Chapter 5.

(*) Mathematical Induction.

Ex 1 $1+2+\dots+n = \dfrac{n(n+1)}{2}, \; n \in \mathbb{Z}^+$

Soln: **Step 1:** $S(n) = 1+2+\dots+n = \dfrac{n(n+1)}{2}$

$S(1)$ L.H.S $= 1$ R.H.S $= 1$ —— (i)

Step 2: Let $S(n)$ is true for $m \in \mathbb{Z}^+$

$S(m) = 1+2+\dots+m = \dfrac{m(m+1)}{2}$ —— (ii)

Step 3: If $S(n)$ is true, then we must show that

$S(m+1) = 1+2+\dots+(m+1) = \dfrac{(m+1)(m+2)}{2}$ —— (iii)

Adding $(m+1)$ on both sides of (ii)

$$1+2+\dots+m+(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$= \frac{m(m+1)+2m+2}{2}$$

$$= \frac{m^2+m+2m+2}{2}$$

$$= \frac{m(m+2)+1(m+2)}{2}$$

$$= \frac{(m+1)(m+2)}{2}$$

Ex 2: If $x \in \mathbb{Z}^+$ show $x < 2^x$

Soln: Step 1: $P(x) = x < 2^x$

$P(1) \longrightarrow 1 < 2^1$ or $1 < 2$. —— (i)

Step 2: $P(m) = m < 2^m$ —— (ii)

Step 3: $P(m+1) = (m+1) < 2^{(m+1)}$ —— (iii) $\rightarrow$

Adding 1 on both sides of (ii)

$$(m+1) < \underline{2^m + 1} < \underline{2^m + 2^m}$$

$$(m+1) < 2^m + 2^m$$

$$(m+1) < 2 \cdot 2^m$$

$$(m+1) < 2^{m+1} \longrightarrow$$

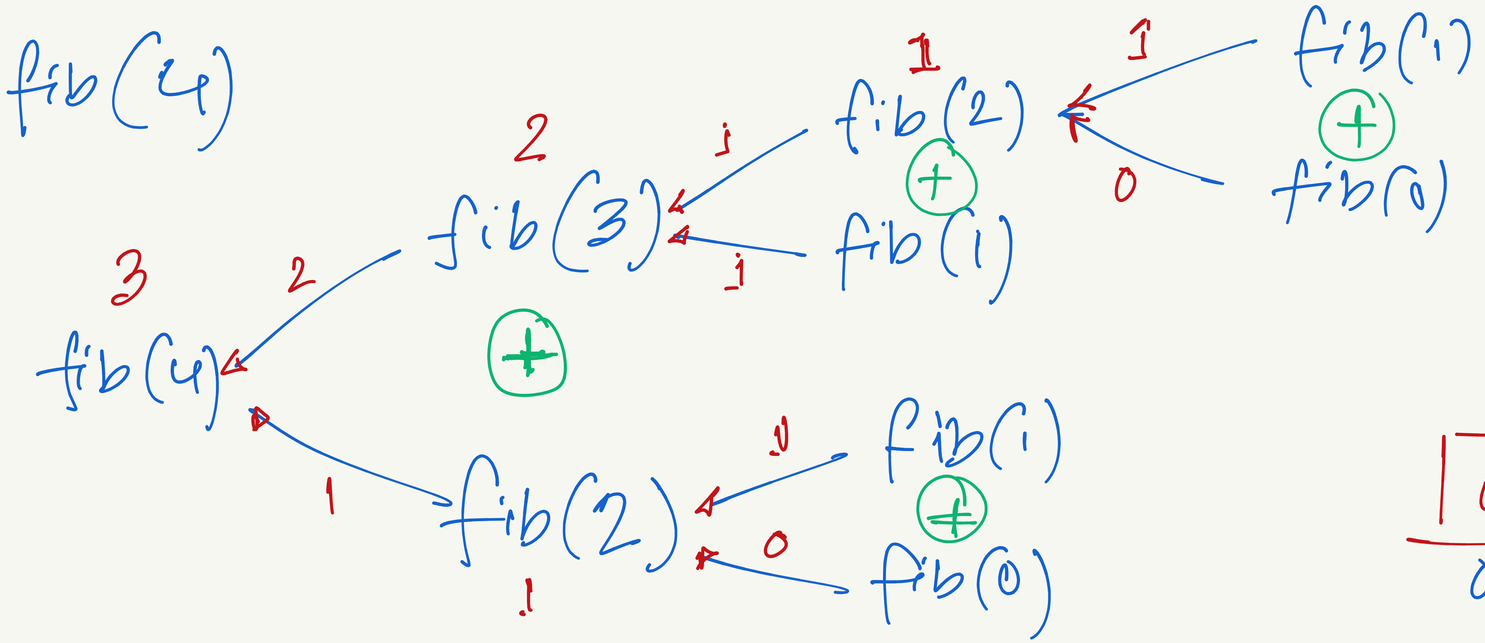
Recursive Algorithms.

