Teória oznamovania 9

Obsah:

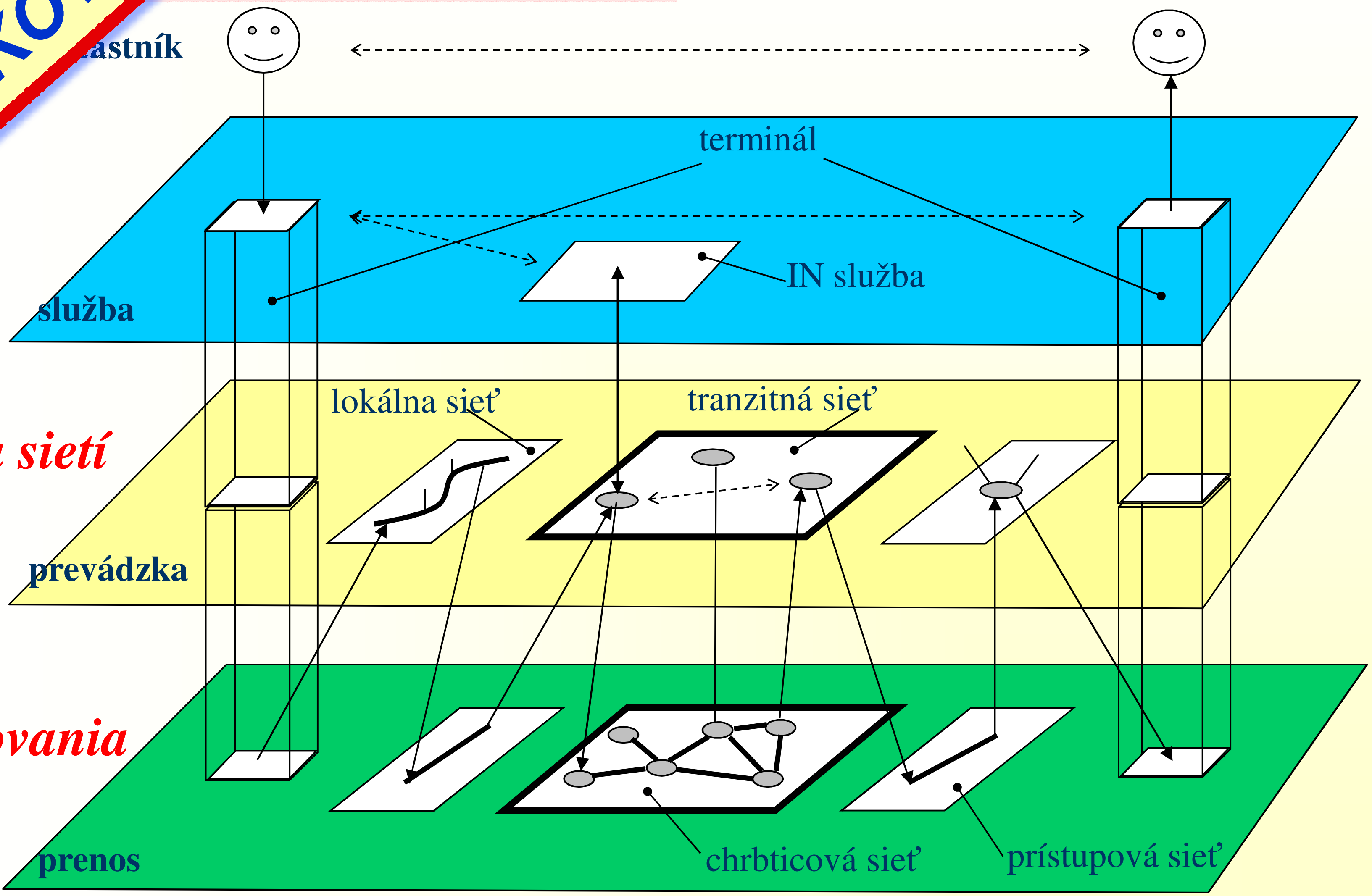
- opakovanie prenosu signálu kanálom
- vlastnosti frekvenčného spektra a prenosu kanála
- ideálny lineárny časovo invariantný kanál
- korekcia frekvenčného prenosu kanála
- optimálny príjem signálu

Základné vrstvy

OPAKOVANIE

Teória sietí

Teória oznamovania

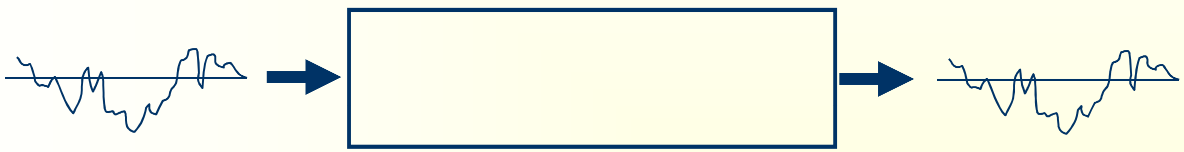


Vrstva prenosu

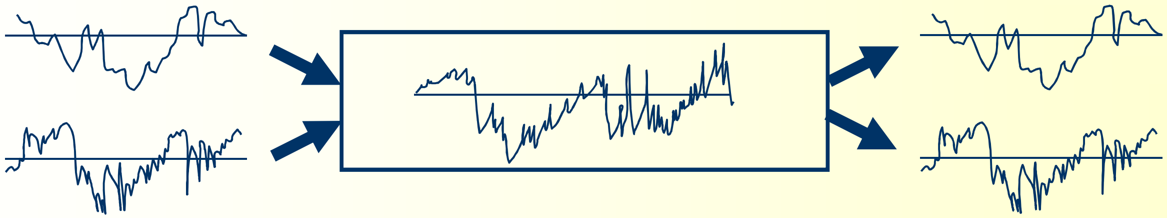
OPAKOVANIE

Hlavné úlohy: ??

