Exercice 3

1) $\Omega = \{A \subset \{1, \ldots, 150\}, card(A) = k\}$

Donc si $\omega \in \Omega$

$\omega = \{i_1, \ldots, i_k\}$

Et $i_1, \ldots, i_k$ ce sont les sièges réservés

2)

a) – Si $k < 50, P(A_1) = 0$

– Si $k \geq 50, P(A_1) = \dfrac{\begin{pmatrix} 100 \\ k-50 \end{pmatrix}}{Card(\Omega)} = \dfrac{\begin{pmatrix} 100 \\ k-50 \end{pmatrix}}{\begin{pmatrix} 150 \\ k \end{pmatrix}}$

b) – Si $k < 100, P(A_1 \cap A_2) = 0$

– Si $k \geq 100, P(A_1 \cap A_2) = \dfrac{\begin{pmatrix} 50 \\ k-100 \end{pmatrix}}{\begin{pmatrix} 150 \\ k \end{pmatrix}}$

c) – Si $k < 150, P(A_1 \cap A_2 \cap A_3) = 0$

– Si $k = 150, P(A_1 \cap A_2 \cap A_3) = 1$

5) $E^c = A_1 \cup A_2 \cup A_3$

6) $P(A_1 \cup A_2 \cup A_3) = P(A_1 \cup (A_2 \cup A_3))$

$= P(A_1) + P(A_2 \cup A_3) - P(A_1 \cap (A_2 U A_3))$

$= P(A_1) + P(A_2) + P(A_3) - P(A_2 \cap A_3) - P(A_1 \cap (A_2 U A_3))$

De plus $P(A_1 \cap (A_2 \cup A_3)) = P((A_1 \cap A_2) \cup (A_1 \cap A_3))$

$= P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$

Donc $P(A_1 \cup A_2 \cup A_3) =$

$P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

7) $P(E) = 1 - P(E^c)$

$$= 1 - P(A_1 \cup A_2 \cup A_3)$$

$$= 1 - 3P(A_1) + 3P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$$

car $P(A_1) = P(A_2) = P(A_3)$

et $P(A_1 \cap A_2) = P(A_1 \cap A_3) = P(A_2 \cap A_3)$

Si $k \geq 148 P(E) = 0$ Si $k \in \{100, \ldots, 147\} P(E) = 1 - \frac{3}{\begin{pmatrix} 150 \\ k \end{pmatrix}} \left(\begin{pmatrix} 100 \\ k-50 \end{pmatrix} - \begin{pmatrix} 50 \\ k-100 \end{pmatrix} \right)$

Si $k \in \{50, \ldots, 99\} P(E) = 1 - \frac{3}{\begin{pmatrix} 150 \\ k \end{pmatrix}} \begin{pmatrix} 100 \\ k-50 \end{pmatrix}$

si $k \leq 49, P(E) = 1$