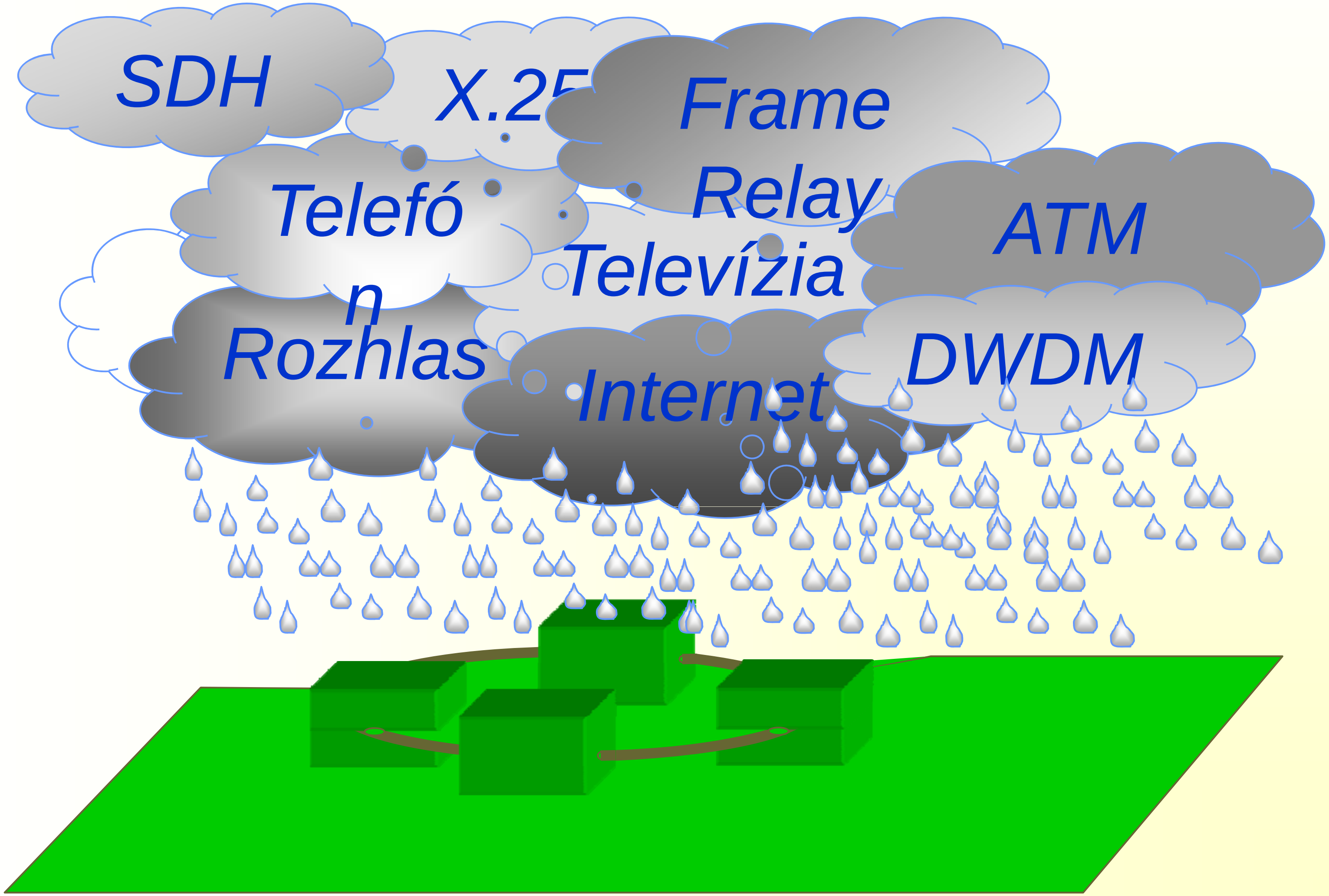
Teória oznamovania 7

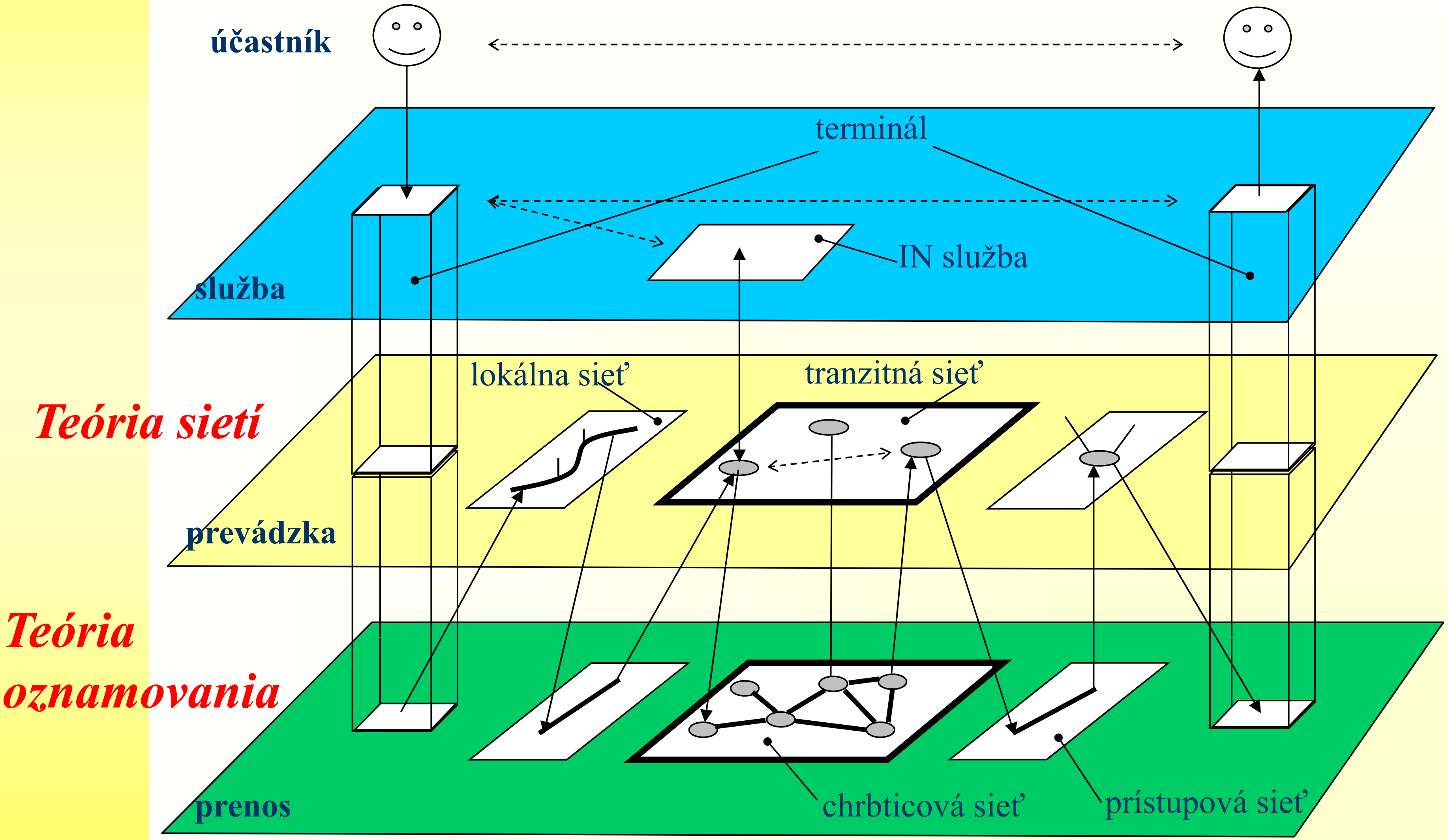
Obsah:

- lineárny kanál
- prechod signálu kanálom
- prechod bázických signálov kanálom
- frekvenčný prenos kanála
- časovo invariantný kanál
- frekvenčný prenos časovo invariantného kanála

