A Successive Interference Cancellation Scheme for an OFDM system

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Abstract - **In this paper we will consider an Orthogonal Frequency Division Multiplexing (OFDM) system with successive interference cancellation to reduce interchannel interference. The system operates in a Rayleigh frequency selective channel and pilot symbols are inserted in each OFDM frame for the purpose of channel estimation. Interchannel interference (ICI) is caused by the time variations in channel gain within one OFDM frame. The ICI leads to an error floor, which is a function of the Doppler frequency. In this study, it was found that by performing successive interference cancellation at the receiver, we were able to significantly reduce the error floor caused by ICI. Furthermore, we introduce in this study a simple channel estimator based on pilot symbols to estimate the multipath fading channel information required to perform the SIC.**

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is an efficient scheme to mitigate the effects of urban delay spreads causing frequency selectivity. It uses the discrete Fourier transform (DFT) technique to multiplex a wideband single-channel signal into numerous narrowband flat subchannels. By making the block period much larger than the delay spread of the channel, the effect of intersymbol interference (ISI) is greatly reduced.

OFDM is spectrally efficient by allowing adjacent subchannels to spectrally overlap, yet remain orthogonal in time [1]. Our study looks at the performance of an OFDM system in a frequency selective, Rayleigh fading channel. By adding a cyclic extension, longer than this impulse response, between consecutive DFT blocks, we can avoid interblock interference and also preserve the orthogonality of the tones [2]. However, time variations of the channel within an OFDM frame also lead to a loss of subchannel orthogonality, resulting in interchannel interference (ICI) [1, 3]. In the research by Stuber [3], the ICI was modeled as an additive Gaussian random process which led to an irreducible error floor. This error floor can be determined analytically as a function of the Doppler frequency.

In this paper, we attempt to reduce the effects of ICI on an OFDM system by using a multistage successive interference cancellation (SIC) scheme similar to that used in DS/CDMA systems [4, 5]. In addition, we consider the effects of channel noise and also use a simple pilot symbol based channel estimator to obtain the channel impulse response necessary to perform successive interference cancellation.

The paper is organized as follows. The system model and the received signal structure is presented in Section 2. Section 3 gives an error analysis of the system. The SIC receiver structure is described in Section 4, and the Channel estimation technique is presented in Section 5. Simulation results for the bit error rates and frame error rates will be presented in section 6, followed by the conclusions of the study in Section 7.

2 System model

Figure 1 shows the discrete-time baseband equivalent model of the system under consideration. The input binary data d_s enters the Quadrature Phase Shift Keying (QPSK) symbol generator, which outputs symbols a_n at a rate of $1/T$. We will assume that these symbols are independent and identically distributed (iid). The serial-to-parallel converter transfers blocks of symbols to the OFDM modulator, which uses an N-point IFFT to modulate them onto the subchannels. A cyclic extension G samples in length is added as a guard interval. The role of the cyclic extension is to avoid interference between consecutive frames and also to convert the linear convolution of the data sequence and the channel impulse response to a circular convolution [6]. A pilot band of length $2G$ is then placed at the beginning of the OFDM frame to provide channel estimation. Details on the pilot band will be given in Section 5. The resulting $N+3G$ complex valued symbols are then transmitted at a rate of $1/T_s$, where $\frac{1}{T_s} = \frac{1}{T}(1+\frac{3G}{N})$. The transmitted sequence corresponding to the samples at $t = kT_s$ is

$$S_k = \begin{cases} 0 & 0 < k < G \\ 1 & k = G \\ 0 & G < k < 2G \\ \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n \exp\left(j\frac{2\pi nk}{N}\right) & 2G \le k \le N+3G-1 \end{cases} \qquad (1)$$

The channel is modeled as a wide sense stationary, uncorrelated scattering (WSSUS), Rayleigh fading channel with maximum delay of MT_s. The received sequence will then have the form

$$R_k^g = \sum_{m=0}^{M-1} h_{m,k} S_{k-m} + f_k \quad 0 \le k \le N+3G-1, \qquad (2)$$

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where $h_{m,k}$ is the value of the channel impulse response at position m and instant k from the dispersive paths, and f_k is the value of additive noise at instant k. The demodulator then removes the pilot band (first $2G$ samples) and also the guard interval according to $R_k = R^g_{3G+(k-G)_N}$, $0 \le k \le N-1$, and performs a DFT on the resulting sequence.

