Lower Bound of BER in M-QAM MIMO System with Ordered ZF-SIC Receiver

Juan HAN[1,2], Xiaofeng TAO[1,2], Qimei CUI[1,2]
1. Wireless Technology Innovation (WTI) Institute
2. Key Laboratory of Universal Wireless Communications, Ministry of Education
Beijing University of Posts & Telecommunications (BUPT), Beijing, P.R. China, 100876
Email: hanjuanbupt@gmail.com

***Abstract*—With the advent of multimedia services featured 4th generation (4G) mobile communications, multiple input multiple output (MIMO) combined with multi-level quadrature amplitude modulation (M-QAM) technique gains great popularity for its capability of supporting high data rate. In this paper, the statistical distribution of post signal to noise plus interference ratio (SINR) in each iterative step of optimal ordered zero forcing perfect successive interference cancellation (ZF-OOPSIC) detection is deduced based on order statistics theory. Resultantly, the bit error rate (BER) of M-QAM MIMO system with ZF-OOPSIC detector is analyzed under flat Rayleigh fading channel. This precise analytical BER result could be taken as the lower bound of various zero-forcing ordered successive interference cancellation (ZF-OSIC) receivers. Monte Carlo simulations validate the analytical results and prove the conclusions.**

***Index Terms*—BER, ZF, ordered SIC, M-QAM MIMO**

I. INTRODUCTION

In recent years, as Fourth-Generation (4G) wireless systems are being designed for offering high-quality multimedia services, which include voice, data, and video, the required data rate is increasing substantially compared with existing services. Thus the conflict between high data rate transmission and limited spectrum resource is more and more serious. In order to solve this problem, considerable research attentions have been focused on improving the spectral efficiency in wireless environment. Among all the newly developed techniques, multiple input multiple output (MIMO) has drawn much attention due to its superiority in exploring spatial resource, enhancing system capacity and increasing the spectral efficiency [1]. Theoretical analysis has proved that the channel capacity of MIMO system is directly proportional to the minor number of transmit antennas and receive antennas in rich scattering propagation environment. However, in practical applications, due to the limited size of equipment, it is difficult to improve the data transmission rate only through increasing the number of antennas. Therefore there has been a trend toward the use of multi-level quadrature amplitude modulation (M-QAM) MIMO to increase transmission rate [2].

Comprehensive evaluation of the system performance is required because the matrix wireless propagation channel may severely degrade the performance of M-QAM MIMO [3]-[5]. In [6], the closed-form analytical expression of averaged bit error rate (BER) is given for MIMO systems with two transmit antennas and multiple receive antennas. For the case of arbitrary number of transmit antennas, the results are not directly available. In [7], for zero-forcing successive interference cancellation (ZF-SIC) receiver, the analytical BER performance for BPSK mode is deduced for the systems with arbitrary number of transmit and receive antennas, but the high order M-QAM mode is not discussed. In the most recent literature on this topic [8], the symbol error rate (SER) of vertical-bell laboratory layered space-time (V-BLAST) with M-QAM is analyzed under rich scattering Rayleigh fading channel for zero-forcing (ZF) receiver and zero-forcing perfect successive interference cancellation (ZF-PSIC) receiver, which two are used respectively as the upper and lower SER bound of ZF-SIC receiver without ordering. It is well known that the BER of M-QAM is not linear with its SER. Therefore, the results in [8] can’t be conveniently applied to the M-QAM BER analysis [9]. Furthermore, ordering is pervasively used in ZF-SIC receiver, and can dramatically enhance the system performance [10][11][12], analytical results for BER of M-QAM MIMO with ordered ZF-SIC (ZF-OSIC) receiver will be discussed in this article.

Among all the ordering criterions, post signal to noise plus interference ratio (SINR) based ordering method applied in [10] is optimal. Resultantly, with perfect interference cancellation, i.e. without error propagation, the optimal ordered ZF-PSIC (ZF-OOPSIC) receiver obviously achieves the lower bound of various ZF-OSIC algorithms. In this paper, by deducing the probability density function (PDF) of post SINR in each iterative step of ZF-OOPSIC detection, analytical BER performance expression for ZF-OOPSIC receiver in M-QAM MIMO system is derived.

