Admission and Power Control for Spectrum Sharing Cognitive Radio Networks

John Tadrous, Ahmed Sultan, and Mohammed Nafie

***Abstract*—We investigate the problem of admission and power control considering a scenario where licensed, or primary, users and cognitive radios, or secondary users, are transmitting concurrently over the same band. The primary users share a common receiver and the interference on this receiver from secondary users should be strictly limited to a certain level. Each secondary link is assumed to have a minimum quality of service (QoS) requirement that should be satisfied together with the interference limit constraint, otherwise the secondary link is not admitted. Under those constraints, admission and power control for secondary users are investigated for two main optimization objectives. First, we maximize the number of admitted secondary links. Second, we maximize the sum throughput of the admitted secondary links. The first problem is NP-hard, hence we provide a distributed close-to-optimal solution based on local measurements at each user and a limited amount of signaling. For the second problem, which is non-convex, we propose a suboptimal algorithm based on sequential geometric programming. The proposed algorithms are compared with previously related work to demonstrate their relative efficiency in terms of outage probability, complexity and achievable throughput.**

***Index Terms*—Cognitive radios, admission and power control, convex optimization.**

I. Introduction

RECENT studies by the FCC show that the current utilization of some spectrum bands is as low as 15% [1]. On the other hand, it is commonly believed that there is a crisis of spectrum availability at frequencies that can be economically used for wireless communications via fixed spectrum allocation. In other words, there is fierce competition for the use of spectra, especially in the bands below 3 GHz while in the same time most of these bands suffer from underutilization. Therefore, there is an increasing interest in developing new efficient techniques for spectrum management and sharing as in [2] which consequently motivates the concept of cognitive radios [3] and dynamic spectrum access.

In this paper we consider the problem of admission and power control for cognitive radios in a spectrum underlay network where cognitive or secondary users share the same spectrum band concurrently with the licensed or primary users under the following constraint. The interference caused by the secondary users on the primary network has to be kept below a maximum allowable limit. We impose an additional constraint that each secondary user has a minimum quality of service (QoS) requirement that should be satisfied. A lot of work addresses power and admission control for cognitive radios in underlay networks under similar constraints (see, for example, [5]-[9]).

In [5], a centralized algorithm based on tree-pruning technique was proposed, the proposed algorithm leads to the largest supported set of secondary links but with extensive computations. Moreover, a distributed game theoretic approach based on sequential play was introduced, but it converges to locally optimal solutions. In [6], a distributed algorithm was introduced that aims at minimizing the total transmit power by primary and secondary links. However, the primary users were allowed to increase their transmission power without bounds. In order to maximize the number of admitted secondary links under QoS and interference constraints, the authors of [7] developed two centralized less complicated, but suboptimal, algorithms namely I-SMIRA and I-SMART(R). The two algorithms are based on permanent removals of links. In [8], joint admission control and rate/power allocation for secondary links was investigated while only mean channel gains are available. The main objective was to admit secondary users so that the sum throughput is maximized under two fairness criteria, namely, proportional fairness and max-min fairness. However, the removal technique used does not necessarily maximize the number of admitted secondary links. In [9], power control for both primary and secondary users is considered for the sake of maximizing the total throughput of both the primary and secondary networks. The QoS of each primary link, however, is guaranteed to be maintained above a minimum limit.

In this paper, our main objective is to **maximize** the sum throughput for the **maximum** number of secondary links that can be admitted to the network under the aforementioned constraints. In order to render this task tractable, we divide it into two subproblems. The first is the maximization of the number of admitted secondary links under the aforementioned constraints, whereas the second is to maximize the sum throughput of the admitted secondary links. The first subproblem becomes NP-hard when not all secondary users can be supported at their QoS requirements [13], and the second subproblem is non-convex [17]. An algorithm based on monotonic optimization has been introduced in [23] and proven to achieve the global optimal solution of sum throughput maximization. However, its computations are still

Manuscript received September 3, 2010; revised December 27, 2010; accepted February 22, 2011. The associate editor coordinating the review of this paper and approving it for publication was S. Mao.

The authors are with the Wireless Intelligent Networks Center (WINC), Nile University, Cairo, Egypt (e-mail: john.tadrous@nileu.edu.eg, {asultan, mnafie}@nileuniversity.edu.eg).

J. Tadrous is also with the Electrical and Computer Engineering Department, Ohio State University, Columbus, OH, USA.

A. Sultan is also with the Electrical Engineering Department, Alexandria University, Egypt.

This work was supported by a grant from the Egyptian National Telecommunications Regulatory Authority.

Digital Object Identifier 10.1109/TWC.2011.040411.101571

rather extensive as the algorithm searches for the optimal solution over the boundary of the feasible set. Moreover, the problem of sum rate maximization in wireless networks resembles the corresponding problem in DSL. There has been considerable research on approximation algorithms for this problem as in [10], [11].

The main contributions of this paper are[1]:

- We propose a distributed online algorithm for maximizing the number of admitted secondary links under QoS and interference constraints. The proposed algorithm can be implemented based on local measurements available at each user with a limited exchange of control signals, because the users do not have to keep track of the channel gains. The proposed algorithm is shown, via numerical simulations, to provide a closer to optimum solution than I-SMIRA proposed in [7], and with less complexity.
- We consider maximizing the sum throughput for the set of admitted secondary links. The optimal solution to this problem is computationally prohibitive because the problem is non-convex. However, we introduce an iterative algorithm based on geometric programming (GP) and prove the convergence to, at least, a local optimum solution. The proposed iterative algorithm requires simpler computations than the single condensation methods proposed in [19] and [22], and the double condensation method described in [22] without a proof of its convergence.
- We show, via simulations, the tradeoff between the number of admitted secondary links and the sum throughput of the secondary network. In general, maximizing the sum throughput of the secondary network with guarantees on the QoS of every admitted link results in less throughput than maximizing the sum throughput without those guarantees. However, we show numerically, that the difference in throughput between the two cases is not significant.

The rest of this paper is organized as follows. In Section II, the system model is introduced, and the problem is formulated. In Section III, an algorithm for maximizing the number of admitted secondary links under the QoS and interference constraints is proposed. In Section IV, maximizing the sum throughput of the admitted secondary links is addressed and an iterative algorithm is developed. Simulation results are presented in Section V, and the paper is concluded in Section VI.

II. System Model and Problem Formulation

We consider an underlay network model where secondary users exist within the coverage area of a primary base station (BS). The primary network is assumed to consist of K primary users communicating with BS. We focus on communication in the uplink direction. The secondary network consists of N secondary links, each secondary link comprises one transmitter and a corresponding receiver as shown in Fig. 1. The secondary users are assumed to communicate in an ad-hoc fashion and they are allowed to transmit such that the interference they create on the BS is below a maximum limit I. All receivers,

[1]Part of this paper has been published in IEEE Globecom'09 [14].

