20. 12. 07   5

s06-022   riešil(a): MIROSLAV DREVEŇÁK

1. $\lim_{x\to-3}\frac{x+2}{x+3} =$ $\lim_{x\to-3^-} = \frac{-1}{0^-} = -1\cdot(-\infty) = \infty$, $\lim_{x\to-3^+} = \frac{-1}{0^+} = -1\cdot\infty = -\infty$ $\Big\}$ $\lim_{x\to-3}\frac{x+2}{x+3}\ \nexists$

$= \lim_{x\to-3}\frac{-1}{0^{\pm}}$

2. $\lim_{x\to-3} \operatorname{arctg}\frac{x+2}{x+3} =$ $\lim_{x\to-3^+} \operatorname{arctg}\frac{-1}{0^+} = \operatorname{arctg} -1\cdot\infty = -\frac{\pi}{2}$, $\lim_{x\to-3^-} \operatorname{arctg}\frac{-1}{0^-} = \operatorname{arctg}\infty = \frac{\pi}{2}$ $\Big\}$ $\nexists$ lebo l^- a l^+ lim sa $\neq$

3. $\lim_{x\to 0}\frac{\operatorname{tg}2x}{\operatorname{tg}3x} =$ $\lim_{x\to0}\left[\frac{\frac{\sin 2x}{\cos 2x}}{\frac{\sin 3x}{\cos 3x}} \quad \frac{\sin 2x\cdot\cos 3x}{\sin 3x\cdot\cos 2x} = \frac{\frac{\sin 2x}{2x}\cdot\frac{\cos 3x}{3}}{\frac{\sin 3x}{3x}\cdot\frac{\cos 2x}{2}}\right] = \frac{1\cdot\frac{1}{3}}{1\cdot\frac{1}{2}} = \frac{2}{3}$

4. $\lim_{x\to0}\frac{\sin 2x}{3x} = \lim_{x\to0}\frac{\frac{\sin 2x}{2}}{1\cdot\frac{3x}{2x}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

subs. $2x = z$, $t\to0$

5. $\lim_{x\to0}\frac{|\sin 2x|}{3x} =$ $\lim_{x\to0^-}\frac{-\sin 2x}{3x} = -\frac{2}{3}$, $\lim_{x\to0^+}\frac{\sin 2x}{3x} = \frac{2}{3}$ $\Big\}$ $\lim_{x\to0}\frac{|\sin 2x|}{3x}\ \nexists$

6. $\lim_{x\to\infty}\frac{x}{2}\sin\frac{3}{x} = \frac{\frac{3}{x}}{\frac{3}{x}} = \lim_{x\to\infty}\frac{3}{2}\cdot\frac{\sin\frac{3}{x}}{\frac{3}{x}} = \frac{3}{2}$

7. $\lim_{x\to0}\frac{\arcsin 2x}{3x} = \frac{\frac{1}{2x}}{\frac{1}{2x}} = \lim_{x\to0}\frac{\frac{\arcsin 2x}{2x}}{\frac{3x}{2x}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

8. $\lim_{x\to0}\frac{\operatorname{arctg}2x}{3x} = \frac{\frac{1}{2x}}{\frac{1}{2x}} = \frac{\frac{\operatorname{arctg}2x}{2x}}{\frac{3x}{2x}}$?. $\frac{1}{\frac{3}{2}} = \frac{2}{3}$

9. $\lim_{x\to0}\frac{1-\cos 2x}{3x} = \frac{\frac{1}{2x}}{\frac{1}{2x}} = \lim_{x\to0}\frac{\frac{1-\cos 2x}{2x}}{\frac{3x}{2x}} = \lim_{x\to0}\frac{\sin^2 x}{\frac{3}{2}x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to0}\frac{\sin^2 x}{\frac{3}{2}x\cdot\frac{1}{x}} = \lim_{x\to0}\frac{\sin 2x}{\frac{3}{2}} = 0$

10. $\lim_{x\to0}(1+\sin 2x)^{\frac{3}{x}} = \lim_{x\to0}\left(e^{\sin 2x}\right)^{\frac{3}{x}} = \lim_{x\to0} e^{\frac{3\sin 2x}{x}} = \lim_{x\to0} e^{\frac{6\sin 2x}{2x}} = \lim_{x\to0} e^{6\cdot\frac{\sin 2x}{2x}} = e^6$?

11. $\lim_{x\to0}(\cos 2x)^{\frac{3}{x}} = \lim_{x\to0}\left(1+\cos 2x-1\right)^{\frac{3}{x}} = \lim_{x\to0}\left(e^{\cos 2x-1}\right)^{\frac{3}{x}} = \lim_{x\to0}\frac{3\cos 2x-3}{x} = e^0 = 1$

$\lim_{x\to0}\frac{(\cos 2x-1)\cdot3}{x} = \lim_{x\to0}\frac{[(\cos^2x-\sin^2x)-(\cos^2x+\sin^2x)]\cdot3}{x} = \lim_{x\to0}\frac{-\sin^2x\cdot3}{x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to0}\frac{-\frac{\sin x}{x}\cdot\sin x\cdot3}{\frac{x}{x}\cdot\frac{1}{x}} = \lim_{x\to0}\frac{-\sin x\cdot3}{1} = \lim_{x\to0} -\sin x = 0$

12. $\lim_{x\to0}\frac{3^x-1}{5^x-1} = \left(\frac{3^{x-1}+3^{x-2}+\dots+3+1}{5^{x-1}+5^{x-2}+\dots+5+1}\right) = \lim_{x\to0}\frac{2}{4}\cdot\ldots = \frac{\ln 3}{\ln 5}$