The paper is organized as follows. In Section II, basic system model considered in this paper is introduced. Section III presents the BER performance lower bound of M-QAM MIMO system with ZF-OSIC receiver. MC simulation results and performance comparisons are followed in Section IV. Finally,

This work is supported by the Specialized Major Science and Technology Project of China (2008ZX03003-004), National 863 project (2007AA01Z261) and NSFC project (60872048).

there come some conclusions in Section V.

II. SYSTEM MODEL

In this paper, a single-user, point to point MIMO system with N_t transmit and N_r receive antennas ($N_t \le N_r$) is considered, which is shown in fig1.

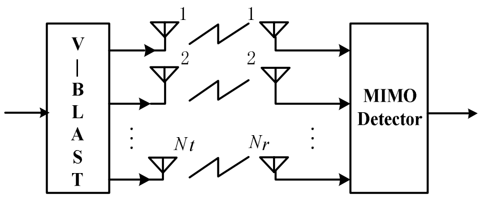

Fig. 1. Model of MIMO systems

Here, V-BLAST scheme [13] is taken as an example to carry out the performance analysis. The transmitted signal is denoted as $\mathbf{s} = [s_1, s_2, \cdots, s_{N_t}]^T$ and the received signal is $\mathbf{r} = [r_1, r_2, \cdots, r_{N_r}]^T$. Channel matrix $\mathbf{H} = (h_{ik})_{N_r \times N_t}$ follows independent identically distributed (i.i.d.) flat Rayleigh fading with zero mean and unit variance, in which fading coefficient $h_{ik} (i = 1, \cdots, N_r; \quad k = 1, \cdots, N_t)$ indicates the complex channel gain from transmit antenna k to receive antenna i. $\mathbf{n} = [n_1, n_2, \cdots, n_{N_r}]^T$ represents the zero-mean additive white Gaussian noise (AWGN) with variance σ^2 .

Further denote $\mathbf{H}$ as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{N_t}]$, in which $\mathbf{h}_k = [h_{1k}, h_{2k}, \cdots, h_{N_r k}]^T \quad (k = 1, \cdots, N_t)$ is the column vector consisting of all the fading coefficients from transmit antenna k. Then the system model could be stated as:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} = \sum_{k=1}^{N_t} \mathbf{h}_k s_k + \mathbf{n} \tag{1}$$

At the receiving end, ZF-OSIC is commonly used for detection of spatial multiplexing based MIMO, and enhances the system performance dramatically. On one hand, SIC could increase the diversity order gradually in each iterative step. On the other hand, proper ordering operation further elevates the capability of ZF-SIC by increasing the post SINR of undetected layers in each iterative step. In current literatures, ordering is operated according to various criterions. In [11], the detection order is decided by the norm of the row of matrix G with the smallest in the first and largest in the last, where $\mathbf{G} = \mathbf{H}^+$ is the moore-penrose pseudo-inverse of channel matrix $\mathbf{H}$. In [12], minors of matrix $\mathbf{H}^H\mathbf{H}$ are utilized to determine the detection order. Among all the ordering criterions, post SINR based ordering method mentioned in [10] is optimal. Only the layer with the highest post SINR is effective in each iterative step, which globally guarantees the lowest BER of each layer. Thereby, this criterion is considered in the following performance analysis.

Furthermore, ZF-OSIC suffers performance degradation when the cancelled estimated symbols are not precise. Besides, the imperfect interference cancellation causes error propagation and results in certain variable distributions changing, which makes the analytical performance highly complicated to deduce. In order to solve these problems and facilitate analysis process conveniently, perfect successive interference cancellation (PSIC) is constructed, which assumes the cancelled estimated symbols are accurate. Thus, in the rest parts of the paper, BER performance of optimal ordered ZF-PSIC (ZF-OOPSIC) is analyzed and taken as the lower BER bound of various ZF-OSIC algorithms.

III. LOWER BER BOUND OF M-QAM MIMO SYSTEM

For ZF-OOPSIC, all the following discussion is developed in the j-th iterative step. Assume that in the j-th iterative step, the transmitted symbol from the k_j-th antenna holds the highest post SINR among all the N_t-j+1 undetected symbols. Furthermore, $\mathbf{r}^{(j)}$ denotes the interference-canceled receive signal obtained in the j-th iterative step. Considering the perfect interference cancellation assumption, $\mathbf{r}^{(j)}$can be written as:

$$\mathbf{r}^{(j)} = \sum_{i=j+1}^{N_t} \mathbf{h}_{k_i} s_{k_i} + \sum_{i=1}^{j} \mathbf{h}_{k_i} \left(s_{k_i} - \hat{s}_{k_i} \right) + \mathbf{n} = \sum_{i=j+1}^{N_t} \mathbf{h}_{k_i} s_{k_i} + \mathbf{n} \tag{2}$$

