t05-052 04 17.04.08 riešil(a): MIROSLAV DREVEŇOK

1. $\int \operatorname{tg}(-3x)\,\mathrm{d}x = \frac{1}{3}\ln|\cos(-3x)| + C$ ✓

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{tg}(-3x)\,\mathrm{d}x = \left[\frac{1}{3}\ln|\cos(-3x)|\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{3}\ln|\cos(-\pi)| - \frac{1}{3}\ln\left|\cos\left(-\frac{\pi}{2}\right)\right| = \frac{1}{3}\ln 1 - \frac{1}{3}\ln 0 = 0-(-\infty) = \infty$$ ✓

$$\int_{0}^{\frac{\pi}{6}} \operatorname{tg}(-3x)\,\mathrm{d}x = \left[\frac{1}{3}\ln|\cos(-3x)|\right]_{0}^{\frac{\pi}{8}} = \frac{1}{3}\ln\left|\cos\left(-\frac{\pi}{2}\right)\right| - \frac{1}{3}\ln|\cos 0| = \frac{1}{3}\ln 0 - \frac{1}{3}\ln 1 = -\infty - 0 = -\infty$$ ✓

$$\int_{-\frac{\pi}{6}}^{0} \operatorname{tg}(-3x)\,\mathrm{d}x = \left[\frac{1}{3}\ln|\cos(-3x)|\right]_{-\frac{\pi}{6}}^{0} = \frac{1}{3}\ln|\cos 0| - \frac{1}{3}\ln\left|\cos\left(\frac{\pi}{2}\right)\right| = \frac{1}{3}\ln 1 - \frac{1}{3}\ln 0 = 0-(-\infty) = \infty$$ ✓

$$\int_{-\frac{\pi}{3}}^{-\frac{\pi}{6}} \operatorname{tg}(-3x)\,\mathrm{d}x = \left[\frac{1}{3}\ln|\cos(-3x)|\right]_{-\frac{\pi}{4}}^{-\frac{\pi}{8}} = \frac{1}{3}\ln\left|\cos\left(\frac{\pi}{2}\right)\right| - \frac{1}{3}\ln|\cos(\pi)| = \frac{1}{3}\ln 0 - \frac{1}{3}\ln 1 = -\infty - 0 = -\infty$$ ✓

$$\int_{0}^{\frac{\pi}{3}} \operatorname{tg}(-3x)\,\mathrm{d}x = \int_0^{\pi/6} \operatorname{tg}(-3x)\,\mathrm{d}x + \int_{\pi/6}^{\pi/3} \operatorname{tg}(-3x)\,\mathrm{d}x = -\infty + \infty \Rightarrow \nexists$$ ✓

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{tg}(-3x)\,\mathrm{d}x = \int_{-\frac{\pi}{6}}^{0} \operatorname{tg}(-3x)\,\mathrm{d}x + \int_{0}^{\frac{\pi}{6}} \operatorname{tg}(-3x)\,\mathrm{d}x = \infty - \infty \Rightarrow \nexists$$ ✓

$$v.p. \int_{-\frac{\pi}{3}}^{0} \operatorname{tg}(-3x)\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{3}}^{-\frac{\pi}{8}-\varepsilon} \operatorname{tg}(-3x)\,\mathrm{d}x + \int_{-\frac{\pi}{8}+\varepsilon}^{0} \operatorname{tg}(-3x)\,\mathrm{d}x\right] = \lim_{\varepsilon\to 0^+} \frac{1}{3}\Big(\underbrace{\ln\left|\cos\left(\tfrac{\pi}{2}+3\varepsilon\right)\right| - \ln|\cos(\pi)|}_{0} + \underbrace{\ln|\cos 0|}_{0} - \underbrace{\ln\left|\cos\left(\tfrac{\pi}{2}-3\varepsilon\right)\right|}_{0}\Big) = 0$$ ✓

$$v.p. \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{tg}(-3x)\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{6}+\varepsilon}^{0} \operatorname{tg}(-3x)\,\mathrm{d}x + \int^{\frac{\pi}{6}+\varepsilon} \operatorname{tg}(-3x)\,\mathrm{d}x\right] = \lim_{\varepsilon\to 0^+} \frac{1}{3}\Big(\underbrace{\ln|\cos 0|}_{0} - \underbrace{\ln\left|\cos\left(\tfrac{\pi}{2}-3\varepsilon\right)\right| + \ln\left|\cos\left(-\tfrac{\pi}{2}+3\varepsilon\right)\right|}_{0} - \ln|\cos 0|\Big) = 0$$ ✓

