



Kódový priestor - príklady

binárny N-rozmerný kódový priestor

$$\mathbf{f} = (f_0, ..., f_{N-1}), \ f_k \in \{0,1\}$$

 $(\mathbf{f}, \mathbf{g}) \in \{0,1\}$

p-nárny N-rozmerný kódový priestor

$$\mathbf{f} = (f_0, ..., f_{N-1}), f_k \in \{0,1,..., p-1\}$$

 $(\mathbf{f}, \mathbf{g}) \in \{0,1\}$

polynomiálny N-rozmerný kódový priestor

$$\mathbf{f} = (f_0, ..., f_{N-1}), \ f_k = a_1 x + a_0, \ a_i \in \{0,1\}$$
$$(\mathbf{f}, \mathbf{g}) = a_1 x + a_0, \ a_i \in \{0,1\}$$



p-nárny kódový priestor

$$\mathbf{f} = (f_0, ..., f_{N-1}), \ f_k \in \{0,1, ..., p-1\}$$
 p - prvočíslo

$$F = \{0, 1, 2, 3, 4\}, p = 5$$

2⁰ = 1, 2¹ = 2, 2² = 4, 2³ = 3

4},
$$p = 5$$

 $2^2 = 4$, $2^3 = 3$

$$f_i = 2^k, f_j = 2^l$$
$$f_i \underset{5}{\otimes} f_j = 2^{k \underset{4}{\otimes} l}$$

 $F = \left\{0, 2^0, 2^1, 2^3, \right.$

$\{0,2^0,2^1,2^3,2^2\}$										
0	1	2	_			\otimes				
0	0	0	0	0		0	0			

1	2	3	4	\otimes	0	2^{o}	21	2^3	2^2
0	0	0	0	0	0	0	0	0	0
1	2	3	4	2^{o}	0	2^{o}	21	2^3	2^2
2	4	1	3	2^{I}	0	2^{I}	2^2	2^{0}	2^3
3	1	4	2	2^3	0	2^3	2^{0}	_	21
4	3	2	1	2^2	0	2^2	2^3	2 ¹	2^{0}

Polynomiálny kódový priestor

 $\mathbf{f} = (f_0, ..., f_{N-1}), f_k \in \{0,1, x, x+1\} \quad q(x) = x^2 + x + 1$

 $F = \{0, 1, x, x + I\}$ $x^0 = 1$, $x^1 = x$, $x^2 = x + 1$ $F = \{0, x^0, x^1, x^2\}$

$f_i = x^k, f_j = x^l$
$f_i \underset{x^2 + x + 1}{\bigotimes} f_j = x^{k \oplus l \atop 3}$

	(,
\otimes	0	1	x	x + 1
0	0	0	0	0
1	0	1	х	x + 1
x	0	x	x+1	1
x+1	0	x+1	1	x

\otimes	0	x^{o}	x^{1}	x^2
0	0	0	0	0
x^{θ}	0	x^{o}	x^{1}	x^2
x^{I}	0	x^{I}	x^2	x^0
x^2	0	x^2	x^0	x^1

KIS – FRI ŽU



Polynomiálny kódový priestor

\oplus	0	1	х	x + I	
0	0	1	х	x + I	
1	1	0	x+1	x	
x	х	<i>x</i> +1	0	1	
x+1	<i>x</i> + <i>1</i>	х	1	0	
\otimes	0	1	х	x+1	
⊗ 0	0	1	<i>x</i> 0	x+1	
0	0	0	0	0	

0	x^{θ}	x^1	x^2
0	x^{θ}	x^{1}	x^2
x^{θ}	0	x^2	x^{1}
x^1	x^2	0	x^0
x^2	x^1	x^{θ}	0
0	x^{0}	x^{1}	x^2
0	0	0	0
0	x^{o}	x^1	x^2
_	1	2	x^{0}
0	x^{I}	x^2	x^1
	$ \begin{array}{c} 0 \\ x^0 \\ x^1 \\ x^2 \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c cc} 0 & x^{0} \\ x^{0} & 0 \\ x^{1} & x^{2} \\ x^{2} & x^{1} \\ 0 & x^{0} \\ 0 & 0 \\ 0 & x^{0} \end{array} $	$\begin{array}{c cccc} 0 & x^{0} & x^{1} \\ x^{0} & 0 & x^{2} \\ \hline x^{1} & x^{2} & 0 \\ x^{2} & x^{1} & x^{0} \\ \hline 0 & x^{0} & x^{1} \\ 0 & 0 & 0 \\ 0 & x^{0} & x^{1} \\ \end{array}$

