On Cognitive Small Cells in Two-Tier Heterogeneous Networks

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***Abstract*—In a two-tier heterogeneous network (HetNet) where small base stations (SBSs) coexist with macro base stations (MBSs), the SBSs may suffer significant performance degradation due to the inter- and intra-tier interferences. Introducing cognition into the SBSs through the spectrum sensing (e.g., carrier sensing) capability helps them detecting the interference sources and avoiding them via opportunistic access to orthogonal channels. In this paper, we use stochastic geometry to model and analyze the performance of two cases of cognitive SBSs in a multichannel environment, namely, the *semi-cognitive* case and the *full-cognitive* case. In the *semi-cognitive* case, the SBSs are only aware of the interference from the MBSs, hence, only inter-tier interference is minimized. On the other hand, in the *full-cognitive* case, the SBSs access the spectrum via a contention resolution process, hence, both the intra- and inter-tier interferences are minimized, but at the expense of reduced spectrum access opportunities. We quantify the performance gain in outage probability obtained by introducing cognition into the small cell tier for both the cases. We will focus on a special type of SBSs called the femto access points (FAPs) and also capture the effect of different admission control policies, namely, the open-access and closed-access policies. We show that a *semi-cognitive* SBS always outperforms a *full-cognitive* SBS and that there exists an optimal spectrum sensing threshold for the cognitive SBSs which can be obtained via the analytical framework presented in this paper.**

***Keywords*:- Cellular networks, heterogeneous networks, cognitive small cells, interference modeling, stochastic geometry.**

I. Introduction

In a typical two-tier heterogeneous wireless network (HetNet), macro base stations (MBSs) characterized by their high transmit power and wide coverage range coexist with small cell base stations (SBSs) characterized by their limited transmit power and small coverage range. Small cells include femtocells, picocells, and microcells. Inter-tier interference is a performance limiting factor for the HetNets that could be mitigated by introducing cognition into SBSs [1]. A cognitive SBS is capable of monitoring the surrounding environment, locate major interference sources and avoid them by opportunistically accessing orthogonal channels. Based on the deployed cognitive technique and a spectrum sensing threshold, an interference source is considered either a major interference source that should be avoided or a minor interference source that can be tolerated.

In this paper, we investigate two types of cognitive SBSs in a multi-channel scenario, namely, the *semi-cognitive* (SC) SBS and the *full-cognitive* (FC) SBS. An SC-SBS considers only nearby MBSs as major interference sources that should be avoided. On the other hand, an FC-SBS considers both nearby MBSs and nearby SBSs as major interference sources that should be avoided. The *spectrum sensing threshold* of a cognitive SBS determines whether a neighboring MBS or SBS is considered a major interference source or not. From a geometric perspective, due to the distance dependent decay of the wireless signal power, the spectrum sensing threshold of a cognitive SBS defines the *spectrum sensing range* (SSR) around the SBS. A cognitive SBS will not reuse any channel used within its SSR, hence, no interference source exists within the SSR (i.e., the *interference exclusion region*). The spectrum sensing threshold is a critical design parameter that should be carefully tuned to achieve the desired tradeoff between the aggregate interference (and hence outage probability) and the spatial frequency reuse efficiency. The lower the spectrum sensing threshold, the larger is the SSR and the lower is the aggregate interference, however, the same frequency channel is reused after larger spatial intervals which results in a poor spatial frequency reuse efficiency. The method used for implementing cognition (e.g., spectrum sensing) into the SBSs determines the major interference sources and highly impacts the overall performance of the HetNet with cognitive small cells.

In this paper, we will focus on a special type of SBSs called the femto access point (FAPs) for three reasons[1]. Firstly, the inter-tier interference is more prominent to FAPs due to the huge transmit power gap between the FAPs and the MBSs. Secondly, we capture the effect of the two types of admission control policies which are uniquely defined for the FAPs, namely, the closed access policy and the open access policy. In the open access policy, a FAP can admit any user. On the contrary, in closed access policy, only users from a closed subscriber group can be admitted. Finally, since the FAPs are mainly deployed by the subscribers, cognition is expected to be an intrinsic characteristic that should be implemented in all FAPs. Quantifying the performance gain in terms of outage probability for downlink transmission to a femtocell user in a two-tier HetNet, and optimization of both the cases of cognitive FAPs considering both open access and closed access FAPs are the main objectives of this paper.

The main contributions of this paper are as follows:

- A performance analysis model to investigate two different cognitive techniques for FAPs for distributed downlink interference management in a two-tier HetNet, and
- Quantifying the performance gain in outage probability for both investigated cognitive techniques and showing the existence of an optimal spectrum sensing threshold for the cognitive SBSs that can be obtained numerically via the proposed model.

The rest of the paper is organized as follows. Section II reviews the related work. The system model and assumptions,

[1]The results presented in this paper, however, apply to any type of SBSs.

and the methodology of analysis are presented in Section III. Section IV calculates the opportunistic spectrum access probability for a cognitive FAP. The outage probability is obtained in Section V. Section VI presents the numerical results before the paper is concluded in Section VII.

II. Related Work

Although the idea of modeling infrastructure-based cellular networks via stochastic geometry dates back to the late 90's [2], [3], much attention has been brought to this modeling technique after more than a decade because of the pioneering work in [4]. In [4], the authors were able to use the Poisson point process (PPP) for the spatial location of the MBSs to derive a tractable yet accurate models for important performance metrics for cellular networks such as the coverage probability and the average achievable rate.

Following [4], many work have been done in the literature using the same methodology to adapt and extend the stochastic geometric approach to different network scenarios. For instance, [5]-[8] extended the model to include offloading via biasing and fractional frequency reuse in HetNets. In [9], the author derived the distribution for the signal-to-interference-plus-noise-ratio (SINR) of a generic user in a HetNet. In [10], the authors developed a capacity extension policy for a two-tier HetNet and determined which type of base stations should be added or switched off to achieve the optimal base station density. In [11], the authors optimized the subchannel allocations when the available subchannels are either aggressively used by both the network tiers or each network tier has its own subset of the available subchannels. Different from the existing work in the literature, in our model, the FAPs are cognitive and we consider the spatial reuse of channels, which is optimized through optimizing the spectrum sensing threshold. Cognitive FAPs were also considered in [12], however, the authors considered a single MBS, with a single user in the system model, derived the outage probability of the macro user only, and the spatial frequency reuse of the cognitive FAPs was not considered.

