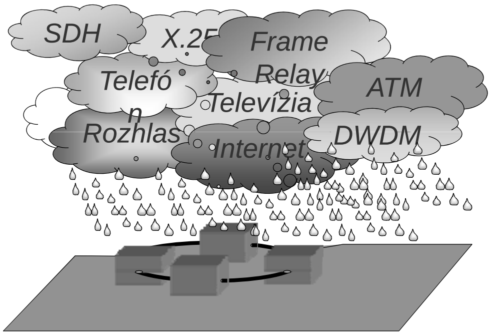
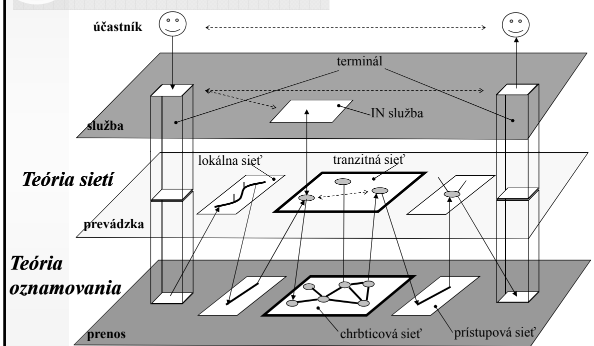
Teória oznamovania 7

Obsah:
- lineárny kanál
- prechod signálu kanálom
- prechod bázických signálov kanálom
- frekvenčný prenos kanála
- časovo invariantný kanál
- frekvenčný prenos časovo invariantného kanála

Všeobecný model siete



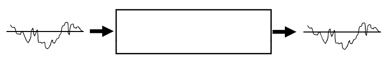
Základné vrstvy



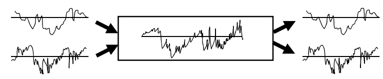
Vrstva prenosu

Hlavné úlohy: ??

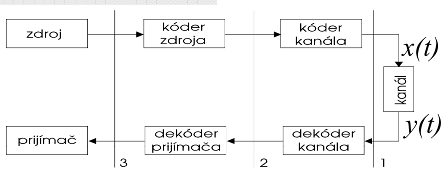
prenos jedného signálu



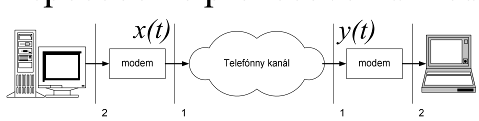
súčasný prenos signálov



Prenos bez skreslenia

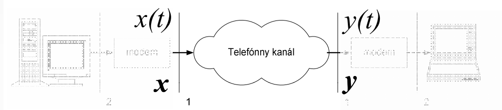


Prispôsobenie prenosovému médiu



Lineárny kanál

Čo je kanál?



$$y = \quad (x)$$

y* – odozva na signál *x

$$\forall \mathbf{x} \in \varphi \Rightarrow \psi(\mathbf{x}) \in \varphi$$

Lineárny kanál

Čo je lineárny kanál?

Lineárny signálový priestor

3. pre všetky $\mathbf{f}_1, \mathbf{f}_2 \in \varphi$ a $k_1, k_2 \in F$

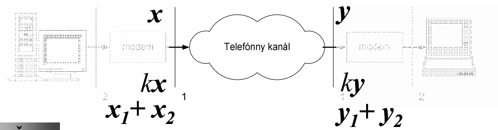
- $1 \cdot \mathbf{f}_1 = \mathbf{f}_1$
- $k_1 \cdot (k_2 \cdot \mathbf{f}_1) = (k_1 \otimes k_2) \cdot \mathbf{f}_1$
- $k_1 \cdot (\mathbf{f}_1 + \mathbf{f}_2) = k_1 \cdot \mathbf{f}_1 + k_1 \cdot \mathbf{f}_2$
- $(k_1 \oplus k_2) \cdot \mathbf{f}_1 = k_1 \cdot \mathbf{f}_1 + k_2 \cdot \mathbf{f}_1$

