20. 12. 07

s10-022

riešil(a): MIROSLAV DREVEŇÁK

1. Vyšetrite priebeh funkcie $f: y = \frac{4x+1}{x+4}$, $x \in D(f)$ a načrtnite jej graf.

$D(f) = \mathbb{R} - \{-4\} = (-\infty, -4) \cup (-4, \infty)$

$H(f) =$

$\lim_{x\to\infty} \left[\frac{4x+1}{x+4} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{4+\frac{1}{x}}{1+\frac{4}{x}}\right] = \frac{4}{1} = 4$ ⇐ 1

$\lim_{x\to\infty} \left[\frac{1-4x}{4-x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{\frac{1}{x}-4}{\frac{4}{x}-1}\right] = \frac{-4}{-1} = 4$ ⇐ 1

$\lim_{x\to -4^+} \left[4 - \frac{15}{x+4}\right] = \left[4 - 15\,\frac{1}{0^+}\right] = 4 - 1500 = -\infty$

$\lim_{x\to -4^-} 4 - \frac{15}{x+4} = \left[4 - 15\,\frac{1}{0^-}\right] = 4 + 1500 = \infty$

$f'(x) = \frac{4(x+4) - (4x+1)}{(x+4)^2} = \frac{15}{(x+4)^2}$

$\frac{4x+1}{x+4}$ – lin lom funkcia ⇒

$x = -\frac{d}{c} = -\frac{4}{1} = -4$ ⇐ 1 $\quad y = \frac{a}{c} = \frac{4}{1} = 4$ ⇐ 1

NB $x=0 \quad y = \frac{1}{4} \quad [0, \frac{1}{4}]$

$y=0 \quad x = -\frac{1}{4} \quad [-\frac{1}{4}, 0]$

$f''(x) = \frac{-15(x+4)^2}{(x+4)^4} = \frac{-15x^2 - 114x - 240}{(x+4)^4}$

$f''(x) = \left[\frac{15}{(x+4)^2}\right]' = \frac{0 \cdot (x+4)^2 - 15 \cdot 2(x+4)\cdot 1}{(x+4)^4} =$

$f^4 = ? \quad = \frac{-30x - 120}{(x+4)^4} = \frac{-30(x+4)}{(x+4)^4} =$

$= -\frac{30}{(x+4)^3}$

stacionárne body nemá, lebo $f'(x) \neq 0$

$f'(-5) = \frac{15}{1} = 15 > 0 \Rightarrow \nearrow$

$f'(-3) = \frac{15}{1^2} = 15 > 0 \Rightarrow \nearrow$

IB: $-15x^2 - 114x - 240 = 0$

D: $114^2 - 4(-15)(-240) = -1404 < 0 \Rightarrow$ NEMÁ IB

Výsledky:

$D(f) = \mathbb{R} - \{-4\}$ $\quad (-\infty, -4) \cup (-4, \infty)$

Nulové body: ~~$[0, \frac{1}{4}]$~~, $[-\frac{1}{4}, 0]$

Párnosť, nepárnosť, periodickosť: ani párna ani nepárna, lebo $x = 4 \in D(f) \wedge x = -4 \notin D(f)$

Spojitosť, body nespojitosti: $x = -4$

Kladnosť, zápornosť: kladná $(-\infty, -4) \cup (-\frac{1}{4}, \infty)$; záporná $(-4, -\frac{1}{4})$