2. $\int \operatorname{cotg}(-3x)\,\mathrm{d}x = -\frac{1}{3}\ln|\sin(-3x)| + c$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \left[-\frac{1}{3}\ln|\sin(-3x)|\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\frac{1}{3}\left(\ln|\sin(-\pi)| - \ln\left|\sin\left(-\frac{\pi}{2}\right)\right|\right) = -\frac{1}{3}(-\infty - 0) = \infty$$ ✓

$$\int_{0}^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \left[-\frac{1}{3}\ln|\sin(-3x)|\right]_{0}^{\frac{\pi}{6}} = -\frac{1}{3}\left(\ln\left|\sin\left(-\frac{\pi}{2}\right)\right| - \ln|\sin 0|\right) = -\frac{1}{3}\cdot(0-(-\infty)) = -\infty$$ ✓

$$\int_{-\frac{\pi}{6}}^{0} \operatorname{cotg}(-3x)\,\mathrm{d}x = \left[-\frac{1}{3}\ln|\sin(-3x)|\right]_{-\frac{\pi}{6}}^{0} = -\frac{1}{3}\left(\ln|\sin 0| - \ln\left|\sin\left(\frac{\pi}{2}\right)\right|\right) = -\frac{1}{3}(-\infty - 0) = \infty$$ ✓

$$\int_{-\frac{\pi}{3}}^{-\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \left[-\frac{1}{3}\ln|\sin(-3x)|\right]_{-\frac{\pi}{3}}^{-\frac{\pi}{6}} = -\frac{1}{3}\left(\ln\left|\sin\left(\frac{\pi}{2}\right)\right| - \ln|\sin(\pi)|\right) = -\frac{1}{3}(0-(-\infty)) = -\infty$$ ✓

$$\int_{0}^{\frac{\pi}{3}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \int_0^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cotg}(-3x)\,\mathrm{d}x = -\infty + \infty \Rightarrow \nexists$$ ✓

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \int_{-\frac{\pi}{6}}^{0} \operatorname{cotg}(-3x)\,\mathrm{d}x + \int_{0}^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \infty + (-\infty) \Rightarrow \nexists$$ ✓

$$v.p. \int_{-\frac{\pi}{3}}^{0} \operatorname{cotg}(-3x)\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{3}+\varepsilon}^{-\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x + \int_{-\frac{\pi}{6}}^{-\varepsilon} \operatorname{cotg}(-3x)\,\mathrm{d}x\right] = \lim_{\varepsilon\to 0^+} \frac{1}{3}\Big(\ln\left|\sin\tfrac{\pi}{2}\right| - \underbrace{\ln|\sin(\pi-3\varepsilon)| + \ln|\sin(3\varepsilon)|}_{0} - \underbrace{\ln\left|\sin\tfrac{\pi}{2}\right|}_{0}\Big) = 0$$ ✓

$$v.p. \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x = \lim_{\varepsilon\to 0^+}\left[\int_{-\frac{\pi}{6}}^{-\varepsilon} \operatorname{cotg}(-3x)\,\mathrm{d}x + \int_{\varepsilon}^{\frac{\pi}{6}} \operatorname{cotg}(-3x)\,\mathrm{d}x\right] = \lim_{\varepsilon\to 0^+} -\frac{1}{3}\Big(\overline{\ln|\sin(3\varepsilon)| - \ln\left|\sin\left(\tfrac{\pi}{2}\right)\right|}^{0} + \underbrace{\ln\left|\sin\left(\tfrac{\pi}{2}\right)\right|}_{0} - \overline{\ln|\sin(-3\varepsilon)|}^{0}\Big) = 0$$

—pokračovanie—

$$\dots 14|\,(x+14)\,\mathrm{d}x = + = \int (x+14)^3\,dx \;\left| \begin{matrix} x+14=t \\ dx=dt \end{matrix} \right| = \int t^3\,dt = \frac{t^4}{4}+c = \frac{(x+14)^4}{4}+c$$

$$-14 \qquad = -\int (x+14)^3\,dx \;\left| \begin{matrix} x+14=t \\ dx=dt \end{matrix} \right| = -\int t^3\,dt = -\frac{t^4}{4}+c = -\frac{(x+14)^4}{4}+c$$

$$\int_{-22}^{-6} |x+14|\,(x+14)^2\,\mathrm{d}x = \left[-\frac{(x+14)^4}{4}\right]_{-22}^{-14} + \left[\frac{(x+14)^4}{4}\right]_{-14}^{-6} = 0 + \frac{8^4}{4} + \frac{8^4}{4} - 0 = \frac{8^4}{2}$$

4. $\int \frac{\mathrm{d}x}{x^2-x-2} = \int \frac{dx}{(x-2)(x+1)} = \int \frac{1}{3(x-2)} - \frac{1}{3(x+1)}\,dx = \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + c$

