



CSE 4205

Digital Logic Design

Gate Level Minimization

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The Map Method

- **Complexity** of the digital logic gates depends on the **complexity** of algebraic expression.
- Simplification – truth table can be used – but no specific rules
- Cut-and-try method – manual but error prone
- Veitch / Karnaugh Map – simple straightforward procedure
- Made of squares – represent minterms

2 or 3 Variables Map

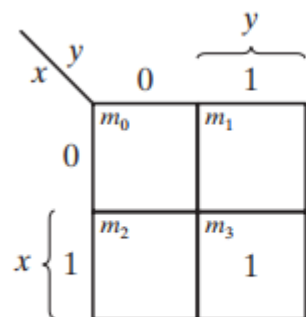
- There are 4 minterms for 2 variables and 8 minterms for 3 variables where variables are appeared in primed or not-primed form.
- Minterms are in binary sequence but appeared in a sequence similar to the reflected code in k-map
- Only one bit changes in value from one adjacent column to the next
 - Any two adjacent squares in the map differ by only one variable
- Any expression in standard form can visualized with K-map

$$x + y = x'y + xy' + xy = m_1 + m_2 + m_3$$

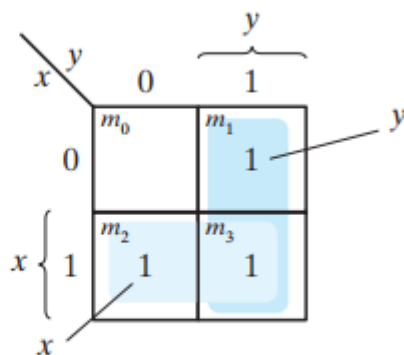
Implicants

- **Implicant** is the **group (adjacent)** of minterms having **2^n 1s** of corresponding minterms in that group
- A ***prime implicant*** is a product term obtained by combining the **maximum possible number of adjacent squares** in the map. (number = 2^n)
- **Essential prime implicant**, is that *prime implicant* that has **at least one minterm** which is not covered by any other prime implicants.

2 or 3 Variables Map

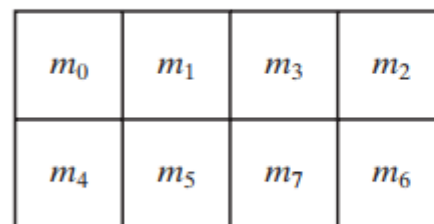


(a) xy

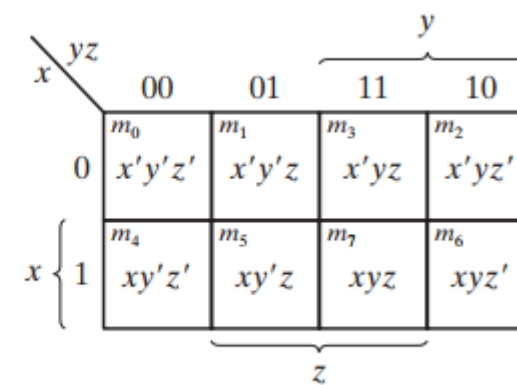


(b) $x + y$

Two variables K-map



(a)



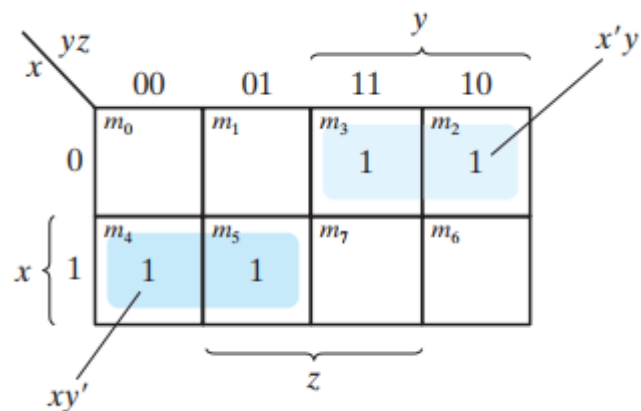
(b)

Three variables K-map

2 or 3 Variables Map...

