29.11.07 28.11.07. 44 OK

**s03-022** riešil(a): MIROSLAV DREVEŇÁK 5Z1012

**1.** Vyjadrite ako zlomok periodické číslo $4,1\overline{006} = 4,1006006006006\ldots$.

$q = 0,001$

$4,1006 = 4,1 + 0,0006 \cdot (1 + q + \ldots + q^n) = 4,1 + 0,0006 \cdot \frac{1-q^{n+1}}{1-q} \quad q \in (0,1) \Rightarrow \frac{1}{1-q}$

$4,1 + 0,0006 \cdot \frac{1}{1-q} = \frac{41}{10} + \frac{6}{10000} \cdot \frac{1}{0,999} = \frac{41}{10} + \frac{6}{9990} = \frac{409590 + 60}{99900} = \frac{409650}{99900} = 4,1\overline{006} =$

$= \frac{40965}{9990} = \frac{2731}{666}$

**2.** Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 4$, $a_{n+1} = \sqrt{29a_n - 54}$, $n \in N$.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$

$\Downarrow$

$a_n = a_{n+1}$

$a_n = \sqrt{29a_n - 54}$

$a_n^2 = 29a_n - 54$

$a_n^2 - 29a_n + 54 = 0$

$D = 29^2 - 4 \cdot 1 \cdot 54$

$D = 625$

$\sqrt{D} = 25$

$a_{n_{1,2}} = \frac{+29 \pm 25}{2}$

$a_{n_1} = +2 \qquad a_{n_2} = 27$

$a_1 = 4$

$a_2 = \sqrt{29 \cdot 4 - 54} = \sqrt{62} \Rightarrow \exists$ / ji bo postupnosť

$\lim_{n\to\infty} a_n = 27$

**3.** Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 3$, $a_{n+1} = \sqrt{29a_n - 54}$, $n \in N$.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$

$a_n = \sqrt{29a_n - 54} \quad /()^2$

$a_n^2 - 29a_n + 54 = 0 \Rightarrow a_n = 27 \Rightarrow \lim_{n\to\infty} a_n = 27$

**4.** Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2$, $a_{n+1} = \sqrt{29a_n - 54}$, $n \in N$.

$a_2 = \sqrt{29 \cdot 2 - 54} = \sqrt{4} = 2$

$a_3 = \sqrt{29 \cdot 2 - 54} = a_2 \Rightarrow a_2 = a_3 \Rightarrow \lim_{n\to\infty} a_n = 2$

$\vdots$

$a_n = a_{n+1} = 2$

**5.** Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 1$, $a_{n+1} = \sqrt{29a_n - 54}$, $n \in N$.

$a_2 = \sqrt{29 \cdot 1 - 54} = \sqrt{-25} \Rightarrow \nexists \Rightarrow \lim_{n\to\infty} a_n = \nexists$

$\Rightarrow$ postupnosť neexistuje nad R $\Rightarrow$ sa nedá vyriešiť

—pokračovanie—

 riešil(a):

6. $\lim\limits_{n\to\infty} \frac{n^3+n^4-4n^2+3}{-2n^2-7n^3+2} = \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim \frac{1+n-4\frac{1}{n}+\frac{3}{n^3}}{-2\frac{1}{n}-7+\frac{2}{n^3}} = \lim \frac{1+n}{-7} = \frac{\infty}{-7} = -\infty$

7. $\lim\limits_{n\to\infty} \frac{-2n^2-7n^3+2}{n^3+n^4-4n^2+3} = \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim \frac{-2\frac{1}{n^2}-7\frac{1}{n}+2\frac{1}{n^4}}{\frac{1}{n}+1-4\frac{1}{n^2}+3\frac{1}{n^4}} = \frac{0}{1} = 0$

8. $\lim\limits_{n\to\infty} \frac{n^3+n^4-4n^2+3}{-2n^2-7n^4+2} = \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim \frac{\frac{1}{n}+1-4\frac{1}{n^2}+3\frac{1}{n^4}}{-2\frac{1}{n^2}-7+2\frac{1}{n^4}} = -\frac{1}{7}$