prenos jedného signálu

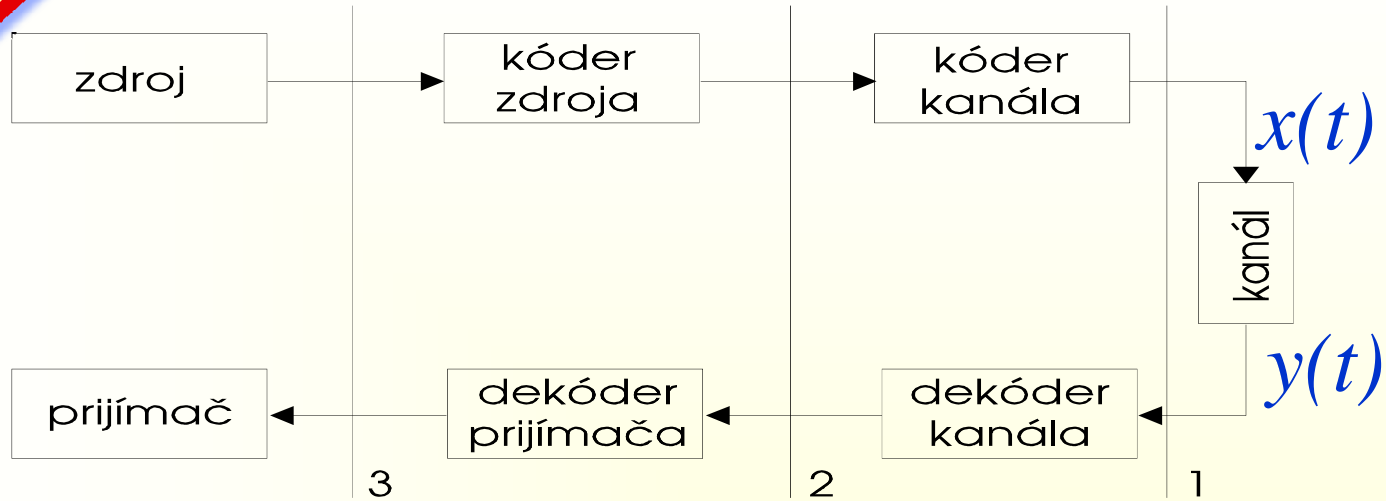


súčasný prenos signálov



OPAKOVANIE

Prenos bez skreslenia



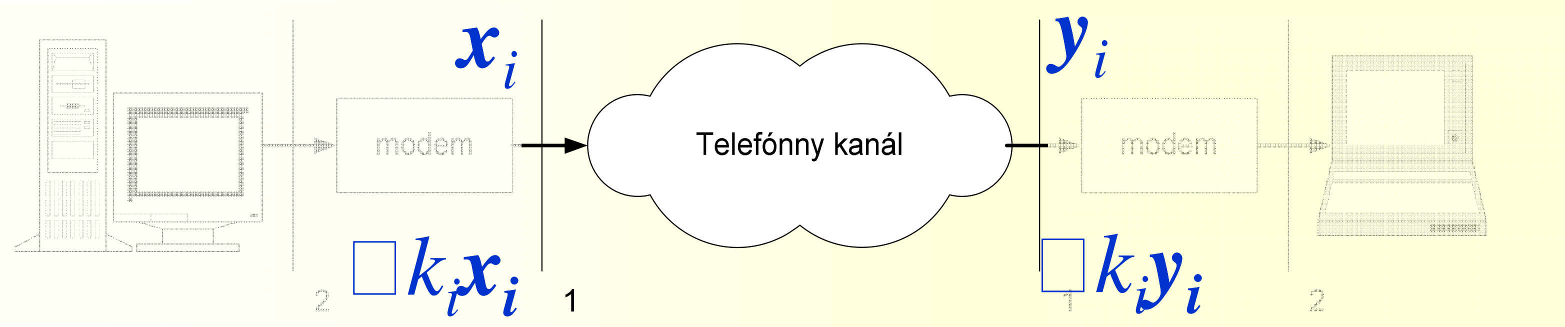
Prispôsobenie prenosovému médiu

Lineárny kanál

OPAKOVANIE

$$\forall\, \mathbf{x}_i \in \varphi, \quad i = 0,1,\ldots,N-1$$

$$\psi\left(\sum_{i=0}^{N-1} k_i.\mathbf{x}_i\right) = \sum_{i=0}^{N-1} k_i.\psi(\mathbf{x}_i) = \sum_{i=0}^{N-1} k_i.\mathbf{y}_i$$

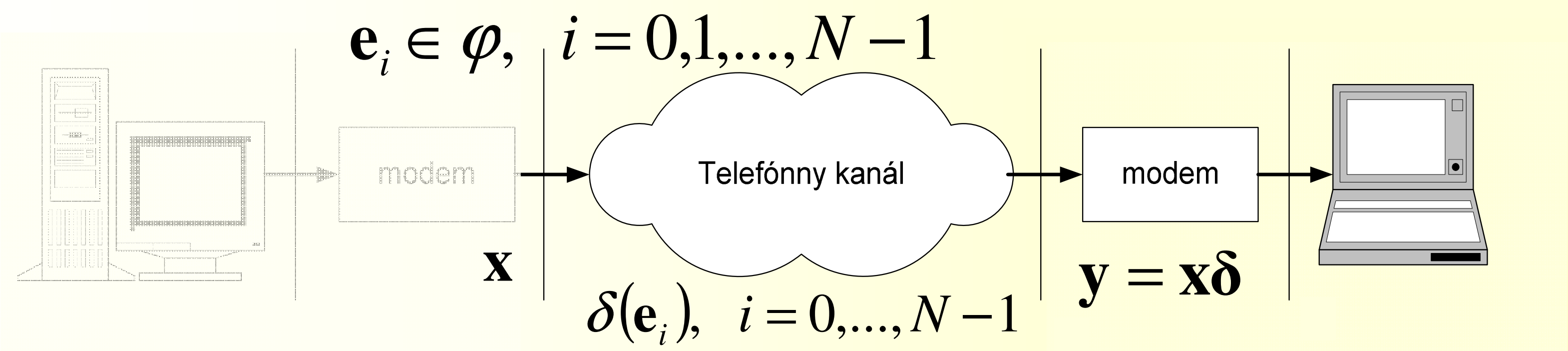


Lineárny kanál

OPAKOVANIE

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=0}^{N-1} x_i.\boldsymbol{\delta}_i$$

$$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$$

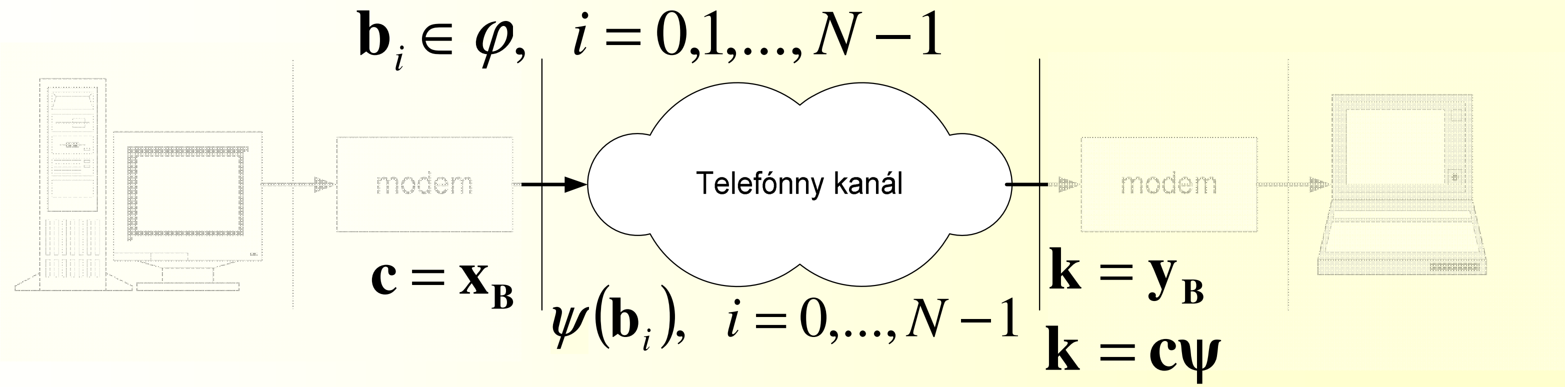


OPAKOVANIE

Lineárny kanál

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \sum_{i=0}^{N-1} c_i . \boldsymbol{\psi}_i$$

$$\mathbf{k} = \mathbf{c}\boldsymbol{\psi}$$



Lineárny kanál

OPAKOVANIE

Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Požadujeme $\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$

$$\mathbf{b}_i \boldsymbol{\psi} = \lambda_i \mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

OPAKOVANIE

Vlastné vektory kanála

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\ldots,N-1$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} k_i.\mathbf{b}_i$$

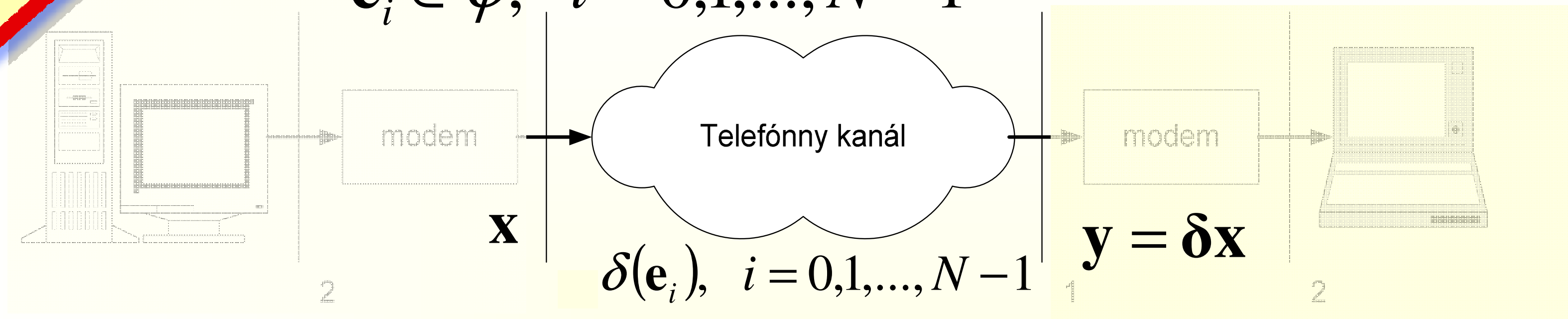
$$\mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\lambda_i \mathbf{b}_i$$

