Chapter 4 Ex 4.7 — Binary System

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1 Introduction

A binary system is a system of two objects in space (usually stars, but also brown dwarfs, planets, neutron stars, black holes, galaxies, or asteroids) which are close enough that their gravitational movement causes them to circle around each other (orbit) around a shared mass. Some definitions (e.g. that of double planet, but not that of binary star) require that this center of mass is not located within the interior of either object. A multiple system is like a binary system but consists of three or more objects. Herein we consider a simplest case where, only the ratio of the masses of two objects varies.

2 Method

If the ratio of masses of the two objects is $r = \frac{m_2}{m1}$, then in the center of mass frame, the simplest circular orbit will be of radii $r_1 = \frac{m_2}{m_1+m_2}d$ and $r_2 = \frac{m_1}{m_1+m_2}d$, where d is the distance between the two objects. From the law of gravitation and Newton's Law, we get (from Ref[1]):

$$\begin{aligned}
\frac{dv_x}{dt} &= -\frac{Gm_1m_2x}{r_1^3} \\
\frac{dx}{dt} &= v_x \\
\frac{dv_y}{dt} &= -\frac{Gm_1m_2y}{r_1^3} \\
\frac{dy}{dt} &= v_y
\end{aligned}$$

And for the other object it has the same equations, where r is the radius of the orbital with respect to the center of mass. Note that it is the so called*Euler-Cromer* method.

3 Data & Verification

If the ratio of the masses of two objects is 1, then the two orbits are overlapped:

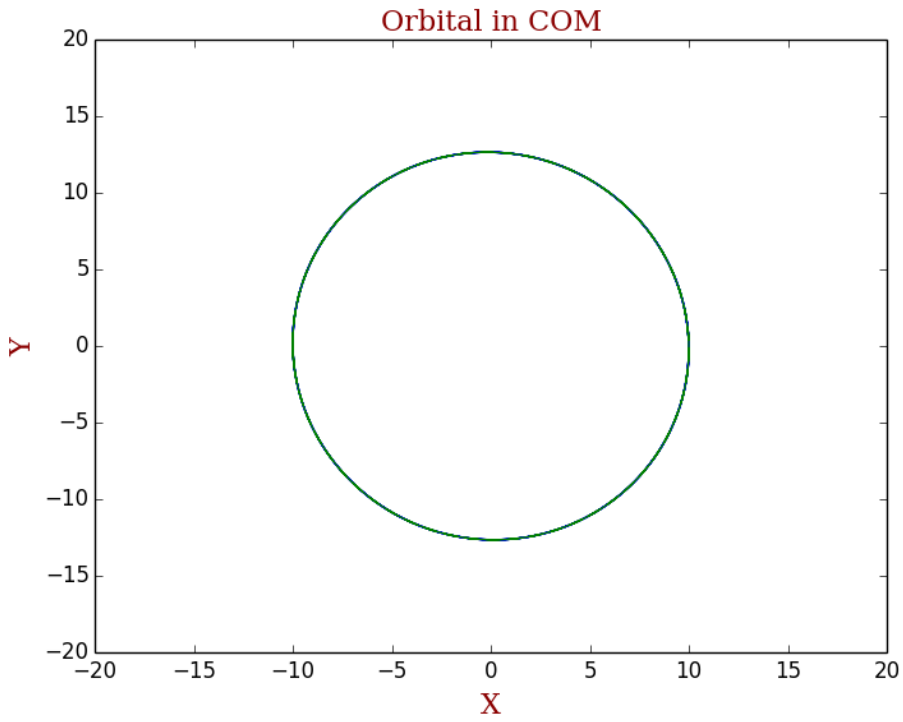

Figure 1: $m_1 = m_2$

and when the ratio of mass is not 1:

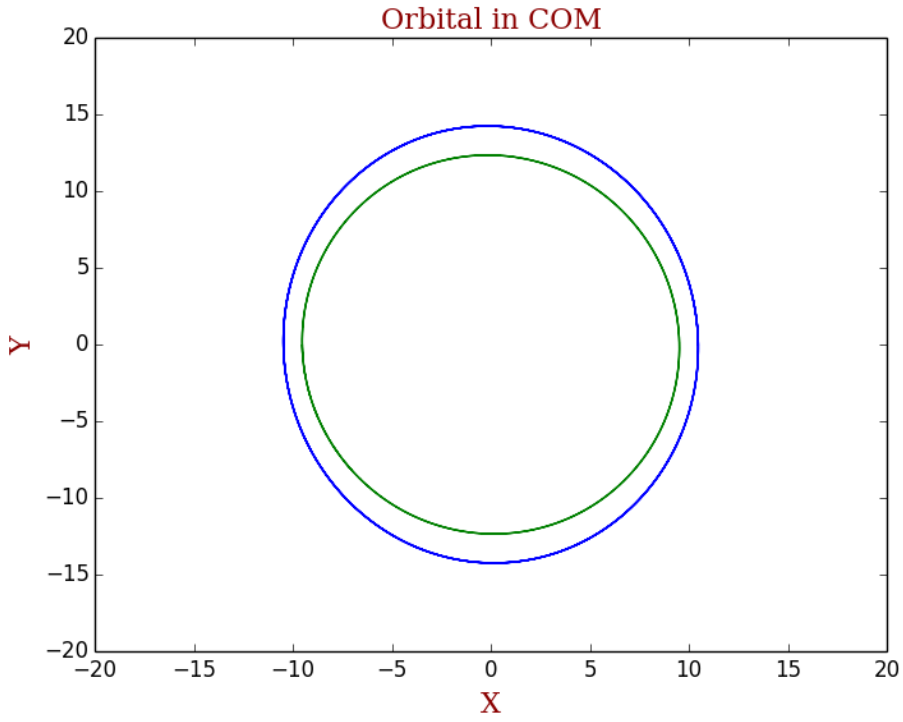

Figure 2: $m_2/m_1 = 0.9$

4 Interpretation & Analysis

As expected, the simplest binary system where the initial total linear momentum is zero obeys a circular orbit.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007, *Computational Physics*.