Všeobecný model siete



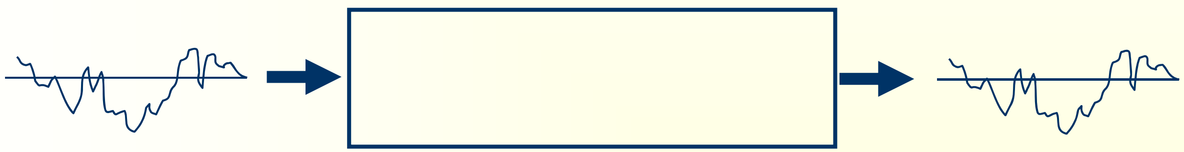
Základné vrstvy



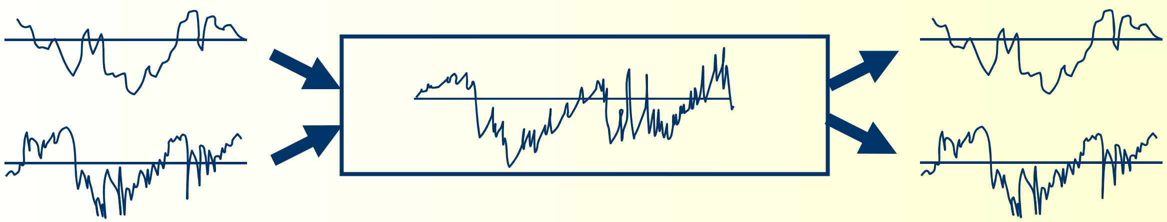
Vrstva prenosu

Hlavné úlohy: ??

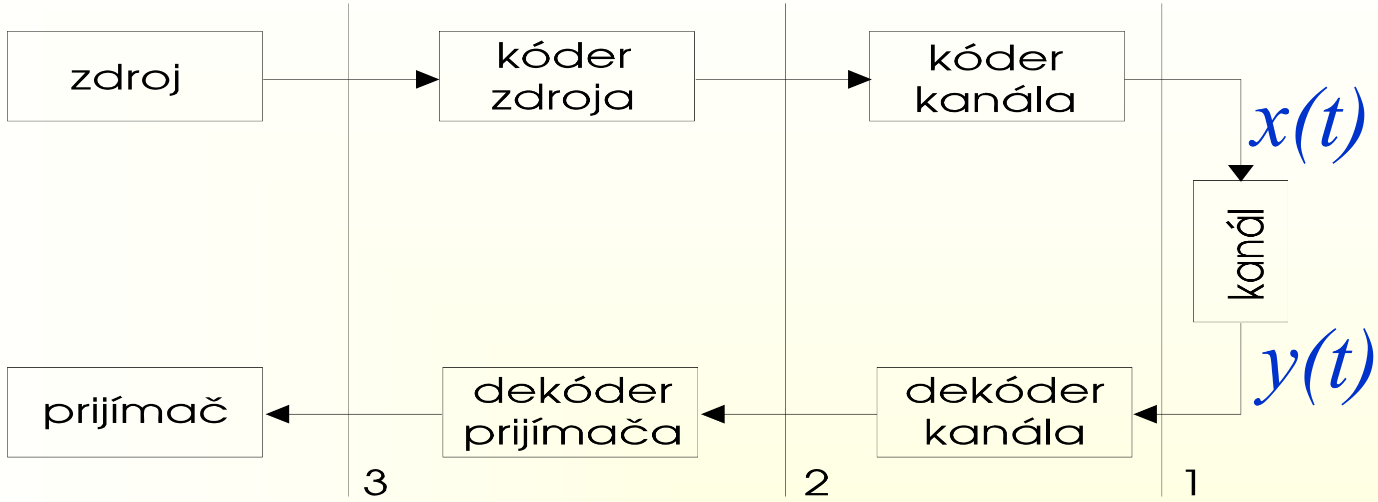
prenos jedného signálu



súčasný prenos signálov



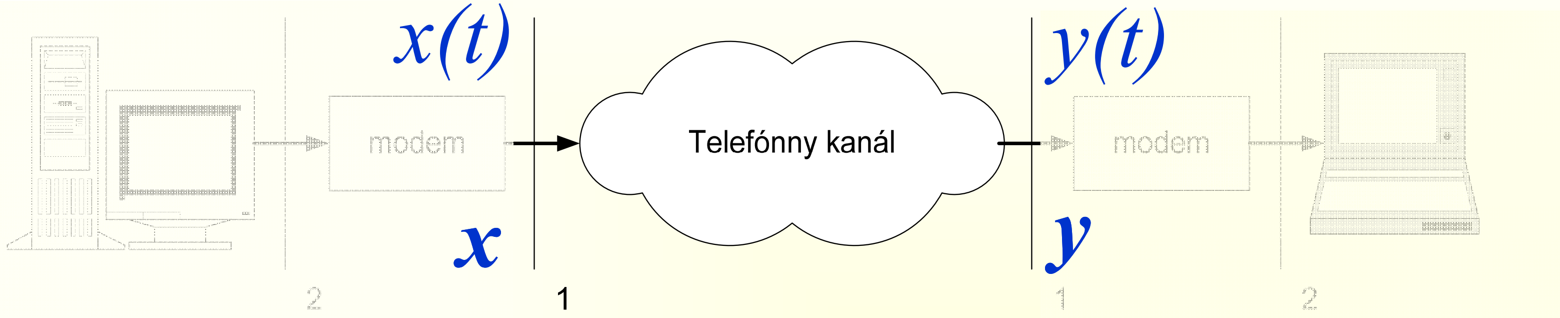
Prenos bez skreslenia



Prispôsobenie prenosovému médiu

Lineárny kanál

Čo je kanál?



$$y = \quad (x)$$

y* – odozva na signál *x

$$\forall \mathbf{x} \in \varphi \Rightarrow \psi\left(\mathbf{x}\right) \in \varphi$$

Lineárny kanál

Čo je lineárny kanál?

Lineárny signálový priestor

3. pre všetky $\mathbf{f}_1, \mathbf{f}_2 \in \varphi$ a $k_1, k_2 \in F$

- $1 \cdot \mathbf{f}_1 = \mathbf{f}_1$
- $k_1 \cdot (k_2 \cdot \mathbf{f}_1) = (k_1 \otimes k_2) \cdot \mathbf{f}_1$
- $k_1 \cdot (\mathbf{f}_1 + \mathbf{f}_2) = k_1 \cdot \mathbf{f}_1 + k_1 \cdot \mathbf{f}_2$
- $(k_1 \oplus k_2) \cdot \mathbf{f}_1 = k_1 \cdot \mathbf{f}_1 + k_2 \cdot \mathbf{f}_1$

Lineárny kanál

$$\forall \mathbf{x}, \mathbf{x}_1, \mathbf{x}_2 \in \varphi$$

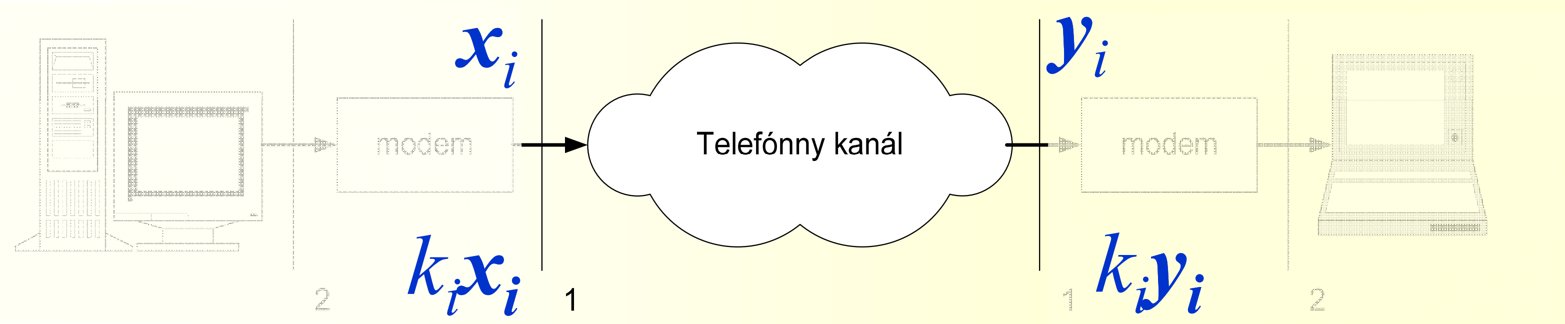
$$\psi(k.\mathbf{x}) = k.\psi(\mathbf{x}) = k.\mathbf{y}$$

$$\psi(\mathbf{x}_1 + \mathbf{x}_2) = \psi(\mathbf{x}_1) + \psi(\mathbf{x}_2) = \mathbf{y}_1 + \mathbf{y}_2$$

