Teória oznamovania 8

Obsah:

- prechod bázických signálov kanálom
- frekvenčný prenos kanála
- časovo invariantný kanál
- frekvenčný prenos časovo invariantného kanála

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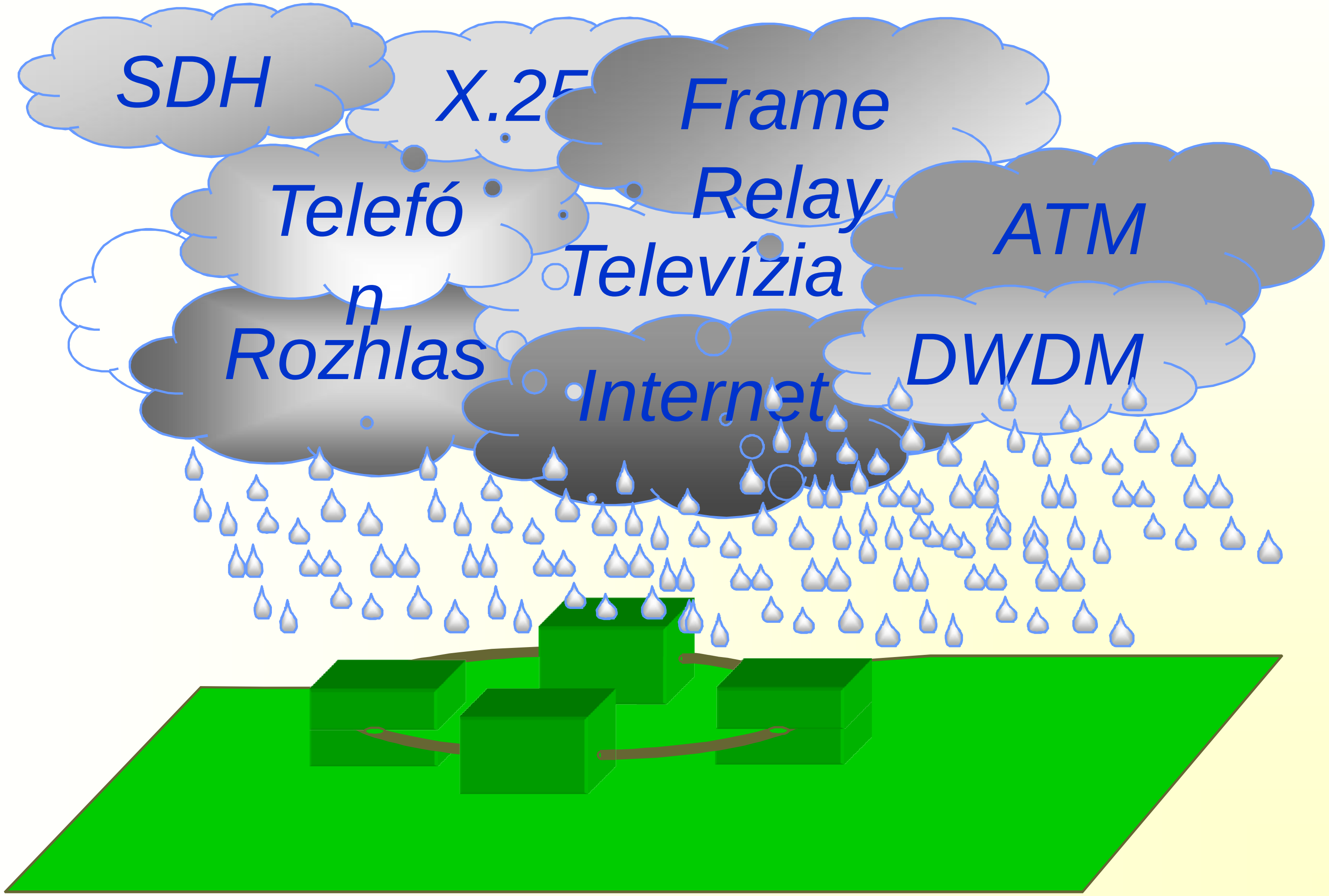
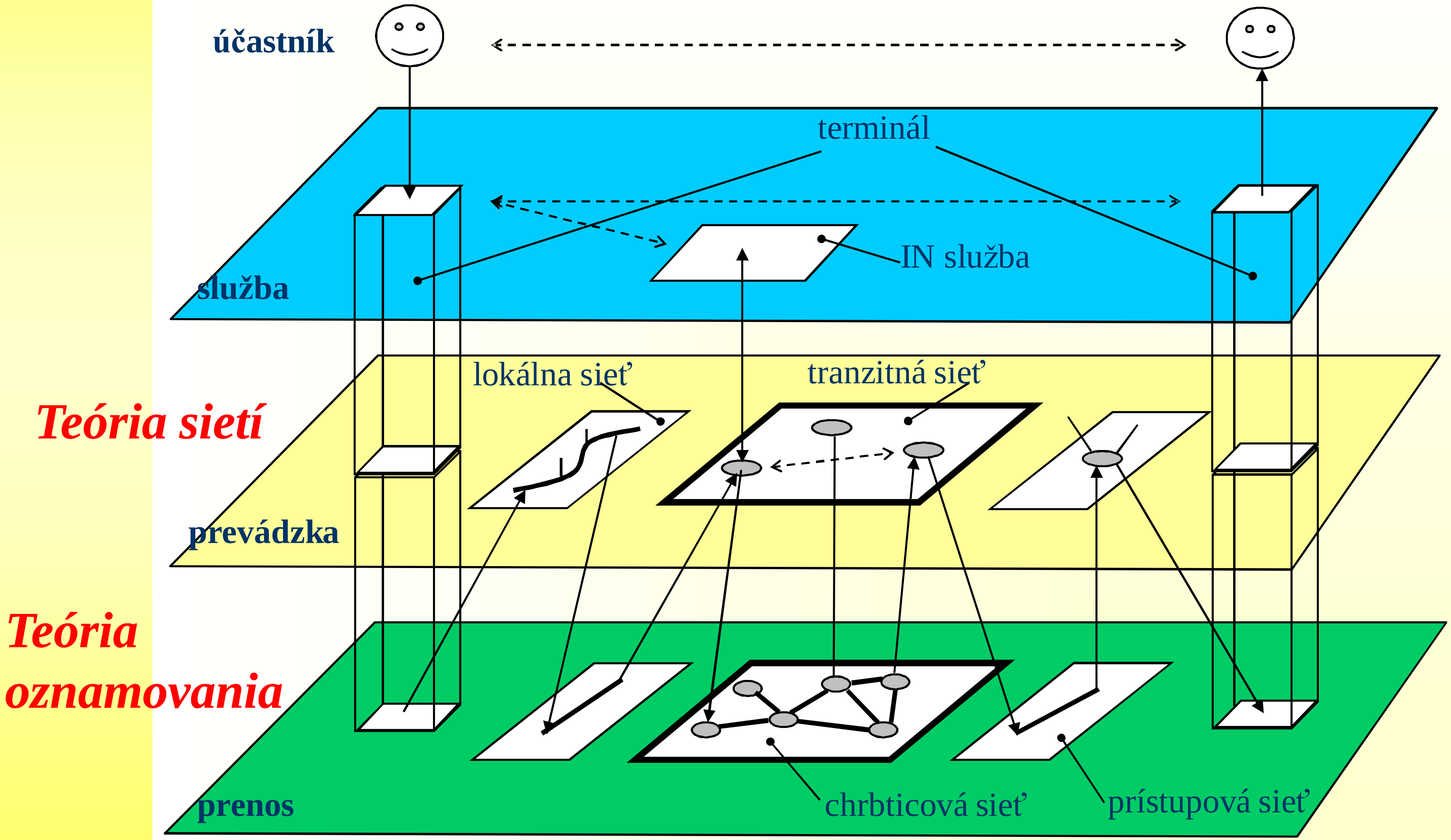
Všeobecný model siete



