5.1 MI

$P(n) \rightarrow$ MI $\quad n \in Z$

1. verify that $P(1)$ is true
2. Show for all $k \in Z^+$ $P(k+1)$ is true iff $P(k)$ is true.

ex $\quad n \in Z^+$, $\quad 1+2+3+\ldots n = \frac{n(n+1)}{2} \rightarrow P(n)$

1. $n=1$ $\quad$ L.H.S $= 1$ $\quad$ | R.H.S $= \frac{1(1+1)}{2} = 1$ ✓

2. $K \in Z^+$ $\quad \underline{P(K)} = \underbrace{1+2+3 \ldots + K}$ $= \boxed{\frac{K(K+1)}{2}}$ $\rightarrow$ Assume true // ??

3// $(K+1) \in Z^+$ $\quad P(K+1) = 1+2+3+\ldots(K+1) = \boxed{\frac{(K+1)(K+2)}{2}} \rightarrow \boxed{\text{must}}$

$\underbrace{1+2+\ldots K}+\underbrace{(K+1)} = \frac{K(K+1)}{2} + (K+1)$ $\quad$ | $\rightarrow \frac{(K+1)(K+2)}{2}$

$= \frac{K(K+1)+2(K+1)}{2}$

ex $\quad n \in Z^+ \quad n < 2^n$

1, $n=1 \rightarrow 1 < 2^1 \rightarrow$ ✓✓

2. $n = k \in Z^+ \rightarrow k < 2^k \longrightarrow (11)$ Assume true ✓

3. $n = (k+1) \in Z^+ \rightarrow (k+1) < 2^{k+1}$

$2 < 3$

$2 < 5 \longrightarrow 3 < 5$

$k < 2^k$

$(k+1) < 2^k + 1$

$(k+1) < 2^k + 2^k \quad 2^k + 1 < 2^k + 2^k$

$(k+1) < 2 . 2^k$

$(k+1) < 2^{\boxed{k+1}}$

Ch. 5.3 Recursive Def^n

$$\sum_{k=0}^{n} a_k$$

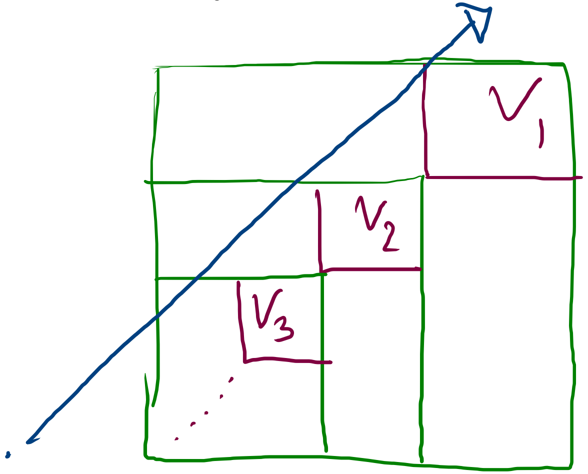
$$0 \to \sum_{k=0}^{0} a_k = a_0$$

$$\sum_{k=0}^{n} a_k = a_n + \sum_{k=0}^{n-1} a_k$$

$$1+2+3+4 = \frac{4(4+1)}{2} = 10$$

$$\sum_{k=1}^{4} k \to 4 + \boxed{\sum_{k=1}^{3} k}$$

$$3 + \sum_{k=1}^{2} k$$



5.4 RA

(*) factorial

$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \rightarrow 4! \cdot 5$

$4! = 1 \cdot 2 \cdot 3 \cdot 4$

$$\left.\begin{aligned} n! &= n \cdot (n-1)! \\ 1! &= 1 \\ 0! &= 1 \end{aligned}\right.$$