Lineárny kanál

$$\forall \mathbf{x}_i \in \varphi, \quad i = 1,2,\ldots,n$$

$$\psi\left(\sum_{i=1}^{n} k_i.\mathbf{x}_i\right) = \sum_{i=1}^{n} k_i.\psi(\mathbf{x}_i) = \sum_{i=1}^{n} k_i.\mathbf{y}_i$$



Lineárny kanál

$$\mathbf{x} = \sum_{i=1}^{n} x_i.\mathbf{e}_i \qquad \mathbf{y} = \sum_{i=1}^{n} y_i.\mathbf{e}_i$$

$$\mathbf{y} = \delta(\mathbf{x}) = \delta\left(\sum_{i=1}^{n} x_i.\mathbf{e}_i\right) = \sum_{i=1}^{n} x_i.\delta(\mathbf{e}_i)$$

$$\mathbf{y} = \sum_{i=1}^{n} x_i.\boldsymbol{\delta}_i$$

$$\boldsymbol{\delta}_i^T = \begin{pmatrix}\delta_{i1} & \dots & \delta_{in}\end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}_\mathbf{B}) = \sum_{i=1}^{n} x_i . \boldsymbol{\delta}_i$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = x_1 \begin{pmatrix} \delta_{11} \\ \delta_{21} \\ \vdots \\ \delta_{n1} \end{pmatrix} + \ldots + x_n \begin{pmatrix} \delta_{1n} \\ \delta_{2n} \\ \vdots \\ \delta_{nn} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=1}^{n} x_i . \boldsymbol{\delta}_i$$

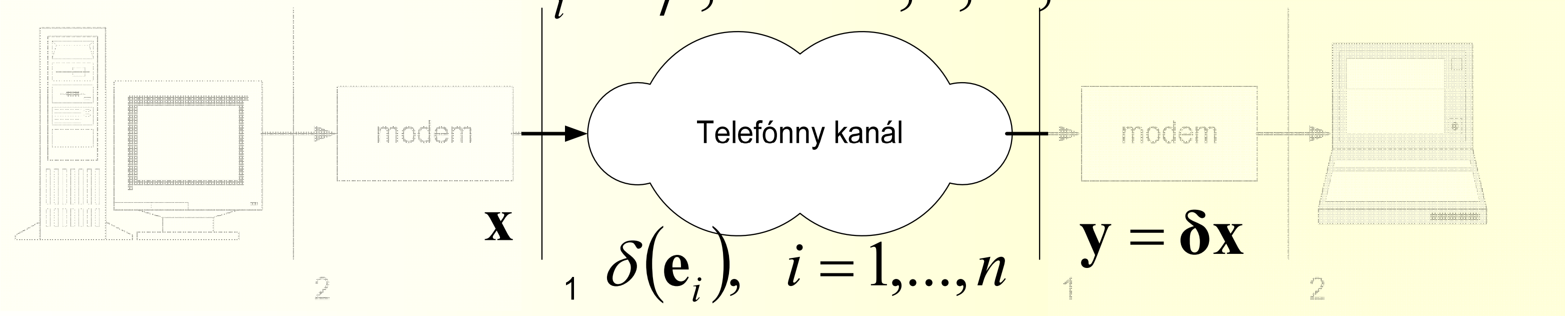
$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ . & . & . & . \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nn} \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=1}^{n} x_i . \boldsymbol{\delta}_i$$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x}$$

$$\mathbf{e}_i \in \varphi, \quad i = 1,2,\ldots,n$$

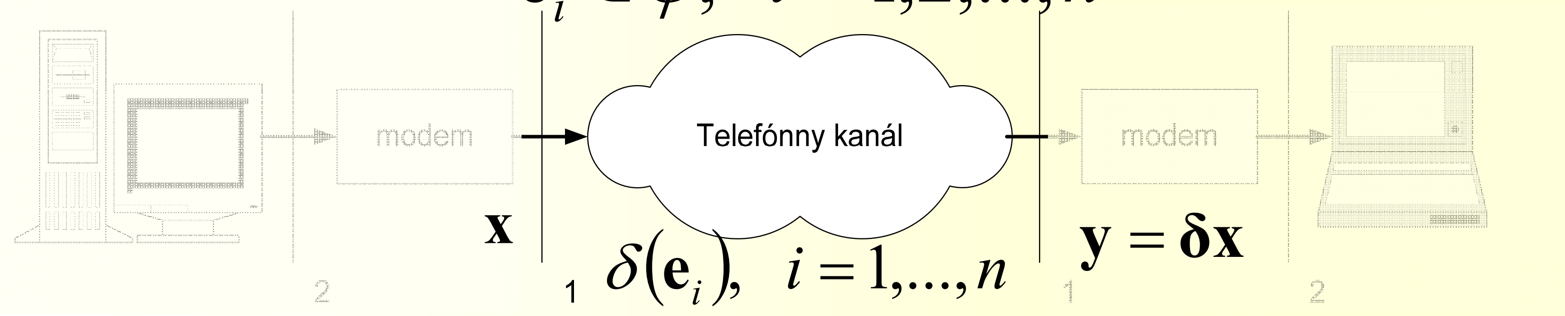


Lineárny kanál

$$\mathbf{e}_1^T = (1,0) \qquad \boldsymbol{\delta}_1^T = (1,2)$$
$$\mathbf{e}_2^T = (0,1) \qquad \boldsymbol{\delta}_2^T = (-1,1) \qquad \mathbf{x}^T = (2,-1)$$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

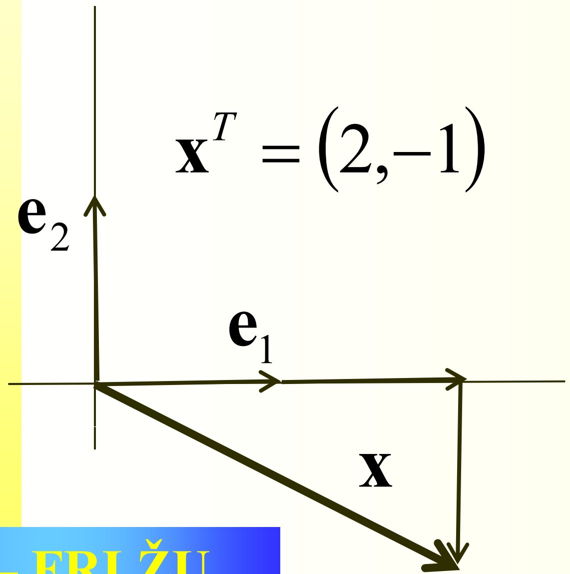
$$\mathbf{e}_i \in \varphi, \quad i = 1,2,\ldots,n$$