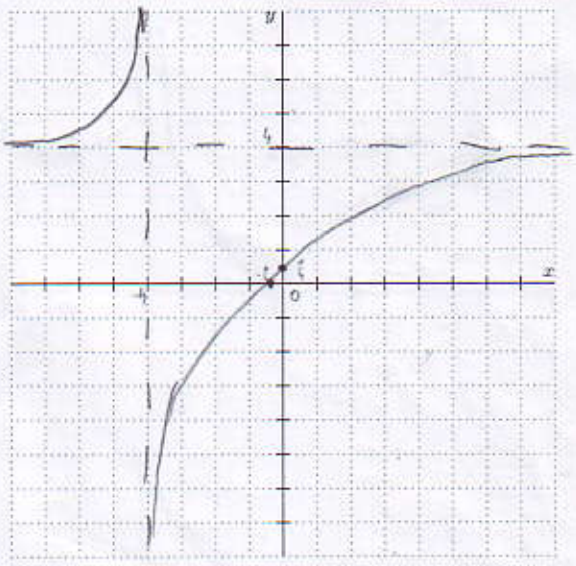
Asymptoty: $x = -4 \quad y = 4$

$f'(x) = \frac{15}{(x+4)^2}$

Extrémy, monotónnosť: extrém nemá, lebo nemá IB, rastúca

$f''(x) = \frac{-15x^2 - 114x - 240}{(x+4)^4} \quad -\frac{30}{(x+4)^3}$

Inflexné body, konvexnosť: IB nemá, ⇒ konvexnosť nemeníme



—pokračovanie—

2. Vyšetrite priebeh funkcie $f: y = \frac{|4x+1|}{|x+4|}$, $x \in D(f)$ a načrtnite jej graf.

$|4x+1|$ $4x+1=0$ $x=-\frac{1}{4}$ $x+4=0$ $x=-4$

$(-\infty,-4) \cup (-4,-\frac{1}{4}) \cup \langle -\frac{1}{4},\infty)$

$\frac{-(4x+1)}{-(x+4)}$ $\frac{-(4x+1)}{x+4}$ $\frac{4x+1}{x+4}$

$D(f) = R - \{-4\}$ $D(f) = R - \{-4\}$ $D(f) = R - \{-4\}$

NB $x=0$ $y=\frac{1}{4}$ $y=0$ $x=-\frac{1}{4}$ $[0,\frac{1}{4}]$ $[-\frac{1}{4},0]$

NB $x=0$ $y=-\frac{1}{4}$ $y=0$ $x=-\frac{1}{4}$ $[0,-\frac{1}{4}]$ $[-\frac{1}{4},0]$

NB $x=0$ $y=\frac{1}{4}$ $y=0$ $x=-\frac{1}{4}$ $[0,\frac{1}{4}]$ $(-\frac{1}{4},0]$

$\lim_{x\to\pm\infty} \frac{-4x-1}{-x+4} = 4$ $\lim_{x\to\pm\infty} \frac{-4x+1}{x+4} = -4$ $\lim_{x\to\pm\infty} \frac{4x+1}{x+4} = 4$

$\lim_{x\to\pm\infty} \frac{4x+1}{x+4} = 4$ $\lim_{x\to\pm\infty} \frac{4x+1}{-x+4} = -4$ $\lim_{x\to\pm\infty} \frac{-4x-1}{-x+4} = 4$

$f'(x) = \left[\frac{-4x-1}{-x-1}\right]' = \frac{-4(-x-4)-(-4x-1)\cdot(-1)}{(-x-4)^2} = \frac{4x+16-4x-1}{(-x-4)^2} = \frac{15}{(-x-4)^2}$

$f''(x) = \left[\frac{15}{(-x-4)^2}\right]' = 15(-x-4)^{-2} = -30(-x-4)^{-3}$

$f'(x) = \left[\frac{-4x-1}{x+4}\right]' = \frac{-4(x+4)-(-4x-1)(1)}{(x+4)^2} = \frac{-4x-16+4x+1}{(x+4)^2} = \frac{-15}{(x+4)^2}$

$f''(x) = -30(-x-4)^{-3}$

$f'(x) = \left[\frac{4x+1}{x+4}\right]' = \frac{4x+16-4x-1}{(x+4)^2} = \frac{15}{(x+4)^2}$