We would like to emphasize that a more in-depth exposition to the cognitive FAPs with a similar system model was provided in our previous work [13]. However, [13] only discussed the *full-cognitive* scheme for the FAPs. In this paper, we show that the gain in terms of the SINR outage of the *full-cognitive* scheme is wasted due to the uncoordinated contention among the FAPs for spectrum access. Therefore, we propose the *semi-cognitive* scheme and show that introducing spectrum awareness with respect to (w.r.t.) the macro-tier only in better than introducing spectrum awareness w.r.t. the two network tiers.

III. System Model and Assumptions

A. Network Model

We consider a two-tier HetNet consisting of MBSs and FAPs. We assume that the two network tiers are independent and each is represented by an independent homogenous PPP in the $\mathbb{R}^2$ plane. That is, the MBSs are spatially distributed according to the homogenous PPP $\mathbf{\Psi}_b = \{b_i; i = 1, 2, 3, \cdots\}$ with intensity $\mathcal{B}$ where b_i is the location of the i^{th} MBS. The FAPs are spatially distributed according to an independent homogenous PPP $\mathbf{\Psi}_a = \{a_i; i = 1, 2, 3, \cdots\}$ with intensity $\mathcal{A}$, where a_i denotes the location of the i^{th} FAP[2]. There are two types of FAPs, namely, open-access FAPs (OFAPs) and closed-access FAPs (CFAPs). The OFAPs can accept any user, however, the CFAPs only accept licensed users. The access policy of the coexisting FAPs is independent of their locations, and a fraction p_c ($0 \leq p_c \leq 1$) of all the FAPs employ the closed access policy. Therefore, the OFAPs constitute a PPP with intensity $\check{\mathcal{A}} = (1 - p_c)\mathcal{A}$, and the CFAPs constitute a PPP with intensity $\hat{\mathcal{A}} = p_c\mathcal{A}$.

In this paper, all the analysis will be performed for unlicensed users, as the licensed users can be treated as a special case by setting $p_c = 0$. The user equipments (UEs) are spatially distributed according to an independent PPP $\mathbf{\Psi}_u = \{u_i; i = 1, 2, 3, \cdots\}$ with intensity $\mathcal{U}$. Each unlicensed user is associated with the geographically closest OFAP or MBS which provides the highest signal power [4], [6]. Note that SIR based association can be also considered as in [5], but at the expense of increased complexity. The highest signal power based association can be justified by looking into the network model as a superposition of two independent Voronoi tessellations, one for the MBSs and the other for the OFAPs (as shown in Fig. 2). By construction, the Voronoi cells belonging to the same tier do not intersect; hence, each user will fall in an intersection between two Voronoi cells belonging to different tiers (i.e., one Voronoi cell for an MBS and the other for an OFAP). Based on the received signal power, each user will associate with either the MBS or the OFAP of the Voronoi cell covering the user. Hence, the set of candidate serving network entities for a generic user reduces to only two network entities, namely, the geographically nearest OFAP and the geographically nearest MBS.

Based on the association of UEs, the set of UEs is divided into three non-overlapping subsets: the subset of users $\mathbf{\Psi}_u^M \subset \mathbf{\Psi}_u$ associated with MBSs, the subset of users $\mathbf{\Psi}_u^{CF} \subset \mathbf{\Psi}_u$ associated with CFAPs, and the subset of users $\mathbf{\Psi}_u^{OF} \subset \mathbf{\Psi}_u$ associated with OFAPs, such that $\mathbf{\Psi}_u^M \cup \mathbf{\Psi}_u^{CF} \cup \mathbf{\Psi}_u^{OF} = \mathbf{\Psi}_u$ and $\mathbf{\Psi}_u^M \cap \mathbf{\Psi}_u^{CF} \cap \mathbf{\Psi}_u^{OF} = \phi$. The intensity of each subset can be calculated through the tier association probability in a way similar to that in [6], [9], [13]. The tier association probability is defined as the probability that a generic user is connected to either an MBS or a FAP. Due to the brevity, calculation of the tier association probability [14] is not included in this paper. We assume that the users associated with the MBSs constitute a PPP with intensity $\mathcal{U}_m$ and each FAP has at least one associated user.

Both the network tiers share the same set of channels[3] $\mathbf{S}$. The channels have a specific order known to all MBSs. All MBSs transmit with the same power P_b, all FAPs transmit with the same power P_a, and the MBSs and FAPs always have packets to transmit to their associated users in the downlink (i.e., saturation conditions are assumed).

[2]With a slight abuse of notation we will use b_i to denote both the location of the i^{th} MBS and the i^{th} MBS itself, and the same for a_i.

[3]A channel can be, for example, a resource block (RB) in LTE/LTE-Advanced systems.

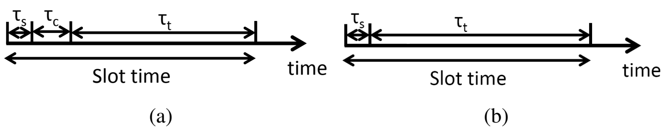

Fig. 1. (a) Time-slot structure for channel access by FC-FAPs, (b) time-slot structure for channel access by SC-FAPs.

B. Cognitive FAP

We consider two types of cognitive FAPs, the SC-FAPs and the FC-FAPs. The SC-FAPs are only aware of the MBSs' transmissions and avoid interfering with nearby MBSs. On the other hand, the FC-FAPs are aware of both the MBSs' and the FAPs' transmissions and avoid interfering with both nearby MBSs and nearby FAPs. Time is assumed to be divided into time slots. The time slot structure for FC-FAPs is shown is Fig. 1(a) and the time slot structure for SC-FAPs is shown is Fig. 1(b). Each MBS assigns channels to its associated users at the beginning of each time slot. The user who arrives to the MBS or requests a session in the middle of a time slot will be assigned a channel in the next time slot. All channels are reused in all MBSs (i.e., universal frequency reuse). Each macro user is assigned only one channel and the MBS assigns the channels to its associated users in a sequential manner (i.e., channel $s_i \in \mathbf{S}$ will be assigned before channel $s_{i+1} \in \mathbf{S}$ and so on until $s_{|\mathbf{S}|}$ is reached). No channel is reused within the cell (i.e., no intra-cell interference). The assumption of the sequential channel assignment within the MBSs keeps the model analytically tractable, maximizes the FAP access probability, and minimizes the inter-tier interference[4].

