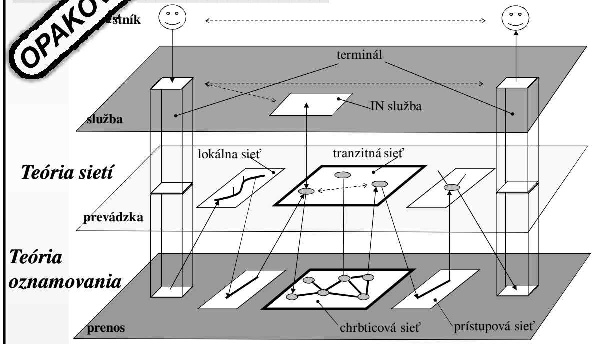
Teória oznamovania 9

Obsah:
- opakovanie prenosu signálu kanálom
- vlastnosti frekvenčného spektra a prenosu kanála
- ideálny lineárny časovo invariantný kanál
- korekcia frekvenčného prenosu kanála
- optimálny príjem signálu

OPAKOVANIE

Základné vrstvy

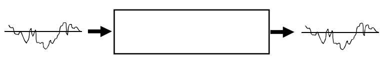


OPAKOVANIE

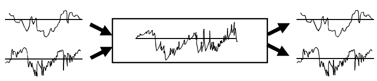
Vrstva prenosu

Hlavné úlohy: ??

prenos jedného signálu

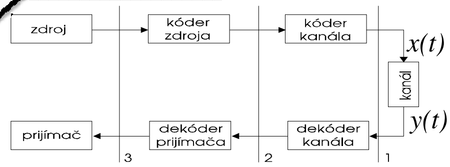


súčasný prenos signálov

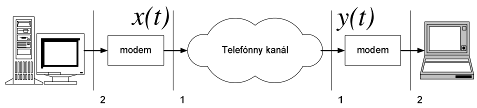


OPAKOVANIE

Prenos bez skreslenia



Prispôsobenie prenosovému médiu



OPAKOVANIE

Lineárny kanál

$$\forall \mathbf{x}_i \in \varphi, \quad i = 0,1,\ldots,N-1$$

$$\psi\left(\sum_{i=0}^{N-1} k_i.\mathbf{x}_i\right) = \sum_{i=0}^{N-1} k_i.\psi(\mathbf{x}_i) = \sum_{i=0}^{N-1} k_i.\mathbf{y}_i$$

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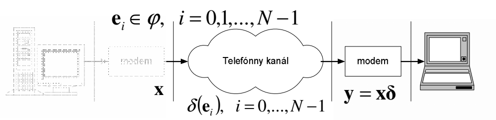
Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=0}^{N-1} x_i.\boldsymbol{\delta}_i$$

$$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$$



Lineárny kanál

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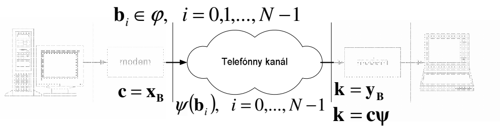
$$\mathbf{y_B}=\psi(\mathbf{x_B})=\sum_{i=0}^{N-1}c_i.\boldsymbol{\psi}_i$$

$$\mathbf{k}=\mathbf{c}\boldsymbol{\psi}$$



Lineárny kanál

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Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Požadujeme $\psi(\mathbf{b}_i)=\lambda_i\mathbf{b}_i$

$$\mathbf{b}_i\boldsymbol{\psi}=\lambda_i\mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