9. $\lim\limits_{n\to\infty} \frac{-2n^2-7n^4+2}{n^3+n^4-4n^2+3} = \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim \frac{-2\frac{1}{n^2}-7+2\frac{1}{n^4}}{\frac{1}{n}+1-4\frac{1}{n^2}+3\frac{1}{n^4}} = -\frac{7}{1} = -7$

10. $\lim\limits_{n\to\infty} \frac{n^{\frac{1}{3}}+n^{\frac{1}{4}}-4n^{\frac{1}{2}}+3}{-2n^{\frac{1}{2}}-7n^{\frac{1}{3}}+2} = \cdot \frac{\frac{1}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{2}}}} = \lim \frac{\frac{1}{n^{\frac{1}{6}}}+\frac{1}{n^{\frac{1}{4}}}-4+\frac{3}{n^{\frac{1}{2}}}}{-2-7\frac{1}{n^{\frac{1}{6}}}+2\frac{1}{n^{\frac{1}{2}}}} = \frac{-4}{-2} = 2$

11. $\lim\limits_{n\to\infty} \frac{-2n^{\frac{1}{2}}-7n^{\frac{1}{3}}+2}{n^{\frac{1}{3}}+n^{\frac{1}{4}}-4n^{\frac{1}{2}}+3} = \cdot \frac{\frac{1}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{2}}}} = \lim \frac{-2-7\frac{1}{n^{\frac{1}{6}}}+\frac{2}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{6}}}+\frac{1}{n^{\frac{1}{4}}}-4+\frac{3}{n^{\frac{1}{2}}}} = \frac{-2}{-4} = \frac{1}{2}$

12. $\lim\limits_{n\to\infty} \frac{n^{\frac{1}{3}}+n^{\frac{1}{4}}-4n^{\frac{1}{2}}+3}{-2n^{\frac{1}{4}}-7n^{\frac{1}{3}}+2} = \cdot \frac{\frac{1}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{2}}}} = \lim \frac{\frac{1}{n^{\frac{1}{6}}}+\frac{1}{n^{\frac{1}{4}}}-4+\frac{3}{n^{\frac{1}{2}}}}{\frac{-2}{n^{\frac{1}{4}}}-\frac{7}{n^{\frac{1}{6}}}+\frac{2}{n^{\frac{1}{2}}}} = \lim \frac{-4}{0} = \cancel{0}$

$\cdot \frac{\frac{1}{n^{\frac{1}{3}}}}{\frac{1}{n^{\frac{1}{3}}}} = \lim \frac{1+\frac{1}{n^{\frac{1}{12}}}-4\frac{n^{\frac{1}{2}}}{n^{\frac{1}{3}}}+\frac{3}{n^{\frac{1}{3}}}}{\frac{-2}{n^{\frac{1}{12}}}-7+\frac{2}{n^{\frac{1}{3}}}} = \frac{-\infty}{-7} = \underline{\underline{\infty}}$

13. $\lim\limits_{n\to\infty} \frac{-2n^{\frac{1}{4}}-7n^{\frac{1}{3}}+2}{n^{\frac{1}{3}}+n^{\frac{1}{4}}-4n^{\frac{1}{2}}+3} = \cdot \frac{\frac{1}{n^{\frac{1}{3}}}}{\frac{1}{n^{\frac{1}{3}}}} = \lim \frac{\frac{-2}{n^{\frac{1}{12}}}-7+\frac{2}{n^{\frac{1}{3}}}}{1+\frac{1}{n^{\frac{1}{12}}}-\frac{4}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{3}}}} = \cancel{-7}$

$\cdot \frac{\frac{1}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{2}}}} = \lim \frac{\frac{-2}{n^{\frac{1}{4}}}-\frac{7}{n^{\frac{1}{6}}}+\frac{2}{n^{\frac{1}{2}}}}{\frac{1}{n^{\frac{1}{6}}}+\frac{1}{n^{\frac{1}{4}}}-4+\frac{3}{n^{\frac{1}{2}}}} = \frac{0}{-4} = \underline{\underline{0}}$

