Teória sietí 5

Úloha vrstvy prevádzky?

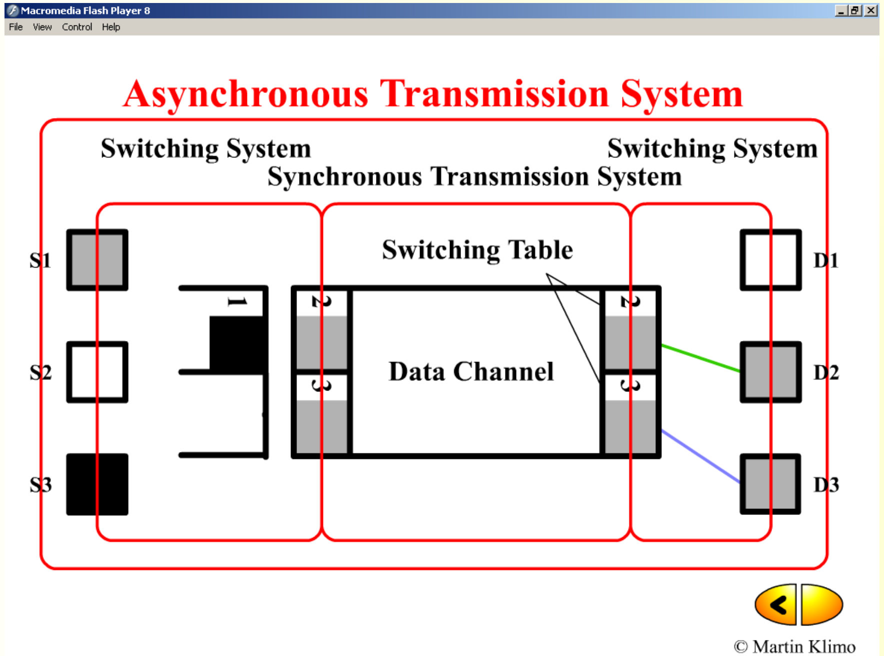
Nájsť kompromis medzi kvalitou a efektívnosťou siete.

1. **z ekonomických dôvodov musí byť kapacita siete menšia než sú možné požiadavky na prenos**
2. **požiadavky na prenos vznikajú náhodne**

Riešenie ?

Policing – odmietnuť záťaž prevyšujúcu kapacitu siete

Shaping – odložiť záťaž prevyšujúcu kapacitu siete na neskôr



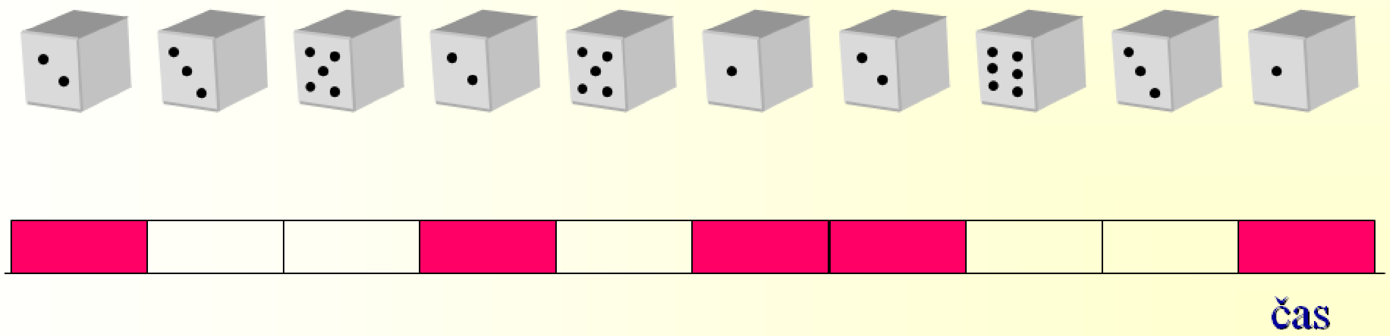
Prvá úloha

Ako popísať proces, ktorý sa v sieti odohráva?

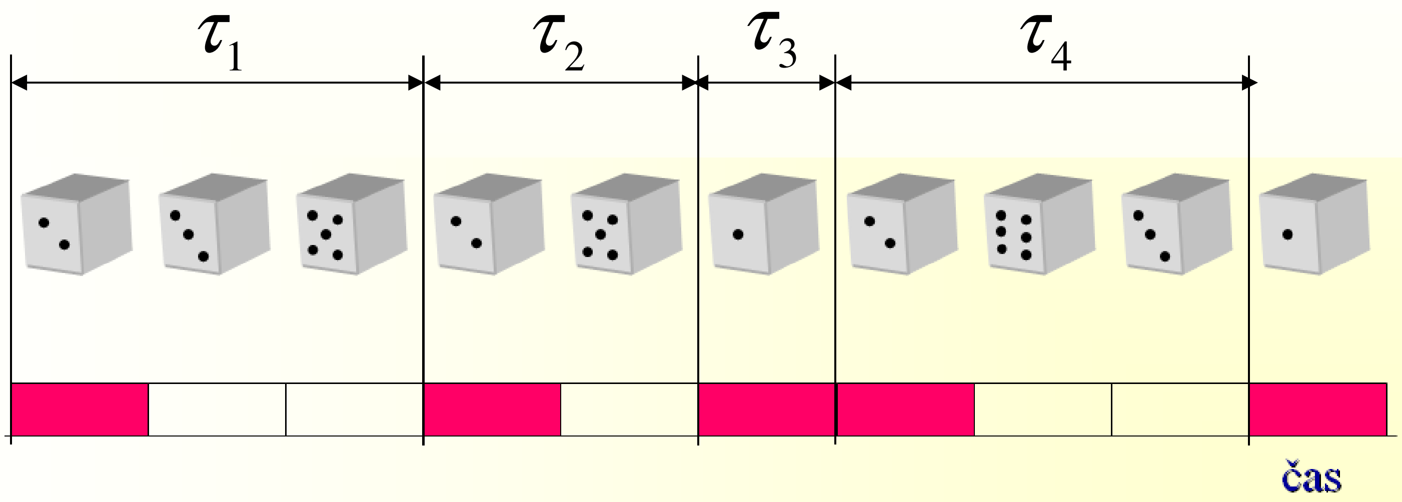
Vlastnosti procesu

1. **javy sú nezávislé**
2. **javy nastávajú s rovnakou pravdepodobnosťou**

Bernoulliho proces



Bernoulliho proces



rozdelenie pravdepodobnosti

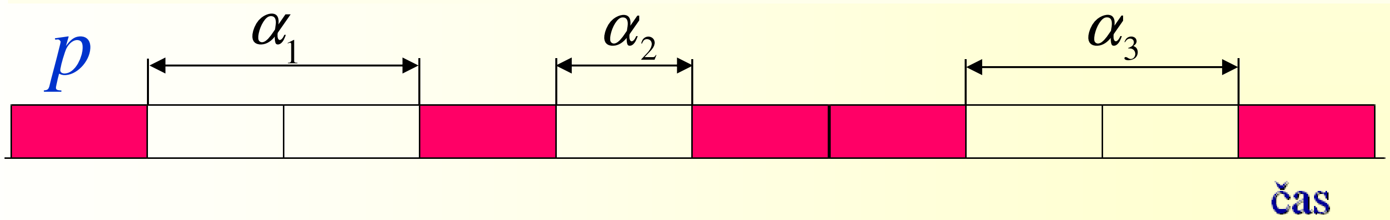
$$P\{\tau_k = n\} = P\{\tau = n\} = p(1-p)^{n-1}$$

$$\forall k,\ n = 1,2,\ldots$$

Proces nie Bernoulliho

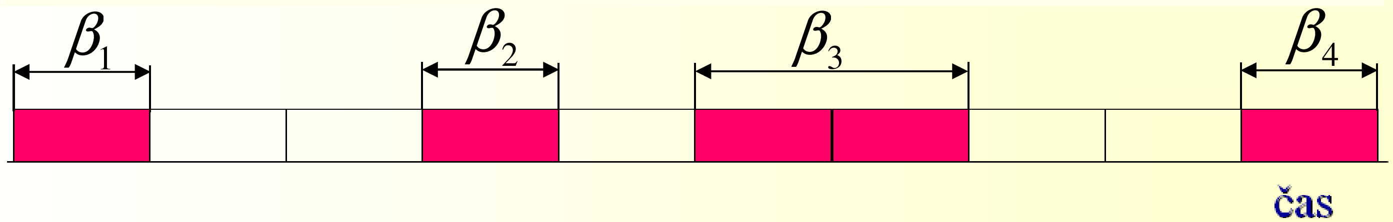
Rozdelenie dĺžok intervalov medzi rámcami

