t05-044 08.04.08 94.08 riešil(a): Smieško

1. $\int \operatorname{tg} 7x\,\mathrm{d}x = \int \frac{\sin 7x}{\cos 7x}\,dx \;\Big|\; \begin{matrix}\cos 7x = t \\ dx = -\frac{dt}{7\sin 7x}\end{matrix} \;\Big|\; = -\frac{1}{7}\int \frac{dt}{t} = -\frac{1}{7}\ln|t| + c = -\frac{1}{7}\ln|\cos 7x| + c$

$\int_{\frac{\pi}{14}}^{\frac{\pi}{7}} \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_{\frac{\pi}{14}}^{\frac{\pi}{7}} = -\frac{1}{7}\ln 1 + \frac{1}{7}\ln 0 = 0 + (-\infty) = \underline{\underline{-\infty}};$

$\int_0^{\frac{\pi}{14}} \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_0^{\frac{\pi}{14}} = -\frac{1}{7}\ln 0 + \frac{1}{7}\ln 1 = +\infty - 0 = \underline{\underline{+\infty}};$

$\int_{-\frac{\pi}{14}}^0 \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_{-\frac{\pi}{14}}^0 = -\frac{1}{7}\ln 1 + \frac{1}{7}\ln 0 = 0 + (-\infty) = \underline{\underline{-\infty}};$

$\int_{-\frac{\pi}{7}}^{-\frac{\pi}{14}} \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_{-\frac{\pi}{7}}^{-\frac{\pi}{14}} = -\frac{1}{7}\ln 0 + \frac{1}{7}\ln 1 = +\infty - 0 = \underline{\underline{+\infty}};$

$\int_0^{\frac{\pi}{7}} \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_0^{\frac{\pi}{14}} + \left[-\frac{1}{7}\ln|\cos 7x|\right]_{\frac{\pi}{14}}^{\frac{\pi}{7}} = -\frac{1}{7}\ln 0 + \frac{1}{7}\ln 1 - \frac{1}{7}\ln 1 + \frac{1}{7}\ln 0 = +\infty + (-\infty) = +\infty - \infty = \nexists$

$\int_{-\frac{\pi}{14}}^{\frac{\pi}{14}} \operatorname{tg} 7x\,\mathrm{d}x = \left[-\frac{1}{7}\ln|\cos 7x|\right]_{-\frac{\pi}{14}}^0 + \left[-\frac{1}{7}\ln|\cos 7x|\right]_0^{\frac{\pi}{14}} = -\frac{1}{7}\ln 1 + \frac{1}{7}\ln 0 - \frac{1}{7}\ln 0 + \frac{1}{7}\ln 1 = 0 + (-\infty) + \infty - 0 = -\infty + \infty = \nexists$

v.p. $\int_{-\frac{\pi}{7}}^0 \operatorname{tg} 7x\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{7}}^{-\frac{\pi}{14}-\varepsilon} \operatorname{tg} 7x\,dx + \int_{-\frac{\pi}{14}+\varepsilon}^{0} \operatorname{tg} 7x\,dx\right] = \lim_{\varepsilon\to 0^+} -\frac{1}{7}\left(\overbrace{\ln|\cos(-\tfrac{\pi}{2}-7\varepsilon)| - \ln|\cos -\pi| + \ln|\cos 0| - \ln|\cos(-\tfrac{\pi}{2}+7\varepsilon)|}^{0}\right) = \underline{0};$

v.p. $\int_{-\frac{\pi}{14}}^{\frac{\pi}{14}} \operatorname{tg} 7x\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{14}+\varepsilon}^{0} \operatorname{tg} 7x\,dx + \int_0^{\frac{\pi}{14}-\varepsilon} \operatorname{tg} 7x\,dx\right] = \lim_{\varepsilon\to 0^+} -\frac{1}{7}\left(\ln|\cos 0| - \underbrace{\ln|\cos(-\tfrac{\pi}{2}+7\varepsilon)| + \ln|\cos(\tfrac{\pi}{2}-7\varepsilon)|}_{0} - \ln|\cos 0|\right) = \underline{0};$

2. $\int \operatorname{cotg} 7x\,\mathrm{d}x = \int \frac{\cos 7x}{\sin 7x} \;\Big|\; \begin{matrix}\sin 7x = t \\ dx = \frac{dt}{7\cos 7x}\end{matrix} \;\Big|\; = \frac{1}{7}\int \frac{dt}{t} = \frac{1}{7}\ln|t| + c = \frac{1}{7}\ln|\sin 7x| + c$

$\int_{\frac{\pi}{14}}^{\frac{\pi}{7}} \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_{\frac{\pi}{14}}^{\frac{\pi}{7}} = \frac{1}{7}\ln 0 - \frac{1}{7}\ln 1 = -\infty - 0 = \underline{\underline{-\infty}};$

