CSE 4205
Digital Logic Design

Gate Level Minimization

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The Map Method

- **Complexity** of the digital logic gates depends on the **complexity** of algebraic expression.
- Simplification – truth table can be used – but no specific rules
- Cut-and-try method – manual but error prone
- Veitch / Karnaugh Map – simple straightforward procedure
- Made of squares – represent minterms

2 or 3 Variables Map

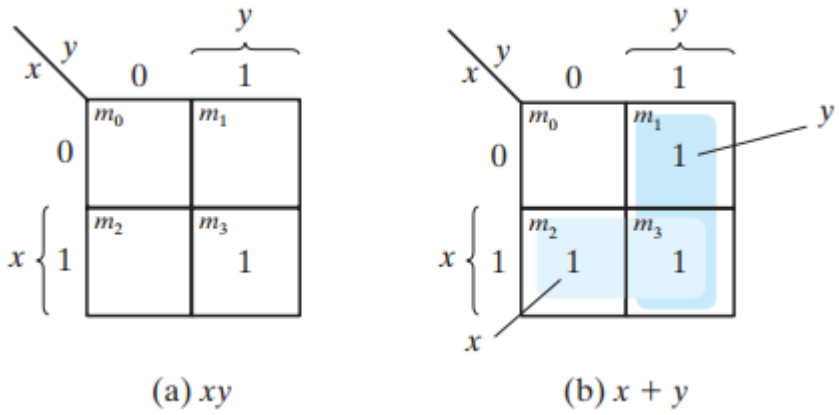
- There are 4 minterms for 2 variables and 8 minterms for 3 variables where variables are appeared in primed or not-primed form.
- Minterms are in binary sequence but appeared in a sequence similar to the reflected code in k-map
- Only one bit changes in value from one adjacent column to the next
  - Any two adjacent squares in the map differ by only one variable
- Any expression in standard form can visualized with K-map

$$x + y = x'y + xy' + xy = m_1 + m_2 + m_3$$

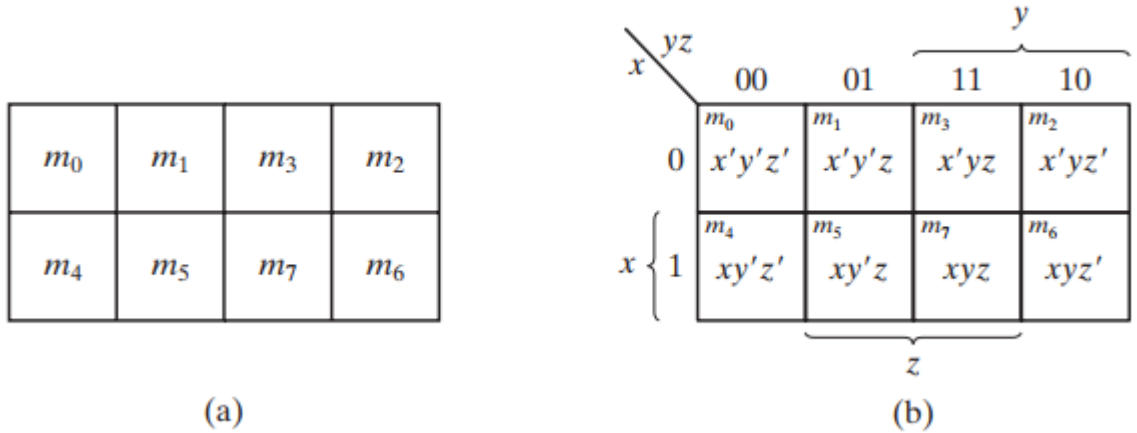
Implicants

- **Implicant** is the group (adjacent) of minterms having **2^n** 1s of corresponding minterms in that group
- A ***prime implicant*** is a product term obtained by combining the **maximum possible number of adjacent squares** in the map. (number = 2^n)
- **Essential prime implicant,** is that *prime implicant* that has **at least one minterm** which is not covered by any other prime implicants.