$$P(\alpha = n) = (1-p)^n p, \qquad n = 0,1,2,\ldots$$

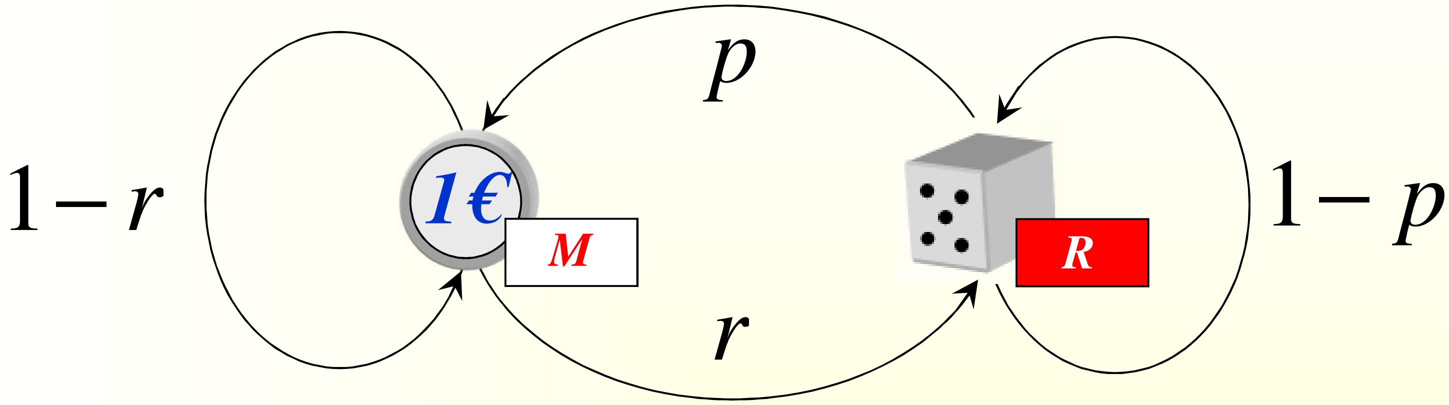


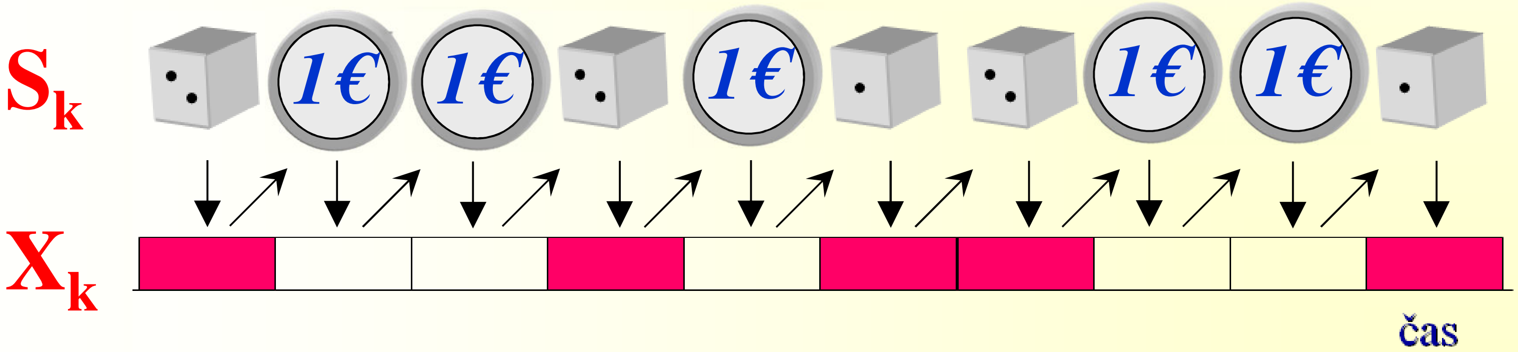
Rozdelenie dĺžok zhlukov rámcov

$$P(\beta = n) = r^n (1-r), \qquad n = 0,1,2,\ldots$$



Stav procesu





Zovšeobecnenie

Proces so stavmi $\{S_1,\ldots,S_n\}$

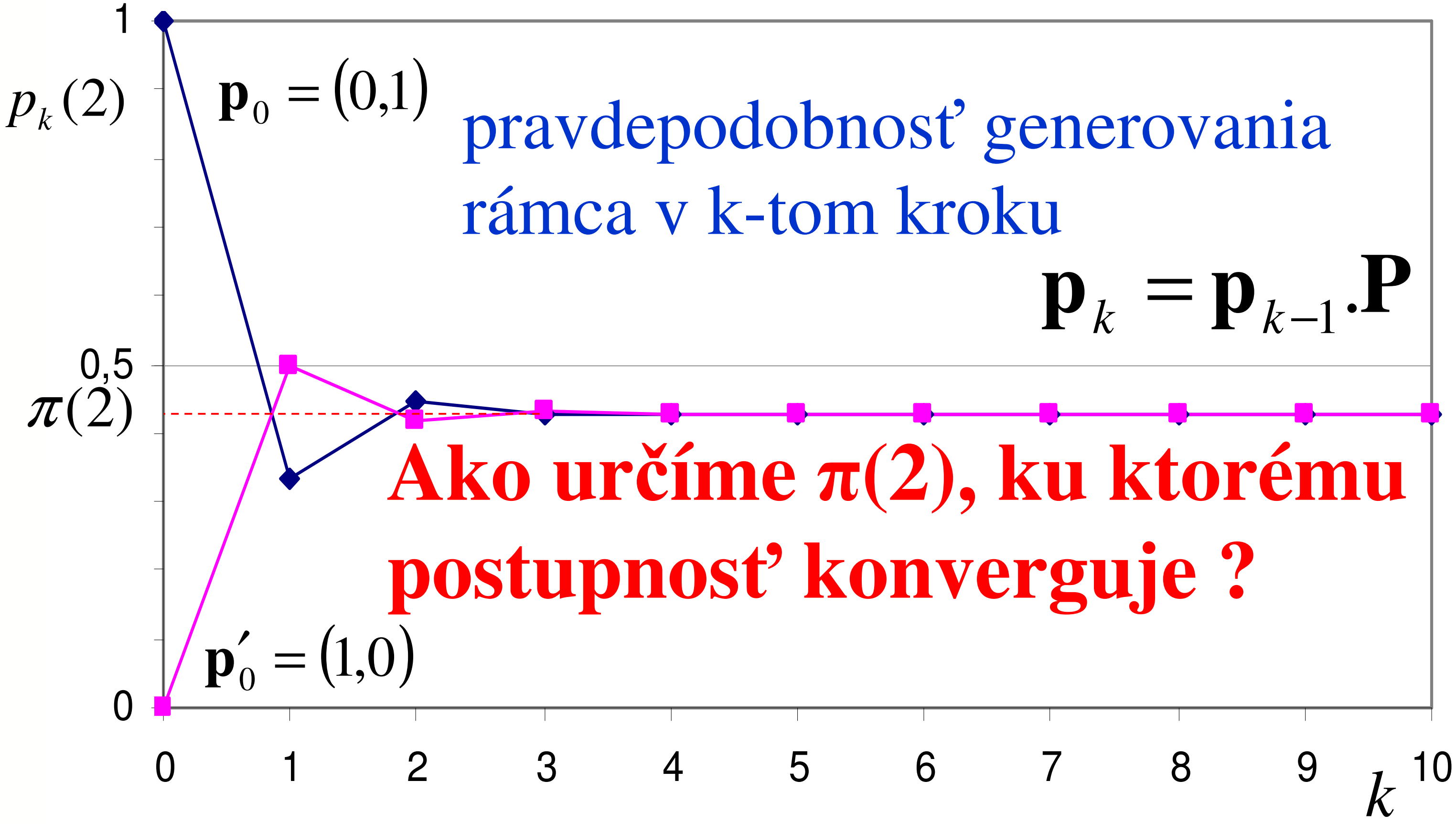
počiatočné rozdelenie pravdepodobnosti

$$\mathbf{p}_0 = (p_0(1),\ldots, p_0(n))$$

matica pravdepodobností prechodov

$$\mathbf{P} = \begin{pmatrix} p_{1,1} & \cdots & p_{1,n} \\ \cdots & \cdots & \cdots \\ p_{n,1} & \cdots & p_{n,n} \end{pmatrix}$$

Rozdelenie pravdepodobnosti stavov



$$\mathbf{p}_k = \mathbf{p}_{k-1}$$

Invariantné rozdelenie

Invariantné rozdelenie pravdepodobnosti

$$\boldsymbol{\pi} = (\pi(1),\dots,\pi(n))$$

procesu so stavmi $\{S_1,\dots,S_n\}$ a maticou pravdepodob-nosti prechodov

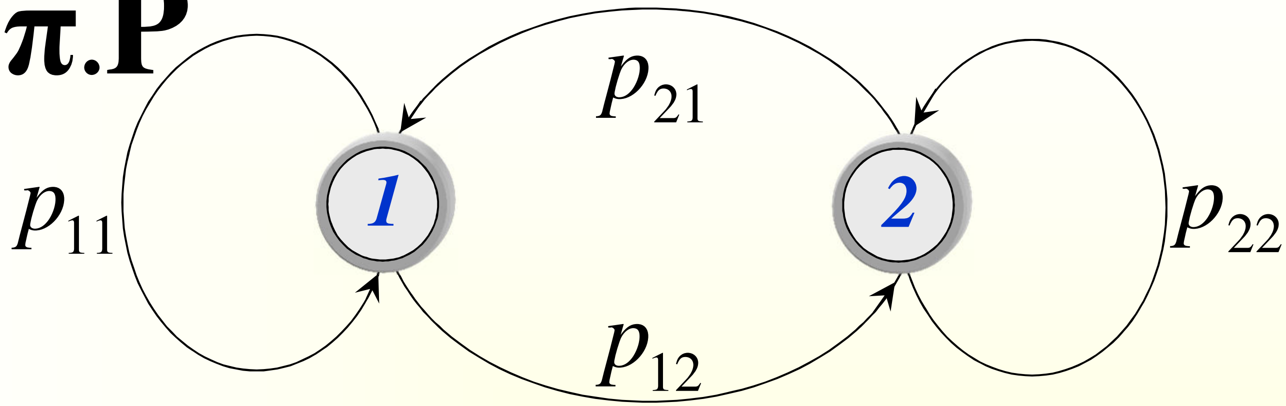
$$\mathbf{P} = \begin{pmatrix} p_{1,1} & \dots & p_{1,n} \\ \dots & \dots & \dots \\ p_{n,1} & \dots & p_{n,n} \end{pmatrix}$$

nájdeme riešením sústavy lineárnych algebraických rovníc

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}\ , \quad \sum_{i=1}^{n} \pi_i = 1$$

