Chapter 3 Ex 3.12 —Chaotic Pendulum

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1 Introduction

A damped driven pendulum is often used as a basic example of a chaotic system. For a chaotic system the future behavior is highly dependent on the exact value of the initial conditions. A tiny change in initial conditions can cause huge changes after a short period of time. In this simulation, we plot the *Poincaré* section to analyze the behavior of the system.

2 Method

The equation of motion for this pendulum with driving force is

$$\begin{aligned} \frac{d\omega}{dt} &= -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t) \\ \frac{d\theta}{dt} &= \omega \end{aligned} \tag{1}$$

where ω and θ is the angular velocity and angular displacement of the pendulum, while Ω_D is the angular frequency of the external driving force, which maybe, for example, resulted from an electric field when the bob is charged. q is the damping coefficient.

From reference [1], the difference scheme is

$$\begin{aligned} \omega_{i+1} &= \omega_i + [-(g/l)\sin\theta - q\omega_i + F_D\sin\Omega_D t_i]\Delta t \\ \theta_{i+1} &= \theta_i + \omega_{i+1}\Delta t \\ t_{i+1} &= t_i + \Delta t \end{aligned} \tag{2}$$

Note that it is already the $Euler - Cromer$ method.

3 Data & Verification

As a test, first let us repeat the result of [1]. The phase diagram with $F_D = 1.2, q = 0.5$ is

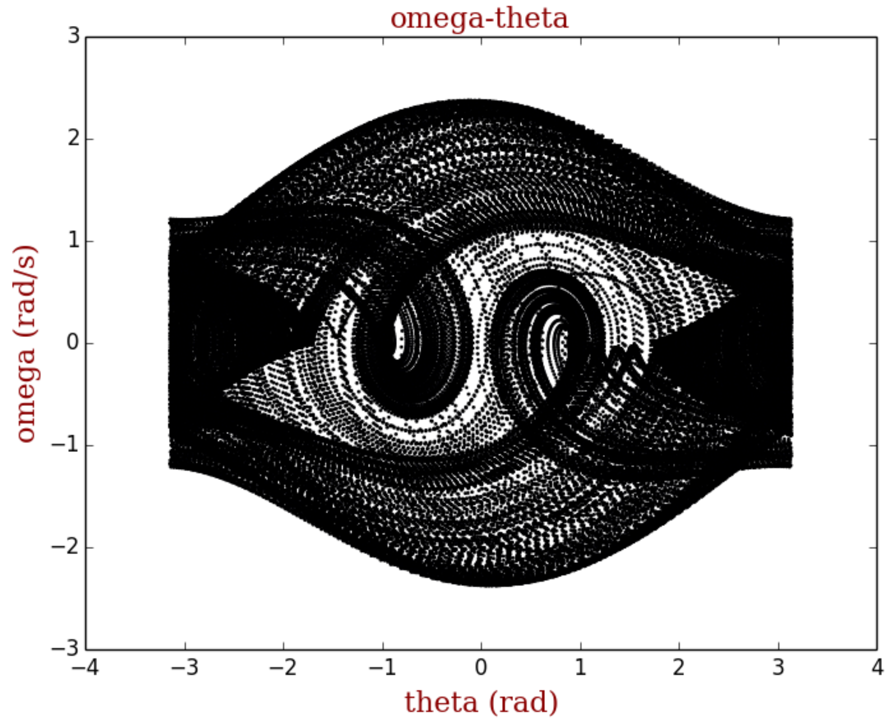

Figure 1: $F_D = 1.2, q = 0.5, \omega vs \theta$

Then we get three *Poincaré* sections corresponding to $\Omega_D t = 2n\pi, 2n\pi + \frac{\pi}{2}$ and $2n\pi + \frac{\pi}{4}, (n = 0, 1, 2, ...)$.

