Chapter 4 Ex 4.20 — Hyperion

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1 Introduction

Hyperion, also known as Saturn VII, is a moon of Saturn discovered by William Cranch Bond, George Phillips Bond and William Lassell in 1848. It is distinguished by its irregular shape, its chaotic rotation, and its unexplained sponge-like appearance. It was the first non-round moon to be discovered.

Hyperion's discovery came shortly after John Herschel had suggested names for the seven previously-known satellites of Saturn in his 1847 publication Results of Astronomical Observations made at the Cape of Good Hope. William Lassell, who saw Hyperion two days after William Bond, had already endorsed Herschel's naming scheme and suggested the name Hyperion in accordance with it. He also beat Bond to publication. Due to its uneven mass distribution and highly elliptical orbit, the motion of Hyperion is chaotic.

2 Method

The mass distribution of Hyperion can be simplified as a dumbbell-like, as pointed out in Ref[1]. The gravitational force on m_1 is

$$\vec{F} = \frac{GM_{sat}m_1}{r^3}(x_1\hat{\imath} + y_1\hat{\jmath}),$$

where M_{sat} is the mass of Saturn, r is the distance from the center of mass to Saturn.

The torque exert on m_1 is

$$\vec{\tau}_1 = [(x_1 - x_c)\hat{\imath} + (y_1 - y_c)\hat{\jmath}] \times \vec{F}_1,$$

which is similar for m_2

So the angular acceleration is

$$\begin{aligned}\frac{d\omega}{dt} &= \frac{\tau_1 + \tau_2}{I} \\ &= \frac{\tau_1 + \tau_2}{m_1 r_1^2 + m_2 r_2^2} \\ &\approx -\frac{3GM_{sat}}{r_c^5}(x_c \sin\theta - y_c \cos\theta)(x_c \cos\theta + y_c \sin\theta)\end{aligned}$$

where θ is the angle between the line connecting m_1 and m_2 and the x-axis. After integrate twice we get the θ from the angular acceleration.

3 Data & Verification

In Hyperion unit, the time unit is Hyperion year, the length unit is the radius of Hyperion's circular orbit. Thus the circular orbit is