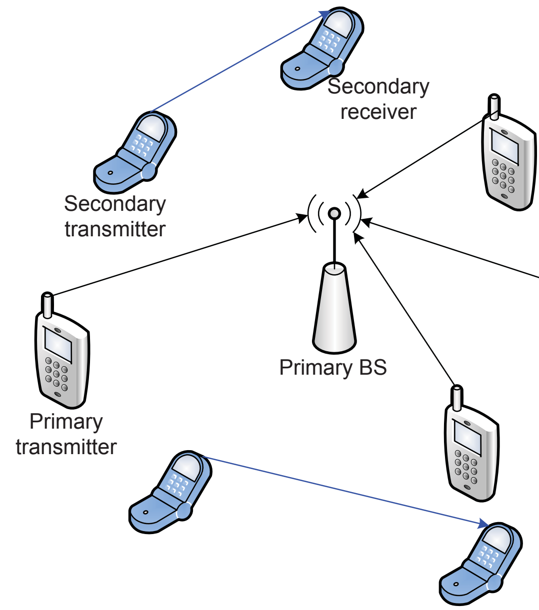


Fig. 1: System model, cognitive underlay network.

primary and secondary, are assumed to treat interference as noise. The K primary users are assumed to be transmitting concurrently to the BS. The transmit power of primary user j is p_j^p, $j = 1, \cdots, K$. The channel gain between primary user j and the BS is $g_{0,j}^{pp}$. Hence the signal-to-interference-plus-noise-ratio (SINR) for each primary link is given by,

$$x_j^p = PG^p \frac{p_j^p g_{0,j}^{pp}}{\sum_{l=1, l\neq j}^{K} p_l^p g_{0,l}^{pp} + \eta + N_0}, \quad j = 1, 2, \cdots, K, \tag{1}$$

where PG^p is the processing gain in case that primary users are employing a spread spectrum technique, N_0 is the noise power at the frequency band of operation, and η is the interference caused by secondary users on the primary receiver where

$$\eta = \sum_{i=1}^{N} p_i^s g_{0,i}^{ps}, \tag{2}$$

the transmit power of secondary link i is p_i^s, $i = 1, \cdots, N$ and $g_{0,i}^{ps}$ is the channel gain from the transmitter of secondary link i to the primary BS.

The primary network has to provide a certain QoS level for its users such that, $x_j^p \geq \gamma^p$, $j = 1, \cdots, K$. Moreover, each primary user cannot increase its transmit power beyond a maximum limit $\hat{p}^p$. Consequently, the primary network can determine the maximum tolerable interference limit, I, as follows.

$$\begin{aligned} &\underset{\mathbf{p}^p, I}{\text{maximize}} \quad I \\ &\text{subject to} \quad \frac{PG^p p_j^p g_{0,j}^{pp}}{\sum_{l=1, l\neq j}^{K} p_l^p g_{0,l}^{pp} + I + N_0} \geq \gamma^p, j = 1, \cdots, K, \\ &\qquad\qquad I \geq 0, \qquad 0 \leq p_j^p \leq \hat{p}^p, \quad j = 1, \cdots, K, \end{aligned} \tag{3}$$

where $\mathbf{p}^p$ is a vector of primary power allocation. The optimization problem in (3) is a linear program and I can directly be obtained using standard algorithms. Furthermore, we assume that the primary network is feasible in the sense that, if the secondary users are not transmitting, i.e., $\eta = 0$, then there exists a primary power allocation vector $\mathbf{p}^p$ such that the constraints of (3) are satisfied. The secondary network has an ad-hoc topology of N links. The channel gain between the receiver of secondary link i and the transmitter of secondary link j is $g^{ss}_{i,j}$, and the channel gain between each secondary receiver i and a primary transmitter l is $g^{sp}_{i,l}$. Thus, the interference caused by primary transmitters on the secondary receiver of link i is given by

$$N_i = \sum_{l=1}^{K} p^p_l g^{sp}_{i,l}, \quad i = 1, \cdots, N. \tag{4}$$

We assume that secondary link i can employ a spread spectrum technique with processing gain factor $PG^s_i, i = 1, \cdots, N$. The SINR of the secondary link i is then given by,

$$x^s_i = PG^s_i \frac{p^s_i g^{ss}_{i,i}}{\sum_{j=1, j\neq i}^{N} p^s_j g^{ss}_{i,j} + N_i + N_0}. \tag{5}$$

Moreover, the secondary links have minimum QoS requirements in terms of SINR. Each secondary link i has then to transmit with power p^s_i such that

$$x^s_i \geq \gamma^s_i, \tag{6}$$

$$0 \leq p^s_i \leq \hat{p}^s_i, \tag{7}$$

$$\eta \leq I, \quad i = 1, \cdots, N, \tag{8}$$

where γ^s_i is the minimum QoS requirement for the ith secondary link, and $\hat{p}^s_i$ is the peak transmit power of this link. Note that the constraints (6)-(8) are convex for x^s_i and η defined in (5) and (2) respectively. If those constraints are feasible, any optimization problem whose constraints are (6)-(8) and its objective function is convex on $\mathbf{p}^s$, where $\mathbf{p}^s$ is the vector of secondary transmit powers, is always convex and can be solved for a global optimum solution [18]. On the other hand, if the constrains are infeasible, i.e., there is no $\mathbf{p}^s$ such that (6)-(8) are satisfied, then not all secondary links can be admitted to the network, and a removal procedure has to take place. Most of the previous work in this area considered removal algorithms that minimize the number of removed links [5]-[13].

By assuming that all users use Gaussian codebooks and the noise samples are i.i.d Gaussian, the throughput of secondary link i is $\log_2(1 + x^s_i)$ bps/Hz, $i = 1, \cdots, N$. Hence, the problem we consider is formulated as follows.

$$\begin{aligned} \underset{\mathbf{p}^s}{\text{maximize}} \quad & \sum_{i=1}^{N} \log_2 (1 + x^s_i) \\ \text{subject to} \quad & (6), (7), (8). \end{aligned} \tag{9}$$

There are two important issues about this problem. First, it may be infeasible and an admission/removal control algorithm is needed to admit the maximum possible number of links. Second, the objective function is non-convex [17] and a global optimum solution is computationally intractable. Consequently, we decompose problem (9) into two subproblems and provide a solution for each of them.

For the first subproblem, in case that not all secondary links can be supported under the given constraints, admitting the maximum possible number of links to the network is an NP-hard problem as has been shown in [7] and [13]. Hence, we propose a suboptimal solution to this problem that can be implemented by secondary links in a distributed manner and achieve a close-to-optimum solution. For the second subproblem, we tackle the non-convexity of the objective function of (9) by introducing an iterative algorithm based on successive solutions to approximate geometric programs of the original problem.

III. Maximizing the Number of Admitted Links

The admission control problem has been addressed before in previous works and was first introduced for conventional networks in [13], then in spectrum underlay cognitive networks, where the additional constraint of the interference limit on primary users is imposed. For this case, new admission control algorithms have been developed to account for the new constraint [5]-[7]. Most of those algorithms use the distributed constrained power control (DCPC) to check the feasibility of the current set of links.

A. DCPC Algorithm

The DCPC algorithm is introduced in [24] and it aims at allocating power to links such that the required SINR for each secondary link is achieved at the minimum possible transmission power, provided that all links can be supported at their QoS requirements. DCPC iteratively allocates power to each secondary link i synchronously, or asynchronously, with other links according to the following formula,

$$p^s_i(T+1) = \min\left\{\hat{p}^s_i, \frac{p^s_i(T)\gamma^s_i}{x^s_i(T)}\right\}, \quad i = 1, \cdots, N, \tag{10}$$

where $p^s_i(T)$ is the transmit power of secondary link i at time T. As proven in [24], the power updates of all links converge to a fixed power vector $\mathbf{p}^{s*}$, regardless of the values of the initial powers. Each element in $\mathbf{p}^{s*}$ denotes the steady state power of one of the transmitters.

If the fixed power vector $\mathbf{p}^{s*}$ is found to contain any elements of maximum transmit power $\hat{p}$, this means the current set of links cannot be supported at its QoS requirements. Alternatively, if $\mathbf{p}^{s*}$ does not contain any $\hat{p}$ value, then it means the current set of requesting links can be supported at exactly their target QoS requirements. In case of cognitive network, any set of secondary links can perform the DCPC algorithm, and after convergence, the SINR of each link can be checked whether it satisfies the constraints or not. Moreover, η can be evaluated based on (2) and the satisfaction of the interference constraint (8) can be checked as well.