The FAPs are cognitive and access the available channels opportunistically. That is, a FAP will use the channel $s_i \in \mathbf{S}$, $i = 1, 2, \cdots, |\mathbf{S}|$ if and only if the received power on this channel is less than the spectrum sensing threshold γ. All coexisting FAPs deploy the same cognitive technique (i.e., either *semi-cognitive* or *full-cognitive*). At the beginning of each time slot, a FAP senses the spectrum during the sensing time τ_s and determines the available channels (i.e., channels not used by the MBSs within its SSR). An SC-FAP will randomly choose one of the available channels and directly use it for downlink transmission. On the other hand, an FC-FAP will access one of the available channels through a process of contention resolution with the coexisting FAPs[5]. That is, a FC-FAP will randomly choose one of the available channels and persistently sense it for a random duration uniformly distributed in the range $[0, \tau_c]$ (Fig. 1(a))[6]. If the channel is still available after the random sensing duration elapses (i.e., not used by a nearby FAP), it will access the channel for downlink transmission. The random sensing time before acquiring a channel minimizes the probability of interference with nearby FAPs (i.e., the probability that two FAPs within the sensing range of each other use the same channel).

Since our focus is on the analysis of outage for femto users, the actual method of sharing the accessed channel among the femto users for downlink transmission is not an important issue here. That is, an outage occurs if it cannot find a channel to access, or if the SINR falls below the threshold defined for correct reception given that the FAP has found a channel to access. In this paper, we assume perfect spectrum sensing. All time slots have the same structure and the same spectrum access procedure is repeated in each time slot.

C. Radio Channel Model

We consider a general power-law path loss model in which the signal power decays at the rate $r^{-\eta}$ with the distance r, where η is the path-loss exponent. All the channel gains are assumed to be independent and the channels have a coherence time greater than or equal to a time slot. For analysis, only Rayleigh fading environment is assumed. General fading environment can be considered by exploiting Plancherel-Parseval Theorem [16], however, the complexity of the analysis increases significantly [4]. Moreover, Rayleigh fading provides tractable results which helps understanding the system behavior. The channel (power) gains from a generic location $x \in \mathbb{R}^2$ to the MBS b_i and the FAP a_i are denoted by $h_{b_i}(x) \sim \text{Exp}(\mu_b)$ and $h_{a_i}(x) \sim \text{Exp}(\mu_a)$, respectively. The channel (power) gains are independent from each other, independent from the locations, and identical distributed (iid). Hence, for brevity of expositions, hereafter, the spatial index x is dropped.

We will use the notation χ to denote the serving network entity for a generic user. That is, $\chi = a$ if the user is associated to a FAP, and $\chi = b$ if the user is associated to a MBS. Without any loss in generality, the analysis is conducted on a typical user located at the origin. According to Slivnyak's theorem [17], conditioning on having a user at the origin does not change the statistical properties of the coexisting PPPs. Hence, the analysis holds for any generic user located at a generic location. Therefore, the SINR at the typical user located at the origin (which also holds for any generic user) served by an MBS or a FAP (MBS/FAP) is given by

$$SINR = \frac{P_\chi h_\chi R_\chi^{-\eta}}{\sum\limits_{b_i \in \tilde{\mathbf{\Psi}}_{b\chi}} P_b h_{b_i} \|b_i\|^{-\eta} + \sum\limits_{a_i \in \tilde{\mathbf{\Psi}}_{a\chi}} P_a h_{a_i} \|a_i\|^{-\eta} + \sigma^2} \tag{1}$$

where R_χ is the distance from the user to the serving network entity (i.e., an MBS or a FAP), $\tilde{\mathbf{\Psi}}_{b\chi}$ denotes the set of interfering MBSs, $\tilde{\mathbf{\Psi}}_{a\chi}$ denotes the set of interfering FAPs, $\|.\|$ denotes the Euclidean norm, and σ^2 is the noise power.

IV. Calculation of the Opportunistic Spectrum Access Probability for Cognitive FAPs

A. Assumptions and Procedure

The MBSs perform the channel assignment at the beginning of each time slot. Therefore, by sensing the spectrum at the beginning of each time slot for a duration τ_s, the FAPs

[4]Note that this may, however, increase the intra-tier interference for the MBSs.

[5]A similar cognitive channel access scheme called the cognitive CSMA (C-CSMA) was proposed in [15].

[6]The random sensing duration is similar to the the random backoff timer generated in a traditional carrier-sense multiple access (CSMA) protocol for collision avoidance and coordination of spectrum access.

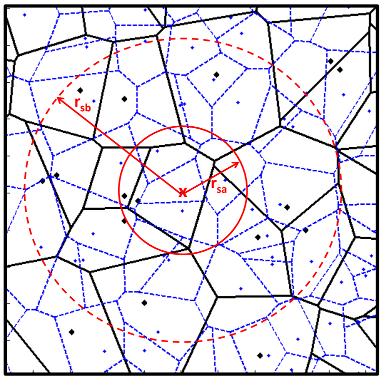

Fig. 2. The network can be represented by the superposition of two independent Voronoi tessellations, the blue for the FAPs and the black for MBSs. The figure shows the two SSRs for a generic FAP (the center red x): the outer dotted circle and the inner solid circle represent, respectively, the average *macro SSR* and the average *femto SSR* radii.

can identify the available spectrum opportunities (i.e., the channels that are not used by the MBSs). An SC-FAP will randomly choose any of the available channels and access it for downlink transmission. On the other hand, an FC-FAP should avoid interfering with any other FAP in its SSR through contention. Due to the low transmission power of the FAPs and the high transmission power of MBSs, and the unified sensing threshold γ, an FC-FAP has two different SSRs, namely, the *femto SSR* and the *macro SSR* as shown in Fig. 2. Hereafter, we will refer to the SSR of a FAP w.r.t. the MBSs as the *macro SSR*, and the SSR of a FAP w.r.t. the FAPs as the *femto SSR*. Each FAP (SC-FAP or FC-FAP) should not reuse any channel used by a MBS within its *macro SSR*. However, only a FC-FAP should avoid reusing any channel used by FAPs within its *femto SSR* (i.e., the SSR of an FC-FAP w.r.t. other FAPs).

B. Opportunistic Spectrum Access for SC-FAPs

Having at least one channel available in the *macro SSR* of a SC-FAP ($\mathbb{P}\{K_f \geq 1\}$) (i.e., there is a channel not used by any of the MBSs in its *macro SSR*) is a necessary and sufficient condition for spectrum access. Therefore, the probability that a generic SC-FAP accesses the spectrum is given by $\mathcal{P}_{sc} = \mathbb{P}\{K_f \geq 1\}$, where the distribution of the number of the free channels within the *macro SSR* of a generic FAP is given by the following Lemma.

