Chapter 5 Ex 5.3 — Symmetry Solution of the Capacitor

Yihua Qiang 2013301220058

1 Introduction

For a plate capacitor shown as below, we choose the potential of the square boundary surrounding the plates to be 0.

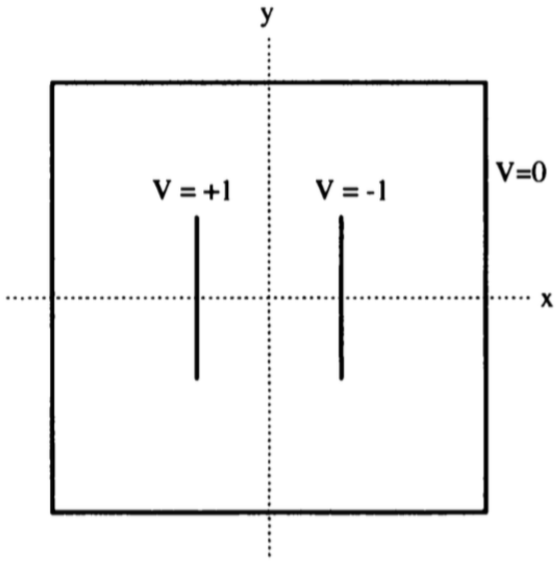

Figure 1: Capacitor

For this 2D problem, the static electric field equation is:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

where V is the potential, x and y denote the coordinate. The boundary condition here is *Dirichlet* condition.

2 Method

For numerical simulation, we choose the so-called, *Gauss-Seidel* method, which is an iterative method used to solve a linear system of equations. The discretized scheme of our *Laplace* equation is

$$V_{i+1}(x,y) = \frac{1}{4}\left[V_i(x+\Delta x, y) + V_i(x-\Delta x, y) + V_i(x, y-\Delta y) + V_i(x, y+\Delta y)\right]$$

Iterative schemes require time to achieve sufficient accuracy and are reserved for large systems of equations where there are a majority of zero elements in the matrix. Often times the algorithms are taylor-made to take advantage of the special structure such as band matrices. Practical uses include applications in circuit analysis, boundary value problems and partial differential equations.

Considering only the geometry, the system possesses 2 axises of symmetry, bisecting the square into 4 equivalent squares. If we consider the upper-right part, we find that the right and upper boundary to satisfy *Dirichlet* condition because of the original condition, and that the left boundary also satisfies *Dirichlet* condition by symmetry consideration.

Something subtle but interesting is that when calculating the electric potential of the horizontal axis of symmetry, we use the data of upper nearest neighbor to substitute the lower neighbor which is outside the region we consider. This meets our symmetry consideration and speed up a little for our program.

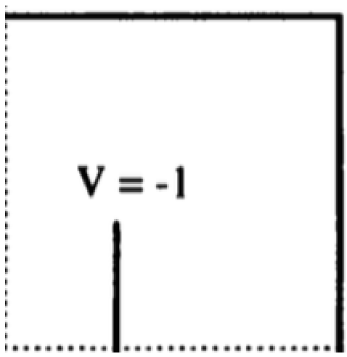

Figure 2: A quater region we consider.

After computing this region, we can quickly *copy* all its value to the whole plane we need to concern, adding some reflection technique and minus sign at most.

3 Data & Verification

The potential is shown below:

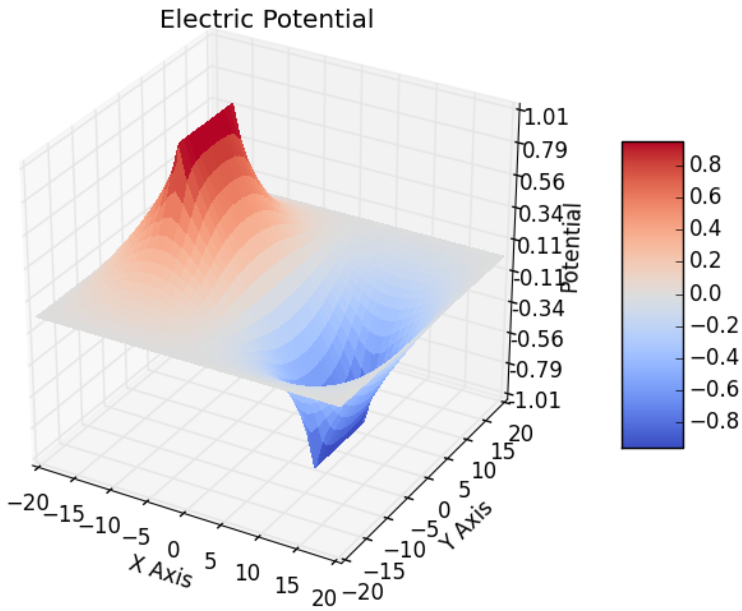

Figure 3: Electric potential

And its contour:

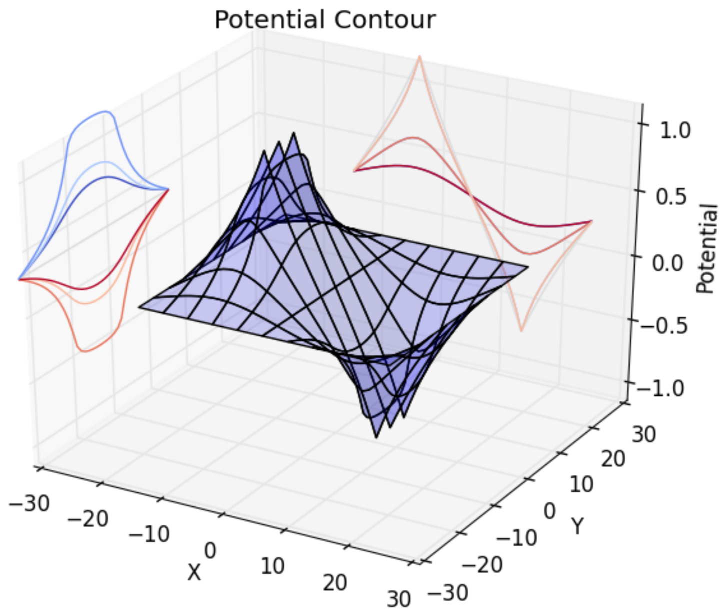

Figure 4: Potential contour

4 Interpretation & Analysis

Since the total time of iteration needed to reach a certain converge is proportional to L^2, the symmetry analysis is necessary when solving large system, as long as some kind of symmetry is possessed by the system.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007,*Computational Physics*.

[2] William H. Press et.al.,1986,*Numerical Recipes*