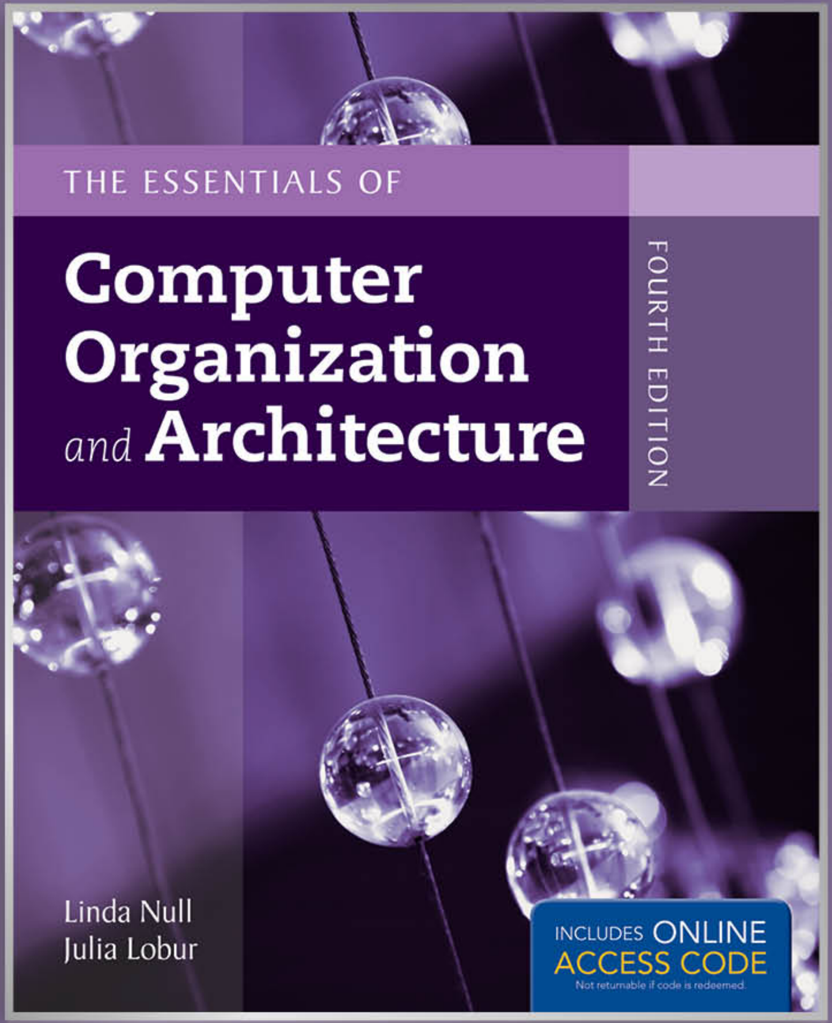
Chapter 3 Special Section

Karnaugh Maps



3A.1 Introduction

- **Simplification of Boolean functions** leads to **simpler and faster circuits**.
- Simplifying functions **using identity laws** is **time-consuming and error-prone**.
- This section presents **Karnaugh Maps** (Kmaps), an **easy, systematic method for reducing Boolean functions**.

3A.2 Description of Kmaps

- A Kmap is a **matrix consisting of rows and columns** that **represent the output values** of a **Boolean function**.
- The **output values** placed in each cell are **derived from the minterms** of a function.
- A **minterm** is a **Boolean product** that contains **all variables exactly once**, either **complemented or not complemented**.