Základné vrstvy



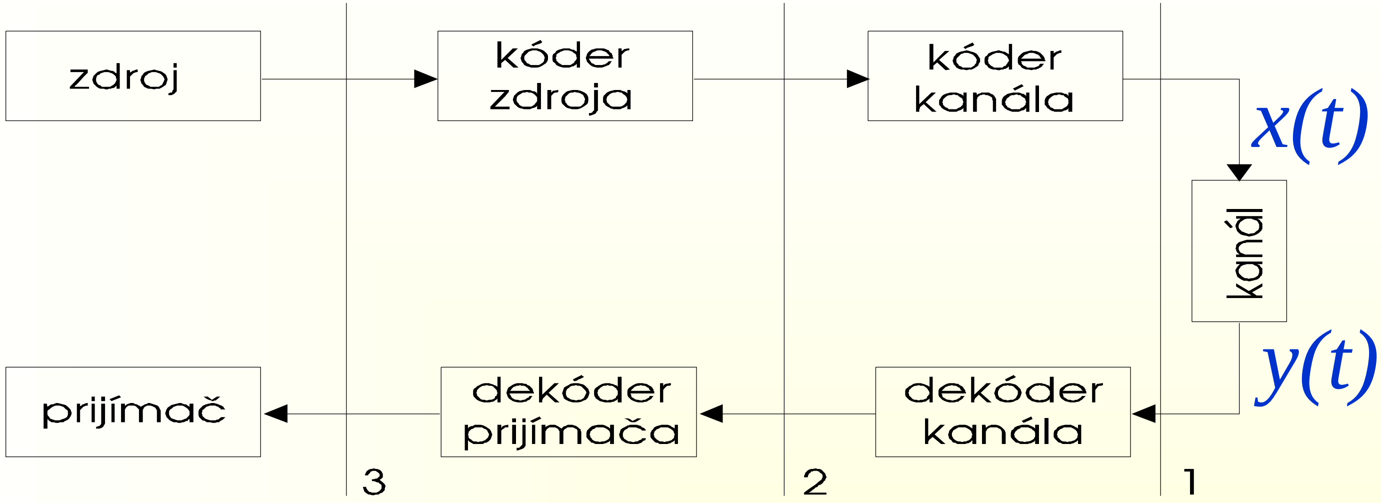
Vrstva prenosu

Hlavné úlohy: ??

prenos jedného signálu

súčasný prenos signálov

Prenos bez skreslenia

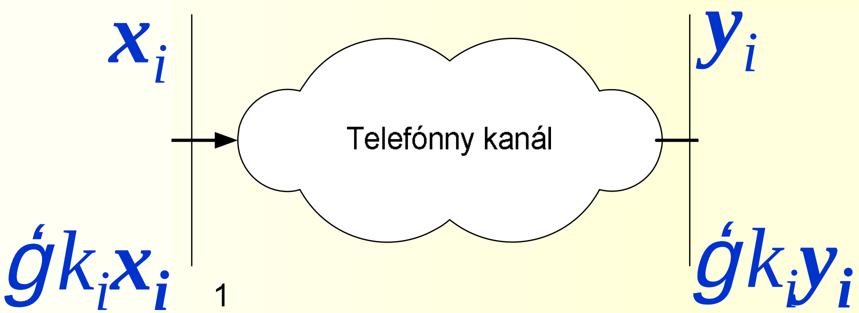


Prispôsobenie prenosovému médiu

Lineárny kanál

$$\forall \mathbf{x}_i \in \varphi, \quad i = 0,1,\ldots, N-1$$

$$\psi\left(\sum_{i=0}^{N-1} k_i.\mathbf{x}_i\right) = \sum_{i=0}^{N-1} k_i.\psi(\mathbf{x}_i) = \sum_{i=0}^{N-1} k_i.\mathbf{y}_i$$



Lineárny kanál

$$\mathbf{x} = \sum_{i=0}^{N-1} x_i . \mathbf{e}_i \qquad \mathbf{y} = \sum_{i=0}^{N-1} y_i . \mathbf{e}_i$$

$$\mathbf{y} = \delta(\mathbf{x}) = \delta\left(\sum_{i=0}^{N-1} x_i . \mathbf{e}_i \right) = \sum_{i=0}^{N-1} x_i . \delta(\mathbf{e}_i)$$

$$\mathbf{y} = \sum_{i=0}^{N-1} x_i . \boldsymbol{\delta}_i$$

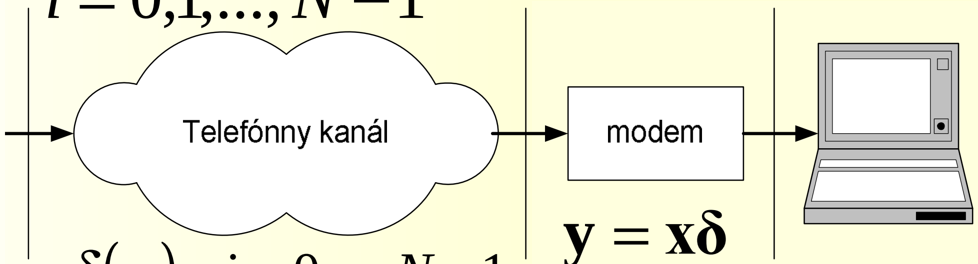
$$\boldsymbol{\delta}_i = \begin{pmatrix} \delta_{i0} & \dots & \delta_{i,N-1} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=0}^{N-1} x_i . \boldsymbol{\delta}_i$$