$x^2-2x+x-2$

$-3 \quad -1 \quad 0 \quad 2 \quad 3$

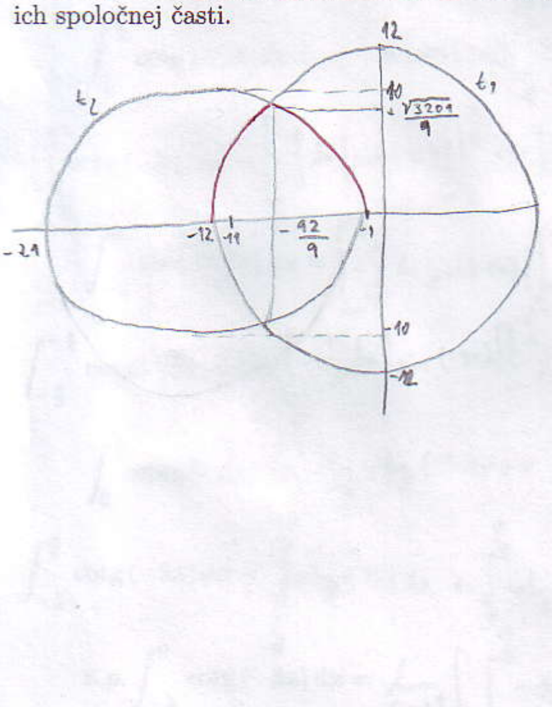
$$\int_{-3}^{5} \frac{\mathrm{d}x}{x^2-x-2} = \int_{-3}^{-1}(\dots)dx + \int_{-1}^{0}(\dots)dx + \int_{0}^{2}(\dots)dx + \int_{2}^{3}(\dots)dx = \frac{1}{3}\big(\ln 3 - \ln 0 - \ln 5 + \ln 2 + \ln 2 - \ln 1 - \ln 3 + \ln 0 + \ln 0 - \ln 3 - \ln 2 + \ln 1 + \ln 1 - \ln 4 - \ln 0 + \ln 3\big) \Rightarrow \nexists$$

$$\int_{-4}^{1} \frac{\mathrm{d}x}{x^2-x-2} = \int_{-4}^{-1}(\dots)dx + \int_{-1}^{1}(\dots)dx = \frac{1}{3}\big(\ln 3 - \ln 0 - \ln 6 + \ln 3 + \ln 1 - \ln 2 + \ln 3 + \ln 0\big) \Rightarrow \nexists$$

$$\int_{8}^{9} \frac{\mathrm{d}x}{x^2-x-2} = \left[\frac{1}{3}\ln|x-2| - \frac{1}{3}|x+1|\right]_8^9 = \frac{1}{3}\big[\ln 7 - \ln 10 - \ln 6 + \ln 9\big]$$

5. $\int_{-6}^{3} \frac{\mathrm{d}x}{x^2+3x+8} = \int_{-6}^{3} \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{\sqrt{23}}{2}\right)^2} = \left[\frac{2\sqrt{23}}{23}\,\mathrm{arctg}\left(\frac{\sqrt{23}}{23}(2x+1)\right)\right]_{-6}^{3} = \frac{2\sqrt{23}}{23}\left(\mathrm{arctg}\left(\frac{2\sqrt{23}}{23}\right) - \mathrm{arctg}\left(-\frac{\sqrt{23}}{23}\right)\right)$

6. Stredy dvoch guli s polomermi 10 cm a 12 cm sú od seba vzdialené 18 cm. Určte (pomocou integrálneho počtu) objem ich spoločnej časti.



$t_1:\; x^2+y^2 = 12^2$ $\qquad$ pr. $\left[-\frac{92}{9};\ \pm\frac{\sqrt{3201}}{9}\right]$

$t_2:\; (x+18)^2+y^2 = 10^2$

$f_1(x): \sqrt{12^2-x^2}, \quad x \in \left\langle -12, -\frac{92}{9}\right\rangle$ $\qquad$ $f_1'(x) = -\frac{x}{\sqrt{12^2-x^2}}$

$f_2(x): \sqrt{10^2-(x+18)^2}, \quad x \in \left\langle -\frac{92}{9}, -1\right\rangle$ $\qquad$ $f_2'(x) = \frac{x+18}{\sqrt{10^2-(x+18)^2}}$

$S = S_1 + S_2$

$$S_1 = 2\pi \int_{-12}^{-\frac{92}{9}} \sqrt{12^2-x^2}\cdot\sqrt{1+\frac{x^2}{12^2-x^2}}\,dx = 24\pi\,[x]_{-12}^{-\frac{92}{9}} = 24\pi\cdot\left(-\frac{92}{9}+12\right) = \frac{130}{3}\pi$$

$$S_2 = 2\pi \int_{-\frac{92}{9}}^{-1} \sqrt{10^2-(x+18)^2}\cdot\sqrt{1+\frac{(x+18)^2}{10^2-(x+18)^2}}\,dx = 24\pi\,[x]_{-\frac{92}{9}}^{-1} = 24\pi\left(-1+\frac{92}{9}\right) = \frac{664}{3}\pi$$

$$S = S_1 + S_2 = \left(\frac{130}{3} + \frac{664}{3}\right)\pi = \frac{794}{3}\pi$$