ZS za01-021 riešil(a): PETER MÁSIAR 51025

1. Dokážte, že $\sqrt{43}$ je iracionálne číslo.

$\sqrt{43} = p/q$ $\quad p, q$ sú nesúdeliteľné

$43 = \frac{p^2}{q^2} \quad / \cdot q^2$

$43q^2 = p^2 \Rightarrow 43 | p \Rightarrow p = 43k$

$43q^2 = 1849k^2$

$q^2 = 43k^2 \Rightarrow 43 | q$ - spor

2. Pomocou matematickej indukcie dokážte pre všetky $n \in N$ rovnosť $\sum_{i=0}^{n} \frac{1}{5^i} = \frac{5-5^{-n}}{4}$.

1) $n = 1$

$L = \sum_{i=0}^{1} \frac{1}{5^i} = 1 + \frac{1}{5} = \frac{6}{5}$

$P = \frac{5-5^{-n}}{4} = \frac{5-5^{-1}}{4} = \frac{5-\frac{1}{5}}{4} = \frac{\frac{24}{5}}{\frac{4}{1}} = \frac{24}{20} = \frac{6}{5};$

2) $\sum_{i=0}^{n+1} \frac{1}{5^i} = \sum_{i=0}^{n} \frac{1}{5^i} + \frac{1}{5^{n+1}} = \frac{5-5^{-n}}{4} + \frac{1}{5^{n+1}} = \frac{5-5^{-n}+4\cdot 5^{-n-1}}{4} = \frac{5-5\cdot 5^{-n-1}+4\cdot 5^{-n-1}}{4} = \frac{5-5^{-n-1}}{4} = \frac{5-5^{-(n+1)}}{4}$

3. Priamo dokážte rovnosť $\sum_{i=1}^{n} \frac{1}{(5i-3)(5i+2)} = \frac{n}{2(5n+2)}$.

$= \frac{1}{5} \sum_{i=1}^{n} \left(\frac{1}{5i-3} - \frac{1}{5i+2} \right) = \frac{1}{5} \left[\left(\frac{1}{2} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{12}\right) + \left(\frac{1}{12} - \frac{1}{17}\right) + \cdots + \left(\frac{1}{5n-3}\right) - \frac{1}{5n+2} \right] =$

$= \frac{1}{5}\left(\frac{1}{2} - \frac{1}{5n+2}\right) = \frac{1}{5} \cdot \left(\frac{5n+2-2}{10n+4}\right) = \frac{5n}{50n+20} = \frac{n}{10n+4} = \frac{n}{2(5n+2)};$

4. Matematickou indukciou dokážte rovnosť $\sum_{i=1}^{n} \frac{1}{(5i-3)(5i+2)} = \frac{n}{2(5n+2)}$.

1.) $n = 1$

$L = \sum_{i=1}^{n} \frac{1}{(5i-3)(5i+2)} = \frac{1}{(5-3)(5+2)} = \frac{1}{14}$

$P = \frac{n}{2(5n+2)} = \frac{1}{2(5+2)} = \frac{1}{14}$

2.) $\sum_{i=1}^{n+1} \frac{1}{(5i-3)(5i+2)} = \sum_{i=0}^{n} \frac{1}{(5i-3)(5i+2)} + \frac{1}{(5(n+1)-3)(5(n+1)+2)} = \frac{n}{2(5n+2)} + \frac{1}{(5n+2)(5n+7)} =$

$= \frac{n(5n+7)+2}{2(5n+2)(5n+7)} = \frac{n+1}{2(5(n+1)+2)};$

(2.) $1+\frac{1}{5}+\frac{1}{25}+\dots+\frac{1}{5^m}$

$F(1)\quad \frac{1}{5^k}=\frac{5-5^{-k}}{4} \Rightarrow \frac{1}{5}=\frac{5-\frac{1}{5}}{4} \Rightarrow \frac{1}{5}=\frac{1}{5}$ $\qquad \forall k\in N:\ F(k)=G(k) \Rightarrow F(k+1)=G(k+1)$

$F(k)=\frac{1}{5^k}=\frac{5-5^{-k}}{4}$

$\Downarrow$

$F(k+1)=1+\frac{1}{5}+\frac{1}{25}+\dots+\frac{1}{5^k}+\frac{1}{5^{k+1}}=\frac{5-5^{-k}}{4}+\frac{1}{5^{k+1}}=\frac{5^{k+2}-5+4}{4\cdot 5^{k+1}}=\frac{5^{k+2}-5^0}{4\cdot 5^{k+1}}=\frac{5(5^{k+1}-5^{-1})}{4\cdot 5^{k+1}}=$

$=\frac{5(5^{k+1}\cdot 5^{-1})\cdot 5^{-k-1}}{4}=\frac{5^{-k}(5^{k+1}-5^{-1})}{4}=\frac{5-5^{-k-1}}{4}=G(k+1)$

(4.) $\frac{1}{14}+\frac{1}{84}+\dots\frac{1}{(5m-3)(5m+2)}=\frac{m}{2(5m+2)}$

$F(1)=\frac{1}{14}=\frac{1}{14}$

$\forall k\in N:\ F(k)=G(k) \Rightarrow F(k+1)=G(k+1)$

$F(k)=\frac{1}{14}+\frac{1}{84}+\dots+\frac{1}{(5k-3)(5k+2)}=G(k)=\frac{k}{2(5k+2)}$

$\Downarrow$

$F(k+1)=\frac{1}{14}+\frac{1}{84}+\dots+\frac{1}{(5k-3)(5k+2)}+\frac{1}{(5(k+1)-3)(5(k+1)+2)}=F(k)+\frac{1}{(5k+2)(5k+7)}=$

$=\frac{k}{2(5k-2)}+\frac{1}{(5k+2)(5k+7)}=\frac{k(5k+7)+2}{2(5k+2)(5k+7)}=\frac{5k^2+7k+2}{2(5k+2)(5k+7)}=\frac{\cancel{(5k+2)}(k+1)}{2\cancel{(5k+2)}(5k+7)}=\frac{k+1}{2(5k+7)}=G(k+1)$