Invariantné rozdelenie - rovnováha

$$\boldsymbol{\pi} = \boldsymbol{\pi}.\mathbf{P}$$

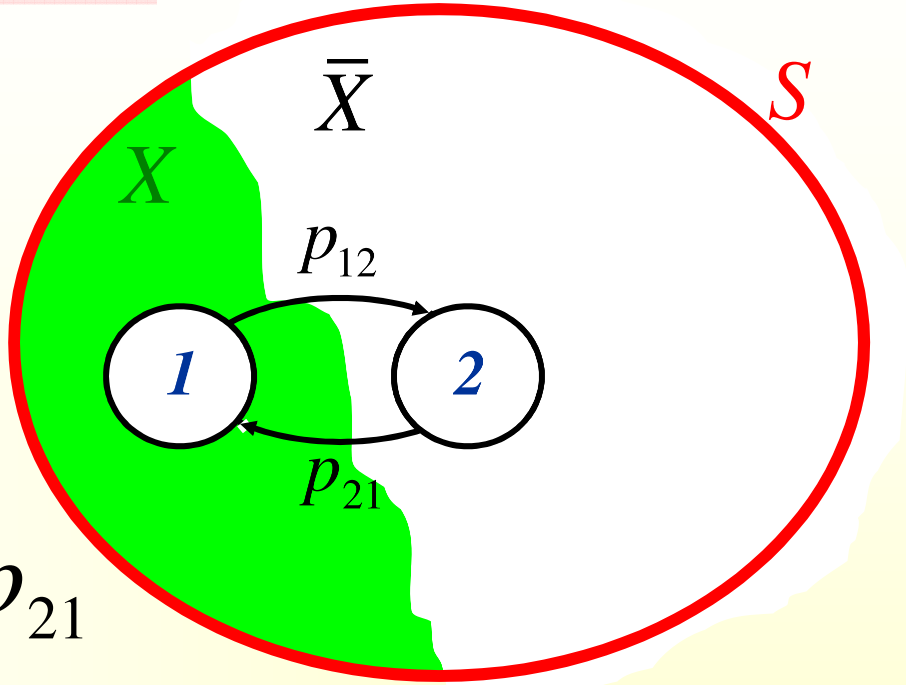


$$\pi(1) = \pi(1)\,p_{11} + \pi(2)\,p_{21}$$

$$\pi(1)\left(1 - p_{11}\right) = \pi(2)\,p_{21}$$

$$\pi(1)\,p_{12} = \pi(2)\,p_{21}$$

Veta o zachovaní toku



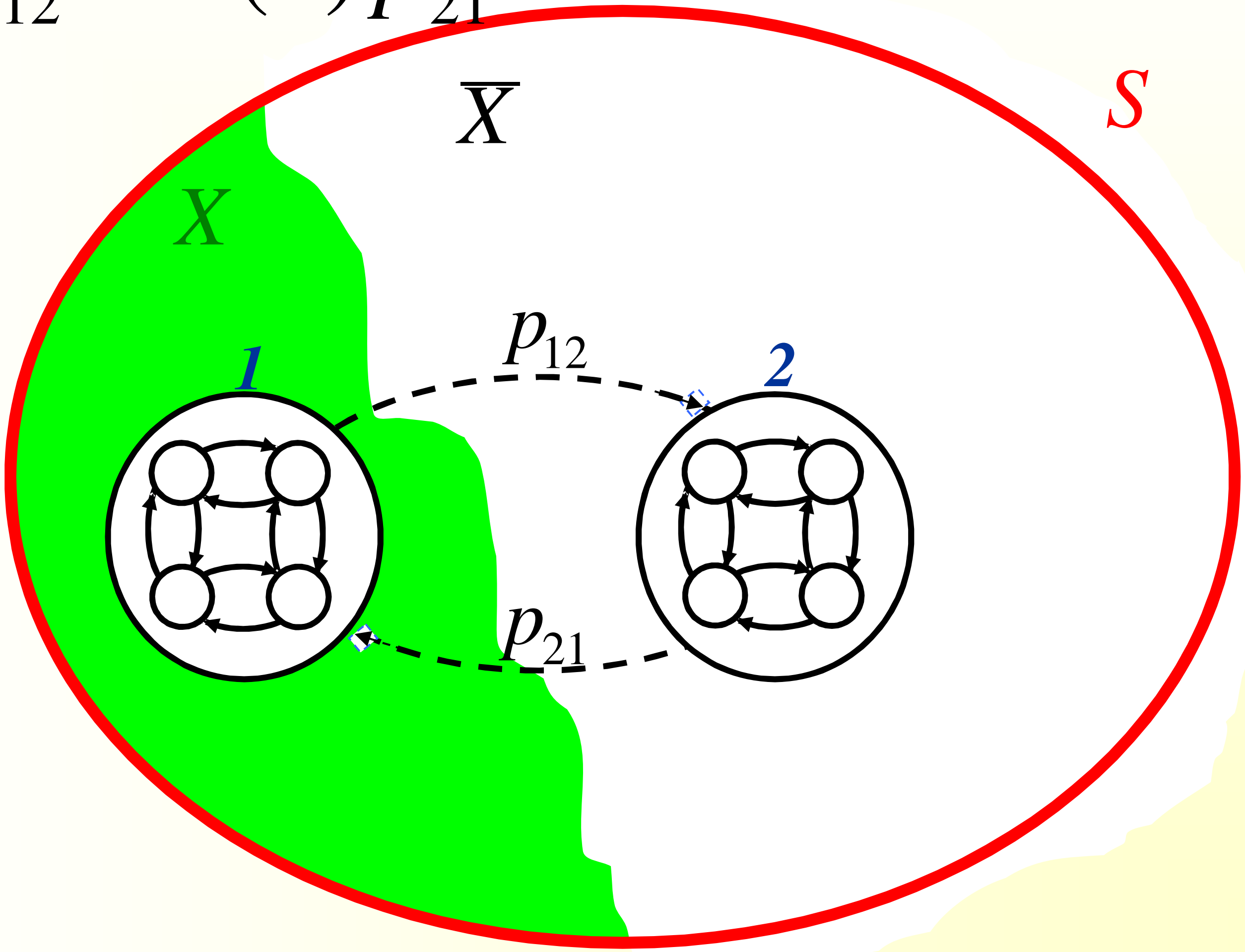
$$\pi(1)\,p_{12} = \pi(2)\,p_{21}$$

$$P\left(x_t \in X \cap x_{t+1} \in \bar{X}\right) = P\left(x_t \in \bar{X} \cap x_{t+1} \in X\right)$$

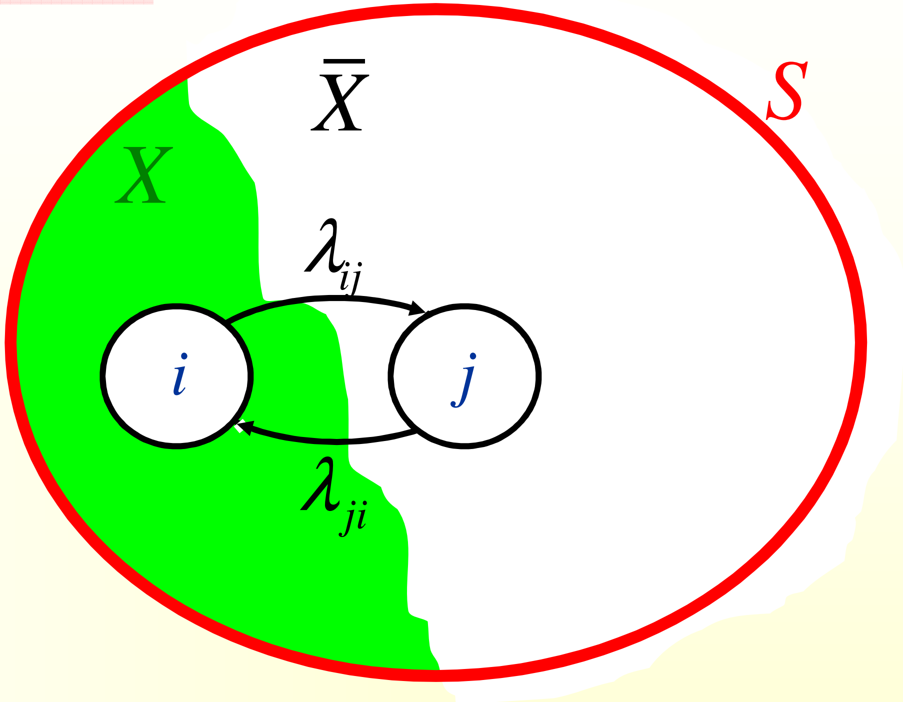
$$\Phi_{X\bar{X}} = \Phi_{\bar{X}X}$$

Veta o zachovaní toku

$$\pi(1)\,p_{12} = \pi(2)\,p_{21}$$



Veta o zachovaní toku



$$\Phi_{X\bar{X}} = \Phi_{\bar{X}X}$$

$$\sum_{i\in X}\sum_{j\in \bar{X}} \pi_i p_{ij} = \sum_{j\in \bar{X}}\sum_{i\in X} \pi_j p_{ji}$$

