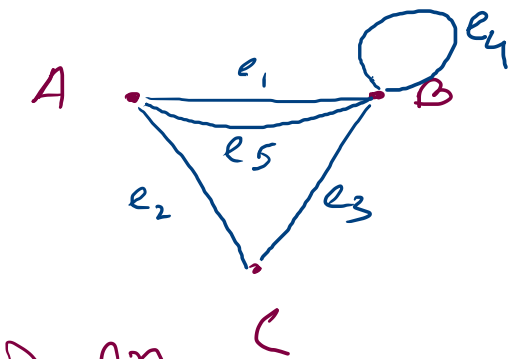
10.1 Graphs & features.

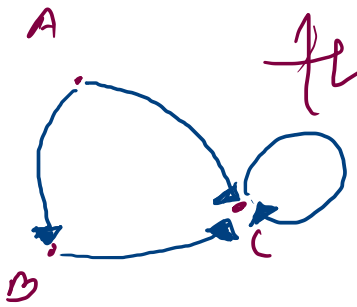
(*) **Def^n-1** $\longrightarrow$ $G(V, E)$



if V < finite $\rightarrow$ finite graph
infinite $\rightarrow$ infinite graph.

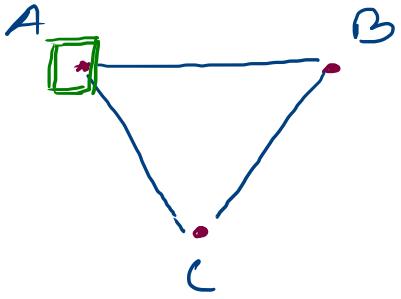
(*) **Def^n-2** $\longrightarrow$ directed graph / di-graph $G(V, E)$

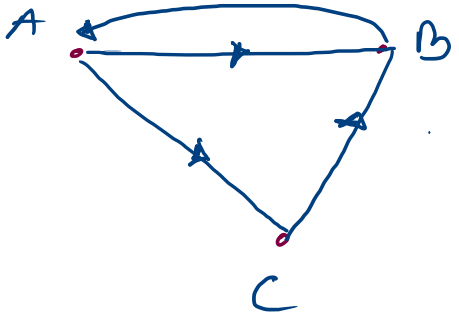
1. $V \longrightarrow$ same as before
2. $E \longrightarrow$ direction, connects 2 endpoints and the end points are ordered pairs $(\vec{u}, \vec{v})$



(*) $V \to$ degree zero $\to$ isolated vertex

$\to$ " one $\to$ pendant "





in-deg

out-deg

(*) Thm-1 : Handshaking Theorem

Let $G(V,E) \to$ undirected graph with m edges, then

$$2m = \sum_{v \in V} \deg(v)$$

(*) Th-2: An UG has an even no. of vertices of odd degree.

$\longrightarrow$ Let $V_1 \longrightarrow$ set of vertices with even degree

$V_2 \longrightarrow$ set of vertices with odd degrees

$1 \quad 3 \longrightarrow 4$

$5 \quad 3 \longrightarrow 8$

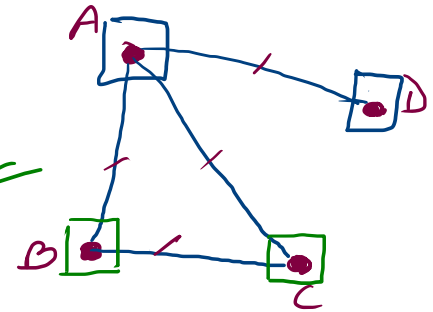
$\underbrace{4} + \underbrace{3+3}_{2} \longrightarrow \not\equiv 10$

We know,

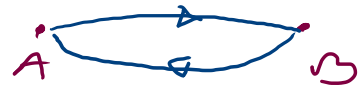
$$\boxed{2m \Rightarrow \sum_{v \in V} \deg(v) \Rightarrow \sum_{v_1 \in V_1} \deg(v_1) + \sum_{v_2 \in V_2} \deg(v_2)}$$

1. $2m$ is even.
2. $\sum_{v_1 \in V_1} \deg(v_1)$ is also even.
3. Since $2m$ is even and $\sum_{v_1 \in V_1} \deg(v_1)$ is even, there MUST be even no. of vertices with odd degree.