Polynomiálny kódový priestor

Každý prvok konečného poľa rádu p^m je koreňom rovnice $X^{p^m} - X = 0$

Dôkaz:

nech $y \in F - \{0\}$ a x je primitívnym prvkom.

Existuje k tak že $y = x^k$. Potom

$$y^{p^m-1} = x^{k \cdot (p^m-1)} = 1$$

 $v^{p^m-1}-1=0$

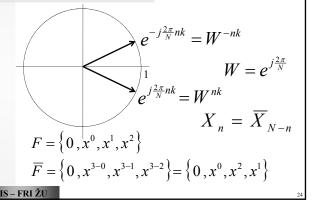
⊗	0	x^{o}	x1	x2
0	0	0	0	0
xθ	0	x^{o}	x^1	x^2
x^{I}	0	x'	x2	x°
x2	0	x^2	x°	x1

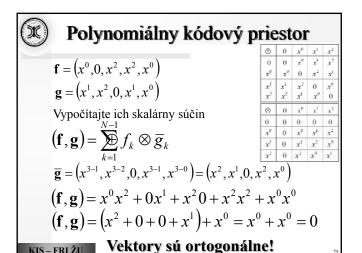
Príklad: p=2, m=2

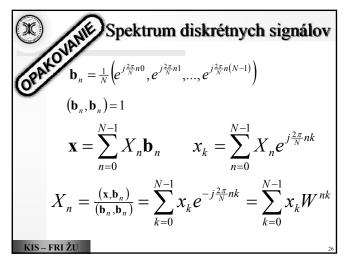
$$y^{2^2-1} = y^3 = 1$$
 $y = x^k \Rightarrow x^{3.k} = 1$

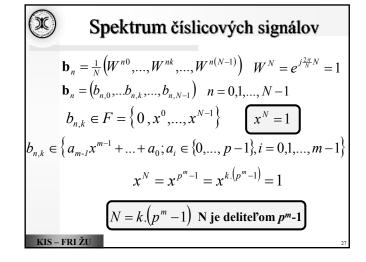


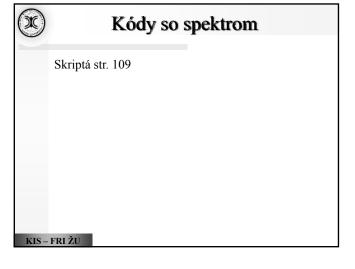
Komplexne združené číslo



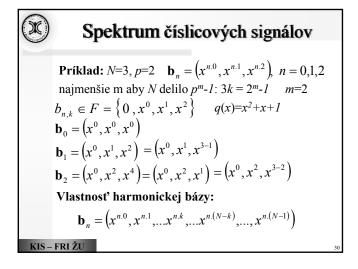














Spektrum číslicových signálov

$$\mathbf{b}_n = (x^{n0}, x^{n1}, ..., x^{n(N-1)}), \ n = 0, 1, ..., N-1$$

$$(\mathbf{b}_n, \mathbf{b}_n) = 1 \quad (\mathbf{b}_n, \mathbf{b}_m) = 1, \ n \neq m$$

$$\mathbf{x} = \sum_{n=0}^{N-1} X_n \mathbf{b}_n$$
 $x_k = \sum_{n=0}^{N-1} X_n x^{nk}$

$$X_n = \frac{(\mathbf{x}, \mathbf{b}_n)}{(\mathbf{b}_n, \mathbf{b}_n)} = \sum_{k=0}^{N-1} x_k x^{-nk}$$