Lemma 4.1: Let $|\mathbf{S}|$ be the total number of channels, K_f be the number of free channels out of $|\mathbf{S}|$ within the *macro SSR* of a generic FAP located at a_i, $a_i \in \mathbb{R}^2$, and γ be the spectrum sensing threshold. Then the distribution of K_f can be obtained from

$$\begin{aligned} F_{K_f}(k) &= \mathbb{P}\{K_f \leq k\} \\ &= 1 - \mathbf{1}_{\{k<|\mathbf{S}|\}} e^{-\varphi_{\gamma b}\left(1-F_{\mathcal{N}_v}(|\mathbf{S}|-k-1)\right)}, \quad k = 0, 1, ..., |\mathbf{S}| \end{aligned} \tag{2}$$

where $\mathbf{1}_{\{.\}}$ is an indicator function which takes the value 1 when the statement $\{.\}$ is true and takes the value 0 otherwise.

The *pmf* of K_f is given by

$$\begin{aligned} f_{K_f}(k) &= e^{-\varphi_{\gamma b}\left(1-F_{\mathcal{N}_v}(|\mathbf{S}|-k)\right)} \mathbf{1}_{\{k>0\}} \\ &\quad - \mathbf{1}_{\{k<|\mathbf{S}|\}} e^{-\varphi_{\gamma b}\left(1-F_{\mathcal{N}_v}(|\mathbf{S}|-k-1)\right)}, \quad k = 0, 1, ..., |\mathbf{S}|. \end{aligned} \tag{3}$$

Although a detailed proof for Lemma 4.1 can be found in our previous work [13], for the sake of completeness, we provide a summarized proof for Lemma 4.1 in **Appendix A**.

C. Opportunistic Spectrum Access for FC-FAPs

Unlike the SC-FAP, due to contention, having at least one channel available in the *macro SSR* of a FC-FAP is a necessary but not sufficient condition for spectrum access. In this section, we will model the contention among different FC-FAPs to access the available channels in order to obtain the conditional opportunistic spectrum access probability for FAPs (i.e., conditioning on the number of free channels). As shown in Fig. 2, the test FC-FAP will contend to access the spectrum with all other FC-FAPs within its *femto SSR*. For analytical tractability, we will assume that all the FC-FAPs coexisting within the *femto SSR* of the test FAP have the same set of free channels. This assumption can be justified by the smaller *femto SSR* w.r.t. the *macro SSR*. Each FAP will generate a random backoff timer and persistently sense a randomly chosen channel out of the K_f free channels. If the chosen channel is still available after the backoff timer elapses, the FAP acquires it and uses it for downlink transmission. Otherwise, it encounters an outage in the current time slot. Due to the contention-based channel access, the cognitive FAPs which simultaneously access the same channel will constitute a Matérn hard core point process (HCPP).

A Matérn HCPP is a repulsive point process where no two points can coexist if their distance is less than the hard core radius r_h. The Matérn HCPP is derived from a PPP via dependent thinning. The dependent thinning is applied in two steps. First, an independent uniformly distributed time mark is applied to the PPP. Then, a point is chosen to be in the Matérn HCPP if and only if it has the lowest mark in its contention domain. The contention domain of a point is defined by a circle of radius r_h around that point. Projecting to our network model, the PPP is the complete set of FAPs contending to access the same channel, the time mark corresponds to the backoff timer generated by each FAP for contention and the HCPP corresponds to the FAPs that succeed to access the spectrum. Similar to Lemma 4.1 (see step 2 in **Appendix A**), the number of FAPs within the *femto SSR* has a Poisson distribution with mean $\varphi_{\gamma a} = \pi \mathcal{A} \left(\frac{P_a}{\mu_a \gamma}\right)^{\frac{2}{\eta}} \Gamma(1+\frac{2}{\eta})$. Since both OFAP and CFAP contend for spectrum access, following [18], [19], [20], it can be shown that, conditioning on the number of available channels (K_f), the opportunistic spectrum access probability for a generic FAP is given by

$$\mathcal{P}_{ac}(K_f) = K_f \frac{1 - e^{-\frac{\varphi_{\gamma a}}{K_f}}}{\varphi_{\gamma a}}. \tag{4}$$

According to the total probability theory, the unconditional opportunistic spectrum access probability can be obtained as $\mathcal{P}_{fc} = \sum_{k=1}^{|\mathbf{S}|} f_{K_f}(k) \mathcal{P}_{ac}(k)$.

$$\mathcal{O}_{sc}^{(SINR)}(K_f) = 1 - \int_0^\infty 2\pi\mathcal{A}r \exp\left\{-\mathcal{A}\pi r^2 - \mathcal{B}_{in}(K_f)\pi r^2\sqrt{p\beta}\arctan\left(\frac{r^2\sqrt{p\beta}}{\max(pr, r_{sb}-r)^2}\right) - \frac{\check{\mathcal{A}}\pi r^2\sqrt{\beta}\arctan\left(\sqrt{\beta}\right)}{K_f} - \frac{\hat{\mathcal{A}}\pi^2 r^2\sqrt{\beta}}{2K_f} - \frac{\beta r^\eta\sigma^2}{P_a}\right\}dr. \tag{5}$$

$$\mathcal{O}_{fc}^{(SINR)}(K_f) \approx 1 - \int_0^\infty 2\pi\check{\mathcal{A}}r \exp\left\{-\check{\mathcal{A}}\pi r^2 - \mathcal{B}_{in}(K_f)\pi r^2\sqrt{p\beta}\arctan\left(\frac{r^2\sqrt{p\beta}}{\max(pr, r_{sb}-r)^2}\right) - (1-p_c)\mathcal{A}_{s_{|\mathbf{S}|-K_f+1}}\pi r^2 \sqrt{\beta}\arctan\left(\frac{r^2\sqrt{\beta}}{\max(r, r_{sa}-r)^2}\right) - p_c\mathcal{A}_{s_{|\mathbf{S}|-K_f+1}}\pi r^2\sqrt{\beta}\arctan\left(\frac{r^2\sqrt{\beta}}{\max(0, r_{sa}-r)^2}\right) - \frac{\beta r^\eta\sigma^2}{P_a}\right\}dr. \tag{6}$$

V. Analysis of Outage Probability

A. Assumptions and Methodology

Due to the sequential assignment of channels in the MBSs and that each user is assigned only one channel, each channel s_i has its own interference statistics. That is, channel s_1 will have all MBSs with one or more associated users causing interference to it. On the other hand, the channel s_i will only have MBSs with i or more associated users causing interference to it. In this section, we consider the worst-case scenario from interference point of view and model the outage on the channels experiencing the highest amount of interference from the macro-tier. That is, for a femto user, given that there are K_f available channels in the *macro SSR* of its serving OFAP, we model the outage on channel $s_{|\mathbf{S}|-K_f+1}$. Note that, the effect of both cognitive schemes on the macro users is out of the scope of this paper for two reasons. Firstly, since both cognitive schemes avoid interfering with the MBSs, they will have equal effects on the macro users' performance. Secondly, the outage probability evaluation for macro users with cognitive FAPs can be found in [12], [13].

