# Low Complexity MMSE-SIC Equalizer Employing Time-Domain Recursion for OFDM Systems

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***Abstract*—In this letter, a novel time-domain recursive algorithm is proposed for a minimum-mean-squared error (MMSE) with successive interference cancellation (SIC) scheme. This algorithm allows us to reduce the complexity of classical MMSE-SIC equalizer for suppressing intercarrier interference (ICI) caused by time-varying multipath channels in orthogonal frequency division multiplexing (OFDM) systems, and it can provide almost the same performance as that of the optimal MMSE-SIC equalizer. It is shown in complexity analysis that the proposed scheme shows better complexity reduction than previously reported low-complexity MMSE-SIC schemes when applied to a channel equalizer for an OFDM system.**

***Index Terms*—Equalization, minimum-mean-squared error with successive interference cancellation (MMSE-SIC), orthogonal frequency division multiplexing (OFDM).**

## I. Introduction

In orthogonal frequency division multiplexing (OFDM) systems, the time selectivity of wireless channel introduces intercarrier interference (ICI), which degrades system performance [1]–[3]. To compensate the ICI effect, a linear minimum-mean-squared error (MMSE) equalizer can be employed with $\mathcal{O}(N^3)$ complexity, where $N$ denotes the number of subcarriers in an OFDM symbol. However, future wireless OFDM systems may support high mobility, which causes highly time-varying channels where a more powerful equalizer is required. In [2], a linear MMSE equalizer incorporated with successive interference cancellation (SIC) in the frequency domain was proposed with $\mathcal{O}(N^4)$ complexity, which shows good performance even at high normalized Doppler frequency ($f_n = f_m T_s \geq 0.1$), where $f_m$ is the maximum Doppler frequency and $T_s$ is the OFDM symbol interval. This scheme is identical to the time domain Vertical Bell Labs Layered Space-Time (V-BLAST) detection. To reduce the complexity and maintain the performance, we can directly exploit one of various low complexity approaches for V-BLAST signal detection with $\mathcal{O}(N^3)$ complexity, e.g., square-root algorithms in [4]–[6] and fast recursive (FR) algorithms in [7]–[9]. However, those schemes are not suitable for further complexity reduction by exploiting the sparseness of time-domain channel matrix when applied to an OFDM equalizer.

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In this letter, a novel time-domain recursive algorithm is proposed to reduce the complexity of the MMSE-SIC scheme for OFDM systems. Even though the ICI effect is shown in a frequency-domain channel matrix after OFDM demodulation, this algorithm exploits the sparseness of a time-domain channel matrix in an OFDM system. The complexity of the proposed algorithm depends not only on $N$ but also on the number of channel taps $L$. Since $L$ is typically much smaller than $N$, the proposed scheme has complexity of $(1/2)N^3 + \mathcal{O}(N^2 \log_2 N)$, which is much simpler than previously reported schemes [4]–[9]. Note that there were several low-complexity equalizers that exploited ICI properties in a frequency-domain channel matrix [1], [3]. However, such a scheme is inferior to the MMSE-SIC scheme in performance and suffers from error floor as SNR increases.

*Notation:* A boldface large and small letter mean a matrix or a vector, respectively. $\mathbf{A}_{m,n}$ denotes the $(m,n)$th element of $\mathbf{A}$. Also, $\mathbf{A}_{m,:}$ and $\mathbf{A}_{:,n}$ denote the $m$th row vector and the $n$th column vector of $\mathbf{A}$, respectively. $\|\cdot\|$ denotes the norm of a vector. $\mathbf{0}_N$ and $\mathbf{I}_N$ represent the $N \times N$ zero and identity matrices, respectively. Also, $(k)_N$ denotes $k$ modulo $N$, $[\cdot]^T$ and $[\cdot]^H$ stand for transpose and Hermitian, respectively, and $\mathbf{F} = (1/\sqrt{N})[\exp^{-j2\pi(m-1)(n-1)/N}]_{n,m=1,\ldots,N}$ is the $N \times N$ fast Fourier transform (FFT) matrix.