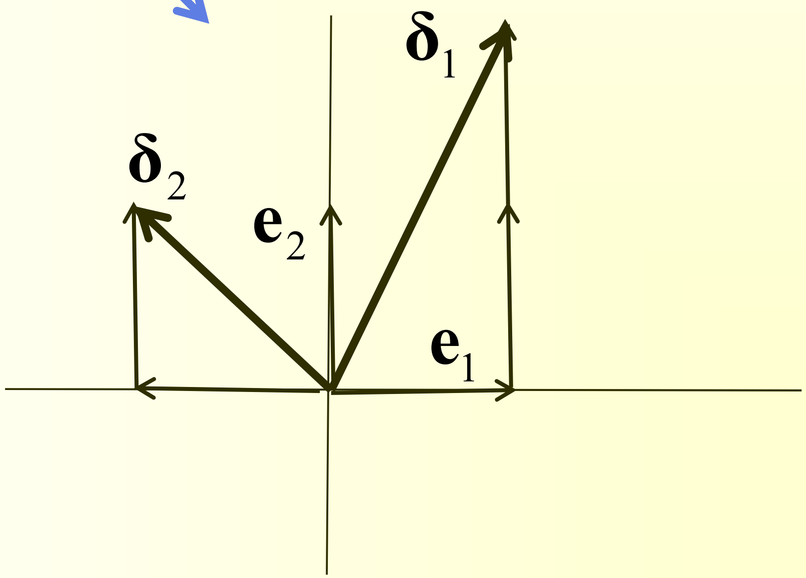


Lineárny kanál

$$\mathbf{e}_1^T = (1,0), \boldsymbol{\delta}_1^T = (1,2)$$
$$\mathbf{e}_2^T = (0,1), \boldsymbol{\delta}_2^T = (-1,1)$$

$$\mathbf{y} = x_1\boldsymbol{\delta}_1 + x_2\boldsymbol{\delta}_2 = 2\begin{pmatrix}1\\2\end{pmatrix} - 1\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$$



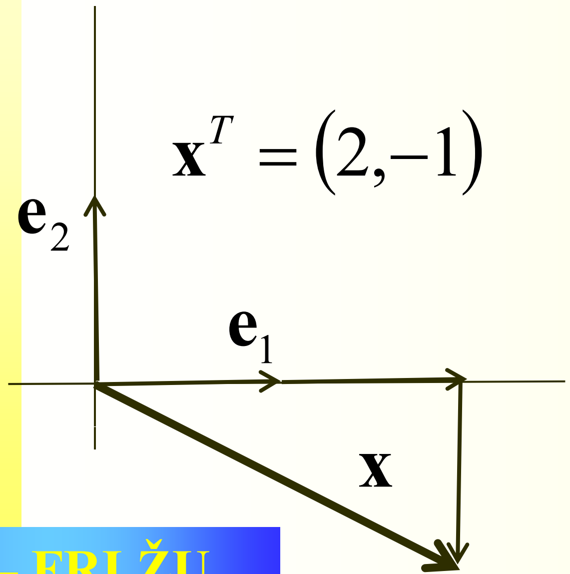


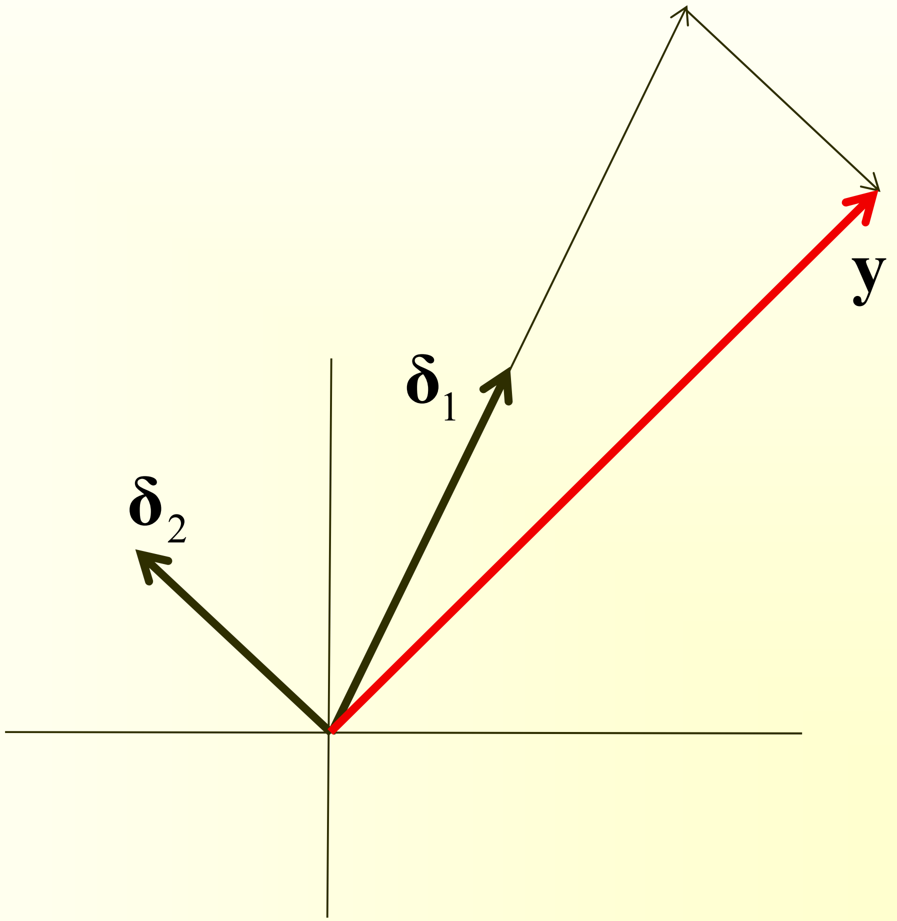
Lineárny kanál

$$\mathbf{e}_1^T = (1,0), \boldsymbol{\delta}_1^T = (1,2) \qquad \mathbf{y} = x_1\boldsymbol{\delta}_1 + x_2\boldsymbol{\delta}_2 = 2\begin{pmatrix}1\\2\end{pmatrix} - 1\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$$