B. Outage Probability for Downlink Transmission in SC-FAPs

Now we calculate the outage probability of a test femto user located at the origin associated with an SC-FAP. As has been discussed previously, a FAP (or equivalently its associated user) experience two kinds of outages. The first is due to the channel unavailability because of the opportunistic channel access. While the second is the SINR outage. Therefore, the outage probability of a femto user under the *semi-cognitive* shceme is given by

$$\mathcal{O}_{sc} = f_{K_f}(0) + \sum_{k=1}^{|\mathbf{S}|} f_{K_f}(k)\mathcal{O}_{sc}^{(SINR)}(k)$$

where $f_{K_f}(0)$ is the probability that there are no available channels in the *macro SSR* of the serving OFAP, and $\mathcal{O}_{sc}^{(SINR)}(K_f)$ is the outage probability due to the SINR falling below the reception threshold β given that there are K_f free channels in the *macro SSR* of the serving OFAP. Given that the user is associated to an OFAP, from the system model, the serving OFAP is the nearest OFAP to that user in the femto network tier. However, there might be a closer CFAP to the test user than his serving OFAP. On the other hand, the interfering MBSs are outside the *macro SSR* of the serving OFAP. Therefore, on average, the nearest interfering MBS is $r_{sb} - R_a$ away from the test femto receiver, where $R_a = \min_i(\|\check{a}_i\|)$, $\check{a}_i$ is the position of the i^{th} OFAP, and $r_{sb} = \left(\frac{P_b\mu_b}{\gamma_s}\right)^{1/\eta}$ is the average *macro SSR* radius. For $\eta = 4$, the SINR outage can be obtained as (5), where $p = \frac{P_b\mu_a}{P_a\mu_b}$.

Due to the brevity of the paper, we will only provide an outline of the proof for (5). Since the OFAPs, the CFAPs, and the MBSs constitute independent PPPs, their aggregate interferences received at the test receiver are independent. Given that there are K_f free channels in the *macro SSR* of the serving OFAP, the set of interfering MBSs on the channel $|\mathbf{S}| - K_f + 1$ (i.e., the worst-case scenario) constitute a PPP outside the *macro SSR* of the serving OFAP with intensity $\mathcal{B}_{in}(K_f) = (1 - F_{\mathcal{N}_v}(|\mathbf{S}| - K_f))\mathcal{B}$. On the other hand, the set of interfering OFAPs and CFAPs constitute independent PPPs with intensities $\frac{\check{\mathcal{A}}}{K_f}$ and $\frac{\hat{\mathcal{A}}}{K_f}$, respectively. Note that, from the system model, the interfering OFAPs cannot be closer than the serving OFAP (i.e., the distance from the test user to the nearest interfering OFAP is greater than R_a), while there are no restrictions on the locations of the CFAPs. Following [4], [17], in a Rayleigh fading environment, the outage probability can be directly obtained from the Laplace transform of the *pdf* of the aggregate interference. The Laplace transform of the *pdf* of the aggregate interference resulting from a PPP existing outside an interference exclusion is given in Sec. 3.7.1 in [17]. Therefore, following the method in [4], [17] and after some mathematical manipulations, (5) is obtained.

C. Outage Probability for Downlink Transmission in FC-FAPs

Similar to the SC-FAPs, the FC-FAPs experience two types of outage and the total outage probability for a femto user associated with an FC-FAP is given by

$$\mathcal{O}_{fc} = f_{K_f}(0) + \sum_{k=1}^{|\mathbf{S}|} f_{K_f}(k)\left[\mathcal{P}_{ac}(k)\mathcal{O}_a^{(SINR)}(k) + (1 - \mathcal{P}_{ac}(k))\right]$$

where $\mathcal{P}_{ac}(K_f)$ is given in (4), and $\mathcal{O}_{fc}^{(SINR)}(K_f)$ is the outage probability due to the SINR falling below the reception threshold β given that there are K_f free channels in the *macro SSR* of the serving OFAP.

When the FAPs use the full cognitive technique, from the system model, the interfering MBSs and FAPs are outside the *macro SSR* and *femto SSR* of the serving FAP, respectively. Therefore, on average, the nearest interfering MBSs and FAPs are, respectively, $r_{sb} - R_a$ and $r_{sa} - R_a$ away from the test femto receiver, where $r_{sa} = \left(\frac{P_a \mu_a}{\gamma_s}\right)^{1/\eta}$ is the average *femto SSR* radius. For $\eta = 4$, the SINR outage probability for the FC-FAPs can be obtained as (6), where $\mathcal{A}_{s_{|\mathbf{S}|-K_f+1}}$ is the intensity of simultaneously active FAPs on the channel $s_{|\mathbf{S}|-K_f+1} \in \mathbf{S}$. The intensity of FC-FAPs accessing the same channel i can be obtained from the probability that a generic FAP will access the channel $s_i \in \mathbf{S}$ as $\mathcal{A}_{s_i} = \mathcal{A} \sum_{k=|\mathbf{S}|-(i-1)}^{|\mathbf{S}|} \mathbb{P}\{K_f = k\} \frac{\mathcal{P}_{ac}(k)}{k}$. Note that (6) can be proved in a way similar to that for (5), however, the only difference is that the interfering FAPs (both the OFAPs and the CFAPs) constitute an HCPP. As in [19], [20], to obtain the Laplace transform of the *pdf* of the aggregate interference of an HCPP, the HCPP should be approximated by a PPP with the same intensity and existing outside the exclusion region of the test transmitter. It was shown in [20] that, for the mean interference from an HCPP, the PPP approximation error never exceeds 1 dB.