Vlastné vektory kanála

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$$\mathbf{b}_i\in\varphi,\ \ i=0,1,\dots,N-1$$

$$\mathbf{x_B}=\sum_{i=0}^{N-1}c_i.\mathbf{b}_i \qquad \mathbf{y_B}=\sum_{i=0}^{N-1}k_i.\mathbf{b}_i$$

$$\mathbf{y_B}=\sum_{i=0}^{N-1}c_i.\lambda_i\mathbf{b}_i$$

$$\boxed{k_i=\lambda_i c_i, i=0,1,\dots,N-1}$$

Časovo-invariantný kanál

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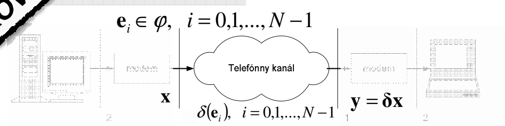
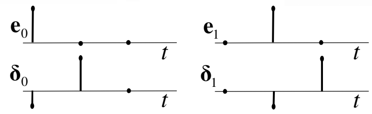




$$\mathbf{e}_0=(1,0,0),\ \boldsymbol{\delta}_0=(\delta_0,\delta_1,\delta_2)$$

$$\mathbf{e}_1=(0,1,0),\ \boldsymbol{\delta}_1=(\delta_2,\delta_0,\delta_1) \qquad (y_0,y_1,y_2)=(x_0,x_1,x_2).\begin{pmatrix}\delta_0&\delta_1&\delta_2\\\delta_2&\delta_0&\delta_1\\\delta_1&\delta_2&\delta_0\end{pmatrix}$$

$$\mathbf{e}_2=(0,0,1),\ \boldsymbol{\delta}_1=(\delta_1,\delta_2,\delta_0)$$

Vlastné signály t-invariantného kanála

OPAKOVANIE

$$\mathbf{e}_0=(1,0,\dots,0), \qquad \boldsymbol{\delta}_0=(\delta_0,\delta_1,\dots,\delta_{N-1})$$

$$\mathbf{e}_1=(0,1,\dots,0) \qquad \boldsymbol{\delta}_1=(\delta_{N-1},\delta_0,\dots,\delta_{N-2})$$

$$\dots \qquad \dots$$

$$\mathbf{e}_{N-1}=(0,0,\dots,1) \qquad \boldsymbol{\delta}_{N-1}=(\delta_1,\delta_2,\dots,\delta_0)$$

$$\mathbf{b}(\boldsymbol{\delta}-\lambda\mathbf{E})=\mathbf{0}$$

$n=0,1,\dots,N-1$

$$\boxed{\begin{aligned}\lambda_n&=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}\\ \mathbf{b}_n&=\left(e^{j\frac{2\pi}{N}n0},e^{j\frac{2\pi}{N}n1},\dots,e^{j\frac{2\pi}{N}n(N-1)}\right)\end{aligned}}$$

Frekvenčný prenos t-invar. kanála

OPAKOVANIE

$$\mathbf{x}=(x_0,x_2,\dots,x_{N-1}) \quad \boldsymbol{\delta}_0=(\delta_0,\delta_2,\dots,\delta_{N-1})$$

$$\lambda_n=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}$$

$n=0,1,\dots,N-1$

$$\mathbf{b}_n=\left(e^{j\frac{2\pi}{N}n0},e^{j\frac{2\pi}{N}n1},\dots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x}=(c_0,c_2,\dots,c_{N-1})_\mathbf{B} \quad \mathbf{y}=(k_0,k_2,\dots,k_{N-1})_\mathbf{B}$$

$$\boxed{\begin{aligned}c_n&=\frac{(\mathbf{x},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)}=\frac{1}{N}\sum_{k=0}^{N-1}x_k e^{-j\frac{2\pi}{N}nk},\quad c_n\in\mathbf{C}\\ k_n&=\lambda_n c_n \quad n=0,1,\dots,N-1\end{aligned}}$$

Vzťah k DFT

$$k_n = \lambda_n c_n, \quad n=0,1,\ldots,N-1$$

$$\cancel{\tfrac{1}{N}}\sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}\,\cancel{\tfrac{1}{N}}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$\boxed{Y_n = F_n X_n, \quad n=0,1,\ldots,N-1}$$

$$\mathbf{X} = DFT(\mathbf{x}) \qquad \mathbf{F} = DFT(\boldsymbol{\delta})$$
$$\mathbf{Y} = DFT(\mathbf{y})$$