$$\mathbf{e}_2^T = (0,1), \boldsymbol{\delta}_2^T = (-1,1)$$

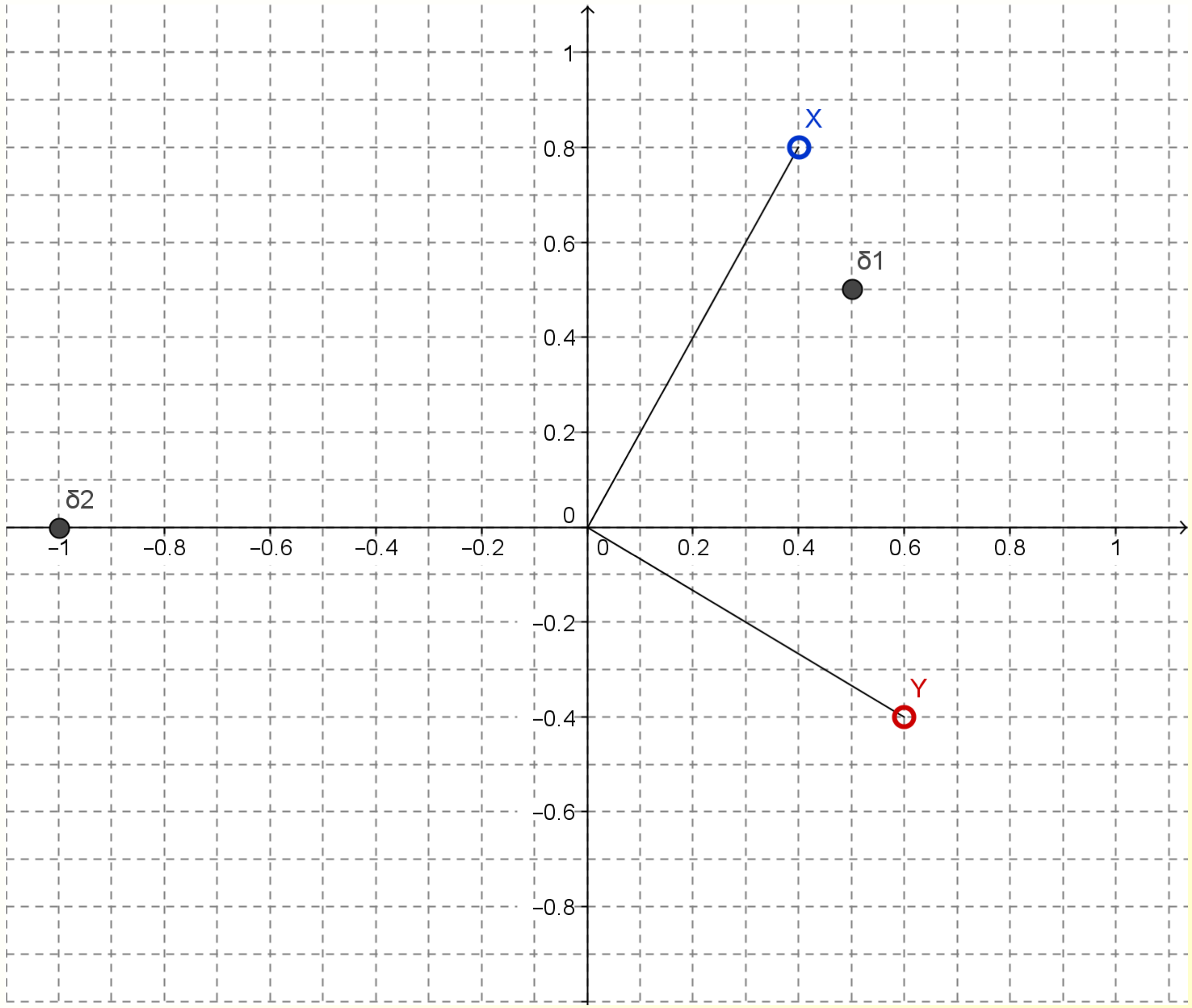
$$\mathbf{x}^T = (2,-1)$$





Lineárny kanál

GeoGebra



Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=1}^{n} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=1}^{n} k_i.\mathbf{b}_i$$

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=1}^{n} c_i.\mathbf{b}_i\right) = \sum_{i=1}^{n} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{k} = \sum_{i=1}^{n} c_i.\boldsymbol{\psi}_i$$

$$\boldsymbol{\psi}_i^T = \begin{pmatrix} \psi_{i1} & \ldots & \psi_{in} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y_B} = \psi\left(\mathbf{x_B}\right) = \sum_{i=1}^{n} c_i . \boldsymbol{\Psi}_i$$

$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = c_1 \begin{pmatrix} \psi_{11} \\ \psi_{21} \\ \vdots \\ \psi_{n1} \end{pmatrix} + \ldots + c_n \begin{pmatrix} \psi_{1n} \\ \psi_{2n} \\ \vdots \\ \psi_{nn} \end{pmatrix}$$

Lineárny kanál

$$\mathbf{y}_{\mathbf{B}} = \psi\left(\mathbf{x}_{\mathbf{B}}\right) = \sum_{i=1}^{n} c_i . \boldsymbol{\Psi}_i$$

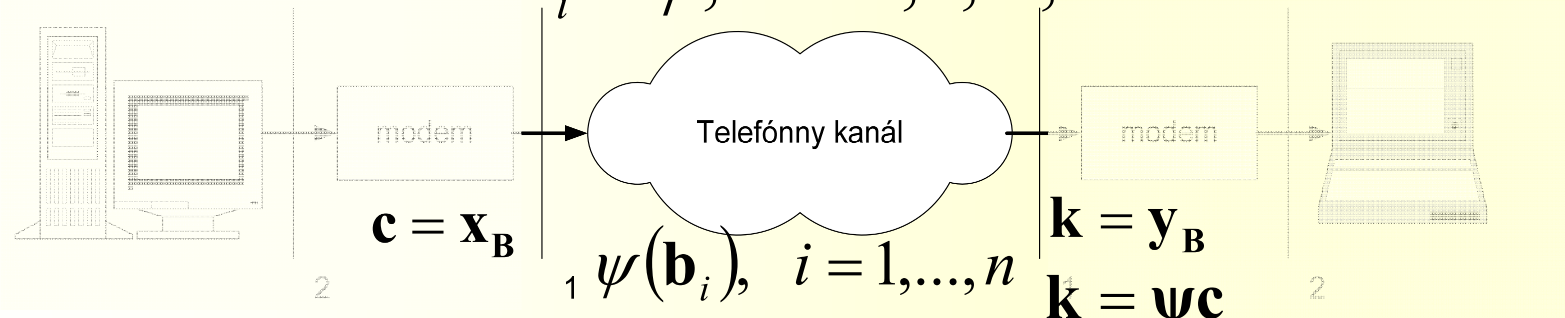
$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}_{\mathbf{B}} = \begin{pmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\ . & . & . & . \\ \psi_{n1} & \psi_{n2} & \cdots & \psi_{nn} \end{pmatrix} . \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}_{\mathbf{B}}$$

Lineárny kanál

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \sum_{i=1}^{n} c_i . \boldsymbol{\psi}_i$$

$$\mathbf{k} = \boldsymbol{\psi}\mathbf{c}$$

$$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$$



$\mathbf{c} = \mathbf{x}_\mathbf{B}$

$\psi(\mathbf{b}_i), \quad i = 1,\ldots,n$

$\mathbf{k} = \mathbf{y}_\mathbf{B}$

$\mathbf{k} = \boldsymbol{\psi}\mathbf{c}$

Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$$

$$\mathbf{y}_{\mathbf{B}} = \psi\left(\mathbf{x}_{\mathbf{B}}\right) = \psi\left(\sum_{i=1}^{n} c_i.\mathbf{b}_i\right) = \sum_{i=1}^{n} c_i.\psi\left(\mathbf{b}_i\right)$$

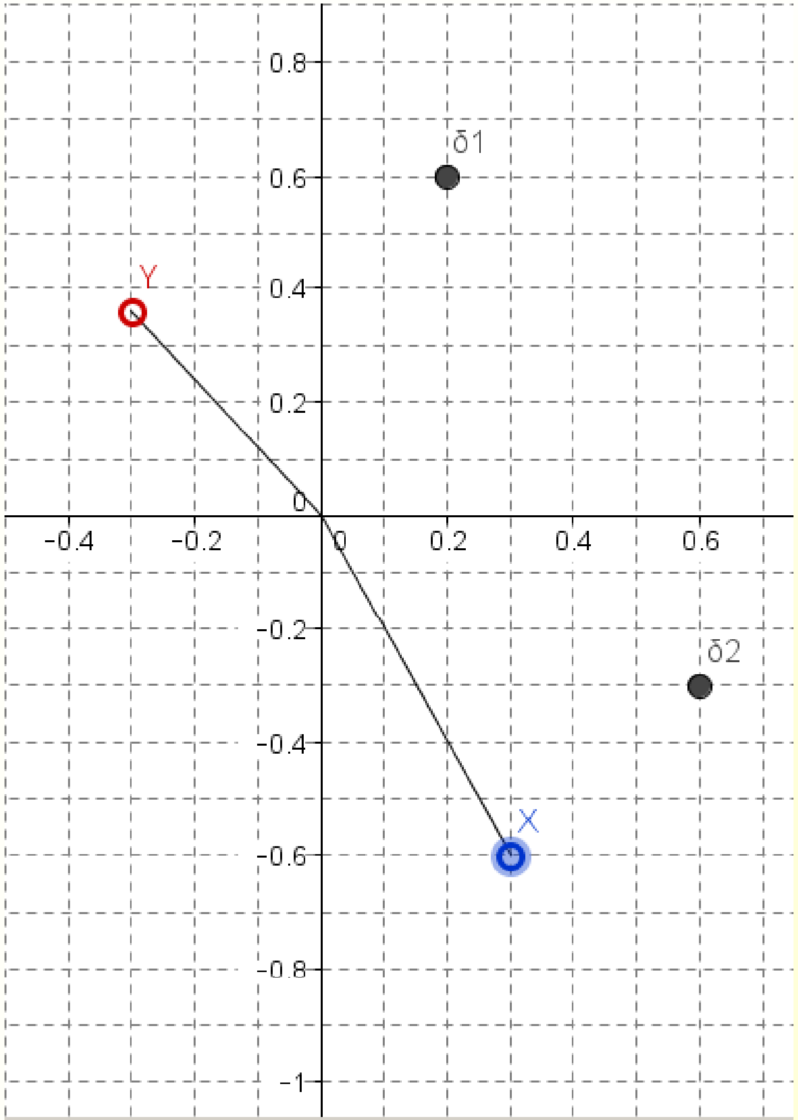
Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Presnejšie, že

$$\psi\left(\mathbf{b}_i\right) = \lambda_i \mathbf{b}_i$$

Lineárny kanál

GeoGebra



Lineárny kanál

Požadujeme $\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$

$$\mathbf{b}_i \boldsymbol{\psi} = \lambda_i \mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

