# Survey of Interference Channel

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***Abstract*—Interference Channel(IC) is one of the fundamental channels studied in the frame work of Information Theory. However, the capacity of this very important channels is still an open question. This paper will introduce the basic models of this channel as well as the best results on the capacity of this channel. We will review different decoding strategies like time division, simultaneous decoding, etc. Most important the paper will summaries Han and Kobayashi strategy, which is the best know inner bound to this day. Also a new outer bound that is with in 1 bit per channel use of capacity will be discussed. Paper will mainly focus on Discrete Memoryless Interference Channel and Gaussian Interference Channel. Paper will also try to make a distinct difference between interference and noise in terms of their structure.**

## I. Introduction

General Interference Channel model with M senders and N receivers is very important problem in Information Theory and still has not been solved. Very good summary of the results on M to N receiver transmitter networks is done in [1].This paper considers Interference Channel with only 2 receivers and 2 transmitters. Fig. 1 shows a simple model of two user Discrete Memoryless Interference Channel(DM-IC), this model was first considered by Shanon in [2]. Two sources generate two statistically independent messages $W_1$ and $W_2$. Encoder $i \in \{1, 2\}$ takes a message $i$ and encodes into string(codeword) $X_i^n$, where $n$ is the length of the codeword. $X_i$ is then transmitter through a memoryless channel with probability transition matrix $P_{Y_1,Y_2|X_1,X_2}(y_1, y_2|x_1, x_2)$. The channel is said to be memoryless if $p(y_1^n, y_2^n|x_1^n, x_2^n) = \prod_{i=n}^{n} p(y_{1i}, y_{2i}|x_{1i}, x_{2i})$. The output of the channel is a string $Y_i^n$. Decoder takes $Y_i^n$ and uses it to estimate message $\hat{W}_i$. The goal of this problem it to find a set of set of achievable rates $(R_1, R_2)$ which force probability of error $P_e = Pr[\{W_1 \neq \hat{W}_1\} \cup \{W_2 \neq \hat{W}_2\}]$ to approach 0 as $n$ goes to infinity. The set of all achievable $(R_1, R_2)$ is the capacity region of Interference Channel. It is emphasized that the capacity is only know in specific cases, which we will consider later.

Interference Channel models many real world problems and finds applications in many areas of communications theory. One of the simple examples of how Interference Channel is used can be seen in analysis of cross talk between a pair of twisted wires. In [3] it is shown importance of Interference Channel as one of the main building blocks of the new developing field of cognitive radio. Another reason for the importance of the Interference Channel is that it is a more general case of other very important practical channels [5].

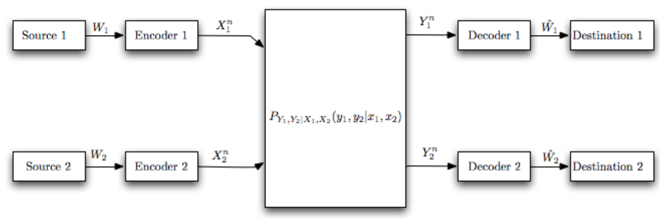
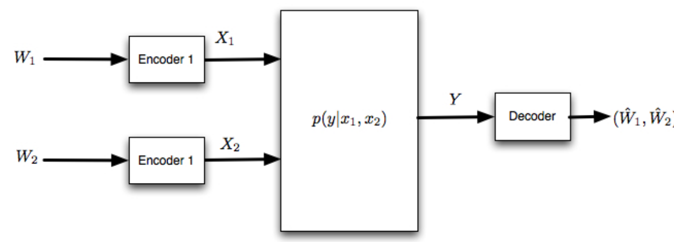
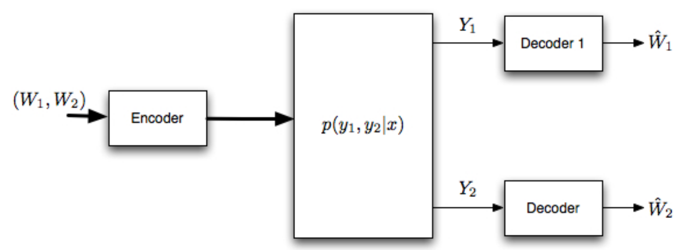



Fig. 1. Model of Interference Channel




(a) Multiple Access Channel




(b) Broadcast Channel

Fig. 2. MAC and Broadcast Channel

For example, MAC( Multiple Access Channel), an Interference Channel with only one receiver, can be used to model cellular communications. Another, example is Broad Cast Channel, an Interference Channel with only one transmitter, this channel for example can model TV or Radio broadcasting. Fig.2 shows MAC and Broad Cast Channel.