Based on the use of DCPC in the way mentioned above, the whole set of requesting secondary links can be tested for the possibility all of the secondary links being admitted. If it is found to be infeasible, there is at least one set of minimum number of secondary links that should be removed from the secondary network such that the remaining set is feasible. The determination of this set of minimum number of removed links is an NP-hard problem, and the optimum solution can be obtained through exhaustive search as follows. Let $\mathcal{N}$ denote the set of requesting secondary links where $|\mathcal{N}| = N$, then for all $\mathcal{A} \subseteq \mathcal{N}$, secondary users in $\mathcal{A}$ perform DCPC algorithm and the optimal solution is $\mathcal{A}^*$ where

$$\begin{aligned} \mathcal{A}^* = \quad & \arg\max_{\mathcal{A}\subseteq\mathcal{N}} \quad |\mathcal{A}| \\ & \text{subject to} \quad x_i^{s*} = \gamma_i^s, \quad i \in \mathcal{A}, \\ & \qquad\qquad\quad p_j^s = 0, \quad j \notin \mathcal{A}, \\ & \qquad\qquad\quad \sum_{i\in\mathcal{A}} p_i^{s*} g_{0,i}^{ps} \le I, \end{aligned} \tag{11}$$

where p_i^{s*} and x_i^{s*} are the stationary transmit power and the corresponding SINR for secondary link $i \in \mathcal{A}$ after the DCPC phase. There are $2^N - 1$ possible $\mathcal{A} \subseteq \mathcal{N}$, hence, to get the optimal solution, the DCPC has to be performed a number of times exponentially proportional to the number of requesting secondary links, which is very computationally exhaustive. However, in [5] it has been proven that, if a set $\mathcal{A} \subset \mathcal{N}$ has been found infeasible, then any other set $\mathcal{A}'$ such that $\mathcal{A} \subset \mathcal{A}' \subseteq \mathcal{N}$ is also infeasible. Although this fact reduces the number of possible sets to be tested via DCPC for the optimal solution, the solution still requires exhaustive search for the minimum number of removable links. This has opened the area for finding suboptimal, but efficient, solutions to the given problem.

B. Distributed Online Algorithm

One of the efficient techniques for link selection is I-SMIRA proposed in [7] which is a removal algorithm that consists of two phases: a DCPC phase and a removal phase. For the removal phase a centralized removal criterion that accounts for the interference constraint is developed, where after the DCPC phase, if a constraint is being violated, the removal criterion selects a single user for permanent removal. By removing one user, the DCPC is performed by the remaining set of users and the constraints are checked so as to call the removal phase if any constraint is violated or to terminate the algorithm otherwise, where all the users performed the last DCPC phase are admitted.

The efficiency of I-SMIRA relies basically on the developed removal criterion. However, the removal criterion introduced necessitates the existence of a central node that knows all the system parameters including the instantaneous channel gains between all nodes, the interference constraint I, the required QoS and the peak transmit power for each secondary link. Here we introduce a distributed algorithm that can be implemented by the secondary links assuming that each secondary link can only measure its local SINR and engage in limited signaling with other secondary users. A generic description of the proposed algorithm goes as follows. The set of requesting secondary links $\mathcal{N}$ is divided into two disjoint subsets, $\mathcal{A}$ and $\mathcal{I}$, where $|\mathcal{A}| + |\mathcal{I}| = |\mathcal{N}| = N$. The set $\mathcal{A}$ is called the active set and it contains all the secondary links with positive transmit power, whereas the second set $\mathcal{I}$ is called the inactive set and it contains all secondary users with zero transmit power. The main steps of the proposed algorithm are as follows.

1) Initially, $\mathcal{A} = \mathcal{N} = \{1, \cdots, N\}$, $\mathcal{I} = \phi$, and the secondary links start with an initial power vector $\mathbf{p}^s(0) = [p_1^s(0), \cdots, p_N^s(0)]$, such that $p_i^s(0) \ll \hat{p}_i$, $i \in \mathcal{N}$.
2) In a round robin fashion one secondary link at a time updates its transmission power according to

$$p_i^s(LN+i) = \begin{cases} \min\left\{\hat{p}_i^s, \frac{\gamma_i^s p_i^s(LN+i-1)}{x_i^s(LN+i-1)}\right\}, & i \in \mathcal{A} \\ 0, & i \in \mathcal{I}, \end{cases} \tag{12}$$

where L is a non-negative integer indexing the cycles of N power updates each.
3) The power updates of (12) continue till one of the following three events occurs:
   a) *Event 1:* The system with active set $\mathcal{A}$ converges to a fixed power vector $\mathbf{p}^{(s*)}$ that does not violate any of the constraints of (11) with the secondary links in $\mathcal{A}$ admitted.
   b) *Event 2:* After a certain power update, the interference constraint on the primary BS is violated.
   c) *Event 3:* After the power update of index $LN + i$, $p_i^s(LN + i) = \hat{p}_i^s$ implying that link i would not reach its target QoS.
4) The system handles the occurrence of any of events 2 or 3 by modifying sets $\mathcal{A}$ and $\mathcal{I}$, and continues from step 2 until convergence at event 1 with the admitted links being in set $\mathcal{A}$.

C. Handling Events 2 or 3

After the occurrence of either of events 2 or 3, the proposed algorithm has to change $\mathcal{A}$ and try another set. We propose here a distributed temporary removal criterion in order to change the set $\mathcal{A}$, such that $|\mathcal{A}|$ is not necessarily reduced by 1. We do this in two steps, the first step is deactivation where one link i, $i \in \mathcal{A}$ is selected to join $\mathcal{I}$, whereas the second is reactivation where a link j, $j \neq i$ and $j \in \mathcal{I}$ is chosen to join $\mathcal{A}$, if possible. We call these two steps the deactivation-reactivation process.

1) Deactivation Step: In this step we choose the secondary link whose power update has resulted in event 2 or 3 as the link to be deactivated. After the power update iteration of index $LN + i$, if event 2 or 3 occurs, then link i, $i \in \mathcal{A}$, is deactivated, i.e., i joins $\mathcal{I}$ by setting its transmit power to zero.

2) Reactivation Step: In this step a previously deactivated secondary link in $\mathcal{I}$ may be reactivated by joining $\mathcal{A}$. If i is the link to be deactivated in the current deactivation-reactivation process, then in the reactivation step our algorithm chooses another link j, $j \neq i$ and $j \in \mathcal{I}$ to join $\mathcal{A}$ if possible. Otherwise, only link i joins $\mathcal{I}$ without reactivating links from $\mathcal{I}$ which yields a reduction in $|\mathcal{A}|$ by 1 and an increase in $|\mathcal{I}|$ by 1. Note that we confine the reactivation process to one link only. For fairness considerations, if there are several candidates for reactivation, we choose the reactivated link at random from the set of candidates. To reactivate the link j we suggest the following two criteria.

a) Previous Inactive Sets: For this reactivation criterion, we assume that each secondary link knows the previous inactive sets. If a link i is to join the current inactive set $\mathcal{I}$, it searches its recorded data for another link j where $j \neq i$ and $j \in \mathcal{I}$ such that if j joins $\mathcal{A}$ and i joins $\mathcal{I}$, the new set $\mathcal{I}$

has not existed previously. This means that each deactivation-reactivation process must result in testing a *new* active set $\mathcal{A}$. This reactivation criterion is suggested based on the fact that under stationary channel conditions, if the DCPC of a set $\mathcal{A}$ is infeasible for some initial power vector, then it is also infeasible for any other initial power vector.

Since the proposed algorithm is to be implemented in a distributed manner, there is no need for each secondary link to maintain a list of all previous inactive sets. Instead, each secondary link i needs to maintain only a list of the previous inactive sets that i was an element of. Moreover, for the memory requirements, when link i can not replace any link j in $\mathcal{I}$ such that the new inactive set has not existed before, then only i joins $\mathcal{I}$ and no reactivation step is done. At this stage, the size of $\mathcal{I}$ is increased by 1, and all secondary links can **clear** the stored inactive sets of the old size.