Where $\hat{s}_{k_i}$ is the hard decision of estimated value of s_{k_i} . If define $\mathbf{g}_l^{(j)}$ as the nulling vector for the l-th $(l = 1, 2, \ldots, N_t - j + 1)$ undetected symbol in the j-th iterative step, $\mathbf{g}_l^{(j)}$ is the l-th row vector of Moore-Penrose pseudo-inverse of $\mathbf{H}^{(j-1)} = [\mathbf{h}_1^{(j-1)}, \mathbf{h}_2^{(j-1)}, \cdots, \mathbf{h}_{N_t - j + 1}^{(j-1)}]$, which is obtained in the (j-1)-th iterative step by extracting the columns indexed by k_1, k_2, $\cdots$, k_{j-1} from channel matrix $\mathbf{H}$. Having $\mathbf{g}_l^{(j)} \boldsymbol{h}_p^{(j-1)} = \begin{cases} 1 \ (l = p) \\ 0 \ (l \ne p) \end{cases}$, the N_t-j+1 estimated symbols derived in the j-th iterative step could be expressed by $\tilde{s}^{(j)}$ as follows:

$$\begin{aligned} \tilde{s}_l^{(j)} &= \mathbf{g}_l^{(j)} \mathbf{r}^{(j-1)} = \mathbf{g}_l^{(j)} \left(\sum_{i=j}^{N_t} \mathbf{h}_{k_i} s_{k_i} + \mathbf{n} \right) \\ &= \mathbf{g}_l^{(j)} \mathbf{h}_l^{(j-1)} s_l + \sum_{i=j, k_i \ne l}^{N_t} \mathbf{g}_l^{(j)} \mathbf{h}_{k_i} s_{k_i} + \mathbf{g}_l^{(j)} \mathbf{n} = s_l + \mathbf{g}_l^{(j)} \mathbf{n} \end{aligned} \tag{3}$$

Specially, among all the N_t-j+1 estimated symbols, the one related to the k_j-th antenna, which obtains the highest post SINR in the current iterative step, is effective. Based on (3), if total power P is divided equally among all the transmit antennas, the post SINR of the l-th undetected layers in the j-th iterative step of ZF-OOPSIC algorithm is

$$SINR_l^{ZF-OOPSIC(j)} = \frac{E\left\{ \left| s_l \right|^2 \right\}}{E\left\{ \left| \mathbf{g}_l^{(j)} \mathbf{n} \right|^2 \right\}} = \frac{\lambda_0}{\left\| \mathbf{g}_l^{(j)} \right\|_2} \tag{4}$$

Where $E\{\bullet\}$ denotes the mathematical expectation, $\|\bullet\|_2$ is the 2-norm of vector, and $\lambda_0 = P / \sigma^2$ represents the averaged transmitted signal to noise ratio (SNR). In (4), $\frac{1}{\left\|\mathbf{g}_l^{(j)}\right\|_2}$ follows independent χ^2 distribution with degree of freedom $2\left(N_r - N_t + j\right)$ and variance 1/2 [14][15]. If define a variable $x_l^{(j)} = \sqrt{\frac{1}{\left\|\mathbf{g}_l^{(j)}\right\|_2}}$, $(l = 1, 2, \cdots, N_t - j + 1)$, for any l, $x_l^{(j)}$ is amenable to generalized Rayleigh distribution with degree of freedom $2\left(N_r - N_t + j\right)$ and variance 1/2 [16], and its PDF meets:

$$p(x^{(j)}) = \frac{2}{\left(N_r - N_t + j - 1\right)!}(x^{(j)})^{2(N_r - N_t + j) - 1} e^{-(x^{(j)})^2} \qquad (5)$$

Since the degree of freedom $2\left(N_r - N_t + j\right)$ is even, the cumulative density function (CDF) of $x_l^{(j)}$ has the closed-form expression as [16]:

$$F(x^{(j)}) = 1 - e^{-(x^{(j)})^2} \sum_{k=0}^{(N_r - N_t + j - 1)} \frac{(x^{(j)})^{2k}}{k!} \qquad (6)$$