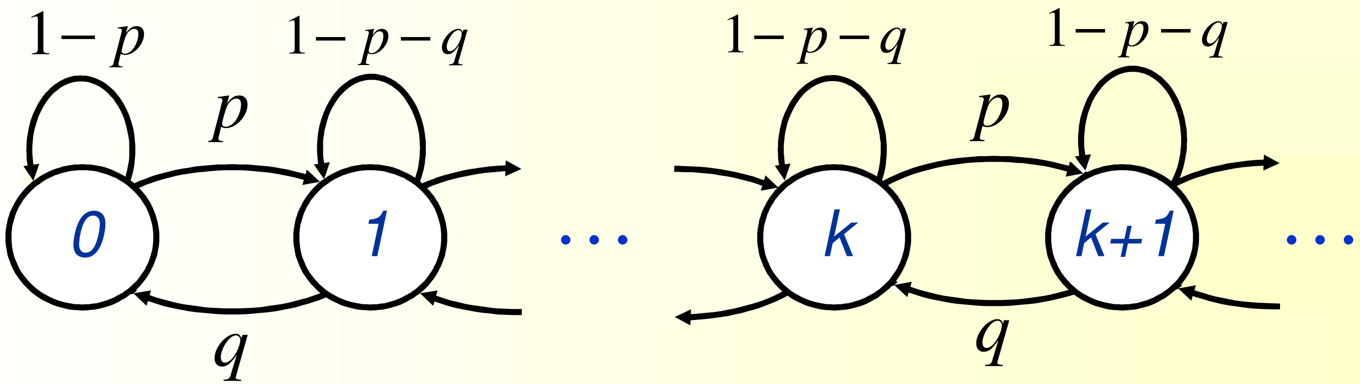
Formálny dôkaz za domácu úlohu

Príklad

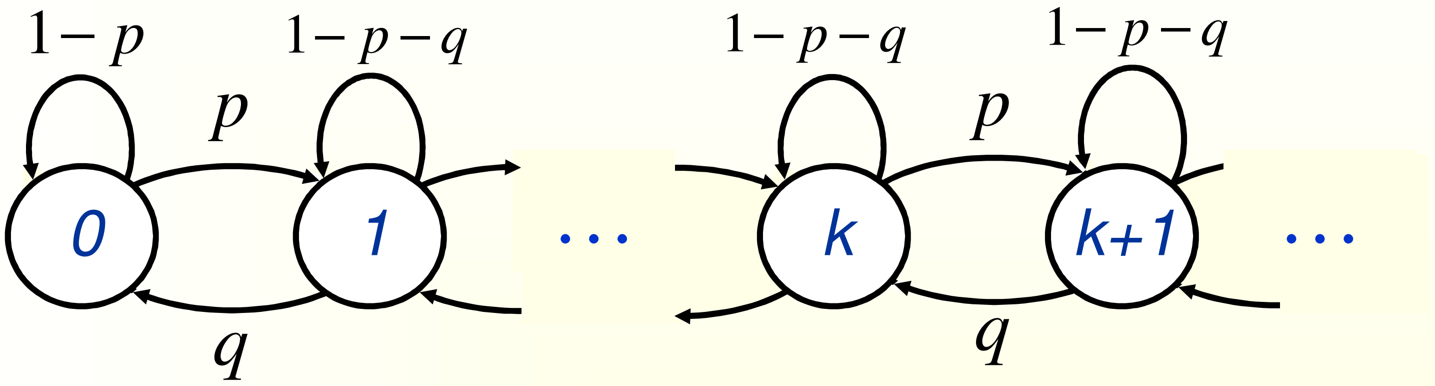
Matica prechodov

$$\mathbf{P}=\begin{pmatrix}1-p & p & 0 & 0 & \dots\\ q & 1-p-q & p & 0 & \dots\\ 0 & q & 1-p-q & p & \dots\\ \dots & \dots & \dots & \dots & \dots\end{pmatrix}$$

Graf prechodov



Invariantné rozdelenie



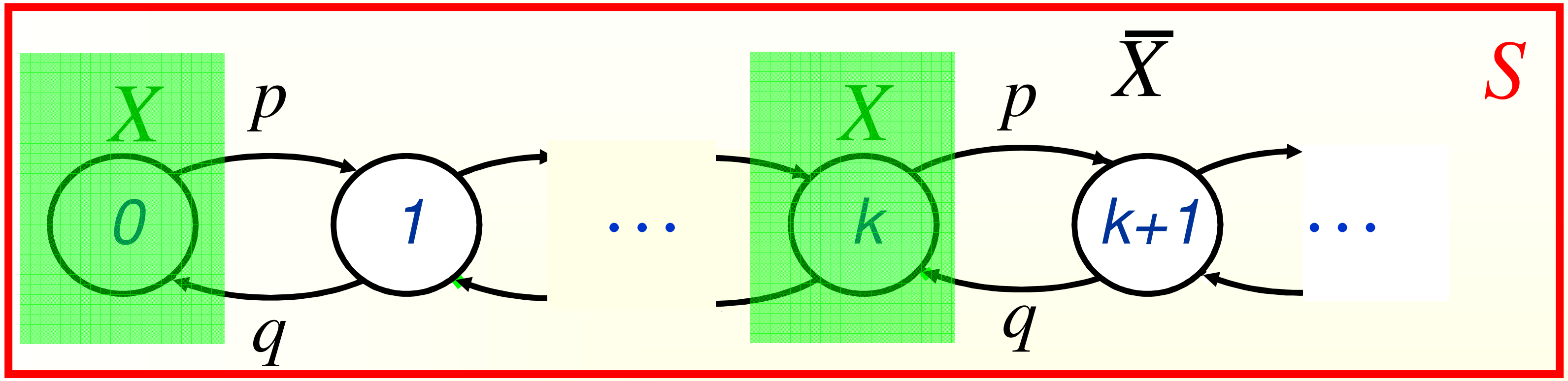
$$\pi_0 = \pi_0(1-p) + \pi_1 q$$

$$\pi_k = \pi_{k-1} p + \pi_k(1-p-q) + \pi_{k+1} q, \quad k = 1,2,\ldots$$

$$\pi_0 p = \pi_1 q$$

$$\pi_k(p+q) = \pi_{k-1} p + \pi_{k+1} q, \quad k = 1,2,\ldots$$

Veta o zachovaní toku

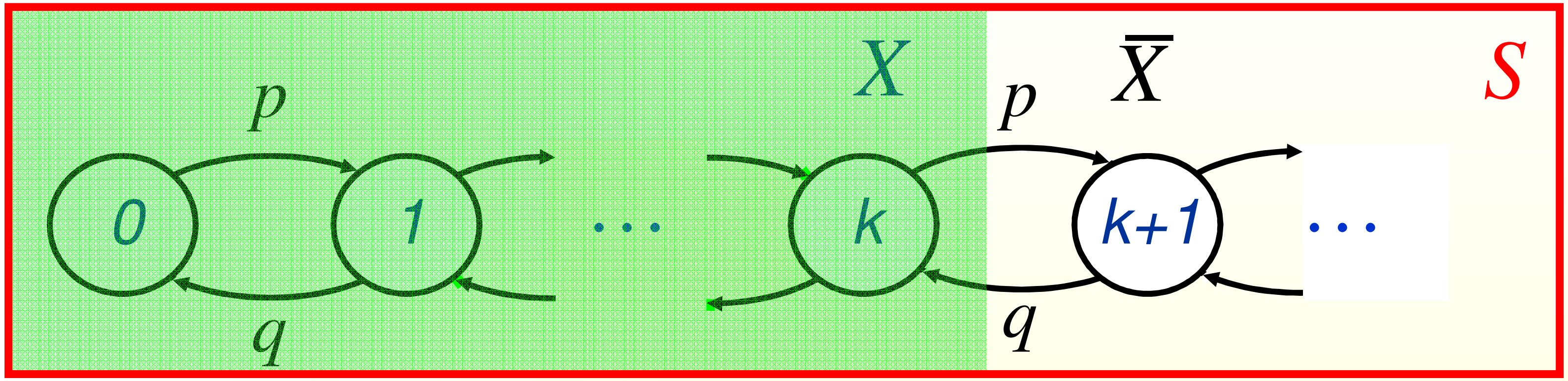


Veta o zachovaní toku pravdepodobnosti

$$\pi_0 p = \pi_1 q$$

$$\pi_k (p+q) = \pi_{k-1} p + \pi_{k+1} q, \quad k = 1,2,...$$

Iné rozdelenie stavov?



$$\pi_k p = \pi_{k+1} q, \quad k = 0,1,\ldots$$

$$\pi_{k+1} = \frac{p}{q}\pi_k = \rho\pi_k, \quad k = 0,1,\ldots$$

Invariantné rozdelenie

$$\pi_k = \rho^k \pi_0, \quad k = 0,1,\ldots$$

$$\sum_{k=0}^{\infty} \pi_k = 1$$

Riešenie

$$\sum_{k=0}^{\infty} \rho^k \pi_0 = 1 \Rightarrow \quad \pi_0 = \left(\frac{1}{1-\rho} \right)^{-1}, \quad \rho < 1$$

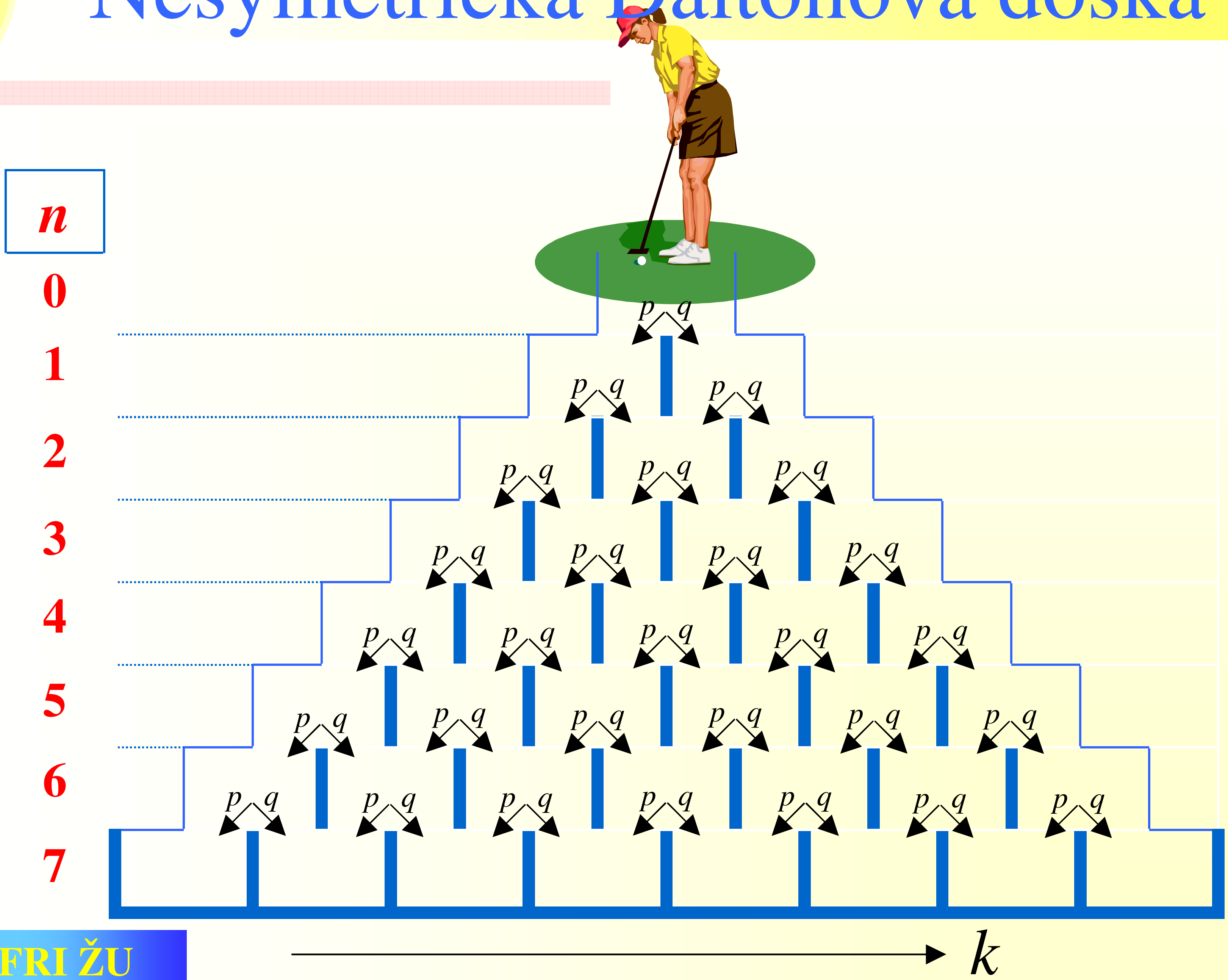
$$\boxed{\pi_{\mathrm{k}} = \rho^k (1 - \rho) \quad ,k = 0,1,\ldots}$$