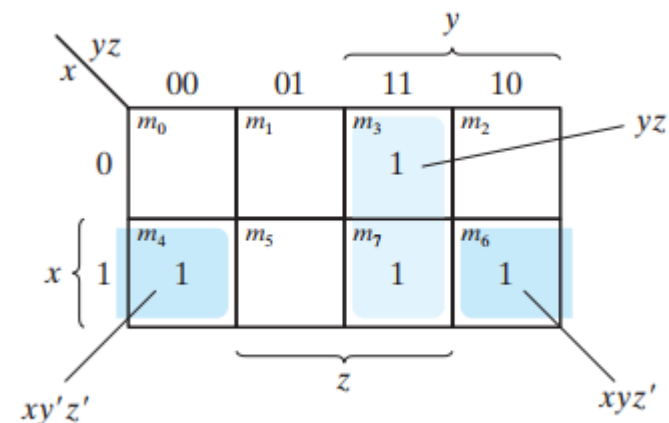
$$F = x'yz + x'yz' + xy'z' + xy'z$$

$$F(x, y, z) = \Sigma(2, 3, 4, 5)$$



$$F = x'yz + xy'z' + xyz + xyz'$$

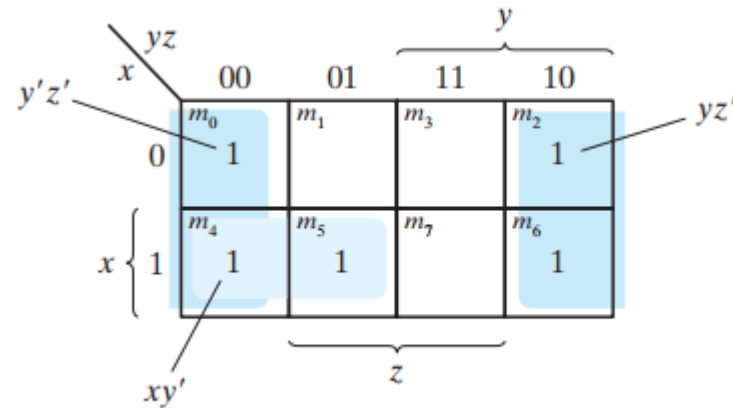
$$F(x, y, z) = \Sigma(3, 4, 6, 7)$$



Note: $xy'z' + xyz' = xz'$

2 or 3 Variables Map...

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



Note: $y'z' + yz' = z'$

Solve these ?

$$F = x'yz + x'yz' + xy'z' + xy'z$$

$$F = A'C + A'B + AB'C + BC$$

4 Variables Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

		y			
		yz			
wx		00	01	11	10
w	00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
	01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
	11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
	10	m_8 $wxy'z'$	m_9 $wxy'z$	m_{11} $wxyz$	m_{10} $wxyz'$
		z			

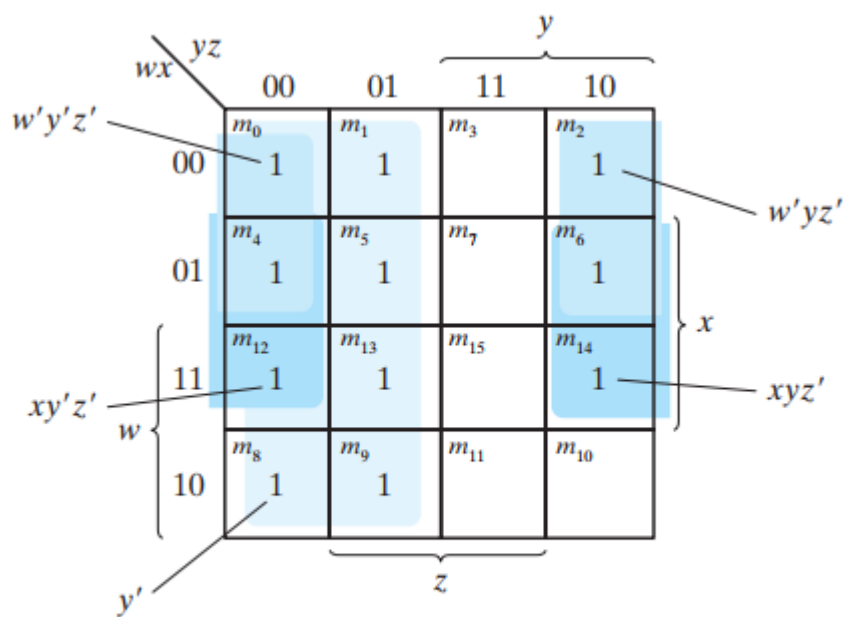
(b)

Remember:

1 square = 4 literals term
 2 squares = 3 literals term
 4 squares = 2 literals term
 8 squares = 1 literal term
 16 squares = 1

4 Variable Map

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$



Note: $w'y'z' + w'yz' = w'z'$
 $xy'z' + xyz' = xz'$

Solve this:

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

5 or 6 Variables Map

CDE									
A	B	000	001	011	010	110	111	101	100
00									
01									
11									
10									

5- variable Karnaugh map (Gray code)

		DEF				D				
		000	001	011	010	110	111	101	100	
ABC	000	0	1	3	2	6	7	5	4	
	001	8	9	11	10	14	15	13	12	
	011	24	25	27	26	30	31	29	28	
	010	16	17	19	18	22	23	21	20	
A	110	48	49	51	50	54	55	53	52	B
	111	56	57	59	58	62	63	61	60	
	101	40	41	43	42	46	47	45	44	
	100	32	33	35	34	38	39	37	36	
		F		E		F				