2 or 3 Variables Map



Two variables K-map

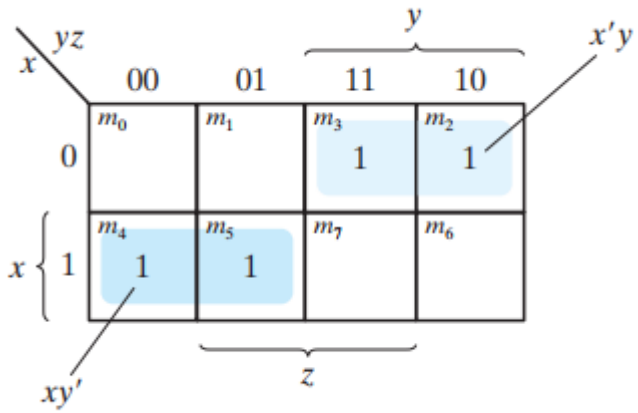


Three variables K-map

2 or 3 Variables Map…

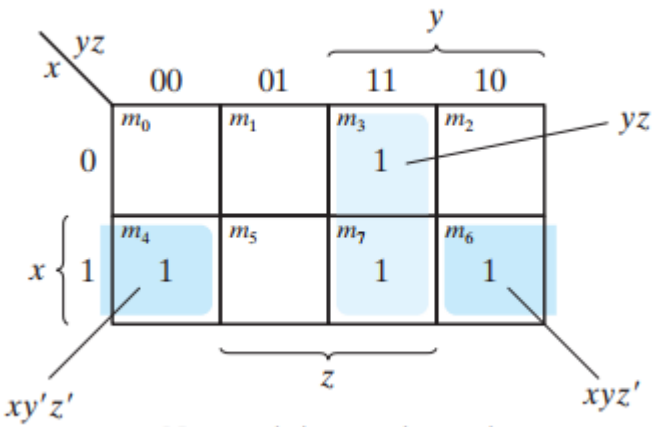
$F = x'yz + x'yz' + xy'z' + xy'z$

$F(x, y, z) = \Sigma(2, 3, 4, 5)$



$F = x'yz + xy'z' + xyz + xyz'$

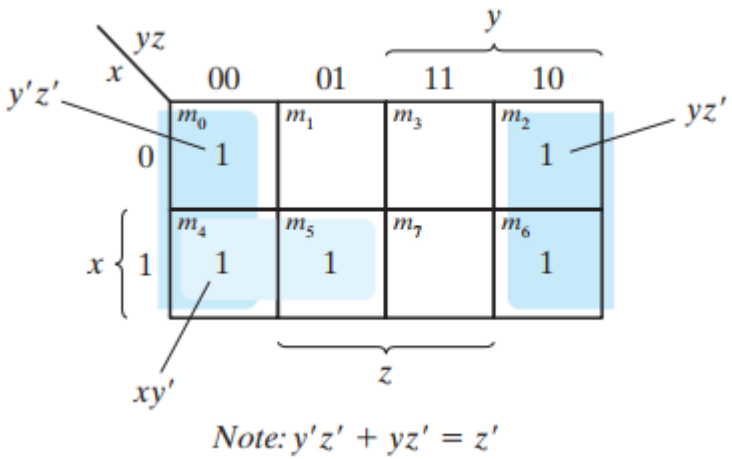
$F(x, y, z) = \Sigma(3, 4, 6, 7)$



Note: $xy'z' + xyz' = xz'$

2 or 3 Variables Map…

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



Note: $y'z' + yz' = z'$

Solve these ?