Vzťah k DFT

$$c_n = \frac{(\mathbf{x},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)} = \frac{1}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}, \quad c_n \in \mathbf{C}$$

$$X_n = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$X_n = \frac{(\mathbf{x},\tilde{\mathbf{b}}_n)}{(\tilde{\mathbf{b}}_n,\tilde{\mathbf{b}}_n)} = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$(\tilde{\mathbf{b}}_n,\tilde{\mathbf{b}}_n)=1 \qquad \tilde{\mathbf{b}}_n = \frac{1}{N}\left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastnosti spektra

$$\tilde{\mathbf{b}}_n = \frac{1}{N}\left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

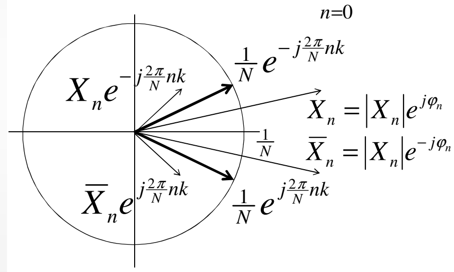
$$\mathbf{x} = \sum_{n=0}^{N-1} X_n \tilde{\mathbf{b}}_n, \quad X_n \in C \qquad ?$$

$$x_k = \frac{1}{N}\sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{N}nk}, \quad x_k \in \Re,\ X_n \in C$$

$$\boxed{X_n = \overline{X}_{N-n}, \quad n=1,\ldots,N-1}$$

Vlastnosti spektra

$$X_n = \overline{X}_{N-n} \qquad x_k = \sum_{n=0}^{N-1} X_n e^{-j\frac{2\pi}{N}nk}$$

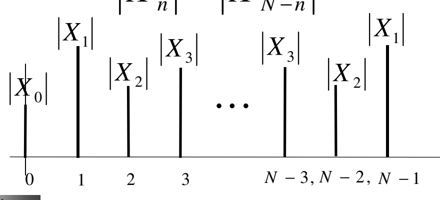


Amplitúdové spektrum

$$X_n = \overline{X}_{N-n}$$

$$|X_n| e^{-j\varphi_n} = |X_{N-n}| e^{j\varphi_{N-n}}$$

$$|X_n| = |X_{N-n}|$$

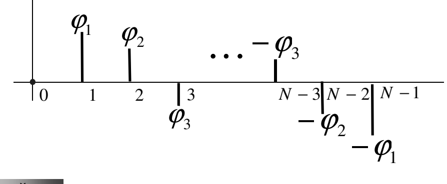


Fázové spektrum

$$X_n = \overline{X}_{N-n}$$

$$|X_n| e^{-j\varphi_n} = |X_{N-n}| e^{j\varphi_{N-n}}$$

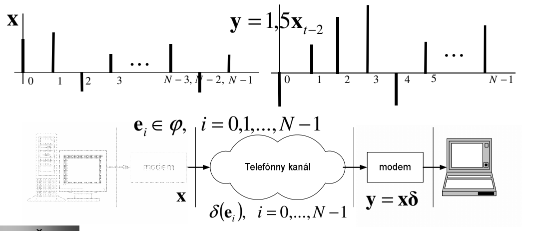
$$\varphi_n = -\varphi_{N-n}$$



Ideálny lineárny t-invariantný kanál

$$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$$

$$\mathbf{y} = \alpha\mathbf{x}_{t-\Delta}$$

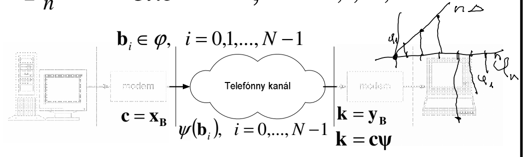


Ideálny lineárny t-invariantný kanál

$$Y_n = F_n X_n, \quad n = 0,1,\ldots,N-1$$

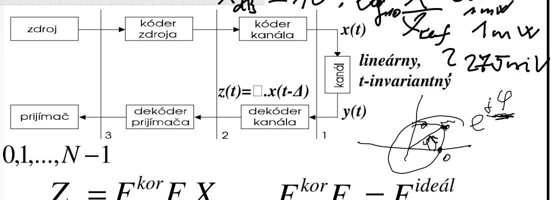
$$\mathbf{y} = \alpha\mathbf{x}_{t-\Delta} \quad Y_n = \alpha X_n e^{-j\frac{2\pi}{N}n\Delta}$$