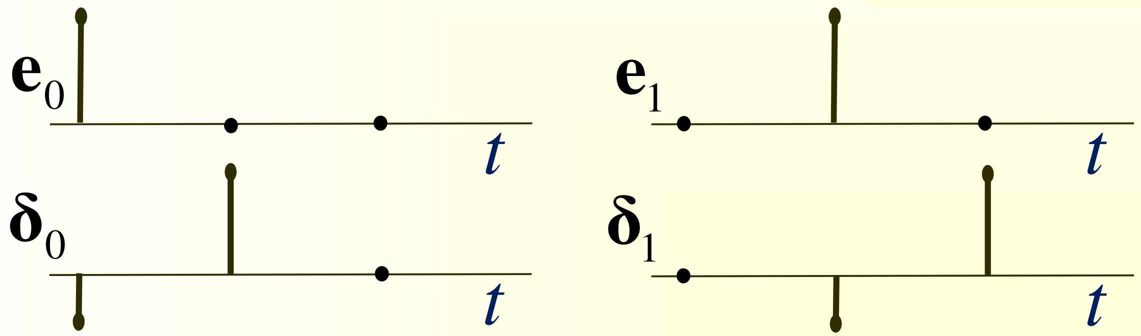
$$\boxed{k_i = \lambda_i c_i, i = 0,1,\ldots,N-1}$$

Časovo-invariantný kanál

OPAKOVANIE

$$\mathbf{e}_i \in \varphi, \quad i = 0,1,\ldots,N-1$$





$$\mathbf{e}_0 = (1,0,0),\ \boldsymbol{\delta}_0 = (\delta_0, \delta_1, \delta_2)$$

$$\mathbf{e}_1 = (0,1,0),\ \boldsymbol{\delta}_1 = (\delta_2, \delta_0, \delta_1) \qquad (y_0, y_1, y_2) = (x_0, x_1, x_2).\begin{pmatrix} \delta_0 & \delta_1 & \delta_2 \\ \delta_2 & \delta_0 & \delta_1 \\ \delta_1 & \delta_2 & \delta_0 \end{pmatrix}$$

$$\mathbf{e}_2 = (0,0,1),\ \boldsymbol{\delta}_1 = (\delta_1, \delta_2, \delta_0)$$

Vlastné signály t-invariantného kanála

OPAKOVANIE

$$\mathbf{e}_0 = (1,0,\ldots,0), \qquad \boldsymbol{\delta}_0 = (\delta_0, \delta_1, \ldots, \delta_{N-1})$$

$$\mathbf{e}_1 = (0,1,\ldots,0) \qquad \boldsymbol{\delta}_1 = (\delta_{N-1}, \delta_0, \ldots, \delta_{N-2})$$

$$\ldots \qquad \ldots$$

$$\mathbf{e}_{N-1} = (0,0,\ldots,1) \qquad \boldsymbol{\delta}_{N-1} = (\delta_1, \delta_2, \ldots, \delta_0)$$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda \mathbf{E}) = \mathbf{0}$$

$$n = 0,1,\ldots,N-1$$

$$\boxed{\begin{aligned} \lambda_n &= \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \\ \mathbf{b}_n &= \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \ldots, e^{j\frac{2\pi}{N}n(N-1)}\right) \end{aligned}}$$

Frekvenčný prenos t-invar. kanála

OPAKOVANIE

$$\mathbf{x} = \left(x_0, x_2, \ldots, x_{N-1}\right) \quad \boldsymbol{\delta}_0 = \left(\delta_0, \delta_2, \ldots, \delta_{N-1}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\ldots,N-1$$

$$\mathbf{b}_n = \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \ldots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x} = \left(c_0, c_2, \ldots, c_{N-1}\right)_{\mathbf{B}} \quad \mathbf{y} = \left(k_0, k_2, \ldots, k_{N-1}\right)_{\mathbf{B}}$$

$$c_n = \frac{(\mathbf{x}, \mathbf{b}_n)}{(\mathbf{b}_n, \mathbf{b}_n)} = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}, \quad c_n \in \mathbf{C}$$

$$k_n = \lambda_n c_n \quad n = 0,1,\ldots,N-1$$

Vzťah k DFT

$$k_n = \lambda_n c_n, \quad n = 0,1,\ldots,N-1$$

$$\cancel{\frac{1}{N}}\sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \cancel{\frac{1}{N}}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$\boxed{Y_n = F_n X_n, \quad n = 0,1,\ldots,N-1}$$

$$\mathbf{X} = DFT(\mathbf{x}) \qquad \mathbf{F} = DFT(\boldsymbol{\delta})$$

$$\mathbf{Y} = DFT(\mathbf{y})$$

Vzťah k DFT

$$c_n = \frac{(\mathbf{x},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)} = \frac{1}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}, \quad c_n \in \mathbf{C}$$

$$X_n = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$X_n = \frac{(\mathbf{x},\tilde{\mathbf{b}}_n)}{(\tilde{\mathbf{b}}_n,\tilde{\mathbf{b}}_n)} = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}nk}$$

$$(\tilde{\mathbf{b}}_n,\tilde{\mathbf{b}}_n) = 1 \qquad \tilde{\mathbf{b}}_n = \tfrac{1}{N}\left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \ldots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastnosti spektra

$$\tilde{\mathbf{b}}_n = \tfrac{1}{N}\left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \dots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x} = \sum_{n=0}^{N-1} X_n \tilde{\mathbf{b}}_n, \quad X_n \in C$$

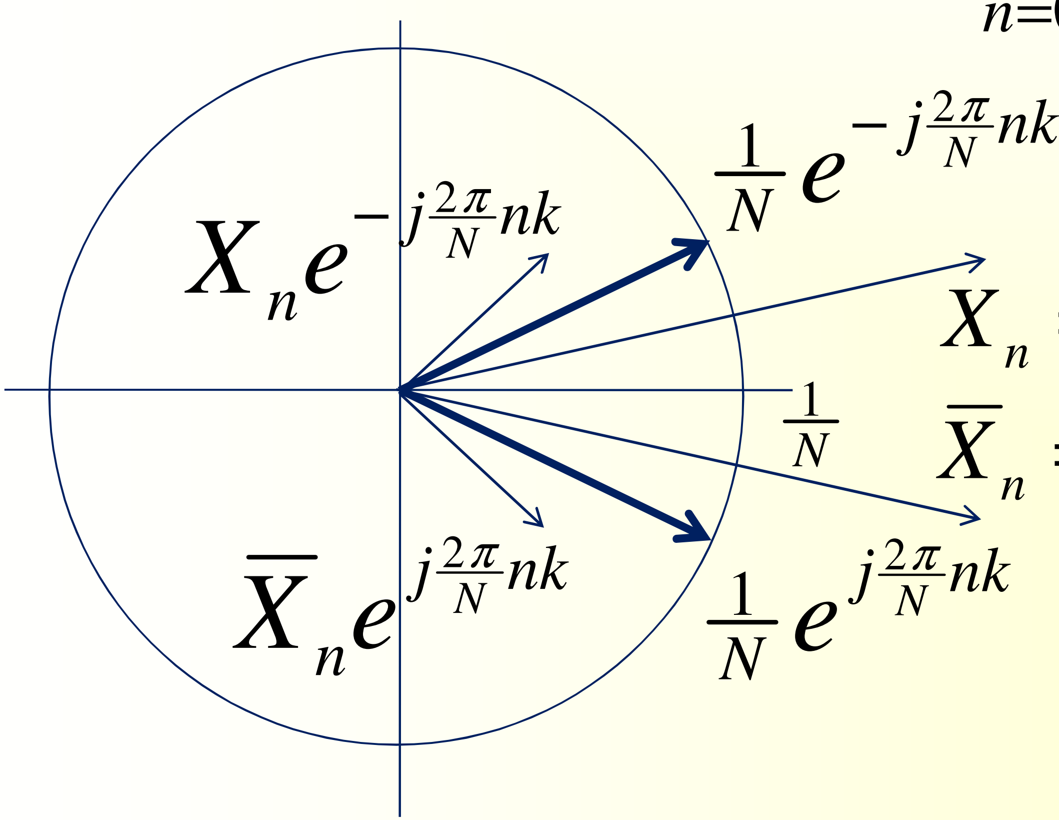
?