14. $\lim\limits_{n\to\infty} \frac{n^3+n^4-4n^2-3\cdot5^n}{-2n^2-7n^4-2\cdot5^n} = \lim\limits_{n\to\infty} \frac{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-\frac{4n^2}{5^n}-3\right)}{5^n\left(-\frac{2n^2}{5^n}-\frac{7n^4}{5^n}-2\right)} = \lim \frac{0+0-0-3}{-0-0-2} = \frac{3}{2}$

15. $\lim\limits_{n\to\infty} \frac{-2n^2-7n^4-2\cdot5^n}{n^3+n^4-4n^2-3\cdot5^n} = \lim\limits_{n\to\infty} \frac{5^n\left(-\frac{2n^2}{5^n}-\frac{7n^4}{5^n}-2\right)}{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-3\right)} = \frac{-0-0-2}{0+0-3} = \frac{2}{3}$

16. $\lim\limits_{n\to\infty} \frac{n^3+n^4-4n^2-3\cdot5^n}{-2n^2-7n^4-2\cdot5^{n+1}} = \lim\limits_{n\to\infty} \frac{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-\frac{4n^2}{5^n}-3\right)}{5\cdot5^n\left(-\frac{2n^2}{5^{n+1}}-\frac{7n^4}{5^{n+1}}-2\right)} = \lim \frac{-3}{-10} = \frac{3}{10}$

17. $\lim\limits_{n\to\infty} \frac{-2n^2-7n^4-2\cdot5^{n+1}}{n^3+n^4-4n^2-3\cdot5^n} = \lim\limits_{n\to\infty} \frac{5\cdot5^n\left(-\frac{2n^2}{5^{n+1}}-\frac{7n^4}{5^{n+1}}-2\right)}{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-\frac{4n^2}{5^n}-3\right)} = \lim \frac{5\cdot(-2)}{-3} = \frac{-10}{-3} = \frac{10}{3}$

18. $\lim\limits_{n\to\infty} \frac{n^3+n^4-4n^2-3\cdot5^n}{-2n^2-7n^3-2\cdot5^{n+1}} = \lim\limits_{n\to\infty} \frac{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-\frac{4n^2}{5^n}-3\right)}{5\cdot5^n\left(-\frac{2n^2}{5^n}-\frac{7n^3}{5^n}-2\right)} = \lim \frac{-3}{-10} = \frac{3}{10}$

19. $\lim\limits_{n\to\infty} \frac{-2n^2-7n^3-2\cdot5^{n+1}}{n^3+n^4-4n^2-3\cdot5^n} = \lim\limits_{n\to\infty} \frac{5\cdot5^n\left(-\frac{2n^2}{5^{n+1}}-\frac{7n^3}{5^{n+1}}-2\right)}{5^n\left(\frac{n^3}{5^n}+\frac{n^4}{5^n}-\frac{4n^2}{5^{n}}-3\right)} = \lim \frac{-10}{-3} = \frac{10}{3}$

—pokračovanie—

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riešil(a):

20. $\lim_{n\to\infty}\left[\sqrt{n^2-2n-1}-\sqrt{n^2+n+2}\right] = \frac{\sqrt{n^2-2n-1}+\sqrt{n^2+n+2}}{\sqrt{n^2-2n-1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{n^2-2n-1-n^2-n-2}{\ldots} = \lim_{n\to\infty}\frac{-3n-3}{\ldots}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = -\frac{3}{2}$

21. $\lim_{n\to\infty}\left[\sqrt[3]{n^3-2n-1}-\sqrt[3]{n^3+n+2}\right] = \frac{(\sqrt[3]{n^3-2n-1})^2+(\sqrt[3]{(n^3-2n-1)(n^3+n+2)})+(\sqrt[3]{n^3+n+2})^2}{-11-} = \lim_{n\to\infty}\frac{n^3-2n-1-n^3-n-2}{-11-} =$

$\lim_{n\to\infty}\frac{-3n-3}{n^2\sqrt[3]{1-\frac{2}{n^2}-\frac{1}{n^3}}+n^2\sqrt[3]{(1-\frac{2}{n^2}-\frac{1}{n^3})(1+\frac{1}{n^2}+\frac{2}{n^3})}+n^2\sqrt[3]{1+\frac{1}{n^2}+\frac{2}{n^3}}} \lim_{n\to\infty}\frac{-3n-3}{3n^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \frac{0}{3} = 0$