Lineárny kanál

$$\forall \mathbf{x}, \mathbf{x}_1, \mathbf{x}_2 \in \varphi$$

$$\psi(k.\mathbf{x}) = k.\psi(\mathbf{x}) = k.\mathbf{y}$$

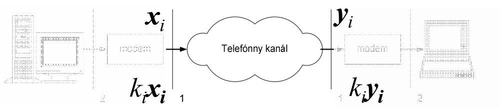
$$\psi(\mathbf{x}_1 + \mathbf{x}_2) = \psi(\mathbf{x}_1) + \psi(\mathbf{x}_2) = \mathbf{y}_1 + \mathbf{y}_2$$



Lineárny kanál

$$\forall \mathbf{x}_i \in \varphi, \quad i = 1,2,\ldots,n$$

$$\psi\left(\sum_{i=1}^{n} k_i.\mathbf{x}_i\right) = \sum_{i=1}^{n} k_i.\psi(\mathbf{x}_i) = \sum_{i=1}^{n} k_i.\mathbf{y}_i$$



Lineárny kanál

$$\mathbf{x} = \sum_{i=1}^{n} x_i.\mathbf{e}_i \qquad \mathbf{y} = \sum_{i=1}^{n} y_i.\mathbf{e}_i$$

$$\mathbf{y} = \delta(\mathbf{x}) = \delta\left(\sum_{i=1}^{n} x_i.\mathbf{e}_i\right) = \sum_{i=1}^{n} x_i.\delta(\mathbf{e}_i)$$

$$\mathbf{y} = \sum_{i=1}^{n} x_i.\boldsymbol{\delta}_i$$

$$\boldsymbol{\delta}_i^T = (\delta_{i1} \quad \ldots \quad \delta_{in})$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x_B}) = \sum_{i=1}^{n} x_i.\boldsymbol{\delta}_i$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = x_1 \begin{pmatrix} \delta_{11} \\ \delta_{21} \\ \vdots \\ \delta_{n1} \end{pmatrix} + \ldots + x_n \begin{pmatrix} \delta_{1n} \\ \delta_{2n} \\ \vdots \\ \delta_{nn} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=1}^{n} x_i.\boldsymbol{\delta}_i$$

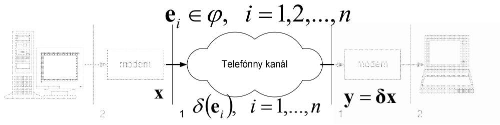
$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \ldots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \ldots & \delta_{2n} \\ . & . & . & . \\ \delta_{n1} & \delta_{n2} & \ldots & \delta_{nn} \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=1}^{n} x_i.\boldsymbol{\delta}_i$$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x}$$

$\mathbf{e}_i \in \varphi, \quad i = 1,2,\dots,n$

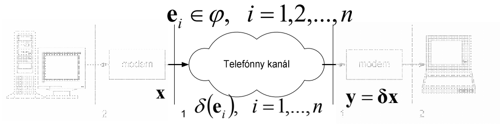


Lineárny kanál

$$\mathbf{e}_1^T = (1,0) \quad \boldsymbol{\delta}_1^T = (1,2) \quad \mathbf{x}^T = (2,-1)$$
$$\mathbf{e}_2^T = (0,1) \quad \boldsymbol{\delta}_2^T = (-1,1)$$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

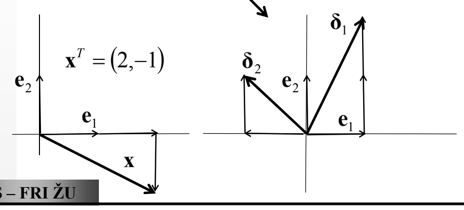
$\mathbf{e}_i \in \varphi, \quad i = 1,2,\dots,n$



Lineárny kanál

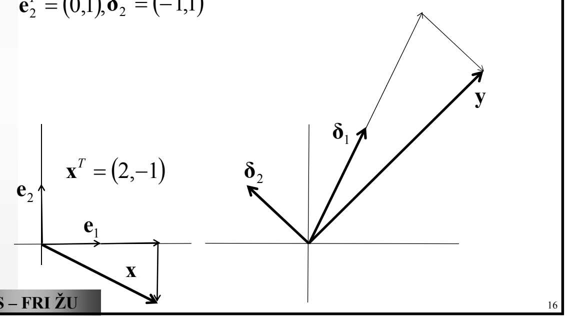
$$\boxed{\begin{matrix} \mathbf{e}_1^T = (1,0), \boldsymbol{\delta}_1^T = (1,2) \\ \mathbf{e}_2^T = (0,1), \boldsymbol{\delta}_2^T = (-1,1) \end{matrix}}$$

$$\mathbf{y} = x_1\boldsymbol{\delta}_1 + x_2\boldsymbol{\delta}_2 = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 1\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$



