03-12-07 30.11.07

s05-022 3 riešil(a): MIROSLAV DREVENÁK 521012

1. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\ldots(f))))$, $n \in N$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = -5 + 3x$, $x \in R$.

$f_2(x) =$ $f_3(x) =$ $f_4(x) =$

$f_n(x) = 3^n x - 5\sum_{i=0}^{n-1} 3^n$ <? $= -5+3(-5+3(-5+3)\ldots(-5+3x))) = 3^n x - 5\frac{3^n-1}{2}$

2. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, ..., $f_n = f(f(f(\ldots(f))))$, $n \in N$, ak funkcia $f_1 = f$ je definovaná predpisom $f(x) = -5 - 3x$, $x \in R$.

$f_2(x) =$ $f_3(x) =$ $f_4(x) =$

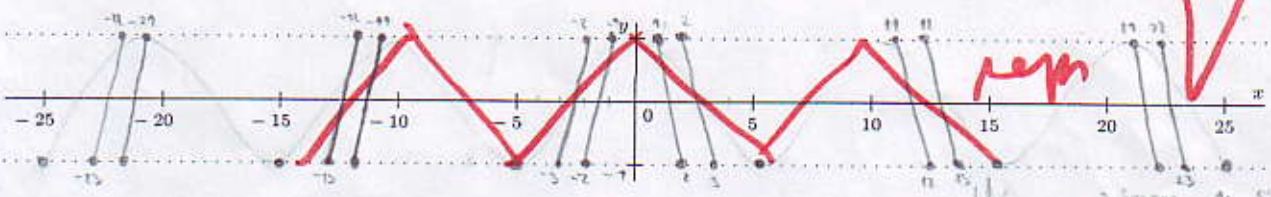
$f_n(x) = -3^n x - 5\sum_{i=0}^{n-1} (-3)^n$ <? $= -5-3(-5-3(-5-3)\ldots(-5-3x))) = (-3)^n x - 5\cdot\frac{(-3)^n-1}{-4}$

3. Nájdite zložené funkcie $f_2 = f(f)$, $f_3 = f(f(f))$, $f_4 = f(f(f(f)))$ a inverznú funkciu f^{-1}, ak funkcia f je definovaná predpisom $f(x) = \frac{x-2}{x-3}$, $x \in R - \{\frac{1}{3}\}$.

$f^{-1}(x) =$ $f_2(x) =$ $f_3(x) =$

$f_4(x) =$

4. Zostrojte periodickú funkciu $y = f(x)$ s primitívnou periódou 10 a načrtnite jej graf tak, aby bola párna, klesajúca na intervale $\langle 1; 2\rangle$, rastúca na intervale $\langle -3; -2\rangle$ a aby $f(5) = -1$.



5. Nájdite s presnosťou $\varepsilon = 0.01$ kladné riešenie rovnice $5\,e^x - 18x^5 - 5 = 0$.

Výpočet počtu krokov n: $0{,}01 = 10^{-2} = \frac{1}{100} \Rightarrow \frac{1}{2^n} < \frac{1}{100} \Rightarrow 100 < 2^n$ $n = 7$ lebo $2^7 = 128$ ..... stačí 7 krokov

n	a_n	x_{n+1}	b_n	$f(x_{n+1})$
0	0	0,5	1	+ 3,68
1	0,5	0,875	1	+ 1,3135
2	0,875	0,8125	1	− 2,2379
3	0,875	0,81125	0,875	− 0,1059
4	0,875	0,78125	0,81125	+ 0,6823
5	0,78125	0,796875	0,81125	+ 0,3090

n	a_n	x_{n+1}	b_n	$f(x_{n+1})$
6	0,796875	0,8046875	0,8125	+ 0,106
7	0,8046875	0,808593[illegible]	0,8125	
8				
9				
10				
11				

Približné riešenie $x_{n+1} \approx$ 0,808593 75

4b do **03.12.07**, 3b do **10.12.07**, 2b do **17.12.07**, 1b do **24.12.07**

/gram.colu

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$$f_1(x) = \frac{-x-2}{x-3}$$

$$f_2(x) = f\left(\frac{-x-2}{x-3}\right) = \frac{-\left(\frac{-x-2}{x-3}\right)-2}{\frac{-x-2}{x-3}-3} = \frac{\frac{x+2}{-3+x}-2}{\frac{-x-2-3x+9}{x-3}} = \frac{\frac{x+2+6-2x}{-3+x}}{\frac{-4x+7}{x-3}} =$$

$$= \frac{\frac{8-x}{-3+x}}{\frac{-4x+7}{x-3}} = \frac{8-x}{\cancel{x-3}} \cdot \frac{\cancel{x-3}}{7-4x} = \frac{8-x}{7-4x}$$

$$f_3(x) = f\left(f\left(\frac{-x-2}{x-3}\right)\right) = f\left(\frac{8-x}{7-4x}\right) = \frac{-\left(\frac{8-x}{7-4x}\right)-2}{\frac{8-x}{7-4x}-3} = \frac{\frac{-8+8x-14}{-4x+7}}{\frac{8-x-21+12x}{7-4x}} = \frac{-22+9x}{11x-13}$$

$$f_4(x) = f\left(f\left(f\left(\frac{-x-2}{x-3}\right)\right)\right) = f\left(\frac{9x-22}{11x-13}\right) = \frac{+\left(\frac{-9x+22}{11x-13}\right)-2}{\frac{9x-22}{11x-13}-3} = \frac{\frac{48x-44-99+22}{11x-13}}{\frac{-27x+66+9x-22}{11x-13}} =$$

$$= \frac{\frac{9x+22}{11x-13}}{\frac{-18x+44}{11x-13}} = \frac{9x+22}{44-18x} = \frac{-9x+22-22x+26}{11x-13} = \frac{48-31x}{9x-22-33x+39} = \frac{48-31x}{17-24x}$$