$$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$$

$\mathbf{e}_i \in \varphi, \quad i = 0,1,\dots, N-1$



$\mathbf{x}$

$\delta(\mathbf{e}_i), \quad i = 0,\dots, N-1$

$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$

Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\dots, N-1$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} k_i.\mathbf{b}_i$$

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=0}^{N-1} c_i.\mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i.\psi(\mathbf{b}_i)$$

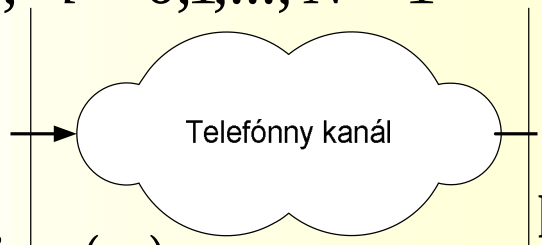
$$\mathbf{k} = \sum_{i=0}^{N-1} c_i.\boldsymbol{\psi}_i \qquad \boldsymbol{\psi}_i = \begin{pmatrix}\psi_{i0} & \dots & \psi_{i,N-1}\end{pmatrix}$$

Lineárny kanál

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \sum_{i=0}^{N-1} c_i . \psi_i$$

$$\mathbf{k} = \mathbf{c}\boldsymbol{\psi}$$

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\dots, N-1$$



$$\mathbf{c} = \mathbf{x_B}$$

$$\psi(\mathbf{b}_i), \quad i = 0,\dots, N-1$$

$$\mathbf{k} = \mathbf{y_B}$$

$$\mathbf{k} = \mathbf{c}\boldsymbol{\psi}$$

Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 0, 1, \ldots, N-1$$

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \psi\left(\sum_{i=0}^{N-1} c_i . \mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i . \psi(\mathbf{b}_i)$$

Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Presnejšie, že

$$\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$$

Lineárny kanál

Požadujeme $\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$

$$\mathbf{b}_i \boldsymbol{\psi} = \lambda_i \mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

Vlastné vektory kanála

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\ldots, N-1$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} k_i.\mathbf{b}_i$$

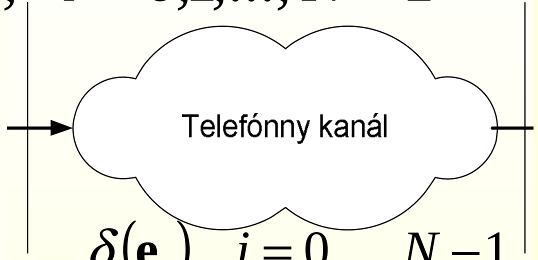
$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=0}^{N-1} c_i.\mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\lambda_i \mathbf{b}_i$$

$$\boxed{k_i = \lambda_i c_i, i = 0,1,\ldots, N-1}$$

Vlastné vektory kanála

$$\mathbf{e}_i \in \varphi, \quad i = 0,1,\dots, N-1$$



$$\mathbf{x} \qquad \delta(\mathbf{e}_i), \quad i = 0,\dots, N-1 \qquad \mathbf{y} = \boldsymbol{\delta}\mathbf{x}$$

$$\mathbf{e}_0 = (1,0), \quad \boldsymbol{\delta}_1 = (0{,}2;0{,}6)$$

$$\mathbf{e}_1 = (0,1), \quad \boldsymbol{\delta}_1 = (0{,}6;-0{,}3)$$

$$\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = (x_0, x_1).\begin{pmatrix} 0{,}2 & 0{,}6 \\ 0{,}6 & -0{,}3 \end{pmatrix}$$

$$\mathbf{b}_0 = (0{,}6;-0{,}9) \qquad \mathbf{x} = (0{,}4;\, 0{,}7) \qquad \mathbf{x} = (0{,}23;\, 0{,}67)_{\mathbf{B}}$$