It is expected that, under stationary channel conditions, preventing the repetition of previously tested sets results in efficient performance in terms of convergence time. But since the proposed algorithm invokes the deactivation-reactivation process before the convergence of the DCPC of the current set $\mathcal{A}$, then testing a previously tested set of users may result in enhanced admission performance at the expense of convergence time. This leads us to suggest another reactivation criterion that allows a set to be repeated for limited number of times where each time a set is repeated it starts from a different power initialization.

b) Vectors with 1/0 *Components:* In this reactivation criterion each secondary link maintains a vector of length N whose components take on binary values. Assume that secondary link i has a vector $\mathbf{v}_i$, $i \in \mathcal{N}$. The jth component of $\mathbf{v}_i$ is denoted by $\mathbf{v}_i(j)$ where $\mathbf{v}_i(j) \in \{0,1\}$ $\forall j = 1, \cdots, N$. If link i, $i \in \mathcal{A}$ is to be deactivated, it searches for a link j, $j \in \mathcal{I}$ such that if $\mathbf{v}_i(j) = 0$ then link i joins $\mathcal{I}$ and link j joins $\mathcal{A}$, then link i sets $\mathbf{v}_i(j) = 1$ and link j sets $\mathbf{v}_j(i) = 1$. However if link i finds that $\mathbf{v}_i(j) = 1$ $\forall j \in \mathcal{I}$, then only link i joins $\mathcal{I}$ where $|\mathcal{I}|$ is increased by 1 and $|\mathcal{A}|$ is decreased by 1. At this stage all secondary links set the components of their vectors to zero.

This reactivation criterion allows each secondary link to replace another link in the inactive set only one time at a certain size of the inactive set, this is because $\mathbf{v}_i(j) \in \{0,1\}$ $\forall i, j \in \mathcal{N}$. Using this method of reactivation, an active set $\mathcal{A}$ may be tested more than once.The memory requirements for this reactivation criterion is limited to the number of links. In other words, each secondary link needs only a vector of size N elements to implement this reactivation method. This is unlike the first reactivation criterion where each secondary link maintains a list of previous inactive sets in which this link was included.

D. Implementation Issues

For the proposed algorithm to be implemented distributively, secondary links should exchange signals to schedule the round robin turns of power updates. Also after each deactivation-reactivation process, the deactivated link should broadcast its identifier and the identifier of the link it has replaced in the inactive set, if it does, so that all links update the current active and inactive sets.

Since we assume that the secondary links do not know the current channel gains between them and the primary BS and the maximum tolerable interference limit I, the BS itself can broadcast a warning signal to the secondary links if the QoS of the primary receivers falls below the target value. Thus, after hearing such a warning, the secondary link who last performed a power update joins $\mathcal{I}$ replacing another link if possible. Here, we assume that the presence of a dedicated control channel which the secondary can access. This control channel can be implemented as a reserved portion of the primary network's bandwidth while the secondary network pays for that.

Under low to moderate mobility conditions, the channel coherence time can be several or tens of milliseconds depending on velocity and carrier frequency (see equation 4.40.c in [26]). Assuming that the QoS measurements occur at a rate of several kHz, our proposed algorithm converges within the coherence time of the channels as demonstrated in Section V for several secondary users. As mobility increases and/or the number of secondary users, the algorithms may not converge within the coherence time. This practical concern concerning high mobility and large secondary networks merits further investigation.

IV. Maximization of the Sum Throughput

Now assume that a feasible set of secondary links $\mathcal{A}$ is obtained by any admission control algorithm. It is required to allocate power to the admitted secondary links such that the sum throughput of all links in $\mathcal{A}$ is maximized without violating the original constraints. This problem can be written as

$$\begin{aligned}
\underset{\mathbf{p}^s}{\text{maximize}} \quad & \sum_{i\in\mathcal{A}} \log_2\left(1 + PG_i^s \frac{p_i^s g_{i,i}^{ss}}{\sum_{j\in\mathcal{A}, j\neq i} p_j^s g_{i,j}^{ss} + N_i + N_0}\right) \\
\text{subject to} \quad & PG_i^s \frac{p_i^s g_{i,i}^{ss}}{\sum_{j\in\mathcal{A}, j\neq i} p_j^s g_{i,j}^{ss} + N_i + N_0} \geq \gamma_i^s, \quad i \in \mathcal{A} \\
& \sum_{i\in\mathcal{A}} p_i^s g_{0,i}^{ps} \leq I \\
& 0 \leq p_i^s \leq \hat{p}_i^s, \quad i \in \mathcal{A}.
\end{aligned} \tag{13}$$

As mentioned earlier, the constraints of (13) are convex as they can be written in a linear form, but the objective function is non-convex making the problem very difficult to solve [17].

Without interference and QoS constraints, the global optimal solution is determined in [20] for $N = 2$ to be in the form of binary power allocation with each user either remaining silent or transmitting with maximum power. Moreover, the authors of [16] showed via numerical simulations that binary power allocation almost always achieves the global optimal solution for more than two users. It is also shown that binary power allocation achieves the global optimal solution when the system is in the interference-limited (the interference power at all receivers is much larger than the noise power) or noise-limited (where the noise power at all receivers is much larger than the interference power) regimes.

The main idea behind binary power allocation is that, the optimal solution often lies on a corner point of the rate region. Those corner points are determined by binary power allocation. For the unconstrained case, the number of those corner points is $2^N - 1$ and obtaining the optimal binary power allocation solution is an exhaustive search problem with exponential complexity. For this reason, several papers addressed reduced complexity binary power allocation algorithms [15], [16], [9]. In the constrained case where we must satisfy QoS requirements for the secondary links as well as interference constraint for the BS, binary power allocation may not achieve an efficient solution. The reason is that the corner points of the new achievable rate region are quite difficult to calculate as they are no longer characterizable by binary power allocation. Moreover, their number can be extremely large. This has led to techniques based on approximating the non-convex problem of (13) into a sequence of geometric programs that can provide efficient suboptimal solutions.

A. Geometric Programming (GP)

A geometric program, as defined in [18], is an optimization problem that takes the form

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \le 1, \quad i = 1, \cdots, l \\ & h_j(\mathbf{x}) = 1, \quad j = 1, \cdots, m, \end{aligned}$$

where, $\mathbf{x} \in \Re^n_{++}$, $f_i(\mathbf{x})$, $i = 0, 1, \cdots, l$ are *posynomial*s and $h_j(\mathbf{x})$, $j = 1, 2, \cdots, m$ are *monomial*s [18]. Since $\sum_i \log(x_i) = \log\left(\prod_i x_i\right)$, and $\log(x)$ is monotonically increasing over x, the optimization problem of (13) can be written as

$$\begin{aligned} \underset{\mathbf{p}^s, \mathbf{x}^s}{\text{maximize}} \quad & \prod_{i\in\mathcal{A}} (1 + x_i^s) \\ \text{subject to} \quad & {x_i^s}^{-1} \gamma_i^s \le 1, \quad i \in \mathcal{A} \\ & \frac{x_i^s \left(\sum_{j\in\mathcal{A}, j\neq i} p_j^s g_{i,j}^{ss} + N_i + N_0\right)}{p_i^s g_{i,i}^{ss}} \le 1, \quad i \in \mathcal{A} \\ & I^{-1} \left(\sum_{i\in\mathcal{A}} p_i^s g_{0,i}^{ps} \right) \le 1 \\ & p_i^s {\hat{p}_i^s}{}^{-1} \le 1, \quad i \in \mathcal{A}. \end{aligned} \tag{14}$$

The inequality constraints of (14) are posynomials, but the objective function $f_0(\mathbf{x}^s) = \prod_{i\in\mathcal{A}} (1 + x_i^s)$ is a posynomial to be maximized. So, to convert it into a minimization problem, $\frac{1}{f_0(\mathbf{x}^s)}$ is not a posynomial, hence, problem (14) is not a geometric program. In high SINR regimes, however, $(1+x_i^s) \approx x_i^s$ which is a monomial. Based on this assumption, the authors of [5] chose the objective function of a problem similar to (14) to be $\prod_{i\in\mathcal{A}} x_i^s$ whose reciprocal is a monomial. The whole problem then became a geometric program that could be solved for a global optimum solution which is an approximate solution for the original problem. This approximation, however, results in inefficient power allocation for low to medium SINR regimes.