$$x_k = \tfrac{1}{N}\sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{N}nk}, \quad x_k \in \Re, \; X_n \in C$$

$$\boxed{X_n = \overline{X}_{N-n}, n = 1, \dots, N-1}$$

Vlastnosti spektra

$$X_n = \overline{X}_{N-n} \qquad x_k = \sum_{n=0}^{N-1} X_n e^{-j\frac{2\pi}{N}nk}$$



$$X_n = |X_n| e^{j\varphi_n}$$

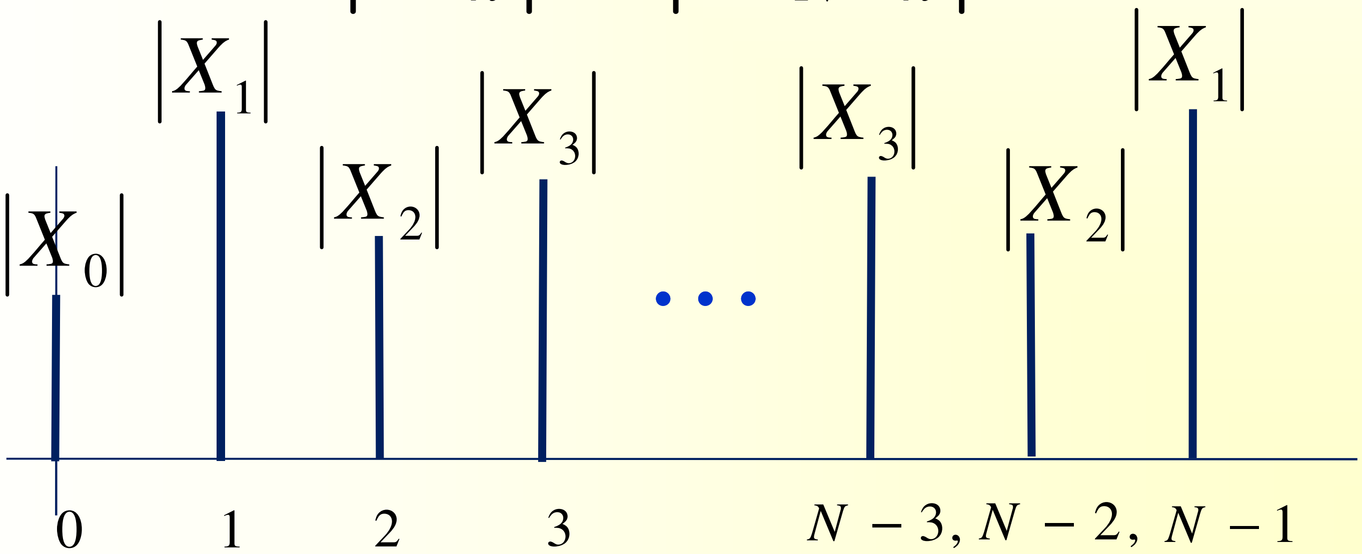
$$\overline{X}_n = |X_n| e^{-j\varphi_n}$$