$$\mathbf{b}_1 = (0{,}9;\; 0{,}6) \qquad c_0 = \frac{(\mathbf{x},\mathbf{b}_0)}{(\mathbf{b}_0,\mathbf{b}_0)} = \frac{((0{,}4;0{,}7),(0{,}6;-0{,}9))}{((0{,}6;-0{,}9),(0{,}6;-0{,}9))} = 0{,}231$$

$$\boldsymbol{\lambda} = (-0{,}7;\, 0{,}6) \qquad c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((0{,}4;0{,}7),(0{,}9;0{,}6))}{((0{,}9;0{,}6),(0{,}9;0{,}6))} = 0{,}667$$

$$k_0 = \lambda_0 c_0 = -0{,}162$$

$$k_1 = \lambda_1 c_1 = 0{,}4 \qquad \mathbf{y} = (0{,}16;\, 0{,}4)_{\mathbf{B}}$$

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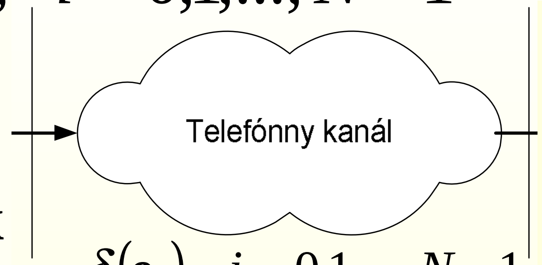
Časovo-invariantný kanál

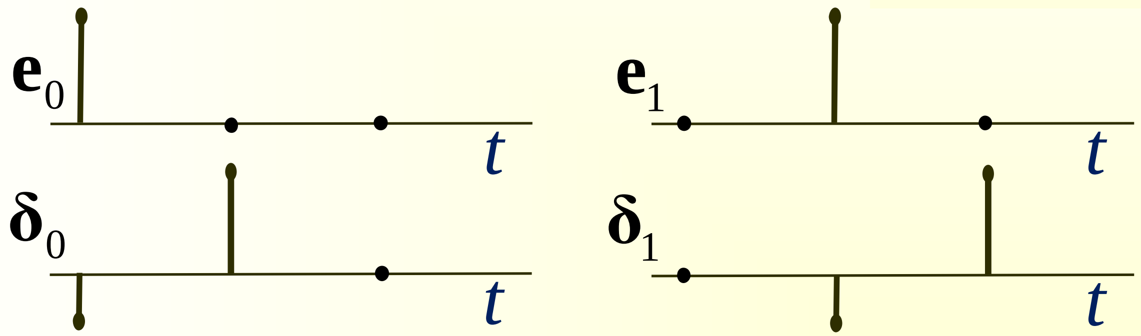
$$\mathbf{e}_i \in \varphi,\quad i = 0,1,\ldots,N-1$$



$\mathbf{x}$

$\delta(\mathbf{e}_i),\quad i = 0,1,\ldots,N-1$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x}$$



$$\mathbf{e}_0 = (1,0,0),\ \boldsymbol{\delta}_0 = (\delta_0, \delta_1, \delta_2)$$

$$\mathbf{e}_1 = (0,1,0),\ \boldsymbol{\delta}_1 = (\delta_2, \delta_0, \delta_1) \qquad (y_0, y_1, y_2) = (x_0, x_1, x_2).\begin{pmatrix} \delta_0 & \delta_1 & \delta_2 \\ \delta_2 & \delta_0 & \delta_1 \\ \delta_1 & \delta_2 & \delta_0 \end{pmatrix}$$

$$\mathbf{e}_2 = (0,0,1),\ \boldsymbol{\delta}_1 = (\delta_1, \delta_2, \delta_0)$$

Časovo-invariantný kanál

$$(y_0, y_1, y_2) = (x_0, x_1, x_2).\begin{pmatrix} \delta_0 & \delta_1 & \delta_2 \\ \delta_2 & \delta_0 & \delta_1 \\ \delta_1 & \delta_2 & \delta_0 \end{pmatrix} \quad y_1 = x_0\delta_1 + x_1\delta_0 + x_2\delta_2$$

$$(y_0, y_1, \ldots, y_{N-1}) = (x_0, x_1, \ldots, x_{N-1}).\begin{pmatrix} \delta_0 & \delta_1 & \ldots\delta_l & \ldots & \delta_{N-1} \\ \delta_{N-1} & \delta_0 & \ldots\delta_{l-1} & \ldots & \delta_{N-2} \\ \delta_{N-k} & \delta_{N-k+1} & \ldots\delta_{l-k} & \ldots & \delta_{N-1-k} \\ \delta_1 & \delta_2 & \ldots\delta_{l+1} & \ldots & \delta_0 \end{pmatrix}$$

$$y_l = (x_0, x_1, \ldots, x_{N-1}).\begin{pmatrix} \delta_l \\ \delta_{l-1} \\ \delta_{l-k} \\ \delta_{l+1} \end{pmatrix} = \sum_{k=0}^{N-1} x_k \delta_{l-k}$$