Vlastné vektory kanála

Príklad $\boldsymbol{\delta}_1^T = (0{,}2;0{,}6)$ $\boldsymbol{\delta}_2^T = (0{,}6;-0{,}3)$ $\boldsymbol{\psi} = \begin{pmatrix} 0{,}2 & 0{,}6 \\ 0{,}6 & -0{,}3 \end{pmatrix}$

Riešenie: $\mathbf{b}\boldsymbol{\psi} = \lambda\mathbf{b}$

$$\mathbf{b}(\boldsymbol{\psi} - \lambda\mathbf{E}) = \mathbf{0}$$

$$\boldsymbol{\psi} - \lambda\mathbf{E} = \mathbf{0}$$

$$\begin{pmatrix} 0{,}2-\lambda & 0{,}6 \\ 0{,}6 & -0{,}3-\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(0{,}2-\lambda).(-0{,}3-\lambda)-0{,}36 = 0$$

Vlastné vektory kanála

$$\lambda_1 = -0,7 \quad \mathbf{b}_1\left(\boldsymbol{\psi} - \lambda_1 \mathbf{E}\right) = \mathbf{0}$$

$$\lambda_2 = 0,6 \quad \mathbf{b}_2\left(\boldsymbol{\psi} - \lambda_2 \mathbf{E}\right) = \mathbf{0}$$

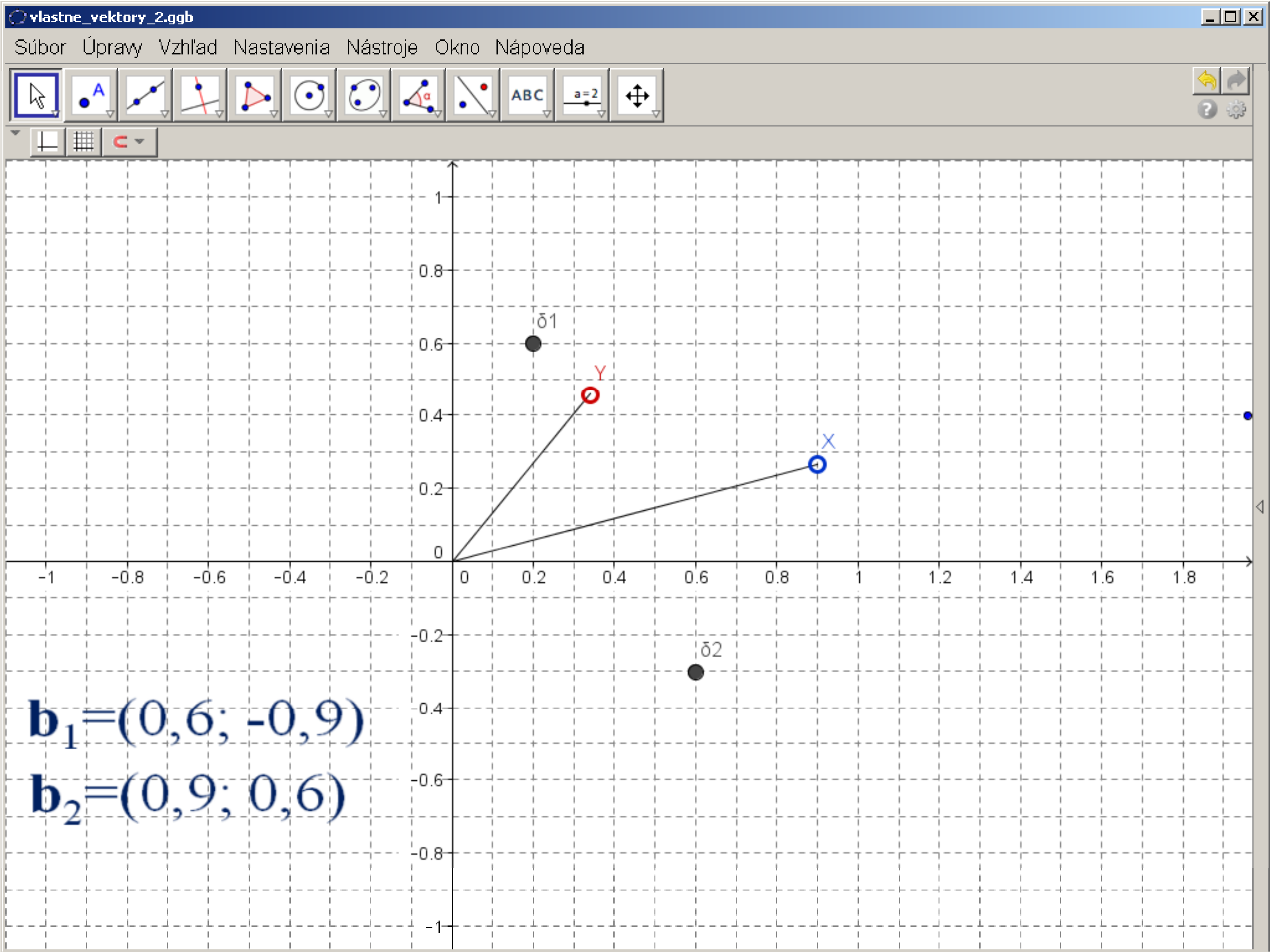
$$\begin{aligned} (0,2-\lambda_1).b_{11} + 0,6.b_{12} &= 0 \\ 0,6.b_{11} + (-0,3-\lambda_1).b_{12} &= 0 \end{aligned} \qquad \begin{aligned} 0,9.b_{11} + 0,6.b_{12} &= 0 \\ 0,6.b_{11} + 0,4.b_{12} &= 0 \end{aligned}$$

$\mathbf{b}_1 = (2t;-3t)$ napr. $\mathbf{b}_1 = (0,6;-0,9)$

$\mathbf{b}_2 = (3t;2t)$ napr. $\mathbf{b}_2 = (0,9;0,6)$

Vlastné vektory kanála

GeoGebra



Vlastné vektory kanála

$$\mathbf{b}_i \in \varphi, \quad i = 1,2,\ldots,n$$

$$\mathbf{x}_{\mathbf{B}} = \sum_{i=1}^{n} c_i.\mathbf{b}_i \qquad \mathbf{y}_{\mathbf{B}} = \sum_{i=1}^{n} k_i.\mathbf{b}_i$$

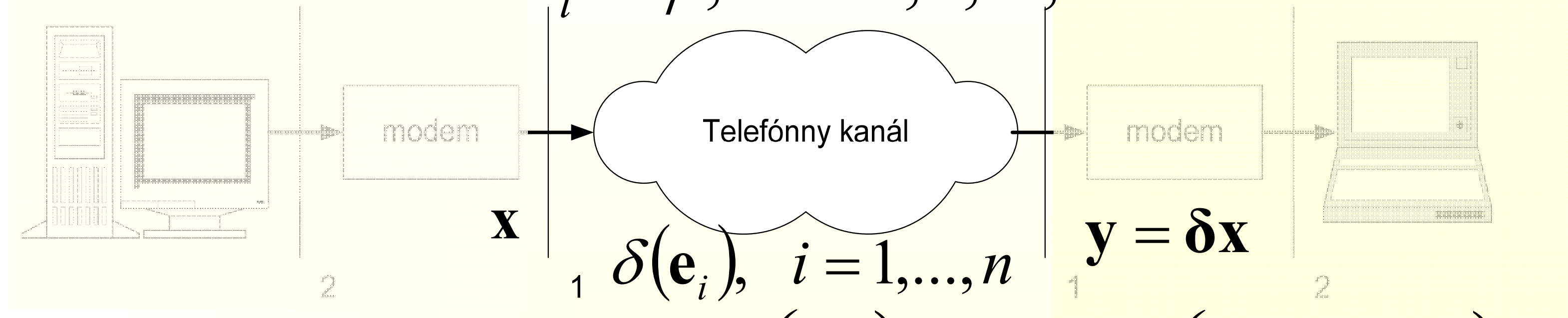
$$\mathbf{y}_{\mathbf{B}} = \psi(\mathbf{x}_{\mathbf{B}}) = \psi\left(\sum_{i=1}^{n} c_i.\mathbf{b}_i\right) = \sum_{i=1}^{n} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{y}_{\mathbf{B}} = \sum_{i=1}^{n} c_i.\lambda_i \mathbf{b}_i$$