$f''(x) = -30(x+4)^{-3}$

$x = -\frac{d}{c} = -\frac{4}{1} = -4$ $y = \frac{a}{c} = \frac{-4}{1} = -4$

$x = -\frac{d}{c} = -\frac{4}{1} = -4$ $y = \frac{a}{c} = \frac{4}{1} = 4$

$f'(x) \neq 0$ nemá SB

$f'(-5) = \frac{15}{(-5-4)^2} = 15 > 0$ ↗

$f''(-5) = \frac{15}{(-5-4)^2} = 15 > 0$ ↗

IB: 0 ⇒ nemá IB

$f'(x) \neq 0$ nemá SB

$f'(-5) = -\frac{15}{1} = -15 < 0$ ↘

$f'(-3) = \frac{-15}{1} = -15 < 0$ ↘

IB: 0 ⇒ nemá IB

$f'(x) \neq 0$ nemá SB

$f'(-5) = \frac{15}{1} = 15 > 0$ ↗

$f'(-3) = \frac{15}{1} = 15 > 0$ ↗

IB: 0 ⇒ nemá IB

OK

Výsledky:

$D(f) = R - \{-4\}$ $(-\infty,-4) \cup (-4,\infty)$

Nulové body: $[0,\frac{1}{4}]$, $[-\frac{1}{4},0]$

Párnosť, nepárnosť, periodickosť: ani párna ani nepárna ani periodická

Spojitosť, body nespojitosti: $x=-4$ nespojité $x=-4$

Kladnosť, zápornosť: nespojité $x=-\frac{1}{4}$

$y=0$ pre $x=-\frac{1}{4}$

Asymptoty: $y=4$ pre $x \in (-\infty,-4) \cup (-\frac{1}{4},0)$

$y=-4$

$f'(x) = \frac{15}{(-x-4)^2}$

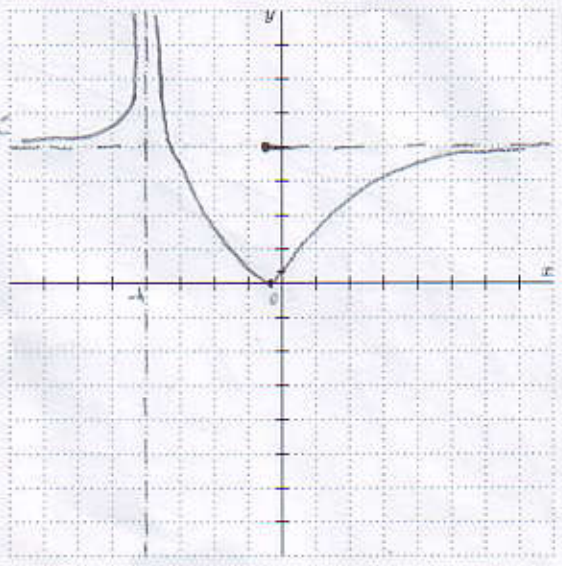
Extrémy, monotónnosť: $x=-\frac{1}{4}$ minimum; $(-\infty,-4)$ rastúca

$(-4,-\frac{1}{4})$ klesajúca

$(-\frac{1}{4},\infty)$ rastúca

$f''(x) = -30(-x-4)^{-3}$

Inflexné body, konvexnosť: nemá IB



—pokračovanie—

3. Vyšetrite priebeh funkcie f: $y = \frac{4x^2+1}{x^2+4}$, $x \in D(f)$ a načrtnite jej graf.

$D(f) = R$ NB $x=0$ $y = \frac{1}{4}$ $[0, \frac{1}{4}]$

$y=0$ $0 = \frac{4x^2+1}{x^2+4}$ $4x^2+1=0$ $4x^2=-1$ $x^2=-\frac{1}{4} \Rightarrow \nexists$ $[\ ,0]$

$\lim_{x\to\infty}\left[\frac{4x^2+1}{x^2+4}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{4+\frac{1}{x^2}}{1+\frac{4}{x^2}}\right] = \frac{4}{1} = 4$

$\lim_{x\to-\infty}\left[\frac{1-4x^2}{4-x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right] = \frac{-4}{-1} = 4$

$f'(x) = \frac{(4x^2+1)'(x^2+4)-(4x^2+1)(x^2+4)'}{(x^2+4)^2} = \frac{(8x)(x^2+4)-(4x^2+1)(2x)}{(x^2+4)^2} = \frac{8x^3+32x-8x^3+2x}{(x^2+4)^2} = \frac{34x}{(x^2+4)^2}$