Amplitúdové spektrum

$$X_n = \overline{X}_{N-n}$$

$$|X_n| e^{-j\varphi_n} = |X_{N-n}| e^{j\varphi_{N-n}}$$

$$|X_n| = |X_{N-n}|$$

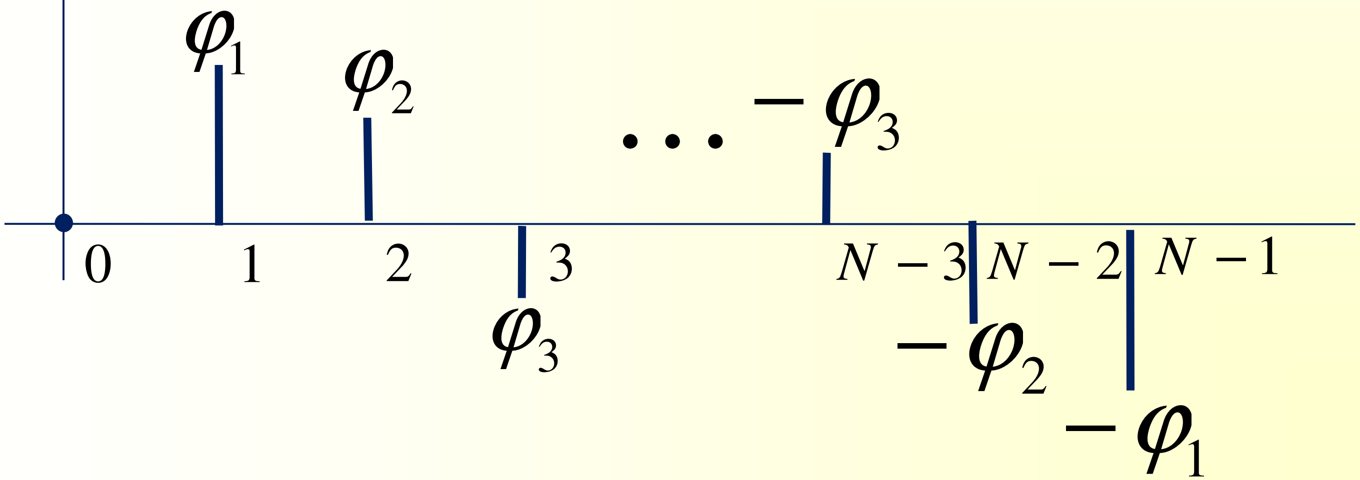


Fázové spektrum

$$X_n = \overline{X}_{N-n}$$

$$|X_n| e^{-j\varphi_n} = |X_{N-n}| e^{j\varphi_{N-n}}$$

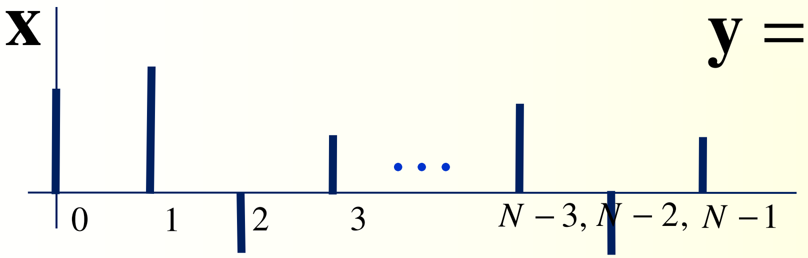
$$\varphi_n = -\varphi_{N-n}$$

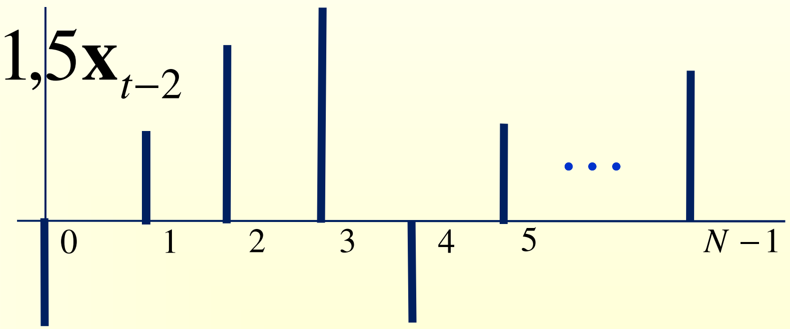


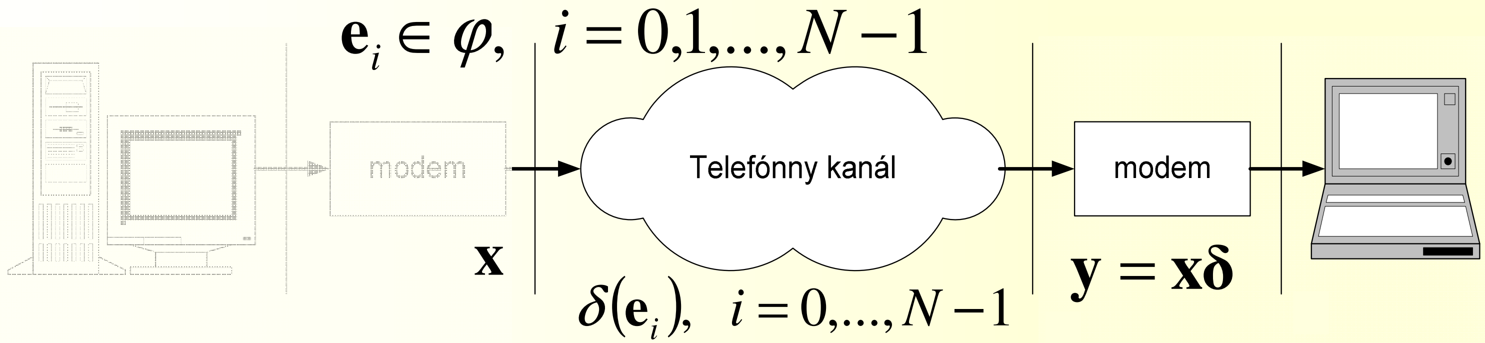
Ideálny lineárny t-invariantný kanál

$$\mathbf{y} = \mathbf{x}\delta$$

$$\mathbf{y} = \alpha \mathbf{x}_{t-\Delta}$$







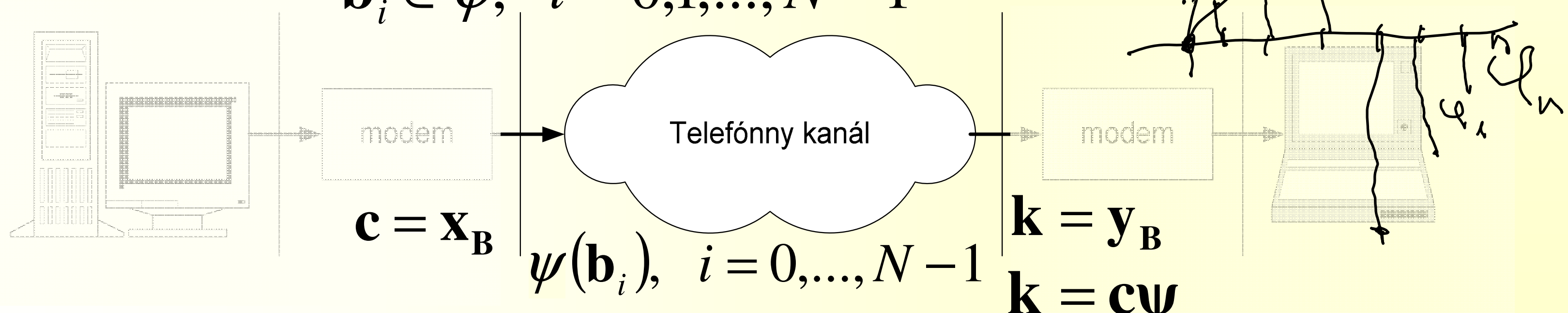
Ideálny lineárny t-invariantný kanál

$$Y_n = F_n X_n, \quad n = 0,1,\ldots,N-1$$

$$\mathbf{y} = \alpha \mathbf{x}_{t-\Delta} \quad Y_n = \alpha X_n e^{-j\frac{2\pi}{N}n\Delta}$$

$$F_n^{ideál} = \alpha . e^{-j\frac{2\pi}{N}n\Delta}, \quad n = 0,1,\ldots,N-1$$

$\mathbf{b}_i \in \varphi, \quad i = 0,1,\ldots,N-1$



$\mathbf{c} = \mathbf{x_B}$

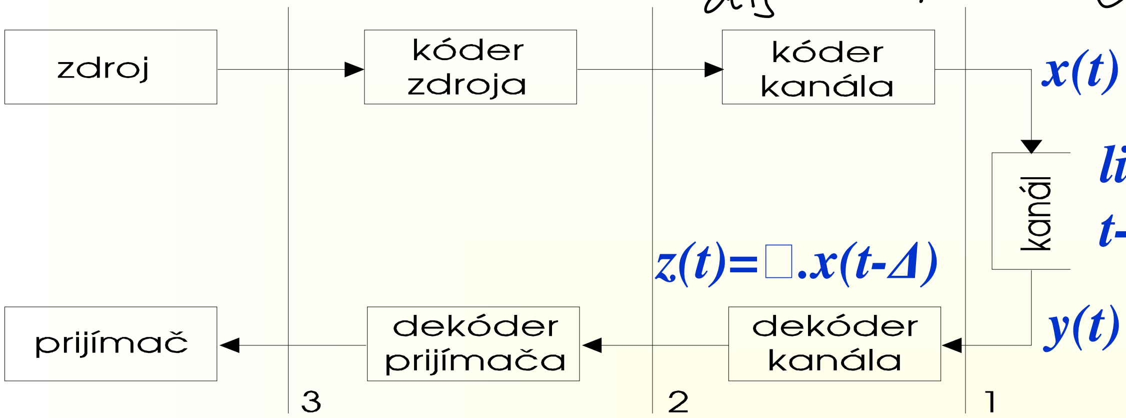
$\psi(\mathbf{b}_i), \quad i = 0,\ldots,N-1$

$\mathbf{k} = \mathbf{y_B}$

$\mathbf{k} = \mathbf{c\psi}$

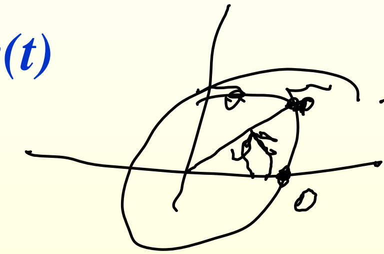
Korekcia kanála

$X_{dB} = 10 \cdot \log_{10} \frac{X}{X_{ref}}$ 600 Ω 1 mW 1 mW ≈ 275 mV



lineárny, t-invariantný

$e^{j\varphi}$



$$n = 0,1,\ldots,N-1$$

$$Z_n = F_n^{kor} F_n X_n, \qquad F_n^{kor} F_n = F_n^{ideál}$$

$$\left|F_n^{kor}\right| e^{-j\varphi_n^{kor}} . \left|F_n\right| e^{-j\varphi_n} = \alpha e^{-j\frac{2\pi}{N} n\Delta}$$

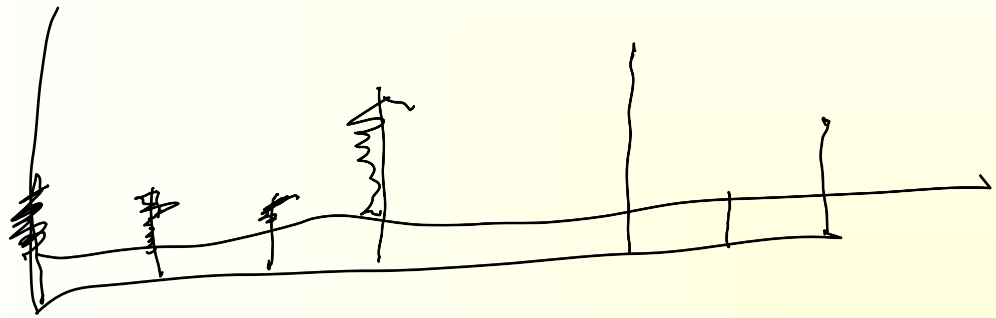
$\varphi_n - \frac{2\pi}{N} n\Delta \approx 0$ φ_n

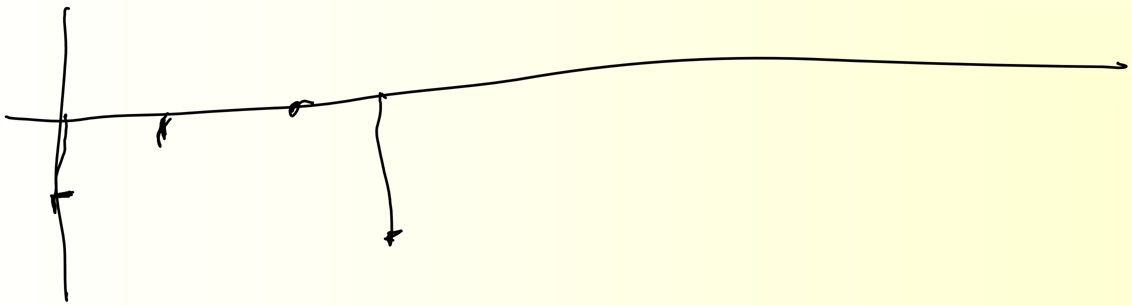
$\Delta > \frac{2\pi}{N} n$

$$\boxed{\left|F_n^{kor}\right| = \alpha . \left|F_n\right|^{-1} \quad \varphi_n^{kor} = \varphi_n + \frac{2\pi}{N} n\Delta}$$