VI. Numerical Results and Discussions

For the numerical evaluations (using Matlab), we choose $P_b = 5$ W, $P_a = 20$ dBm, $\mathcal{B} = 1$ MBS/km^2, $\mathcal{A} = 10$ FAP/km^2, $\mathcal{U}_m = 10$ user/km^2, $|\mathbf{S}| = 30$ channels, $\beta = 1$, all the channel gains to have unit mean (i.e., $\frac{1}{\mu_a} = \frac{1}{\mu_b} = 1$), and $\eta = 4$. The effect of noise is ignored (i.e., the network is interference-limited) [4], [5].

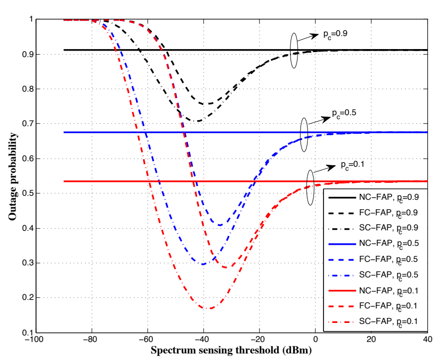

Fig. 3. The overall femto user outage probability vs. spectrum sensing threshold for cognitive techniques and different values p_c.

Fig. 3 shows the performance gain in outage probability obtained by introducing cognition into the femto-tier (i.e., into the SC-FAPs and FC-FAPs) in comparison with that for the non-cognitive FAPs (NC-FAPs) for different values of p_c. The figure shows that the outage probability can be significantly decreased for both the *semi-cognitive* and the *full-cognitive*) cases. There exist an optimal spectrum sensing threshold that maximizes the gain in terms of the outage probability. That is, the optimal spectrum sensing threshold minimizes the total outage probability for the SC-FAPs by up to 35% and up to 23% for the FC-FAPs. For higher values of spectrum sensing threshold, the spectrum opportunities for FAPs increase, however, the aggregate interference increases and dominates the outage probability which results in a degraded outage performance. For very high values of spectrum sensing threshold, the cognitive FAPs become very aggressive and the performance approaches to that of the non-cognitive FAPs. The figure also shows that cognition may deteriorate the network performance in terms of the outage probability if the sensing threshold is set to a very conservative value. As expected, Fig. 3 shows that as the percentage of CFAPs increases, the outage probability also increases.

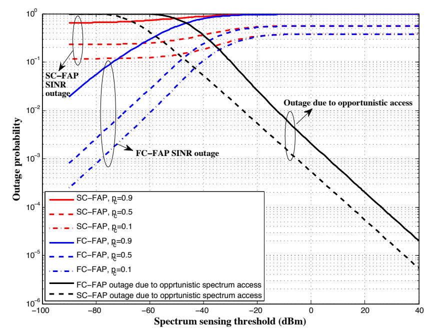

Fig. 4. The SINR outage performance and the outage due to opportunistic spectrum access for femto users vs. spectrum sensing threshold for cognitive techniques and different values p_c.

Interestingly, Fig. 3 shows that the SC-FAPs always outperforms the FC-FAPs despite the fact that the aggregate interference in minimized in the FC-FAPs. To explain this behavior we plot Fig. 4. For FC-FAPs, at lower values of the spectrum sensing threshold, despite that the aggregate interference will be very low, the effect of channel unavailability dominates the outage probability and results in a degraded outage performance. That is, the performance gain in terms of the SINR outage for the FC-FAPs is wasted by the degraded opportunistic spectrum access probability due to contention. On the other hand, the SC-FAC has a higher opportunistic spectrum access probability which compensates its degraded SINR outage. As the spectrum sensing threshold of the FC-FAP decreases, more FC-FAPs are included in the contention, and therefore, opportunities for spectrum access decrease. Note that as the spectrum sensing threshold for the SC-FAP or FC-FAP decreases, the number of MBSs in their *macro SSR* increases and the probability that a MBS in the *macro SSR* is using a high number of channels increases. Hence, the opportunistic spectrum access probability for the SC-FAPs decreases with decreasing the spectrum sensing threshold.

To show that the spectrum awareness w.r.t. the MBSs only always outperforms the spectrum awareness w.r.t. both the MBSs and FAPs, we plot Fig. 5 and Fig. 6. Since both of the cognition techniques avoid interference with MBSs, the intensity of MBSs and transmission parameters will have equal effects on both the schemes. Therefore, in Fig. 5 and Fig. 6 we show the effect of the intensity and transmission power of the FAPs on the cognitive schemes. These figures show that the SC-FAPs always perform better than the FC-FAPs in terms of the outage probability. One reason for this is that, as proven in [5], changing the intensity and the transmission powers of the BSs belonging to the same network tier (assuming that all BSs have the same transmission power) will not change the SIR statistics in that network tier. Hence, increasing the intensity or the transmission powers of the FAPs, the SIR statistics within the femto tier does not change. Hence, the *semi-cognitive* scheme is not affected. On the other hand, the *full-cognitive* scheme will have higher outage probability due to spectrum unavailability.

As shown Fig. 5 and Fig. 6, the overall outage performance of the FAPs improves as their intensity increases and/or their transmission power increases. This is because, increasing the intensity and transmission powers of the FAPs, while keeping the parameters of the MBSs constant, enhances the useful signal w.r.t. the inter-tier interference. Finally, the results (see Fig. 3, Fig. 5, Fig. 6) show that there exists an optimal spectrum sensing threshold which depends on the system parameters.

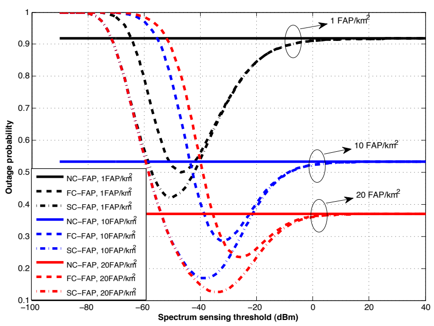

Fig. 5. The effect of FAP intensity on the outage probability.

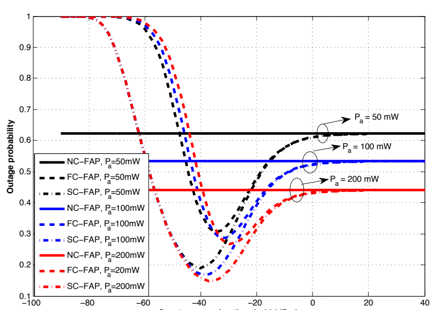

Fig. 6. The effect of FAP transmission power on the outage probability.