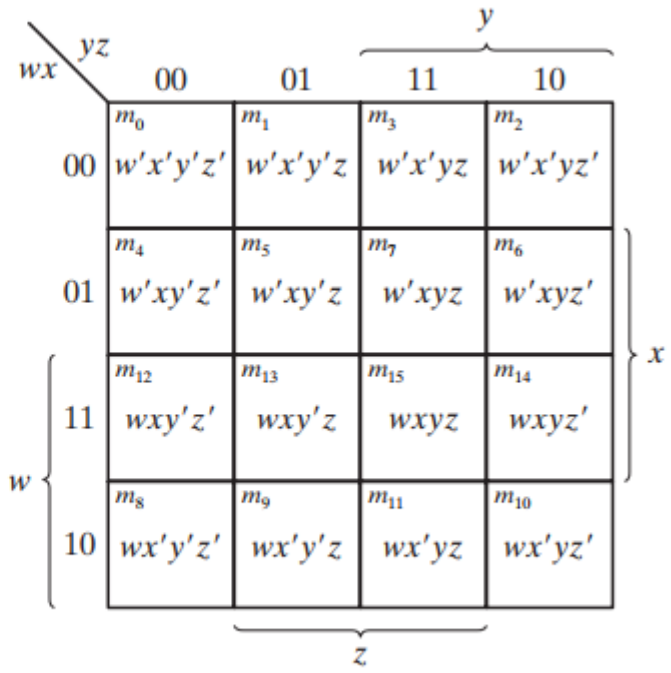
$F = x'yz + x'yz' + xy'z' + xy'z$

$F = A'C + A'B + AB'C + BC$

4 Variables Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)



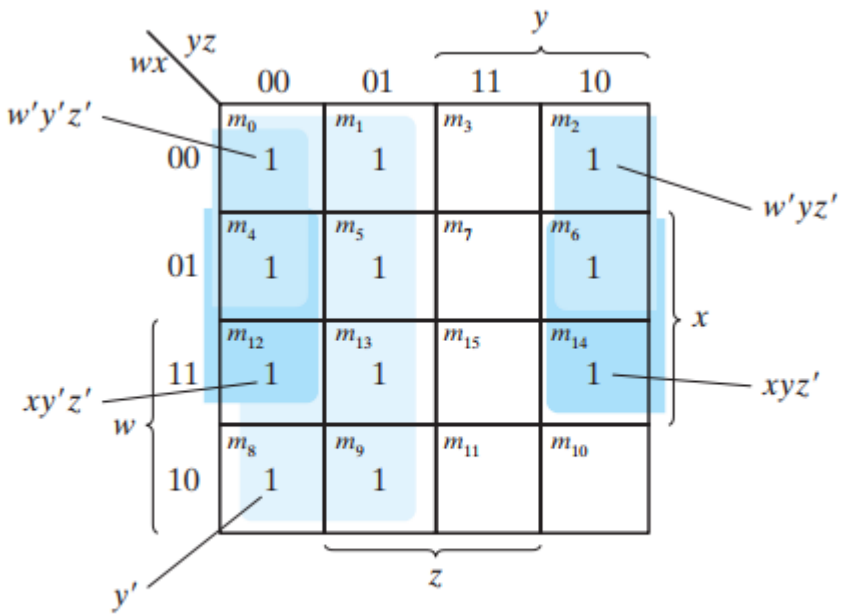
(b)

Remember:

1 square = 4 literals term
2 squares = 3 literals term
4 squares = 2 literals term
8 squares = 1 literal term
16 squares = 1