3A.2 Description of Kmaps

- The minterms for a function having the inputs *x* and *y* are **`x'y'`**, **`x'y`**, **`xy'`**, and **`xy`**.
- Consider the Boolean function, **`F(x,y)= xy + xy'`**
- **Minterm table**:

Minterm	X	Y
X'Y'	0	0
X'Y	0	1
X Y'	1	0
X Y	1	1

3A.2 Description of Kmaps

- Minterm table of a function with variables x, y, z:

Minterm	X	Y	Z
X'Y'Z'	0	0	0
X'Y'Z	0	0	1
X'Y Z'	0	1	0
X'Y Z	0	1	1
X Y'Z'	1	0	0
X Y'Z	1	0	1
X Y Z'	1	1	0
X Y Z	1	1	1

3A.2 Deriving Kmaps from Truth Tables

$$F(x, y) = (x'y) + (xy') + (xy)$$

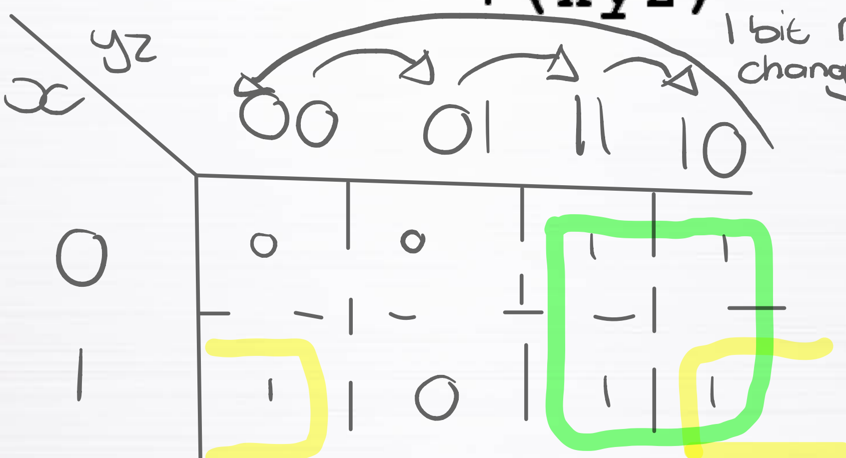
x \ y	0	1
0	$x'y'$ 0	$x'y$ 1
1	xy' 1	xy 1

X	Y	F(x,y)
0	0	0
0	1	1
1	0	1
1	1	1

3A.2 Deriving Kmaps from Truth Tables

$$F(x,y,z) = (x'yz') + (x'yz) + (xy'z') + (xyz') + (xyz)$$

x	y	z	F(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



1 bit may change at a time

x can be 0 or 1 so x does not matter

z can be 0 or 1 so z does ...

$f = y + xz'$

3A.2 Description of Kmaps

- A Kmap has a **cell for each minterm**.
- This means that it has a **cell for each line of the truth table** of a function.
- **Example:**

F(X,Y) = XY

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X \ Y	0	1
0	0	0
1	0	1

3A.3 Kmap Simplification for Two Variables

- We can **reduce the function** of the Kmap to its simplest terms **by finding neighbored 1s** that can be collected into groups **that are powers of two**.
- Possible group sizes are 1, 2, 4, 8, 16, …

- In our example, we have two such groups.
  – Can you find them?

X \ Y	0	1
0	0	1
1	1	1

3A.3 Kmap Simplification for Two Variables

- We see that **both groups are powers of two** and that the **groups overlap**.
- The next slide gives guidance for selecting Kmap groups.

green = $y=1$

Pink = $x=1$

$$f = y + x$$

X \ Y	0	1
0	0	1
1	1	1

3A.3 Kmap Simplification for Two Variables

The **rules** of Kmap simplification are:

- Groups can contain **1s only**; no 0s.
- Groups can be formed only at **right angles**; diagonal groups are not allowed.
- The **group sizes** must be a **power of 2** – even if it contains a single 1.
- The **groups must** be made **as large as possible.**
- **Groups can overlap** and **wrap around** the sides of the Kmap.

3A.3 Kmap Simplification for Three Variables

$$F(x,y,z) = (x'yz') + (x'yz) + (xy'z') + (xyz') + (xyz)$$

x	y	z	xz' + y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

– Notice that the values for the **yz** combination is not a normal binary sequence.

x \ yz	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x\ y\ z$	$x'y\ z'$
1	$x\ y'z'$	$x\ y'z$	$x\ y\ z$	$x\ y\ z'$

3A.3 Kmap Simplification for Three Variables

$$F(X,Y,Z) = X'Y'Z + X'YZ + XY'Z + XYZ$$

– What is the largest group of 1s that is a power of 2?