Eg. $\mathcal{M}_2$

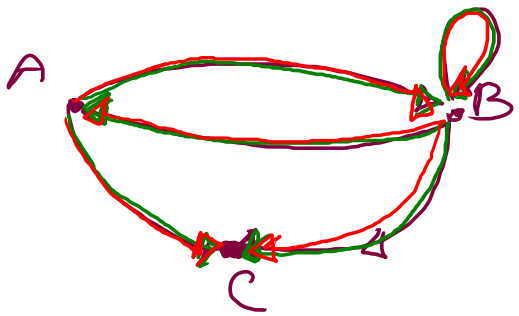


- m = # of edges = 4
- # of even vertices = 2
- # of odd " → 2 → even.



Def^n - 5 : In a DG then the vertices have 2 types of degrees associated.

1. In-degree $[\deg^-(v)]$ → v → has to be the terminal vertex of the edge
2. Out-degree $[\deg^+(v)]$ → v → has to be the initial vertex of the edge.



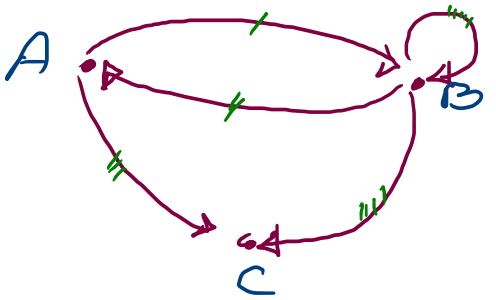
$\deg^-(A) = 1$ $\quad$ $\deg^+(A) = 2$

$\deg^-(B) = 2$ $\quad$ $\deg^+(B) = 3$

$\deg^-(C) = 2$ $\quad$ $\deg^+(C) = 0$

(*) Theo - 3 → in a dg $G = (V, E)$

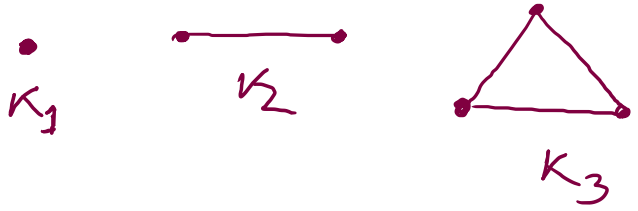
$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$



v	$\deg^-(v)$	$\deg^+(v)$
A	1	2
B	2	3
C	2	0
	5	5

(*) Types of graphs

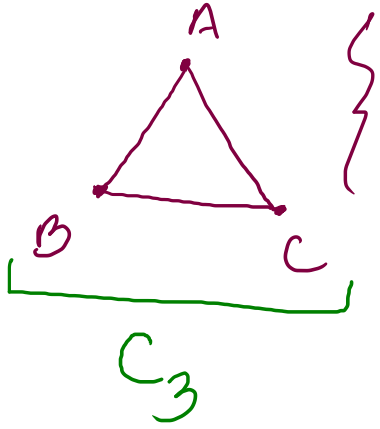
(1) Complete graph $\to K_n$ → Exactly one edge between each $\boxed{\text{pair}}$ of distinct vertices.