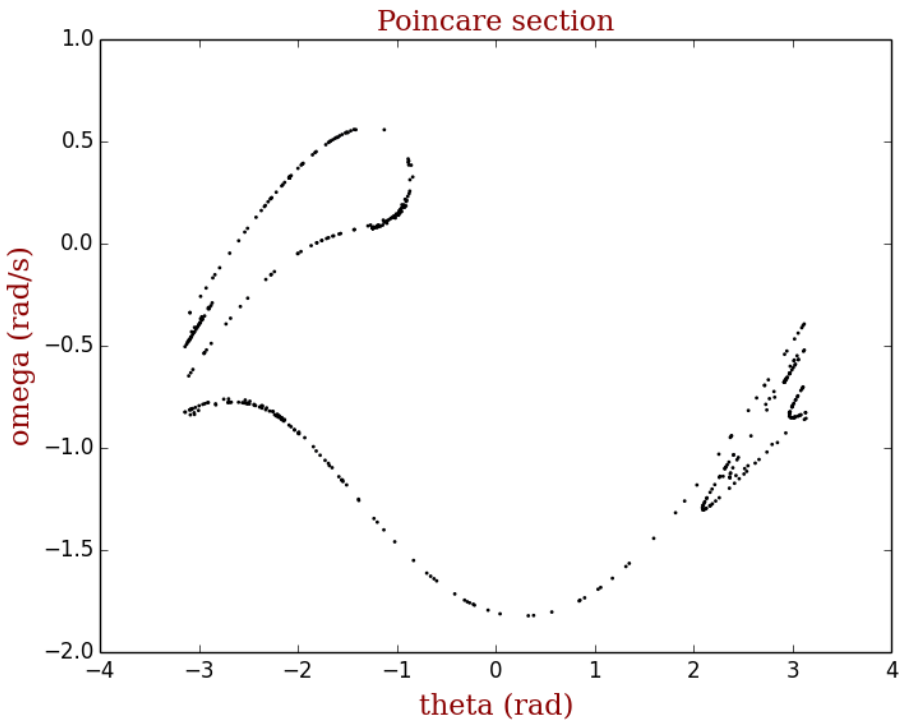

Figure 2: $\Omega_D t = 2n\pi$

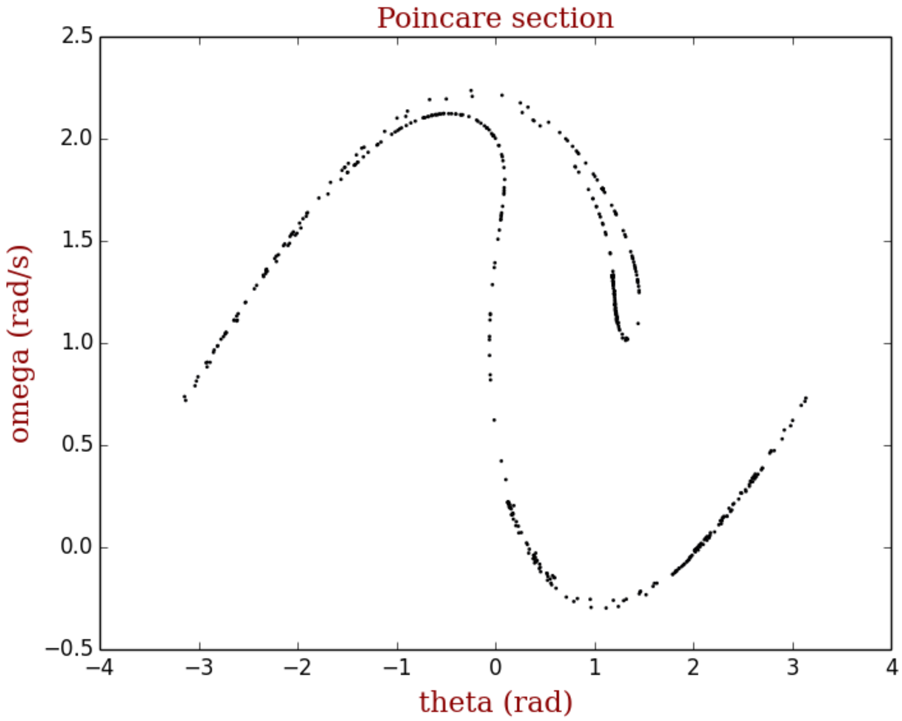

Figure 3: $\Omega_D t = 2n\pi + \frac{\pi}{2}$

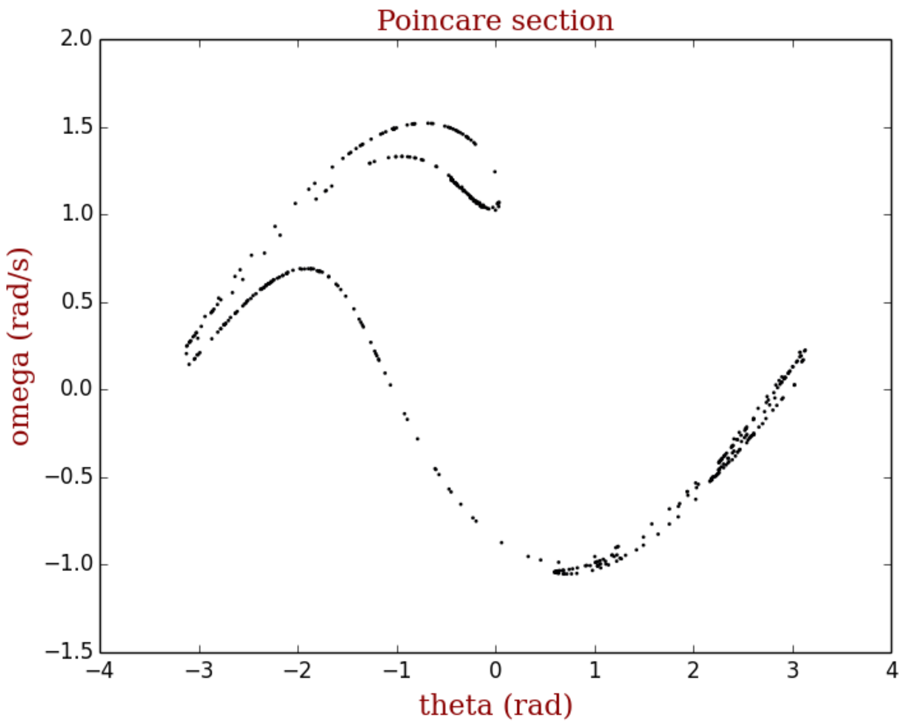

Figure 4: $\Omega_D t = 2n\pi + \frac{\pi}{4}$

Here $0, \frac{\pi}{2}$ and $\frac{\pi}{4}$ are also called *Poincaré* phases.

4 Interpretation & Analysis

Compare the latter two *Poincaré* sections with the first one, one may find the resemblance in the different sections with similar sub-structure all displaying the chaotic feature of the system.

References

[1] Nicholas J. Giordano, Hisao Nakanishi, 2007,*Computational Physics*.