$$F_n^{ideál} = \alpha.e^{-j\frac{2\pi}{N}n\Delta}, \quad n = 0,1,\ldots,N-1$$



Korekcia kanála

$X_{dB} = 10 . \log_{10} \frac{X}{X_{ref}}$ 600 Ω 1 mW ? 275 mV



$$n = 0,1,\ldots,N-1$$

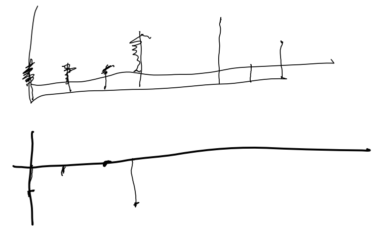
$$Z_n = F_n^{kor} F_n X_n, \qquad F_n^{kor} F_n = F_n^{ideál}$$

$$\left|F_n^{kor}\right| e^{-j\varphi_n^{kor}} . \left|F_n\right| e^{-j\varphi_n} = \alpha e^{-j\frac{2\pi}{N}n\Delta}$$

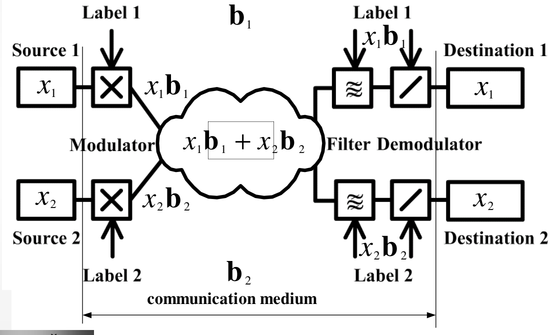
$$\boxed{\left|F_n^{kor}\right| = \alpha.\left|F_n\right|^{-1} \quad \varphi_n^{kor} = -\varphi_n + \tfrac{2\pi}{N}n\Delta}$$

$X . X^{kor} = \alpha \qquad X_{dB} + X_{dB}^{kor} = \alpha_{dB}$

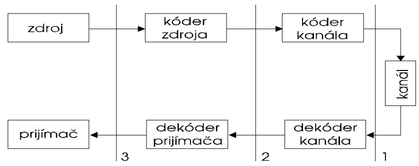
$X_{dB}^{kor} = \alpha_{dB} - X_{dB} < 0$



Príznakový multiplex

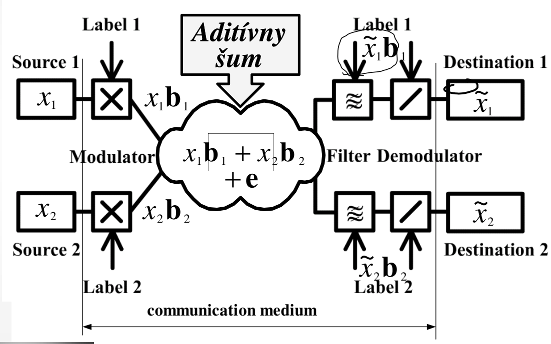


Prenos bez skreslenia



Zabránenie vplyvu šumu

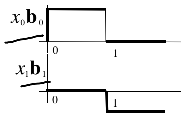
Príznakový multiplex so šumom



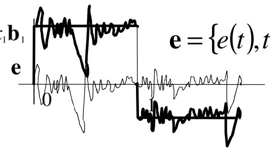
Šum v signálovom priestore

$x_0 = 2 \quad \mathbf{b}_0 = (1,0)$

$x_1 = -1 \quad \mathbf{b}_1 = (0,1)$



$\mathbf{e} + \; x_0\mathbf{b}_0 + x_1\mathbf{b}_1$



$\mathbf{e} = \{e(t), t \in [0,2T)\}$

spektrum:

$$c_n = \frac{1}{2T}\int_0^{2T} e(t).e^{j\frac{2\pi}{2T}nt}dt, \quad n = \ldots,-1,0,1,\ldots$$