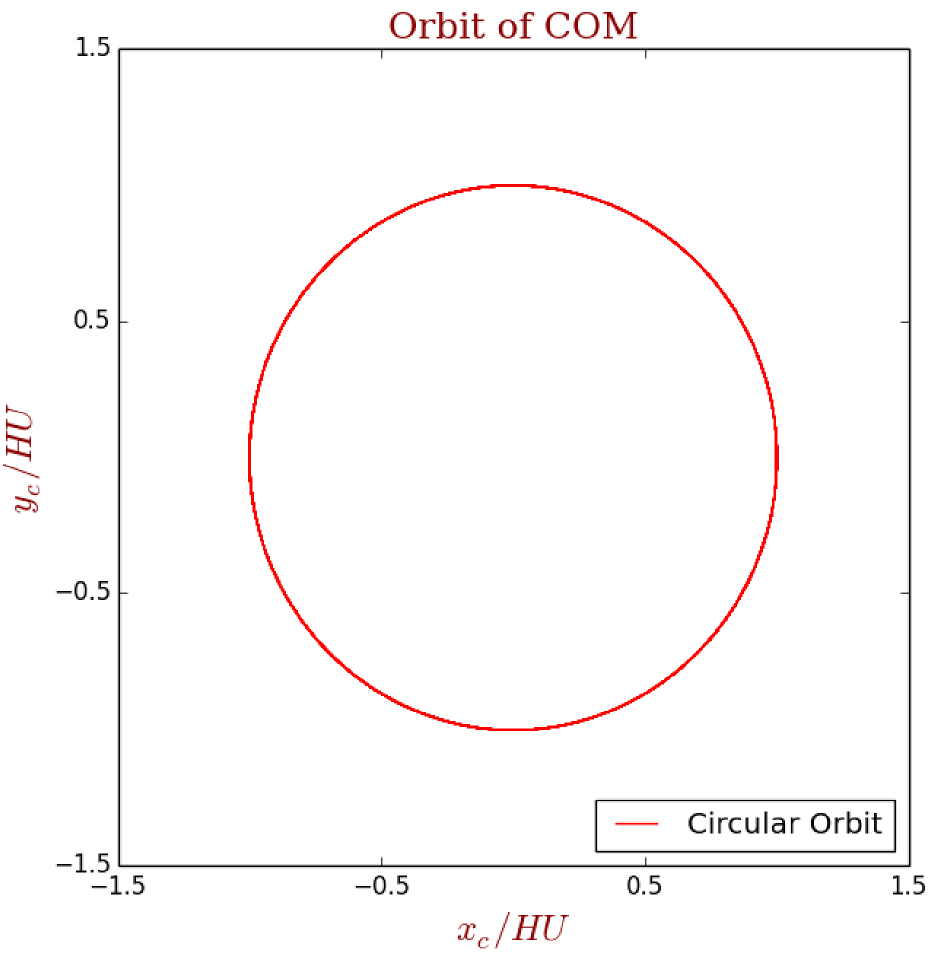

Figure 1: Circular orbit of center of mass

the angular velocity is

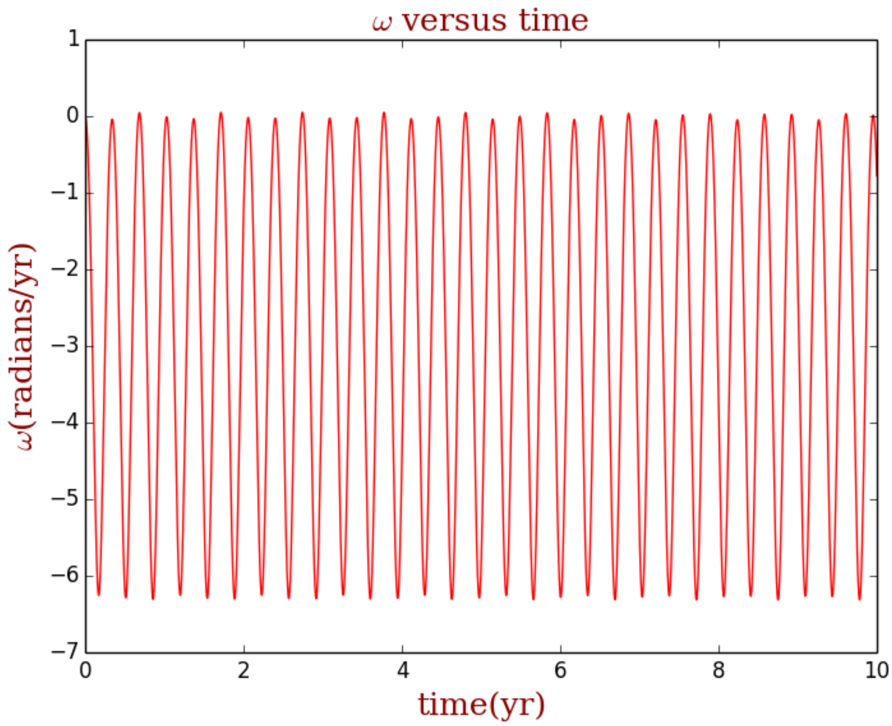

Figure 2: Non chaotic angular velocity

while when the center of mass is under elliptical motion,

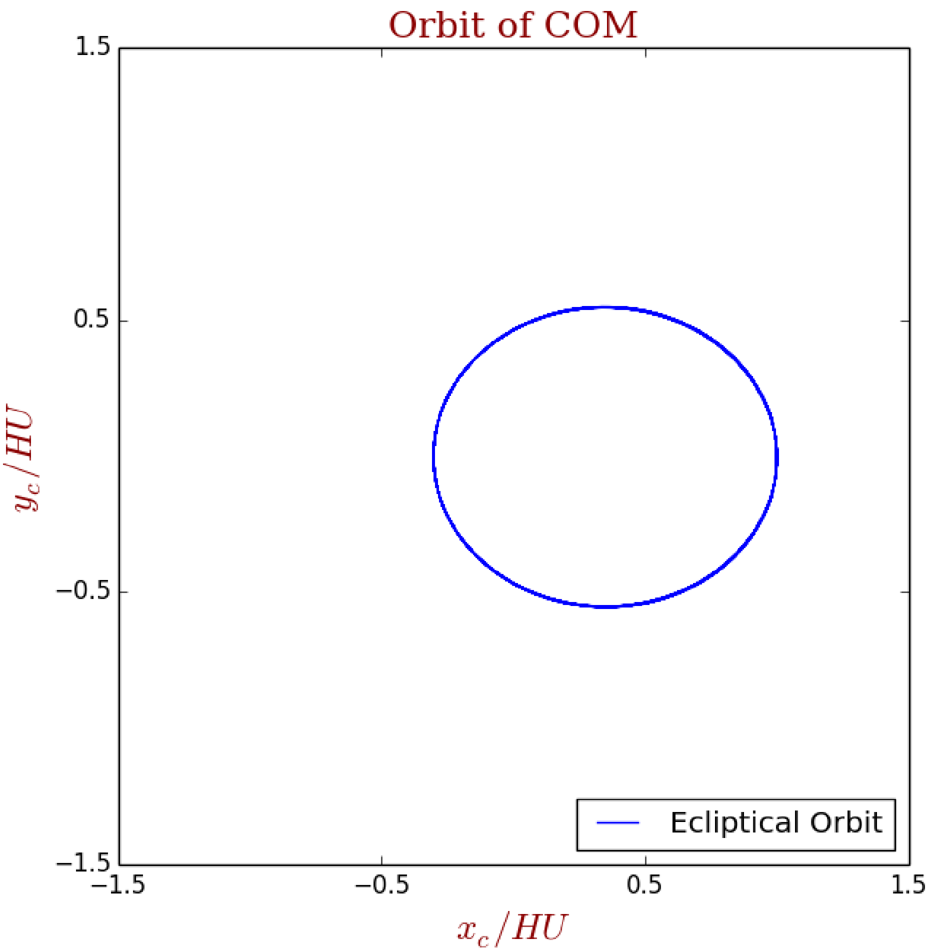

Figure 3: Circular orbit of center of mass

the angular velocity of the "spin" of Hyperion is chaotic:

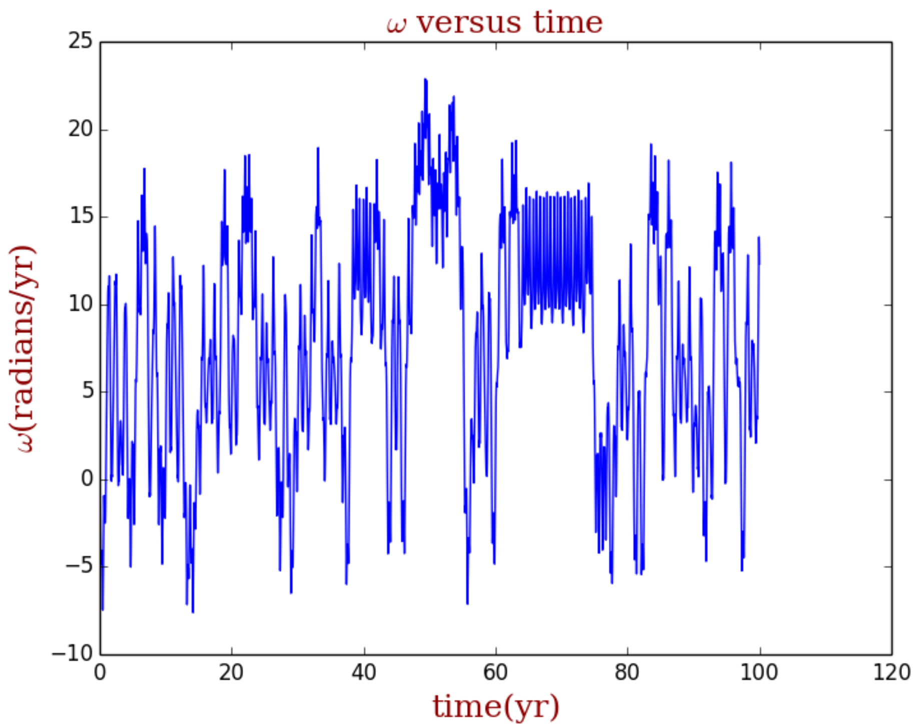

Figure 4: Chaotic angular velocity

If we put two initial θ differ only 0.1 radians, we find $\Delta\theta$ vs time as

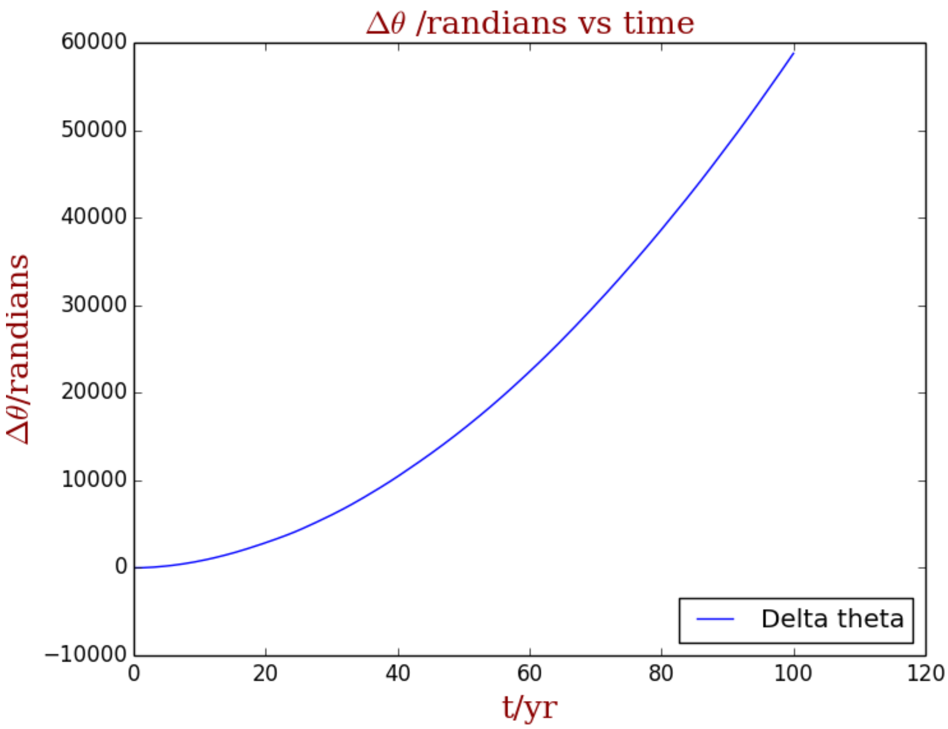

Figure 5: $\Delta\theta$ vs time

4 Interpretation & Analysis

As tidal dissipation drives Hyperion's spin toward a nearly synchronous value, Hyperion necessarily enters the large chaotic zone. At this point Hyperion becomes attitude unstable and begins to tumble.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007,*Computational Physics*.

[2] Jack Wisdom et.al.,1983, *ICARUS* **58**, 137-152.