## II. System Description

In an OFDM transmitter, the frequency-domain data stream is divided into blocks of length $N$ and modulated by $N$-point inverse FFT (IFFT). At the receiver, the received blocks are demodulated by $N$-point FFT. For each block, the input-output relation can be described as

$$\mathbf{y} = \mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{x} + \mathbf{F}\mathbf{w}_t = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$ are the $N \times 1$ transmitted and received symbol vectors, respectively, and $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_N$. Also, $\mathbf{w}_t$ and $\mathbf{w}$ denote the vectors of additive complex white Gaussian noise (AWGN) with variance $\sigma^2$ in time-domain and frequency-domain, respectively. The matrix $\mathbf{H}_t$ denotes the $N \times N$ time-domain channel matrix, whose $(n,l)$th element is $h_{n,(n-l)_N}$, $1 \leq n \leq N$, $1 \leq l \leq L$, where $h_{n,l}$ denotes the complex channel gain of the $l$th tap at the $n$th time instance. Since $L \ll N$, $\mathbf{H}_t$ is a sparse matrix with $NL$ nonzero elements. The matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_N] = \mathbf{F}\mathbf{H}_t\mathbf{F}^H$ denotes the $N \times N$ frequency-domain channel matrix. In time-selective channels, $\mathbf{H}$ is typically not a diagonal matrix and the off-diagonal elements in $\mathbf{H}$ cause the occurrence of the ICI, which may result in severe performance degradation as $f_n$ increases.

## III. MMSE-SIC Equalization

### A. Classical MMSE-SIC Scheme

The MMSE-SIC scheme [2] requires $N$ iterations for each OFDM symbol. Each iteration is comprised of three steps for the symbol detection: optimal ordering, linear MMSE filtering, and ICI cancellation. The detailed procedure is described as follows. The MMSE filter is given in [2] as

$$\mathbf{G} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_N)^{-1} = \mathbf{H}^H\mathbf{R}^{-1}. \tag{2}$$

Assume that an ordering sequence vector $\mathbf{z}$ is defined as $\mathbf{z} = [z_1, z_2, \ldots, z_N]$, where $z_k$ denotes the symbol index to be detected at the $k$th iteration. Let $\mathbf{H}^{[1]} = \mathbf{H}$ and $\mathbf{H}^{[k]}$, $k = 2, 3, \ldots, N$, be the modified channel matrices after nulling column vectors $\mathbf{h}_{z_1}, \mathbf{h}_{z_2}, \ldots, \mathbf{h}_{z_{k-1}}$ from $\mathbf{H}^{[1]}$. Then, the MMSE filter at the $k$th iteration is given as $\mathbf{G}^{[k]} = \mathbf{H}^{[k],H}\mathbf{R}^{[k],-1}$.[1] At the $k$th iteration, the first step is to choose the current best symbol $x_{z_k}$ with the smallest error variance among undetected data symbols. This ordering can reduce performance degradation due to the error propagation. By computing $\mathbf{R}^{[k],-1}$, $z_k$ can be determined as

$$z_k = \arg\min_m \mathbf{R}^{[k],-1}_{m,m}. \tag{3}$$

In the second step, the symbol is detected as

$$\hat{x}_{z_k} = \mathbf{g}^{[k],H}\mathbf{y}^{[k]} \tag{4}$$

where $\mathbf{g}^{[k],H}$ denotes the $k$th row vector of $\mathbf{G}^{[k]}$ and $\mathbf{y}^{[k]}$ is the ICI cancelled received signal vector at the $k$th iteration. Then, the hard-decision value $\bar{x}_{z_k}$ is obtained. Finally, the third step is to remove the ICI of the detected symbol $\bar{x}_{z_k}$ from the received signal vector as

$$\mathbf{y}^{[k+1]} = \mathbf{y}^{[k]} - \mathbf{h}_{z_k}\bar{x}_{z_k} \tag{5}$$

and replace the column vector $\mathbf{h}_{z_k}$ with a zero vector (nulling $\mathbf{h}_{z_k}$). This procedure is repeated for $k = 1, 2, \ldots, N$.

It is well known that a suboptimal ordering technique can be used to reduce the computational complexity of the MMSE-SIC with negligible performance degradation [10]. However, the MMSE filter should be recalculated at each iteration, which is computationally intensive due to matrix inversion. In the next subsection, a new recursive algorithm is proposed to compute the MMSE filter at each iteration.