22. $\lim_{n\to\infty}\left[\sqrt{n^2-2n-1}-n+1\right] = \lim_{n\to\infty}\frac{\sqrt{n^2-2n-1}-\sqrt{n^2+1}}{1}\cdot\frac{\sqrt{n^2-2n-1}+\sqrt{n^2+1}}{\sqrt{n^2-2n-1}+\sqrt{n^2+1}} = \frac{n^2-2n-1-n^2-1}{\sqrt{n^2-2n-1}+\sqrt{n^2+1}} = \lim_{n\to\infty}\frac{-2n-2}{-11-}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \frac{-2}{2} = -1$

?!

23. $\lim_{n\to\infty}\left[\sqrt[3]{n^3-2n-1}-n+1\right] = \frac{(\sqrt[3]{n^3-2n-1})^2+\sqrt[3]{(n^3+1)(n^3-2n-1)}+(\sqrt[3]{n^3+1})^2}{-11-} = \lim_{n\to\infty}\frac{n^3-2n-1-n^3-1}{-11-} =$

?!

$= \lim_{n\to\infty}\frac{-2n-2}{n^2\sqrt[3]{1-\frac{2}{n^2}-\frac{1}{n^3}}+n^2\sqrt[3]{(1+\frac{1}{n^3})(1-\frac{2}{n^2}-\frac{1}{n^3})}+\sqrt[3]{1+\frac{1}{n^3}}} = \lim_{n\to\infty}\frac{-2n-2}{3n^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \frac{-\frac{2}{n}-\frac{1}{n^2}}{3} = 0$

24. $\lim_{n\to\infty}\left[\sqrt{n^2-2n-1}-\sqrt{n-1}\right] = n\sqrt{1-\frac{2}{n}-\frac{1}{n^2}} - n\cdot\sqrt{\frac{1}{n}-\frac{1}{n^2}} = \lim n\cdot\left(\sqrt{1-\frac{2}{n}-\frac{1}{n^2}}-\sqrt{\frac{1}{n}-\frac{1}{n^2}}\right) =$

$= n\left(\sqrt{1}-\sqrt{0}\right) = n = \infty$

25. $\lim_{n\to\infty}\left[\sqrt[3]{n^3-2n-1}-\sqrt[3]{n-1}\right] = \lim_{n\to\infty} n\sqrt[3]{1-\frac{2}{n^2}-\frac{1}{n^3}} - n\sqrt[3]{\frac{1}{n^2}-\frac{1}{n^3}} \lim_{n\to\infty} n\left(\sqrt[3]{1-\frac{2}{n^2}-\frac{1}{n^3}}-\sqrt[3]{\frac{1}{n^2}-\frac{1}{n^3}}\right)$

$= n\cdot\left(\sqrt[3]{1}-\sqrt[3]{0}\right) = n = \infty$

26. $\lim_{n\to\infty}\left[\frac{3n-4}{3n-7}\right]^3 = \lim_{n\to\infty}\left[\frac{3n-4-7+7}{3n-7}\right]^3 = \lim_{n\to\infty}\left[1+\frac{3}{3n-7}\right]^3 = 1$

27. $\lim_{n\to\infty}\left[\frac{3n-4}{3n-7}\right]^{n+3} = \lim_{n\to\infty}\left(\frac{3n-4-7+7}{3n-7}\right)^{n+3} = \lim_{n\to\infty}\left(1+\frac{3}{3n-7}\right)^{n+3} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n-7}{3}}\right)^{\frac{3n-7}{3}}\right]^{n+3\cdot\frac{3}{3n-7}} = e^{\lim\frac{3n+9}{3n-7}} = e$

28. $\lim_{n\to\infty}\left[\frac{3n-4}{3n-7}\right]^{n^2+3} = \lim_{n\to\infty}\left(\frac{3n-4-7+7}{3n-7}\right)^{n^2+3} = \lim_{n\to\infty}\left(1+\frac{3}{3n-7}\right)^{n^2+3} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n-7}{3}}\right)^{\frac{3n-7}{3}}\right]^{n^2+3\cdot\frac{3}{3n-7}} = e^{\lim\frac{3n^2+9}{3n-7}} = e^{\infty} = \infty$