At the output of the DFT demodulator, the sequence can be written as

$$Z_l = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} a_n H_m(n-l)\exp\left(-j\frac{2\pi nm}{N}\right) + n_l \quad (3)$$

for $0 \le l \le N-1$, where

$$H_m(n-l) = \frac{1}{N}\sum_{k=0}^{N-1} h_{m,G+(k-G)_N}\exp\left(j\frac{2\pi k}{N}(n-l)\right) \quad (4)$$

and

$$n_l = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} f_k\exp\left(-j\frac{2\pi lk}{N}\right) \qquad 0 \le l \le N-1. \quad (5)$$

The Z_l's of different frames are fed to a SIC detector, which will be disussed later in Section 4. We can see that the Z_l term in (3) contains information from all the transmitted symbols a_n for $0 < n < N$. In fact, as we will show in Section 3, all the terms a_n where $n \neq l$ contrubute to interchannel interference, and only exists when the channel $h_{m,k}$ varies within the OFDM frame. In the extreme case where the Doppler frequency = 0, ICI becomes non-existent and the orthoganality of the received signal is preserved.

3 Error Analysis for a Conventional Detector

For the Rayleigh channel described above, the channel signal to noise ratio (SNR) is defined as

$$\mathrm{SNR} = \frac{\frac{1}{2}\mathrm{E}\{h \cdot h^*\}}{\frac{1}{2}\mathrm{E}\{f \cdot f^*\}} = \frac{\sum_{m=0}^{M-1}\sigma^2_{h_m}}{\sigma_f^2} \quad (6)$$

where $\sum_{m=0}^{M-1}\sigma^2_{h_m}$ is the signal power from the dispersive paths, and σ_f^2 is the noise power of the channel. By normalizing σ_f^2 and thus σ_n^2 to 1, our SNR becomes

$$\mathrm{SNR} = \sum_{m=0}^{M-1}\sigma^2_{h_m} \quad (7)$$

For the conventional receiver in [3], the ICI is modeled as a white Gaussian process (for a sufficiently large N). Let us rewrite (3) as

$$\begin{aligned} Z_l &= \zeta_l a_l + c_l + n_l \\ &= \zeta_l a_l + e_l. \end{aligned} \quad (8)$$

where

$$\zeta_l = \sum_{m=0}^{M-1} H_m(0)\exp\left(-j\frac{2\pi lm}{N}\right), \quad (9)$$

is a complex Gaussian variable with a variance of

$$\begin{aligned} \sigma_u^2 &= \frac{1}{2}\mathrm{E}\{\zeta_l \cdot \zeta_l^*\} \\ &= \sum_{m=0}^{M-1}\sigma^2_{h_m}\cdot\Omega \;=\; \mathrm{SNR}\cdot\Omega \end{aligned} \quad (10)$$

which we derive using the WSSUS channel model, and

$$\Omega = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{k'=0}^{N-1} J_0\Big(2\pi f_D T_s((k-G)_N - (k'-G)_N)\Big). \quad (11)$$

Also, e_l in (8) is the sum of the noise term n_l and the term c_l where

$$c_l = \sum_{n\neq l}\sum_{m=0}^{M-1} a_n H_m(n-l)\exp\left(-j\frac{2\pi nm}{N}\right), \quad (12)$$

is the random ICI term, with variance

$$\sigma_c^2 = \sum_{m=0}^{M-1}\sigma^2_{h_m}\cdot(1-\Omega) \;=\; \mathrm{SNR}\cdot(1-\Omega). \quad (13)$$

e_l can then be treated as a combined additive white Gaussian noise term with power

$$\sigma_e^2 = \sigma_c^2 + \sigma_n^2 = \mathrm{SNR}\cdot(1-\Omega) + 1 \quad (14)$$

Note that when there are no time variations in gain during a block period, $h_{m,k} = h_m$ and $H_m(n-l) = h_m\delta_{nl}$, where δ_{nl} is the Kronecker delta function. As a result, $c_l = 0$ and there is no ICI.