Časovo-invariantný kanál

$$\mathbf{e}_0 = (1,0), \qquad \boldsymbol{\delta}_0 = (\delta_0, \delta_1)$$

$$\mathbf{e}_1 = (0,1), \qquad \boldsymbol{\delta}_1 = (\delta_1, \delta_0)$$

$$(y_0, y_1) = (x_0, x_1).\begin{pmatrix} \delta_0 & \delta_1 \\ \delta_1 & \delta_0 \end{pmatrix}$$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda\mathbf{E}) = \mathbf{0}$$

$$\begin{pmatrix} \delta_0 - \lambda & \delta_1 \\ \delta_1 & \delta_0 - \lambda \end{pmatrix} = \mathbf{0}$$

$$(\delta_0 - \lambda)^2 - \delta_1^2 = 0$$

$$\lambda_{0,1} = \delta_0 \pm \delta_1$$

Časovo-invariantný kanál

$$\mathbf{b}(\boldsymbol{\psi} - \lambda\mathbf{E}) = \mathbf{0}$$

$$(b_{00}, b_{01}).\begin{pmatrix} \delta_0 - (\delta_0 + \delta_1) & \delta_1 \\ \delta_1 & \delta_0 - (\delta_0 + \delta_1) \end{pmatrix} = (0,0)$$

$$(b_{00}, b_{01}).\begin{pmatrix} -\delta_1 & \delta_1 \\ \delta_1 & -\delta_1 \end{pmatrix} = (0,0)$$

$$-\delta_1 b_{00} + \delta_1 . b_{01} = 0$$

$$b_{00} = b_{01}$$

$$\mathbf{b}_0 = (1, 1)$$

Časovo-invariantný kanál

$$\mathbf{b}(\boldsymbol{\psi} - \lambda \mathbf{E}) = \mathbf{0}$$

$$(b_{21}, b_{22}).\begin{pmatrix} \delta_1 - (\delta_1 - \delta_2) & \delta_2 \\ \delta_2 & \delta_1 - (\delta_1 - \delta_2) \end{pmatrix} = (0,0)$$

$$(b_{21}, b_{22}).\begin{pmatrix} \delta_2 & \delta_2 \\ \delta_2 & \delta_2 \end{pmatrix} = (0,0)$$