### B. Proposed Recursive Algorithm

In (2), the $k$th frequency-domain channel matrix $\mathbf{H}^{[k]}$ can be described as $\mathbf{H}^{[k]} = \mathbf{H}^{[1]}\mathbf{P}^{[k]}$, where $\mathbf{P}^{[k]}$ is an $N \times N$ diagonal matrix with the $i$th diagonal entry defined as

$$\mathbf{P}^{[k]}_{i,i} = \begin{cases} 0, & i \in \{z_1, z_2, \ldots z_{k-1}\} \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

Note that $\mathbf{P}^{[1]} = \mathbf{I}_N$. From (1) and (2), the $k$th MMSE filter $\mathbf{G}^{[k]}$ can be rewritten as

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\mathbf{M}^{[k],-1}\mathbf{H}^{-1} \tag{7}$$

where

$$\mathbf{M}^{[k]} = \mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_\mathbf{t}^H\mathbf{H}_\mathbf{t}\mathbf{F}^H\right)^{-1}. \tag{8}$$

[1]Although such a nulling has been widely used in many MMSE-SIC schemes, $\mathbf{G}^{[k]}$ is not the exact MMSE filter unless all preceding symbol detections are correct, which may cause performance degradation. However, in this letter, we use $\mathbf{G}^{[k]}$ as did in most literature because an appropriate ordering can greatly reduce such degradation.

The derivations of (7) and (8) are given in Appendix A. From (7) and (8), a recursive expression can be invoked by using the Sherman–Morrison formula [11] as follows.

Suppose that $\mathbf{M}^{[k-1],-1}$ is known. Then, $\mathbf{M}^{[k],-1}$ can be expressed as

$$\begin{aligned}\mathbf{M}^{[k],-1} &= \left(\mathbf{M}^{[k-1]} + \mathbf{c}_k\mathbf{d}_k^T\right)^{-1} \\ &= \mathbf{M}^{[k-1],-1} - \frac{\mathbf{M}^{[k-1],-1}\mathbf{c}_k\mathbf{d}_k^T\mathbf{M}^{[k-1],-1}}{1 + \mathbf{d}_k^T\mathbf{M}^{[k-1],-1}\mathbf{c}_k}\end{aligned} \tag{9}$$

where $\mathbf{c}_k$ and $\mathbf{d}_k$ are $N \times 1$ vectors with only one nonzero element at the $z_{k-1}$th entry, satisfying $\mathbf{P}^{[k]} = \mathbf{P}^{[k-1]} + \mathbf{c}_k\mathbf{d}_k^T$. For convenience, set the $z_{k-1}$th entries of $\mathbf{c}_k$ and $\mathbf{d}_k$ to $-1$ and 1, respectively, and let $\alpha_k = (1 + \mathbf{d}_k^T\mathbf{M}^{[k-1],-1}\mathbf{c}_k)$. Then, (9) can be rewritten as

$$\mathbf{M}^{[k],-1} = \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{M}^{[k-1],-1} \tag{10}$$

where $\mathbf{A}^{[k]} = (1/\alpha_k)\mathbf{M}^{[k-1],-1}\mathbf{c}_k\mathbf{d}_k^T$ is a sparse matrix with nonzero elements only at the $z_{k-1}$th column. By substituting (10) into (7), $\mathbf{G}^{[k]}$ can be expressed as

$$\mathbf{G}^{[k]} = \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{G}^{[k-1]}. \tag{11}$$

Therefore, the MMSE filter $\mathbf{G}^{[k]}$, for $k = 2, 3, \ldots, N$, can be recursively calculated as

$$\mathbf{G}^{[k]} = \mathbf{T}^{[k]}\mathbf{G}^{[1]} \tag{12}$$

where

$$\begin{aligned}\mathbf{T}^{[k]} &= \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\left[\mathbf{I}_N - \mathbf{A}^{[k-1]}\right]\cdots\left[\mathbf{I}_N - \mathbf{A}^{[1]}\right] \\ &= \left[\mathbf{I}_N - \mathbf{A}^{[k]}\right]\mathbf{T}^{[k-1]},\end{aligned} \tag{13}$$

$$\mathbf{A}^{[k]}_{:,z_{k-1}} = -(1/\alpha_k)\mathbf{P}^{[k]}\mathbf{T}^{[k-1]}\mathbf{M}^{[1],-1}_{:,z_{k-1}}. \tag{14}$$