VII. Conclusion

We have provided a framework for modeling and analysis of two-tier HetNets with two cognitive techniques for FAPs employing open and closed access policies. We have shown that cognition (e.g., opportunistic spectrum access) is an important feature that can boost up the overall femtocell network performance. The results have showed that the performance gain in outage probability introduced by the full cognition is wasted by the degraded opportunistic access performance due to channel unavailability. Hence, the semi-cognitive FAPs have a better optimal outage performance than the full-cognitive FAPs. One way to enhance the tradeoff between SINR performance and the opportunistic spectrum access for full-cognitive FAPs is to introduce cooperation among the coexisting FAPs, which will be investigated in our future work. The results have shown that the spectrum sensing threshold is a critical parameter for both full-cognitive FAPs and semi-cognitive FAPs which should be tuned carefully.

Appendix A
Proof of Lemma 4.1

To find the distribution of the number of the available channels within the *macro SSR* of a generic FAP, we will exploit Slivnyak's theorem and find the number of the available channels within the *macro SSR* of the test FAP located at the origin. The proof is divided into four steps. In the first step, we find the distribution of the number of occupied channels within a generic MBS. In the second step, the distribution of the number of the MBSs within the *macro SSR* of the test FAP is obtained. In the third step, we find the distribution of the total number of occupied channels within the *macro SSR* of the test FAP. Finally, in the fourth step we find the distribution of free channels within the *macro SSR* of the test FAP. Note that in the first 3 steps we will assume that we can have infinite number of channels, then in step four, we will bring the maximum number of available channels into the picture.

Since each MBS assigns orthogonal channels to its associated users, the number of occupied channels in each MBS is equal to the number of users associated to that MBS. Let V be the area of the Voronoi cell. Then, conditioning on a Voronoi cell area V, the number of macro users $\mathcal{N}_v$ in that Voronoi cell is a Poisson random variable with the probability mass function (*pmf*) given by

$$\mathbb{P}\left\{\mathcal{N}_v = n | V = v\right\} = \frac{(\mathcal{U}_m v)^n e^{-\mathcal{U}_m v}}{n!}, \qquad n = 0, 1, 2, \cdots . \tag{7}$$

Note that in (7) we account only for macro users by considering $\mathcal{U}_m$. Following [10], [21], the Voronoi cell area V is a random variable that is approximated by the gamma distribution $f_V(v) \approx \frac{(\mathcal{B}c)^c v^{c-1} e^{-c\mathcal{B}v}}{\Gamma(c)}$, $0 \leq v < \infty$, where $c = 3.575$ is a constant defined for the Voronoi tessellation in the $\mathbb{R}^2$. Therefore, the unconditional *pmf* of $\mathcal{N}_v$ is given by

$$\begin{aligned} f_{\mathcal{N}_v}(n) &= \int_0^\infty \frac{(\mathcal{U}_m v)^n \, e^{-\mathcal{U}_m v}}{n!} f_V(v) dv \\ &= \frac{\Gamma(n+c)}{\Gamma(n+1)\Gamma(c)} \frac{(\mathcal{U}_m)^n \, (\mathcal{B}c)^c}{(c\mathcal{B} + \mathcal{U}_m)^{n+c}}, \qquad n = 0, 1, 2, \cdots . \end{aligned}$$

The *cdf* of $\mathcal{N}_v$ is given by

$$F_{\mathcal{N}_v}(k) = \sum_{n=0}^{k} \frac{\Gamma(n+c)}{\Gamma(n+1)\Gamma(c)} \frac{(\mathcal{U}_m)^n \, (\mathcal{B}c)^c}{(c\mathcal{B} + \mathcal{U}_m)^{n+c}}, \; k = 0, 1, 2, \cdots . \tag{8}$$

The second step in the proof is to calculate the distribution of the number of MBSs within the *macro SSR* of the test FAP. An MBS is considered within the *macro SSR* of a generic FAP if and only if the received power from this MBS at the test FAP is greater than or equal to spectrum sensing threshold γ. Let $\mathcal{N}_{\gamma b}$ be the number of MBSs within the *macro SSR* of the test FAP, then $\mathcal{N}_{\gamma b}$ can be written as

$$\mathcal{N}_{\gamma b} = \sum_{b_i \in \mathbf{\Psi}_b} \mathbf{1}_{\{P_b h_{b_i} \|b_i\|^{-\eta} > \gamma\}}. \tag{9}$$

The moment generating function (*mgf*) of $\mathcal{N}_{\gamma b}$ can be written as

$$\begin{aligned}
\mathbb{E}\left[e^{t\mathcal{N}_{\gamma b}}\right] &= \mathbb{E}\left[e^{t\sum\limits_{b_i \in \mathbf{\Psi}_b} \mathbf{1}_{\{P_b h_{b_i} \|b_i\|^{-\eta} > \gamma\}}}\right] \\
&= \mathbb{E}_{\Psi_b}\left[\prod_{b_i \in \mathbf{\Psi}_b} \mathbb{E}_{h_{b_i}}\left[e^{t\mathbf{1}_{\{P_b h_{b_i} \|b_i\|^{-\eta} > \gamma\}}}\right]\right] \\
&\overset{(*)}{=} \exp\left\{-\mathbb{E}_{h_b}\left[\int_0^{2\pi}\int_0^{\left(\frac{P_b h_b}{\gamma}\right)^{\frac{1}{\eta}}} (1-e^t)\mathcal{B} r dr d\theta\right]\right\} \\
&= \exp\left\{-(1-e^t)\pi\mathcal{B}\left(\frac{P_b}{\mu_b \gamma}\right)^{\frac{2}{\eta}} \Gamma(1+\frac{2}{\eta})\right\}
\end{aligned} \tag{10}$$

where $\mathbb{E}_{\mathbf{\Psi}_b}[.]$ is the expectation w.r.t. the point process $\mathbf{\Psi}_b$ and $\mathbb{E}_{h_{b_i}}[.]$ is the expectation w.r.t. the channel gain h_{b_i}. The equality $(*)$ is obtained by the probability generating functional of the PPP and switching the order of the integration and the expectation [17]. Since the distribution of a random variable is uniquely characterized by its *mgf*, we can see that $\mathcal{N}_{\gamma b}$ has a Poisson distribution with mean $\varphi_{\gamma b} = \mathbb{E}[\mathcal{N}_{\gamma b}] = \pi\mathcal{B}\left(\frac{P_b}{\mu_b \gamma}\right)^{\frac{2}{\eta}} \Gamma(1+\frac{2}{\eta})$.