Limity počítajte bez použitia L'Hospitalovho pravidla!   —pokračovanie—

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13. $\lim\limits_{x\to 4} \frac{x^5-4^5}{x^6-4^6} = \frac{(x-4)(x^4+4x^3+16x^2+64x+256)}{(x-4)(x^5+4x^4+16x^3+64x^2+256x+1024)} = \frac{256+256+256+256+256}{1024+1024+1024+1024+1024+1024} = \frac{5\cdot 256}{6\cdot 256\cdot 4} = \underline{\underline{\frac{5}{24}}}$

14. $\lim\limits_{x\to 4} \frac{\sqrt[5]{x}-\sqrt[5]{4}}{\sqrt[6]{x}-\sqrt[6]{4}} = \frac{\sqrt[5]{x}-\sqrt[5]{4}}{\sqrt[6]{x}-\sqrt[6]{4}} = \lim\limits_{x\to 4} \frac{\left((\sqrt[5]{x})^2-(\sqrt[5]{4})^2\right)\cdot(\sqrt[5]{x}+\sqrt[5]{4})}{\left((\sqrt[6]{x})^2-(\sqrt[6]{4})^2\right)\cdot(\sqrt[3]{x}+\sqrt[3]{4})} \cdot \frac{(\sqrt{x}+\sqrt{4})\left((\sqrt[3]{x})^2+\sqrt[3]{x}\sqrt[3]{4}+(\sqrt[3]{4})^2\right)}{(\sqrt{x}+\sqrt{4})\left((\sqrt[3]{x})^2+\sqrt[3]{x}\cdot\sqrt[3]{4}+(\sqrt[3]{4})^2\right)} = \ldots \frac{6}{5}\cdot 2^{\frac{1}{15}}$

ostatné členy sú nepodstatné

15. $\lim\limits_{x\to\infty} \frac{(3x-3)^{600}(2x+3)^{200}}{(4x-2)^{800}} = \lim\limits_{x\to\infty} \frac{(3^{600}\cdot x^{600}+\ldots)(2^{200}\cdot x^{200}+\ldots)}{4^{800}\cdot x^{800}} = \frac{5^{50}\cdot x^{800}\ldots}{4^{800}\cdot x^{800}\ldots} = \frac{3^{50}}{4^{800}}$

16. $\lim\limits_{x\to\infty} \frac{(3x-3)^{600}(2x+3)^{200}}{(4x-2)^{700}} = \lim\limits_{x\to\infty} \frac{(3^{600}\cdot x^{600}+\ldots)\cdot(2^{200}\cdot x^{200}+\ldots)}{4^{700}\cdot x^{700}} = \lim\limits_{x\to\infty} \frac{3^{400}\cdot x^{800}}{4^{700}\cdot x^{700}} = \lim\limits_{x\to\infty} \frac{x^{100}}{1} = \underline{\infty}$

17. $\lim\limits_{x\to\infty} \frac{(3x-3)^{600}(2x+3)^{200}}{(4x-2)^{900}} = \lim\limits_{x\to\infty} \frac{(3^{600}\cdot x^{600}+\ldots)(2^{200}\cdot x^{200}+\ldots)}{4^{900}} = \lim\limits_{x\to\infty} \frac{3^{600}\cdot 2^{200}\cdot x^{800}}{4^{900}\cdot x^{900}} = \lim\limits_{x\to\infty} \frac{3^{600}\cdot 2^{200}}{4^{900}\cdot x^{100}} = \underline{\underline{0}}$

18. $\lim\limits_{x\to\infty} \frac{2x^3+4x^4-x^2-2}{3x^2-5x^4-1} = \lim\limits_{x\to\infty} \frac{\frac{2}{x}+4-\frac{1}{x^2}-\frac{2}{x^4}}{\frac{3}{x^2}-5-\frac{1}{x^4}} = \underline{\underline{-\frac{4}{5}}}$

19. $\lim\limits_{x\to\infty} \left[\sqrt[3]{x^3+3x-3} - \sqrt[3]{x^3+4x-2}\right] = \frac{\ldots}{\left(\sqrt[3]{x^3+3x-3}\right)^2 + \sqrt[3]{(x^3+3x-3)(x^3+4x-2)} + \left(\sqrt[3]{x^3+4x-2}\right)^2} =$

$= \lim\limits_{x\to\infty} \frac{x^3+3x-3-x^3-4x-2}{-\,''\,-} = \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{3-4}{\infty} = \underline{\underline{0}}$

20. $\lim\limits_{x\to\infty} \left[\sqrt[3]{x^3+3x-3} - \sqrt[3]{x-4}\right] = \lim\limits_{x\to\infty} x\sqrt[3]{1+\frac{3}{x^2}-\frac{3}{x^3}} - x\sqrt[3]{\frac{1}{x^2}-\frac{4}{x^3}} = \lim\limits_{x\to\infty} x\left(\sqrt[3]{1+\frac{3}{x^2}-\frac{3}{x^3}} - \sqrt[3]{\frac{1}{x^2}-\frac{4}{x^3}}\right)$

$= \infty\cdot(1-0) = \underline{\underline{\infty}}$

21. $\lim\limits_{x\to\infty} \left[\frac{3x-3}{3x+4}\right]^{x+3} = \lim\limits_{x\to\infty} \left[\frac{3x-3+4-4}{3x+4}\right]^{x+3} = \left(1+\frac{-4}{3x+4}\right)^{x+3} = \left[\left(1+\frac{-3}{3x+4}\right)^{3x+4}\right]^{\frac{x+3}{3x+4}} = \left(e^{-4}\right)^{\frac{1}{3}} = e^{-\frac{4}{3}}$