Priemet do podpriestoru

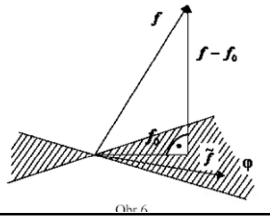
Veta :

Nech je Hilbertov priestor a nech φ je uzavretý lineárny podpriestor priestoru . Nech signál a $\delta = \inf$

Potom existuje práve jeden signál $f_0 \in \varphi$ tak, že $d(\mathbf{f},\mathbf{f}_0) = \delta$

Navyše $\mathbf{f} - \mathbf{f}_0 \perp \varphi$, t.j. pre všetky $\tilde{f} \in \varphi$ platí $(\mathbf{f} - \mathbf{f}_0, \tilde{\mathbf{f}}) = 0$

Pritom f_0 je jediným signálom priestoru φ s vlastnosťou $\mathbf{f} - \mathbf{f}_0 \perp \varphi$



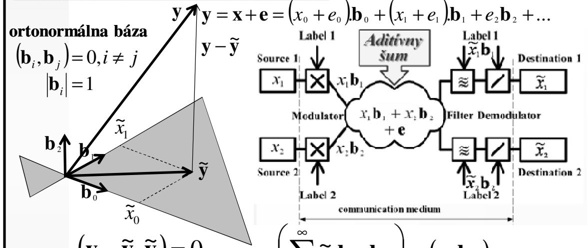
Optimálny prijímač

$\mathbf{y} = \mathbf{x} + \mathbf{e} = (x_0 + e_0).\mathbf{b}_0 + (x_1 + e_1).\mathbf{b}_1 + e_2\mathbf{b}_2 + \ldots$

ortonormálna báza

$(\mathbf{b}_i, \mathbf{b}_j) = 0, i \neq j$

$|\mathbf{b}_i| = 1$



$(\mathbf{y} - \tilde{\mathbf{y}}, \tilde{\mathbf{y}}) = 0$

$(\mathbf{y} - \tilde{\mathbf{y}}, \mathbf{b}_n) = 0$

$(\tilde{\mathbf{y}}, \mathbf{b}_n) = (\mathbf{y}, \mathbf{b}_n)$

$\left(\sum_{-\infty}^{\infty} \tilde{x}_k \mathbf{b}_k, \mathbf{b}_n\right) = (\mathbf{y}, \mathbf{b}_n)$

$\sum_{-\infty}^{\infty} \tilde{x}_k (\mathbf{b}_k, \mathbf{b}_n) = (\mathbf{y}, \mathbf{b}_n),$

$(\mathbf{b}_k, \mathbf{b}_n) = 0, k \neq n$

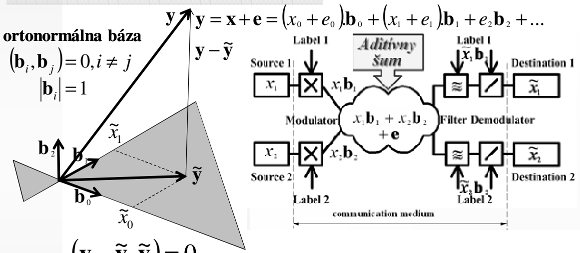
Optimálny prijímač

$\mathbf{y} = \mathbf{x} + \mathbf{e} = (x_0 + e_0).\mathbf{b}_0 + (x_1 + e_1).\mathbf{b}_1 + e_2\mathbf{b}_2 + \ldots$

ortonormálna báza

$(\mathbf{b}_i, \mathbf{b}_j) = 0, i \neq j$

$|\mathbf{b}_i| = 1$



$(\mathbf{y} - \tilde{\mathbf{y}}, \tilde{\mathbf{y}}) = 0$

$$\boxed{\tilde{x}_n = \frac{(\mathbf{y}, \mathbf{b}_n)}{(\mathbf{b}_n, \mathbf{b}_n)}}$$

Ďakujem za Vašu pozornosť