## II. Two User Gaussian Interference Channel

This section introduce Gaussian Interference Channel. The reason this model is so important is that it models real world behavior of the channel under noise and interference. Fig.3 depicts Gaussian IC with channel inputs $X_{i,k}$ where $i \in \{1, 2\}$ and $k \in \{1, 2, ..., n\}$ is time instance and channel

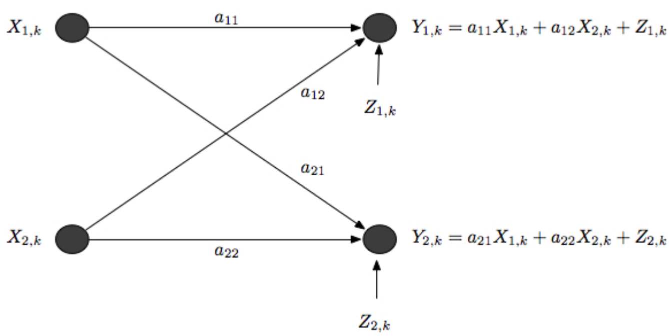



Fig. 3. Model of Interference Channel

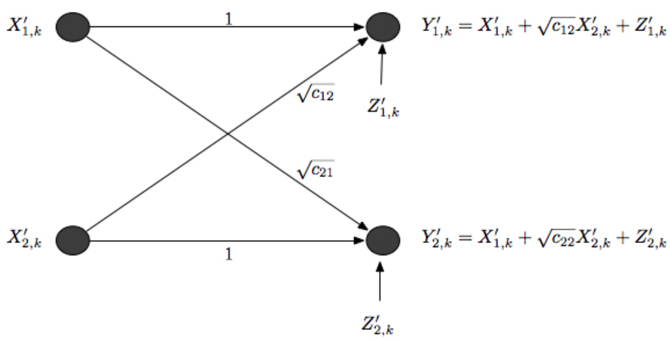



Fig. 4. Standard model of Interference Channel

outputs $Y_{i,k}$. Channel inputs have average power constrain $\frac{1}{n}\sum_{k=1}^{n}|X_{i,k}|^2 \leq P_i$. The channel is subject to Gaussian noise $Z_i \sim \mathcal{N}(0, N_i)$, $N_i > 0$. Where $\{a_{11}, a_{12}, a_{21}, a_{22}\}$are channel gains. The channel outputs at time k is shown below:

$$Y_{1,k} = a_{11}X_{1,k} + a_{12}X_{2,k} + Z_{1,k} \tag{1}$$
$$Y_{2,k} = a_{22}X_{1,k} + a_{22}X_{2,k} + Z_{2,k} \tag{2}$$

### A. Standard Model

It is often useful to standardize the model of the channel, meaning set variance of noise to be 1, $Z'_i \sim \mathcal{N}(0, 1)$, trick often used in statistics. Standardized variables are marked with prime symbol, "$\prime$". Standardization is done by dividing both sides of equations by $\sqrt{N_i}$, the step are shown below.

$$\frac{Y_{1,k}}{\sqrt{N_1}} = \frac{a_{11}X_{1,k} + a_{12}X_{2,k} + Z_{1,k}}{\sqrt{N_1}} \tag{3}$$
$$\frac{Y_{2,k}}{\sqrt{N_2}} = \frac{a_{22}X_{1,k} + a_{22}X_{2,k} + Z_{2,k}}{\sqrt{N_2}} \tag{4}$$