(2) Cycles, $n \geq 3$ $\boxed{C_n}$

$V = \{v_1, v_2, \ldots v_{n-1}, v_n\}$

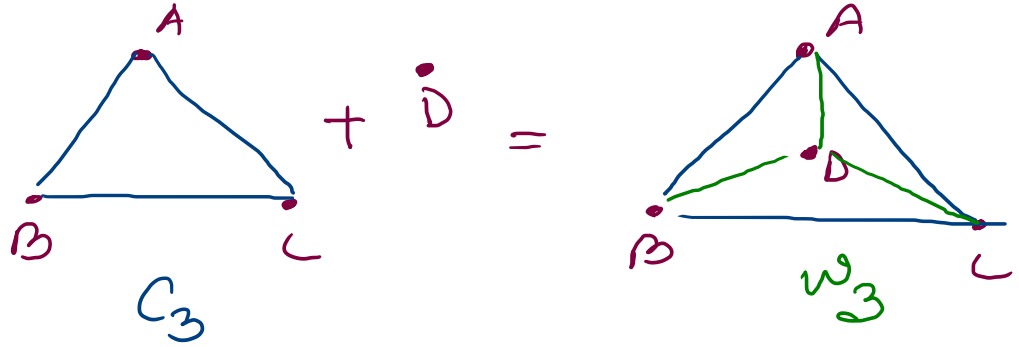
$E = \{\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_{n-1}, v_n\} \{v_n, v_1\}\}$

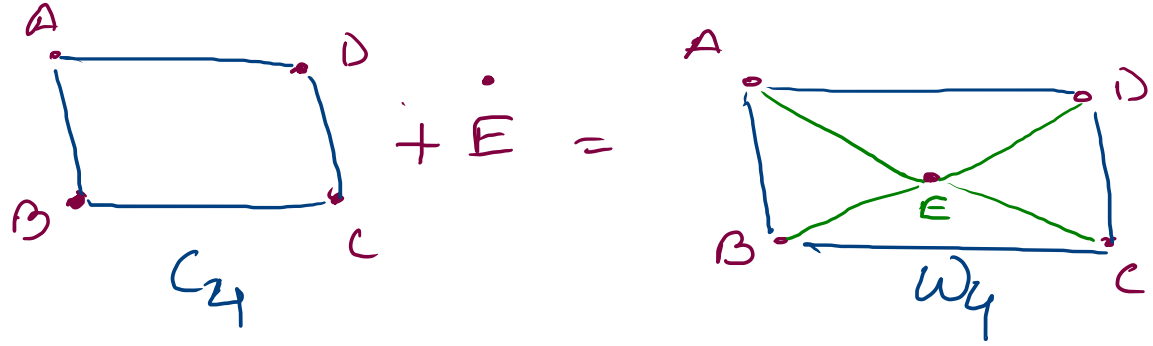


$\{\{A, B\} \{B, C\} \{C, A\}\}$

3. Wheels

$$\boxed{W_n = C_n + 1}$$





4. n-cubes. Q_n

↳ n-dimensional hypercube

- Vertices are represented by 2^n bit strings of length n.
- the adjacent vertices will have bit strings that differ exactly by 1 bit.

$Q_n = 3$

$\boxed{Q_3}$

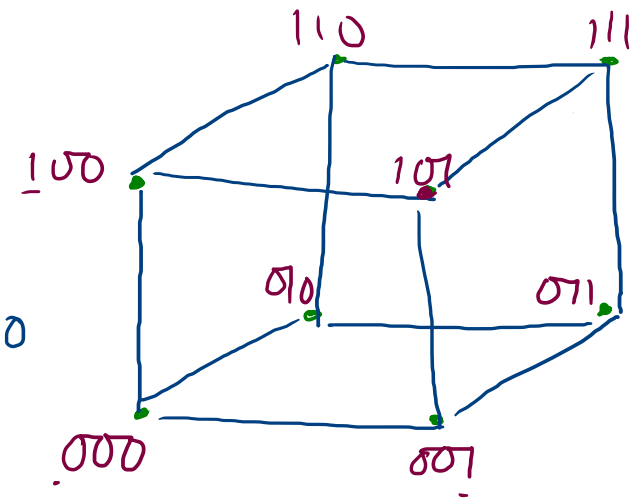
↳ $2^{n} \boxed{3}$

↳ 8

000

⋮

111



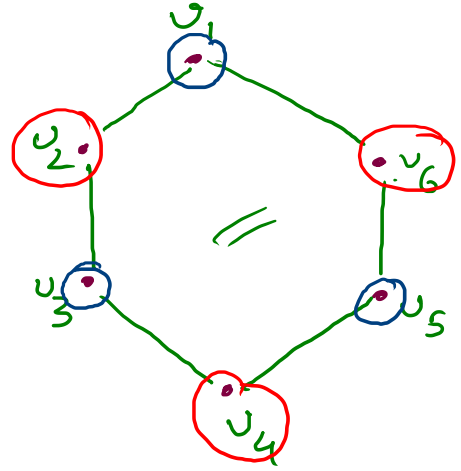
2 Bipartite graphs

- Divide set of vertices into 2 subsets V_1, V_2 such no vertices in V_1 or V_2 are adjacent