Ex 1: procedure fib(n: nonneg int)

→ if n = 0 return 0

→ else if n = 1 return 1

else return fib(n-1) + fib(n-2)



$f_1 = 0$, $f_2 = 1$

$f_3 = f_1 + f_2$

$f_4 = f_2 + f_3$

$f_n = f_{n-2} + f_{n-1}$

$f_3 = 1$

$f_4 = 1 + 1 = 2$

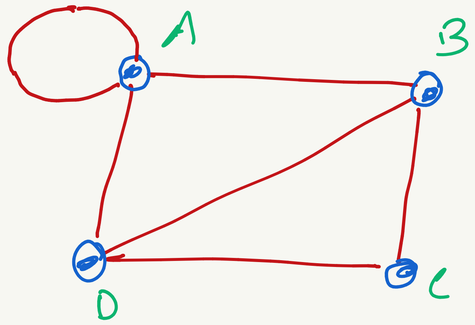
$f_5 = 2 + 1 = 3.$

0	1	1	2	3
0	1	2	3	4

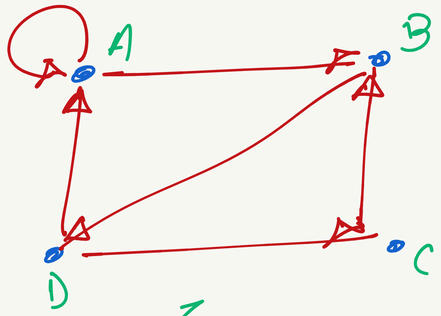
Chapter - 10

Graphs and features.

→ $G(V, E)$ — vertices, edges.



G_1



G_2

→ (1) Undirected "

(2) Directed → (u, v) — starts at u, ends at v

→ Adjacent vertices: u, v → e in G

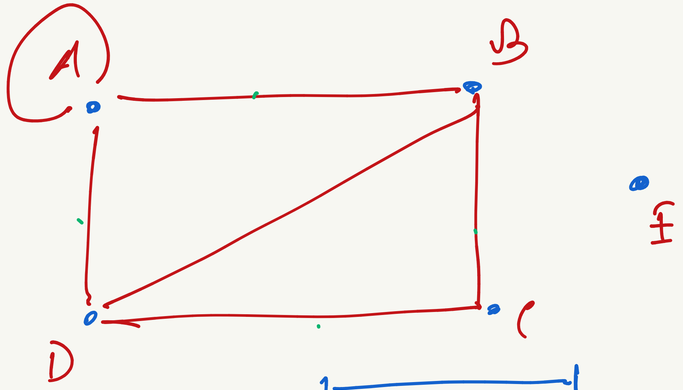
→ Degree of vertices:

(*) Self loop adds (2) to the overall degree of ~~any~~ vertex

G_1 → $\deg(A) \to 4$

$\deg(B) \to 3$

$\deg(C) \to 2$ $\deg(D) \to 3$



$|E| = 6$

$N(A) = \{A, B, D\}$ $\quad \deg(A) = 4$

$N(B) = \{A, D, C\} \longleftrightarrow \deg(B) = 3$

$N(C) = \{B, D\} \longleftrightarrow \deg(C) = 2$

$N(D) = \{A, B, C\} \longleftrightarrow \deg(D) = 3$

$N(E) = \emptyset \longleftrightarrow \deg(E) = 0$

$\overline{}$ 12

$2+2 \longrightarrow 4$

$2+3 \longrightarrow 5$

$\rightarrow 3+3+3 \rightarrow 9 \leftarrow$

$\rightarrow 3+3+3+3 \rightarrow 12 \leftarrow$

Theorem - 1 if $G = (V, E)$ and is undirected with m edges.

$$2m = \sum_{v \in V} \deg(v)$$

$$2 \cdot 6 = 4+3+2+3$$

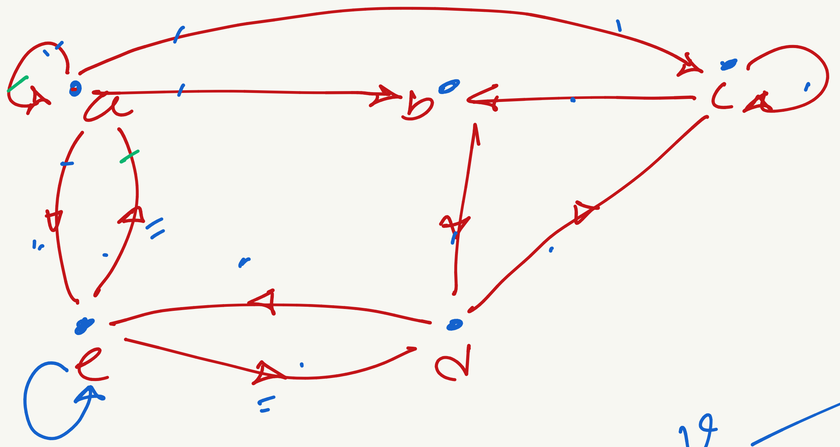
$$\boxed{12 = 12}$$

Theorem 2 An undirected Graph has an even number of vertices of odd degree.

$V \begin{cases} V_E \\ V_O \end{cases}$

$$2m = \sum_{v \in V} \deg(v)$$

$$= \sum_{v_E \in V_E} \deg(v_E) + \sum_{v_O \in V_O} \deg(v_O)$$



$\deg^-(v) \rightarrow v$ is terminal vertex $\rightarrow$ in-degree

$\deg^+(v) \rightarrow v$ is initial vertex $\rightarrow$ out-degree

v — edges emerge out of v
v — edges terminate in v

$\deg^-(a) = 2$
$\deg^-(b) = 2$
$\deg^-(c) = 3$
$\deg^-(d) = 2$
$\deg^-(e) = 3$

$\sum \deg^-(v) = 12$

$\deg^+(a) = 4$
$\deg^+(b) = 1$
$\deg^+(c) = 2$
$\deg^+(d) = 2$
$\deg^+(e) = 3$

$\sum \deg^+(v) = 12$

$$\sum \deg^-(v) = \sum \deg^+(v) = |E|$$