Invariantné rozdelenie

Postup:

1. **určenie stavov**
2. určenie rezov
3. napísanie rovníc o zachovaní toku
4. vyriešenie rovníc

Nesymetrická Daltonova doska



Binomické rozdelenie

$$P\{x(n)=k\}=\binom{n}{k}p^k q^{n-k}, \qquad k=0,1,\ldots,n$$

n							
0	1						
1	$1p$	$1q$					
2	$1p^2$	$2pq$	$1q^2$				
3	$1p^3$	$3p^2q$	$3pq^2$	$1q^3$			
4	$1p^4$	$4p^3q$	$6p^2q^2$	$4pq^3$	$1q^4$		
5	$1p^5$	$5p^4q$	$10p^3q^2$	$10p^2q^3$	$5pq^4$	$1q^5$	
6	$1p^6$	$6p^5q$	$15p^4q^2$	$20p^3q^3$	$15p^2q^4$	$6pq^5$	$1q^6$

$\longrightarrow k$

Stredná hodnota

$$P\{x(n)=k\}=\binom{n}{k}p^k q^{n-k}, \qquad k=0,1,\dots,n$$

$$E\{X(n)\}=\overline{m}=\sum_{k=0}^{n} kP\{x(n)=k\}=\sum_{k=0}^{n} k\binom{n}{k}p^k q^{n-k}$$

$$\overline{m}=\sum_{k=0}^{n} k\,\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1}p^k q^{n-k}=$$

$$=\sum_{k=1}^{n} k\,\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1}p^k q^{n-k}$$

Stredná hodnota

$$\overline{m} = \sum_{k=1}^{n} \frac{n(n-1)\dots(n-k+1)}{(k-1)\dots 2} p^k q^{n-k} =$$

$$r = k - 1$$

$$= np \underbrace{\sum_{r=0}^{n-1} \frac{(n-1)\dots(n-1-r+1)}{r(r-1)\dots 1} p^r q^{n-1-r}}_{1}$$

$$\boxed{\overline{m} = np}$$

Veľká Daltonova doska

$$P\{x(n)=k\}=\binom{n}{k}p^k q^{n-k}, \qquad k=0,1,\ldots,n$$

$$\overline{m}=np \Rightarrow p=\frac{\overline{m}}{n}$$

$$P\{x(n)=k\}=\binom{n}{k}\left(\frac{\overline{m}}{n}\right)^k\left(1-\frac{\overline{m}}{n}\right)^{n-k}$$

$$P\{x(n)=k\}=\frac{\prod_{j=0}^{k-1}(n-j)}{k!}\left(\frac{\overline{m}}{n}\right)^k\left(1-\frac{\overline{m}}{n}\right)^{n-k}$$

Veľká Daltonova doska

$$P\{x(n)=k\}=\frac{\prod_{j=0}^{k-1}(n-j)}{k!}\left(\frac{\overline{m}}{n}\right)^{k}\left(1-\frac{\overline{m}}{n}\right)^{n-k}$$

$$P\{x(n)=k\}=\frac{\overline{m}^{k}}{k!}\left(1-\frac{\overline{m}}{n}\right)^{n-k}\prod_{j=0}^{k-1}\left(1-\frac{j}{n}\right)$$

Veľká Daltonova doska

$n \to \infty$

$$P\{x=k\} = \lim_{n\to\infty} \frac{\overline{m}^k}{k!}\left(1-\frac{\overline{m}}{n}\right)^{n-k} \prod_{j=0}^{k-1}\left(1-\frac{j}{n}\right)$$

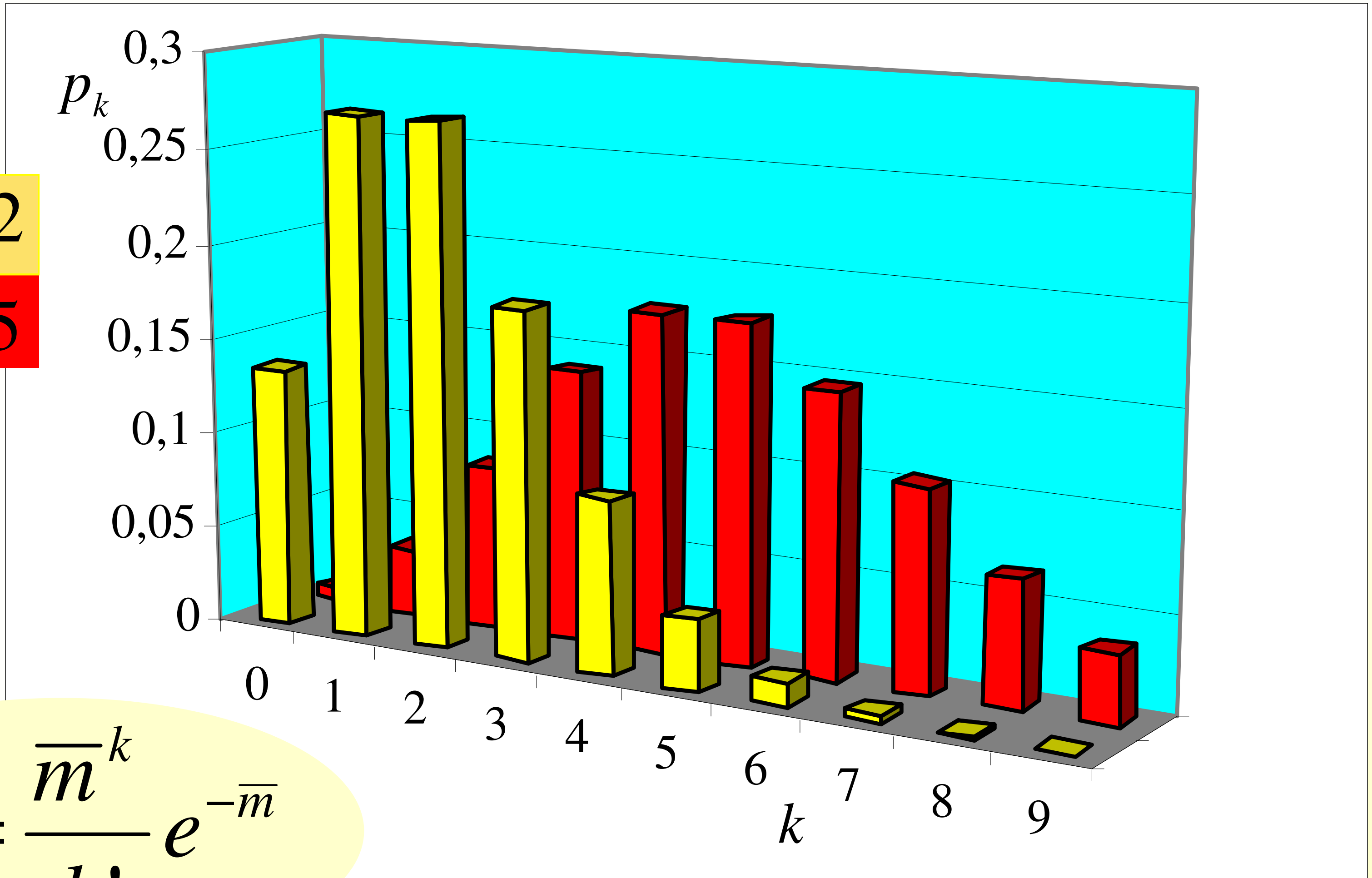
$$P\{x=k\} = \frac{\overline{m}^k}{k!} \underbrace{\lim_{n\to\infty}\left(1-\frac{\overline{m}}{n}\right)^{n-k}}_{e^{-\overline{m}}} \underbrace{\lim_{n\to\infty}\prod_{j=0}^{k-1}\left(1-\frac{j}{n}\right)}_{1}$$