For ordering operation, the maximum post SINR in each iterative step should be selected, thus the largest one is identified among the N_t-j+1 i.i.d. random variables $x_l^{(j)}(l = 1, 2, \cdots, N_t - j + 1)$. This issue could be modelled as an order statistics problem [17]. Let $x_{(1)}^{(j)}, x_{(2)}^{(j)}, \ldots,\ x_{(N_t - j + 1)}^{(j)}$ denote the order statistics of the random sample $x_l^{(j)}$ from the continuous population with CDF $F(x^{(j)})$ and PDF $p(x^{(j)})$, where $x_{(1)}^{(j)} < x_{(2)}^{(j)} < \ldots, < x_{(N_t - j + 1)}^{(j)}$. Then, the PDF of $x_{(l)}^{(j)}$ is

$$p_l(x^{(j)}) = \frac{n!}{(l-1)!\ (n-l)!} F(x^{(j)})^{l-1} [1 - F(x^{(j)})]^{n-l}\ p(x^{(j)}) \quad (7)$$

Where $n = N_t$-j+1. According to (5) (6) (7), in the j-th iterative step, the PDF for the maximum $x^{(j)}$ (i.e. $x_{(N_t - j + 1)}^{(j)}$) is derived as (8) at the top of next page.

Under AWGN channel, the BER of optimized receiver utilizing M-QAM with rectangular signal constellations could be expressed as (9) when $M = 4^t (t = 1, 2, \ldots)$ [9][18]-[20]:

$$P_M = \left(1 - \frac{1}{\sqrt{M}}\right) \cdot \frac{1}{\log_2 \sqrt{M}} \cdot erfc\left(\sqrt{\frac{3 \bullet SINR}{2(M-1)}}\right) \qquad (9)$$

Defining $A = \left(1 - \frac{1}{\sqrt{M}}\right)$, $B = \frac{1}{\log_2 \sqrt{M}}$, $\xi = \sqrt{\frac{3\lambda_0}{2(M-1)}}$, then using (8) and (9), the averaged BER performance of ZF-OOPSIC algorithm under flat Rayleigh fading channel can be derived as (10) at the top of next page.

IV. Simulation Results and Analysis

In this section, extensive simulations are carried out to support the analytical results derived above. BER performance of M-QAM MIMO systems with different antenna configurations and modulation modes are investigated.

In Fig.2, the results of analytical and MC simulations for 2x4 with 16QAM, 2x4 with 64QAM and 4x8 with 16QAM MIMO systems are demonstrated. It can be read from this figure that the analytical results achieved by (10) have no significant difference compared with the MC simulation results of corresponding ZF-OOPSIC receiver. Thus the conclusion can be crisply drawn that the analytical BER for ZF-OOPSIC receiver in (10) is reasonable. Additionally, all the MC simulation results for optimal ordered ZF-SIC (ZF-OOSIC) detector are tightly approaching the deduced lower bound in (10), which illustrates the slight but confirmed performance degradation caused by error propagation. Since curves for ZF-OOSIC detector is above the analytical results of (10), meanwhile, ZF-OOSIC outperforms all other ZF-OSIC detectors, it can be concluded that (10) could be used as the lower BER bound for all kinds of ZF-OSIC detectors.

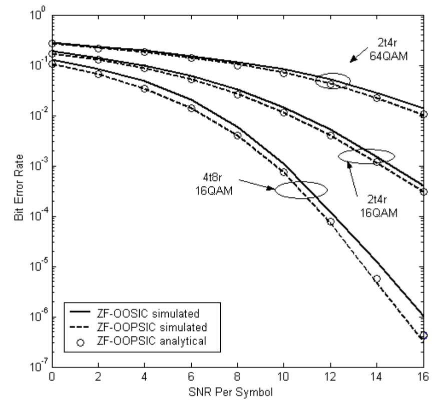

Fig 2. BER performance of M-QAM MIMO system by analytical and MC simulations

In Fig.3, analytical BER lower bound deduced in this paper is utilized directly for further discussion. Results of numerical simulations for 2x4, 4x6 and 4x8 MIMO systems with 16QAM are demonstrated here. For each system configuration, three curves of analytical BER for ZF, ZF-PSIC and ZF-OOPSIC detectors respectively, are laid out for comparison. Here, the curves for ZF and ZF-PSIC detectors are obtained from (9) and the corresponding SINR distribution deduced in [8]， in the similar way of obtaining the ZF-OOPSIC result in (10). As the performance of ZF receiver is only related to the difference between transmit and receive antenna number[8], the ZF curves of 2x4 and 4x6 MIMO systems are identical. However, compared with the results in 2x4 MIMO system, the gap between ZF-OOPSIC and ZF-PSIC detectors is even larger especially in high SNR region when antenna configuration is 4x6. Since $N_r - N_t$ are the same for the two groups of curves, It can be concluded that ordering operation benefits from a bigger N_t. The prime cause of this is that iteration time is decided by N_t. More iteration time holds more potential in elevating detection