B. Iterative Solutions

Another approach to find a suboptimal solution to (14) is successive or sequential GP. The main idea of successive GP algorithms is to solve a sequence of approximate problems to (14) such that the series of solutions to those approximate problems converges to at least a local maximum point of the original problem [19]. Let the approximate problem of the kth iteration be the same as the original problem but with the objective function $\tilde{f}_0^{(k)}(\mathbf{x}^s)$ instead of $f_0(\mathbf{x}^s)$, where $\tilde{f}_0^{(k)}(\mathbf{x}^s)$ is a monomial. And let $\mathbf{x}_0^{s(k-1)}$ be the solution to the approximate problem at the $(k-1)$th iteration. Then for the next iteration, the kth iteration, to have $f_0(\mathbf{x}_0^{s(k)}) \ge f_0(\mathbf{x}_0^{s(k-1)})$, the new approximate objective function, $\tilde{f}_0^{(k)}(\mathbf{x}^s)$, has to satisfy the following three conditions:

1) $f_0(\mathbf{x}^s) \ge \tilde{f}_0^{(k)}(\mathbf{x}^s) \quad \forall \mathbf{x}^s \succcurlyeq \mathbf{0}$.
2) $f_0(\mathbf{x}_0^{s(k-1)}) = \tilde{f}_0^{(k)}(\mathbf{x}_0^{s(k-1)})$.
3) $\nabla f_0(\mathbf{x}_0^{s(k-1)}) = \nabla \tilde{f}_0^{(k)}(\mathbf{x}_0^{s(k-1)})$.

As shown in [19], [21], the above three conditions are sufficient to guarantee that the solution of each approximate problem increases the objective function, and after convergence, the Karush-Kuhn-Tucker conditions of the original problem will be satisfied.

To simplify notation, let $\tilde{f}_0(\mathbf{x}^s)$ denote the approximate objective function for the next iteration and $\mathbf{x}_0$ denote the solution to the previous approximate problem. Hence, we assume the following approximate objective function

$$\tilde{f}(\mathbf{x}^s) = c \prod_{i\in\mathcal{A}} {x_i^s}^{\lambda_i},$$

and the approximate GP is given by

$$\underset{\mathbf{p}^s, \mathbf{x}^s}{\text{maximize}} \quad c \prod_{i\in\mathcal{A}} x_i^{s\lambda_i} \quad \text{subject to} \quad \text{constraints of (14)} \tag{15}$$

where the objective function is a monomial to be maximized, so its reciprocal is also monomial to be minimized so it fits in the GP definition introduced earlier in this section. Suppose that $\mathbf{x}_0^s$ is known. In order to determine $\tilde{f}(\mathbf{x}^s)$ we use the aforementioned three conditions as follows. From the third condition we have

$$\lambda_i = \frac{x_{0i}^s}{1 + x_{0i}^s}, \quad i \in \mathcal{A}. \tag{16}$$

And from the second condition

$$c = \frac{\prod_{i\in\mathcal{A}} (1 + x_{0i}^s)}{\prod_{i\in\mathcal{A}} (x_{0i}^s)^{\lambda_i}}. \tag{17}$$

To show that the first condition is satisfied, it is required to prove that

$$\prod_{i\in\mathcal{A}} (1 + x_i^s) \ge c \prod_{i\in\mathcal{A}} {x_i^s}^{\lambda_i}.$$

Equivalently, by substituting for c and λ_i from (17) and (16), the first condition will be

$$\prod_{i\in\mathcal{A}} (1 + x_i^s) \ge \prod_{i\in\mathcal{A}} \frac{(1 + x_{0i}^s)}{{x_{0i}}^{s\left(\frac{x_{0i}^s}{1+x_{0i}^s}\right)}} . x_i^{s\left(\frac{x_{0i}^s}{1+x_{0i}^s}\right)}. \tag{18}$$

Theorem 1: Fix i, for all $x_i^s \ge 0$ the following inequality is satisfied

$$1 + x_i^s \ge \frac{(1 + x_{0i}^s)}{{x_{0i}^s}^{\left(\frac{x_{0i}^s}{1+x_{0i}^s}\right)}} . x_i^{s\left(\frac{x_{0i}^s}{1+x_{0i}^s}\right)}, \quad x_i^s \ge 0. \tag{19}$$

Proof: By combining both sides of (19), and taking the log we define

$$g(x_i^s) \triangleq \log \frac{1 + x_{0i}^s}{1 + x_i^s} + \frac{x_{0i}^s}{1 + x_{0i}^s} \log \frac{x_i^s}{x_{0i}^s}. \tag{20}$$

Then we prove that $g(x_i) \leq 0$ by showing that the maximum of $g(x_i^s)$ is zero. First, the stationary point is obtained via differentiation

$$\frac{\partial g(x_i^s)}{\partial x_i^s}|_{x_i^s=x_i^{s*}} = 0 \Rightarrow \frac{-1}{1+x_i^{s*}} + \frac{x_{0i}^s}{x_i^{s*}(1+x_{0i}^s)} = 0$$
$$\Rightarrow \quad x_i^{s*} = x_{0i}^s$$

By substituting with x_i^{s*} in (20), $g(x_i^{s*} = x_{0i}^s) = 0$. The second derivative of $g(x_i^s)$ is

$$\frac{\partial^2 g(x_i^{s*})}{\partial x_i^{s*2}} = \frac{-1/x_{0i}^s}{(1+x_{0i})^{s^2}}$$

which is non-positive for $x_{0i}^s \geq 0$. Therefore, $g(x_i^s) \leq 0$ for $x_i^s \geq 0$ and, consequently, the inequality of (19) is satisfied [2]. ■

Since the inequality of (19) is satisfied $\forall i \in \mathcal{A}$, inequality (18) is also satisfied, and the first condition on the approximate function is maintained.

The proposed iterative algorithm is then described in Algorithm 1. The algorithm starts with any initial feasible vector of SINRs, i.e. $\mathbf{x}_0^s(0)$, which has a corresponding sum throughput of $\log_2 \prod_{i\in\mathcal{A}}(1+x_{0i}^{s(0)})$, then after each iteration the new sum throughput will be greater than the sum throughput of the previous iteration. For example, if we choose $x_{0i}^{s(0)}$ such that all corresponding λ_i are equal, this is equivalent to the case of high SINR assumption where iterative GP can start with these values of λ_i and get improved solution than that based on high SINR.

Algorithm 1 Successive GP

1: For all $i \in \mathcal{A}$, initialize $x_{0i}^s \geq 0$ and calculate λ_i from (16), then calculate c from (17).
2: **while** Target accuracy is not reached **do**
3: Solve (15).
4: Update λ_i, $i \in \mathcal{A}$ from (16) and c from (17).
5: **end while**

There are other algorithms based on successive GP introduced in [19], [22]. Those algorithms use different approximate objective functions to the original throughput maximization. However, the single condensation method introduced in [19] and used in [16] requires about $(|\mathcal{A}|+1)^{|\mathcal{A}|}$ variables to be updated each iteration, whereas the double condensation method proposed in [22] requires very complex computations to update each of $|\mathcal{A}|$ variables per iteration. In addition, the condensation technique of this double condensation method is not proven to satisfy the above three conditions and to converge to at least a local minimum. Note that our proposed technique requires only $|\mathcal{A}|+1$ parameters to be updated per iteration, namely, $\lambda_i, i \in \mathcal{A}$ and c.