The error analysis above provides the necessary information to produce analytical results for the conventional OFDM receiver. The analysis method used is based on a method presented by Cavers and Ho in [7] and have been used to analyze both an uncoded and a TCM-coded OFDM system in [8] by the authors of this paper.

We see that even in the absence of channel noise ($\sigma_n^2 = 0$), the presence of ICI will lead to an irreducible error floor from (14) and this is why we need an SIC detector.

4 SIC Detector

We consider in this study a successive interference canceller (SIC) for data detection. We first consider a system with perfect channel state information (CSI), and consider channel estimation in the next section. Recall from equation 8 that ICI is considered as additive noise. If we can subtract the ICI terms (c_l) from the symbols Z_l, we can eliminate ICI. To accurately compute c_l, we have to know all the transmitted symbols a_n ($n \neq l$). The symbols a_n are what we need to find for detection but we also need them for ICI cancellation. This indicates a need for a decision feedback system. At the receiver, we can make decisions on the received symbols Z_l ($0 < l < N$) to produce the detected symbols $\hat{a}_l$. But if we feed back incorrectly detected symbol information, the ICI effects may worsen. Hence, we should only feed back ICI information if our "confidence" level of the particular $\hat{a}_l$ is high. As a result, a ranking criteria is needed to determine the confidence level.

4.1 SIC First Stage

We rank the "confidence" level of the received symbols based on the likelihood criteria, where we compute the Euclidean distance (ED) for all 4 possible constellation points (QPSK) P_i (Equation 15).

$$D_l = \min_{i=1,\dots,4} |Z_l - \zeta_l P_i|^2 \tag{15}$$

D_l is computed for all N symbols to obtain the ranking list, with the symbol containing the smallest D_l ranked first. Then, starting with the highest ranked symbol, call it symbol $r1$, we detect the symbol $\hat{a}_{r1}$ with no ICI information fed back. That is,

$$\hat{a}_{r1} = P_i \tag{16}$$

if P_i minimizes

$$D_{r1} = \min_{i=1,\dots,4} |Z_{r1} - \zeta_{r1} P_i|^2 \tag{17}$$

For the 2nd ranked symbol, to compute $\hat{a}_{r2}$, the Euclidean distance criteria is performed on

$$D_{r2} = \min_{i=1,\dots,4} |Z_{r2} - \zeta_{r2} P_i - \mathrm{ICI}(\hat{a}_{r1})|^2, \tag{18}$$

where the ICI term $\mathrm{ICI}(\hat{a}_{r1})$ is computed according to (12) based on the only detected term $\hat{a}_{r1}$. Specifically,

$$\mathrm{ICI}(\hat{a}_{r1}) = \sum_{m=0}^{M-1} \hat{a}_{r1} H_m(r_1 - r_2) \exp\left(-j\frac{2\pi r_1 m}{N}\right). \tag{19}$$

For the 3rd ranked symbol, once again, we choose the constellation point P_i that minimizes

$$D_{r3} = \min_{i=1,\dots,4} |Z_{r3} - \zeta_{r3} P_i - \mathrm{ICI}(\hat{a}_{r1}, \hat{a}_{r2})|^2, \tag{20}$$

with $\mathrm{ICI}(\hat{a}_{r1}, \hat{a}_{r2})$ calculated as

$$\begin{aligned}\mathrm{ICI}(\hat{a}_{r1}, \hat{a}_{r2}) &= \sum_{m=0}^{M-1} \hat{a}_{r1} H_m(r_1 - r_3) \exp\left(-j\frac{2\pi r_1 m}{N}\right) + \\ &\quad \sum_{m=0}^{M-1} \hat{a}_{r2} H_m(r_2 - r_3) \exp\left(-j\frac{2\pi r_2 m}{N}\right)\end{aligned} \tag{21}$$

Similarly, the rest of the symbols in the ranking list are decoded while more and more ICI terms are fed back to the canceller as more $\hat{a}$ terms become available. Figure 2 shows an illustration of the SIC first stage.

