

1. $\lim_{x \rightarrow -1} \frac{x-4}{x+1} =$

$\lim_{x \rightarrow -1} \frac{x-4}{x+1} = \frac{-5}{0} = -5(-\infty) = \infty$

$\lim_{x \rightarrow -1^+} \frac{x-4}{x+1} = \frac{-5}{0^+} = -5(\infty) = -\infty$

$\Rightarrow \neq \checkmark$

2. $\lim_{x \rightarrow -1} \operatorname{arctg} \frac{x-4}{x+1} =$

$\lim_{x \rightarrow -1^-} \operatorname{arctg} \frac{x-4}{x+1} = \operatorname{arctg} -5(-\infty) = \frac{\pi}{2}$

$\lim_{x \rightarrow -1^+} \operatorname{arctg} \frac{x-4}{x+1} = \operatorname{arctg} -5 \cdot \infty = -\frac{\pi}{2}$

$\neq \checkmark$

3. $\lim_{x \rightarrow 0} \frac{\operatorname{tg}(-4x)}{\operatorname{tg} x} =$

$\lim_{x \rightarrow 0} \frac{\frac{\sin(-4x)}{\cos(-4x)}}{\frac{\sin x}{\cos x}} = \lim_{x \rightarrow 0} \frac{\sin(-4x) \cdot \cos x}{\sin x \cdot \cos(-4x)} = \lim_{x \rightarrow 0} \frac{\frac{\sin(-4x)}{-4x} \cdot \frac{\cos x}{1}}{\frac{\sin x}{x} \cdot \frac{\cos(-4x)}{-4}} = \frac{1}{1 \cdot -\frac{1}{4}} = -4$

4. $\lim_{x \rightarrow 0} \frac{\sin(-4x)}{x} = \lim_{x \rightarrow 0} \frac{\frac{\sin(-4x)}{-4x}}{\frac{x}{-4x}} = \frac{1}{\frac{1}{4}} = -4$

5. $\lim_{x \rightarrow 0} \frac{|\sin(-4x)|}{x} =$

$\lim_{x \rightarrow 0} \frac{-\sin(-4x)}{x} = 4$

$\lim_{x \rightarrow 0} \frac{\sin(-4x)}{x} = -4$

$\neq \checkmark$

6. $\lim_{x \rightarrow \infty} \left(-\frac{x}{4}\right) \sin \frac{1}{x} =$

$\lim_{x \rightarrow \infty} -\frac{1}{4} \cdot \frac{\sin \frac{1}{x}}{\frac{1}{x}} = -\frac{1}{4}$

7. $\lim_{x \rightarrow 0} \frac{\arcsin(-4x)}{x} =$

$\lim_{x \rightarrow 0} \frac{\frac{\arcsin(-4x)}{-4x}}{\frac{x}{-4x}} = \frac{1}{-\frac{1}{4}} = -4$

8. $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}(-4x)}{x} =$

$\lim_{x \rightarrow 0} \frac{\frac{\operatorname{arctg}(-4x)}{-4x}}{\frac{x}{-4x}} = \frac{1}{-\frac{1}{4}} = -4$

9. $\lim_{x \rightarrow 0} \frac{1 - \cos(-4x)}{x} =$

$\lim_{x \rightarrow 0} \frac{\frac{1 - \cos(-4x)}{-4x}}{\frac{x}{-4x}} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos(-4x)}{-4x} \cdot \frac{1}{1}}{\frac{x}{-4x} \cdot \frac{1}{x}} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos(-4x)}{-4x}}{-\frac{1}{4}} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos(-4x)}{-4x}}{-\frac{1}{4}} = 0$

10. $\lim_{x \rightarrow 0} (1 + \sin(-4x))^{\frac{1}{x}} = \lim_{x \rightarrow 0} (1 - \sin 4x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} (\cos 4x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} (1 + \cos 4x - 1)^{\frac{1}{x}} = \lim_{x \rightarrow 0} (e^{\cos 4x - 1})^{\frac{1}{x}} = e^0 = 1$