$$\boxed{k_i = \lambda_i c_i \quad i = 1,2,\ldots,n}$$

Vlastné vektory kanála

$$\mathbf{e}_i \in \varphi, \quad i = 1,2,\ldots,n$$



$$\mathbf{e}_1 = (1,0), \quad \boldsymbol{\delta}_1^T = (0{,}2;0{,}6) \qquad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (x_1, x_2).\begin{pmatrix} 0{,}2 & 0{,}6 \\ 0{,}6 & -0{,}3 \end{pmatrix}$$

$$\mathbf{e}_2^T = (0,1), \quad \boldsymbol{\delta}_2^T = (0{,}6;-0{,}3)$$

$$\mathbf{b}_1^T = (0{,}6;-0{,}9) \qquad \mathbf{x} = (0{,}4;\, 0{,}7) \qquad \mathbf{x} = (0{,}23;\, 0{,}67)_{\mathbf{B}}$$

$$\mathbf{b}_2^T = (0{,}9;\ 0{,}6) \qquad c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((0{,}4;0{,}7),(0{,}6;-0{,}9))}{((0{,}6;-0{,}9),(0{,}6;-0{,}9))} = 0{,}231$$

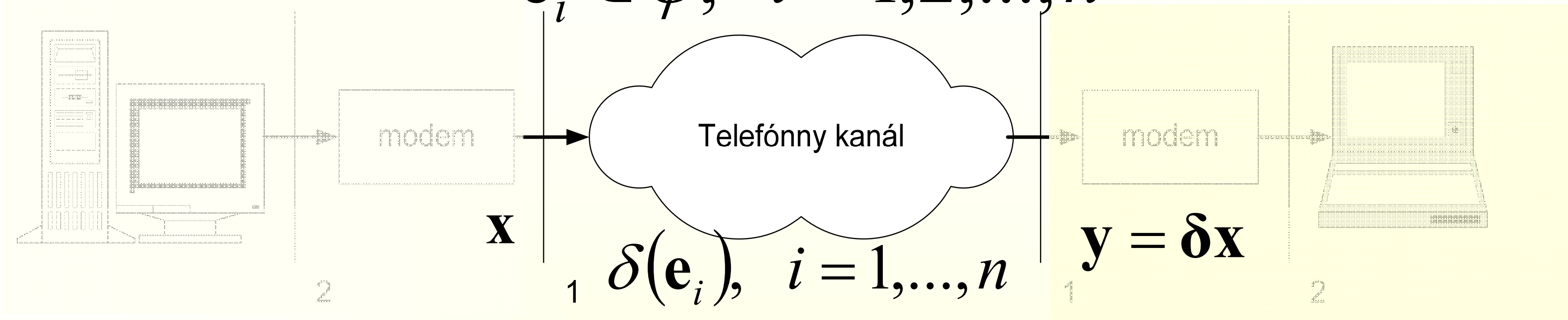
$$\boldsymbol{\lambda} = (-0{,}7;\, 0{,}6) \qquad c_2 = \frac{(\mathbf{x},\mathbf{b}_2)}{(\mathbf{b}_2,\mathbf{b}_2)} = \frac{((0{,}4;0{,}7),(0{,}9;0{,}6))}{((0{,}9;0{,}6),(0{,}9;0{,}6))} = 0{,}667$$

$$k_1 = \lambda_1 c_1 = 0{,}162 \qquad \mathbf{y} = (0{,}16;\, 0{,}4)_{\mathbf{B}}$$

$$k_2 = \lambda_2 c_2 = 0{,}4$$

časovo-invariantný kanál

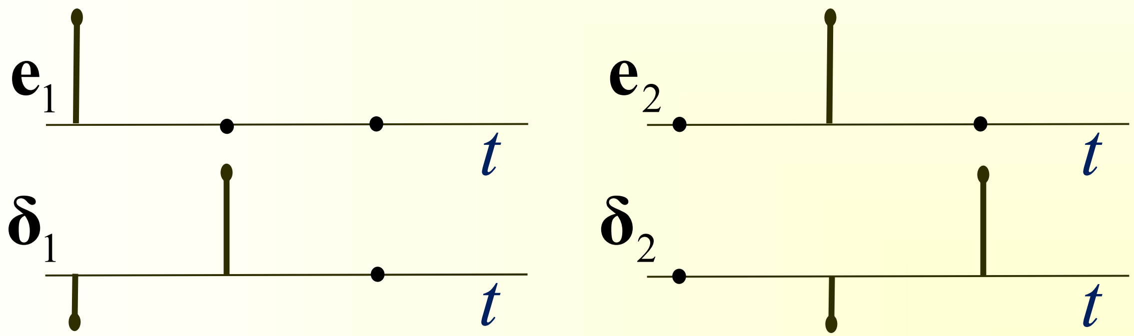
$$\mathbf{e}_i \in \varphi, \quad i = 1,2,\dots,n$$



$$\mathbf{x}$$

$$\delta(\mathbf{e}_i), \quad i = 1,\dots,n$$

$$\mathbf{y} = \boldsymbol{\delta}\mathbf{x}$$



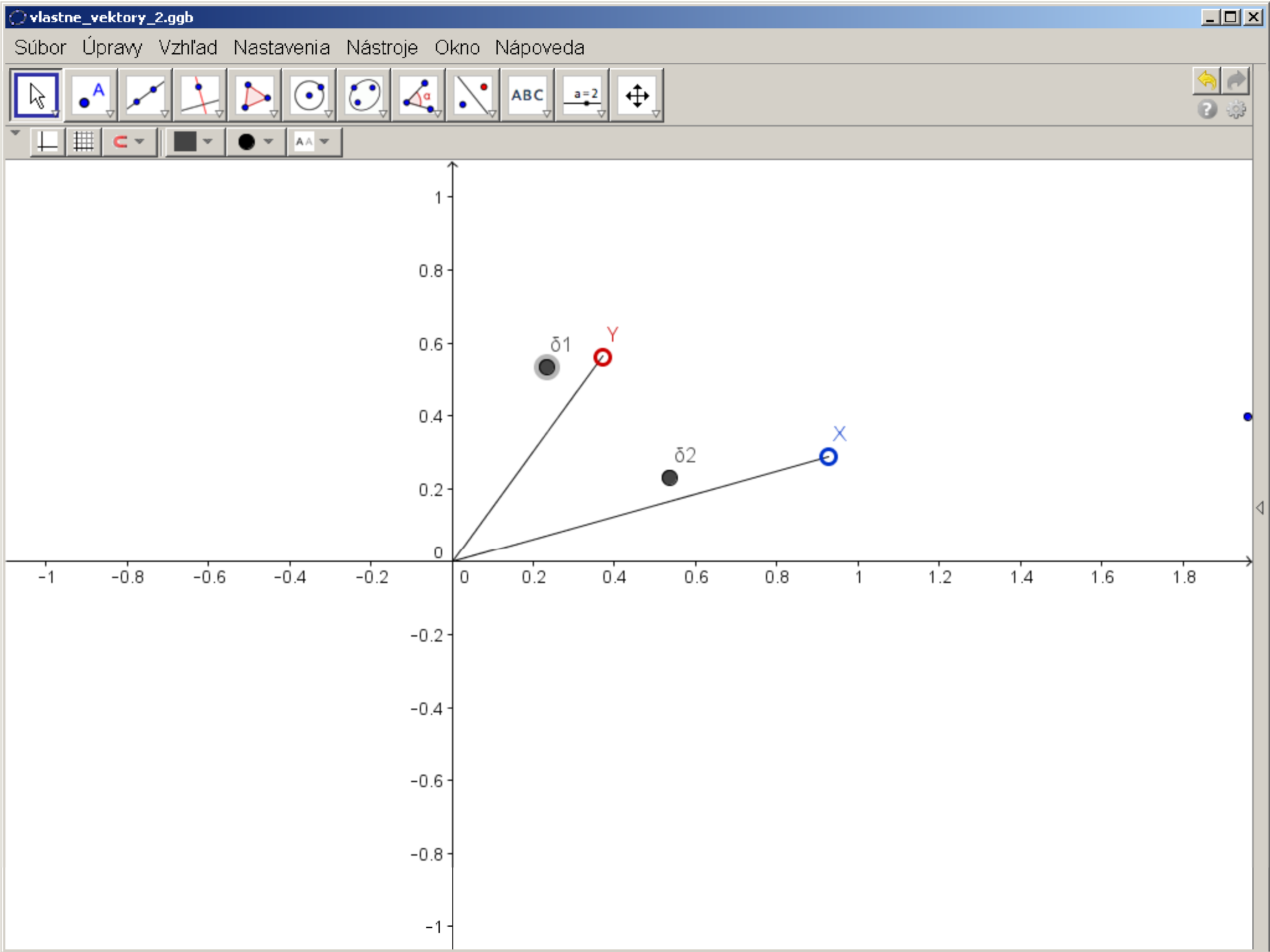
$$\mathbf{e}_1 = (1,0), \qquad \boldsymbol{\delta}_1^T = (\delta_1; \delta_2)$$