4.2 SIC (2nd stage and above)

After all the $\hat{a}_l$ symbols are decoded using the SIC procedure described above, we have completed one stage of the interference cancellation. Recall that the top ranked symbols had little to no ICI removed from them. Therefore, in the second stage, we feed back all the ICI information, obtained from the previous stage, when we make decisions on symbols, hoping to get a better interference cancellation. Subsequent SIC stages (k^{th} stages) are identical to the 2nd stage, using the ICI information from the $(k-1)^{\mathrm{th}}$ stage to perform cancellation. Figure 3 gives and illustration of these SIC stages.

5 Channel Estimation

The SIC detector, besides needing feedback information on the $\hat{a}$ symbols, requires information on the channel impulse response (refer to Equation 9). In our study, channel estimation is done using a pilot band at the beginning of each OFDM frame. Figure 4 illustrates how the pilot band is inserted in the OFDM frame.

At the beginning of each OFDM frame, a period of null transmission of a duration equivalent to that of the guard interval (G samples) is inserted. A pilot tone, one sample duration in length, is then transmitted, followed by another G samples of null transmission. Recall that the duration of the guard interval is chosen to be bigger than the maximum delay spread. Hence this allows the receiver to extract the (noise corrupted) delay power profile from the pilot band. In order to track the time varying channel gain, we need to perform gain interpolation and provide the ζ_l terms to the SIC detector. Gain interpolation is performed using 9 consecutive frames (from the 9 pilot bands) by resampling the pilot data at a higher rate using lowpass interpolation. Each sample in the pilot band represents a possible delay path. But it is quite likely that a number of those paths do not exist, and are simply noise samples. To reduce complexity, we do not wish to interpolate all the delay paths in the pilot band, but rather try to identify which paths are relevant to be interpolated.

To determine which paths are present, we observe all the samples within the pilot band for all 9 frames. An average of the 9 frames is then taken for each possible delay position. Furthermore, we take the average of the 10 smallest averages and consider that the noise floor level. In our study, using the 10 smallest averages has proven to work quite well. To then determine which delay paths are present, we compare each delay average with the noise floor level. If the delay average is greater than 10 times the noise floor level, we consider the delay path to exist at that location. If not, we disregard the path as non-existent, or too weak to be relevant. Once we obtain the delay power profile, we perform gain interpolation on the paths we consider to exist.

To obtain the gain for frame k, we use the pilot bands from frames $(k-4)$ to $(k+4)$ to perform the interpolation. Hence there is a 4 frame delay at the output of the receiver.

6 Simulation Results

In our simulation we were considering an OFDM system operating in a frequency selective fading environment. Uncoded QPSK modulation using coherent detection was assumed, and each OFDM frame contains $N = 256$ data symbols. The normalized fade rate considered was $f_d T_f = 3.5\%$, where T_f is the period of an OFDM frame. The guard interval G was chosen to be $7\mu s$, sufficient to cover the $5\mu s$ maximum delay spread in a typical urban environment.

For multipath, we chose to use a two path model, with equal power split, one at zero delay, and the other at $5\mu s$ delay. We will present two sets of results, one for an OFDM system using the SIC detector with perfect channel state information (CSI). The other set is for an OFDM system using the SIC detector, with the channel estimated by the proposed channel estimation scheme. We consider both the bit error rate (BER) and frame error rate (FER) for each system. A frame is considered erroneous if a single bit within the frame has an error.

Figure 5 shows the average BER performance of the SIC detector with perfect CSI knowledge. The results are compared with a conventional detector with no ICI cancellation, and up to 4 stages of SIC were used. It can be seen that the SIC receiver performs slightly better at the regions of lower SNR. For example, there is an approximately 2dB gain for the 4-stage SIC detector at 10^{-3} BER. For higher SNR, the SIC receivers show a performance improvement over the conventional detector and reduce the error floor. The more stages there are, the more reduction in BER, but with diminishing return. In general, the BER performance is modest because we only see about a 2-fold improvement in the irreducible error floor for the 4-stage SIC receiver. The reason is because most errors come in bursts. Many OFDM frames have little to no errors, and those frames are easily correctable using the SIC detector. However, frames with large errors occasionally occur, and performing SIC, like any other decision feedback receivers, will worsen the BER. These error bursts will dramatically degrade the BER. Hence we also study the frame error rate (FER) performance.