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3A.3 Kmap Simplification for Three Variables

- This group tells us that changes in the variables *x* and *y* have no influence on the value of the function: They are irrelevant.
- This means that the function,

F(X,Y,Z)=X′Y′Z + X′YZ + XY′Z + XYZ

reduces to **F(z) = z***.*

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3A.3 Kmap Simplification for Three Variables

- A more complicated Kmap:

$$F(X,Y,Z) = X'Y'Z' + X'Y'Z + X'YZ + X'YZ' + XY'Z' + XYZ'$$

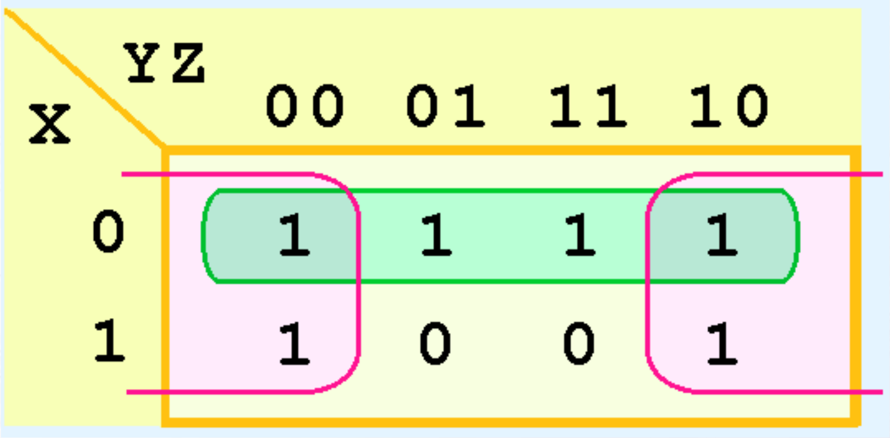
- Its Kmap is shown below. There are (only) two groupings of 1s.
  - Can you find them?

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

3A.3 Kmap Simplification for Three Variables

- The **pink group wraps around** the sides of a Kmap.
- This group tells us that the **values of *x* and *y*** are **not relevant** to the term of the function that is represented by the group.
- The **reduced term** of this group is **z'**

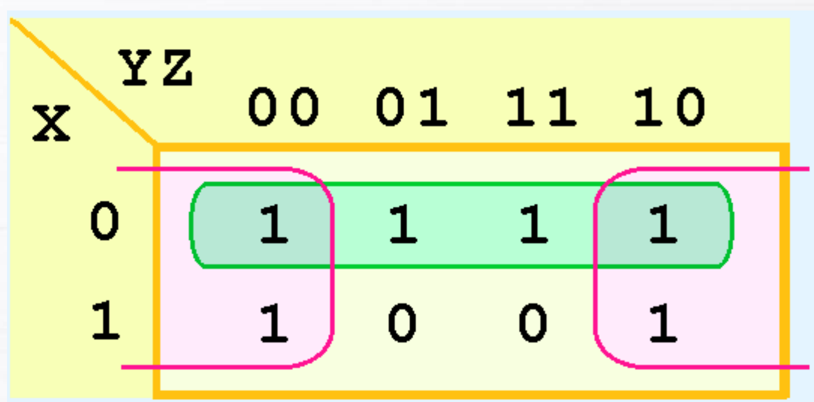
What about the green group in the top row?



3A.3 Kmap Simplification for Three Variables

- The **green group** tells us that **only the value of *x* is relevant** in that group.
- **x** is complemented in that row, so the **reduced term** of corresponding to this group is **X′** .
- The reduced function the sum of terms of all groups:
  F(X,Z) = X′+ Z′

Recall that we had six minterms in our original function!



