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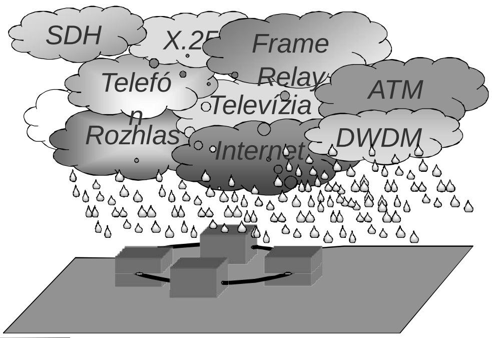
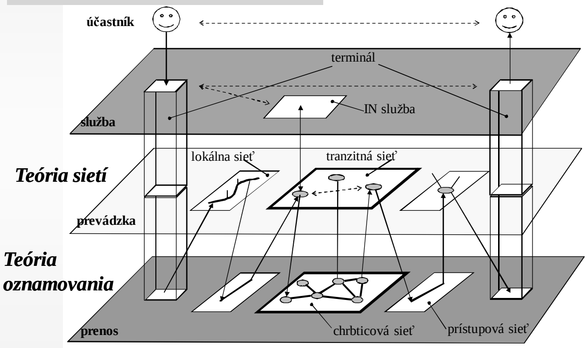
Obsah:
- prechod bázických signálov kanálom
- frekvenčný prenos kanála
- časovo invariantný kanál
- frekvenčný prenos časovo invariantného kanála

Všeobecný model siete



Základné vrstvy



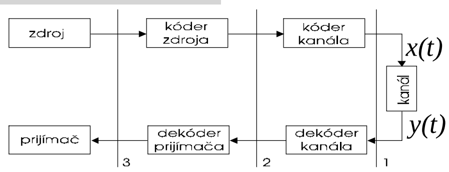
Vrstva prenosu

Hlavné úlohy: ??

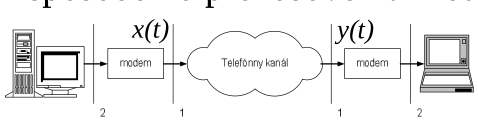
prenos jedného signálu

súčasný prenos signálov

Prenos bez skreslenia



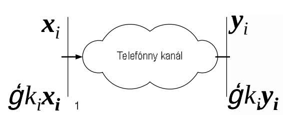
Prispôsobenie prenosovému médiu



Lineárny kanál

$$\forall \mathbf{x}_i \in \varphi, \quad i = 0,1,\ldots,N-1$$

$$\psi\left(\sum_{i=0}^{N-1} k_i.\mathbf{x}_i\right) = \sum_{i=0}^{N-1} k_i.\psi\left(\mathbf{x}_i\right) = \sum_{i=0}^{N-1} k_i.\mathbf{y}_i$$



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Lineárny kanál

$$\mathbf{x} = \sum_{i=0}^{N-1} x_i.\mathbf{e}_i \qquad \mathbf{y} = \sum_{i=0}^{N-1} y_i.\mathbf{e}_i$$

$$\mathbf{y} = \delta(\mathbf{x}) = \delta\left(\sum_{i=0}^{N-1} x_i.\mathbf{e}_i\right) = \sum_{i=0}^{N-1} x_i.\delta(\mathbf{e}_i)$$

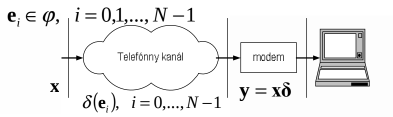
$$\mathbf{y} = \sum_{i=0}^{N-1} x_i.\boldsymbol{\delta}_i$$

$$\boldsymbol{\delta}_i = \left(\delta_{i0} \quad \ldots \quad \delta_{i,N-1}\right)$$

Lineárny kanál

$$\mathbf{y} = \delta(\mathbf{x}) = \sum_{i=0}^{N-1} x_i.\boldsymbol{\delta}_i$$

$$\mathbf{y} = \mathbf{x}\boldsymbol{\delta}$$



Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\ldots, N-1$$

$$\mathbf{x}_\mathbf{B} = \sum_{i=0}^{N-1} c_i.\mathbf{b}_i \qquad \mathbf{y}_\mathbf{B} = \sum_{i=0}^{N-1} k_i.\mathbf{b}_i$$