procedure

```
fact(n)
    if (n==0) return 1;
    else n.fact(n-1)
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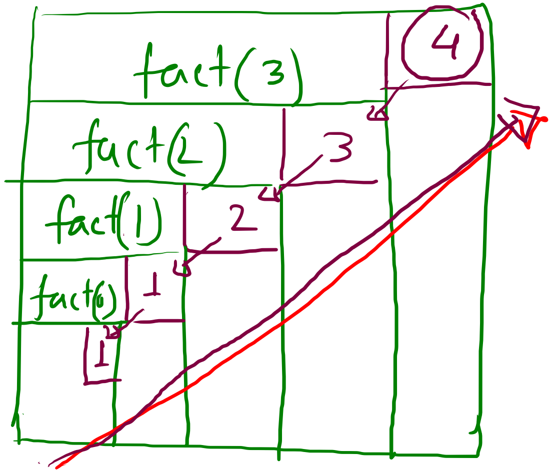
fact(4)



power(a, n) → a^n

$2^3 \to 2\cdot 2^2 \to 2\cdot 2^1 \to \boxed{2\cdot 2^0} \to \boxed{2\cdot 1}$

Procedure

pow(a, n)

if (n == 0) return 1

else return a. pow(a, n-1)

fibonacci numbers.

1 2 3 4 5

→ 0, 1, 1, 2, 3, 5 . . .

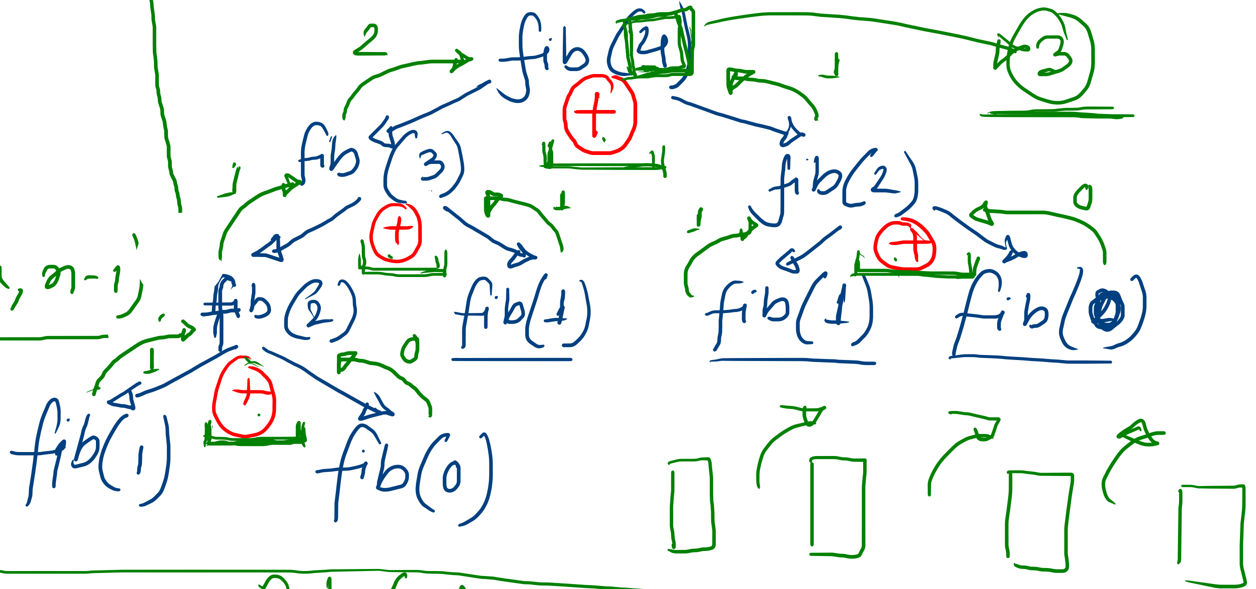
procedure

fib(n)

if n == 0 return 0

elseif n == 1 return 1

else return fib(n-1) + fib(n-2)



fib(n) → n = (n+1)th fib no.