4 Variable Map

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$



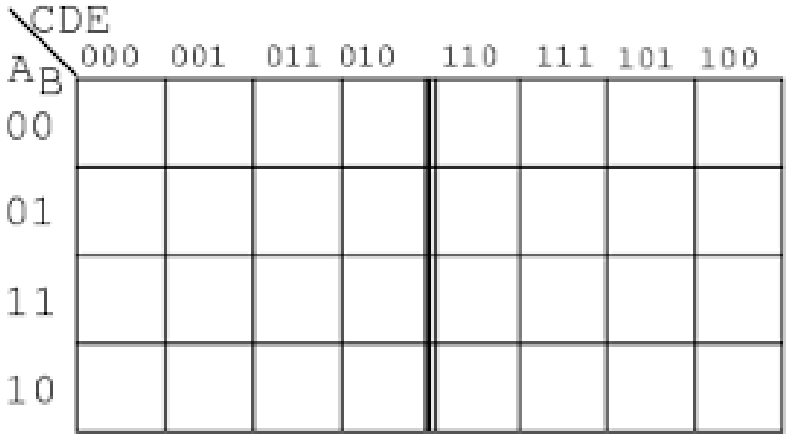
Note: $w'y'z' + w'yz' = w'z'$

$xy'z' + xyz' = xz'$

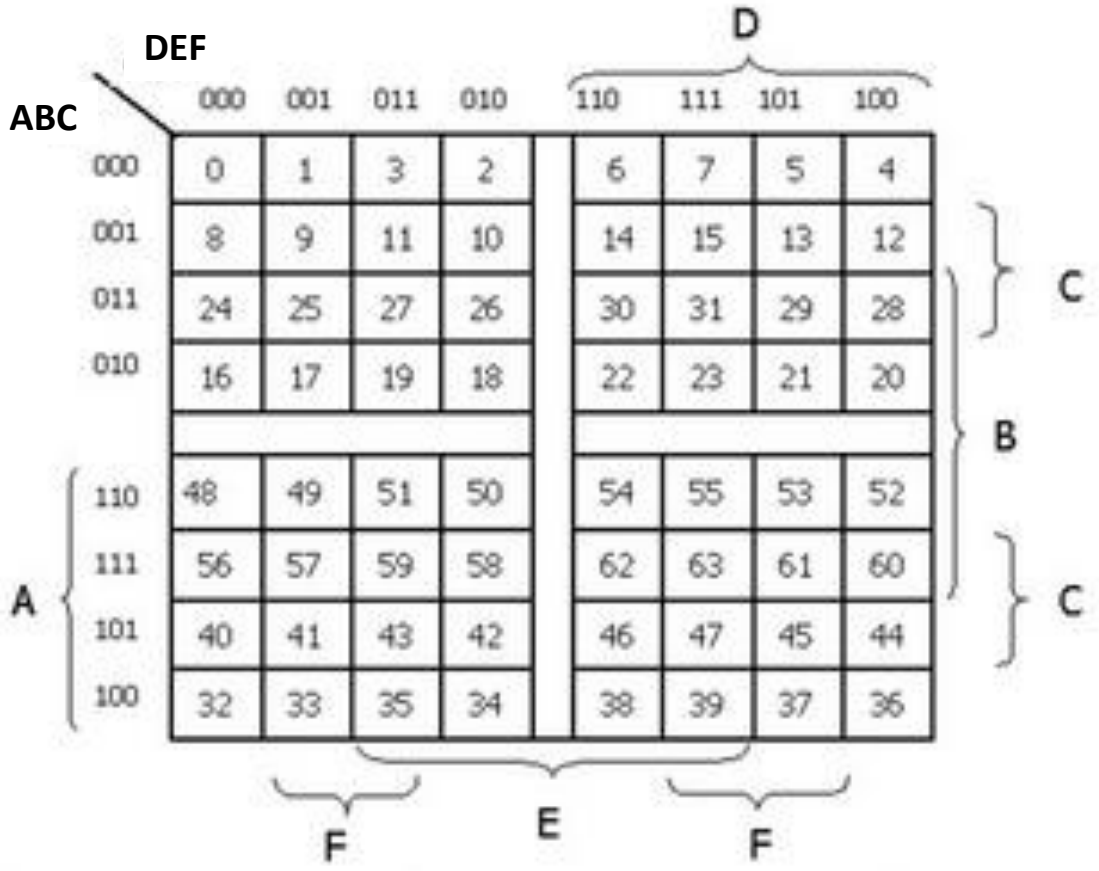
Solve this:

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

5 or 6 Variables Map



5- variable Karnaugh map (Gray code)

