习题一 P.22

1. 证重言式

(a) $A \to (\neg A \to B)$ $\cancel{\neg A \vee}$

$= \neg(\neg A) \to (A \vee B)$

$= (\neg A \vee A) \vee B$

$= 1 \vee B = 1$

(b) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

$= [(\neg A) \vee (\neg B) \vee C] \to [((\neg A) \vee B) \to ((\neg A) \vee C)]$

$= [(\neg A) \vee (\neg B) \vee C] \to [A \vee (\neg B) \vee (\neg A) \vee C]$

$= A \vee \overset{B}{\cancel{(\neg B)}} \vee (\neg C) \vee A \vee (\neg B) \vee (\neg A) \vee C$

$= A \vee (\neg A) \vee B \vee (\neg B) \vee C \vee (\neg C) = 1$

(c) $(\neg A \to \neg B) \to (B \to A)$

$= (A \vee (\neg B)) \to ((\neg B) \vee A)$

$= (\neg A) \cancel{\vee (\neg B)} \vee B \vee (\neg B) \vee A$

$= A \vee (\neg A) \vee B \vee (\neg B) = 1$

2. 证明公式成立

(a) $(A \vee B) \to C = (A \to B) \wedge (B \to C)$

① $(A \vee B) \to C = \neg(A \vee B) \vee C = (\neg A \wedge \neg B) \vee C$

$= (\neg A \vee C) \wedge (\neg B \vee C)$

② $(A \to B) \wedge (B \to C) = (\neg A \vee C) \wedge (\neg B \vee C)$

① $\neq$ ②

(b) $(A\wedge B)\rightarrow C = (A\rightarrow C)\vee(B\rightarrow C)$

① $(A\wedge B)\rightarrow C = \neg(A\wedge B)\vee C = (\neg A\vee\neg B)\vee C$

② $(A\rightarrow C)\vee(B\rightarrow C) = (\neg A\vee C)\vee(\neg B\vee C) = \neg A\vee\neg B\vee C$

①=② 得证、

(c) $A\rightarrow(B\rightarrow C) = B\rightarrow(A\rightarrow C)$

① $A\rightarrow(B\rightarrow C) = A\rightarrow(\neg B\vee C) = \neg A\vee\neg B\vee C$

② $B\rightarrow(A\rightarrow C) = B\rightarrow(\neg A\vee C) = \neg B\vee\neg A\vee C$

①=② 得证、

3、$(\neg p_1\rightarrow p_2)\rightarrow p_3$

$= (p_1\vee p_2)\rightarrow p_3 = \neg(p_1\vee p_2)\vee p_3$

$= (\neg p_1\wedge\neg p_2)\vee p_3 = (p_3\vee\neg p_1)\wedge(p_3\vee\neg p_2)$

析取范式：$(\neg p_1\wedge\neg p_2)\vee p_3$

合取范式：$(p_3\vee\neg p_1)\wedge(p_3\vee\neg p_2)$

4、真度

(a) $(p_1\vee p_2)\rightarrow p_3 = (\neg p_1\wedge\neg p_2)\vee p_3$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	1
1	0	1
1	1	1

时命题为真；真度为 $\frac{5}{8}$

(b) $(p_1 \to p_2) \vee (p_3 \to p_4) = \neg p_1 \vee p_2 \vee \neg p_3 \vee p_4$

仅当$(p_1, p_2, \cancel{\text{系}} p_3, p_4) = (1, 0, 1, 0)$时命题为假

故真度 $\tau(A) = \dfrac{15}{16}$

(c) $(\neg p_1 \to p_2) \to p_3 = (p_1 \wedge \neg p_2) \vee p_3$

当

p_1	p_2	p_3
0	0	1
0	1	1
1	0	0
1	0	1
1	1	1

时，命题为真，

故真度为 $\dfrac{5}{8}$

5、证明

$A \downarrow A = \neg(A \vee A) = \neg A$

$(A \downarrow B) \downarrow (A \downarrow B) = \neg(A \downarrow B) = \neg(\neg(A \vee B)) = A \vee B$

$(A \downarrow A) \downarrow (B \downarrow B) = \neg A \downarrow \neg B = \neg(\neg A \vee \neg B) = A \wedge B$

$((A \downarrow A) \downarrow B) \downarrow ((A \downarrow A) \downarrow B) = (\neg A \downarrow B) \downarrow (\neg A \downarrow B)$

$= \neg(\neg A \downarrow B) = \neg(\neg(\neg A \vee B)) = \neg A \vee B = A \to B$.

$\therefore$ 可以用 $\{\downarrow\}$ 表示 $\{\neg, \vee, \wedge, \to\}$，得证.

6、不存在

$\because m = 2^k$，故 m 不可能为除 1 以外的奇数.

故不存在这样的命题 A