The new standardized channel gains and power constrains are

$$X'_1 = \frac{a_1 1}{\sqrt{N_1}}X_1 \qquad X'_2 = \frac{a_2 2}{\sqrt{N_2}}X_2 \tag{5}$$
$$Y'_1 = \frac{1}{\sqrt{N_1}}Y_1 \qquad Y'_2 = \frac{1}{\sqrt{N_2}}Y_2 \tag{6}$$
$$c_{12} = \frac{a_{12}^2 N_1}{a_{11}^2 N_2} \qquad c_{21} = \frac{a_{21}^2 N_2}{a_{22}^2 N_1} \tag{7}$$
$$c_{11} = 1 \qquad c_{22} = 1 \tag{8}$$
$$P'_1 = \frac{a_{11}^2}{N_1}P_1 \qquad P'_2 = \frac{a_{22}^2}{N_2}P_2 \tag{9}$$

Thus the new expressions for the standard model are:

$$Y'_1 = X'_1 + \sqrt{c_{21}}X'_2 + Z'_1 \tag{10}$$
$$Y'_2 = \sqrt{c_{12}}X'_1 + X'_2 + Z'_2 \tag{11}$$

Standard model for Gaussian Interference Channel is show on Fig.4.

### B. What is a useful Metric?

Since two user Interference Channel has many parameters it is very difficult to come up with one metric that will be used to characterize its behavior. One of the most common and very intuitive metrics is strength of interference. There are four main interference regimes: weak interference, mixed interference, strong interference, very strong interference. The four regimes are defined based on the behavior of Interference Channel and are often sub-divided even further.

*1) Very Strong Interference:* Case of a very strong interference regime for DM-IC can be define in terms of mutual information terms as follows:

$$I(X_2; Y_1) \geq I(X_2; Y_2|X_1) \tag{12}$$
$$I(X_1; Y_2) \geq I(X_1; Y_1|X_2) \tag{13}$$

This inequalities tell us how much information is being transmitted in desired direction versus undesired direction [6]. In the normalized Gaussian Interference Channel very strong interference regime can also be defined in this manner:

$$c_{12} \geq (P'_1 + 1) \tag{14}$$
$$c_{21} \geq (P'_2 + 1) \tag{15}$$

The plus one terms is variance of noise. Intuitively this means that Interference must be stronger than power of the signal plus power of the noise.

*2) Strong Interference:* The case of Strong Interference for DM-IC is defined as following:

$$I(X_2; Y_1|X_1) \geq I(X_2; Y_2|X_1) \tag{16}$$
$$I(X_1; Y_2|X_2) \geq I(X_1; Y_1|X_2) \tag{17}$$

We see that Very Strong Interference also fits this definition, however it is not true the other way around. In the normalized Gaussian Interference Channel strong interference regime can also be defined as follows:

$$c_{12} \geq 1 \tag{18}$$
$$c_{21} \geq 1 \tag{19}$$

Where one is noise variance(power). This means in order for the interference to be strong its power seen at the receiver must be greater than noise power.

*3) Weak Interference and Mixed Interference:* The case of Weak and Mixed Interference for DM-IC is defined as following:

$$I(X_2;Y_1|X_1) \leq I(X_2;Y_2|X_1) \tag{20}$$
$$I(X_1;Y_2|X_2) \leq I(X_1;Y_1|X_2) \tag{21}$$

This again can be interpreted by think of flow of the information. From inequalities it is seen that the flow of information between direct links is greater than between cross links. This is the opposite of strong interference case. In case of Gaussian Interference Channel weak and mixed interference can be defined as following:

$$c_{12} \leq 1 \tag{22}$$
$$c_{21} \leq 1 \tag{23}$$

Where one is noise variance(power). This means in order for the interference to be weak its power seen at the receiver must be less than noise power. Further more, one can further define weak interference in terms of Gaussian channel as follows. Define $\rho_1 \in [0,1]$ and $\rho_2 \in [0,1]$. Then condition for weak interference is shown below:

$$\sqrt{c_{12}}(1 + c_{21}P_1) \leq \rho_2\sqrt{1-\rho_1^2} \tag{24}$$
$$\sqrt{c_{21}}(1 + c_{12}P_2) \leq \rho_1\sqrt{1-\rho_2^2} \tag{25}$$

Where mixed interference is in between weak and strong.