$f''(x) = \left(\frac{34x}{(x^2+4)^2}\right)' = \frac{34(x^2+4)^2-34x(4x^2+16)}{(x^2+8x^2+16)^2} = \frac{34x^4+272x^2+544-136x^4-544x^2}{(x^4+8x^2+16)^2} = \frac{34x^4-272x^2-544x^2+544}{(x^4+8x^2+16)^2}$

$IB = -90x^4-240x^2+480=0$

$x^2=a \quad -3a^2-8a+16=0$

$f(x) = \frac{4x^2+1}{x^2+4}$ $f(-x) = \frac{4x^2+1}{x^2+4}$ } párna

$f'(x)=0$; $30x=0 \Rightarrow x=0$ $SB[0,\frac{1}{4}]$

$f'(-1) = \frac{-30}{1+8+16} = -\frac{30}{25} < 0 \Rightarrow \searrow$

$f''(0) = \frac{544}{162} > 0 \Rightarrow$ Lok. min.

$f'(1) = \frac{30}{1+8+16} = \frac{30}{25} > 0 \Rightarrow \nearrow$

$f'(x) = \frac{(4x^2+1)'(x^2+4)-(4x^2+1)(x^2+4)'}{(x^2+4)^2} = \frac{8x(x^2+4)-2x(4x^2+1)}{(x^2+4)^2} =$ $f', f'' = ?$

$= \frac{8x^3+32x-8x^3-2x}{(x^2+4)^2} = \frac{30x}{(x^2+4)^2}$

$f''(x) = \left(\frac{30x}{(x^2+4)^2}\right)' = \frac{30(x^2+4)^2-30x\cdot 2(x^2+4)\cdot 2x}{(x^2+4)^4} = \frac{30x^4+240x^2+480-(120x^4+480x^2)}{(x^2+4)^4} = \frac{-90x^4-240x^2+480}{(x^2+4)^4}$

Výsledky:

$D(f) = R$ Nulové body: $[0, \frac{1}{4}]$ ~~$\nexists$~~

Párnosť, nepárnosť, periodickosť: f je párna

Spojitosť, body nespojitosti: f je spojitá

Kladnosť, zápornosť: všade $(-\infty, \infty)$

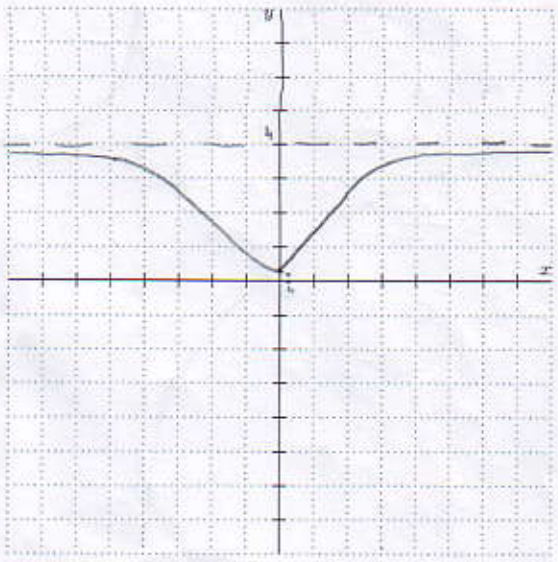
Asymptoty: $y=4$

$f'(x) = \frac{34x}{(x^2+4)^2}$ $\frac{30x}{(x^2+4)^2}$ $(-\infty,0)$ klesajúca

Extrémy, monotónnosť: lokálne minimum $[0,\frac{1}{4}]$; $(0,\infty)$ rastúca

$f''(x) = \frac{34x^4-272x^2-544x^2+544}{(x^4+8x^2+16)^2}$ $\frac{-90x^4-240x^2+480}{(x^2+4)^4}$

Inflexné body, konvexnosť: IB nemá; f je konvexná



4b do **28.12.07**, 3b do **04.01.08**, 2b do **11.01.08**, 1b do **18.01.08**