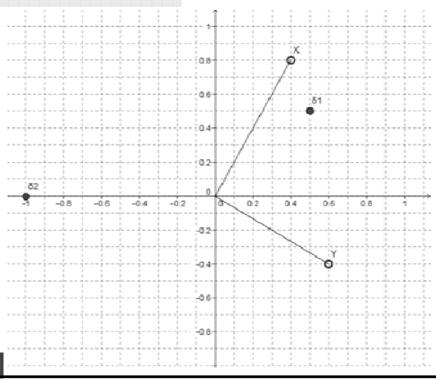
Lineárny kanál

$$\mathbf{e}_1^T = (1,0), \boldsymbol{\delta}_1^T = (1,2) \qquad \mathbf{y} = x_1\boldsymbol{\delta}_1 + x_2\boldsymbol{\delta}_2 = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 1\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$
$$\mathbf{e}_2^T = (0,1), \boldsymbol{\delta}_2^T = (-1,1)$$



Lineárny kanál

GeoGebra



Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 1,2,\dots,n$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=1}^{n} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=1}^{n} k_i.\mathbf{b}_i$$

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=1}^{n} c_i.\mathbf{b}_i\right) = \sum_{i=1}^{n} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{k} = \sum_{i=1}^{n} c_i.\boldsymbol{\psi}_i$$

$$\boldsymbol{\psi}_i^T = (\psi_{i1} \quad \dots \quad \psi_{in})$$

Lineárny kanál

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \sum_{i=1}^{n} c_i.\boldsymbol{\psi}_i$$

$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = c_1 \begin{pmatrix} \psi_{11} \\ \psi_{21} \\ \vdots \\ \psi_{n1} \end{pmatrix} + \ldots + c_n \begin{pmatrix} \psi_{1n} \\ \psi_{2n} \\ \vdots \\ \psi_{nn} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \sum_{i=1}^{n} c_i.\boldsymbol{\psi}_i$$

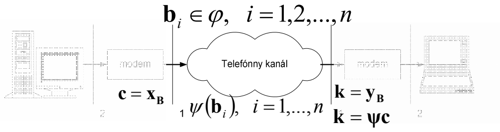
$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}_{\mathbf{B}} = \begin{pmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\ . & . & . & . \\ \psi_{n1} & \psi_{n2} & \cdots & \psi_{nn} \end{pmatrix} . \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}_{\mathbf{B}}$$

Lineárny kanál

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \sum_{i=1}^{n} c_i.\boldsymbol{\psi}_i$$

$$\mathbf{k} = \boldsymbol{\psi}\mathbf{c}$$

$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$



Lineárny kanál

$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \psi\left(\sum_{i=1}^{n} c_i.\mathbf{b}_i\right) = \sum_{i=1}^{n} c_i.\psi(\mathbf{b}_i)$$

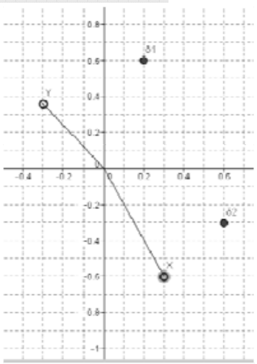
Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Presnejšie, že

$$\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$$

Lineárny kanál

GeoGebra



Lineárny kanál

Požadujeme $\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$

$$\mathbf{b}_i \boldsymbol{\psi} = \lambda_i \mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

Vlastné vektory kanála

Príklad $\boldsymbol{\delta}_1^T=(0{,}2;0{,}6)$ $\boldsymbol{\delta}_2^T=(0{,}6;-0{,}3)$ $\boldsymbol{\psi}=\begin{pmatrix}0{,}2 & 0{,}6\\ 0{,}6 & -0{,}3\end{pmatrix}$