Note that $\mathbf{T}^{[1]} = \mathbf{I}_N$ and $\mathbf{A}^{[1]} = \mathbf{0}_N$. According to (2) and (7), the $z_{k-1}$th column vector of $\mathbf{M}^{[1],-1}$ can be calculated as $\mathbf{M}^{[1],-1}_{:,z_{k-1}} = \mathbf{G}^{[1]}\mathbf{H}_{:,z_{k-1}}$. Also, note that $\alpha_k \neq 0$ because $\mathbf{G}^{[1]}_{z_{k-1},:}\mathbf{H}_{:,z_{k-1}} \neq 1$. Using (12)–(14), the estimate of the $k$th symbol can be described as

$$\hat{x}_{z_k} = \mathbf{T}^{[k]}_{z_k,:}\mathbf{G}^{[1]}\mathbf{y}^{[k]} = \mathbf{T}^{[k]}_{z_k,:}\bar{\mathbf{y}}^{[k]} \tag{15}$$

where $\mathbf{y}^{[k]} = \mathbf{y}^{[k-1]} - \mathbf{h}_{z_{k-1}}\bar{x}_{z_{k-1}}$ and $\mathbf{y}^{[1]} = \mathbf{y}$. It is shown that the column vectors, $\bar{\mathbf{y}}^{[k]} = \mathbf{G}^{[1]}\mathbf{y}^{[k]}$ and $\mathbf{M}^{[1],-1}_{:,z_{k-1}} = \mathbf{G}^{[1]}\mathbf{H}_{:,z_{k-1}}$, are the outputs of the first MMSE filter $\mathbf{G}^{[1]}$ when the inputs are $\mathbf{y}^{[k]}$ and $\mathbf{H}_{:,z_{k-1}}$, respectively, which can be easily calculated as follows. From (1) and (2), we can describe $\mathbf{G}^{[1]}$ with the time-domain channel matrix $\mathbf{H}_t$ as

$$\begin{aligned}\mathbf{G}^{[1]} &= \mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\left(\mathbf{F}\mathbf{H}_t\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right)^{-1} \\ &= \mathbf{F}\mathbf{H}_t^H\left(\mathbf{H}_t\mathbf{H}_t^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{F}^H.\end{aligned} \tag{16}$$

Clearly, $\mathbf{R}_t = (\mathbf{H}_t\mathbf{H}_t^H + \sigma^2\mathbf{I}_N)$ is a Hermitian and sparse matrix. By performing the LDL$^H$ factorization [11], $\mathbf{R}_t$ is represented as $\mathbf{R}_t = \mathbf{L}\mathbf{D}\mathbf{L}^H$, where $\mathbf{D}$ is a diagonal matrix and $\mathbf{L}$ is a lower triangular matrix with unit diagonal elements. Then,

TABLE I
PROPOSED RECURSIVE ALGORITHM

1) Compute $\mathbf{R}_t$ and perform the LDL$^H$ factorization of $\mathbf{R}_t$.
2) Compute $\bar{\mathbf{y}}^{[1]}$ by forward and backward substitutions.
3) Make a hard decision on $\hat{x}_{z_1}$.
4) For k=2,3,...,N
   a) Compute the ICI cancelled signal vector $\mathbf{y}^{[k]}$ from (5).
   b) Compute $\mathbf{M}^{[1],-1}_{:,z_{k-1}}$ by forward and backward substitutions.
   c) Compute $\mathbf{A}^{[k]}_{:,z_{k-1}}$ from (14).
   d) Compute $\mathbf{T}^{[k]}$ from (13).
   e) Compute $\bar{\mathbf{y}}^{[k]}$ by forward and backward substitutions.
   f) Compute decision statistics from (15) and make a hard decision.
   End

*Initialization*: $\mathbf{P}^{[1]}=\mathbf{I}_N$, $\mathbf{A}^{[1]}=\mathbf{0}_N$, $\mathbf{T}^{[1]}=\mathbf{I}_N$ and $\mathbf{z}=[z_1, z_2, ..., z_N]$

$\mathbf{R}_t^{-1}$ can be solved by forward and backward substitutions. The steps for computing $\bar{\mathbf{y}}^{[k]}$ is described as follows.