The third step in the proof is to calculate the distribution of the total number of occupied channels (K_o) within the *macro SSR* of a generic FAP. Since that all of the MBSs assign their channels in the same sequential manner, the total number of occupied channels within the *macro SSR* is equal to the number of occupied channels by the MBS with the highest number of associated users (i.e., the most congested MBS). Since each MBS $b_i \in \mathbf{\Psi}_b$ has N_{v_i} users[7], conditioning on the number of MBS $(\mathcal{N}_{\gamma b})$ within the *macro SSR* of the test FAP, we have

$$\mathbb{P}\left\{K_o \leq k | \mathcal{N}_{\gamma b} = n\right\} = \mathbb{P}\left\{\max_{i=1,2,\cdots,n}(N_{v_i}) \leq k\right\} = \left(F_{\mathcal{N}_v}(k)\right)^n. \tag{11}$$

Then, according to the theory of total probability, we have

$$\begin{aligned}
\mathbb{P}\left\{K_o \leq k\right\} &= \sum_{j=0}^{\infty} \mathbb{P}\left\{\mathcal{N}_{\gamma b} = j\right\} \mathbb{P}\left\{\max_{i=1,2,\ldots,j}(N_{v_i}) \leq k\right\} \\
&= e^{-\varphi_{\gamma b}(1-F_{\mathcal{N}_v}(k))}, \qquad k = 0,1,2,\cdots.
\end{aligned} \tag{12}$$

Finally, having the *pdf* of the number of occupied channels (K_o) within the *macro SSR* of the test FAP, the *pmf* of the number of free channels within the *macro SSR* of the test FAP is calculated by exploiting the fact that the probability for k or fewer channels being free is equal to the probability that $|\mathbf{S}| - k$ or more channels are occupied, and the lemma is proved.

[7]The random variables N_{v_i} are i.i.d. and follow the distribution of $\mathcal{N}_v$.

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References

[1] S. Cheng, S. Lien, F. hu, and K. Chen, "On exploiting cognitive radio to mitigate interference in macro/femto heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 40–47, June 2011.

[2] F. Baccelli, M. Klein, M. Lebourges, and S. Zuyev, "Stochastic geometry and architecture of communication networks," *Journal of Telecommunication Systems*, vol. 7, no. 1, pp. 209–227, 1997.

[3] T. X. Brown, "Cellular performance bounds via shotgun cellular systems," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 11, pp. 2443–2455, November 2000.

[4] J. Andrews, F. Baccelli, and R. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, November 2011.

[5] H. Dhillon, R. Ganti, F. Baccelli, and J. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE Journal on Sel. Areas in Comm.*, vol. 30, no. 3, pp. 550–560, April 2012.

[6] H. Jo, Y. Sang, P. Xia, and J. Andrews, "Outage probability for heterogeneous cellular networks with biased cell association," *IEEE Global Communications Conference (Globecom 2011)*, 5-9 December, Houston, TX, USA, 2011.

[7] T. Novlan, R. Ganti, A. Ghosh, and J. Andrews, "Analytical evaluation of fractional frequency reuse for OFDMA cellular networks," *IEEE Transactions on Wireless Communications*, vol. 10, no. 12, pp. 4294–4305, December 2011.

[8] T. Novlan, R. Ganti, A. Ghosh, and J. Andrews, "Analytical evaluation of fractional frequency reuse for heterogeneous cellular networks," *IEEE Transactions on Communications*, vol. 60, no. 7, pp. 2029–2039, July 2012.

[9] S. Mukherjee, "Distribution of downlink SINR in heterogeneous cellular networks," *IEEE Journal on Sel. Areas in Comm.*, vol. 30, no. 3, pp. 575–585, April 2012.

[10] D. Cao, S. Zhou, and Z. Niu, "Optimal base station density for energy-efficient heterogeneous cellular networks," in Proc. of *IEEE Int. Conf. on Communications (ICC 2012)*, Ottawa, Canada, 10-15 June 2012.

[11] W. Cheung, T. Quek, and M. Kountouris, "Throughput optimization, spectrum allocation, and access control in two-tier femtocell networks," *IEEE Journal on Sel. Areas in Comm.*, vol. 30, no. 3, pp. 561–574, April 2012.

[12] C. Lima, M. Bennis, and M. Latva-aho, "Coordination mechanisms for self-organizing femtocells in two-tier coexistence scenarios," *IEEE Transactions on Wireless Communications*, vol. 11, no. 6, pp. 2212–2223, June 2012.

[13] H. ElSawy and E. Hossain, "Two-tier HetNets with cognitive femtocells: Downlink performance modeling and analysis in a multi-channel environment," *IEEE Transactions on Mobile Computing*, accepted.

[14] H. ElSawy, E. Hossain, and S. Camorlinga, "Traffic offloading techniques in two-tier femtocell networks," to be presented in *IEEE ICC'13*, Budapest, Hungary, 9-13 June 2013.

[15] T. Nguyen and F. Baccelli, "A probabilistic model of carrier sensing based cognitive radio," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*, pp. 1–12, April 2010.

[16] F. Baccelli, B. Blaszczyszyn, and P. Mühlethaler, "Stochastic analysis of spatial and opportunistic ALOHA," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1105–1119, September 2009.

[17] M. Haenggi and R. Ganti, "Interference in large wireless networks," in Foundations and Trends in Networking. NOW Publishers, 2008, vol. 3, no. 2, pp. 127–248.

[18] H. ElSawy and E. Hossain, "Modeling random CSMA wireless networks in general fading environments," in Proc. of *IEEE Int. Conf. on Communications (ICC 2012)*, Ottawa, Canada, 10-15 June 2012.

[19] H. Nguyen, F. Baccelli, and D. Kofman, "A stochastic geometry analysis of dense IEEE 802.11 networks," in Proc. of *26th IEEE International Conference on Computer Communications (INFOCOM'07)*, May 2007, pp. 1199–1207.

[20] M. Haenggi, "Mean interference in hard-core wireless networks," *IEEE Communications Letters*, vol. 15, pp. 792–794, August 2011.

[21] A. Okabe, B. Boots, and K. Sugihara, *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. John Wiley, 1992.