11. $\lim_{x \rightarrow 0} (\cos(-4x))^{\frac{1}{x}} = \lim_{x \rightarrow 0} (1 + \cos(-4x) - 1)^{\frac{1}{x}} = \lim_{x \rightarrow 0} (e^{\cos(-4x) - 1})^{\frac{1}{x}} = e^0 = 1$

12. $\lim_{x \rightarrow 0} \frac{3^x - 1}{x - 1} =$

$\frac{1}{0}$

13. $\lim_{x \rightarrow 7} \frac{x^5 - 7^5}{x^6 - 7^6} =$

~~$\lim_{x \rightarrow 7} \frac{x^5 - 7^5}{x^6 - 7^6} = \lim_{x \rightarrow 7} \frac{x^5(1 - \frac{7^5}{x^5})}{x^6(1 - \frac{7^6}{x^6})} = \lim_{x \rightarrow 7} \frac{1}{x} = \frac{1}{7} = \frac{1}{7}$~~

14. $\lim_{x \rightarrow 7} \frac{\sqrt[5]{x} - \sqrt[5]{7}}{\sqrt[6]{x} - \sqrt[6]{7}} =$

~~$\lim_{x \rightarrow 7} \frac{\sqrt[5]{x}(1 - \frac{\sqrt[5]{7}}{\sqrt[5]{x}})}{\sqrt[6]{x}(1 - \frac{\sqrt[6]{7}}{\sqrt[6]{x}})} = \lim_{x \rightarrow 7} \frac{1}{\sqrt[6]{x}} = \frac{1}{\sqrt[6]{7}}$~~

15. $\lim_{x \rightarrow \infty} \frac{(2x-3)^{350}(4x+2)^{150}}{(3x+1)^{500}} =$

$\lim_{x \rightarrow \infty} \frac{(2^{350} \cdot x^{350} \dots)(4^{150} \cdot x^{150} \dots)}{(3^{500} \cdot x^{500} \dots)} = \lim_{x \rightarrow \infty} \frac{2^{350} \cdot x^{500}}{3^{500} \cdot x^{500}} = \frac{2^{350}}{3^{500}}$

16. $\lim_{x \rightarrow \infty} \frac{(2x-3)^{350}(4x+2)^{150}}{(3x+1)^{400}} =$

$\lim_{x \rightarrow \infty} \frac{(2^{350} \cdot x^{350} \dots)(4^{150} \cdot x^{150} \dots)}{(3^{400} \cdot x^{400} \dots)} = \lim_{x \rightarrow \infty} \frac{2^{350} \cdot x^{500}}{3^{400} \cdot x^{400}} = \lim_{x \rightarrow \infty} \frac{x^{100}}{1} = \infty$

17. $\lim_{x \rightarrow \infty} \frac{(2x-3)^{350}(4x+2)^{150}}{(3x+1)^{600}} =$

$\lim_{x \rightarrow \infty} \frac{(2^{350} \cdot x^{350} \dots)(4^{150} \cdot x^{150} \dots)}{(3^{600} \cdot x^{600} \dots)} = \lim_{x \rightarrow \infty} \frac{2^{350} \cdot 4^{150} \cdot x^{500}}{3^{600} \cdot x^{600}} = \lim_{x \rightarrow \infty} \frac{2^{350} \cdot 4^{150}}{3^{600} \cdot x^{100}} = 0$

18. $\lim_{x \rightarrow \infty} \frac{2x^4 + 2x^5 + x^3 - 2}{3x^3 - 5x^5 - 1} =$

$\lim_{x \rightarrow \infty} \frac{x^5(\frac{2}{x} + 2 + \frac{1}{x^2} - \frac{2}{x^5})}{x^5(\frac{3}{x^2} - 5 - \frac{1}{x^5})} = -\frac{2}{5}$

19. $\lim_{x \rightarrow \infty} [\sqrt[4]{x^4 + 3x - 5} - \sqrt[4]{x^4 + 2x - 2}] =$