$$p^{(j)}_{(N_t-j+1)}(x)=\frac{2(N_t-j+1)}{\left(N_r-N_t+j-1\right)!}\left[1-e^{-x^2}\sum_{k=0}^{(N_r-N_t+j-1)}\frac{x^{2k}}{k!}\right]^{N_t-j} x^{2(N_r-N_t+j)-1}e^{-x^2} \tag{8}$$

$$\begin{aligned}\tilde{P}_M^{ZF-OOPSIC} &= \frac{1}{N_t}\sum_{j=1}^{N_t}\tilde{P}_M^{ZF-OOPSIC(j)} = \frac{1}{N_t}\sum_{j=1}^{N_t}\left(\int_0^{\infty}P_M(x)\,p^{(j)}_{(N_t-j+1)}(x)dx\right) \\ &= \frac{1}{N_t}\sum_{j=1}^{N_t}\left(\frac{2AB(N_t-j+1)}{\left(N_r-N_t+j-1\right)!}\int_0^{\infty}erfc\left(\xi x\right)\bullet\left[1-e^{-x^2}\sum_{k=0}^{\left(N_r-N_t+j-1\right)}\frac{x^{2k}}{k!}\right]^{N_t-j} x^{2(N_r-N_t+j)-1}e^{-x^2}dx\right)\end{aligned} \tag{10}$$

performance, which could be realized by proper ordering operations.

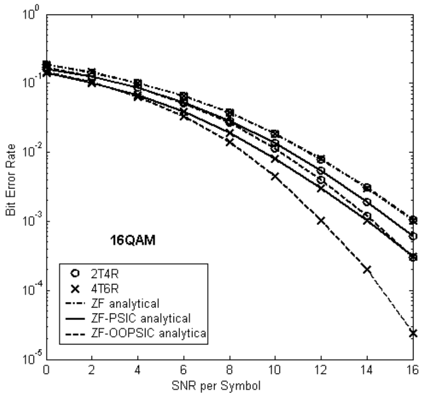

Fig 3. Analytical BER performance of M-QAM MIMO systems with different receivers (1)

Fig 4 shows the case of 4x6 and 4x8 MIMO with 16QAM. Obviously, on the premise of identical transmit antenna number, larger receive antenna number induces performance elevation for all kinds of detectors. Especially, with the same SNR, 4x8 MIMO sees more performance promotion caused by ordering operation than that of 4x6 MIMO.

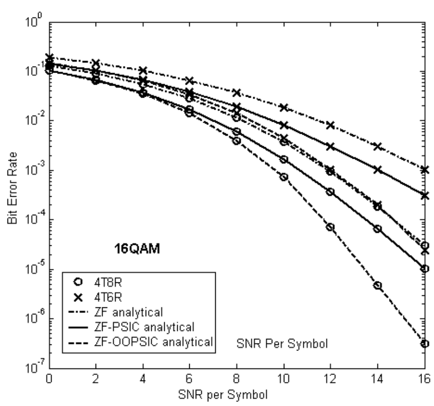

Fig 4. Analytical BER performance of M-QAM MIMO systems with different receivers (2)

V. Conclusion

In recent years, as demanded transmission data rate of wireless communications is rocketing, MIMO system draws much attention due to its high spectrum efficiency and capability of exploring spatial resource. With high order M-QAM, MIMO system seizes even more superiority in supporting high data rate transmission. Therefore, solid contribution towards theoretical analysis of M-QAM MIMO system is needed. In this paper, BER performance of M-QAM MIMO systems with ZF-OSIC detector is investigated under Rayleigh fading channel. The statistical distribution of post SINR in each iterative step of ZF-OOPSIC detection is derived here, which is afterward employed in deducing the analytical BER for ZF-OOPSIC receiver. Taking the resultant analytical BER of ZF-OOPSIC detector as lower bound, it is convenient to roughly predicate the performance of various ZF-OSIC receivers for link optimization and system configuration. Extensive MC simulations are carried out to verify the validity of our analytical results.