Riešenie: $\mathbf{b}\boldsymbol{\psi}=\lambda\mathbf{b}$

$$\mathbf{b}(\boldsymbol{\psi}-\lambda\mathbf{E})=\mathbf{0}$$

$$\boldsymbol{\psi}-\lambda\mathbf{E}=\mathbf{0}$$

$$\begin{pmatrix}0{,}2-\lambda & 0{,}6\\ 0{,}6 & -0{,}3-\lambda\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$(0{,}2-\lambda).(-0{,}3-\lambda)-0{,}36=0$$

Vlastné vektory kanála

$$\lambda_1=-0{,}7\quad \mathbf{b}_1(\boldsymbol{\psi}-\lambda_1\mathbf{E})=\mathbf{0}$$
$$\lambda_2=0{,}6\quad \mathbf{b}_2(\boldsymbol{\psi}-\lambda_2\mathbf{E})=\mathbf{0}$$

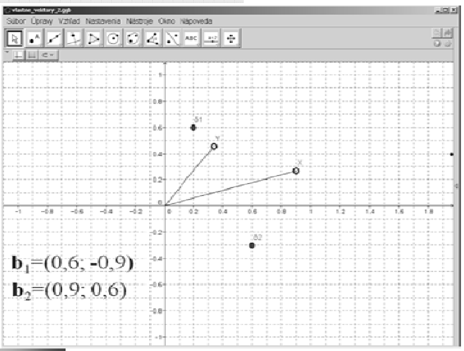
$$(0{,}2-\lambda_1).b_{11}+0{,}6.b_{12}=0\qquad 0{,}9.b_{11}+0{,}6.b_{12}=0$$
$$0{,}6.b_{11}+(-0{,}3-\lambda_1).b_{12}=0\qquad 0{,}6.b_{11}+0{,}4.b_{12}=0$$

$\mathbf{b}_1=(2t;-3t)$ napr. $\mathbf{b}_1=(0{,}6;-0{,}9)$

$\mathbf{b}_2=(3t;2t)$ napr. $\mathbf{b}_2=(0{,}9;0{,}6)$

Vlastné vektory kanála

GeoGebra



Vlastné vektory kanála

$$\mathbf{b}_i\in\varphi,\quad i=1,2,\dots,n$$

$$\mathbf{x_B}=\sum_{i=1}^{n}c_i.\mathbf{b}_i\qquad \mathbf{y_B}=\sum_{i=1}^{n}k_i.\mathbf{b}_i$$

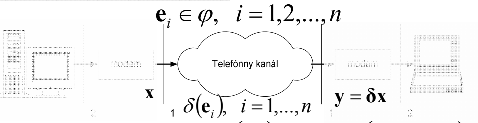
$$\mathbf{y_B}=\psi(\mathbf{x_B})=\psi\left(\sum_{i=1}^{n}c_i.\mathbf{b}_i\right)=\sum_{i=1}^{n}c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{y_B}=\sum_{i=1}^{n}c_i.\lambda_i\mathbf{b}_i$$

$$\boxed{k_i=\lambda_i c_i\quad i=1,2,\dots,n}$$

Vlastné vektory kanála

$\mathbf{e}_i\in\varphi,\quad i=1,2,\dots,n$



$\mathbf{x}$ $\delta(\mathbf{e}_i),\ i=1,\dots,n$ $\mathbf{y}=\boldsymbol{\delta}\mathbf{x}$

$\mathbf{e}_1=(1{,}0),\ \boldsymbol{\delta}_1^T=(0{,}2;0{,}6)$ $\begin{pmatrix}y_1\\y_2\end{pmatrix}=(x_1,x_2).\begin{pmatrix}0{,}2 & 0{,}6\\0{,}6 & -0{,}3\end{pmatrix}$

$\mathbf{e}_2^T=(0{,}1),\ \boldsymbol{\delta}_2^T=(0{,}6;-0{,}3)$

$\mathbf{b}_1^T=(0{,}6;-0{,}9)$ $\mathbf{x}=(0{,}4;\,0{,}7)$ $\mathbf{x}=(0{,}23;\,0{,}67)_{\mathbf{B}}$

$\mathbf{b}_2^T=(0{,}9;\ 0{,}6)$ $c_1=\frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)}=\frac{((0{,}4;0{,}7),(0{,}6;-0{,}9))}{((0{,}6;-0{,}9),(0{,}6;-0{,}9))}=0{,}231$

