tce 4,5

ZS z08-021

riešil(a): PETER MÁSIAR 51025

1. Funkciu $f(x) = x^7 + x^5 + 2x^4 - x^3 - 2x^2 - 2$ rozviňte do Taylorovho polynómu stupňa 7 so stredom v bode $x_0 = 1$.

		$\frac{f^{(i)}(x_0)}{i!}$
$f(x) = x^7 + x^5 + 2x^4 - x^3 - 2x^2 - 2$	$f(1) = -1$	-1
$f'(x) = 7x^6 + 5x^4 + 8x^3 - 3x^2 - 4x$	$f'(1) = 13$	13
$f''(x) = 42x^5 + 20x^3 + 24x^2 - 6x - 4$	$f''(1) = 76$	38
$f^{(3)}(x) = 210x^4 + 60x^2 + 48x - 6$	$f^{(3)}(1) = 312$	52
$f^{(4)}(x) = 840x^3 + 120x + 48$	$f^{(4)}(1) = 1008$	42
$f^{(5)}(x) = 2520x^2 + 120$	$f^{(5)}(1) = 2640$	22
$f^{(6)}(x) = 5040x$	$f^{(6)}(1) = 5040$	7
$f^{(7)}(x) = 5040$	$f^{(7)}(1) = 5040$	1

$$T_7(x) = -1 + 13(x-1) + 38(x-1)^2 + 52(x-1)^3 + 42(x-1)^4 + 22(x-1)^5 + 7(x-1)^6 + (x-1)^7$$

2. Funkciu $f(x) = x^7 + x^5 + 2x^4 - x^3 - 2x^2 - 2$ rozviňte do Taylorovho polynómu stupňa 7 so stredom v bode $x_0 = -1$.

	$\frac{f^{(i)}(x_0)}{i!}$
$f(-1) = -3$	-3
$f'(-1) = 5$	5
$f''(-1) = -36$	-18
$f^{(3)}(-1) = 216$	36
$f^{(4)}(-1) = -912$	-38
$f^{(5)}(-1) = 2640$	22
$f^{(6)}(-1) = -5040$	-7
$f^{(7)}(-1) = 5040$	1

$$T_7(x) = -3 + 5(x+1) - 18(x+1)^2 + 36(x+1)^3 - 38(x+1)^4 + 22(x+1)^5 - 7(x+1)^6 + (x+1)^7$$

ZS z08-021 riešil(a): PETER MÁSIAR 51025

3. Funkciu $f(x) = x^7 + x^5 + 2x^4 - x^3 - 2x^2 + 2$ rozviňte do Taylorovho polynómu stupňa 7 so stredom v bode $x_0 = 2$.

	$\frac{f^{(i)}(x_0)}{i!}$
$f(2) = 174$	174
$f'(2) = 572$	572
$f''(2) = 1584$	792
$f^{(3)}(2) = 3690$	615
$f^{(4)}(2) = 7008$	292
$f^{(5)}(2) = 10200$	85
$f^{(6)}(2) = 10080$	14
$f^{(7)}(2) = 5040$	1

$$T_7(x) = 174 + 572(x-2) + 792(x-2)^2 + 615(x-2)^3 + 292(x-2)^4 + 85(x-2)^5 + 14(x-2)^6 + (x-2)^7$$

4. Určte Maclaurinov polynóm stupňa $n \in N$ pre funkciu $f(x) = \frac{1}{\sqrt[4]{1-2x-x^6}}$.

$$f(x) = \frac{1}{\sqrt[4]{1-2x-x^6}}$$

$$f'(x) = \frac{6x^5+2}{4(-x^6-2x+1)^{\frac{5}{4}}}$$

$$f''(x) = \frac{30x^4(-x^6-2x+1)^{\frac{5}{4}} - \frac{5(-6x^5-2)(6x^5+2)(-x^6-2x+1)^{\frac{1}{4}}}{4}}{4(-x^6-2x+1)^{\frac{5}{2}}}$$

$$f'''(x) = -\frac{15(7x^{15} - 65x^{10} + 58x^9 + 31x^5 - 31x^4 + 16x^3 + 3)}{8(-x^6-2x+1)^{\frac{1}{4}}(x^6+2x-1)^3}$$

	$\frac{f^{(i)}(x_0)}{i!}$
$f'(0) = \frac{1}{2}$	$\frac{1}{2}$
$f''(0) = \frac{5}{4}$	$\frac{5}{8}$
$f'''(0) = \frac{45}{8}$	$\frac{15}{16}$

$$T_n(x) = 1 + \frac{1}{2}x + \cancel{\frac{5}{8}}x^2 + \cancel{\frac{15}{16}}x^3 + \dots +$$

