Math 385
Section 000
Semester winter23
Carlos Tapia Assignment HW

Section 1.3:

Problem 1. Find the forward and backward error for the following functions, where the root is 3/4 and the approximate root is $x_a = 0.74$

Solution: forward error = 0.01
Backward error

$$(a) f(x) = 4x - 3 : \quad x = 0.74 \quad = 0.04$$
$$(b) f(x) = (4x - 3)^2 : \quad x = 0.74 \quad = 0016$$
$$(c) f(x) = (4x - 3)^3 : \quad x = 0.74 \quad = 000064$$
$$(d) f(x) = \sqrt{4x - 3} : \quad x = 0.74 \quad = 0.34199518933534$$

□

Problem 3. (a) Find the multiplicity of the root r = 0 of $f(x) = 1 - cosx$. (b) Find the forward and backward errors of the approximate root xa = 0.0001.

Solution:

$$(a)\ f\prime(x) = sin(x) \ \ : x = 0 = 1$$
$$f\prime\prime(x) = cos(x) \ \ : x = 0 = 1$$

$$Foward\ = 0.0001$$
$$Backward\ = f(x_a) = 5x10^{-9}$$

□

Computer

Problem 5. Use 1.21 to approximate the root of $f(x) = (x-1)(x-2)(x-3)(x-4)-10-6x6$ near r = 4. Find the error magnification factor. Use fzero to check your approximation

```
(env) ~/Documents/_School/Depaul/MAT385/code > python3 accuracy.py
New root =  [4.00068251]
Change in root =  0.0006826666666090866
Error magnification factor =  170.66666665227163
```

Section 1.5:

Problem 1a. Apply two steps of the Secant Method to the following equations with initial guesses x0=1 and x1=2

Solution:

$$x_2 = x_1 - \frac{(x_1 - x_0)f(x_1)}{f(x_1) - f(x_0)} = 8/5$$

$$x_2 = x_1 - \frac{(x_1 - x_0)f(x_1)}{f(x_1) - f(x_0)} = 1.742268$$

□

Problem 7. Consider the following four methods for calculating $2^{1/4}$, the fourth root of 2. (a) Rank them for speed of convergence, from fastest to slowest. Be sure to give reasons for your ranking.

Solution: Answer = SECANT, FPI_2, BISECTION, FPI_1 doesn't converge.
1.BISECTION :step is 0.5.
2.SECANT :$f(2^{1/4}) = 0; f\prime(2^{1/4}) = 0$
3.FPI_1 :$g(2^{1/4}) = \frac{2^{1/4}}{2} + \frac{1}{2^{3/4}} = 2^{1/4} > 1$
4.FPI_2 :$g\prime(2^{1/4}) = 1/3 - \frac{1}{(2^{1/4})^4} = -1/6$

□

Computer

Problem 1a. Use the Secant Method to find the (single) solution of each equation in Exercise 1.

```
(env) ~/Documents/_School/Depaul/MAT385/code > python3 secant.py
a. iterations: 5         solution: 1.7692921651959501
b. iterations: 5         solution: 1.672821698053204
c. iterations: 4         solution: 1.12998009605245
```

Section 3.1:

Problem 1a. Use Lagrange interpolation to find a polynomial that passes through the points. $(a)(0,1),(2,3),(3,0)$ $(b)(-1,0),(2,1),(3,1),(5,2)$ $(c)(0,-2),(2,1),(4,4)$

Solution:

$$P(x) = \frac{(x-2)(x-3)}{-2(-3)} + \frac{x(x-3)}{2(3)} + \frac{x(x-2)}{3(1)}$$
$$(-4/3)x^2 + (11/3)x + 1$$

□

Problem 1b.