$$\mathbf{e}_2^T = (0,1), \qquad \boldsymbol{\delta}_2^T = (\delta_2; \delta_1)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (x_1, x_2) \cdot \begin{pmatrix} \delta_1 & \delta_2 \\ \delta_2 & \delta_1 \end{pmatrix}$$

Báza t-invariantného kanála

GeoGebra



časovo-invariantný kanál

$$\mathbf{e}_1 = (1,0), \quad \boldsymbol{\delta}_1^T = (\delta_1; \delta_2)$$

$$\mathbf{e}_2^T = (0,1), \quad \boldsymbol{\delta}_2^T = (\delta_2; \delta_1)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (x_1, x_2).\begin{pmatrix} \delta_1 & \delta_2 \\ \delta_2 & \delta_1 \end{pmatrix}$$

$$\mathbf{b}(\boldsymbol{\psi} - \lambda\mathbf{E}) = \mathbf{0}$$

$$\delta_1 - (\delta_1 + \delta_2).b_{11} + \delta_2.b_{12} = 0$$

$$b_{11} = b_{12}$$

$$\begin{pmatrix} \delta_1 - \lambda & \delta_2 \\ \delta_2 & \delta_1 - \lambda \end{pmatrix} = \mathbf{0}$$

$$\delta_1 - (\delta_1 - \delta_2).b_{21} + \delta_2.b_{22} = 0$$

$$b_{21} = -b_{22}$$

$$(\delta_1 - \lambda)^2 - \delta_2^2 = 0$$

$$\mathbf{b}_1^T = (1; 1)$$

$$\mathbf{b}_2^T = (1; -1)$$

$$\lambda_{1,2} = \delta_1 \pm \delta_2$$

$$\boldsymbol{\lambda} = (\delta_1 + \delta_2; \delta_1 - \delta_2)$$

Frekvenčný prenos t-invar. kanála

$$\mathbf{x} = (x_1; x_2) \quad \mathbf{b}_1^T = (1; 1) \qquad \boldsymbol{\lambda} = (\delta_1 + \delta_2; \delta_1 - \delta_2)$$

$$\mathbf{b}_2^T = (1; \ -1)$$

$$c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((x_1;x_2),(1;1))}{((1;1),(1;1))} = \frac{x_1+x_1}{2}$$

$$c_2 = \frac{(\mathbf{x},\mathbf{b}_2)}{(\mathbf{b}_2,\mathbf{b}_2)} = \frac{((x_1;x_2),(1;-1))}{((1;-1),(1;-1))} = \frac{x_1-x_1}{2}$$

$$\mathbf{x} = \left(\tfrac{1}{2}(x_1 + x_2); \tfrac{1}{2}(x_1 - x_2)\right)_{\mathbf{B}}$$

$$\mathbf{y} = \left(\tfrac{1}{2}(x_1 + x_2).(\delta_1 + \delta_2); \tfrac{1}{2}(x_1 - x_2).(\delta_1 - \delta_2)\right)_{\mathbf{B}}$$

Vlastné signály t-invariantného kanála

$$\mathbf{e}_0 = (1,0,\dots,0), \qquad \boldsymbol{\delta}_0^T = (\delta_0, \delta_1, \dots, \delta_{N-1})$$

$$\mathbf{e}_1^T = (0,1,\dots,0) \qquad \boldsymbol{\delta}_1^T = (\delta_{N-1}, \delta_0, \dots, \delta_{N-2})$$

$$\dots \qquad \dots$$

$$\mathbf{e}_{N-1}^T = (0,0,\dots,1) \qquad \boldsymbol{\delta}_0^T = (\delta_1, \delta_2, \dots, \delta_0)$$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda \mathbf{E}) = \mathbf{0}$$

$$n = 0,1,\dots, N-1$$

$$\boxed{\begin{aligned} \lambda_n &= \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \\ \mathbf{b}_n &= \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \dots, e^{-j\frac{2\pi}{N}n(N-1)}\right) \end{aligned}}$$

Frekvenčný prenos t-invar. kanála

$$\mathbf{x} = (x_0, x_2, \ldots, x_{N-1}) \quad \boldsymbol{\delta} = (\delta_0, \delta_2, \ldots, \delta_{N-1})$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\ldots,N-1$$

$$\mathbf{b}_n = \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \ldots, e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x} = (c_0, c_2, \ldots, c_{N-1})_{\mathbf{B}} \quad \mathbf{y} = (k_0, k_2, \ldots, k_{N-1})_{\mathbf{B}}$$

$$\boxed{\begin{gathered} c_n = \frac{(\mathbf{x}, \mathbf{b}_n)}{(\mathbf{b}_n, \mathbf{b}_n)} = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}, \quad c_n \in \mathbf{C} \\ k_n = \lambda_n c_n \quad n = 0,1,\ldots,N-1 \end{gathered}}$$

Vlastné signály – skúška správnosti

$$\mathbf{b}_n = \left(e^{-j\frac{2\pi}{N}n0}, e^{-j\frac{2\pi}{N}n1}, \ldots, e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\ldots,N-1$$

$$\mathbf{b}_n \boldsymbol{\delta} = \lambda_n \mathbf{b}_n$$

$$\boxed{\left(1,\ldots,e^{-j\frac{2\pi}{N}nk},\ldots,e^{-j\frac{2\pi}{N}n(N-1)}\right)} \begin{pmatrix} \delta_0, & \ldots, & \delta_l, & \ldots, & \delta_{N-1} \\ \delta_{N-1}, & \ldots, & \delta_{l+1}, & \ldots, & \delta_0 \end{pmatrix} =$$

$$= \boxed{\sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk}} \left(1,\ldots,\boxed{e^{-j\frac{2\pi}{N}nl}},\ldots,e^{-j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastné signály – skúška správnosti

$$\left(1,\ldots,e^{-j\frac{2\pi}{N}nk},\ldots,e^{-j\frac{2\pi}{N}n(N-1)}\right)\cdot\begin{pmatrix}\delta_l\\ \delta_{l-1}\\ \\ \delta_{l+1}\end{pmatrix}=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}e^{-j\frac{2\pi}{N}nl}$$

$$\sum_{k=0}^{N-1}\delta_{l-k}e^{-j\frac{2\pi}{N}nk}=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}n(k+l)}$$

+ súčet *mod(N)*

$$k+l=\hat{k}$$

$$\sum_{k=0}^{N-1}\delta_{l-k}e^{-j\frac{2\pi}{N}nk}=\sum_{\hat{k}=l}^{l-1}\delta_{l-\hat{k}}e^{-j\frac{2\pi}{N}n\hat{k}}$$

Ďakujem za Vašu pozornosť