$$V_1 = \{v_{11}, v_{12}, v_{13} \ldots\}$$

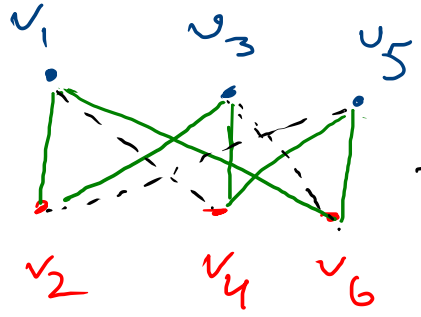
$$V_2 = \{v_{21}, v_{22}, v_{23} \ldots\}$$

$G =$



$$V_1 = \{v_1, v_3, v_5\}$$

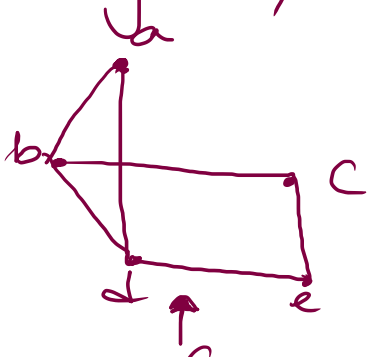
$$V_2 = \{v_2, v_4, v_6\}$$

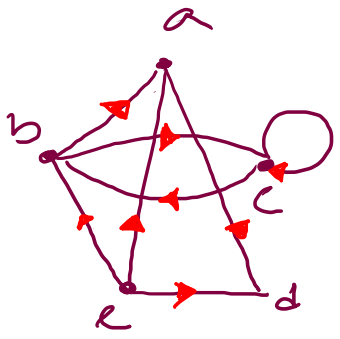


$\rightarrow$ if all edges are present then Complete BG

10.3 Graph Representation

① Adjacency List





uG

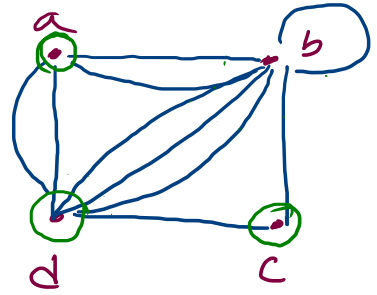
v_i	Adj v_i
a	b, d.
b	a, c, d
c	b, e
d	a, b, e
e	c, d

dG

v_i	v_T
a	d
b	a, c
c	b, c
d	–
e	a, b, d

② Adjacency matrix

$|V| \times |V|$

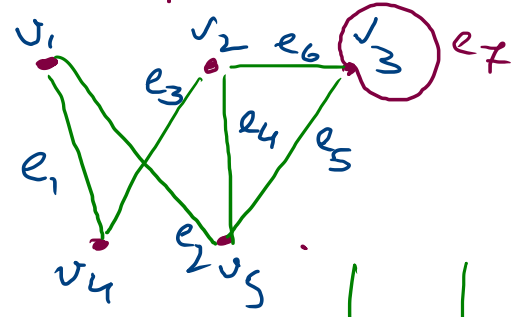


$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
0\ a & 0 & 2 & 0 & 2 \\
1\ b & 2 & 1 & 1 & 3 \\
2\ c & 0 & 1 & 0 & 1 \\
d & 2 & 3 & 1 & 0
\end{array}
\qquad
\begin{array}{l}
\to 4 \\
\to 7+1 \\
\to 2 \\
\to 6
\end{array}
$$

$M_{i,j} \to$ if $i == j$ and $M_{ij} > 0$

③ Incidence Matrix

$|V| \times |E|$



$$
\begin{array}{c|ccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\
\hline
v_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
v_3 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
v_4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
v_5 & 0 & 1 & 0 & 1 & 1 & 0 & 0
\end{array}
$$