$$\boxed{P\{x=k\} = \frac{\overline{m}^k}{k!} e^{-\overline{m}}}$$

Poissonovo rozdelenie

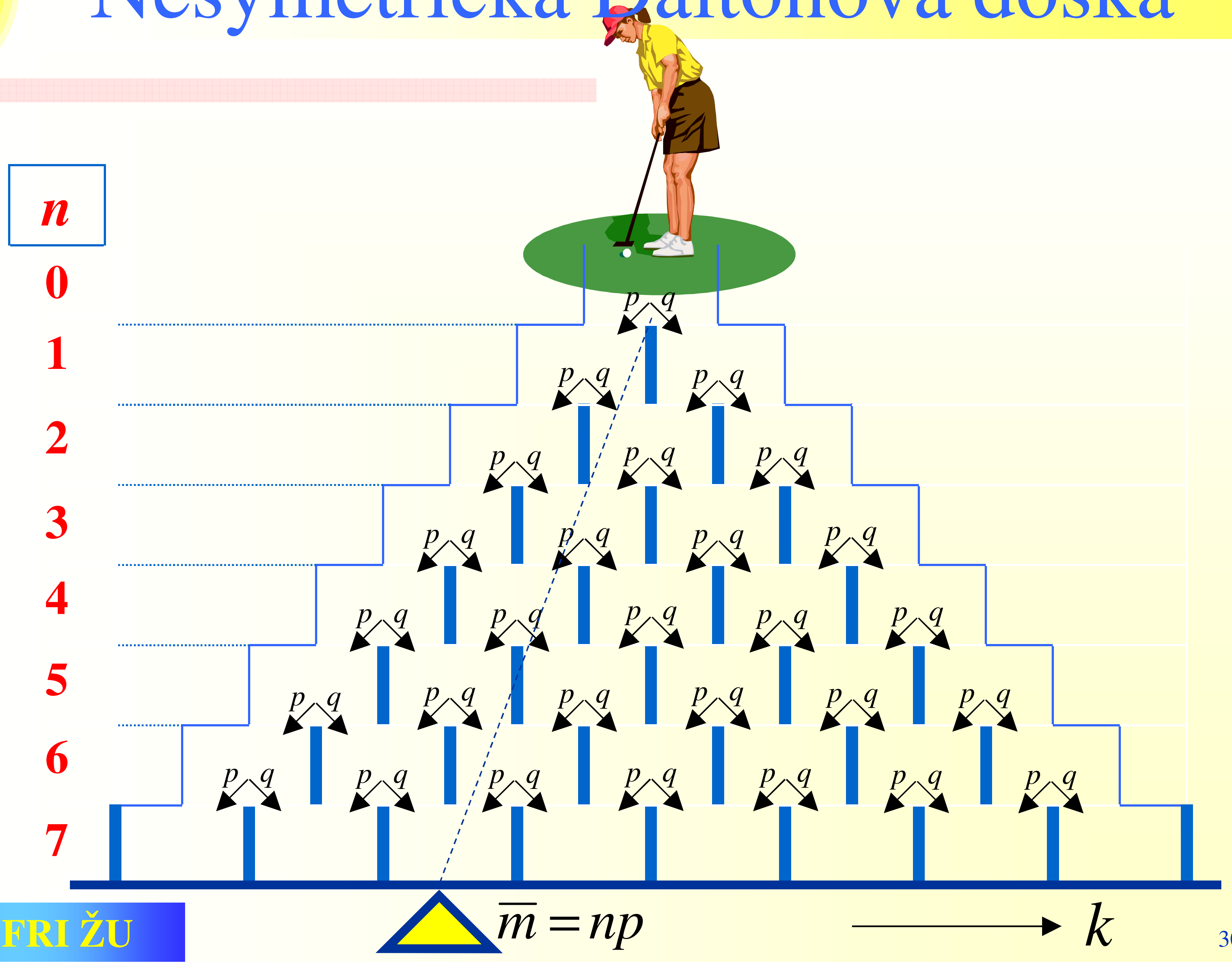
$\overline{m} = 2$

$\overline{m} = 5$



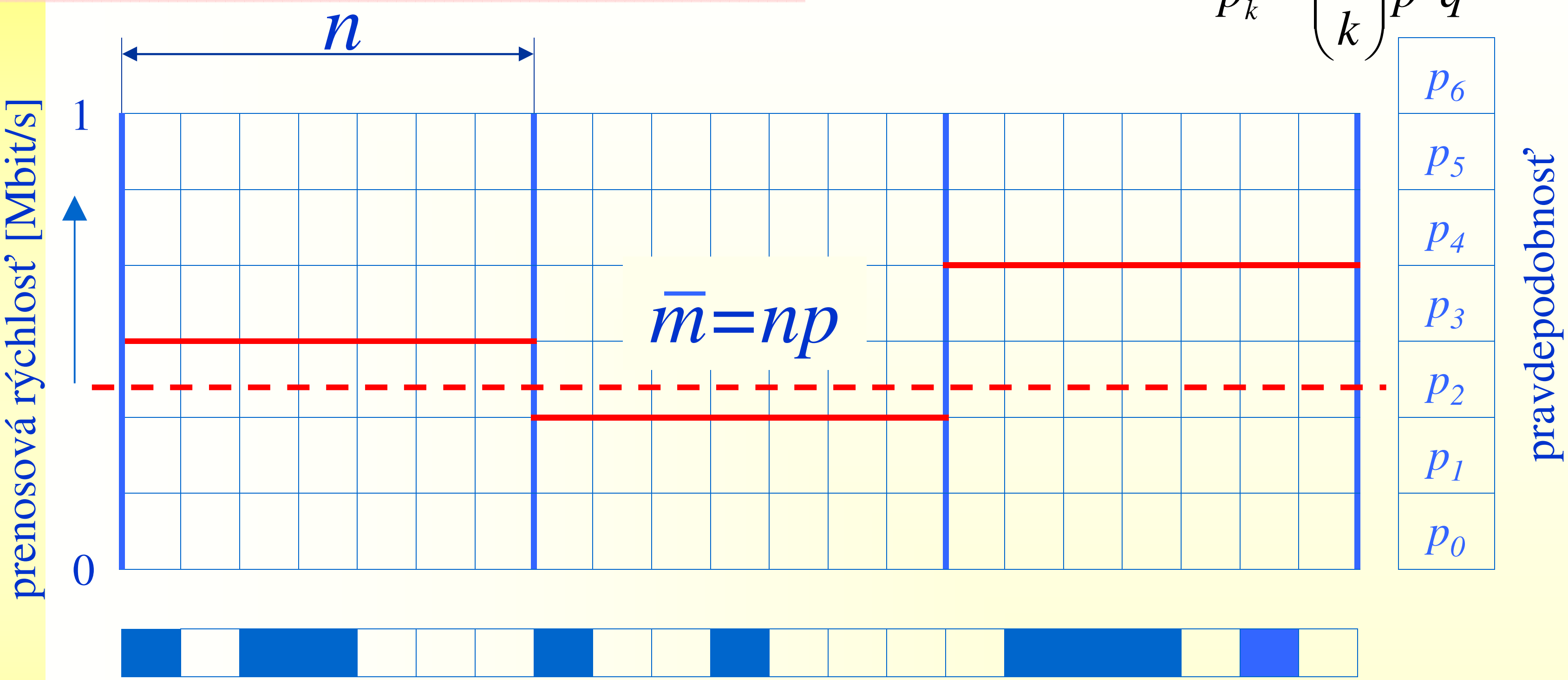
$$p_k = \frac{\overline{m}^k}{k!} e^{-\overline{m}}$$

