CSE 4203 - 4.2 - Integer representations & Algorithms.

⋆ Addition Algorithm.

$a = (1110)_2$ $\quad n = 4$

$b = (1011)_2$ $\quad i = 0 \ldots (n-1)$

$3\ 2\ 1\ 0$

L R

$a_n + b_n + c_{n-1} = c_n \cdot 2 + s_n$

$a_0 + b_0 + 0 = c_0 \cdot 2 + s_0$

$0 + 1 + 0 = 0 \cdot 2 + 1$

$a_1 + b_1 + c_0 = c_1 \cdot 2 + s_1$

$1 + 1 + 0 = 1 \cdot 2 + 0$

$a_2 + b_2 + c_1 = c_2 \cdot 2 + s_2$

$1 + 0 + 1 = 1 \cdot 2 + 0$

$a_3 + b_3 + c_2 = c_3 \cdot 2 + s_3$

$\rightarrow s_4$

$1 + 1 + 1 = 1 \cdot 2 + 1$

L

R

R

L

$a + b = (11001)_2$

```
  111
  1110
  1011
 11001
```

L R

$c := 0$

for $i = 0 \rightarrow n-1$ {

$c_n \equiv d := \lfloor (a_i + b_i + c)/2 \rfloor$

$s_n \equiv s_i := a_i + b_i + c - 2d$

$c_{n-1} \equiv c := d$

}

$s_n := c$

$d = \lfloor (a_0 + b_0 + c)/2 \rfloor = \lfloor 1/2 \rfloor = 0$

$s_0 = 0 + 1 + 0 - 2 \cdot 0 = 1$

$c = d = 0$

(*) Multiplication Algorithm.

LSB
3 2 1 0

$$ab = a(b_0 \cdot 2^0 + b_1 2^1 + b_2 \cdot 2^2 + \cdots b_{n-1} \cdot 2^{n-1})$$
$$= a \cdot b_0 \cdot 2^0 + a b_1 2^1 + a b_2 \cdot 2^2 + \cdots a b_{n-1} \cdot 2^{n-1}$$

(*) If multiplied by 2, the binary expansion is shifted 1 bit to the left for each multiplication by 2 and add a 0 to the right

$a = (110)_2$ $b = (101)_2$

$ab_0 2^0 = (110)_2 \cdot 1 \cdot 2^0 \leftrightarrow (110)_2$

$ab_1 2^1 = (110)_2 \cdot 0 \cdot 2^1 = (0000)_2$

$ab_2 2^2 = (110)_2 \cdot 1 \cdot 2^2 = (11000)_2$

$(11110)_2$

$$\begin{array}{r} 110 \\ \times\ 101 \\ \hline 110 \\ 000\times \\ +\ 110\times\times \\ \hline (11110)_2 \end{array}$$

⊛ Division Algorithm

$a, d \in \mathbb{Z}$, $d > 0$, $d \mid a$

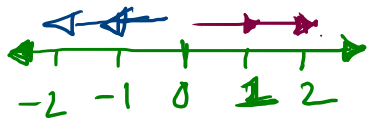
a → Dividend

d → Divisor

$q = a \text{ div } d = d \mid a$

$r = a \bmod d = a \,\%\, d$

```
q := 0
r := |a|
while (r ≥ d){
    r := r - d
    q := q + 1
}
if a < 0 & r > 0 then,
    r := d - r
    q := -(q+1)
```



$3 \mid -11 \mid -4$

-12

1

$a = -11 \quad d = 3$

$q = 0$, $r = |a| = |-11| = 11$

while

$r \geq d \rightarrow 11 > 3$? ✓ → $r = 11 - 3 = 8$, $q = 1$

$r \geq d \rightarrow 8 > 3$? ✓ → $r = 8 - 3 = 5$, $q = 2$

$r \geq d \rightarrow 5 > 3$? ✓ → $r = 5 - 3 = 2$, $q = 3$

$r \geq d \rightarrow 2 > 3$? ✗ → (exit)

$a < 0 \rightarrow -11 < 0$? ✓

$r > 0 \rightarrow 2 > 0$? ✓

$r = 3 - 2 = 1$

$q = -(3+1) = -4$