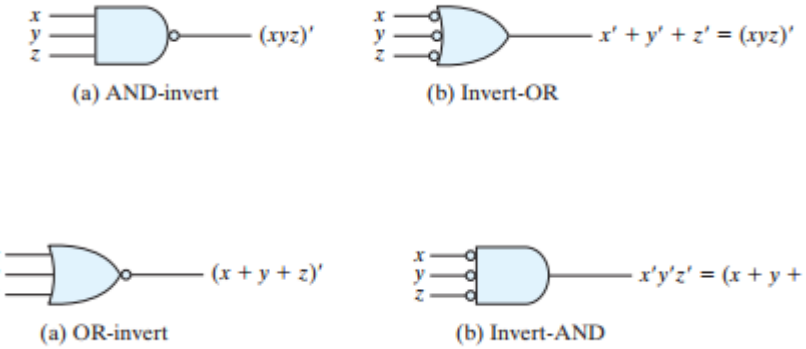


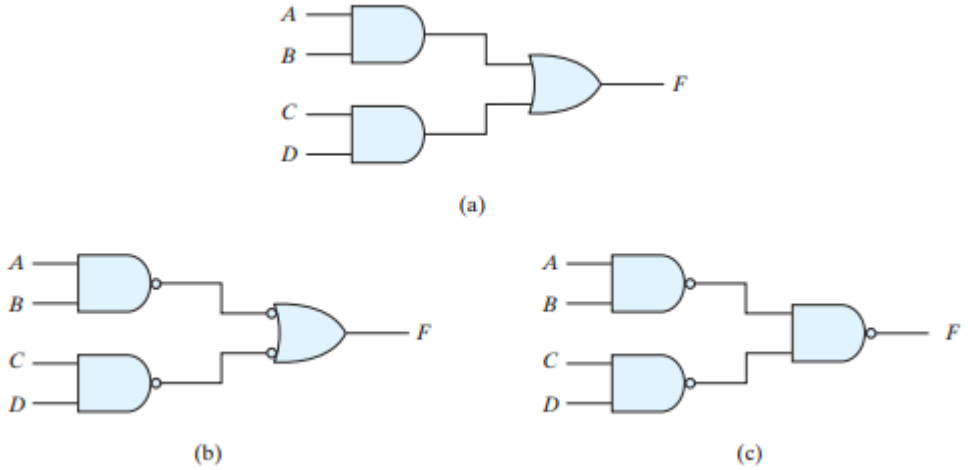
POS & SOP Simplification

Last Class

NAND & NOR implementation

- NAND and NOR are easier to implement





NAND implementation

Home Task

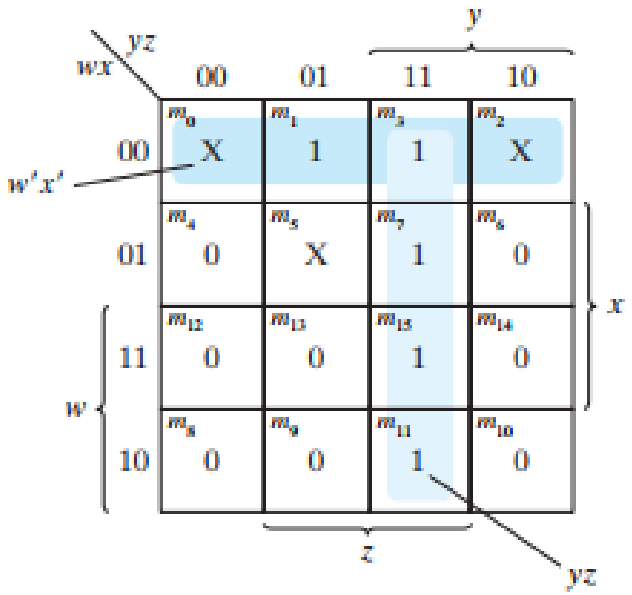
NOR Implementation

Home Task

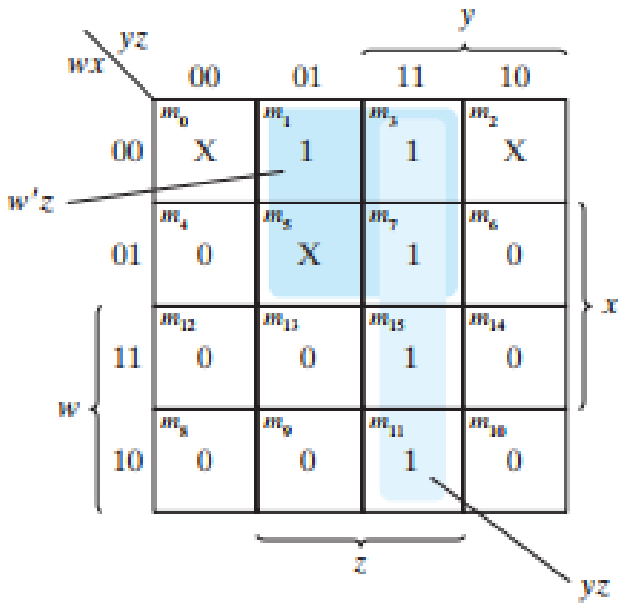
Don’t Care Condition

$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

$$d(w, x, y, z) = \Sigma(0, 2, 5)$$



(a) $F = yz + w'x'$



(b) $F = yz + w'z$

The Tabular Method / Quine-McCluskey Method

$$F = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$$

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 –	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

(c)

	w x y z
0, 2, 8, 10	– 0 – 0
0, 8, 2, 10	– 0 – 0
10, 11, 14, 15	1 – 1 –
10, 14, 11, 15	1 – 1 –

$$F = w'x'y' + x'z' + wy$$

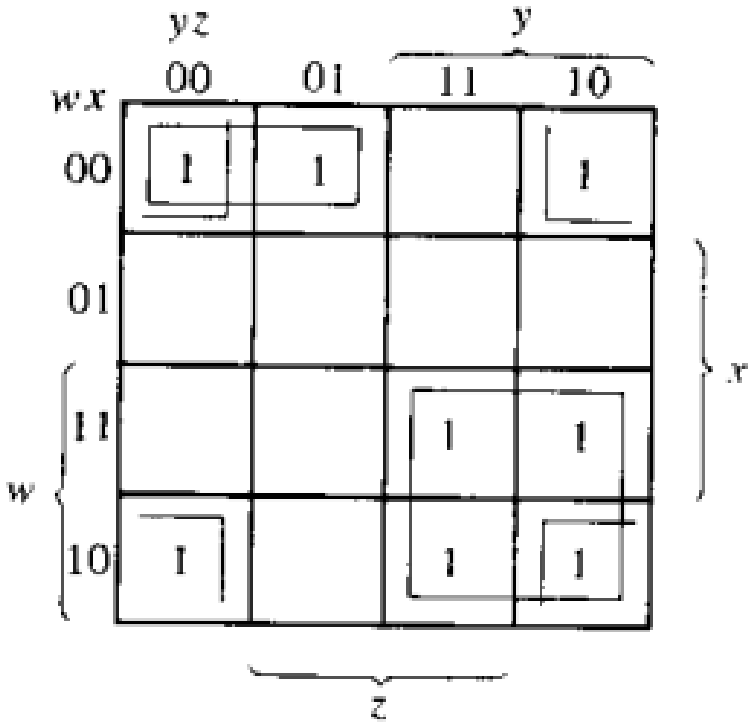
The Tabular Method (Alternative Way)