$\int_0^{\frac{\pi}{14}} \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_0^{\frac{\pi}{14}} = \frac{1}{7}\ln 1 - \frac{1}{7}\ln 0 = 0 - (-\infty) = \underline{\underline{+\infty}};$

$\int_{-\frac{\pi}{14}}^0 \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_{-\frac{\pi}{14}}^0 = \frac{1}{7}\ln 0 - \frac{1}{7}\ln 1 = -\infty - 0 = \underline{\underline{-\infty}};$

$\int_{-\frac{\pi}{7}}^{-\frac{\pi}{14}} \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_{-\frac{\pi}{7}}^{-\frac{\pi}{14}} = \frac{1}{7}\ln 1 - \frac{1}{7}\ln 0 = 0 - (-\infty) = \underline{\underline{+\infty}};$

$\int_0^{\frac{\pi}{7}} \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_0^{\frac{\pi}{14}} + \left[\frac{1}{7}\ln|\sin 7x|\right]_{\frac{\pi}{14}}^{\frac{\pi}{7}} = \frac{1}{7}\ln 1 - \frac{1}{7}\ln 0 + \frac{1}{7}\ln 0 - \frac{1}{7}\ln 1 = 0 - \infty + \infty - 0 = \nexists$

$\int_{-\frac{\pi}{14}}^{\frac{\pi}{14}} \operatorname{cotg} 7x\,\mathrm{d}x = \left[\frac{1}{7}\ln|\sin 7x|\right]_{-\frac{\pi}{14}}^0 + \left[\frac{1}{7}\ln|\sin 7x|\right]_0^{\frac{\pi}{14}} = \frac{1}{7}\ln 0 - \frac{1}{7}\ln 1 + \frac{1}{7}\ln 1 - \frac{1}{7}\ln 0 = -\infty - (-\infty) = \nexists$

v.p. $\int_{-\frac{\pi}{7}}^0 \operatorname{cotg} 7x\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{7}+\varepsilon}^{-\frac{\pi}{14}} \operatorname{cotg} 7x\,dx + \int_{-\frac{\pi}{14}}^{-\varepsilon} \operatorname{cotg} 7x\,dx\right] = \lim_{\varepsilon\to 0^+} \frac{1}{7}\left[\ln|\sin -\tfrac{\pi}{2}| - \underbrace{\ln|\sin(-\pi+7\varepsilon)| + \ln|\sin -7\varepsilon|}_{0} - \ln|\sin -\tfrac{\pi}{2}|\right] = \underline{\underline{0}}.$

v.p. $\int_{-\frac{\pi}{14}}^{\frac{\pi}{14}} \operatorname{cotg} 7x\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{14}}^{-\varepsilon} \operatorname{cotg} 7x\,dx + \int_{\varepsilon}^{\frac{\pi}{14}} \operatorname{cotg} 7x\,dx\right] = \lim_{\varepsilon\to 0^+} \frac{1}{7}\left[\overbrace{\ln|\sin -7\varepsilon| - \ln|\sin -\tfrac{\pi}{2}| + \ln|\sin \tfrac{\pi}{2}| - \ln|\sin 7\varepsilon|}\right] = 0$

—pokračovanie—

3. $\int |x+14|\,(x+14)\,dx = + \int (x+14)^3 dx \;\Big|\; \begin{matrix} x+14=t \\ dx=dt \end{matrix} \;\Big| = \int t^3 dt = \frac{t^4}{4} + c = \frac{(x+14)^4}{4} + c$

NB = −14

$= - \int (x+14)^3 dx \;\Big|\; -\|- \;\Big| = -\int t^3 dt = -\frac{t^4}{4} + c = -\frac{(x+14)^4}{4} + c$

$\int_{-19}^{-9} |x+14|\,(x+14)^2\,dx = \left[-\frac{(x+14)^4}{4} \right]_{-19}^{-14} + \left[\frac{(x+14)^4}{4} \right]_{-14}^{-9} = 0 + \frac{5^4}{4} + \frac{5^4}{4} - 0 = \frac{5^4}{2}$

4. $\int \frac{dx}{x^2-9x+14} = \int \frac{dx}{(x-2).(x-7)} = -\frac{1}{5}\int \frac{dx}{x-2} + \frac{1}{5}\int \frac{dx}{x-7} = -\frac{1}{5}\ln|x-2| + \frac{1}{5}\ln|x-7| + c = \boxed{\frac{1}{5}\ln|x-7| - \frac{1}{5}\ln|x-2| + c}$