Nesymetrická Daltonova doska



Stredná hodnota

$$p_k = \binom{n}{k} p^k q^{n-k}$$



$$\overline{m} = \lambda t$$

Parameter Poissonovho procesu

Stredný počet príchodov za čas *t*

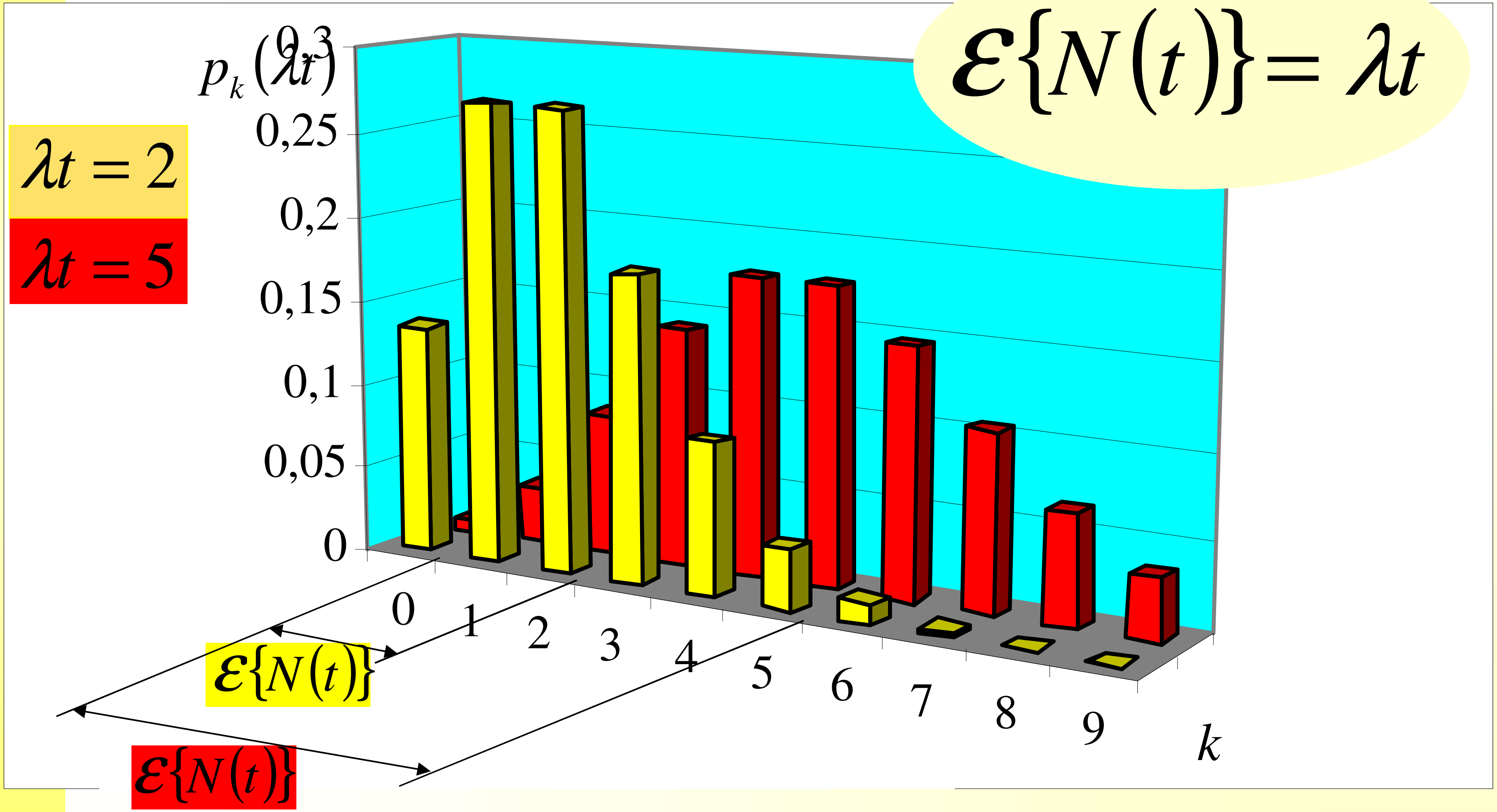
$$\mathcal{E}\{N(t)\} = \lambda t$$

Stredný počet príchodov za jednotku času

$$\mathcal{E}\{N(1)\} = \lambda$$

Stredná hodnota počtu

$$\mathcal{E}\{N(t)\} = \lambda t$$



Intenzita Poissonovho procesu

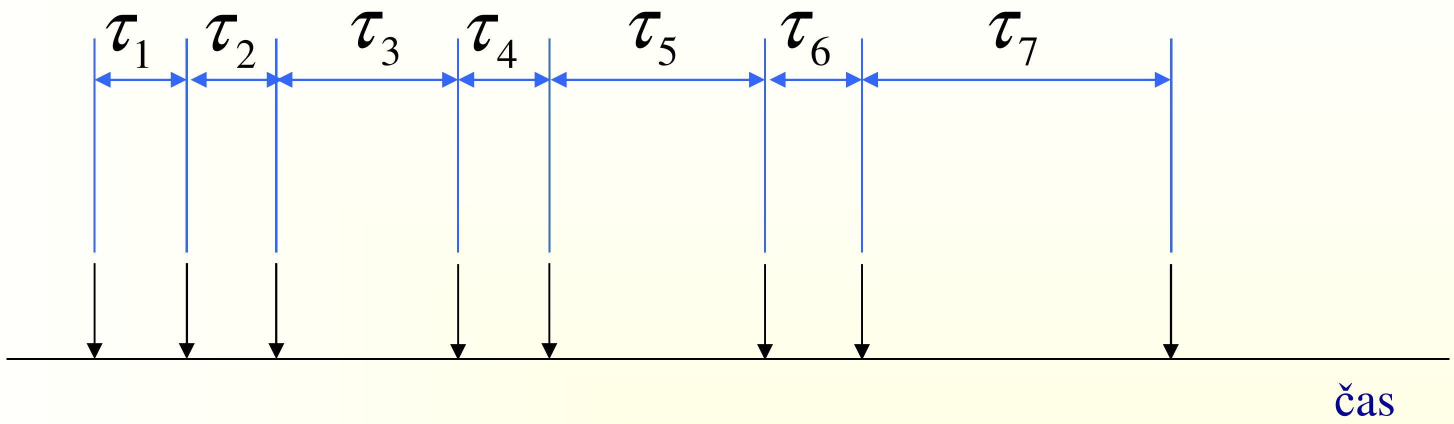
Intenzita príchodu

$$\lim_{t \to 0^+} \frac{p_k(t)}{t} = \lim_{t \to 0^+} \frac{\lambda(\lambda t)^{k-1}}{k!} e^{-\lambda t} = \begin{cases} \lambda, & k = 1 \\ 0, & k = 2,3,\ldots \end{cases}$$

Intenzita procesu

$$\lim_{t \to 0^+} \frac{1}{t} \sum_{k=1}^{\infty} p_k(t) = \sum_{k=1}^{\infty} \lim_{t \to 0^+} \frac{p_k(t)}{t} = \lambda + 0 + 0 + \ldots = \lambda$$

Popis procesu v čase

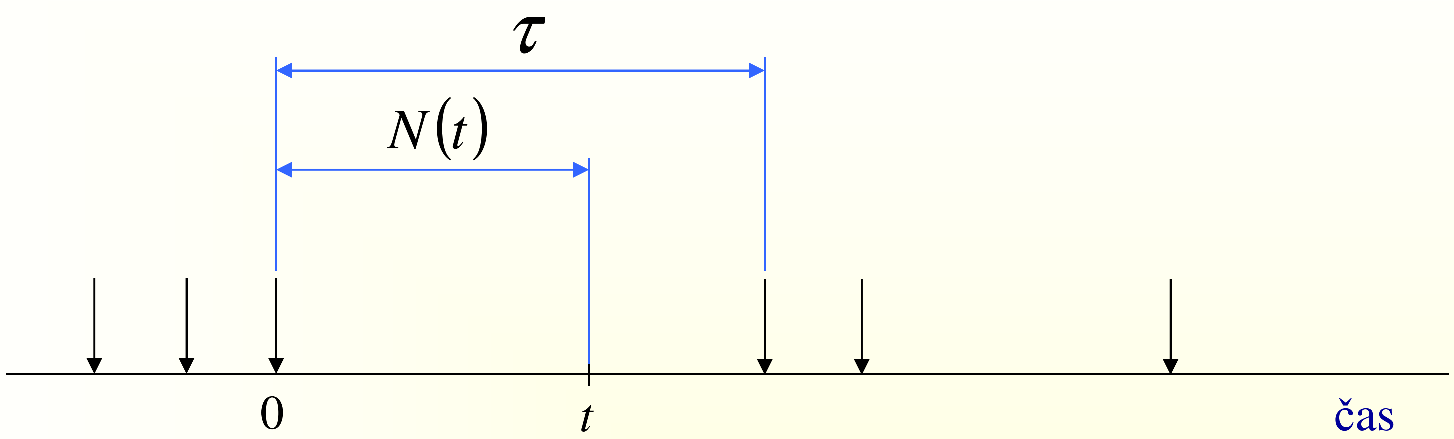


Distribučná funkcia

$$F_k(t) = P\{\tau_k < t\} = F(t),\ \forall k$$

Proces je homogénny

Interval medzi príchodmi



Distribučná funkcia

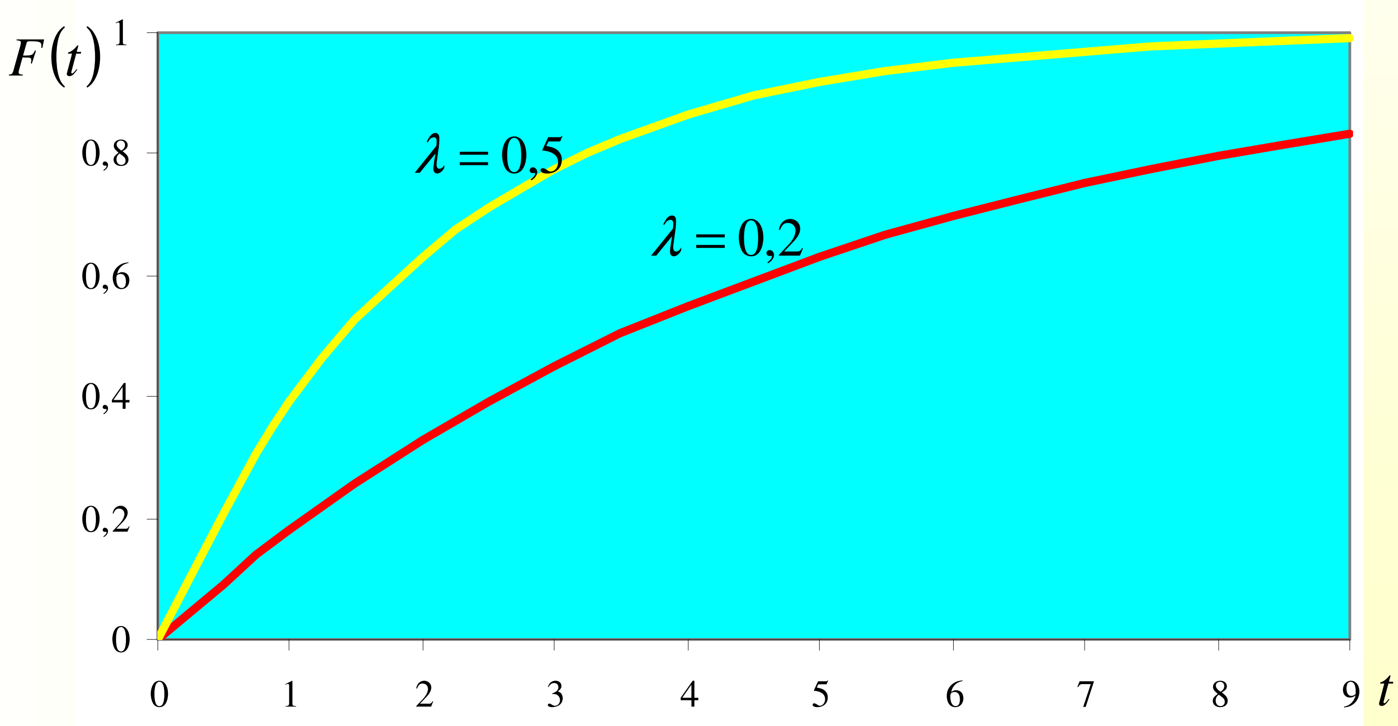
$$F(t) = P\{\tau < t\} = 1 - \boxed{P\{\tau \geq t\} = ?}$$