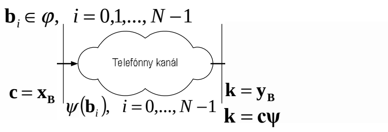
$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=0}^{N-1} c_i.\mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{k} = \sum_{i=0}^{N-1} c_i.\boldsymbol{\psi}_i \qquad \boldsymbol{\psi}_i = \left(\psi_{i0} \quad \ldots \quad \psi_{i,N-1}\right)$$

Lineárny kanál

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \sum_{i=0}^{N-1} c_i.\boldsymbol{\psi}_i$$

$$\mathbf{k} = \mathbf{c}\boldsymbol{\psi}$$



Lineárny kanál

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\ldots, N-1$$

$$\mathbf{y}_\mathbf{B} = \psi(\mathbf{x}_\mathbf{B}) = \psi\left(\sum_{i=0}^{N-1} c_i.\mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i.\psi(\mathbf{b}_i)$$

Existuje taká báza, že sa po prechode lineárnym kanálom nezmení?

Presnejšie, že

$$\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$$

Lineárny kanál

Požadujeme $\psi(\mathbf{b}_i) = \lambda_i \mathbf{b}_i$

$$\mathbf{b}_i \boldsymbol{\psi} = \lambda_i \mathbf{b}_i$$

Riešenie:

$\mathbf{b}_i$ sú vlastné vektory $\boldsymbol{\psi}$

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Vlastné vektory kanála

$$\mathbf{b}_i \in \varphi, \quad i = 0,1,\dots,N-1$$

$$\mathbf{x_B} = \sum_{i=0}^{N-1} c_i.\mathbf{b}_i \qquad \mathbf{y_B} = \sum_{i=0}^{N-1} k_i.\mathbf{b}_i$$

$$\mathbf{y_B} = \psi(\mathbf{x_B}) = \psi\left(\sum_{i=0}^{N-1} c_i.\mathbf{b}_i\right) = \sum_{i=0}^{N-1} c_i.\psi(\mathbf{b}_i)$$

$$\mathbf{y_B} = \sum_{i=0}^{N-1} c_i.\lambda_i\mathbf{b}_i$$

$$\boxed{k_i = \lambda_i c_i, i = 0,1,\dots,N-1}$$

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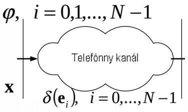
Vlastné vektory kanála

$\mathbf{e}_i \in \varphi, \quad i = 0,1,\dots,N-1$



$\mathbf{x}$ $\quad \delta(\mathbf{e}_i), \quad i = 0,\dots,N-1$ $\quad \mathbf{y} = \boldsymbol{\delta}\mathbf{x}$

$$\mathbf{e}_0 = (1,0), \quad \boldsymbol{\delta}_1 = (0,2;0,6) \qquad \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = (x_0, x_1).\begin{pmatrix} 0,2 & 0,6 \\ 0,6 & -0,3 \end{pmatrix}$$
$$\mathbf{e}_1 = (0,1), \quad \boldsymbol{\delta}_1 = (0,6;-0,3)$$

$$\mathbf{b}_0 = (0,6;-0,9) \qquad \mathbf{x} = (0,4;\ 0,7) \qquad \mathbf{x} = (0,23;\ 0,67)_\mathbf{B}$$

$$\mathbf{b}_1 = (0,9;\ 0,6) \qquad c_0 = \frac{(\mathbf{x},\mathbf{b}_0)}{(\mathbf{b}_0,\mathbf{b}_0)} = \frac{((0,4;0,7),(0,6;-0,9))}{((0,6;-0,9),(0,6;-0,9))} = 0,231$$

$$\boldsymbol{\lambda} = (-0,7;\ 0,6) \qquad c_1 = \frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)} = \frac{((0,4;0,7),(0,9;0,6))}{((0,9;0,6),(0,9;0,6))} = 0,667$$

$$k_0 = \lambda_0 c_0 = -0,162 \qquad \mathbf{y} = (0,16;\ 0,4)_\mathbf{B}$$
$$k_1 = \lambda_1 c_1 = 0,4$$