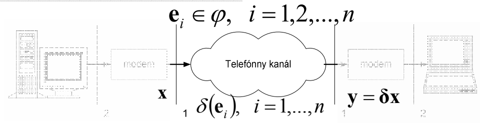
$\boldsymbol{\lambda}=(-0{,}7;0{,}6)$ $c_2=\frac{(\mathbf{x},\mathbf{b}_2)}{(\mathbf{b}_2,\mathbf{b}_2)}=\frac{((0{,}4;0{,}7),(0{,}9;0{,}6))}{((0{,}9;0{,}6),(0{,}9;0{,}6))}=0{,}667$

$k_1=\lambda_1c_1=0{,}162$

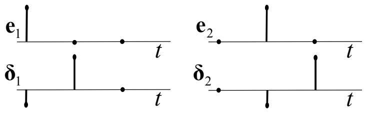
$k_2=\lambda_2c_2=0{,}4$ $\mathbf{y}=(0{,}16;\,0{,}4)_{\mathbf{B}}$

časovo-invariantný kanál

$\mathbf{e}_i\in\varphi,\quad i=1,2,\dots,n$



$\mathbf{x}$ $\delta(\mathbf{e}_i),\ i=1,\dots,n$ $\mathbf{y}=\boldsymbol{\delta}\mathbf{x}$

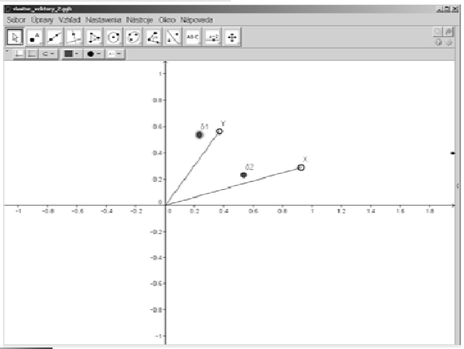


$\mathbf{e}_1=(1{,}0),\ \boldsymbol{\delta}_1^T=(\delta_1;\delta_2)$ $\begin{pmatrix}y_1\\y_2\end{pmatrix}=(x_1,x_2).\begin{pmatrix}\delta_1 & \delta_2\\ \delta_2 & \delta_1\end{pmatrix}$

$\mathbf{e}_2^T=(0{,}1),\ \boldsymbol{\delta}_2^T=(\delta_2;\delta_1)$

Báza t-invariantného kanála

GeoGebra



časovo-invariantný kanál

$\mathbf{e}_1 = (1,0),\quad \boldsymbol{\delta}_1^T = (\delta_1;\delta_2)$

$\mathbf{e}_2^T = (0,1),\quad \boldsymbol{\delta}_2^T = (\delta_2;\delta_1)$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (x_1, x_2).\begin{pmatrix} \delta_1 & \delta_2 \\ \delta_2 & \delta_1 \end{pmatrix}$$

$$\mathbf{b}(\boldsymbol{\psi} - \lambda\mathbf{E}) = \mathbf{0}$$

$$\begin{pmatrix} \delta_1 - \lambda & \delta_2 \\ \delta_2 & \delta_1 - \lambda \end{pmatrix} = \mathbf{0}$$

$$(\delta_1 - \lambda)^2 - \delta_2^2 = 0$$

$$\lambda_{1,2} = \delta_1 \pm \delta_2$$

$$\delta_1 - (\delta_1 + \delta_2).b_{11} + \delta_2.b_{12} = 0$$

$$b_{11} = b_{12}$$

$$\delta_1 - (\delta_1 - \delta_2).b_{21} + \delta_2.b_{22} = 0$$

$$b_{21} = -b_{22}$$

$\mathbf{b}_1^T = (1;1)$

$\mathbf{b}_2^T = (1;\ -1)$

$\boldsymbol{\lambda} = (\delta_1 + \delta_2;\ \delta_1 - \delta_2)$

Frekvenčný prenos t-invar. kanála

$\mathbf{x} = (x_1;\, x_2)\quad \mathbf{b}_1^T = (1;1)\quad \boldsymbol{\lambda} = (\delta_1 + \delta_2;\ \delta_1 - \delta_2)$