Figure 6 shows the average FER performance of the SIC detector. Once again, a conventional detector with no ICI cancellation is used for comparison. We see that the error floor has been reduced by almost a decade. This confirms our earlier observation about error bursts, and we see that the SIC scheme does in fact operate well for frames with small amounts of error, caused by either ICI or by channel noise.

We then look at the performance of the SIC detector using our channel estimator. Figure 7 shows the preliminary results for the average BER performance of the detector with channel estimation. The performance curve of the receiver with perfect CSI knowledge and no ICI cancellation is included for comparison purposes. We see that although in general the system does not perform as well as the system with CSI knowledge, the trend appears to be the same. A higher SIC stage turns out better performance. The simulation results for the FER were also done for this system, and is shown in Figure 8. We also see that although the system does not work as well as that with perfect CSI knowledge, the results indicate that the SIC scheme does in fact improve the error performance at high SNR values and reduce the error floor considerably.

7 Conclusions

We presented in this paper an SIC receiver structure for OFDM in a frequency selective Rayleigh fading channel. The first part of the receiver performs channel estimation in order to provide the channel impulse response needed for proper detection. The second part of the receiver consists of a successive interference canceller that exploits the channel state information provided by the channel estimator, along with decision feedback symbols, for the purpose of signal regeneration and cancellation.

The results indicate that the SIC receiver indeed provides improvement over the conventional receiver with no ICI cancellation. The decision feedback SIC receiver works well when the number of errors in the frame is small, which is indicated by the better FER performance over the BER performance. Increasing the number of SIC stages also proved to reduce the error rates, up to a limit, and 4 stages seem to be quite sufficient to achieve that limit.

Preliminary results of the OFDM system with channel estimator also indicate that the channel estimation scheme works well in this system to provide the necessary channel impulse response information to the SIC receiver.

Further improvements on the channel estimator is currently under investigation, along with the study of different ranking criteria used in the first SIC stage.

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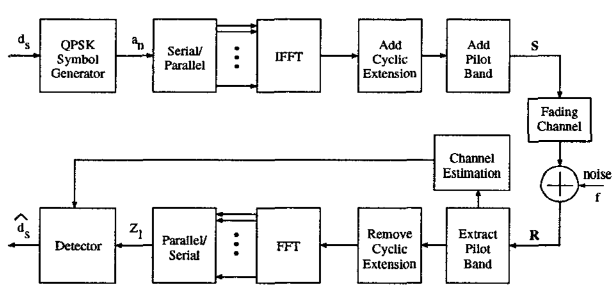

Figure 1: System model for the OFDM system

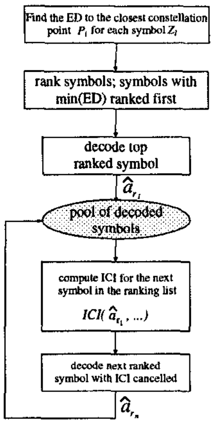

Figure 2: Block diagram of the SIC detector (First stage)

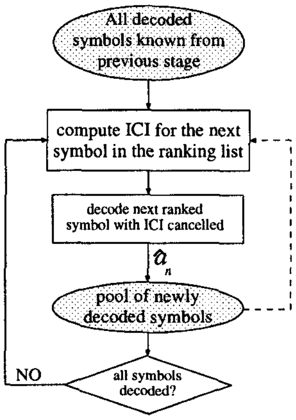

Figure 3: Block diagram of the SIC detector (Other stages)

