Math 385
Section 000
Semester winter23
Carlos Tapia Assignment HW1

0.5

Problem 1a. Use the Intermediate Value Theorem to prove that $f(c) = 0$ for some $0 < c < 1$.
$f(x) = x^3 - 4x + 1$.

Solution: if $f(c) = 0$ then $f(0) < f(c) < f(1)$;
Since $f(0) = 1$ and $f(1) = -2$, $f(c) = 0$ for some c between 0 and 1/2 by the intermediate value theorem. □

Problem 2c. Find c satisfying the Mean Value Theorem for $f(x)$ on the interval $[0, 1]$.
$f(x) = 1/(x + 1)$

Solution:

$$f\prime(x) = \frac{-1}{(x+1)^2}$$

let $a = 0, b = 1$, then $f\prime(c) = \frac{f(b)-f(a)}{b-a}$

$$\begin{aligned} f\prime(x) &= \frac{-1}{(c+1)^2} = \frac{\frac{1}{2}-1}{1-0} \\ &= \frac{-1}{(c+1)^2} = \frac{1}{2} \\ &= (c+1)^2 = 2 \\ &= c = \pm\sqrt{2}-1 \\ &= c = \sqrt{2}-1 \end{aligned}$$

$c = \sqrt{2} - 1$ satisfies [0,1] □

Problem 4a. Find the Taylor polynomial of degree 2 about the point $x = 0$ for the following function: $f(x) = e^{x^2}$

Solution:

$$1 + x^2 + \frac{x^4}{2} + \cdots$$

□

Problem 7. (a) Find the Taylor polynomial of degree 4 for $f(x) = ln(x)$ about the point $x = 1$. (b) Use the result of (a) to approximate f(0.9) and f(1.1). (c) Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part (b). Which of the two approximations in part (b) do you expect to be closer to the correct value? (d) Use a calculator to compare the actual error in each case with your error bound from part (c).

Solution:

$$f(x) = ln(x)$$

(a) Taylor series of $ln(x)$ @ x = 1

$$f(a) + \frac{f\prime(a)}{1!}(x-a)^1 + \frac{f\prime(a)}{2!}(x-a)^2 + \frac{f\prime(a)}{3!}(x-a)^3 + \frac{f\prime(a)}{4!}(x-a)^4$$

$$f(1) + \frac{f\prime(1)}{1!}(x-1)^1 + \frac{f\prime\prime(1)}{2!}(x-1)^2 + \frac{f\prime\prime\prime(1)}{3!}(x-1)^3 + \frac{f\prime\prime\prime\prime(1)}{4!}(x-1)^4$$

$$(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

(b)
$f(0.9) = -0.105358333333333$
$f(1.1) = +0.095308333333333$
(c)

$$R_4(x) = f(x) - P_4(x) = \frac{f^5(c)}{5}(x-1)^5$$

$c \in (1, 1.1)$

$$R_4 = \frac{24(0.1)^5}{5!} \approx 2\text{e}{-6}$$

$c \in (0.9, 1)$

$$R_4 = \frac{\frac{24}{0.9^5}(0.1)^5}{5!} \approx 3.3870\text{e}{-6}$$

Max Error : $f(1.1) \approx 2\text{e}{-6}$ and $f(0.9) \approx 3.3870\text{e}{-6}$ □

1.1

Problem 2a. Use the Intermediate Value Theorem to find an interval of length one that contains a root of the equation. $x^5 + x = 1$

Solution:

$$f(x) = x^5 + x - 1$$
$$f(0) = -1$$
$$f(1) = 1$$

According to the Intermediate Value Theorem there is a solution on interval [0,1]. □

Problem 2c. Use the Intermediate Value Theorem to find an interval of length one that contains a root of the equation.$lnx + x^2 = 3$

Solution:

$$f(x) = ln(x) + x^2 - 3$$
$$f(1) = -2$$
$$f(2) = 1.6...$$

According to the Intermediate Value Theorem there is a solution on interval [1,2] □

Problem 4a. Consider the equations in Exercise 2. Apply two steps of the Bisection Method to find an approximate root within 1/8 of the true root. $x^5 + x = 1$

Solution: $f(x) = x^5 + x - 1$

Interval	x	f(x)	bisection
[0]	0	-1	
[0, 1]	1	1	0.5
[0, 1]	0.5	-4.6875	0.75
[0.5, 1]	0.75	.009521	0.675
[0.5, 0.75]	0.675	-.2796...	.6875

□

Problem 4c. Consider the equations in Exercise 2. Apply two steps of the Bisection Method to find an approximate root within 1/8 of the true root. $lnx + x^2 = 3$

Solution: $f(x) = ln(x) + x^2 - 3$

Interval	x	f(x)	bisection
[1]	1	-2	
[1, 2]	2	1.693	1.5
[1, 2]	1.5	-0.3445...	1.75
[1.5, 2]	1.75	0.6221...	1.625
[1.5, 1.75]	1.625	.1261...	1.5625

□

Problem 5. Consider the equation $x^4 = x^3 + 10$. (a) Find an interval $[a, b]$ of length one inside which the equation has a solution. (b) Starting with $[a, b]$, how many steps of the Bisection Method are required to calculate the solution within 10^-10? Answer with an integer.

Solution:

$$f(x) = x^4 - x^3 + 10 = 0$$

(a)$f(2) = -2$ and $f(3) = 44$; Interval [2,3] has a solution.
(b) $|C_n - r| < \frac{b-a}{2^{n+1}}$

$$10^{-10} < \frac{3-2}{2^{n+1}}$$

$$10^{10} < 2^{n+1}$$

$$log_2(10^{10}) - 1 < n$$

Since $n > 32.2192$, you would require 33 iterations of Bisection to be within 10^{-10} □

Computer

Problem 2a. Use the Bisection Method to find the root to eight correct decimal places. $x^5 + x = 1$

Solution: See attached file. (0.7548776641488075, 27) □

Problem 2c. Use the Bisection Method to find the root to eight correct decimal places. $(b) sinx = 6x + 5$

Solution: See attached file. (-0.9708989188075066, 27) □

Problem 3b. Use the Bisection Method to locate all solutions of the following equations. Sketch the function by using MATLAB's plot command and identify three intervals of length one that contain a root. Then find the roots to six correct decimal places. $(b)ex-2+x3-x=0$

Solution: See attached file. (0.8194315209984779, 27) □

Problem 6. Use the Bisection Method to calculate the solution of $cos(x)=sin(x)$ in the interval $[0,1]$ within six correct decimal places.

Solution: See attached file. (0.7853975296020508, 20) □