1) Compute $\mathbf{R}_t$ and perform the LDL$^H$ factorization.
2) Solve the linear equation $\mathbf{R}_t\mathbf{a} = (\mathbf{F}^H\mathbf{y}^{[k]})$ by solving $\mathbf{Lb} = (\mathbf{F}^H\mathbf{y}^{[k]})$, $\mathbf{Dc} = \mathbf{b}$, and $\mathbf{L}^H\mathbf{a} = \mathbf{c}$, respectively.
3) Calculate $\bar{\mathbf{y}}^{[k]} = \mathbf{F}\mathbf{H}_t^H\mathbf{a}$.

Step 1) is done only once at the first iteration. By replacing $\mathbf{y}^{[k]}$ with $\mathbf{H}_{:,z_{k-1}}$, we can also compute $\mathbf{M}^{[1],-1}_{:,z_{k-1}}$. It is known that the complexity of the FFT (or IFFT) operation is $(1/2)N\log_2 N$. Since $\mathbf{L}$ is sparse due to the spareness of $\mathbf{R}_t$, the complexity of steps 2) and 3) is substantially reduced. The proposed algorithm is summarized in Table I. The major computational complexity at the $k$th iteration is in calculating $\mathbf{A}^{[k]}_{:,z_{k-1}}$ and $\mathbf{T}^{[k]}$, whose overall complexity will be shown as $(1/2)N^3 + \mathcal{O}(N^2\log_2 N)$ in complexity analysis. Thus, the proposed algorithm can reduce the complexity of the MMSE-SIC scheme by a factor of $N$.

In the optimal MMSE-SIC scheme, the ordering sequence $\mathbf{z}$ should be recalculated at each iteration. However, it is already known in [10] that the performance degradation is negligible if the initially optimal ordering sequence is used throughout the whole iterations. Thus, such a suboptimal ordering is adopted in the proposed scheme. The MMSE filter coefficients are designed to maximize the signal-to-interference-plus-noise ratio (SINR) between the output of the filter and the transmitted signal. At the first iteration, the SINR of the $k$th symbol is given in [2] as

$$SINR_k = \frac{\left|\mathbf{M}^{[1],-1}_{k,k}\right|^2}{\sum\limits_{m,m\neq k}\left|\mathbf{M}^{[1],-1}_{m,k}\right|^2 + \sigma^2\left\|\mathbf{G}^{[1]}_{k,:}\right\|^2}. \tag{17}$$

We approximate the SINRs as the signal terms $|[\mathbf{M}^{[1],-1}_{k,k}|^2$ and determine a ordering vector $\mathbf{z}$ as the descending order of the signal terms. To obtain $\mathbf{z}$, it is sufficient to compute $\mathbf{M}^{[1],-1}$ before entering the iteration loop in Table I.

## IV. Complexity and Performance Evaluation

### A. Complexity Analysis

We evaluate the complexity of the proposed recursive algorithm by counting the number of complex multiplications (CMs) and the number of complex additions (CAs). The LDL$^H$ factorization of an $N \times N$ Hermitian matrix $\mathbf{A}$ is described in [11]. The complexity to compute $\mathbf{R}_t$ and perform LDL$^H$ factorization depends on the number of channel taps $L$ and the maximum delay of the channel $L_{max}$. To evaluate the upper bound of the complexity, we set $L = L_{max} + 1$. The sparse matrix $\mathbf{R}_t$ contains $(2L_{max}+1)N$ nonzero elements. The complexity to compute $\mathbf{R}_t$ is $(1/2)(L^2_{max}+3L_{max}+2)N$ CMs and $(1/2)(L^2_{max} + L_{max})N$ CAs. The LDL$^H$ factorization of $\mathbf{R}_t$ requires roughly $(2L^2_{max} + 7L_{max} + 2)N$ CMs and $(2L^2_{max} - 2L_{max} + 3)N$ CAs, where $\mathbf{L}$ has $(2L_{max})N$ nonzero elements except the unit diagonal elements. The complexity of computing $\mathbf{y}^{[k]}$, for $k = 1, 2, \ldots, N$, is $N^2 - N$ CMs and $N^2 - N$ CAs. The main computational complexity of the proposed algorithms is summarized in Table II, where $C[\cdot]$ denotes the number of CMs or the number of CAs to calculate a matrix or a vector. The total numbers of CMs and CAs required for the proposed algorithm are, respectively, given as

