Chapter 6 Ex 6.6 — Superposition of Two Wavepackets

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1 Introduction

An important feature of a linear equation is a that the sum of two solutions is also a solution of the origin function. Here we demonstrate this feature by setting up a string with an profile such that there are two Gaussian wavepackets located at different places on the string and observe the motion of the two wavepackets.

2 Method

The one dimensional wave equation is

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}. \tag{1}$$

Its discrete form is

$$y(i, n+1) = 2[1 - r^2]y(i, n) - y(i, n-1) + r^2[y(i+1, n) + y(i-1, n)], \tag{2}$$

where $i = 1, 2, ..., M$ denotes the horizontal position of points on the string, n denotes the time, and $r = c\frac{\Delta t}{\Delta x} = 1$. Here we set the ends of the string to be fixed, that is $\forall n, y(0, n) = y(M, n) = 0$. The initial condition is that we set two gaussian wavepackets on the string: $y(x, 0) = y(x_1, 0) + y(x_2, 0) = \exp[-k(x - x_1)^2] + \exp[-k(x - x_2)^2]$.

3 Data & Verification

We use $c = 300m/s, \Delta x = 0.005m, x_1 = 0.3m \text{and} x_2 = 0.6m$ to set two wavepackets:

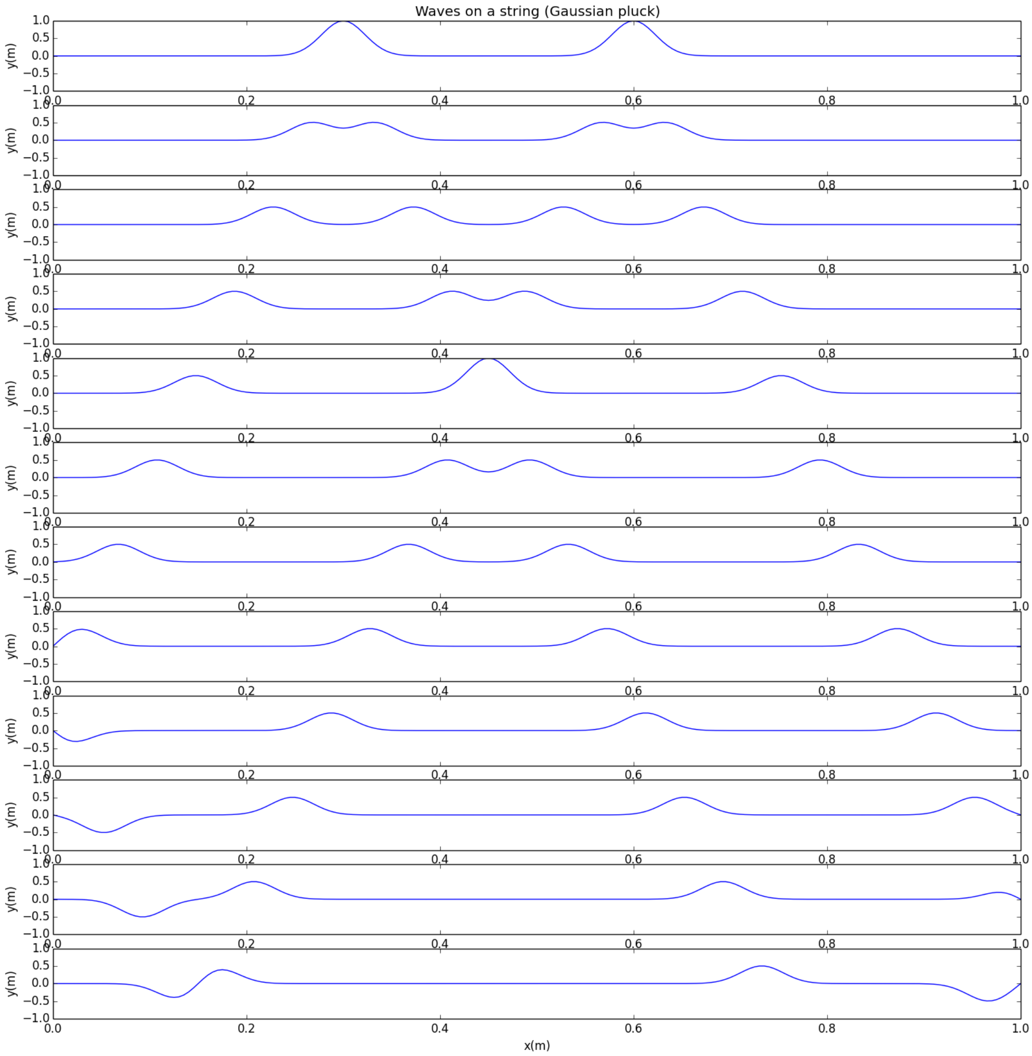

Figure 1: Motion of two packets.

Clearly, their motions meet superposition principle.

4 Interpretation & Analysis

The linear equation has superposition properties, which guarantees the independency of the propagation of two waves.

5 Acknowledgements

I thank Roach for his code performing single wave pack.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007, *Computational Physics*.