ZS z03-021 02.11.09 riešil(a): PETER MASIAR 51025 09.11.09

1. Vyjadrite ako zlomok periodické číslo $4,86\overline{76} = 4,86767676\ldots$.

OK 1kg

$a_1 = 0,0076$

$q = 0,01$

$4,86\overline{76} = 4,86 + 0,0076 + 0,000076 \ldots = \frac{486}{100} + \frac{0,0076}{1-0,01} = \frac{486}{100} + \frac{76}{9900} = \frac{48114+76}{9900} = \frac{4819}{990}$;

2. Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 4$, $a_{n+1} = \sqrt{26a_n - 48}$, $n \in N$.

$a_1 = 4$, $a_{n+1} = \sqrt{26a_n - 48}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$

$a_n = \sqrt{26a_n - 48}$

$a_n^2 = 26a_n - 48$

$a_n^2 - 26a_n + 48 = 0$

$D = 26^2 - 4 \cdot 1 \cdot 48$

$D = 484$

$\sqrt{D} = \pm 22$

$a_1 = 4$

$a_2 = \sqrt{26 \cdot 4 - 48} = \sqrt{56}$

$a_{n_{1,2}} = \frac{26 \pm 22}{2}$

$a_{n_1} = 2$

$a_{n_2} = 24$

$\lim_{n\to\infty} a_n = 24$

3. Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 3$, $a_{n+1} = \sqrt{26a_n - 48}$, $n \in N$.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$

$a_n^2 = 26a_n - 48 \Rightarrow a_n = 24$ $\lim_{n\to\infty} a_n = 24$

4. Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2$, $a_{n+1} = \sqrt{26a_n - 48}$, $n \in N$.

$a_1 = 2$

$a_2 = \sqrt{26 \cdot 2 - 48} = \sqrt{4} = 2$

$a_3 = \sqrt{26 \cdot 2 - 48} = a_2 \Rightarrow \lim_{n\to\infty} a_n = 2$

5. Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 1$, $a_{n+1} = \sqrt{26a_n - 48}$, $n \in N$.

$a_2 = \sqrt{26 \cdot 1 - 48} = \sqrt{-22} \Rightarrow \nexists \Rightarrow \lim_{n\to\infty} a_n = \nexists$

6. $\lim\limits_{n\to\infty} \frac{n^5-2n^6-4n^4-3}{-8n^4-4n^5+2} = \lim\limits_{n\to\infty} \frac{\cancel{n^6}\left(\frac{1}{n}-2-\frac{4}{n^2}-\frac{3}{n^6}\right)}{\cancel{n^5}\left(-\frac{8}{n}-4+\frac{2}{n^5}\right)} = \lim\limits_{n\to\infty} \frac{n-2}{-4} = -\infty$

7. $\lim\limits_{n\to\infty} \frac{-8n^4-4n^5+2}{n^5-2n^6-4n^4-3} = \lim\limits_{n\to\infty} \frac{\cancel{n^5}\left(-\frac{8}{n}-4+\frac{2}{n^5}\right)}{\cancel{n^6}\left(\frac{1}{n}-2-\frac{4}{n^2}-\frac{3}{n^6}\right)} = \lim\limits_{n\to\infty} \frac{-4}{n-2} = 0$

8. $\lim\limits_{n\to\infty} \frac{n^5-2n^6-4n^4-3}{-8n^4-4n^6+2} = \lim\limits_{n\to\infty} \frac{\cancel{n^6}\left(\frac{1}{n}-2-\frac{4}{n^2}-\frac{3}{n^6}\right)}{\cancel{n^6}\left(-\frac{8}{n^2}-4+\frac{2}{n^6}\right)} = \frac{1}{2}$

9. $\lim\limits_{n\to\infty} \frac{-8n^4-4n^6+2}{n^5-2n^6-4n^4-3} = \lim\limits_{n\to\infty} \frac{\cancel{n^6}\left(-\frac{8}{n^2}-4+\frac{2}{n^6}\right)}{\cancel{n^6}\left(\frac{1}{n}-2-\frac{4}{n^2}-\frac{3}{n^6}\right)} = 2$

10. $\lim\limits_{n\to\infty} \frac{\sqrt[5]{n}-2\sqrt[6]{n}-4\sqrt[4]{n}-3}{-8\sqrt[4]{n}-4\sqrt[5]{n}+2} = \lim\limits_{n\to\infty} \frac{n^{\frac15}-2n^{\frac16}-4n^{\frac14}-3}{-8n^{\frac14}-4n^{\frac15}+2} = \lim\limits_{n\to\infty} \frac{\cancel{n^{\frac14}}\left(n^{-\frac{1}{20}}-2n^{-\frac{1}{12}}-4-3n^{-\frac14}\right)}{\cancel{n^{\frac14}}\left(-8-4n^{-\frac{1}{20}}+2n^{-\frac14}\right)} = \frac{1}{2}$