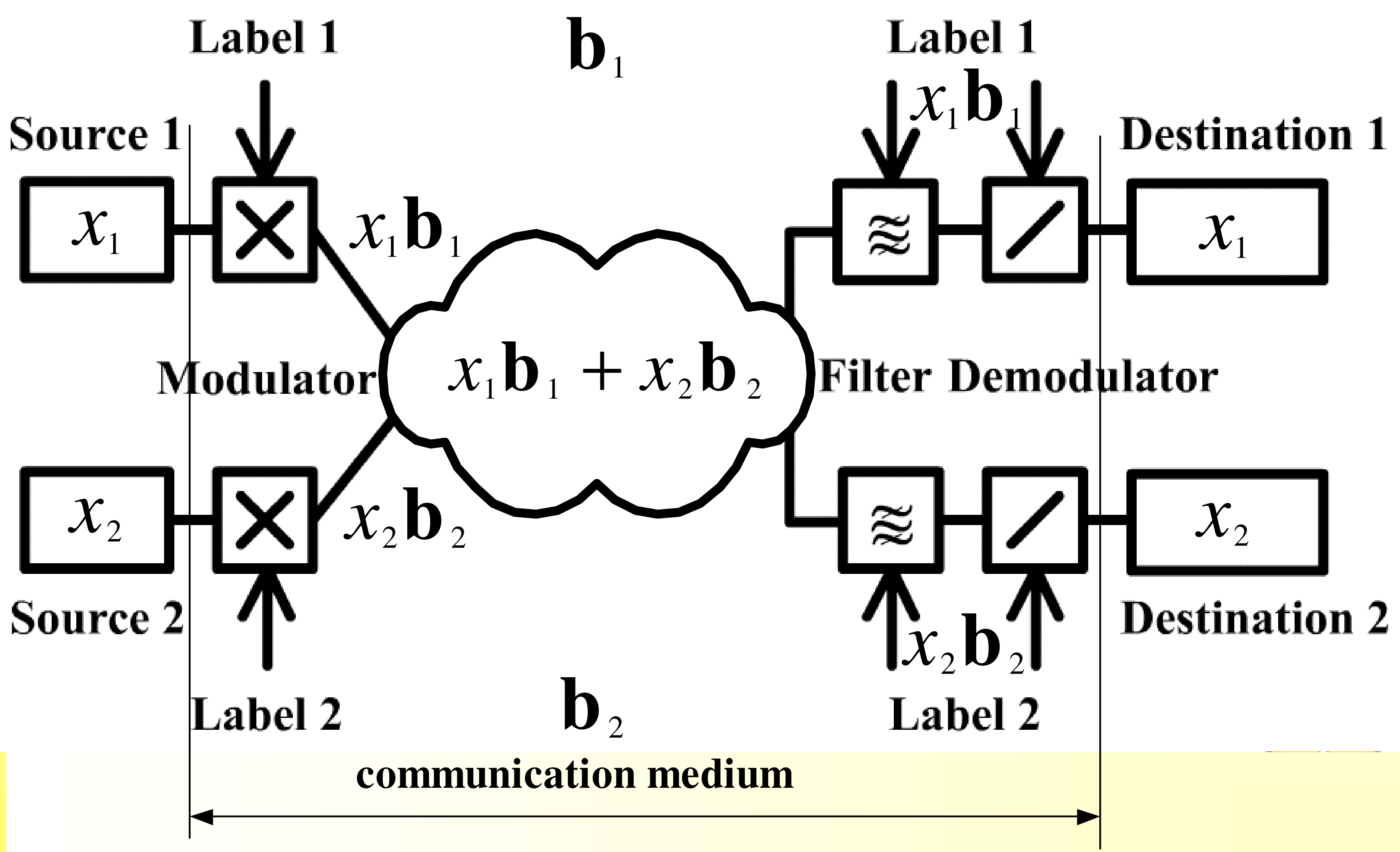
$$X \cdot X^{kor} = \alpha \qquad X_{dB} + X_{dB}^{kor} = \alpha_{dB}$$