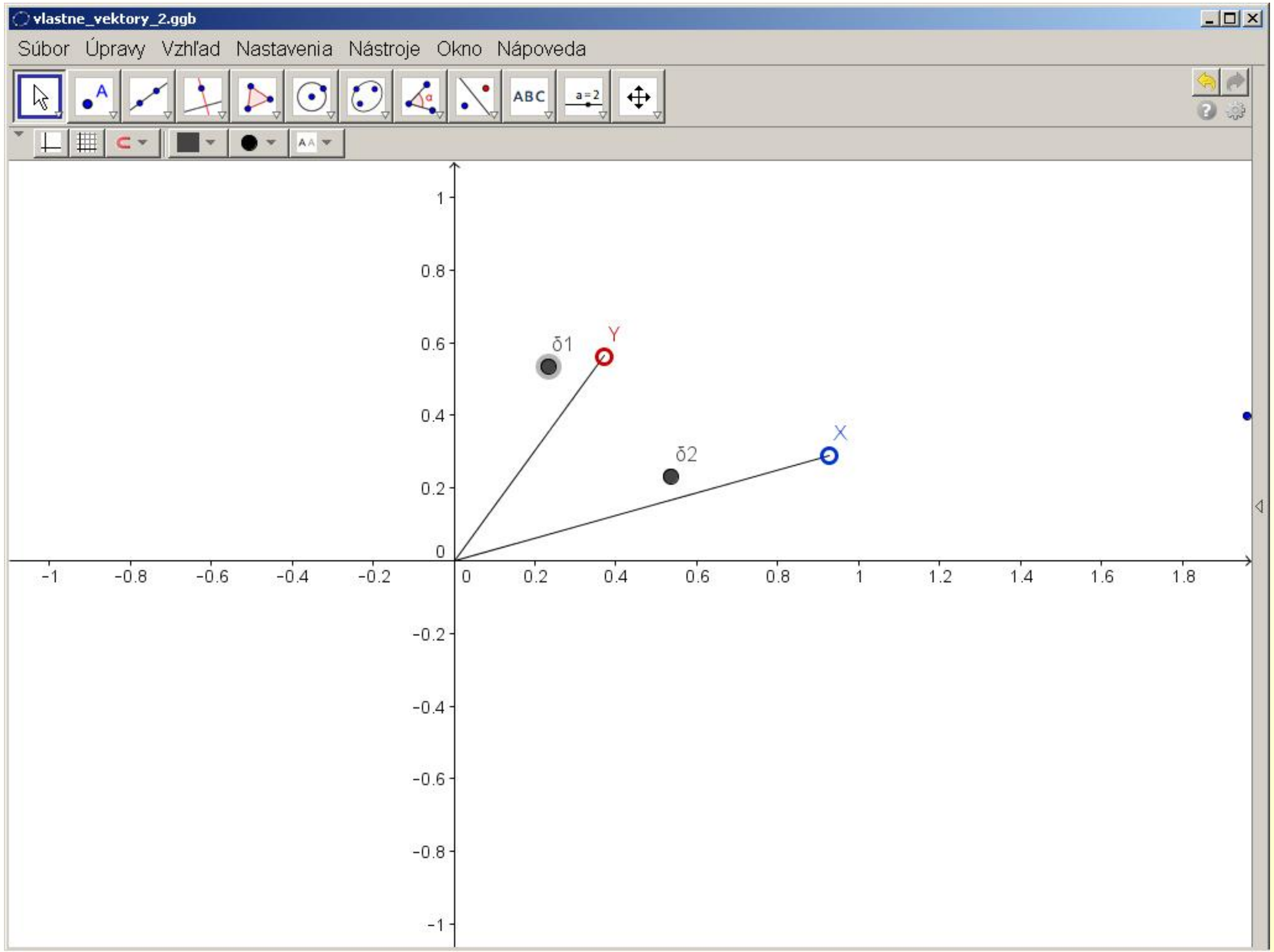
$$\delta_2 b_{11} + \delta_2 . b_{12} = 0$$

$$b_{11} = -b_{12}$$

$$\mathbf{b}_2 = (1, -1)$$

Báza t-invariantného kanála

GeoGebra



Frekvenčný prenos t-invar. kanála

$$\mathbf{x} = (x_0, x_1) \quad \mathbf{b}_0 = (1;1) \qquad \boldsymbol{\lambda} = (\delta_0 + \delta_1; \delta_0 - \delta_1)$$

$$\mathbf{b}_1 = (1;\ -1)$$

$$c_0 = \frac{(\mathbf{x},\mathbf{b}_0)}{(\mathbf{b}_0,\mathbf{b}_0)} = \frac{((x_0;x_1),(1;1))}{((1;1),(1;1))} = \frac{x_0+x_1}{2}$$

$$c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((x_0;x_1),(1;-1))}{((1;-1),(1;-1))} = \frac{x_0-x_1}{2}$$

$$\mathbf{x} = \left(\tfrac{1}{2}(x_0 + x_1); \tfrac{1}{2}(x_0 - x_1)\right)_{\mathbf{B}}$$

$$\mathbf{y} = \left(\tfrac{1}{2}(x_0 + x_1).(\delta_0 + \delta_1); \tfrac{1}{2}(x_0 - x_1).(\delta_0 - \delta_1)\right)_{\mathbf{B}}$$

Vlastné signály t-invariantného kanála

$$\mathbf{e}_0 = (1,0,\dots,0), \qquad \boldsymbol{\delta}_0 = (\delta_0, \delta_1, \dots, \delta_{N-1})$$

$$\mathbf{e}_1 = (0,1,\dots,0) \qquad \boldsymbol{\delta}_1 = (\delta_{N-1}, \delta_0, \dots, \delta_{N-2})$$

$$\dots \qquad \dots$$

$$\mathbf{e}_{N-1} = (0,0,\dots,1) \qquad \boldsymbol{\delta}_{N-1} = (\delta_1, \delta_2, \dots, \delta_0)$$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda \mathbf{E}) = \mathbf{0}$$

$$n = 0,1,\dots,N-1$$

$$\boxed{\begin{aligned} \lambda_n &= \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \\ \mathbf{b}_n &= \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \dots, e^{j\frac{2\pi}{N}n(N-1)}\right) \end{aligned}}$$