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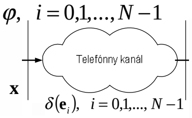
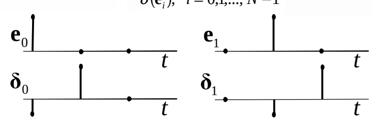
Časovo-invariantný kanál

$\mathbf{e}_i \in \varphi, \quad i = 0,1,\dots,N-1$



$\mathbf{x}$ $\quad \delta(\mathbf{e}_i), \quad i = 0,1,\dots,N-1$ $\quad \mathbf{y} = \boldsymbol{\delta}\mathbf{x}$



$$\mathbf{e}_0 = (1,0,0), \ \boldsymbol{\delta}_0 = (\delta_0, \delta_1, \delta_2)$$
$$\mathbf{e}_1 = (0,1,0), \ \boldsymbol{\delta}_1 = (\delta_2, \delta_0, \delta_1) \qquad (y_0, y_1, y_2) = (x_0, x_1, x_2).\begin{pmatrix} \delta_0 & \delta_1 & \delta_2 \\ \delta_2 & \delta_0 & \delta_1 \\ \delta_1 & \delta_2 & \delta_0 \end{pmatrix}$$
$$\mathbf{e}_2 = (0,0,1), \ \boldsymbol{\delta}_1 = (\delta_1, \delta_2, \delta_0)$$

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Časovo-invariantný kanál

$$(y_0, y_1, y_2) = (x_0, x_1, x_2).\begin{pmatrix} \delta_0 & \delta_1 & \delta_2 \\ \delta_2 & \delta_0 & \delta_1 \\ \delta_1 & \delta_2 & \delta_0 \end{pmatrix} \quad y_1 = x_0\delta_1 + x_1\delta_0 + x_2\delta_2$$

$$(y_0, y_1, \dots, y_{N-1}) = (x_0, x_1, \dots, x_{N-1}).\begin{pmatrix} \delta_0 & \delta_1 & \dots\delta_l & \dots & \delta_{N-1} \\ \delta_{N-1} & \delta_0 & \dots\delta_{l-1} & \dots & \delta_{N-2} \\ \delta_{N-k} & \delta_{N-k+1} & \dots\delta_{l-k} & \dots & \delta_{N-1-k} \\ \delta_1 & \delta_2 & \dots\delta_{l+1} & \dots & \delta_0 \end{pmatrix}$$

$$y_l = (x_0, x_1, \dots, x_{N-1}).\begin{pmatrix} \delta_l \\ \delta_{l-1} \\ \delta_{l-k} \\ \delta_{l+1} \end{pmatrix} = \sum_{k=0}^{N-1} x_k \delta_{l-k}$$

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Časovo-invariantný kanál

$$\mathbf{e}_0 = (1,0), \quad \boldsymbol{\delta}_0 = (\delta_0, \delta_1) \qquad (y_0, y_1) = (x_0, x_1).\begin{pmatrix} \delta_0 & \delta_1 \\ \delta_1 & \delta_0 \end{pmatrix}$$
$$\mathbf{e}_1 = (0,1), \quad \boldsymbol{\delta}_1 = (\delta_1, \delta_0)$$

$$\mathbf{b}(\boldsymbol{\delta} - \lambda\mathbf{E}) = \mathbf{0}$$

$$\begin{pmatrix} \delta_0 - \lambda & \delta_1 \\ \delta_1 & \delta_0 - \lambda \end{pmatrix} = \mathbf{0}$$

$$(\delta_0 - \lambda)^2 - \delta_1^2 = 0$$

$$\lambda_{0,1} = \delta_0 \pm \delta_1$$

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Časovo-invariantný kanál

$$\mathbf{b}(\boldsymbol{\psi}-\lambda\mathbf{E})=\mathbf{0}$$

$$(b_{00},b_{01}).\begin{pmatrix}\delta_0-(\delta_0+\delta_1) & \delta_1\\ \delta_1 & \delta_0-(\delta_0+\delta_1)\end{pmatrix}=(0,0)$$

$$(b_{00},b_{01}).\begin{pmatrix}-\delta_1 & \delta_1\\ \delta_1 & -\delta_1\end{pmatrix}=(0,0)$$