TABLE II
NUMBER OF CMs AND CAs

| Calculations | Computational Complex |
|---|---|
| $C[\sum_{k=1,\ldots,N} \bar{\mathbf{y}}^{[k]}]$ | $(log_2 N + 5L_{max} + 2)N^2$ CMs<br>$(5L_{max} - 2)N^2$ CAs |
| $C[\sum_{k=1,\ldots,N} \mathbf{M}^{[1],-1}_{:,k}]$ | $(log_2 N + 5L_{max} + 2)N^2$ CMs<br>$(5L_{max} - 2)N^2$ CAs |
| $C[\sum_{k=1,\ldots,N} \mathbf{A}^{[k]}]$ | $(1/6)N^3 - N^2 - (13/6)N + 1$ CMs<br>$2N - 3$ CAs |
| $C[\sum_{k=1,\ldots,N} \mathbf{T}^{[k]}]$ | $(1/6)N^3 - (1/2)N^2 + (1/3)N$ CMs<br>$(1/6)N^3 - (1/2)N^2 + (1/3)N$ CAs |
| $C[\sum_{k=1,\ldots,N} \hat{x}_{z_k}]$ | $(1/2)N^2 - (1/2)N$ CMs<br>$(1/2)N^2 - (3/2)N + 1$ CAs |

$$C_{CM} = \frac{1}{3}N^3 + (2log_2 N + 10L_{max} + 4)N^2 + \left(\frac{5}{2}L^2_{max} + \frac{17}{2}L_{max} - \frac{1}{3}\right)N + 1 \tag{18}$$

$$C_{CA} = \frac{1}{6}N^3 + (10L_{max} - 3)N^2 + \left(\frac{5}{2}L^2_{max} - \frac{3}{2}L_{max} + \frac{17}{6}\right)N - 2. \tag{19}$$

When $L_{max} \ll N$, the complexity of the proposed algorithm is roughly $(1/2)N^3 + \mathcal{O}(N^2\log_2 N)$. To compute the upper bound on the complexity for outdoor OFDM systems such as IEEE 802.16e [12], $L_{max}$ is set to $L_{max} = L_{CP} = (1/8)N$, where $L_{CP}$ is the length of cyclic prefix (CP). Then, the total numbers of CMs and CAs are, respectively, upper-bounded as

$$C_{CM} \leq \left(\frac{19}{12} + \frac{5}{128}\right)N^3 + \left(2log_2 N + \frac{81}{16}\right)N^2 - \frac{1}{3}N + 1, \tag{20}$$

$$C_{CA} \leq \left(\frac{17}{12} + \frac{5}{128}\right)N^3 - \frac{51}{16}N^2 + \frac{17}{6}N - 2. \tag{21}$$

Then, the total number of complex operations (COs) is upper bounded as $C_{Total} = (3 + (5/64))N^3 + \mathcal{O}(N^2\log_2 N)$. The complexity of previously reported low-complexity schemes in

TABLE III
COMPLEXITY OF PREVIOUSLY REPORTED ALGORITHMS

| Algorithm | Computational Complexity |
|---|---|
| $C_{[4]}$ | $(5+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CMs, $(5+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CAs |
| $C_{[5]}$ | $(4+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CMs, $(4+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CAs |
| $C_{[6]}$ | $(3+\frac{1}{2})N^3+\mathcal{O}(N^2)$ CMs, unknown CAs |
| $C_{[7]}$ | $(3+\frac{2}{3})N^3+\mathcal{O}(N^2)$ CMs, $3N^3+\mathcal{O}(N^2)$ CAs |
| $C_{[8]}$ | $(2+\frac{11}{12})N^3+\mathcal{O}(N^2)$ CMs, $(3+\frac{1}{6})N^3+\mathcal{O}(N^2)$ CAs |
| $C_{[9]}$ | $(3+\frac{1}{3})N^3+\mathcal{O}(N^2)$ COs |

