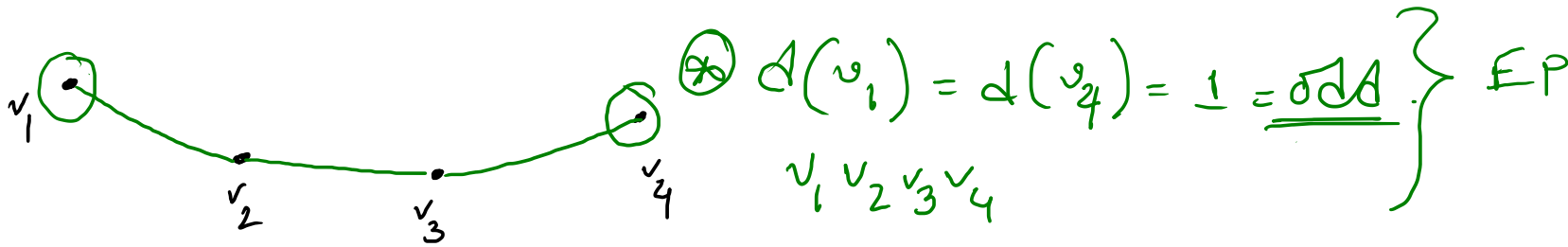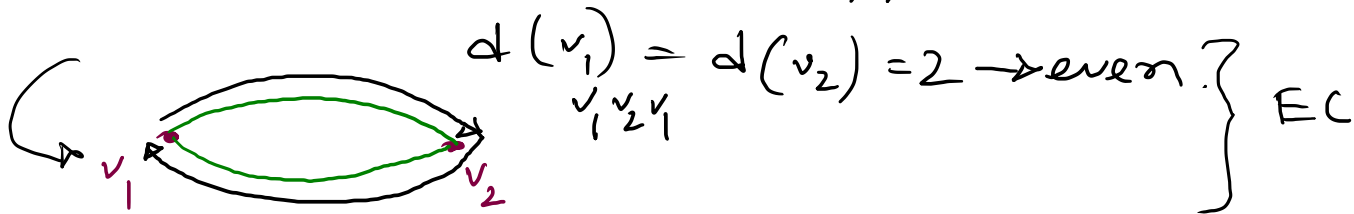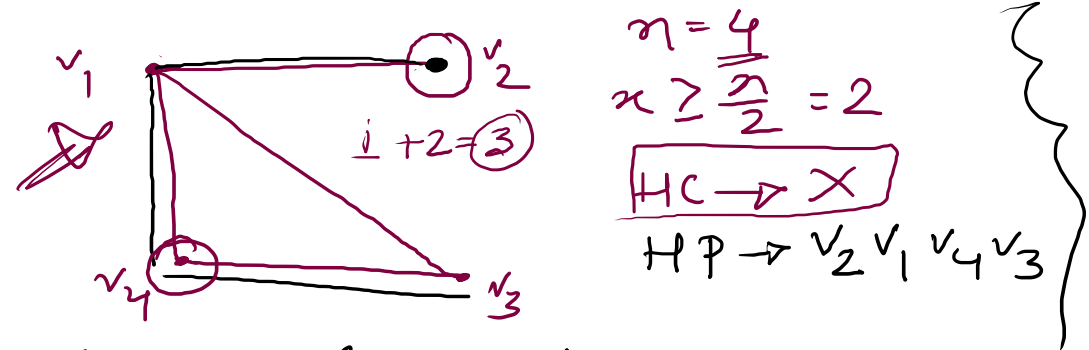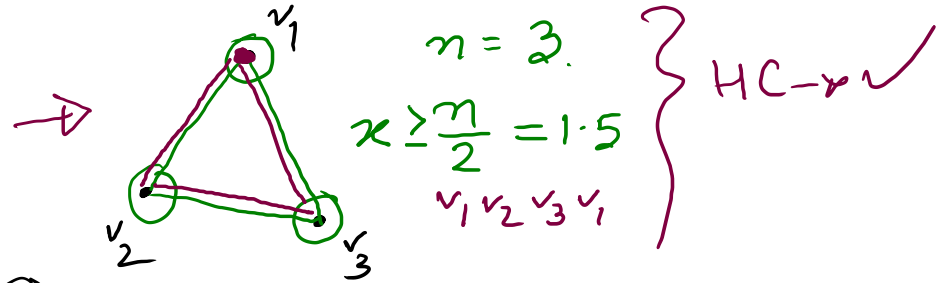## 10.5. Euler + Hamiltonian path

(*) __Theorem -1__ A connected graph has an Euler path and not an Euler circuit iff there are exactly 2 vertices of odd degree.

(*) __Theorem-2__ A connected graph with at least 2 vertices has an Euler circuit iff each vertex has even degree.

$$d(v_1) = d(v_2) = 2 \rightarrow even. \Big\} \; EC$$

$$v_1 v_2 v_1$$

(*) $d(v_1) = d(v_4) = 1 = \underline{odd} . \Big\} \; EP$

$$v_1 v_2 v_3 v_4$$

⊛ **Diracs Theorem** : If G is a simple graph, with n vertices and $n \geq 3$ such that the degree of each vertex is at least $n/2$ then $G \to$ has Hamiltonian circuit.

$\to$

$n = 3$.

$x \geq \dfrac{n}{2} = 1.5$

$v_1 v_2 v_3 v_1$

$\Big\}$ HC $\to$ ✓

$i + 2 = ③$

$n = 4$

$x \geq \dfrac{n}{2} = 2$

HC $\to$ ✗

HP $\to v_2 v_1 v_4 v_3$

⊛ **Ore's theorem:** If G is a simple graph, with n vertices and $n \geq 3$ such that for every pair of non-adjacent vertices $(u,v) \to$ $\boxed{\deg(u) + \deg(v) \geq n}$ $\to$ the G has HC

$n = 4$

$(v_1, v_3) \to \deg(v_1) + \deg(v_3) = 2 + 2 = 4 = n$ $\Big\}$ $G \to$ HC

$(v_2, v_4) \to \deg(v_2) + \deg(v_4) = 2 + 2 = 4 = n$