References

[1] G. J. Foschini and M. J. Gans, “On limits of wireless communications in a fading environment when using multiple antennas,” Wireless Personal Communications, vol. 6, no. 3, pp. 311-335, 1998.

[2] Ping Zhang, Xiaofeng Tao, Jianhua Zhang, Ying Wang, Lihua Li and Yong Wang, “The visions from FuTURE beyond 3G TDD,” IEEE Communications Magazine, vol. 43, no. 1, pp. 38-44, Jan. 2005.

[3] S. L. Loyka and J. R. Mosig, “Channel capacity of N-antenna BLAST architecture,” Electron. Lett., vol. 36, no. 7, pp. 660–661, Mar. 2000.

[4] S. L. Loyka, “Channel capacity of MIMO architecture using the exponential correlation matrix,” IEEE Commun. Lett., vol. 5, pp. 369–371, Sept. 2001.

[5] S. Loyka and G. Tsoulos, “Estimating MIMO system performance using the correlation matrix approach,” IEEE Commun. Lett., vol. 6, pp. 19–21, Jan. 2002.

[6] S. Loyka and F. Gagnon, “Performance analysis of the V-BLAST algorithm: an analytical approach,” IEEE Transactions on Wireless Communications, vol. 3, no. 4, pp. 1326-1337, July. 2004.

[7] Cong Shen, Yan Zhu, Shidong Zhou and Jinjing Jiang, “On the performance of V-BLAST with zero-forcing successive interference cancellation receiver,” Proceedings of IEEE GLOBECOM ’04, Dallas USA, 2004, pp. 2818-2822.

[8] Jin Xu, Xiaofeng Tao and Ping Zhang, “Analytical SER Performance Bound of M-QAM MIMO System with ZF-SIC Receiver,” IEEE ICC '08. 19-23 May 2008, pp. 5103 – 5107.

[9] Dongweon Yoon; Kyongkuk Cho; Jinsock Lee, “Bit error probability of M-ary quadrature amplitude modulation,” 52nd IEEE Vehicular Technology Conference, Boston, MA, Sep. 24-28, 2000. Vol. 5, pp. 2422 – 2427.

[10] Wolniansky P.W., Foschini G.J., Golden G.D. and Valenzuela R. A., “V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel,” International Symposium on Signals, Systems, and Electronics, 29 Sept.-2 Oct. 1998, pp. 295 – 300.

[11] Tao Xiaofeng, Yu Zhuizhuan, Qin Haiyan, Zhang Ping, Haas H. and Costa E., “New sub-optimal detection algorithm of layered space-time code,” VTC Spring 2002. vol. 4, 6-9 May 2002, pp. 1791 – 1794.

[12] GUO Zhi-heng; LI Li-hua and TAO Xiao-feng, “Fast Ordering Detection Algorithm for V-BLAST System,” Journal of Beijing University of Posts and Telecommunications, 2007, vol. 30, no. 04, pp. 83-87.
[13] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas,” Bell Labs Technical Journal, vol. 1, no. 2, pp. 41-59, 1996.
[14] Jinshan Liu, “The Introduction of Wishart Distribution,” Beijing: Science Press, 2005, pp. 70-97.
[15] Xin Li and Zaiping Nie, “Performance losses in V-BLAST due to correlation,” IEEE Antennas and Wireless Propagation Letters, vol. 3, no. 1, pp. 291-294, 2004.
[16] J. G. Proakis, “Digital Communications,” 4th ed., New York: McGrawHill, 2001, pp.45-46, 278.
[17] G. Casella and R. L. Berger, “Statistical Inference,” Duxbury Advanced Series; Second Edition, 2002
[18] LL Yang, L Hanzo, “A recursive algorithm for the error probability evaluation of M-QAM,” IEEE Communications Letters, Oct.2000, Vol.4, pp. 304-306.
[19] J Lu, KB Letaief, JCI Chuang, ML Liou, “M-PSK and M-QAM BER computation using signal-space concepts,” IEEE Transactions on Communications, Feb 1999, Vol. 47, pp. 181-184.
[20] K Cho, D Yoon, W Jeong, M Kavehrad, “BER analysis of arbitrary rectangular QAM,” IEEE Transactions on Communications, Jul 2002, Vol. 50, no.7, pp. 1074- 1080.