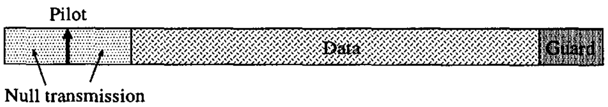

Figure 4: Illustration of Pilot band in an OFDM frame

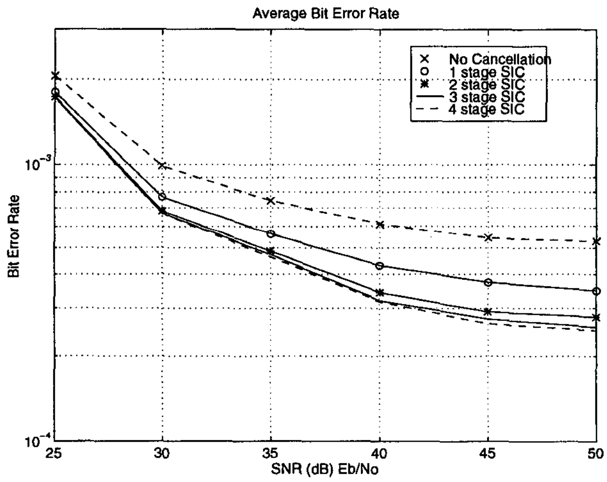

Figure 5: BER of SIC detector with perfect CSI

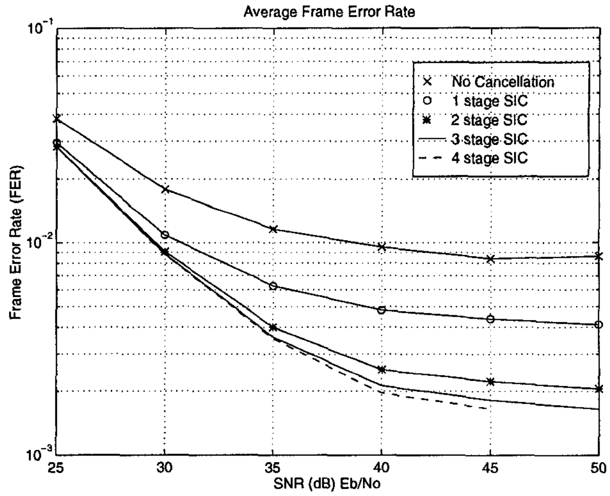

Figure 6: FER of SIC detector with perfect CSI

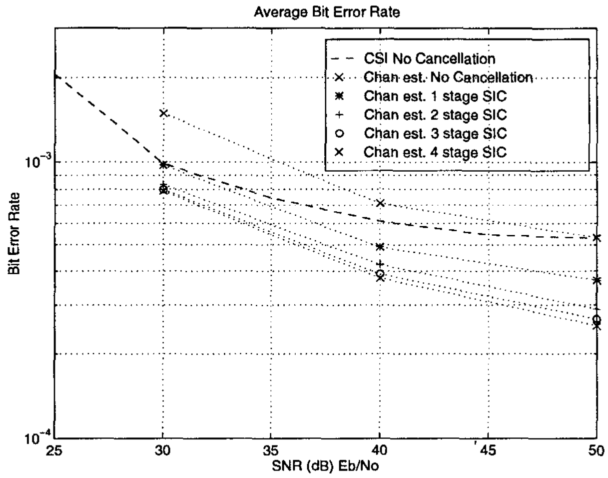

Figure 7: BER of SIC detector with channel estimation

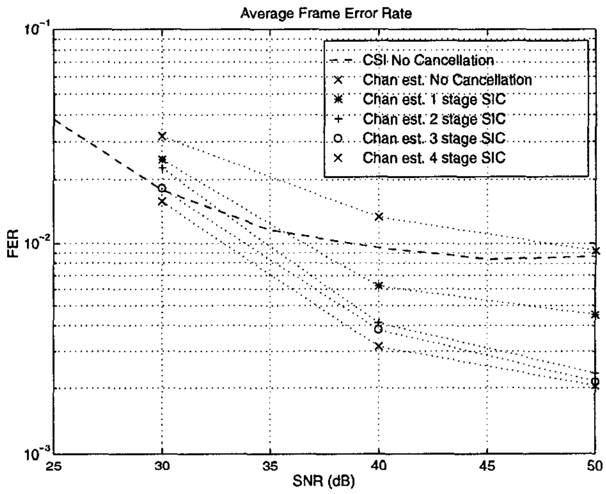

Figure 8: FER of SIC detector with channel estimation