

03.12.07

30.11.07

s05-022

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1. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\dots(f))))$, $n \in \mathbb{N}$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = -5 + 3x$, $x \in \mathbb{R}$.

$$f_2(x) = f(-5 + 3x) = -5 + 3(-5 + 3x) = -5 - 15 + 9x = 9x - 20$$

$$f_3(x) = f(f_2(x)) = f(9x - 20) = -5 + 3(9x - 20) = -5 + 27x - 60 = 27x - 65$$

$$f_4(x) = f(f_3(x)) = f(27x - 65) = -5 + 3(27x - 65) = -5 + 81x - 195 = 81x - 200$$

$$f_n(x) = -5 + 3^n x - 5 \sum_{k=0}^{n-1} 3^k$$

$$f_2(x) = 9x - 20 \quad f_3(x) = 27x - 65 \quad f_4(x) = 81x - 200$$

$$f_n(x) = 3^n x - 5 \sum_{k=0}^{n-1} 3^k = -5 + 3(-5 + 3(-5 + 3 \dots (-5 + 3x) \dots)) = 3^n x - 5 \frac{3^n - 1}{2}$$

2. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\dots(f))))$, $n \in \mathbb{N}$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = -5 - 3x$, $x \in \mathbb{R}$.

$$f_2(x) = -5 - 3(-5 - 3x) = -5 + 15 + 9x = 10 + 9x$$

$$f_3(x) = -5 - 3(10 + 9x) = -5 - 30 - 27x = -35 - 27x$$

$$f_4(x) = -5 - 3(-35 - 27x) = -5 + 105 + 81x = 100 + 81x$$

$$f_n(x) = -5 - 3^n x + 5 \sum_{k=0}^{n-1} 3^k$$

$$f_2(x) = 10 + 9x$$

$$f_3(x) = -35 - 27x$$

$$f_4(x) = 100 + 81x$$

$$f_n(x) = -3^n x - 5 \sum_{k=0}^{n-1} (-3)^k = -5 - 3(-5 - 3(-5 - 3 \dots (-5 - 3x) \dots)) = -3^n x - 5 \frac{(-3)^n - 1}{-4}$$

3. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, $f_4 = f(f(f(f)))$ a inverznú funkciu f^{-1} , ak funkcia f je definovaná predpisom $f(x) = \frac{x-2}{x-3}$, $x \in \mathbb{R} - \{3\}$.

$$f^{-1}(x) = \frac{x-2}{x-3}$$

$$f_2(x) = \frac{f(x)-2}{f(x)-3} = \frac{\frac{x-2}{x-3}-2}{\frac{x-2}{x-3}-3} = \frac{x-2-2(x-3)}{x-2-3(x-3)} = \frac{x-2-2x+6}{x-2-3x+9} = \frac{-x+4}{-2x+7} = \frac{x-4}{2x-7}$$

$$f_3(x) = f(f_2(x)) = \frac{\frac{x-4}{2x-7}-2}{\frac{x-4}{2x-7}-3} = \frac{x-4-2(2x-7)}{x-4-3(2x-7)} = \frac{x-4-4x+14}{x-4-6x+21} = \frac{-3x+10}{-5x+17} = \frac{3x-10}{5x-17}$$

$$f_4(x) = f(f_3(x)) = \frac{\frac{3x-10}{5x-17}-2}{\frac{3x-10}{5x-17}-3} = \frac{3x-10-2(5x-17)}{3x-10-3(5x-17)} = \frac{3x-10-10x+34}{3x-10-15x+51} = \frac{-7x+24}{-12x+41} = \frac{7x-24}{12x-41}$$

