CS 760: Machine Learning Spring 2024

Homework 3: Logistic Regression

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Problem 3.1

(a) I used good ol' fashioned guess and check. I set the iterations to 3000 and printed out the weights after every 100 iterations. Once the values started to level off, I considered that convergence

(b) It took me about 3000 iterations to converge.

(c) I performed min-max normalization to avoid running into a RuntimeWarning (when the exponential got WAY too big), so my results for $\hat{\theta}$ were:

$$[-6.75837555, 4.55726066, -14.28891882, -3.05890586, -0.50213527, 7.82165471]$$

(d) The maximum log-likelihood of $\hat{\theta}$ is: -470.15901001623024

(e) From Theorem 6.2 in the Logistic Regression notes, we can see that $\hat{\theta} \xrightarrow{\text{d}} \mathcal{N}(\theta^*, I_{\theta^*}^{-1})$ where the Fisher Information is shown as:

$$I_{\theta^*} = \sum_{i=1}^{N} \frac{e^{-\theta^{*T}\mathsf{x}_\mathsf{i}}}{(1+e^{-\theta^{*T}\mathsf{x}_\mathsf{i}})^2}\mathsf{x}_\mathsf{i}\mathsf{x}_\mathsf{i}^\mathsf{T}$$

Problem 3.2

(a) Borrowing from the Logistic Regression notes again, we can see that the MLE of the log-odds $\hat{\omega} := \hat{\theta}^\mathsf{T}\mathsf{x}$ where $\hat{\theta}$ are the true parameters, θ^*.

(b) Furthermore, the asymptotic distribution of $\hat{\omega}$ is defined as $\hat{\omega} \xrightarrow{\text{d}} \mathcal{N}(\theta^{*\mathsf{T}}\mathbf{x}, \mathbf{x}^\mathsf{T} I_{\theta^*}^{-1}\mathbf{x})$

Problem 3.3

(a) I maximized my feature vector, having my entire family on board in the cheapest class and fare I could find. WIth that, my feature vector looked like so:

$$[Pclass = 3, male = 0, age = 24, siblings = 7, parents = 2, fare = 8]$$

When I run this through my model, unfortunately I do not survive. As I tested other values, I found that affluent women were far more likely to survive.

(b) Given $\tau = \Phi_{\mathcal{N}}^{-1}(\frac{\alpha}{2}|0, \mathbf{x}^{\mathsf{T}} I_{\theta^*}^{-1} \mathbf{x})$

(c)

Problem 3.4

(a)

(b)

(c)