11. $\lim\limits_{n\to\infty} \frac{-8\sqrt[4]{n}-4\sqrt[5]{n}+2}{\sqrt[5]{n}-2\sqrt[6]{n}-4\sqrt[4]{n}-3} = \lim\limits_{n\to\infty} \frac{-8n^{\frac14}-4n^{\frac15}+2}{n^{\frac15}-2n^{\frac16}-4n^{\frac14}+3} = \lim\limits_{n\to\infty} \frac{\cancel{n^{\frac14}}\left(-8-4n^{-\frac{1}{20}}+2n^{-\frac14}\right)}{\cancel{n^{\frac14}}\left(n^{-\frac{1}{20}}-2n^{-\frac{1}{12}}-4+3n^{-\frac14}\right)} = 2$

12. $\lim\limits_{n\to\infty} \frac{\sqrt[5]{n}-2\sqrt[6]{n}-4\sqrt[4]{n}-3}{-8\sqrt[6]{n}-4\sqrt[5]{n}+2} = \lim\limits_{n\to\infty} \frac{n^{\frac15}-2n^{\frac16}-4n^{\frac14}-3}{-8n^{\frac16}-4n^{\frac15}+2} = \lim\limits_{n\to\infty} \frac{n^{\frac14}\left(n^{-\frac{1}{20}}-2n^{-\frac{1}{12}}-4-3n^{-\frac14}\right)}{\cancel{n^{\frac15}}\left(-8n^{-\frac{1}{30}}-4+2n^{-\frac15}\right)} = \infty$

13. $\lim\limits_{n\to\infty} \frac{-8\sqrt[6]{n}-4\sqrt[5]{n}+2}{\sqrt[5]{n}-2\sqrt[6]{n}-4\sqrt[4]{n}-3} = \lim\limits_{n\to\infty} \frac{-8n^{\frac16}-4n^{\frac15}+2}{n^{\frac15}-2n^{\frac16}-4n^{\frac14}-3} = \lim\limits_{n\to\infty} \frac{\cancel{n^{\frac15}}\left(-8n^{-\frac{1}{30}}-4+2n^{-\frac15}\right)}{n^{\frac14}\left(n^{-\frac{1}{20}}-2n^{-\frac{1}{12}}-4-3n^{-\frac14}\right)} = 0$

14. $\lim\limits_{n\to\infty} \frac{n^5-2n^6-4n^4+3\cdot3^n}{-8n^4-4n^6-2\cdot3^n} = \lim\limits_{n\to\infty} \frac{\cancel{3^n}\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)}{\cancel{3^n}\left(-\frac{8n^4}{3^n}-\frac{4n^6}{3^n}-2\right)} = -\frac{3}{2}$

15. $\lim\limits_{n\to\infty} \frac{-8n^4-4n^6-2\cdot3^n}{n^5-2n^6-4n^4+3\cdot3^n} = \lim\limits_{n\to\infty} \frac{3^n\left(-\frac{8n^4}{3^n}-\frac{4n^6}{3^n}-2\right)}{\cancel{3^n}\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)} = -\frac{2}{3}$

16. $\lim\limits_{n\to\infty} \frac{n^5-2n^6-4n^4+3\cdot3^n}{-8n^4-4n^6-2\cdot3^{n+1}} = \lim\limits_{n\to\infty} \frac{3^n\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)}{3\cdot\cancel{3^n}\left(-\frac{8n^4}{3^{n+1}}-\frac{4n^6}{3^{n+1}}-2\right)} = -\frac{1}{2}$

17. $\lim\limits_{n\to\infty} \frac{-8n^4-4n^6-2\cdot3^{n+1}}{n^5-2n^6-4n^4+3\cdot3^n} = \lim\limits_{n\to\infty} \frac{3\cdot\cancel{3^n}\left(-\frac{8n^4}{3^{n+1}}-\frac{4n^6}{3^{n+1}}-2\right)}{\cancel{3^n}\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)} = -2$

18. $\lim\limits_{n\to\infty} \frac{n^5-2n^6-4n^4+3\cdot3^n}{-8n^4-4n^5-2\cdot3^{n+1}} = \lim\limits_{n\to\infty} \frac{\cancel{3^n}\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)}{3\cdot\cancel{3^n}\left(-\frac{8n^4}{3^{n+1}}-\frac{4n^5}{3^{n+1}}-2\right)} = -\frac{1}{2}$

