

1. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\dots(f))))$, $n \in \mathbb{N}$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = 6 + 3x$, $x \in \mathbb{R}$.

$$\begin{array}{r} 6 \\ 18 + 6 \\ 54 + 18 + 6 \\ 162 + 54 + 18 + 6 \\ \hline 6(3^3) + 6(3^2) + 6(3^1) + 6 \cdot 3^0 \end{array}$$

$$f_2(x) = 24 + 9x$$

$$f_3(x) = 78 + 27x$$

$$f_4(x) = 240 + 81x$$

$$f_n(x) = 3^n x + 6 \sum_{k=0}^{n-1} 3^k = 3^n x + 6 \frac{3^n - 1}{2}$$

2. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\dots(f))))$, $n \in \mathbb{N}$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = 6 - 3x$, $x \in \mathbb{R}$.

$$\begin{array}{r} 6 \\ -18 + 6 \\ 54 - 18 + 6 \\ -162 + 54 - 18 + 6 \\ \hline -6(3^3) + 6(3^2) - 6(3^1) + 6 \cdot 3^0 \end{array}$$

$$f_2(x) = -12 + 9x$$

$$f_3(x) = 42 - 27x$$

$$f_4(x) = -120 + 81x$$

$$f_n(x) = (-3)^n x + 6 \sum_{k=0}^{n-1} (-3)^k = (-3)^n x + 6 \frac{(-3)^n - 1}{-4}$$

3. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, $f_4 = f(f(f(f)))$ a inverznú funkciu f^{-1} , ak funkcia f je definovaná predpisom $f(x) = \frac{x+3}{2x-1}$, $x \in \mathbb{R} - \{\frac{1}{2}\}$.

$$y = \frac{x+3}{2x-1} \quad | 2x-1 \quad x = \frac{y+3}{2y-1}$$

$$\begin{aligned} y(2x-1) &= x+3 \\ 2xy-y &= x+3 \\ 2xy-x &= y+3 \end{aligned}$$

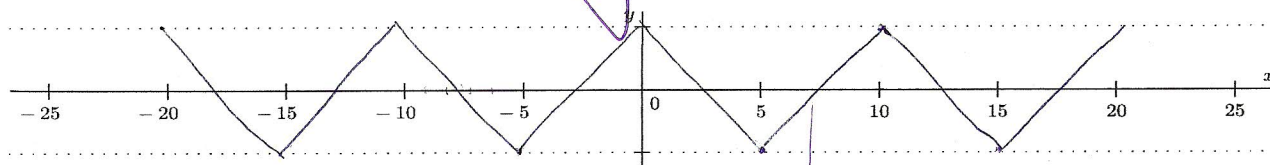
$$f^{-1}(x) = \frac{x+3}{2x-1}$$

$$f_2(x) = \frac{5x-6}{x+4}$$

$$f_3(x) = \frac{-8x+18}{6x-10}$$

$$f_4(x) = \frac{26x-48}{-14x+28}$$

4. Zostrojte periodickú funkciu $y = f(x)$ s primitívnou periódou 10 a načrtnite jej graf tak, aby bola párna, klesajúca na intervale $(-9; -8)$, rastúca na intervale $(18; 19)$ a aby $f(5) = -1$.



5. Nájdite s presnosťou $\varepsilon = 0.01$ kladné riešenie rovnice $4e^x - 16x^5 - 4 = 0$.

$$\text{Výpočet počtu krokov } n: 0.01 = 10^{-2} = \frac{1}{100} \Rightarrow \frac{1}{2^n} < \frac{1}{100} \Rightarrow 100 < 2^n \quad m=7 \quad 2^7=128$$

| n | a_n | x_{n+1} | b_n | $f(x_{n+1})$ |
|---|---------|-----------|---------|--------------|
| 0 | 0 | 0.5 | 1 | 2.09489 |
| 1 | 0.5 | 0.75 | 1 | 0.67113 |
| 2 | 0.75 | 0.875 | 1 | -2.61104 |
| 3 | 0.875 | 0.9375 | 0.875 | -0.65134 |
| 4 | 0.9375 | 0.96875 | 0.9375 | 0.08019 |
| 5 | 0.96875 | 0.984375 | 0.96875 | -0.26689 |

| n | a_n | x_{n+1} | b_n | $f(x_{n+1})$ |
|----|----------|-----------|----------|--------------|
| 6 | 0.984375 | 0.990625 | 0.984375 | -0.08882 |
| 7 | 0.984375 | 0.990625 | 0.984375 | -0.0032 |
| 8 | 0.984375 | 0.990625 | 0.984375 | 0.0388 |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |

Približné riešenie $x_{n+1} \approx 0.983203125$