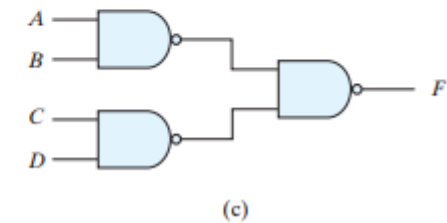
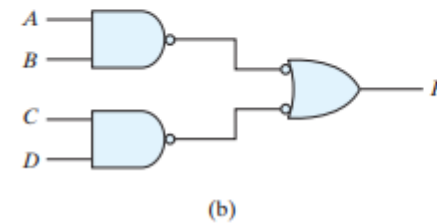
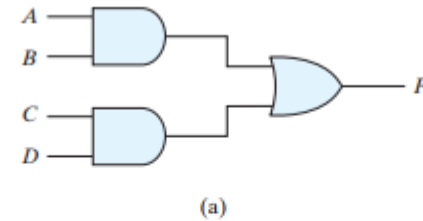
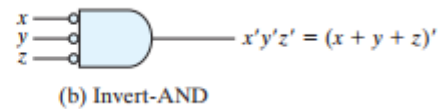
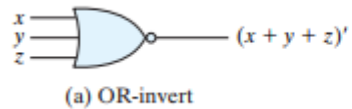
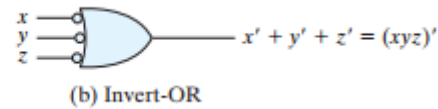
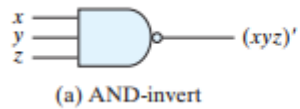


POS & SOP Simplification

Last Class

NAND & NOR implementation

- NAND and NOR are easier to implement





NAND implementation

Home Task



NOR Implementation

Home Task

Don't Care Condition

$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

$$d(w, x, y, z) = \Sigma(0, 2, 5)$$

		y			
		00	01	11	10
wx	yz	m ₀	m ₁	m ₃	m ₂
	00	X	1	1	X
w'x'	01	0	X	1	0
	11	0	0	1	0
w	10	0	0	1	0
	00	0	0	1	0

Diagram (a) shows a 4x4 Karnaugh map for the function F(w, x, y, z). The map is labeled with variables w, x, y, and z. The top row is labeled yz, and the right column is labeled y. The bottom row is labeled wx, and the left column is labeled w'x'. The map shows the function F(w, x, y, z) = yz + w'x'. The cells containing 1s are m₁, m₃, m₇, m₁₁, and m₁₅. The cells containing Xs are m₀, m₂, m₅, and m₁₀. The map is grouped into four 2x2 blocks, each representing a prime implicant: yz (m₁, m₃, m₅, m₇), w'x' (m₀, m₁, m₄, m₅), wx (m₁₂, m₁₃, m₁₄, m₁₅), and w'z (m₁, m₃, m₉, m₁₁).

(a) $F = yz + w'x'$

		y			
		00	01	11	10
wx	yz	m ₀	m ₁	m ₃	m ₂
	00	X	1	1	X
w'z	01	0	X	1	0
	11	0	0	1	0
w	10	0	0	1	0
	00	0	0	1	0

Diagram (b) shows a 4x4 Karnaugh map for the function F(w, x, y, z). The map is labeled with variables w, x, y, and z. The top row is labeled yz, and the right column is labeled y. The bottom row is labeled wx, and the left column is labeled w'z. The map shows the function F(w, x, y, z) = yz + w'z. The cells containing 1s are m₁, m₃, m₇, m₁₁, and m₁₅. The cells containing Xs are m₀, m₂, m₅, and m₁₀. The map is grouped into four 2x2 blocks, each representing a prime implicant: yz (m₁, m₃, m₅, m₇), w'z (m₁, m₃, m₉, m₁₁), wx (m₁₂, m₁₃, m₁₄, m₁₅), and w'x' (m₀, m₁, m₄, m₅).

(b) $F = yz + w'z$

The Tabular Method / Quine-McCluskey Method

$$F = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$$

(a)						(b)						(c)					
$w x y z$						$w x y z$						$w x y z$					
0	0	0	0	0	✓	0, 1	0	0	0	-		0, 2, 8, 10	-	0	-	0	
						0, 2	0	0	-	0	✓	0, 8, 2, 10	-	0	-	0	
1	0	0	0	1	✓	0, 8	-	0	0	0	✓	10, 11, 14, 15	1	-	1	-	
2	0	0	1	0	✓							10, 14, 11, 15	1	-	1	-	
8	1	0	0	0	✓	2, 10	-	0	1	0	✓						
						8, 10	1	0	-	0	✓						
10	1	0	1	0	✓	10, 11	1	0	1	-	✓						
11	1	0	1	1	✓	10, 14	1	-	1	-	✓						
14	1	1	1	0	✓												
15	1	1	1	1	✓	11, 15	1	-	1	1	✓						
						14, 15	1	1	1	-	✓						