$$X_{dB}^{kor} = \alpha_{dB} - X_{dB} \leq 0$$

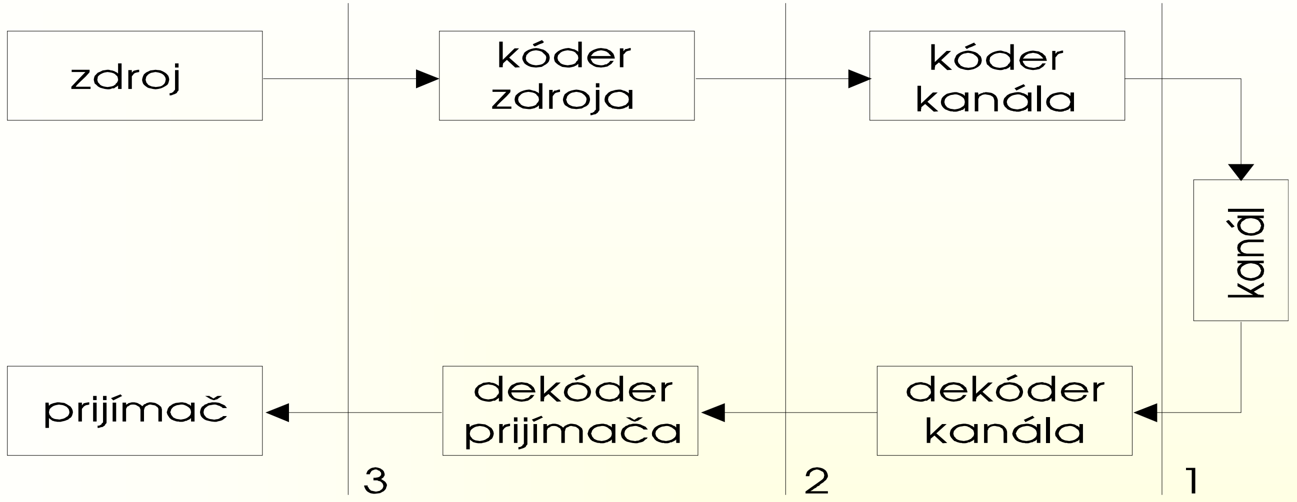




Príznakový multiplex

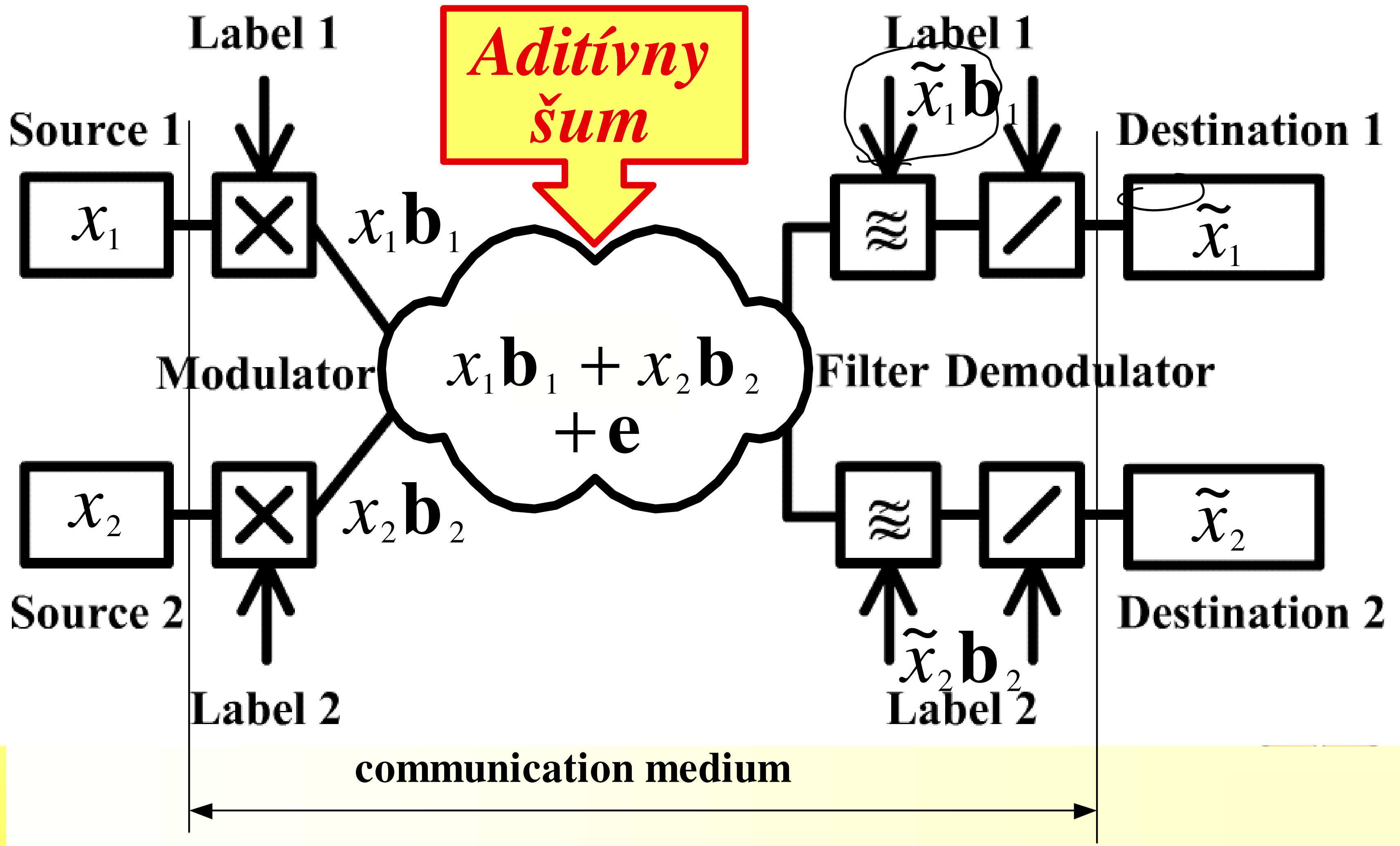


Prenos bez skreslenia



Zabránenie vplyvu šumu

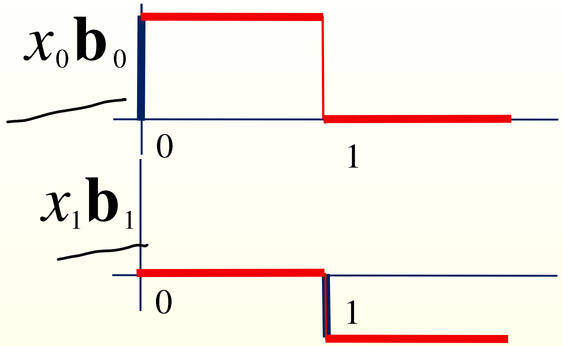
Príznakový multiplex so šumom



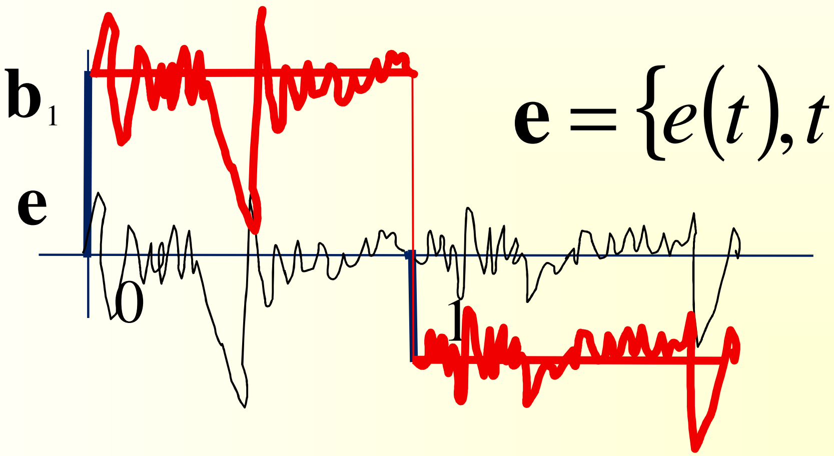
Šum v signálovom priestore

$$x_0 = 2 \quad \mathbf{b}_0 = (1,0)$$

$$x_1 = -1 \quad \mathbf{b}_1 = (0,1)$$



$\mathbf{e} + \; x_0\mathbf{b}_0 + x_1\mathbf{b}_1$



$$\mathbf{e} = \{e(t), t \in [0,2T)\}$$

spektrum:

$$c_n = \frac{1}{2T}\int_0^{2T} e(t).e^{j\frac{2\pi}{2T}nt}\,dt, \quad n = \ldots,-1,0,1,\ldots$$

Priemet do podpriestoru

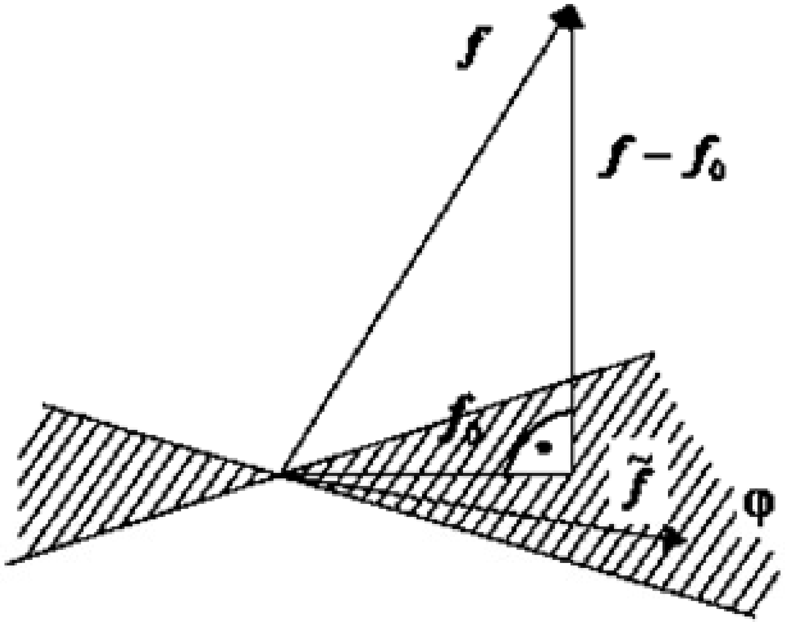
Veta :

Nech je Hilbertov priestor a nech φ je uzavretý lineárny podpriestor priestoru . Nech signál a $\delta = \inf$

Potom existuje práve jeden signál $f_0 \in \varphi$ tak, že $d(\mathbf{f}, \mathbf{f}_0) = \delta$

Navyše $\mathbf{f} - \mathbf{f}_0 \perp \varphi$, t.j. pre všetky $\tilde{f} \in \varphi$ platí $(\mathbf{f} - \mathbf{f}_0, \tilde{\mathbf{f}}) = 0$

Pritom f_0 je jediným signálom priestoru φ s vlastnosťou $\mathbf{f} - \mathbf{f}_0 \perp \varphi$