19. $\lim\limits_{n\to\infty} \frac{-8n^4-4n^5-2\cdot3^{n+1}}{n^5-2n^6-4n^4+3\cdot3^n} = \lim\limits_{n\to\infty} \frac{3\cdot\cancel{3^n}\left(-\frac{8n^4}{3^{n+1}}-\frac{4n^5}{3^{n+1}}-2\right)}{\cancel{3^n}\left(\frac{n^5}{3^n}-\frac{2n^6}{3^n}-\frac{4n^4}{3^n}+3\right)} = -2$

20. $\lim_{n\to\infty} [\sqrt{n^2-1} - \sqrt{n^2-3n}] = \cdot \frac{\sqrt{n^2-1}+\sqrt{n^2-3n}}{\sqrt{n^2-1}+\sqrt{n^2-3n}} = \lim_{n\to\infty} \frac{n^2-1-n^2+3n}{n^2-1^{\frac{1}{2}}+n-3n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{3n-1}{2n-3n^{\frac{1}{2}}-1} =$

$= \lim_{n\to\infty} \frac{n(3-\frac{1}{n})}{n(2-3n^{-\frac{1}{2}}-\frac{1}{n})} = \frac{3}{2}$

21. $\lim_{n\to\infty} [\sqrt[4]{n^4-1} - \sqrt[4]{n^4-3n}] = \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2}\cdot\sqrt[4]{n^4-3n} + \sqrt[4]{n^4-1}\cdot\sqrt[4]{(n^4-3n)^2} + \sqrt[4]{(n^4-3n)^3}}{-\,\|\,-} =$

$\lim_{n\to\infty} \frac{n(3-\frac{1}{n})}{n^3\sqrt[4]{(1-\frac{1}{n^4})^3} + \sqrt[4]{(\frac{1}{n^2}+\frac{1}{n^4})^2}\cdot\sqrt[4]{1-\frac{3}{n^3}} + \sqrt[4]{1-\frac{1}{n^4}}\cdot\sqrt[4]{(1-\frac{3}{n^3})^2} + \sqrt[4]{(1-\frac{3}{n^3})^3}} = 0$

22. $\lim_{n\to\infty} [\sqrt{n^2-1} - n - 3] = \cdot \frac{\sqrt{n^2-1}+(n-3)}{-\,\|\,-} = \lim_{n\to\infty} \frac{n^2-1-(n-3)^2}{\sqrt{n^2-1}+(n-3)} = \lim_{n\to\infty} \frac{n^2-1-n^2+6n-9}{n(\sqrt{\frac{1}{n}-\frac{1}{n^2}}+1-\frac{3}{n})} = \lim_{n\to\infty} \frac{n(+6-\frac{10}{n})}{n\sqrt{1-0}+1-0} =$

$= 3$

23. $\lim_{n\to\infty} [\sqrt[4]{n^4-1} - n - 3] = \lim_{n\to\infty} \frac{n^4(1-\frac{1}{4}-(\frac{1}{n^3}-\frac{3}{n^4})^4)}{n^3\sqrt[4]{(1-\frac{1}{n^4})^3} + \sqrt[4]{(1-\frac{1}{n^4})^2}\cdot(\frac{1}{n^3}-\frac{3}{n^4}) + \sqrt[4]{(1-\frac{1}{n^4})}\cdot(\frac{1}{n^3}-\frac{3}{n^4})^2 + (\frac{1}{n^3}-\frac{3}{n^4})^2} = \infty$

24. $\lim_{n\to\infty} [\sqrt{n^2-1} - \sqrt{n+3}] = \cdot \frac{\sqrt{n^2-1}+\sqrt{n+3}}{-\,\|\,-} = \lim_{n\to\infty} \frac{(n^2-1)-(n+3)}{\sqrt{n^2-1}+\sqrt{n+3}} = \lim_{n\to\infty} \frac{n^2(1-\frac{1}{n}+\frac{2}{n^2})}{n(\sqrt{1-\frac{1}{n}}+\sqrt{\frac{1}{n}+\frac{3}{n^2}})} = \infty$

25. $\lim_{n\to\infty} [\sqrt[4]{n^4-1} - \sqrt[4]{n+3}] = \lim_{n\to\infty} \frac{(n^4-1)-(n+3)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2}\cdot\sqrt[4]{n^4+3n} + \sqrt[4]{n^4-1}\cdot\sqrt[4]{(n^4+3)^2} + \sqrt[4]{(n^4+3)^3}} =$