29. $\lim_{n\to\infty}\left[\frac{3n^2-4}{3n^2-7}\right]^{n+3} = \lim_{n\to\infty}\left(\frac{3n^2-7-4+7}{3n^2-7}\right)^{n+3} = \lim_{n\to\infty}\left(1+\frac{3}{3n^2-7}\right)^{n+3} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n^2-7}{3}}\right)^{\frac{3n^2-7}{3}}\right]^{n+3\cdot\frac{3}{3n^2-7}} = e^{\lim\frac{3n+9}{3n^2-7}} = e^0 = 1$

30. $\lim_{n\to\infty}\left[\frac{3n^2-4}{3n^2-7}\right]^{n^2+3} = \lim_{n\to\infty}\left(\frac{3n^2-4-7+7}{3n^2-7}\right)^{n^2+3} = \lim_{n\to\infty}\left(1+\frac{3}{3n^2-7}\right)^{n^2+3} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n^2-7}{3}}\right)^{\frac{3n^2-7}{3}}\right]^{n^2+3\cdot\frac{3}{3n^2-7}} =$

$= e^{\lim_{n\to\infty}\frac{3n^2+9}{3n^2-7}} = e$

4b do **21.11.07**, 3b do **28.11.07**, 2b do **05.12.07**, 1b do **12.12.07**

28.11.2007

22. $\lim\limits_{n\to\infty}\left[\sqrt{n^2-2n-1}-n-1\right]=\dfrac{\sqrt{n^2-2n-1}+(n-1)}{\cdots}=\dfrac{n^2-2n-1-(n^2-2n+1)}{\sqrt{n^2-2n-1}+(n-1)}=\dfrac{-2}{n\left(\sqrt{1-0-0}+1-0\right)2n}=-\dfrac{1}{n}=0$

$\underline{\underline{0}}$

23. $\lim\limits_{n\to\infty}\left[\sqrt[3]{n^3-2n-1}-n+1\right]\cdot\dfrac{\sqrt[3]{(n^3-2n-1)^2}+\sqrt[3]{n^3-2n-1}\cdot(n-1)+(n-1)^2}{\sqrt[3]{(n^3-2n-1)^2}+\sqrt[3]{n^3-2n-1}\cdot(n-1)+(n-1)^2}=\dfrac{n^3-2n-1-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3-2n-1)^2}+\sqrt[3]{n^3-2n-1}\cdot(n-1)+(n-1)^2}$

$\left(\cancel{=\dfrac{3n^2-5}{n^2\cdot\sqrt[3]{n^3-2n-1}}}\right)$

$=\dfrac{3n^2-5n}{n^2\sqrt[3]{\left(1-\frac{2}{n^2}-\frac{1}{n^3}\right)^2}+n\sqrt[3]{1-\frac{2n}{n^2}-\frac{1}{n^3}}\cdot n\left(1-\frac{1}{n}\right)+\left(n^2-2n+1\right)}=$

$=\dfrac{n^2\left(3-\frac{5}{n}\right)}{n^2\sqrt[3]{\left(1-\frac{2}{n^2}-\frac{1}{n^3}\right)^2}+n^2\left(\sqrt[3]{1-\frac{2n}{n^2}-\frac{1}{n^3}}\cdot\left(1-\frac{1}{n}\right)\right)+n^2\left(1-\frac{2}{n}+\frac{1}{n^2}\right)}=$

$=\lim\dfrac{n^2\left(3-\frac{5}{n}\right)}{n^2\left(\sqrt[3]{\left(1-\frac{2}{n^2}-\frac{1}{n^3}\right)^2}+\sqrt[3]{1-\frac{2}{n^2}-\frac{1}{n^3}}\cdot\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}+\frac{1}{n^2}\right)\right)}=\dfrac{3}{\sqrt[3]{1^2}+\sqrt[3]{1}\cdot 1+1}=\dfrac{3}{1+1+1}=\underline{\underline{1}}$