$$f(x-5) = -x-2$$

$$7x-5y = -x-2$$

$$7x+x = -x-2$$

$$x = \frac{-2}{8} = -\frac{1}{4}$$

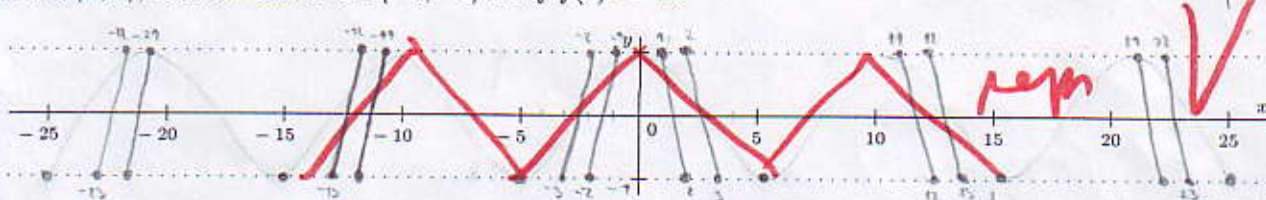
$$f^{-1}(x) = \frac{3x-10}{5x-17}$$

$$f_2(x) = \frac{x-4}{2x-7}$$

$$f_3(x) = \frac{3x-10}{5x-17}$$

$$f_4(x) = \frac{7x-24}{12x-41}$$

4. Zostrojte periodickú funkciu $y = f(x)$ s primitívnou periódou 10 a načrtnite jej graf tak, aby bola párna, klesajúca na intervale $(1; 2)$, rastúca na intervale $(-3; -2)$ a aby $f(5) = -1$.



5. Nájdite s presnosťou $\varepsilon = 0.01$ kladné riešenie rovnice $5e^x - 18x^5 - 5 = 0$.

$$\text{Výpočet počtu krokov } n: 0.01 = 10^{-2} = \frac{1}{100} \Rightarrow \frac{1}{2^n} < \frac{1}{100} \Rightarrow 100 < 2^n \Rightarrow n \geq 7 \text{ krokov}$$

n	a_n	x_{n+1}	b_n	$f(x_{n+1})$
0	0	0.5	1	+ 2.68
1	0.5	0.75	1	+ 1.3125
2	0.75	0.875	1	- 2.2379
3	0.875	0.8925	0.875	- 0.1059
4	0.8925	0.8975	0.8925	+ 0.6823
5	0.8975	0.89875	0.8975	+ 0.2090

n	a_n	x_{n+1}	b_n	$f(x_{n+1})$
6	0.89875	0.899375	0.89875	+ 0.106
7	0.899375	0.8996875	0.899375	
8				
9				
10				
11				

Približné riešenie $x_{n+1} \approx 0.89859375$

4b do 03.12.07, 3b do 10.12.07, 2b do 17.12.07, 1b do 24.12.07

$$f_1(x) = \frac{-x-2}{x-3}$$

$$f_2(x) = f\left(\frac{-x-2}{x-3}\right) = \frac{-\left(\frac{-x-2}{x-3}\right) - 2}{\frac{-x-2}{x-3} - 3} = \frac{\frac{x+2}{-3+x} - 2}{\frac{-x-2-3x+9}{x-3}} = \frac{\frac{x+2+6-2x}{-3+x}}{\frac{-4x+7}{x-3}} =$$

$$= \frac{\frac{8-x}{-3+x}}{\frac{-4x+7}{x-3}} = \frac{8-x}{x-3} \cdot \frac{x-3}{7-4x} = \frac{8-x}{7-4x}$$

$$f_3(x) = f\left(f\left(\frac{-x-2}{x-3}\right)\right) = f\left(\frac{8-x}{7-4x}\right) = \frac{-\left(\frac{8-x}{7-4x}\right) - 2}{\frac{8-x}{7-4x} - 3} = \frac{\frac{-8+x+14}{-4x+7}}{\frac{8-x-21+12x}{7-4x}} = \frac{-22+x}{11x-13}$$

$$f_4(x) = f\left(f\left(f\left(\frac{-x-2}{x-3}\right)\right)\right) = f\left(\frac{9x-22}{11x-13}\right) = \frac{+\left(\frac{-9x+22}{11x-13}\right) - 2}{\frac{9x-22}{11x-13} - 3} = \frac{\frac{-9x+22-22x+22}{11x-13}}{\frac{9x-22-33x+39}{11x-13}} = \frac{-31x+44}{-24x+17}$$

$$= \frac{9x+22}{11x-13} \cdot \frac{11x-13}{44-24x} = \frac{9x+22}{44-24x}$$

$$= \frac{-9x+22-22x+22}{11x-13} = \frac{-31x+44}{-24x+17} = \frac{48-31x}{17-24x}$$