$= \lim_{n\to\infty} \frac{n^4(1-\frac{1}{n^3}-\frac{4}{n^4})}{n^3(\sqrt[4]{(1-\frac{1}{n^4})^3} + \sqrt[4]{(\frac{1}{n^2}-\frac{1}{n^4})^2}\cdot\sqrt[4]{(1+\frac{3}{n^4})} + \sqrt[4]{(1-\frac{1}{n^4})}\cdot\sqrt[4]{(1+\frac{3}{n^4})^2} + \sqrt[4]{(1+\frac{3}{n^4})^3}} = \infty$

26. $\lim_{n\to\infty} \left[\frac{4n}{4n-9}\right]^3 = \lim_{n\to\infty} \left(1+\frac{9}{4n-9}\right)^3 = 1$

27. $\lim_{n\to\infty} \left[\frac{4n}{4n-9}\right]^{n+3} = \lim_{n\to\infty} \left(1+\frac{9}{4n-9}\right)^{n+3} = \lim_{n\to\infty} \left[\underbrace{\left(1+\frac{9}{4n-9}\right)^{4n-9}}_{\to e^9}\right]^{\frac{n+3}{4n-9}} = e^9$

28. $\lim_{n\to\infty} \left[\frac{4n}{4n-9}\right]^{n^2+3} = \lim_{n\to\infty} \left(1+\frac{9}{4n-9}\right)^{n^2+3} = \lim_{n\to\infty} \left[\left(1+\frac{9}{4n-9}\right)^{4n-9}\right]^{\frac{n^2+3}{4n-9}} = \infty$

29. $\lim_{n\to\infty} \left[\frac{4n^2}{4n^2-9}\right]^{n+3} = \lim_{n\to\infty} \left(1+\frac{9}{4n^2-9}\right)^{n+3} = \lim_{n\to\infty} \left[\left(1+\frac{9}{4n^2-9}\right)^{4n^2-9}\right]^{\frac{n+3}{4n^2-9}} = 1$

30. $\lim_{n\to\infty} \left[\frac{4n^2}{4n^2-9}\right]^{n^2+3} = \lim_{n\to\infty} \left(1+\frac{9}{4n^2-9}\right)^{n^2+3} = \lim_{n\to\infty} \left[\left(1+\frac{9}{4n^2-9}\right)^{4n^2-9}\right]^{\frac{n^2+3}{4n^2-9}} = e^{9(\frac{1}{4})} = e^{\frac{9}{4}}$

(6) $\lim\limits_{m\to\infty} \dfrac{m^5-2m^6-4m^4-3}{-8m^4-4m^5+2} = \lim\limits_{m\to\infty} \dfrac{m^6\left(\frac{1}{m}-2-\frac{4}{m^2}-\frac{3}{m^6}\right)}{m^5\left(-\frac{8}{m}-4+\frac{2}{m^5}\right)} = \lim\limits_{m\to\infty} \dfrac{-2m}{-4} = \infty$

(22) $\lim\limits_{m\to\infty}\left[\sqrt{m^2-1}-m-3\right] = \lim\limits_{m\to\infty}\left[\sqrt{m^2-1}-(m+3)\right]\cdot \dfrac{\sqrt{m^2-1}+(m+3)}{-\,\|\,-} = \lim\limits_{m\to\infty} \dfrac{m^2-1-(m+3)^2}{\sqrt{m^2-1}+(m+3)} =$

$= \lim\limits_{m\to\infty} \dfrac{\cancel{m^2}-1-(\cancel{m^2}+6m+9)}{\sqrt{m^2-1}+m+3} = \lim\limits_{m\to\infty} \dfrac{-6m-10}{\sqrt{m^2-1}+m+3} = \lim\limits_{m\to\infty} \dfrac{m\left(-6-\frac{10}{m}\right)}{m\sqrt{1-\frac{1}{m^2}}+1+\frac{3}{m}} = -3$

(27) $\lim\limits_{m\to\infty}\left[\dfrac{4m}{4m-9}\right]^{m+3} = \lim\limits_{m\to\infty}\left(1+\dfrac{9}{4m-9}\right)^{m+3} = \lim\limits_{m\to\infty}\left[\left(1+\dfrac{9}{4m-9}\right)^{4m-9}\right]^{\frac{m+3}{4m-9}} = e^{\left(\frac{9}{4}\right)}$

