Section 3.5:

Problem 1b. Find the one-piece Bézier curve (x(t), y(t)) defined by the given four points. (b) (1,1), (0,0), (–2,0) (-2,1)

Solution:

$$\begin{aligned}
B_x &= 3(X_2 - X_1) &&= 3(0-1) = -3\\
C_x &= 3(X_3 - X_2) - B_x &&= 3(-2-0) = -3\\
D_x &= X_4 - X_1 - B_x - C_x &&= -2-1-(-3)-(-3) = 3\\
\\
B_y &= 3(y_2 - y_1) &&= 3(0-1) = -3\\
C_y &= 3(y_3 - y_2) - B_y &&= 3(0-0)-(-3) = 3\\
D_y &= y_4 - y_1 - B_y - C_y &&= 1-1-(-3)-(3) = 0
\end{aligned} \tag{1}$$

Answer =
x(t)= $1 - 3t - 3t^2 + 3t^3$
y(t)=$1 - 3t + 3t^2$

□

Problem 2b. Find the first endpoint, two control points, and last endpoint for the following one-piece Bézier curves.
$x(t) = 3 + 4t - t^2 + 2t^3$
$y(t) = 2 - t + t^2 + 3t^3$

Solution:

$$\begin{aligned}
a_x &= 3 = X_1\\
b_x &= 4 = 3(x_2 - x_1)\\
c_x &= -1 = 3(x_3 - x_2) - b_x\\
d_x &= 2 = x_4 - x_1 - b_x - c_x\\
\\
a_y &= 3 = y_1\\
b_y &= 4 = 3(y_2 - y_1)\\
c_y &= -1 = 3(y_3 - y_2) - b_y\\
d_y &= 2 = y_4 - y_1 - b_y - c_y
\end{aligned} \tag{2}$$

Answer=
$x1, x2, x3, x4 = \{3, 13/3, 16/3, 8\}$
$y1, y2, y3, y4 = \{2, 5/3, 5/3, 5\}$ points = $(3,2)$,$(13/3, 5/3)$,$(16/3, 5/3)$,$(8, 5)$ □

Problem 7. Find a one-piece Bézier spline that has vertical tangents at its endpoints (-1, 0) and (1, 0) and that passes through (0,1).

Solution:

$$
\begin{array}{ll}
 & b_x = 3(X_2 - x_1) = 0 \\
 & c_x = 3(X_3 - x_2) - b_x = 6 \\
(X_1, Y_1) = -1, 0; & d_x = X_4 - x_1 - b_x - c_x = -4 \\
(X_2, Y_2) = -1, y_2; & \\
(X_3, Y_3) = -1, y_3; & b_y = 3(y_2 - y_1) = 3U_1 \\
(X_4, Y_4) = 1, 0; & c_y = 3(y_3 - y_2) - b_y = -3U_2 \\
 & d_y = y_4 - y_1 - b_y - c_y = 0
\end{array} \tag{3}
$$

Answer =
X(t)=$-1 + 6t^2 - 4t^3$
Y(t)=$0 + 3yt - 3yt^2 + 0$
If you set it equal to 0, you find a root at 1/2.
$1 = 3y(1/2) - 3y(1/2)^2$ gives you y = 4/3
Therefore
X(t)=$-1 + 6t^2 - 4t^3$
Y(t) = $4t - 4t^2$ □

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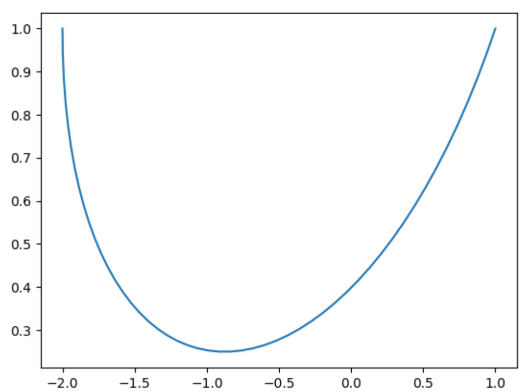
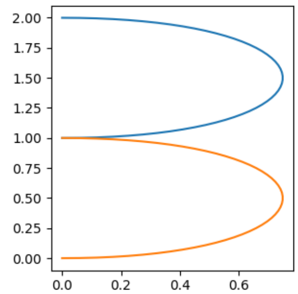
Problem 1b.



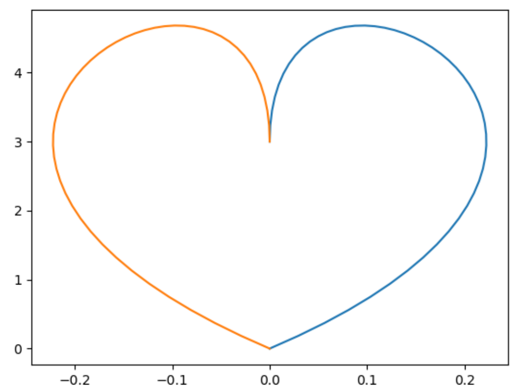
Solution: □

Problem 5.



Solution: □

Problem EXTRA.



Solution: □