C. Implementation Issues

The proposed iterative algorithm requires the availability of the channel gains between each secondary transmitter and receiver and between each secondary transmitter and the primary BS. Moreover, it requires the value of the maximum tolerable interference limit I. This problem can be solved by a capable central controller that collects the required information from the secondary nodes and a cooperating BS. In the case of a secondary network supplying revenue to the BS, it can be assumed that the BS does these calculations[3]. The complexity of solving the sum throughput maximization algorithm depends on the GP solver used. In [5], it is claimed that there exist convex optimization solvers with reasonably small complexity, $O(\sqrt{n})$, where n is the size of the problem.

V. Numerical Results

In this section we provide simulation results[4] to investigate the performance of proposed algorithm. For all simulations we assume that primary users are communicating with a BS in the uplink direction. The secondary transmitters are located randomly in an area of size $2000m \times 2000m$ with the primary BS located at the center. The receiving node of each secondary link is placed randomly in a $1000m \times 1000m$ square with its transmitting node at the center. The channel gain between any transmitter j and receiver i is modeled as $g_{i,j} = K_0 \times 10^{\beta_{i,j}/10} \times d_{i,j}^{-4}$, where $d_{i,j}$ is the corresponding distance, $\beta_{i,j}$ is a random Gaussian variable with zero mean and a standard deviation of 6 dB to account for shadowing effects, and $K_0 = 10^3$ is a factor that includes some system parameters such as antenna gain and carrier frequency. The total noise and interference at the receiving node of all secondary links is $N_i + N_0 = 10^{-10}$ W, the maximum transmission power on secondary links is $\hat{p}_i^s = 0.1$ W $\forall i \in \mathcal{N}$. Moreover, we assume that the secondary links use processing gain $PG_i^s = 80\ \forall i \in \mathcal{N}$. For each simulation run, the locations of secondary links are generated randomly. This simulation model is close to that used by the authors of [7] to evaluate the performance of I-SMIRA. We use the CVX–a package for specifying and solving convex programs [25]–to implement Algorithm 1.

A. Admission Control Algorithm

The proposed admission control algorithm is compared to I-SMIRA and the optimal removal technique. In Fig. 2, outage probability is plotted versus γ_i^s for $N = 15$ links and $I = 5(N_i + N_0)$. Outage probability is the ratio of the number of removed links to the number of requesting links. The figure shows the performance of optimal, I-SMIRA and the proposed removal algorithms where the proposed algorithm is implemented using the two reactivation techniques (lists of previous inactive sets and vectors with 1/0 components). The optimal algorithm is implemented by exhaustive search for the largest set of admitted links. It is clear from Fig. 2 that both reactivation techniques achieve better performance than I-SMIRA and are closer to the optimum solution. Moreover, the second reactivation technique has slightly more efficient outage performance than the first one. These results are obtained by averaging over 10^3 simulations. The performance of

[2]Also, the theorem can be proved using the arithmetic mean-geometric mean inequality.

[3]The solution of (3) requires the availability of the peak transmit power of each primary transmitter as well as the channel gains between primary transmitters and the BS. However, there exists an online learning algorithm that can be used to determine I without requiring such parameters, but we omitted it for the space limitation.

[4]Simulations are carried out on MATLAB platform.

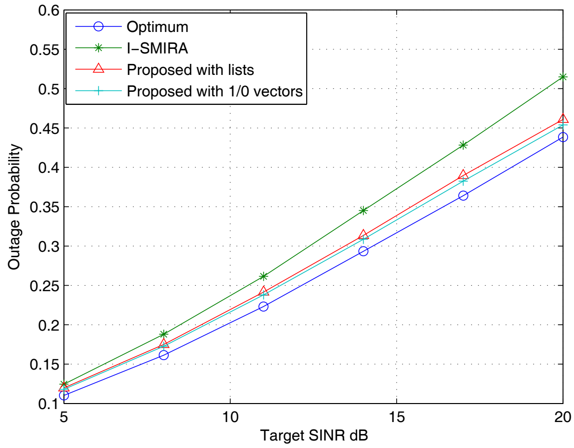


Fig. 2: Outage probability versus target SINR, γ_i^s, for $I = 5(N_i + N_0)$ and $N = 15$ links.

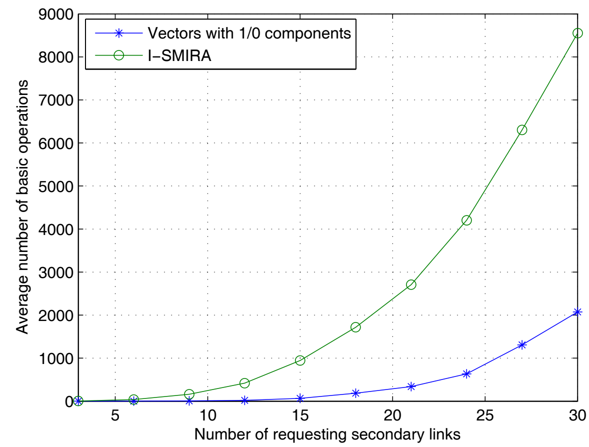


Fig. 4: Complexity of I-SMIRA and the proposed admission control algorithm implemented by vectors of 1/0 components for $\gamma_i = 15$ dB and $I = 5(N_i + N_0)$.

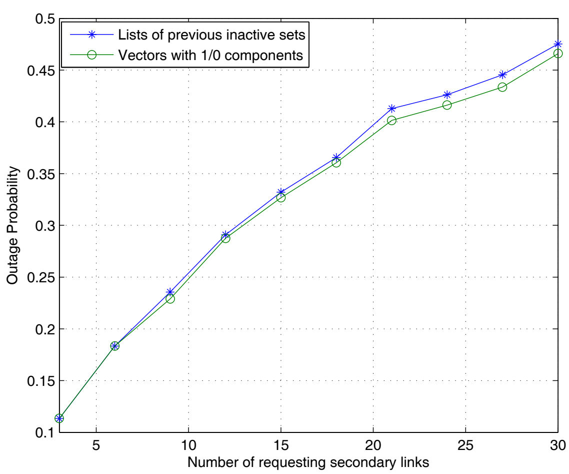


Fig. 3: Outage probability versus the number of requesting links, N, for $\gamma_i^s = 15$ dB $\forall i \in \mathcal{N}$ and $I = 5(N_i + N_0)$.

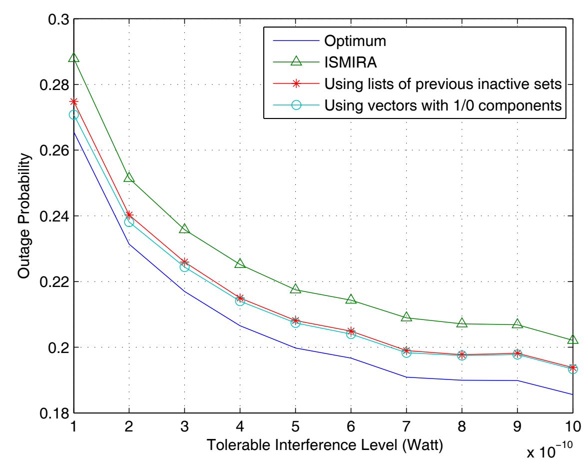


Fig. 5: Effect of interference constraint, I, on the outage probability for $\gamma_i = 15$ dB $\forall i \in \mathcal{N}$ and $N = 7$.

outage probability versus the number of requesting secondary links is evaluated for the two reactivation techniques of the proposed distributed algorithm. Results are obtained assuming that $\gamma_i^s = 15$ dB $\forall i \in \mathcal{N}$. Fig. 3 shows the results of this simulation, it is clear that outage probability increases with the number of requesting secondary links where the performance of the second reactivation technique goes better than the first one as N increases. Moreover, the average complexity of the second removal technique is compared to that of I-SMIRA for different values of N and under the same target SINR $= 15$ dB for all links. We consider the number of basic operations to calculate the complexity of each technique. For the second technique (vectors of 1/0 components are used), the basic operation is the comparison through which a link checks whether it can replace another inactive link. For I-SMIRA, based on the argument about its complexity in [7], the number of basic operations per simulation is $\sum_{i=0}^{R-1} (N-i)^2$, where R is the number of removed links. Fig. 4 shows the complexity performance where it is obvious that the complexity of the proposed algorithm using the second technique is smaller than the complexity of I-SMIRA. The results of Figs. 3 and 4 are obtained by averaging over 10^2 simulation runs.