$\frac{x^4 + 3x - 5 - (x^4 + 2x - 2)}{\sqrt[4]{(x^4 + 3x - 5)^3} + \sqrt[4]{(x^4 + 3x - 5)^2} \cdot \sqrt[4]{x^4 + 2x - 2} + \sqrt[4]{x^4 + 3x - 5} \cdot \sqrt[4]{(x^4 + 2x - 2)^2} + \sqrt[4]{(x^4 + 2x - 2)^3}}$
 $= \frac{x(1 - \frac{2}{x})}{x^3 + \dots} = \frac{1}{\infty} = 0$

20. $\lim_{x \rightarrow \infty} [\sqrt[4]{x^4 + 3x - 5} - \sqrt[4]{x - 2}] =$

$\frac{x^4 + 3x - 5 - (x - 2)}{\sqrt[4]{(x^4 + 3x - 5)^3} + \sqrt[4]{(x^4 + 3x - 5)^2} \cdot \sqrt[4]{x - 2} + \sqrt[4]{x^4 + 3x - 5} \cdot \sqrt[4]{(x - 2)^2} + \sqrt[4]{(x - 2)^3}}$
 $= \frac{x^4(1 + \frac{3}{x} - \frac{5}{x^4})}{x^3 + \dots} = \infty$

21. $\lim_{x \rightarrow \infty} \left[\frac{3x-5}{3x+2} \right]^{x+4} =$

$\lim_{x \rightarrow \infty} \left(1 + \frac{-7}{3x+2} \right)^{x+4} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-7}{3x+2} \right)^{3x+2} \right]^{\frac{x+4}{3x+2}} = e^{(-7)(\frac{1}{3})} = e^{(-\frac{7}{3})}$

$\lim_{x \rightarrow \infty} \frac{x+4}{3x+2} = \frac{x(1 + \frac{4}{x})}{x(3 + \frac{2}{x})} = \frac{1}{3}$

$$(12) \lim_{x \rightarrow 0} \frac{3^x - 1}{7^x - 1} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{\frac{3^x - 1}{x}}{\frac{7^x - 1}{x}} = \frac{\ln 3}{\ln 7}$$

$$(13) \lim_{x \rightarrow 7} \frac{x^5 - 7^5}{x^6 - 7^6} = \lim_{x \rightarrow 7} \frac{(x^4 + 7x^3 + 49x^2 + 343x + 2401)(x-7)}{(x^5 + 7x^4 + 49x^3 + 343x^2 + 2401x + 16807)(x-7)} = \frac{12005}{100842} = \frac{5}{42}$$

$$(14) \lim_{x \rightarrow 7} \frac{\sqrt[5]{x} - \sqrt[5]{7}}{\sqrt[6]{x} - \sqrt[6]{7}} = \frac{6 \cdot 7^{\frac{1}{30}}}{5}$$

$$\frac{6 \cdot 7^{\frac{1}{30}}}{5}$$

$$\frac{6 \cdot 7^{\frac{1}{30}}}{5}$$

$$x = a^{30} \Rightarrow a = x^{\frac{1}{30}}$$

$$b = 7^{\frac{1}{30}}$$

$$x \rightarrow 7 \Rightarrow a \rightarrow 7^{\frac{1}{30}} \Rightarrow a \rightarrow b$$

$$\lim_{a \rightarrow b} \frac{a^6 - b^6}{a^5 - b^5}$$

$$\lim_{a \rightarrow b} \frac{a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 + b^5}{a^4 + a^3b + a^2b^2 + ab^3 + b^4}$$

$$\frac{6a^5}{5a^4} = \frac{6a}{5} = \frac{6 \cdot 7^{\frac{1}{30}}}{5}$$

$$(10) \lim_{x \rightarrow 0} (1 + \sin(-4x))^{\frac{1}{x}} = e^{\lim_{x \rightarrow 0} \ln(1 + \sin(-4x)) \cdot \frac{1}{x}} = e^{-4}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1 + \sin(-4x))}{x} = \lim_{x \rightarrow 0} \frac{\ln(1 + \sin(-4x)) \cdot \sin(-4x)}{\sin(-4x) \cdot x}$$

$$\lim_{x \rightarrow 0} \frac{\sin(-4x)}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{(\sin(-4x))}{x} \cdot \frac{-4}{-4} = -4$$