$\to e^9$

$\lim\limits_{m\to\infty} \dfrac{m+3}{4m-9} = \lim\limits_{m\to\infty} \dfrac{m\left(1+\frac{3}{m}\right)}{m\left(4-\frac{9}{m}\right)} = \dfrac{1}{4}$

(23.) $\lim\limits_{m\to\infty}\left[\sqrt[4]{m^4-1}-m-3\right] = \lim\limits_{m\to\infty}\left[\sqrt[4]{m^4-1}-(m+3)\right] = \lim\limits_{m\to\infty} \dfrac{m^4-1-(m+3)^4}{\sqrt[4]{(m^4-1)^3}+\sqrt[4]{(m^4-1)^2}\cdot(m-3)+\sqrt[4]{(m^4-1)}\cdot(m+3)^2+(m+3)^3}$

$= \lim\limits_{m\to\infty} \dfrac{\cancel{m^4}-1-\cancel{m^4}-12m^3-54m^2-81m-81}{\sqrt[4]{(m^4-1)^3}+\sqrt[4]{(m^4-1)^2}\cdot(m-3)+\sqrt[4]{(m^4-1)}\cdot(m+3)^2+(m+3)^3} = \lim\limits_{m\to\infty} \dfrac{1-12m^3-54m^2-81m-81}{\sqrt[4]{(m^4-1)^3}+\sqrt[4]{(m^4-1)^2}\cdot(m-3)+\sqrt[4]{(m^4-1)}\cdot(m^2+6m+9)+(m^3+9m^2+27m+27)}$

$= \lim\limits_{m\to\infty} \dfrac{\cancel{m^3}\left(\frac{1}{m^3}-1-\frac{54}{m}-\frac{81}{m^2}-\frac{81}{m^3}\right)}{\cancel{m^3}\left(\underbrace{\sqrt[4]{\left(1-\frac{1}{m^4}\right)^3}}_{1}+\sqrt[4]{\left(1-\frac{1}{m^4}\right)^2}\cdot\underbrace{\left(\frac{1}{m^2}-\frac{3}{m^3}\right)}_{0}+\sqrt[4]{\left(1-\frac{1}{m^4}\right)}\cdot\underbrace{\left(\frac{1}{m}+\frac{6}{m^2}+\frac{9}{m^3}\right)}_{6}+\underbrace{\left(1+\frac{9}{m}+\frac{27}{m^2}+\frac{27}{m^3}\right)}_{1}\right)} = \cancel{-\dfrac{1}{2}}$

$\lim\limits_{m\to\infty} \dfrac{\cancel{m^4}-1-\cancel{m^4}-12m^3-54m^2-108m-81}{\sqrt[4]{(m^4-1)^3}+\sqrt[4]{(m^4-1)^2}\cdot(m-3)+\sqrt[4]{(m^4-1)}\cdot(m+3)^2+(m+3)^3} = \lim\limits_{m\to\infty} \dfrac{m^3\left(-12-\frac{54}{m}-\frac{108}{m^2}-\frac{82}{m^3}\right)}{m^3\left(\underbrace{\sqrt[4]{\left(1-\frac{1}{m^4}\right)^3}}_{1}+\sqrt[4]{\left(1-\frac{1}{m^4}\right)^2}\cdot\underbrace{\left(\frac{1}{m^2}-\frac{3}{m^3}\right)}_{0}+\sqrt[4]{1-\frac{1}{m^4}}\cdot\underbrace{\left(\frac{1}{m}+\frac{6}{m^2}+\frac{9}{m^3}\right)}_{0}+\left(1+\frac{9}{m}+\frac{27}{m^2}+\frac{27}{m^3}\right)\right)}$

$= -\dfrac{12}{2} = -\cancel{6}$

(23) $\lim\limits_{m\to\infty} \dfrac{-12m^3-54m^2-108m-82}{\sqrt[4]{m^{12}-3m^8+3m^4-1}+\sqrt[4]{m^8-2m^4+1}\,(m+3)+\sqrt[4]{m^4-1}\,(m^2+6m+9)+(m^3+9m^2+27m+27)} = \lim\limits_{m\to\infty} \dfrac{-12m^3-\ldots}{\sqrt[4]{m^{12}(1-\underbrace{\odot}_{0}}+\sqrt[4]{m^{12}\cdot(1-\underbrace{\odot}_{0}}+\sqrt[4]{m^{12}\cdot(1-\underbrace{\odot}_{0}}+(m^3\ldots}$

$= \lim\limits_{m\to\infty} \dfrac{m^3(-12-\odot^{\to 0})}{m^3(4+\odot)\to 0} = -3$