Fig. 5 shows the effect of the tolerable interference level at the primary BS on the outage probability for $N = 7$ and $\gamma_i = 15$ dB $\forall i \in \mathcal{N}$. The outage probability of the proposed algorithm with its two techniques of reactivation is compared to that of I-SMIRA and the optimal admission algorithms. It is clear that increasing the tolerable interference level at the primary network reduces the outage probability. The performance may, however, saturate with more increase in I because the system becomes more limited by the SINR constraints. It is also obvious that, the proposed distributed algorithm with its reactivation techniques still achieves better performance than I-SMIRA. This result is obtained by averaging over 10^4 simulations.

B. Throughput Performance

The convergence of the proposed GP algorithm is shown in Fig. 6 for the following setup. The number of secondary links $N = 7$, $\gamma_i^s = 0$, $\forall i \in \mathcal{N}$, and $I = 5(N_i + N_0)$ W.

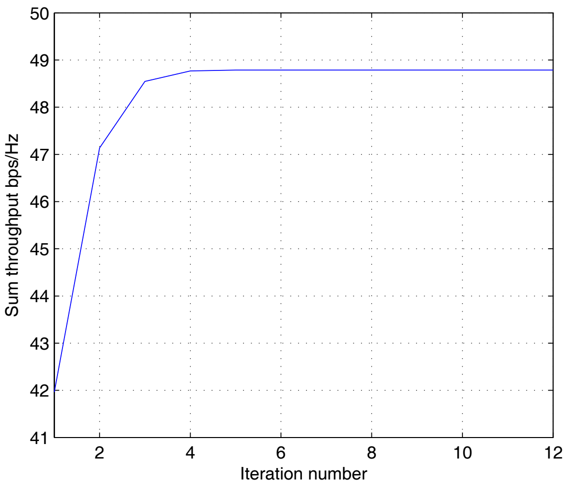


Fig. 6: Convergence of the proposed successive GP algorithm. Starting from the high SINR approximation as an initial point, the proposed algorithm converges to a point of higher sum throughput.

Initially, all values of λ_i are set to $1\ \forall i \in \mathcal{N}$, meaning that, the first iteration of the proposed successive GP algorithm is the solution to the high SINR approximation. It is obvious that the iterations of the proposed algorithm result in increasing sum throughput till convergence to a local optimum point. The main idea behind setting the QoS requirements for the secondary users to 0 is to give the system an opportunity to exploit low and medium SINR encountered by some users to enhance the sum throughput. With a small number of iterations, the system converges to a point with more sum throughput than that obtained with the high SINR assumption. The potential enhancement by the proposed sum throughput maximization technique is quantified in Fig. 7. The figure depicts the sum throughput of the admitted secondary links versus the number of requesting secondary links, N, for the different admission algorithms discussed in this paper. For each admission algorithm, two values of sum throughput are calculated, the first one is obtained when the admitted links by that algorithm are exactly supported at their target SINR, i.e., $\sum_{i\in\mathcal{A}} \log_2(1+\gamma_i^s)$. The second sum throughput is obtained through the proposed sum throughput maximization algorithm, i.e., from Algorithm 1 assuming the initial values of $x_{0i}^s = \gamma_i^s\ \forall i \in \mathcal{A}$. It is clear that Algorithm 1 significantly enhances the sum throughput of the secondary network and yields more utilization. Moreover, there are no remarkable differences in sum throughputs obtained by the different admission algorithms with maximizing the sum throughput after admission. Furthermore, as N increases, the number of admitted links becomes limited by the interference constraint which results in bounding the sum throughput of the secondary network. For this simulation we assume that $\gamma_i^s = 15$ dB $\forall i \in \mathcal{A}$ and $I = N_i + N_0$. The results are obtained by averaging over 10^2 simulations. Finally the effect of the guarantees on the QoS of the admitted secondary links on the system is evaluated. We assume that a secondary system without QoS guarantees (opportunistic) is such that

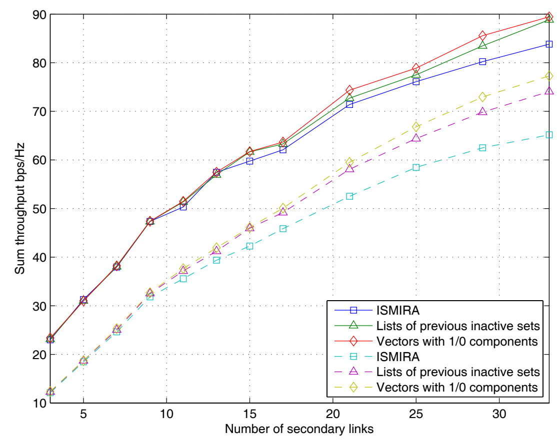
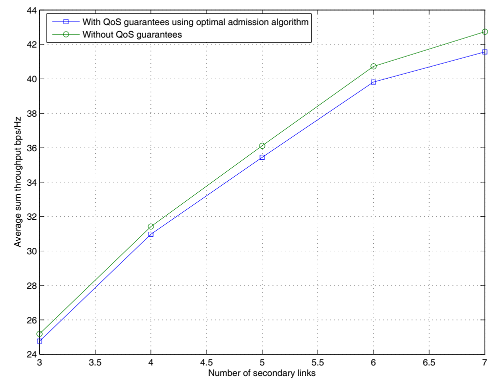


Fig. 7: Sum throughput of the secondary network versus the number of requesting secondary links, N, where $\gamma_i^s = 15$ dB $\forall i \in \mathcal{N}$ and $I = N_i + N_0$. The solid lines represent the sum throughput obtained using the proposed iterative GP algorithm after the admission process, whereas the dashed lines show the sum throughput when all admitted links are supported exactly at the target SINR. For large values of N, the sum throughput tends to saturate.

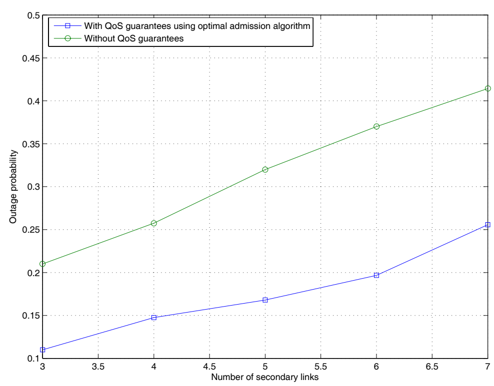
the secondary links are not guaranteed any QoS. In this scenario the sum throughput of the secondary network can be maximized using Algorithm 1 directly without the constraints $x_i^{s-1}\gamma_i^s \leq 1,\ i \in \mathcal{N}$. Furthermore, we assume that the secondary links operating at SINR below the target are in outage. By comparing the performance of this opportunistic system to the original system for $\gamma_i = 15$ dB $\forall i \in \mathcal{N}$ and $I = 5(N_i + N_0)$, it is found from Fig. 8a that the sum throughput of the opportunistic secondary system is slightly better than the sum throughput of the original secondary system. However, in terms of outage probability, Fig. 8b shows that the original system has much better performance than the opportunistic system. Hence, it can be noted that providing guarantees on the SINR of secondary links will not significantly reduce the network utilization, instead, more secondary links can operate reliably. Results of Fig. 8 are obtained by averaging over 10^2 simulations.

