16.11.07 12.11.07

s01-022 riešil(a): MIROSLAV DREVEŇÁK

1. Dokážte, že $\sqrt{43}$ je iracionálne číslo.

Dokážem sporom:
Nech $\sqrt{43}$ je racionálne číslo t.j. $\sqrt{43} = p/q$, kde $p, q \in N$ sú nesúdeliteľné čísla. Potom
$43 = p^2/q^2$, t.j. $43q^2 = p^2 \Rightarrow 43/p \Rightarrow p = 43k \Rightarrow 43q^2 = p^2 = 43^2k^2 \Rightarrow q^2 = 43k^2 \Rightarrow 43/q$

$$43/p \wedge 43/q \Rightarrow \sqrt{43} \notin Q$$

2. Pomocou matematickej indukcie dokážte rovnosť $\sum_{i=0}^{n} \frac{1}{6^i} = \frac{6-6^{-n}}{5}$.

$1 + \frac{1}{6} + \frac{1}{36} + \dots + \frac{1}{6^n}$

$F(1): \frac{1}{6^k} = \frac{6-6^{-k}}{5} \Rightarrow \frac{1}{6} = \frac{6-\frac{1}{6}}{5} \Rightarrow \frac{1}{6} = \frac{1}{6}$

$\forall k \in N: F(k) = G(k) \Rightarrow F(k+1) = G(k+1)$

$F(k): \frac{1}{6^k} = \frac{6-6^{-k}}{5}$

$F(k+1) = 1 + \frac{1}{6} + \frac{1}{36} + \dots + \frac{1}{6^k} + \frac{1}{6^{k+1}} = \frac{6-6^{-k}}{5} + \frac{1}{6^{k+1}} = \frac{6^{k+2} - 6 + 5}{5 \cdot 6^{k+1}} = \frac{6^{k+2} - 6^0}{5 \cdot 6^{k+1}} =$

$= \frac{6(6^{k+1} - 6^{-1})}{5 \cdot 6^{k+1}} = \frac{6(6^{k+1} \cdot 6^{-1}) \cdot 6^{-k-1}}{5} = \frac{6^{-k}(6^{k+1} - 6^{-1})}{5} = \frac{6 - 6^{-k-1}}{5} = G(k+1)$

3. Priamo dokážte rovnosť $\sum_{i=1}^{n} \frac{1}{(9i-3)(9i+6)} = \frac{n}{6(9n+6)}$.

$\sum_{i=1}^{n} \frac{1}{(9i-3)(9i+6)} = \frac{n}{6(9n+6)}$

$\underbrace{\frac{1}{6 \cdot 15} + \frac{1}{15 \cdot 24}} + \frac{1}{24 \cdot 33} + \dots + \frac{1}{(9n-3)(9n+6)} = \frac{n}{6(9n+6)}$

$\underbrace{\frac{24+6}{6 \cdot 15 \cdot 24} + \frac{1}{24 \cdot 33}} + \dots + \frac{1}{(9n-3)(9n+6)} = \frac{n}{6(9n+6)}$

$\underbrace{\frac{66+6}{6 \cdot 24 \cdot 33} + \dots + \frac{1}{(9n-3)(9n+6)}} = \frac{n}{6(9n+6)}$

$\frac{n}{6(9n+6)} = \frac{n}{6(9n+6)}$

4. Matematickou indukciou dokážte rovnosť $\sum_{i=1}^{n} \frac{1}{(9i-3)(9i+6)} = \frac{n}{6(9n+6)}$.

$\frac{1}{90} + \frac{1}{360} + \dots + \frac{1}{(9n-3)(9n+6)} = \frac{n}{6(9n+6)}$

$F(1) = \frac{1}{90} = \frac{1}{90}$ ✓

$\forall k \in N: F(k) = G(k) \Rightarrow F(k+1) = G(k+1)$

$F(k) = \frac{1}{90} + \frac{1}{360} + \dots + \frac{1}{(9k-3)(9k+6)} = G(k) = \frac{k}{6(9k+6)}$ potom,

$F(k+1) = \frac{1}{90} + \frac{1}{360} + \dots + \frac{1}{(9k-3)(9k+6)} + \frac{1}{(9(k+1)-3)(9(k+1)+6)} = F(k) + \frac{1}{(9(k+1)-3)(9(k+1)+6)} =$

$= \frac{k}{6(9k+6)} + \frac{1}{(9(k+1)-3)(9(k+1)+6)} = \frac{9k^2+15k+6}{6(9k+6)(9k+15)} = \frac{(9k+6)(k+1)}{6(9k+6)(9k+15)} = \frac{k+1}{6 \cdot (9k+15)} = G(k+1)$

Týmto je dané tvrdenie na základe princípu mat. indukcie dokázané.

4b do **12.11.07**, 3b do **19.11.07**, 2b do **26.11.07**, 1b do **03.12.07**

(3) $$\sum_{i=1}^{n} \frac{1}{(9i-3)(9i+6)} = \frac{n}{6(9n+6)}$$

$$\frac{1}{(9i-3)(9i+6)} = \frac{A}{(9i-3)} + \frac{B}{(9i+5)} = \frac{A(9i+6)+B(9i-3)}{(9i-3)(9i+6)} = \frac{9Ai+6A+9Bi-3B}{(9i-3)(9i+6)} =$$

$$= \frac{\frac{1}{9}}{9i-3} - \frac{\frac{1}{9}}{9i+6}$$

$$9Ai+6A+9Bi-3B=1$$
$$9i(A+B)+6A-3B=1$$
$$9i\cdot 0+6A-3B=1$$
$$6A-3B=1$$

$$A+B=0 \Rightarrow A=-B$$

$$6A-3B=1$$
$$-6A+3B=1$$
$$-9B=1$$
$$B=-\frac{1}{9} \Rightarrow A=\frac{1}{9}$$

$$\sum_{i=1}^{n} \frac{1}{(9i-3)(9i+6)} = \sum_{i=1}^{n}\left(\frac{\frac{1}{9}}{9i-3} - \frac{\frac{1}{9}}{9i+6}\right)$$

$$\left(\frac{\frac{1}{9}}{9-3} - \cancel{\frac{\frac{1}{9}}{9+6}}\right) + \left(\cancel{\frac{\frac{1}{9}}{18-3}} - \cancel{\frac{\frac{1}{9}}{18+6}}\right) + \left(\cancel{\frac{\frac{1}{9}}{27-3}} - \cancel{\frac{\frac{1}{9}}{27+6}}\right) + \dots + \left(\cancel{\frac{\frac{1}{9}}{(9n-3)}} - \frac{\frac{1}{9}}{(9n+6)}\right) =$$

$$= \frac{\frac{1}{9}}{6} - \frac{\frac{1}{9}}{9n+6} = \frac{\frac{1}{9}(9n+6) - \frac{1}{9}6}{6(9n+6)} = \frac{n}{6(9n+6)}$$