

Exerice 7

$$\begin{aligned} 1) \mathbb{P}(\{|X| \leq 1\}) &= \mathbb{P}(X = -1) + P(X = 0) + P(X = 1) \\ &= \frac{4}{13} + \frac{1}{13} + \frac{2}{13} = \frac{7}{13} \end{aligned}$$

$$\begin{aligned} 2) E[X] &= \sum_{k=-3}^1 k\mathbb{P}(X = k) \\ &= -3\frac{3}{13} + -2\frac{3}{13} + -1\frac{4}{13} + 1\frac{2}{13} \\ &= \frac{-9}{13} + \frac{-6}{13} + \frac{-4}{13} + \frac{2}{13} \\ &= \frac{-17}{13} \end{aligned}$$

$$\begin{aligned} Var(X) &= E[X^2] - E[X]^2 \\ E[X^2] &= 9\frac{3}{13} + 4\frac{3}{13} + 1\frac{4}{13} + 1\frac{2}{13} \\ &= \frac{45}{13} \end{aligned}$$

$$\text{Donc } Var(X) = E[X^2] - E[X]^2 = \frac{45}{13} - \left(\frac{17}{13}\right)^2 = \frac{296}{169}$$