5. Určte Maclaurinov polynóm stupňa $n \in N$ pre funkciu $f(x) = \ln\left|\frac{(1-2x+x^2)^2}{(1-2x-x^2)^3}\right|$.

$$f(x) = \ln \frac{(1-2x+x^2)^2}{(1-2x+x^2)^3} = \ln \frac{-(1-2x+x^2)^2}{-(1-2x+x^2)^3}$$

$$\frac{f^{(k)}(x_0)}{k!}$$

$$f'(x) = -\frac{2x+2}{x^2+2x-1} \qquad f'(x) = 2 \qquad 2$$

$$f''(x) = \frac{2(x^2+2x+3)}{(x^2+2x-1)^2} \qquad f''(x) = 6 \qquad 3$$

$$f^{(3)}(x) = \frac{4(x+1)(x^2+2x+7)}{(x^2+2x-1)^3} \qquad f'''(x) = 28 \qquad \frac{14}{3}$$

$$f^{(4)}(x) = \frac{12(x^4+4x^3+18x^2+28x+17)}{(x^2+2x-1)^4} \qquad f^{(4)}(x) = 204 \qquad \frac{17}{2}$$

$$f^{(5)}(x) = \frac{48(x+1)(x^4+4x^3+26x^2+44x+41)}{(x^2+2x-1)^5} \qquad f^{(5)}(x) = 1968 \qquad \frac{82}{5}$$

$$T_5(x) = 2x + \cancel{3x^2} + \cancel{\frac{14}{3}x^3} + \cancel{\frac{17}{2}x^4} + \cancel{\frac{82}{5}x^5}$$

$$T_n(x) =$$

6. Určte Maclaurinov polynóm stupňa $n \in N$ pre funkciu $f(x) = \sin x$.

$$f'(x) = \cos x \qquad f'(0) = 1$$

$$f''(x) = -\sin x \qquad f''(0) = 0$$

$$f'''(x) = -\cos x \qquad f^{(3)}(0) = -1$$

$$f^{(4)}(x) = \sin x \qquad f^{(4)}(0) = 0$$

$$f^{(5)}(x) = \cos x \qquad f^{(5)}(0) = 1$$

$$f^{(6)}(x) = -\sin x \qquad f^{(6)}(0) = 0$$

$$T_n(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$= 0 + x + \frac{0 \cdot x^2}{2!} + \frac{-1x^3}{3!} + \frac{0 \cdot x^4}{4!} + \dots = x - \frac{x^2}{6} + \frac{x^5}{120} + \dots + \frac{(-1)^2 \cdot x^{k+1}}{(2k)!}$$

ZS z08-021

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7. Určte Maclaurinov polynóm stupňa 6 pre funkciu $f(x) = \sin^3 x$. [Úlohu najprv riešte priamo pre funkciu $f(x)$ a potom pomocou Maclaurinovho radu funkcie $g(x) = \sin x$, resp. $g(x) = \cos x$. Oba výsledky porovnajte (musia byť rovnaké).]

		$\frac{f^{(i)}(x_0)}{i!}$
$f'(x) = 3\cos x \sin^2 x$	$f'(0) = 0$	0
$f''(x) = (9\cos^2(x) - 3)\sin x$	$f''(0) = 0$	0
$f^{(3)}(x) = 27\cos^3(x) - 21\cos(x)$	$f'''(0) = 6$	1
$f^{(4)}(x) = (21 - 81\cos^2(x))\sin x$	$f^{(4)}(0) = 0$	0
$f^{(5)}(x) = 183\cos x - 243\cos^3 x$	$f^{(5)}(0) = -60$	$-\frac{1}{2}$
$f^{(6)}(x) = (729\cos^2 x - 183)\sin x$	$f^{(6)}(0) = 0$	0

$g(x) = \sin x$

$g'(x) = \cos x$	1
$g''(x) = -\sin x$	0
$g^{(3)}(x) = -\cos x$	-1
$g^{(4)}(x) = \sin x$	0
$g^{(5)}(x) = \cos x$	1
$g^{(6)}(x) = 0$	0

$g(x) = \cos x$

$g'(x) = -\sin x$	0
$g''(x) = -\cos x$	-1
$g'''(x) = \sin x$	0
$g^{(4)}(x) = \cos x$	1
$g^{(5)}(x) = -\sin x$	0
$g^{(6)}(x) = -\cos x$	-1

$T_6(x) = x^3 - \frac{1}{2}x^5$

$\left(x - \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right) =$

8. Určte Maclaurinov polynóm stupňa 6 pre funkciu $f(x) = \sin(x^3)$. [Úlohu najprv riešte priamo pre funkciu $f(x)$ a potom pomocou Maclaurinovho radu funkcie $g(x) = \sin x$, resp. $g(x) = \cos x$. Oba výsledky porovnajte (musia byť rovnaké).]

		$\frac{f^{(i)}(x_0)}{i!}$
$f'(x) = 3x^2\cos(x^2)$	$f'(0) = 0$	0
$f''(x) = 6x\cos(x^3) - 9x^4\sin(x^3)$	$f''(0) = 0$	0
$f'''(x) = (6 - 27x^6)\cos(x^3) - 54x^3\sin(x^3)$	$f'''(0) = 6$	1
$f^{(4)}(x) = (81x^8 - 180x^2)\sin(x^3) - 324x^5\cos(x^3)$	$f^{(4)}(0) = 0$	0
$f^{(5)}(x) = (1620x^7 - 360x)\sin(x^3) + (243x^{10} - 2160x^4)\cos(x^3)$	$f^{(5)}(0) = 0$	0
$f^{(6)}(x) = (-729x^{12} + 17820x^6 - 360)\sin(x^3) + (7290x^9 - 9720x^3)\cos(x^3)$	$f^{(6)}(0) = 0$	0

$T_6(x) = x^3$

$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!}$