$$F = w'x'y' + x'z' + wy$$

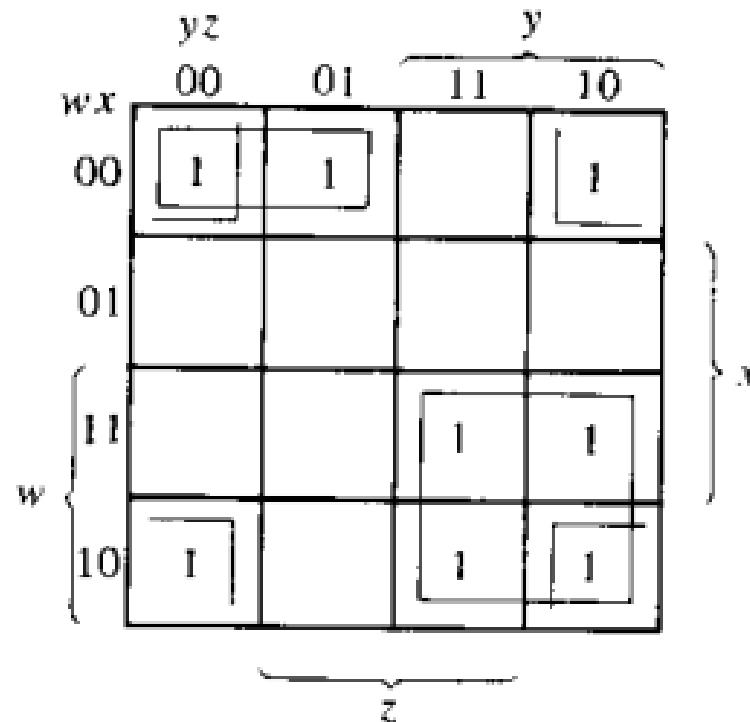
The Tabular Method (Alternative Way)

TABLE 3-6
Determination of Prime Implicants of Example 3-13 with Decimal Notation

(a)	(b)	(c)
0 ✓	0, 1 (1)	0, 2, 8, 10 (2, 8)
	0, 2 (2) ✓	0, 2, 8, 10 (2, 8)
1 ✓	0, 8 (8) ✓	
2 ✓		10, 11, 14, 15 (1, 4)
8 ✓	2, 10 (8) ✓	10, 11, 14, 15 (1, 4)
	8, 10 (2) ✓	
10 ✓		
	10, 11 (1) ✓	
11 ✓	10, 14 (4) ✓	
14 ✓		
	11, 15 (4) ✓	
15 ✓	14, 15 (1) ✓	

The Tabular Method

- Verification using K-map:



Prime Implicants Determination

- Another Example:

$$F(w, x, y, z) = \Sigma(1, 4, 6, 7, 8, 9, 10, 11, 15)$$

Determination of Prime Implicants for Example 3-14

(a)			(b)		(c)
0001	1	✓	1, 9	(8)	8, 9, 10, 11 (1, 2)
0100	4	✓	4, 6	(2)	8, 9, 10, 11 (1, 2)
1000	8	✓	8, 9	(1) ✓	
			8, 10	(2) ✓	
0110	6	✓			
1001	9	✓	6, 7	(1)	
1010	10	✓	9, 11	(2) ✓	
			10, 11	(1) ✓	
0111	7	✓			
1011	11	✓	7, 15	(8)	
			11, 15	(4)	
1111	15	✓			

Prime Implicants Selections

Prime implicants					
Decimal	Binary				Term
	w	x	y	z	
1, 9 (8)	—	0	0	1	$x'y'z$
4, 6 (2)	0	1	—	0	$w'xz'$
6, 7 (1)	0	1	1	—	$w'xy$
7, 15 (8)	—	1	1	1	xyz
11, 15 (4)	1	—	1	1	wyz
8, 9, 10, 11 (1, 2)	1	0	—	—	wx'

Prime Implicant Table for Example 3-15

		1	4	6	7	8	9	10	11	15
✓ $x'y'z$	1, 9	X					X			
✓ $w'xz'$	4, 6		X	X						
$w'xy$	6, 7			X	X					
xyz	7, 15				X					X
wyz	11, 15								X	X
✓ wx'	8, 9, 10, 11					X	X	X	X	
		✓	✓	✓		✓	✓	✓	✓	

The Tabular Method

Verification using K-map:

		yz				
		00	01	11	10	
wx	00		1			
	01	1		1	1	x
	11			1		
w	10	1	1	1	1	

$$F = x'y'z + w'xz' + wx' + xyz$$





Prime Implicants Selections





Administration



Administration



Administration



Administration



Administration



Administration



Administration