$$-\delta_1 b_{00}+\delta_1.b_{01}=0$$

$$b_{00}=b_{01}$$

$$\mathbf{b}_0=(1,1)$$

Časovo-invariantný kanál

$$\mathbf{b}(\boldsymbol{\psi}-\lambda\mathbf{E})=\mathbf{0}$$

$$(b_{21},b_{22}).\begin{pmatrix}\delta_1-(\delta_1-\delta_2) & \delta_2\\ \delta_2 & \delta_1-(\delta_1-\delta_2)\end{pmatrix}=(0,0)$$

$$(b_{21},b_{22}).\begin{pmatrix}\delta_2 & \delta_2\\ \delta_2 & \delta_2\end{pmatrix}=(0,0)$$

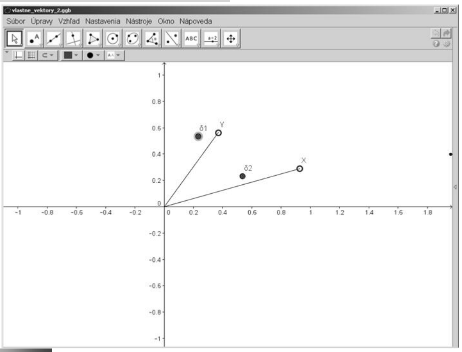
$$\delta_2 b_{11}+\delta_2.b_{12}=0$$

$$b_{11}=-b_{12}$$

$$\mathbf{b}_2=(1,-1)$$

Báza t-invariantného kanála

GeoGebra



Frekvenčný prenos t-invar. kanála

$$\mathbf{x}=(x_0,x_1)\quad \mathbf{b}_0=(1;1)\quad \boldsymbol{\lambda}=(\delta_0+\delta_1;\delta_0-\delta_1)$$

$$\mathbf{b}_1=(1;\ -1)$$

$$c_0=\frac{(\mathbf{x},\mathbf{b}_0)}{(\mathbf{b}_0,\mathbf{b}_0)}=\frac{((x_0;x_1),(1;1))}{((1;1),(1;1))}=\frac{x_0+x_1}{2}$$

$$c_1=\frac{(\mathbf{x},\mathbf{b}_1)}{(\mathbf{b}_1,\mathbf{b}_1)}=\frac{((x_0;x_1),(1;-1))}{((1;-1),(1;-1))}=\frac{x_0-x_1}{2}$$

$$\mathbf{x}=\left(\tfrac{1}{2}(x_0+x_1);\tfrac{1}{2}(x_0-x_1)\right)_{\mathbf{B}}$$

$$\mathbf{y}=\left(\tfrac{1}{2}(x_0+x_1).(\delta_0+\delta_1);\tfrac{1}{2}(x_0-x_1).(\delta_0-\delta_1)\right)_{\mathbf{B}}$$

Vlastné signály t-invariantného kanála

$\mathbf{e}_0=(1,0,\ldots,0),\qquad \boldsymbol{\delta}_0=(\delta_0,\delta_1,\ldots,\delta_{N-1})$

$\mathbf{e}_1=(0,1,\ldots,0)\qquad \boldsymbol{\delta}_1=(\delta_{N-1},\delta_0,\ldots,\delta_{N-2})$

$\ldots\qquad\qquad \ldots$

$\mathbf{e}_{N-1}=(0,0,\ldots,1)\qquad \boldsymbol{\delta}_{N-1}=(\delta_1,\delta_2,\ldots,\delta_0)$

$$\mathbf{b}(\boldsymbol{\delta}-\lambda\mathbf{E})=\mathbf{0}$$

$n=0,1,\ldots,N-1$

$$\boxed{\begin{aligned}\lambda_n&=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}\\ \mathbf{b}_n&=\left(e^{j\frac{2\pi}{N}n0},e^{j\frac{2\pi}{N}n1},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)\end{aligned}}$$