Solution:

$$P(x) = \frac{(x+1)(x-3)(x-5)}{(2+1)(2-3)(2-5)} + \frac{(x+1)(x-2)(x-5)}{(3+1)(3-2)(3-5)} + \frac{(x+1)(x-3)(x-2)}{(5+1)(5-2)(5-3)}$$

$$(1/24)x^3 - (1/4)x^2 + (11/24)x + 3/4$$

□

Problem 2a. Use Newton's divided differences to find the interpolating polynomials of the points in Exercise 1, and verify agreement with the Lagrange interpolating polynomial.

Solution:

x	f(x)	1st	2nd
0	1	1	-
2	3	-	-4/3
3	0	-3	-

Answer $= (-4/3)x^2 + (11/3)x + 1$ □

Problem 2b.

Solution:

x	f(x)	1st	2nd	3rd
-1	0	(1/3)	-(1/12)	(1/24)
2	1	0	(1/6)	-
3	1	(1/2)	-	-
5	2	-	-	-

Answer $= (1/24)x^3 - (1/4)x^2 + (11/24)x + 3/4$ □

Problem 5. (a) Find a polynomial P(x) of degree 3 or less whose graph passes through the four data points (-2,8),(0,4),(1,2),(3,-2). (b) Describe any other polynomials of degree 4 or less which pass through the four points in part (a).

Solution:

x	f(x)	1st	2nd	3rd
-2	8	-2	0	0
0	4	-2	0	-
1	2	-2	-	-
3	-2	-	-	-

Answer $= 8 - 2(x+2) + 0 + 0$ □

Problem 12. Can a degree 3 polynomial intersect a degree 4 polynomial in exactly five points? Explain.

Solution: The max number of points that would be the same is 4; therefore, 5 points is impossible. Setting both polynomial equal to each other we get a 4th degree polynomial; meaning that the roots will only intersect 4 times.
$ax^4 + bx^3 + cx^2 + dx + e = bx^3 + cx^2 + dx + e$ □

Problem 17. The estimated mean atmospheric concentration of carbon dioxide in earth's atmosphere is given in the table that follows, in parts per million by volume. Find the degree 3 interpolating polynomial of the data and use it to estimate the CO2 concentration in (a) 1950 and (b) 2050. (The actual concentration in 1950 was 310 ppm.)

Solution:

x	f(x)	1st	2nd	3rd
1800	280	(3/50)	(1/1000)	(1/62500)
1850	283	(4/25)	(21/5000)	-
1950	291	(79/100)	-	-
2000	370	-	-	-

f(x) = $(1/62500)x^3 - (439/5000)x^2 + (3212/20)x - 97730$
a. f(1950) = 316.
b. f(2050) = 465. □

Computer

Problem 1c. Apply the following world population figures to estimate the 1980 population, using (a) the straight line through the 1970 and 1990 estimates; (b) the parabola through the 1960, 1970, and 1990 estimates; and (c) the cubic curve through all four data points. Compare with the 1980 estimate of 4452584592.

```
(env) ~/Documents/_School/Depaul/MAT385/code > python3 newtdd.py
Newton interpolating polynomial:
  3039585530.0 + 66789035.7(x-1960) + 397328.6983333334(x-1960)(x-1970) + -9028.152083333345(x-1960)(x-1970)(x-1990)

 y value at x=1980 is 4,472,888,287.833333
Difference is 4,472,888,287.833333 - 4,452,584,592 = 20,303,695.833333015
```

Problem EXTRA. Build a "cosine" calculator function on the interval $[0, \pi/2]$ from an interpolating polynomial at nodes

$$[0, 1], [\pi/6, \sqrt{3}/2], [\pi/3, 1/2], [\pi/2, 0]$$

```
(env) ~/Documents/_School/Depaul/MAT385/code > python3 Extra.py
Newton interpolating polynomial:
  1.0 + -0.255872630837368(x-0) + -0.42320992478470193(x-0)(x-0.5235987755982988) + 0.11387189907141194(x-0)(x-0.5235987755982988)(x-1.0471975511965976)

 y value at x=0.6283185307179586 is 0.8082459488697549
```