3A.3 Kmap Simplification for Four Variables

- General format of a **Kmap** of a function with **4 variables and 16 minterms**:

wx \ yz	00	01	11	10
00	w′x′y′z′	w′x′y′z	w′x y z	w′x′y z′
01	w′x y′z′	w′x y′z	w′x y z	w′x y z′
11	w x′y′z′	w x′y′z	w x y z	w x′y z′
10	w x y′z′	w x y′z	w x y z	w x y z′

3A.3 Kmap Simplification for Four Variables

- Example (only non-zero minterms shown):

$$F(W,X,Y,Z) = W'X'Y'Z' + W'X'Y'Z + W'X'YZ' + W'XYZ' + WX'Y'Z' + WX'Y'Z + WX'YZ'$$

  – Can you identify (only) three groups in this Kmap?

Y.x + Y.Z.W + x.Z

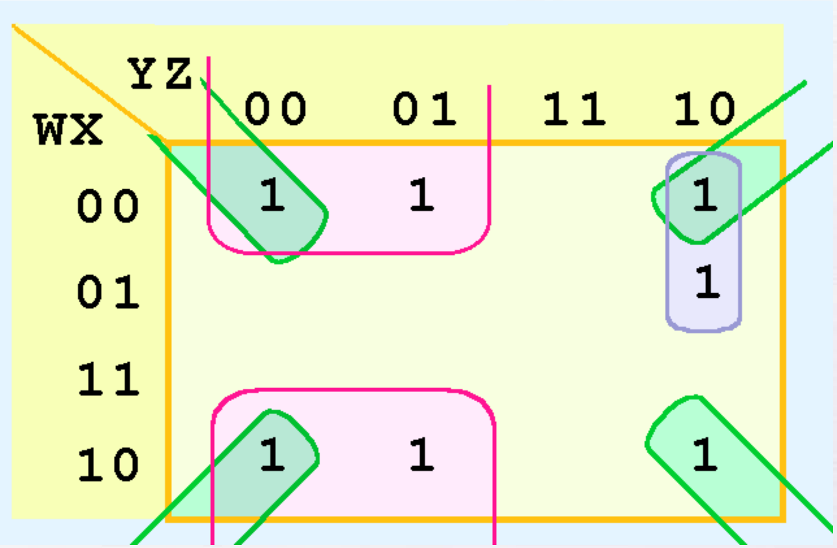
Recall that groups can overlap and wrap around.

WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

3A.3 Kmap Simplification for Four Variables

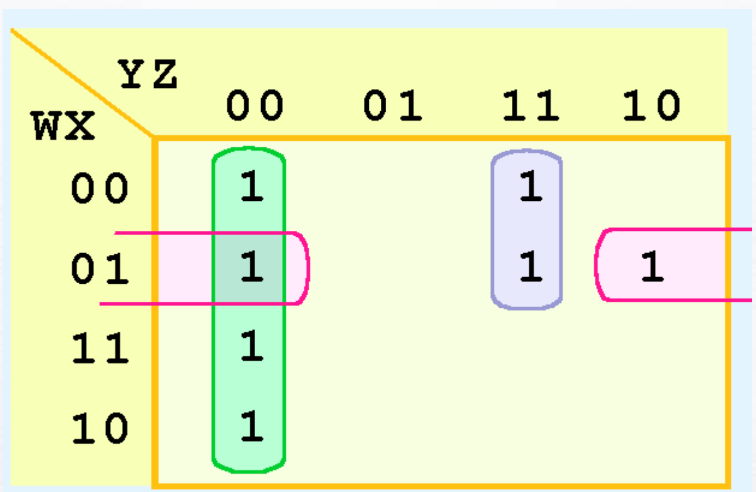
- The three groups consist of:
  - A **purple group** entirely within the Kmap at the right.
  - A **pink group** that wraps the top and bottom.
  - A **green group** that spans the corners.
- We have three terms in our final function:

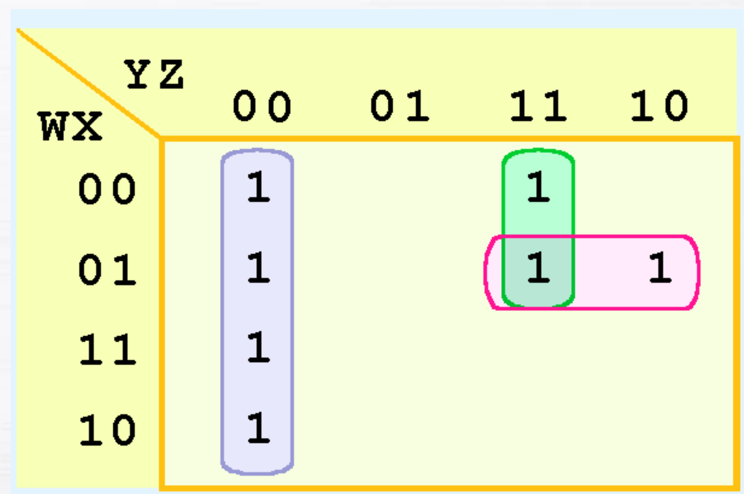
$$F(W,X,Y,Z) = W'Y' + X'Z' + W'YZ'$$



3A.3 Kmap Simplification for Four Variables

- It is **possible to have a choice** as to how to pick groups within a Kmap, while keeping the groups as large as possible.
- The **different functions** that result from the groupings below are **logically equivalent**.





3A.6 Don't Care Conditions

- Consider the function F(w,x,y,z) that returns whether the **quotient wx/yz has a remainder**
- Examples:
  - $01_2/10_2$ = 1/2 has a remainder, output: 1
  - $11_2/11_2$ = 3/3 has no remainder, output: 0
  - $10_2/00_2$ = 2/0 is an **invalid input**, output: undefined
- Circuits must be designed such that **invalid inputs** will **never happen**
- In a Kmap, we **don't need to care** whether the output corresponding to an invalid input is 0 or 1

3A.6 Don’t Care Conditions

- Consider the function F(w,x,y,z) that returns whether the **quotient wx/yz has a remainder**

WX \ YZ	00	01	11	10
00	X			
01	X		1	1
11	X			1
10	X		1	

- The **cross *X*** indicates **‘don’t care’ cases** (division by 0)
- When building groups, we are **free to include or ignore the *X*’s**

3A.6 Don’t Care Conditions

- Consider the function F(w,x,y,z) that returns whether the **quotient wx/yz has a remainder**

WX \ YZ	00	01	11	10
00	X			
01	X		1	1
11	X			1
10	X		1	

- Simplified function F(w,x,y,z) = xz’ + w’xy + wx’yz
- We included **some** *X*’s in order to get larger groups

3A.6 Don't Care Conditions

- Second example:

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don’t Care Conditions

- One possible solution:

$$F(W,X,Y,Z) = W'Y' + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don't Care Conditions

- Another possible solution:

$$F(W,X,Y,Z) = W'Z + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don’t Care Conditions

- The truth table of: $F(W,X,Y,Z) = W'Y' + YZ$

  differs from the truth table of:
  $F(W,X,Y,Z) = W'Z + YZ$

- However, the values for which they differ, are the inputs for which we have **don’t care** conditions.

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A Summary

- **Kmaps** provide an easy graphical method for **simplifying Boolean functions**.
- A Kmap is a **matrix** consisting **of the outputs** of the minterms of a Boolean function.
- In this section, we have discussed 2- 3- and 4-input Kmaps. This method **can be extended to any number** of inputs through the use of multiple tables.

3A Summary

Recapping the **rules of Kmap simplification**:

- Groupings can contain only 1s; no 0s.
- Groups can be formed only at right angles; diagonal groups are not allowed.
- The number of 1s in a group must be a power of 2 – even if it contains a single 1.
- The groups must be made as large as possible.
- Groups can overlap and wrap around the sides of the Kmap.
- Use don’t care conditions when you can.

3A Exercise

- Build a Kmap corresponding to the following truth table
- Derive the simplified function via building groups

A	B	C	D	Out
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1