TABLE 3-6
Determination of Prime Implicants of Example 3-13 with Decimal Notation

(a)	(b)	(c)
0 ✓	0, 1 (1)	0, 2, 8, 10 (2, 8)
	0, 2 (2) ✓	0, 2, 8, 10 (2, 8)
1 ✓	0, 8 (8) ✓	
2 ✓		10, 11, 14, 15 (1, 4)
8 ✓	2, 10 (8) ✓	10, 11, 14, 15 (1, 4)
	8, 10 (2) ✓	
10 ✓		
	10, 11 (1) ✓	
11 ✓	10, 14 (4) ✓	
14 ✓		
	11, 15 (4) ✓	
15 ✓	14, 15 (1) ✓	

The Tabular Method

- Verification using K-map:



Prime Implicants Determination

- **Another Example:**

$$F(w, x, y, z) = \Sigma(1, 4, 6, 7, 8, 9, 10, 11, 15)$$

Determination of Prime Implicants for Example 3-14

(a)			(b)			(c)
0001	1	✓	1, 9	(8)		8, 9, 10, 11 (1, 2)
0100	4	✓	4, 6	(2)		8, 9, 10, 11 (1, 2)
1000	8	✓	8, 9	(1)	✓	
			8, 10	(2)	✓	
0110	6	✓				
1001	9	✓	6, 7	(1)		
1010	10	✓	9, 11	(2)	✓	
			10, 11	(1)	✓	
0111	7	✓				
1011	11	✓	7, 15	(8)		
			11, 15	(4)		
1111	15	✓				

Prime Implicants Selections

Prime Implicants

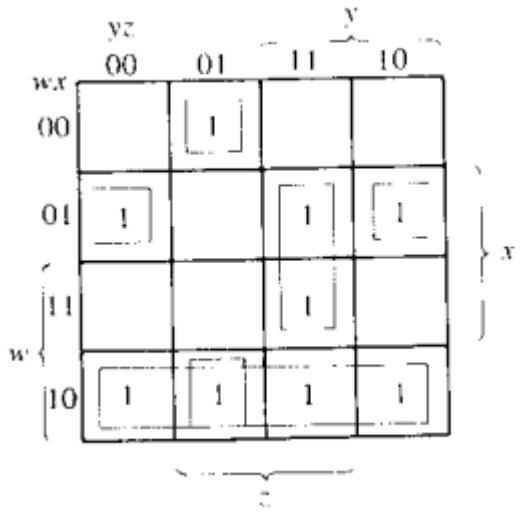
Decimal	Binary *w*	*x*	*y*	*z*	Term
1, 9 (8)	–	0	0	1	$x'y'z$
4, 6 (2)	0	1	–	0	$w'xz'$
6, 7 (1)	0	1	1	–	$w'xy$
7, 15 (8)	–	1	1	1	xyz
11, 15 (4)	1	–	1	1	wyz
8, 9, 10, 11 (1, 2)	1	0	–	–	wx'

Prime Implicant Table for Example 3-15

		1	4	6	7	8	9	10	11	15
✓ $x'y'z$	1, 9	X					X			
✓ $w'xz'$	4, 6		X	X						
$w'xy$	6, 7			X	X					
xyz	7, 15				X					X
wyz	11, 15								X	X
✓ wx'	8, 9, 10, 11					X	X	X	X	
		✓	✓	✓		✓	✓	✓	✓	

The Tabular Method

Verification using K-map:



$$F = x'y'z + w'xz' + wx' + xyz$$

Prime Implicants Selections

Administration

Administration

Administration

Administration

Administration

Administration

Administration