$$= 1 - P\{N(t) = 0\} = 1 - \frac{(\lambda t)^0}{0!} e^{-\lambda t}$$

Interval medzi príchodmi

Distribučná funkcia

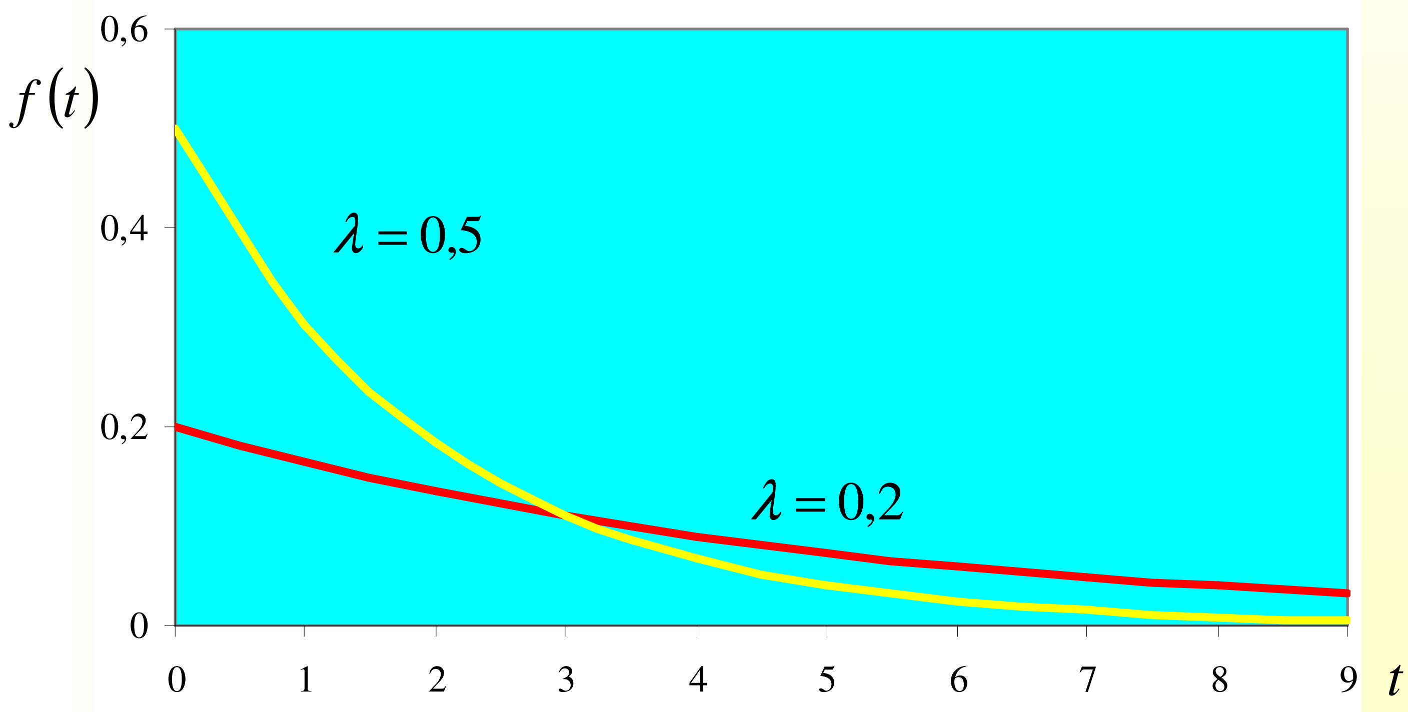
$$F(t) = 1 - e^{-\lambda t},\ t \geq 0, \lambda > 0$$



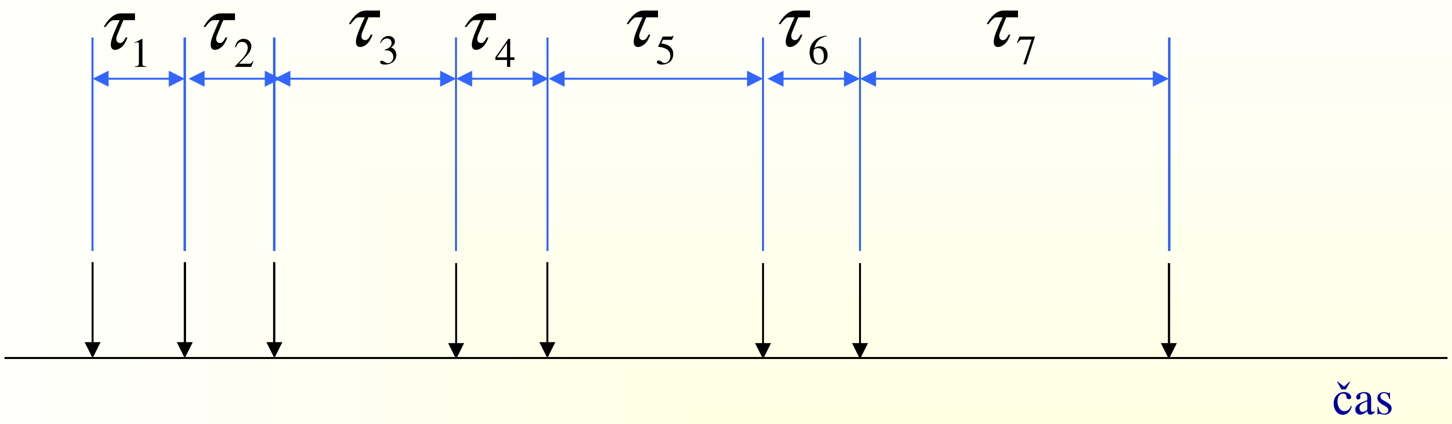
Interval medzi príchodmi

Hustota rozdelenia pravdepodobnosti

$$f\left(t\right)=F'\left(t\right)=\lambda e^{-\lambda t}$$

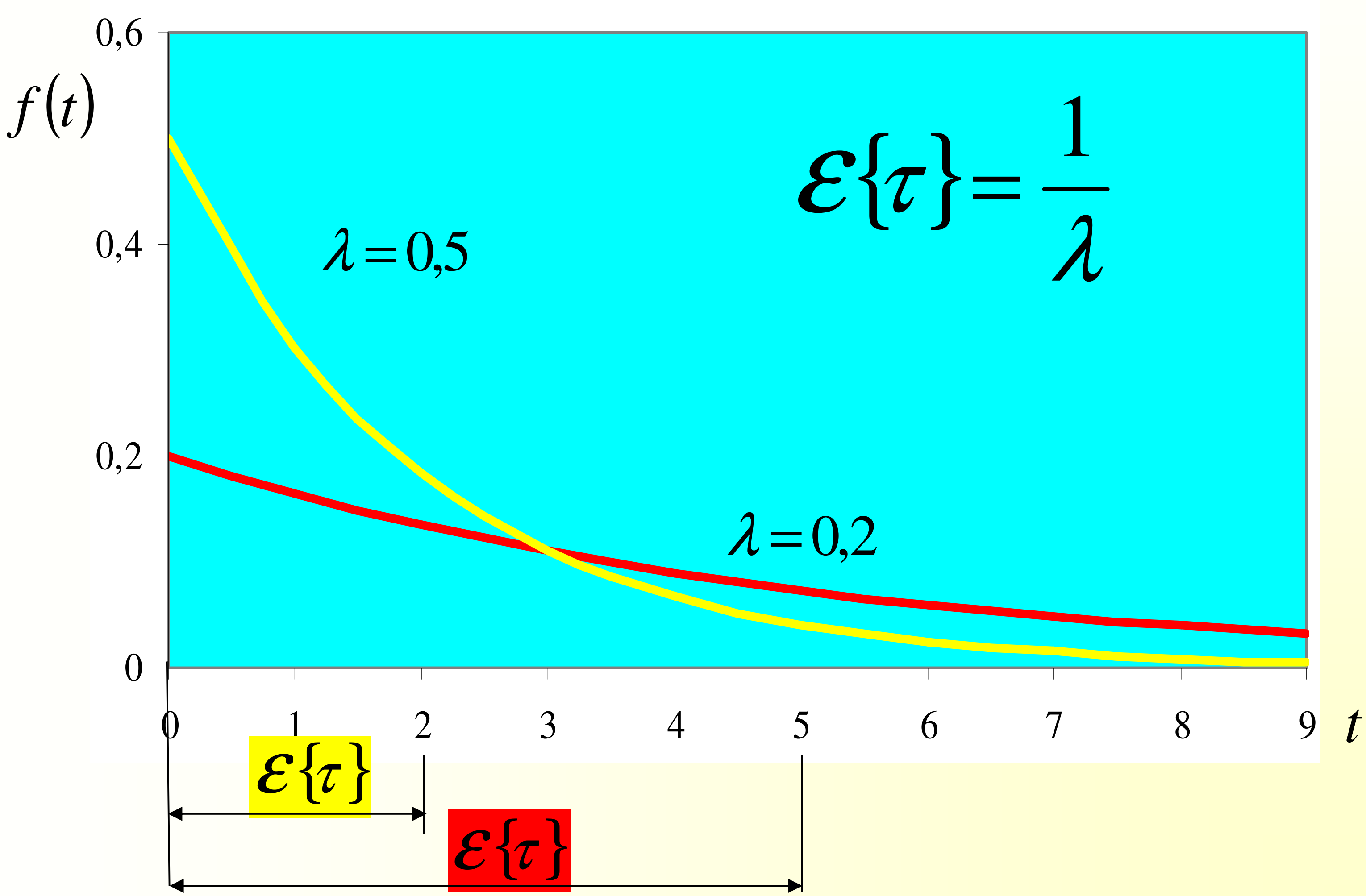


Stredná hodnota intervalu



$$\mathcal{E}\{\tau\} = \int_0^\infty t f(t)\,dt = \int_0^\infty t\lambda e^{-\lambda t}\,dt = \frac{1}{\lambda}$$

Interval medzi príchodmi



Ďakujem za pozornosť