Frekvenčný prenos t-invar. kanála

$$\mathbf{x} = \left(x_0, x_2, \ldots, x_{N-1}\right) \quad \boldsymbol{\delta}_0 = \left(\delta_0, \delta_2, \ldots, \delta_{N-1}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\ldots, N-1$$

$$\mathbf{b}_n = \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \ldots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x} = \left(c_0, c_2, \ldots, c_{N-1}\right)_{\mathbf{B}} \quad \mathbf{y} = \left(k_0, k_2, \ldots, k_{N-1}\right)_{\mathbf{B}}$$

$$c_n = \frac{(\mathbf{x}, \mathbf{b}_n)}{(\mathbf{b}_n, \mathbf{b}_n)} = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}, \quad c_n \in \mathbf{C}$$

$$k_n = \lambda_n c_n \quad n = 0,1,\ldots, N-1$$

Vlastné signály – skúška správnosti

$$\mathbf{b}_n = \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \dots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\dots,N-1$$

$$\mathbf{b}_n \boldsymbol{\delta} = \lambda_n \mathbf{b}_n$$

$$\left(1,\dots,e^{j\frac{2\pi}{N}nk},\dots,e^{j\frac{2\pi}{N}n(N-1)}\right) \begin{pmatrix} \delta_0, & \dots, & \delta_l, & \dots, & \delta_{N-1} \\ \delta_{N-1}, & \dots, & \delta_{l+1}, & \dots, & \delta_0 \end{pmatrix} =$$

$$= \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \left(1,\dots,e^{j\frac{2\pi}{N}nl},\dots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastné signály – skúška správnosti

$$\left(e^{j\frac{2\pi}{N}n0},\dots,e^{j\frac{2\pi}{N}nk},\dots,e^{j\frac{2\pi}{N}n(N-1)}\right)\cdot\begin{pmatrix}\delta_l\\ \cdots\\ \delta_{l-k}\\ \cdots\\ \delta_{l+1}\end{pmatrix}=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}e^{j\frac{2\pi}{N}nl}$$

$$\sum_{k=0}^{N-1}\delta_{l-k}e^{j\frac{2\pi}{N}nk}=\sum_{k=0}^{N-1}\delta_k e^{j\frac{2\pi}{N}n(l-k)}$$

Vlastné signály – skúška správnosti

N=4, l=2

$$\sum_{k=0}^{N-1}\delta_{l-k}e^{j\frac{2\pi}{N}nk}=\sum_{k=0}^{N-1}\delta_{k}e^{j\frac{2\pi}{N}n(l-k)}$$

$$\delta_{2-0}e^{j\frac{2\pi}{N}n0}+\delta_{2-1}e^{j\frac{2\pi}{N}n1}+\delta_{2-2}e^{j\frac{2\pi}{N}n2}+\delta_{2-3}e^{j\frac{2\pi}{N}n3}=$$

$$=\delta_{0}e^{j\frac{2\pi}{N}n(2-0)}+\delta_{1}e^{j\frac{2\pi}{N}n(2-1)}+\delta_{2}e^{j\frac{2\pi}{N}n(2-2)}+\delta_{3}e^{j\frac{2\pi}{N}n(2-3)}$$

$$\delta_{2}e^{j\frac{2\pi}{N}n0}+\delta_{1}e^{j\frac{2\pi}{N}n1}+\delta_{0}e^{j\frac{2\pi}{N}n2}+\delta_{3}e^{j\frac{2\pi}{N}n3}=$$

$$=\delta_{0}e^{j\frac{2\pi}{N}n2}+\delta_{1}e^{j\frac{2\pi}{N}n1}+\delta_{2}e^{j\frac{2\pi}{N}n0}+\delta_{3}e^{j\frac{2\pi}{N}n3}$$

Vlastné signály – skúška správnosti

$$\sum_{k=0}^{N-1} \delta_{l-k} e^{j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{j\frac{2\pi}{N}n(l-k)}$$

$$(l-k)+k=l \qquad k+(l-k)=l$$

Ďakujem za Vašu pozornosť