Spektrum číslicových signálov

Príklad: N=3, p=2, m=2

$$\mathbf{b}_0 = (1,1,1)$$
$$\mathbf{b}_1 = (1,x,x^2)$$

 $\mathbf{b}_{2} = (1, x^{2}, x)$

0	0	x°	x^1	x^2		0	0	xº	x ¹	x2
0		xº				0	0	0	0	0
x^{o}	x°				П	x°	-0	xo		
x^{I}	x1	x2	0 .rº	x^{θ}	П	x^{I}	0	x^{l}	x2	x ⁰
x^2	x^2	x^1	x	0	П	χ^2	0	χ^2	X ⁰	χi

Vypočítajte spektrum slov $\mathbf{x} = (1,0,1) \mathbf{x}' = (1,1,1)$

$$X_0 = \frac{(\mathbf{x}, \mathbf{b}_0)}{(\mathbf{b}_0, \mathbf{b}_0)} = ((1, 0, 1), (1, 1, 1)) = 0$$

$$X_1 = \frac{(\mathbf{x}, \mathbf{b}_1)}{(\mathbf{b}_1, \mathbf{b}_1)} = ((1, 0, 1), (1, x, x^2)) = x$$

$$X_2 = \frac{(\mathbf{x}, \mathbf{b}_2)}{(\mathbf{b}_2, \mathbf{b}_2)} = ((1, 0, 1), (1, x^2, x)) = x^2$$



Spektrum číslicových signálov

Príklad: N=3, p=2, m=2

$$\mathbf{b}_0 = (1,1,1)$$

$$\mathbf{b}_0 = (1,1,1)$$
$$\mathbf{b}_1 = (1,x,x^2)$$

$$\mathbf{b}_1 = (1, x, x^2)$$

$$\mathbf{b}_2 = \left(1, x^2, x\right)$$

Vypočítajte spektrum slov $\mathbf{x} = (1,0,1) \mathbf{x}' = (1,1,1)$

$$X'_0 = \frac{(\mathbf{x}', \mathbf{b}_0)}{(\mathbf{b}_0, \mathbf{b}_0)} = ((1, 1, 1), (1, 1, 1)) = 1$$

$$X'_1 = \frac{(x', b_1)}{(b_1, b_1)} = ((1, 1, 1), (1, x, x^2)) = 0$$

$$X'_2 = \frac{(x', b_2)}{(b_2, b_2)} = ((1,1,1), (1, x^2, x)) = 0$$



Spektrum cyklického kódu

$$\mathbf{X} = (X_0, X_1, 0, X_3, \overline{X}_3, 0, \overline{X}_1)$$

Príklad: N=3, p=2, m=2 **X** = $(X_0, 0, :0)$

$$\mathbf{b}_0 = (1,1,1)$$

$$\mathbf{b}_1 = \left(1, x, x^2\right)$$

Spektrá slov kódu X = $(X_0, 0, :0)$, $X_0 \in \{0,1, x, x^2\}$

$$\begin{array}{l} \textbf{Spektrá slov nepatriacich do kódu} \\ \textbf{X} = \left(X_0, X_1, \vdots \overline{X}_1\right), \ X_0 \in \left\{0, 1, x, x^2\right\}, X_1 \in \left\{1, x, x^2\right\} \end{array}$$



Kódový multiplex

$$\mathbf{X} = (0, X_1, 0, 0, \vdots 0, 0, \overline{X}_1)$$

$$\mathbf{X}' = (0,0, X_2, 0, \vdots 0, \overline{X}_2 0)$$

Spektrálny multiplex

$$\mathbf{X''} = (0,0,0,X_3, \overline{X}_3,0,0)$$

 $\mathbf{X} + \mathbf{X}' + \mathbf{X}'' = (0, X_1, X_2, X_3, \overline{X}_2, \overline{X}_2, \overline{X}_1)$

Poznámka: uvedené spektrá kódových slov sú len ilustratívne a nemusia patriť realizovateľnému kódu



Teória oznamovania 12

Dakujem za Vašu pozornosť