Chapter 3 Ex 3.8 — Simple Gravity Pendulum

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1 Introduction

A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. The time for one complete cycle, a left swing and a right swing, is called the period. The period depends on the length of the pendulum and also to a slight degree on the amplitude, the width of the pendulum's swing.

This simulation is intended to explore the relation between the natural period and amplitude of a specific pendulum.

2 Method

For pendulum without friction but with big amplitude, its equation of motion is

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \tag{1}$$

where θ is the angular displacement, g is the gravity acceleration, and l is the length of the rope hanging the mass.

Write this SODE into two FODEs:

$$\frac{d\theta}{dt} = \omega \tag{2}$$

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin\theta \tag{3}$$

and translate to finite difference scheme:

$$\omega_{i+1} = \omega_i - \frac{g}{l}\sin\theta_i\Delta t \tag{4}$$

$$\theta_{i+1} = \omega_{i+1}\Delta t + \theta_i \tag{5}$$

$$t_{i+1} = t_i + \Delta t \tag{6}$$

Note that this is *semi-implicit Euler method* or so-called *Euler-Cromer method*.

3 Data & Verification

First let us see the influence of **large** angular amplitude. (I set $g/l = 1$)

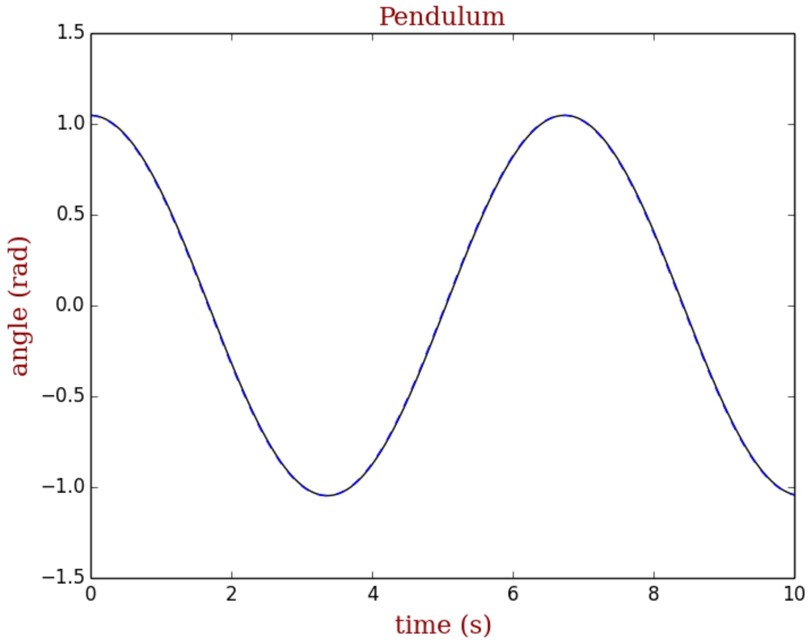

Figure 1: $\theta_{max} = \frac{\pi}{3}$

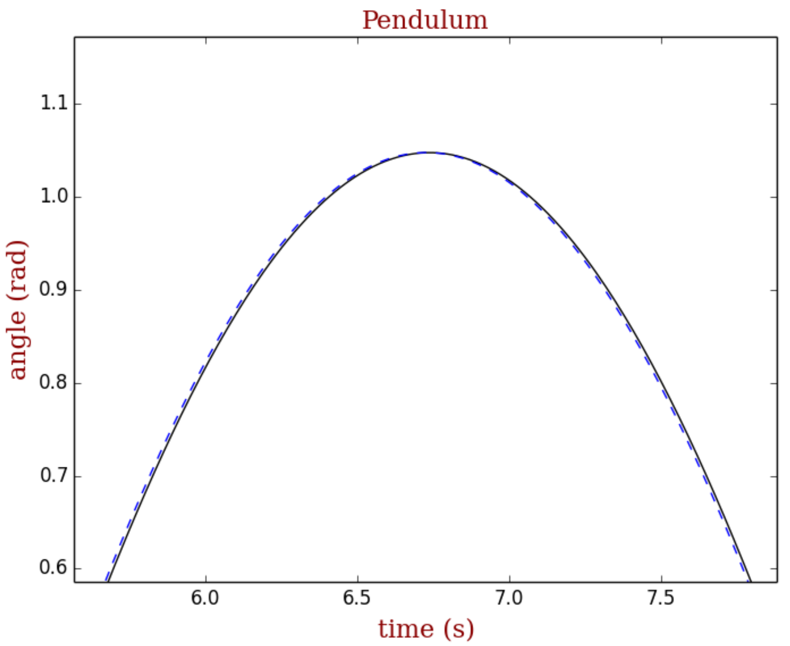

Figure 2: A magnified region around $t = 6.7s$. Dashed lines denote result of the small angle approximation.

And the relation between amplitude and period is shown as

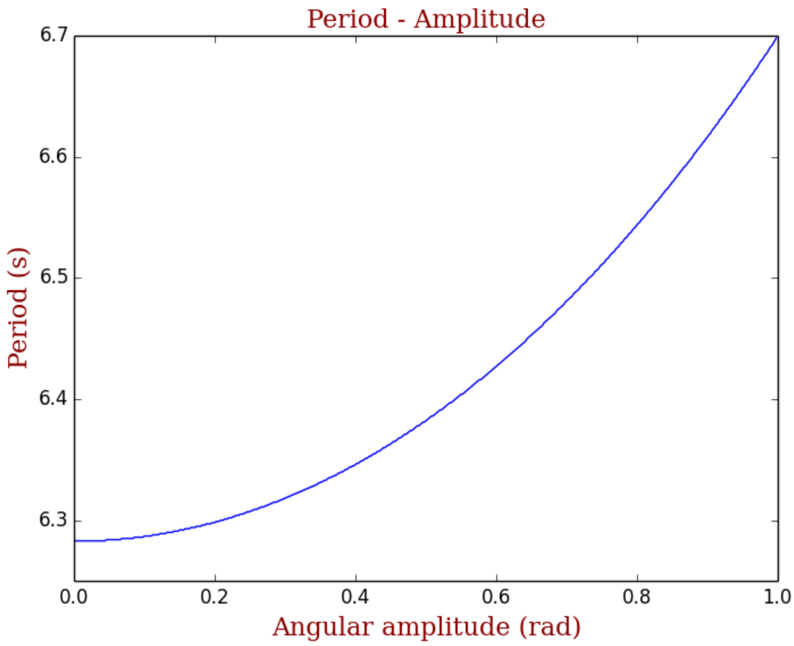

Figure 3: period vs amplitude

4 Interpretation & Analysis

As expected, for small angular amplitude the period is independent of the amplitude and is determined as $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi$. But as the amplitude increases, the small angle approximation failed. The first 2 terms of *Maclaurin series* for $\sin\theta$ is $\theta - \frac{\theta^3}{3!}$ which diminish the restoring force. Thus, similar to the damping oscillation, the period is enlarged. The extend has a close correlation with the angular amplitude, or explicitly, its cube.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007,*Computational Physics*

[2] Saul A. Teukolsky et.al, 1997,*Numerical Recipes*