1}{p} \mid \frac{1-p}{p^2} \mid \frac{pe^t}{1-(1-p)e^t}$$

$$\mathrm{Uniform}(a,b)=\frac{1}{b-a}$$

$$\frac{a+b}{2} \mid \frac{(b-a)^2}{12} \mid \frac{e^{tb}-e^{ta}}{t(b-a)}$$

$$\mathrm{Normal}(\mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathrm{E}(Z^{2r+1})=0 \mid \mathrm{E}(Z^{2r})=\frac{(2r)!}{(2^r r!)}$$

$$\mu \mid \sigma^2 \mid e^{\mu t+\frac{1}{2}\sigma^2t^2}$$

$$\mathrm{Gamma}(\alpha,\beta)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$$

$$\frac{\alpha}{\beta} \mid \frac{\alpha}{\beta^2} \mid \left(\frac{\beta}{\beta-t}\right)^\alpha$$

$$\mathrm{E}(\ln(X))=\frac{\Gamma'(\alpha)}{\Gamma(\alpha)}-\ln(\beta)$$

$$\theta=\frac{1}{\beta} \mid \sum\Gamma(\alpha_i,\beta)=\Gamma(\sum\alpha_i,\beta)$$

$$\Gamma(1,\beta)=\mathrm{Exponential}(\beta) \mid \Gamma(\frac{n}{2},\frac{1}{2})=\chi^2_n$$

$$\mathrm{Beta}(\alpha,\beta)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

$$\frac{\alpha}{\alpha+\beta} \mid \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \mid \varnothing$$

$$\mathrm{B}(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$\mathrm{Beta}(1,1)=\mathrm{U}[0,1] \mid \mu_r'=\mathrm{E}(X^r)$$

$$\mathrm{ZIP}(\phi,\lambda):=ZX|(1-\phi)\lambda|(1-\phi)\lambda(1+\phi\lambda)$$

$$Z=\mathrm{Bernoulli}(1-\phi),X=\mathrm{Poisson}(\lambda)$$

$$\mathrm{M_{ZIP}}(t)=\phi+(1-\phi)e^{\lambda(e^t-1)}$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2}-2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+\frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$$

$$X_1|X_2 \mid \mu_1+\rho\frac{\sigma_1}{\sigma_2}(x_2-\mu_2) \mid \sigma_1^2(1-\rho^2)$$

$$\mathrm{Skewness}=\frac{\mu_3}{\sigma^3} \mid \mathrm{Kurtosis}=\frac{\mu_4}{\sigma^4}$$

$$\mathrm{P}(g(X)\ge c)\le\frac{\mathrm{E}(g(x))}{C}$$

$$\mathrm{P}(|X-\mu|\ge c\sigma)\le\frac{1}{c^2}$$

$$\mathrm{P}(|X-\mu|\ge c)\le\frac{\mathrm{E}(|X|^r)}{c^r}$$

$$\mathrm{E}(XY)^2\le\mathrm{E}(X^2)\mathrm{E}(Y^2)$$

$$\mathrm{E}(g(X))\ge g(\mathrm{E}(X)),\forall\text{凸函数 g}(x^2)$$

$$\mathrm{E}(X)=\mathrm{E}[\mathrm{E}(X|Y)]$$

$$\mathrm{Var}(X)=\mathrm{E}[\mathrm{Var}(X|Y)]+\mathrm{Var}[\mathrm{E}(X|Y)]$$

$$\text{q 分位数}\xi_q:=\mathrm{F}(\xi_q)=\mathrm{P}(X\le\xi_q)\ge q$$

$$\mathrm{pgf}:=\mathrm{G}_X(z)=\mathrm{E}(z^X)=\sum_{x\in\mathbb{S}}z^x\mathrm{P}_X(X=x)$$

Additivity: N, Poisson, NB, Binomial

$$IBF:=f_X(x)\propto\frac{f_{X|Y}(x|y_0)}{f_{Y|X}(y_0|x)}$$

2 Sampling Distributions

$$g(y_1,y_2)=f(x_1,x_2)\times|\mathbf{J}|$$

$$\mathbf{J}=\begin{matrix}\frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2}\\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2}\end{matrix}$$

$$\overline{X}=\frac{1}{n}\sum X_i \mid S^2=\frac{1}{n-1}\sum(X_i-\overline{X})^2$$

$$\overline{X}\sim\mathrm{N}(\mu,\frac{\sigma^2}{n}) \mid \overline{X}\perp S^2$$

$$\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$$

$$T=\frac{Z}{\sqrt{\frac{Y}{n}}}\sim\mathrm{t}(n),Y\sim\mathrm{X}^2(n)$$

$$T=\frac{\sqrt{n}(\overline{X}-\mu)}{S}\sim\mathrm{t}(n-1)$$

$$W=\frac{\frac{U}{M}}{\frac{V}{n}}\sim\mathrm{F}(m,n),U\sim\mathrm{X}^2(m),V\sim\mathrm{X}^2(n)$$

$$F=\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}=\frac{\sigma_2^2S_1^2}{\sigma_1^2S_2^2}\sim F(n_1-1,n_2-1)$$

$$f(1-\alpha/2,\nu_1,\nu_2)=\frac{1}{f(\alpha/2,\nu_2,\nu_1)}$$

$$\frac{\sqrt{n}(\overline{X}_n-\mu)}{\sigma(\mu)}\xrightarrow{d}N(0,1),\forall\text{单参数族}X$$

$$\text{Wald-tyoe CI}\left[\overline{X}_n-z_{\alpha/2}\frac{\hat\sigma}{\sqrt{n}},\quad\overline{X}_n+z_{\alpha/2}\frac{\hat\sigma}{\sqrt{n}}\right]$$

$$\left[\overline{X}_n-z_{\alpha/2}\frac{\hat\sigma}{\sqrt{n}},\quad\overline{X}_n+z_{\alpha/2}\frac{\hat\sigma}{\sqrt{n}}\right],S(\theta;x)=\frac{\partial}{\partial\theta}\ln L(\theta),nI(\theta)=Var(S(\theta;x))$$

$$\sqrt{nI(\theta)}(\hat\theta_n-\theta)\xrightarrow{d}N(0,1)$$

3 Confidence Interval

$$2\theta n\overline{X}\sim\chi^2(2n),X\sim\mathrm{Exponential}(\theta)$$

$$-2\ln F(X;\theta)\sim\chi^2(2),\forall\text{连续 CDF}F(X;\theta)$$

$$Z=\frac{\sqrt{n}(\overline{X}-\mu)}{\sigma_0}\sim N(0,1),\text{已知}\sigma_0,\text{未知}\mu$$

$$T=\frac{\sqrt{n}(\overline{X}-\mu)}{S}\sim t(n-1),\mu,\sigma\text{均未知}$$

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}\sim N(0,1),\sigma_1,\sigma_2\text{已知}$$

$$T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\sim t(n_1+n_2-2)$$

$$S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2},\text{假设}\sigma_1=\sigma_2$$

$$T_{Welch}=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}}\approx t(\nu)$$

$$\nu=\left(\frac{c^2}{n_1-1}+\frac{(1-c)^2}{n_2-1}\right)^{-1},c=\frac{S_1^2/n_1}{S_1^2/n_1+S_2^2/n_2}$$

$$P=\sum_{i=1}^n\left(\frac{X_i-\mu_0}{\sigma}\right)^2=\frac{\sum_{i=1}^n(X_i-\mu_0)^2}{\sigma^2}\sim\chi^2(n),\text{已知}\mu_0$$

$$P=\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1),\mu\text{未知}$$