Exerice 6

$X : \Omega \to \{0, \dots, N\}$ $N \geq 1$

<u>**Méthode 1**</u>**:** $\displaystyle\sum_{k=0}^{N-1} P(X > k)$

$$= \sum_{k=0}^{N-1} \sum_{j=k+1}^{N} P(X = j)$$

$$= \sum_{0 \leq k < j \leq N} P(X = j)$$

$$= \sum_{j=1}^{N} \sum_{k=0}^{j-1} P(X = j) = \sum_{j=1}^{N} jP(X = j)$$

<u>**Méthode 2**</u>**:**

$$\sum_{k=1}^{N} kP(X = k) = \sum_{k=1}^{N} k(P(X > k-1) - P(X > k))$$

$$= \sum_{k=1}^{N} k \left(P(X > k-1) - \sum_{k=1}^{N} P(X > k) \right)$$

$$= \sum_{k=0}^{N-1} (k+1)P(X > k) - \sum_{k=1}^{N} kP(X > k)$$

$$= \sum_{k=0}^{N-1} (k+1)P(X > k) - \sum_{k=0}^{N-1} kP(X > k) \text{ (*)}$$

$$= \sum_{k=0}^{N-1} P(X > k)$$

(*) on a $\displaystyle\sum_{k=1}^{N} kP(X > k) = \sum_{k=0}^{N-1} kP(X > k) - 0(P(X > 0)) + NP(X > N)$
or $P(X > N) = 0$ car X ne peut pas être pas plus grand que N
Donc $\displaystyle\sum_{k=1}^{N} kP(X > k) = \sum_{k=0}^{N-1} kP(X > k)$

1- $\Omega = \{1, \ldots, N\}^n$

$Card(\Omega) = N^n$

$P(X \leq k) = \left(\frac{k}{N}\right)^n$

$P(X > k) = 1 - P(X \leq k) = 1 - \left(\frac{k}{N}\right)^n$, $\forall k \geq 1$

$\sum_{k=0}^{N-1} P(X > k) = \sum_{k=0}^{N-1} \left(1 - \left(\frac{k}{N}\right)^n\right) = N - \sum_{k=1}^{N-1} \left(\frac{k}{N}\right)^n$

2- $P(X = k) = P(X \leq k) - P(X \leq k-1) = \left(\frac{k}{N}\right)^n - \left(\frac{k-1}{N}\right)^n$, $\forall k \in \{1, \ldots, N\}$