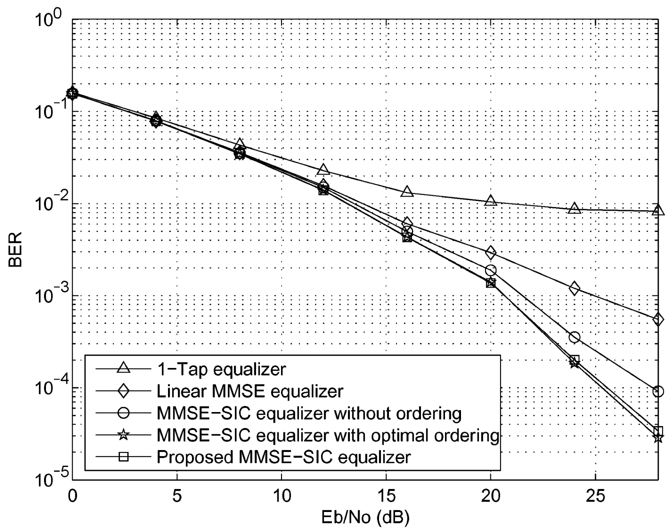


Fig. 1. Performance comparisons of the proposed MMSE-SIC equalizer with the classical MMSE-SIC equalizer when $f_n = 0.1$.

[4]–[9] is summarized in Table III.[2] It is seen that even the upper-bound of the complexity of the proposed scheme is smaller than the complexity of existing schemes. Furthermore, $L_{max}$ is smaller than the CP length and $L$ is typically much smaller than $L_{max}$. Thus, the complexity of the proposed scheme is typically much smaller than the upper bounds.

### B. Simulation Results

To evaluate the BER performance of the proposed algorithm, an uncoded OFDM system with $N = 64$ and QPSK constellation is used. The length of CP is set to eight samples. The wide-sense stationary uncorrelated scattering (WSSUS) tapped-delay-line channel model is used with six paths $(L = 6)$ and an exponential power delay profile whose decay factor is 1 dB/sample. The path gain for each channel tap is independently generated from the Jakes' model, and perfect channel estimation is assumed at the receiver.

Fig. 1 shows the BER performance comparison when $f_n = 0.1$. There is no great difference in performance at low SNR regime $(E_b/N_o \leq 8)$. However, as the SNR increases, the classical 1-tap equalizer shows an error floor quickly. The MMSE-SIC techniques outperform the linear MMSE equalizer due to the ICI cancellation. The difference between the MMSE-SIC equalizer and the proposed equalizer comes from the suboptimal ordering, which is negligible as expected.

[2]The additional complexity (in CM) for the optimal ordering is $(2/3)N^3$ [4], [5] or $(1/2)N^3$ [6], which is much smaller than the complexity for computing MMSE filter coefficients.

## V. CONCLUSION

In this letter, we proposed a novel time-domain recursive algorithm for reducing the complexity of the MMSE-SIC scheme in OFDM systems. Taking the advantage of the spareness of the time-domain channel matrix, the proposed scheme can achieve a significant complexity reduction, and it can provide an MMSE-SIC equalizer for OFDM systems with complexity much smaller than existing low-complexity MMSE-SIC schemes. Also, the performance loss due to the suboptimal ordering was confirmed to be negligible.

## APPENDIX A
## DERIVATIONS OF (7) AND (8)

Using properties of the IFFT/FFT matrix, ($\mathbf{F}^H = \mathbf{F}^{-1}$ and $\mathbf{F}^H\mathbf{F} = \mathbf{F}\mathbf{F}^H = \mathbf{I}_N$), $\mathbf{G}^{[k]}$ can be described as

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right]^{-1}. \tag{22}$$

For convenience, let $\mathbf{J} = \left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{P}^{[k]}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H + \sigma^2\mathbf{I}_N\right]^{-1}$. Then, $\mathbf{J}$ can be rewritten as

$$\mathbf{J} = \left[\mathbf{F}\mathbf{H}_t\mathbf{F}^H\left\{\mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_t^H\mathbf{H}_t\mathbf{F}^H\right)^{-1}\right\}\mathbf{F}\mathbf{H}_t^H\mathbf{F}^H\right]^{-1}. \tag{23}$$

Substituting (23) into (22) gives

$$\mathbf{G}^{[k]} = \mathbf{P}^{[k]}\left[\mathbf{P}^{[k]} + \sigma^2\left(\mathbf{F}\mathbf{H}_t^H\mathbf{H}_t\mathbf{F}^H\right)^{-1}\right]^{-1}\mathbf{F}\mathbf{H}_t^{-1}\mathbf{F}^H. \tag{24}$$

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