VI. Conclusion

Our aim in this paper has been to maximize the sum throughput for the maximum number of secondary links admitted to an underlay cognitive network under two main constraints: interference limit on the primary receiver and minimum QoS guarantees for each secondary link. We have divided the problem into two parts. In the first part we have considered the admission of maximum number of links that can be supported under the two constraints. We have devised a suboptimal distributed algorithm that has been numerically evaluated and shown to produce closer-to-optimal results than previously known ones and with less complexity. The second part is concerned with maximizing the sum throughput of the admitted secondary links under the same constraints. This problem is intractable, so we have suggested an iterative



(a) Effect of QoS guarantees on the sum throughput of the secondary network. Opportunistic secondary network does not significantly enhance the sum throughput.



(b) Effect of QoS guarantees on the outage probability. Opportunistic system has significantly worse performance.

Fig. 8: Under no QoS guarantees, the outage probability increases significantly with no remarkable gain in the performance of the sum throughput. $\gamma_i^s = 15$ dB $\forall i \in \mathcal{N}$ and $I = 5(N_i + N_0)$.

algorithm based on geometric programming. The proposed algorithm converges to at least a local optimum solution with simpler computations between iterations than previously proposed techniques such as single and double condensation. Finally, we have found numerically that the QoS guarantees do not result in a significant loss in the sum throughput of the secondary network, instead, they allow more secondary links to operate reliably.

References

[1] FCC, Spectrum policy task force report, FCC 02-155, Nov. 2002.

[2] FCC, Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order, FCC 03-322, Dec. 2003.

[3] J. Mitola III, "Cognitive radio: an integrated agent architecture for software defined radio," Doctor of Technology dissertation, Royal Institute of Technology (KTH), Sweden, May 2000.

[4] I. Akyildiz, W. Lee, M. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Comput. Netw. J.*, vol. 50, no. 13, pp. 2127-2159, Sep. 2006.

[5] Y. Xing, C. Mathur, M. Haleem, R. Chandramouli, and K. Subbalakshmi, "Dynamic spectrum access with QoS and interference temperature constraints," *IEEE Trans. Mobile Comput.*, vol. 6, no. 4, pp. 423-433, Apr. 2007.

[6] M. H. Islam, Y. C. Liang, and A. T. Hoang, "Distributed power and admission control for cognitive radio networks using antenna arrays," in *Proc. IEEE DySPAN'07*, pp. 250-253, 17-20, Apr. 2007.

[7] L. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306-5315, Dec. 2008.

[8] D. Kim, L. Le, and E. Hossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 5517-5527, Dec. 2008.

[9] J. Tadrous, A. Sultan, and M. Nafie, "Power control for constrained throughput maximization in spectrum shared networks," in *Proc. IEEE Global Telecommun. Conf.*, 2010.

[10] J. Papandriopoulos and J. S. Evans, "SCALE: a low complexity distributed protocol for spectrum balancing in multiuser DSL networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 8, pp. 3711-4843, Aug. 2009.

[11] P. Tsiaflakis, M. Diehl, and M. Moonen, "Distributed spectrum management algorithms for multiuser DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4825-4843, Oct. 2008.

[12] Mehdi, A. R. Sharafat, and J. Zander, "Improved distributed power control algorithms with gradual removal in wireless networks," *European Wireless 2008*.

[13] SM. Andersin, Z. Rosberg, and J. Zander, "Gradual removals in cellular PCS with constrained power control and noise," *ACM/Baltzer Wireless Netw. J.*, vol. 2, no. 1, pp. 27-43, 1996.

[14] J. Tadrous, A. Sultan, and M. Nafie, "Distributed admission and power control for cognitive radios in spectrum underlay networks," in *Proc. IEEE Global Telecommun. Conf.*, pp. 1-6, 2009.

[15] B. Zayen, M. Haddad, A. Hayar, and G. Oien, "Binary power allocation for cognitive radio networks with centralized and distributed user selection strategies," *Physical Commun.*, vol. 1, no. 3, pp. 183-193, Sep. 2008.

[16] A. Gjendemsjo, D. Gesbert, G. Oien, and S. Kiani, "Binary power control for sum rate maximization over multiple interfering links," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3164-3173, Aug. 2008.

[17] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: complexity and duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 57-72, Feb. 14, 2008.

[18] S. Boyd and L. Vandenberge, *Convex Optimization*. Cambridge University Press, 2004.

[19] M. Chiang, C. W. Tan, D. P. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640-2651, July 2007.

[20] M. Charafeddine, "Communication over N-user interference channel," Ph.D. thesis, Stanford University, Dec. 2008.

[21] B. Marks and G. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Operations Research*, vol. 26, no. 4, pp. 681-683, 1978.

[22] M. Charafeddine and A. Paulraj, "Sequential geometric programming for 2 × 2 interference channel power control," in *Proc. 41st Annual Conf. Inf. Sciences Syst.*, pp. 185-189, Mar. 2007.

[23] L. P. Qian and Y. Jun, "Monotonic optimization for non-concave power control in multiuser multicarrier network systems," in *Proc. IEEE INFOCOM 2009.*

[24] S. A. Grandhi and J. Zander, "Constrained power control," *Wireless Personal Commun.*, vol. 1, no. 4, 1995.

[25] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming." Available: http://stanford.edu/~boyd/cvx, June 2009.

[26] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 1st edition. Prentice Hall, 1996.

John Tadrous received the B.S. degree in electrical engineering from Cairo University, Egypt, in 2008 and M.S. degree in wireless technology from the Wireless Intelligent Networks Center (WINC), Nile University, Egypt, in 2010. He is currently pursuing the Ph.D. degree in the Electrical and Computer Engineering department at Ohio State university, Ohio, USA. His research interests include cognitive radios, wireless networks and predictive wireless communications. He is also a student member of the IEEE.

Ahmed Sultan (Member, IEEE) received the B.S. and M.S. degrees in electrical engineering from Alexandria University, Egypt. He finished his Ph.D. from the Electrical Engineering Department at Stanford University in 2007. His thesis work was focused on the development of new models and signal analysis techniques to infer planetary surface parameters from data acquired by earth-based and space borne remote sensing experiments.

He is an assistant professor at Nile University's Wireless Intelligent Networks Center (WINC), Egypt. His current research interests are dynamic spectrum access, cognitive radio networks, physical layer-based secrecy, low-complexity MIMO receivers, and OFDM for optical communications.

Mohammed Nafie received his B.Sc. degree with honors from Cairo University in 1991. He received his M.Sc. degree with Distinction from King's College, University of London in 1993. He received his Ph.D. degree from University of Minnesota in 1999 where his research interests were in the area of low complexity detection algorithms. After his graduation he joined the Wireless Communications lab of the DSPS Research and Development Center of Texas Instruments. He was a primary contributor in TI's effort in the IEEE wireless personal area networks standardization process. In 2001, Dr. Nafie joined Ellipsis Digital Systems, where he was responsible for the development of the Physical Layer (PHY) of 802.11b wireless LAN standard. In 2002, Dr. Nafie co-founded SySDSoft, a company specializing in the design of wireless digital communication systems. He was the Chief Technology Officer of SySDSoft till December 2006.

Before joining the Wireless Intelligent Networks Center of Nile University as an Associate Professor and Director where he currently serves, Dr. Nafie was an Associate Professor at Cairo University, Faculty of Engineering. His research interests are in the area of wireless and wireline digital communications and digital signal processing, and include cognitive radio systems, equalization, and MIMO systems among others. Dr. Nafie has published numerous Journal and Conference papers, and has eighteen issued US patents as well as a few others pending.