Obr 6

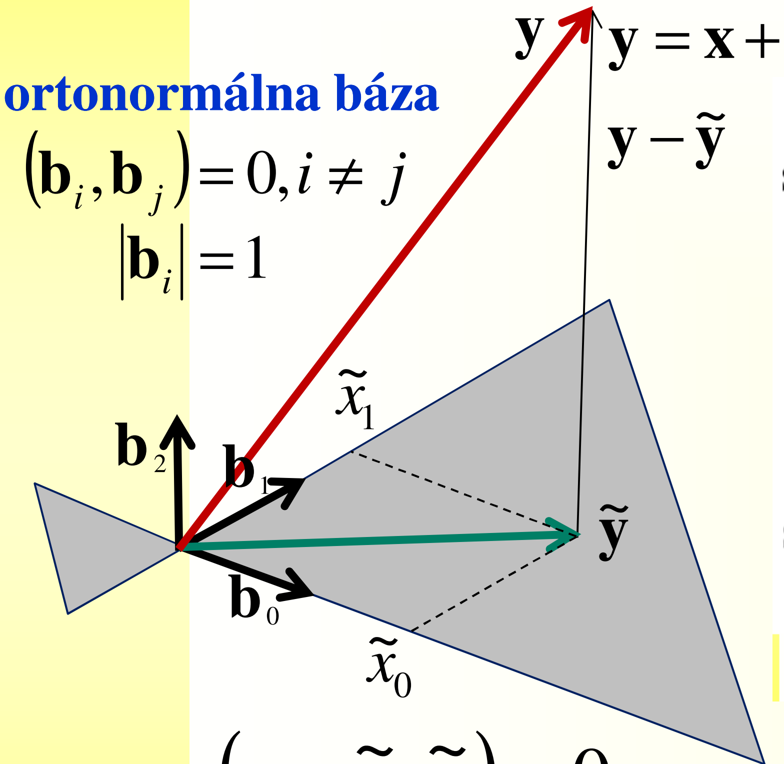
Optimálny prijímač

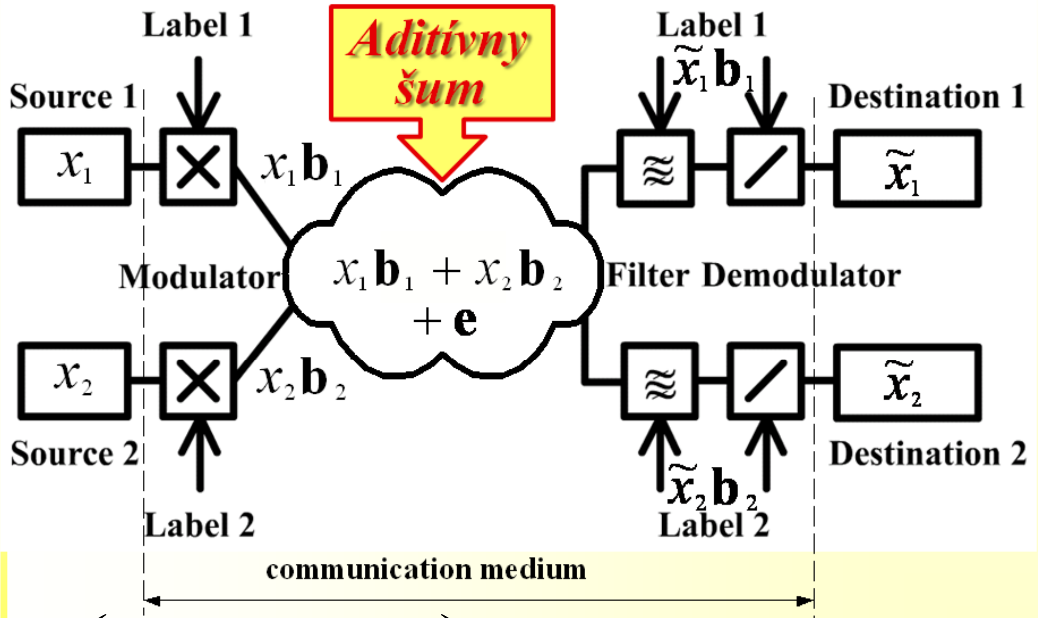
$$\mathbf{y} = \mathbf{x} + \mathbf{e} = (x_0 + e_0).\mathbf{b}_0 + (x_1 + e_1).\mathbf{b}_1 + e_2\mathbf{b}_2 + \ldots$$

ortonormálna báza

$$(\mathbf{b}_i, \mathbf{b}_j) = 0, i \neq j$$

$$|\mathbf{b}_i| = 1$$





$$(\mathbf{y} - \tilde{\mathbf{y}}, \tilde{\mathbf{y}}) = 0$$

$$(\mathbf{y} - \tilde{\mathbf{y}}, \mathbf{b}_n) = 0$$

$$(\tilde{\mathbf{y}}, \mathbf{b}_n) = (\mathbf{y}, \mathbf{b}_n)$$

$$\left(\sum_{-\infty}^{\infty} \tilde{x}_k \mathbf{b}_k, \mathbf{b}_n\right) = (\mathbf{y}, \mathbf{b}_n)$$

$$\sum_{-\infty}^{\infty} \tilde{x}_k (\mathbf{b}_k, \mathbf{b}_n) = (\mathbf{y}, \mathbf{b}_n),$$

$$(\mathbf{b}_k, \mathbf{b}_n) = 0, k \neq n$$

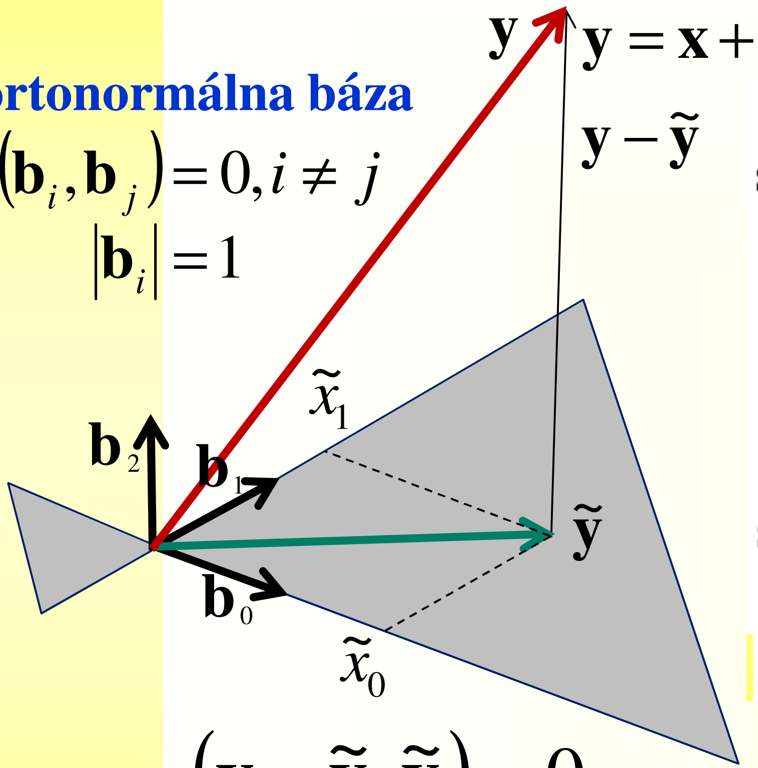
Optimálny prijímač

ortonormálna báza

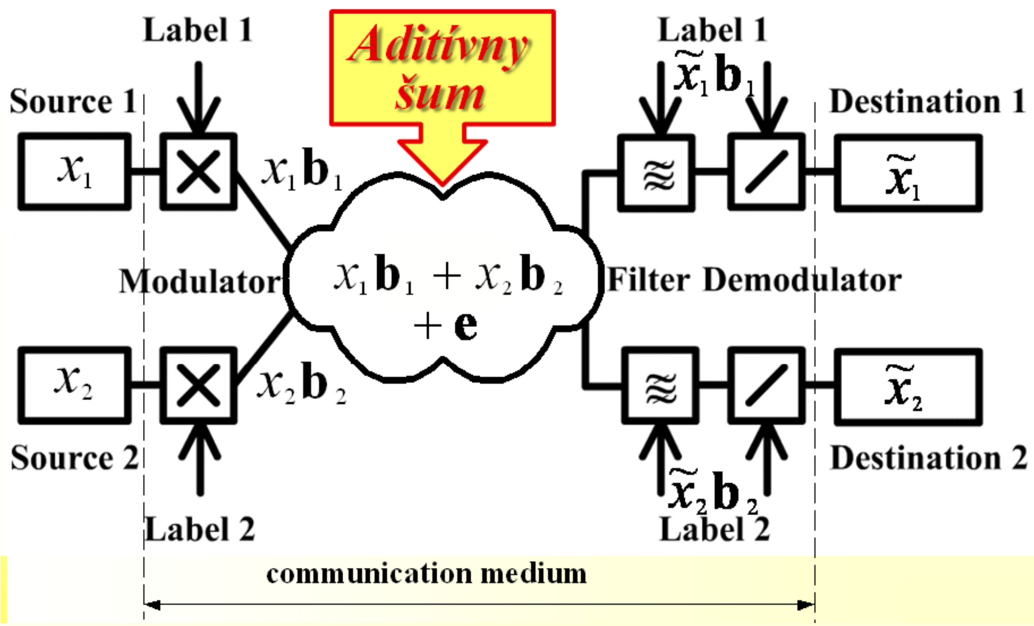
$(\mathbf{b}_i,\mathbf{b}_j)=0, i \neq j$

$|\mathbf{b}_i|=1$

$$\mathbf{y}=\mathbf{x}+\mathbf{e}=(x_0+e_0)\mathbf{b}_0+(x_1+e_1)\mathbf{b}_1+e_2\mathbf{b}_2+\dots$$



$$(\mathbf{y}-\tilde{\mathbf{y}},\tilde{\mathbf{y}})=0$$



$$\boxed{\tilde{x}_n=\frac{(\mathbf{y},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)}}$$

Ďakujem za Vašu pozornosť