Chapter 2 Projectile motion

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1 Introduction

Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only. The only force of significance that acts on the object is gravity, which acts downward to cause a downward acceleration. Because of the object's inertia, no external horizontal force is needed to maintain the horizontal motion.

Herein, the cannon shooting problem is investigated considering the air drag force, gravitation, *Coriolis* force and the change of air density and gravitation with respect to flying height.

2 Method

Follow the spirit of ref[1], we write all forces in its three Cartesian components with $\hat{\imath}$ directs south, $\hat{\jmath}$ east and $\hat{k}$ upward. Before repeating the similar procedure in ref[1] for decomposition of the gravitation and air drag force $\boldsymbol{F}_{drag}$, we first look at the *Coriolis* force $\boldsymbol{F}_{Cor}$ involved in this specific problem.

The form of *Coriolis* force is

$$\begin{aligned} \boldsymbol{F}_{Cor} &= -2m\boldsymbol{\omega} \times \boldsymbol{v} \\ &= \hat{\imath}2m(v_y\omega_z - v_z\omega_y) + \hat{\jmath}2m(v_z\omega_x - v_x\omega_z) + \hat{k}2m(v_x\omega_y - v_y\omega_x), \end{aligned} \tag{1}$$

Common cannon shell has a range of 20km to 30km, which is far less than the radius of our earth, thus it is reasonable to consider the direction of $\boldsymbol{\omega}$ does not change. If the cannon shell is shoot eastward, i.e. $\omega_y = 0$, Eq.(1) becomes

$$\boldsymbol{F}_{Cor} = \hat{\imath}2mv_y\omega_z + \hat{\jmath}2m(v_z\omega_x - v_x\omega_z) - \hat{k}2mv_y\omega_x \tag{2}$$

Together with F_{drag},

$$\begin{aligned} \boldsymbol{F}_{drag} &= \hat{\imath}F_{drag_x} + \hat{\jmath}F_{drag_y} + \hat{k}F_{drag_z} \\ &= \hat{\imath}(-B_2vv_x) + \hat{\jmath}(-B_2vv_y) + \hat{k}(-B_2vv_z), \end{aligned} \tag{3}$$

where B_2 is the same as in ref[1], v is the speed of cannon shell at given moment,

And with gravitation,

$$m\boldsymbol{g} = \hat{\jmath} mg \tag{4}$$

where g changes with height as $g = 9.78049 \cdot (1 + 0.005288 \cdot \sin^2 lat - 0.000006 \cdot \sin(2lat^2)) - 0.0003086 \cdot z$ and lat is the latitude of your artillery. We have

$$a_x = 2v_y\omega_z - \frac{B_2}{m}vv_x \tag{5}$$

$$a_y = 2(v_z\omega_x - v_x\omega_z) - \frac{B_2}{m}vv_y \tag{6}$$

$$a_z = -2v_y\omega_x - \frac{B_2}{m}vv_z - g \tag{7}$$

Following the acceleration, we get the finite difference form of $\boldsymbol{v}$ and $\boldsymbol{x}$.

$$v_{\alpha i+1} = a_{\alpha i}\Delta t \tag{8}$$

$$x_{\alpha i+1} = v_{\alpha i}\Delta t \tag{9}$$

where $\alpha = x, y, z$.

As for the initial condition,

$$\begin{aligned} v_{x0} &= v_0 \sin\theta \cos\phi \\ v_{y0} &= v_0 \sin\theta \sin\phi \\ v_{z0} &= v_0 \cos\theta \end{aligned} \tag{10}$$

where v_0 is the initial speed, θ is the shooting angle, ϕ describes the direction, both with respect to y axis.

3 Data & Verification

We choose our target $30km$ away, and explore the feasible combination of shooting angle and initial speed in a given range say $30° < \theta < 40°, 700m/s <$ initial speed $< 900m/s$, and leave the ϕ correction to the computer.

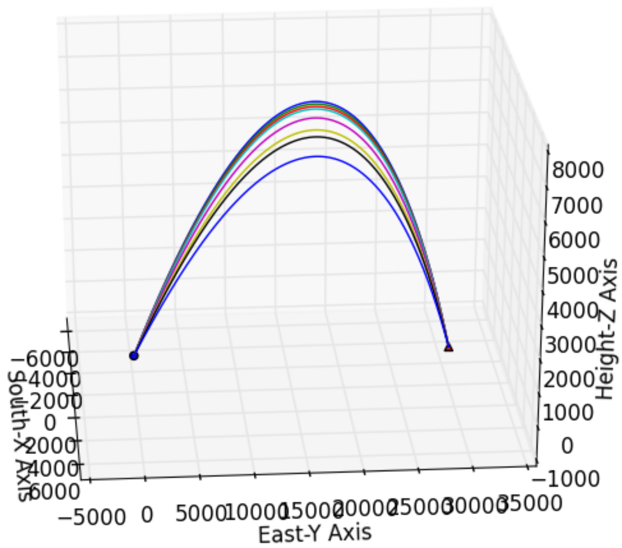

Figure 1: ϕ correction about $-0.1°$ with target 30000m away

where red dot is the starting point and blue triangle symbols the target To see a specific trajectory,

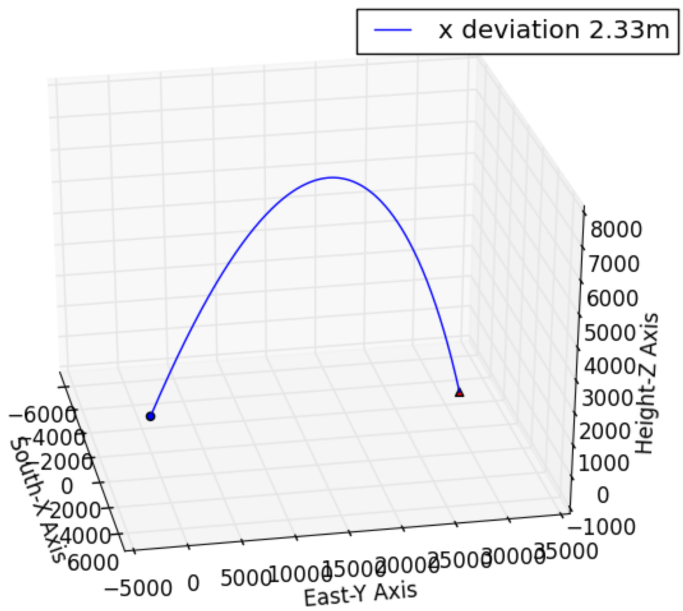

Figure 2: ϕ correction about $-0.09°$, y-range 30000.39633m, x-deviation 2.33937m

That means our ϕ correction works well, without which the deviation will be over $40m$ due south when shooting eastward.

4 Interpretation & Analysis

We see that there are in principle infinitely many combinations of initial speed and firing angle to hit a given target, but optimization can lead to a few best combinations (usually one), if the error of initial condition is to be considered. One of the approach is to write the functional of the mean deviation:

$$S = F[f(\boldsymbol{r}, t)],$$

where $f(\boldsymbol{r}, t)$ is the trajectory of cannon shell, with ends represent the starting point and the landing point. But here the latter point is not fixed, so this is a little complicated.

Another way is just the simulation, which may not give exactly the same result but converge to the best one(s) when you repeating infinite times of experiments.

Hopefully, even you consider the shape of earth within $30km$'s range, this do not raise too much difficulty, and *Gaussian distribution* is easy to generate using *numpy* module in Python, detailed realization may be conducted a few moment later.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007,*Computational Physics*

[2] Wikipedia – *Projectile motion*