Frekvenčný prenos t-invar. kanála

$$\mathbf{x}=(x_0,x_2,\ldots,x_{N-1})\quad \boldsymbol{\delta}_0=(\delta_0,\delta_2,\ldots,\delta_{N-1})$$

$$\lambda_n=\sum_{k=0}^{N-1}\delta_k e^{-j\frac{2\pi}{N}nk}\qquad n=0,1,\ldots,N-1$$

$$\mathbf{b}_n=\left(e^{j\frac{2\pi}{N}n0},e^{j\frac{2\pi}{N}n1},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\mathbf{x}=(c_0,c_2,\ldots,c_{N-1})_{\mathbf{B}}\quad \mathbf{y}=(k_0,k_2,\ldots,k_{N-1})_{\mathbf{B}}$$

$$\boxed{\begin{aligned}c_n&=\frac{(\mathbf{x},\mathbf{b}_n)}{(\mathbf{b}_n,\mathbf{b}_n)}=\frac{1}{N}\sum_{k=0}^{N-1}x_k e^{-j\frac{2\pi}{N}nk},\quad c_n\in\mathbf{C}\\ k_n&=\lambda_n c_n\quad n=0,1,\ldots,N-1\end{aligned}}$$

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Vlastné signály – skúška správnosti

$$\mathbf{b}_n = \left(e^{j\frac{2\pi}{N}n0}, e^{j\frac{2\pi}{N}n1}, \ldots, e^{j\frac{2\pi}{N}n(N-1)}\right)$$

$$\lambda_n = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} \qquad n = 0,1,\ldots,N-1$$

$$\mathbf{b}_n \boldsymbol{\delta} = \lambda_n \mathbf{b}_n$$

$$\boxed{\left(1,\ldots,e^{j\frac{2\pi}{N}nk},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)} \begin{pmatrix} \delta_0, & \ldots, & \boxed{\begin{matrix}\delta_l, \\ \delta_{l+1},\end{matrix}} & \ldots, \delta_{N-1} \\ \delta_{N-1}, & \ldots, & & \ldots, \delta_0 \end{pmatrix} =$$

$$= \boxed{\sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk}} \left(1,\ldots,\boxed{e^{j\frac{2\pi}{N}nl},\ldots},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right)$$

Vlastné signály – skúška správnosti

$$\left(e^{j\frac{2\pi}{N}n0},\ldots,e^{j\frac{2\pi}{N}nk},\ldots,e^{j\frac{2\pi}{N}n(N-1)}\right) \begin{pmatrix} \delta_l \\ \vdots \\ \delta_{l-k} \\ \vdots \\ \delta_{l+1} \end{pmatrix} = \sum_{k=0}^{N-1} \delta_k e^{-j\frac{2\pi}{N}nk} e^{j\frac{2\pi}{N}nl}$$

$$\sum_{k=0}^{N-1} \delta_{l-k} e^{j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{j\frac{2\pi}{N}n(l-k)}$$

Vlastné signály – skúška správnosti

N=4, l=2 $\quad \sum_{k=0}^{N-1} \delta_{l-k} e^{j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{j\frac{2\pi}{N}n(l-k)}$

$$\delta_{2-0} e^{j\frac{2\pi}{N}n0} + \delta_{2-1} e^{j\frac{2\pi}{N}n1} + \delta_{2-2} e^{j\frac{2\pi}{N}n2} + \delta_{2-3} e^{j\frac{2\pi}{N}n3} =$$

$$= \delta_0 e^{j\frac{2\pi}{N}n(2-0)} + \delta_1 e^{j\frac{2\pi}{N}n(2-1)} + \delta_2 e^{j\frac{2\pi}{N}n(2-2)} + \delta_3 e^{j\frac{2\pi}{N}n(2-3)}$$

$$\delta_2 e^{j\frac{2\pi}{N}n0} + \delta_1 e^{j\frac{2\pi}{N}n1} + \delta_0 e^{j\frac{2\pi}{N}n2} + \delta_3 e^{j\frac{2\pi}{N}n3} =$$

$$= \delta_0 e^{j\frac{2\pi}{N}n2} + \delta_1 e^{j\frac{2\pi}{N}n1} + \delta_2 e^{j\frac{2\pi}{N}n0} + \delta_3 e^{j\frac{2\pi}{N}n3}$$

Vlastné signály – skúška správnosti

$$\sum_{k=0}^{N-1} \delta_{l-k} e^{j\frac{2\pi}{N}nk} = \sum_{k=0}^{N-1} \delta_k e^{j\frac{2\pi}{N}n(l-k)}$$

$$(l-k)+k=l \qquad k+(l-k)=l$$

Ďakujem za Vašu pozornosť