Exerice 5

1- On note S la somme des valeurs des 2 dés

Alors $E[S] = \sum_{k=2}^{12} k\mathbb{P}(S=k)$

$$\mathbb{P}(S=k) = \begin{cases} \frac{k-1}{36} \text{ si } k \leq 7 \\ \frac{13-k}{36} \text{ si } k \geq 7 \end{cases}$$

$$\begin{aligned} \text{Donc } E[S] &= \sum_{k=2}^{7} \frac{k(k-1)}{36} + \sum_{k=8}^{12} \frac{13k-k^2}{36} \\ &= \frac{1}{6}\left(\sum_{k=2}^{7} k^2 - k + \sum_{k=8}^{12} 13k - k^2\right) \\ &= \frac{1}{6}(2+6+12+20+30+42+40+36+30+22+12) \\ &= 7 \end{aligned}$$

2. $E[S] = E[x_1 + x_2] = E[x_1] + E[x_2]$

$$\begin{aligned} &= 2E[x_1] \\ &= 2\sum_{k=1}^{6} k\mathbb{P}(X=k) \\ &= 2\sum_{k=1}^{6} k\frac{1}{6} \\ &= \frac{1}{3}\sum_{k=1}^{6} k \\ &= \frac{1}{3} * \frac{6*7}{2} = 7 \end{aligned}$$