$\int_0^{16} \frac{dx}{x^2-9x+14} = \int_0^2 (..)dx + \int_2^4 (..)dx + \int_4^7 (..)dx + \int_7^{16} (..)dx = \frac{1}{5}(\ln 5 - \ln 0 - \ln 7 + \ln 2 + \ln 3 - \ln 1 - \ln 5 + \ln 0 + \ln 0 - \ln 5 -$

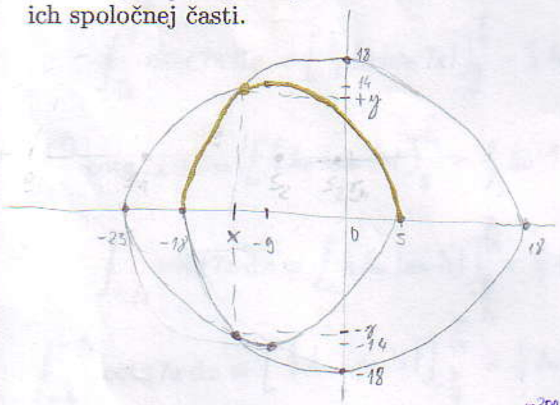
$- \ln 5 + \ln 2 + \ln 9 - \ln 14 - \ln 0 + \ln 5 = -(-\infty) - \infty - \infty - (-\infty) + \dots = \nexists$

$\int_{-1}^{5} \frac{dx}{x^2-9x+14} = \int_{-1}^{2} (..)dx + \int_2^5 (..)dx = \frac{1}{5}(\ln 5 - \ln 0 - \ln 8 + \ln 3 + \ln 2 - \ln 3 - \ln 5 + \ln 0 = -(-\infty) - \infty + \dots = \nexists$

$\int_{16}^{31} \frac{dx}{x^2-9x+14} = \left[\frac{1}{5}\ln|x-7| - \frac{1}{5}\ln|x-2| \right]_{16}^{31} = \frac{1}{5}(\ln 24 - \ln 29 - \ln 9 + \ln 14)$

5. $\int_{-5}^{-1} \frac{dx}{x^2-3x+9} = \int_{-5}^{-1} \frac{dx}{(x-\frac{3}{2})^2 + (\frac{\sqrt{27}}{2})^2} = \left[\frac{2\sqrt{27}}{27} \operatorname{arctg}\left(\frac{\sqrt{27}}{27}(2x-3) \right) \right]_{-5}^{-1} = \frac{2\sqrt{27}}{27}\left(\operatorname{arctg}\left(-\frac{5\sqrt{27}}{27}\right) - \operatorname{arctg}\left(-\frac{8\sqrt{27}}{27}\right) \right)$

6. Stredy dvoch gulí s polomermi 14 cm a 18 cm sú od seba vzdialené 9 cm. Určte (pomocou integrálneho počtu) povrch ich spoločnej časti.



$K_1 = x^2 + y^2 = 18^2$

$K_2 = (x+9)^2 + y^2 = 14^2$

priesečníky $\left[-\frac{209}{18}, \pm \frac{\sqrt{18^4 - 209^2}}{18} \right]$

$f_1(x) = \sqrt{18^2 - x^2}$ $x \in \langle -18, -\frac{209}{18} \rangle$ $f_1'(x) = -\frac{x}{\sqrt{18^2-x^2}}$

$f_2(x) = \sqrt{14^2 - (x+9)^2}$ $x \in \langle \frac{209}{18}, 5 \rangle$ $f_2'(x) = -\frac{(x+9)}{\sqrt{14^2 - (x+9)^2}}$

$x^2 + y^2 = 18^2$

$-x^2 - 18x - 81 - y^2 = -14^2$

$-18x - 81 = 18^2 - 14^2$

$-18x = 128 + 81$

$-18x = 209$

$x = -\frac{209}{18}$

$y = \pm \frac{\sqrt{18^4 - 209^2}}{18} \doteq 13{,}75$

$S = S_1 + S_2$

$S_1 = 2\pi \int_{-18}^{-209/18} \sqrt{18^2 - x^2} \cdot \sqrt{1 + \frac{x^2}{18^2 - x^2}}\, dx = 36\pi \left[x \right]_{-18}^{-209/18} = 36\pi\left(-\frac{209}{18} + 18 \right) = 230\pi$

$S_2 = 2\pi \int_{-209/18}^{5} \sqrt{14^2 - (x+9)^2} \cdot \sqrt{1 + \frac{(x+9)^2}{14^2 - (x+9)^2}} = 18\pi \left[x \right]_{-209/18}^{5} = 18\pi\left(5 + \frac{209}{18} \right) = 299\pi$

$S = S_1 + S_2 = 230\pi + 299\pi = \underline{\underline{529\pi}}$

4b do **18.04.08**, 3b do **25.04.08**, 2b do **02.05.08**, 1b do **27.06.08**