$\mathbf{b}_2^T = (1;\ -1)$

$$c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((x_1;x_2),(1;1))}{((1;1),(1;1))} = \frac{x_1+x_1}{2}$$

$$c_2 = \frac{(\mathbf{x},\mathbf{b}_2)}{(\mathbf{b}_2,\mathbf{b}_2)} = \frac{((x_1;x_2),(1;\text{-}1))}{((1;\text{-}1),(1;\text{-}1))} = \frac{x_1-x_1}{2}$$

$$\mathbf{x} = \left(\tfrac{1}{2}(x_1 + x_2);\, \tfrac{1}{2}(x_1 - x_2)\right)_{\mathbf{B}}$$

$$\mathbf{y} = \left(\tfrac{1}{2}(x_1 + x_2).(\delta_1 + \delta_2);\, \tfrac{1}{2}(x_1 - x_2).(\delta_1 - \delta_2)\right)_{\mathbf{B}}$$

Vlastné signály t-invariantného kanála

$\mathbf{e}_0 = (1,0,\dots,0),\quad \boldsymbol{\delta}_0^T = (\delta_0, \delta_1, \dots, \delta_{N-1})$

$\mathbf{e}_1^T = (0,1,\dots,0)\quad \boldsymbol{\delta}_1^T = (\delta_{N-1}, \delta_0, \dots, \delta_{N-2})$

$\dots$

$\mathbf{e}_{N-1}^T = (0,0,\dots,1)\quad \boldsymbol{\delta}_0^T = (\delta_1, \delta_2, \dots, \delta_0)$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda\mathbf{E}) = \mathbf{0}$$

$n = 0,1,\dots, N-1$

$$\boxed{\begin{aligned} \lambda_n &= \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \\ \mathbf{b}_n &= \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right) \end{aligned}}$$

Frekvenčný prenos t-invar. kanála

$\mathbf{x} = (x_0, x_2, \dots, x_{N-1})\quad \boldsymbol{\delta} = (\delta_0, \delta_2, \dots, \delta_{N-1})$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk}\qquad n = 0,1,\dots, N-1$$

$$\mathbf{b}_n = \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

$\mathbf{x} = (c_0, c_2, \dots, c_{N-1})_{\mathbf{B}}\quad \mathbf{y} = (k_0, k_2, \dots, k_{N-1})_{\mathbf{B}}$

$$\boxed{\begin{aligned} c_n &= \frac{(\mathbf{x},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)} = \frac{1}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk},\quad c_n \in \mathbf{C} \\ k_n &= \lambda_n c_n\quad n = 0,1,\dots, N-1 \end{aligned}}$$

Vlastné signály – skúška správnosti

$$\mathbf{b}_n = \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk}\qquad n = 0,1,\dots, N-1$$

$$\mathbf{b}_n\boldsymbol{\delta} = \lambda_n\mathbf{b}_n$$

$$\boxed{\left(1,\dots, e^{-j\frac{2\pi}{N}nk}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right)}\begin{pmatrix} \delta_0, & \dots, & \boxed{\delta_l,} & \dots, \delta_{N-1} \\ \delta_{N-1}, & \dots, & \boxed{\delta_{l+1},} & \dots, \delta_0 \end{pmatrix} =$$

$$= \boxed{\sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk}}\left(1,\dots, \boxed{e^{-j\frac{2\pi}{N}nl},} \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastné signály – skúška správnosti

$$\left(1, \dots, e^{-j\frac{2\pi}{N}nk}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right) \cdot \begin{pmatrix} \delta_l \\ \delta_{l-1} \\ \delta_{l+1} \end{pmatrix} = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} e^{-j\frac{2\pi}{N}nl}$$

$$\sum_{k=0}^{N-1} \delta_{l-k} e^{-j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}n(k+l)}$$

+ súčet *mod(N)*

$k + l = \hat{k}$

$$\sum_{k=0}^{N-1} \delta_{l-k} e^{-j\frac{2\pi}{N}nk} = \sum_{\hat{k}=l}^{l-1} \delta_{l-\hat{k}} e^{-j\frac{2\pi}{N}n\hat{k}}$$

Ďakujem za Vašu pozornosť