### C. Other useful types of Gaussian Channels

Many times when there is a need to find bounds on capacity of Gaussian Interference Channel and other classes of this channel can become very useful. This section present of several out of many classes of Gaussian Interference Channel and explain their importance.

*1) No Interference Channel:* No interference channel is defined as following, $c_{12} = 0$ and $c_{12} = 0$. This channel is shown on Fig.7. This channel is useful because its capacity is the most general outer bound for all other Gaussian Channels. Moreover, no interference is the base case scenario, thus it is impossible to do better than this.

*2) Z-Channel:* This channel is also very useful as an outer bound and is defined as follows. Either $c_{12} = 0$ or $c_{21} = 0$, since one of the cross channel gains is gone channel topology resembles letter z. This channel is shown on Fig.5

*3) Symmetric Channel:* Symmetric channel is class of interference channel when cross channel gains are the same, $\sqrt{c_{12}} = \sqrt{c_{21}}$. Symmetric channel is very useful when one is trying to generalize behavior of the channel. For example, symmetric channel simplifies work when one is trying to come up with a communications strategy. Also because of the symmetry rates at both receiver are equal which means a lot parameters can be "omitted", this greatly simplifies mental visualization of how interference channel works.

## III. Known Result on Capacity

This section will present know results on the capacity of the Interference Channel. The paper will present results in the same sequence as they were developed historically.

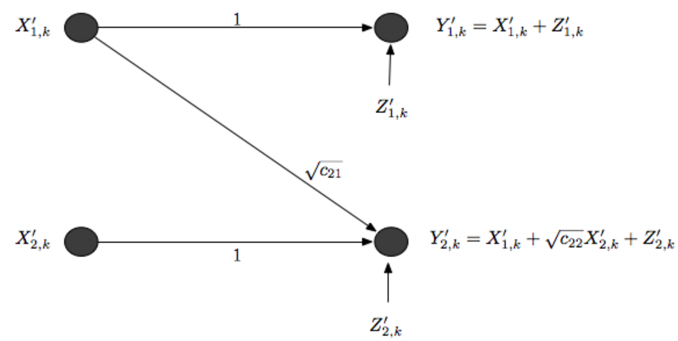



Fig. 5. Z channel model of Interference Channel

### A. Capacity under No Interference

It would be natural to start with the capacity results for the interference channel under no interference. No interference regime is descirbed as folows in terms of mutual information:

$$I(X_2;Y_1|X_1) = 0 \tag{26}$$
$$I(X_1;Y_2|X_2) = 0 \tag{27}$$

This condition implies that there is no flow of information between an output and interfering input, which is exactly what one thinks of no interference. In the case of Gaussina Interference Channel no interference regime can be also describe as follows:

$$c_{12} = 0 \tag{28}$$
$$c_{21} = 0 \tag{29}$$

This implies that cross link channel gains are 0, this also has a very intuitive feel for no interference regime. Thus in the case of no interference Gaussian Interference Channel Collapses to two point to point channels, show in Fig.6. Capacity of point to point channel is well know and was first shown by Shanon. The subject point to point interference is well explained in [7]. Capacity of point point channel is :

$$C_i = \max_{E[X_i^2] \leq P'_i} I(X_i;Y_i) = \frac{1}{2}\log(1+P') \tag{30}$$

where maximum is achieved when $X \sim \mathcal{N}(0, P')$. The capacity region of Interference Channel under on interference is show on Fig.7. Naturally, this is the best case scenario and capacity of all other regimes is a subset of capacity for no interference.

### B. Capacity in Very Strong Interference

Very strong interference is defined in eq. 13, capacity for this regime was shown buy Carleial in [1]. What was proven might seem very counterintuitive at first. One would expect, achievable rates to approach 0 as interference approaches infinity, however, this is not the cases. What might seem even more surprising is that capacity at very high interference is exactly the same as capacity if there was no interference. The

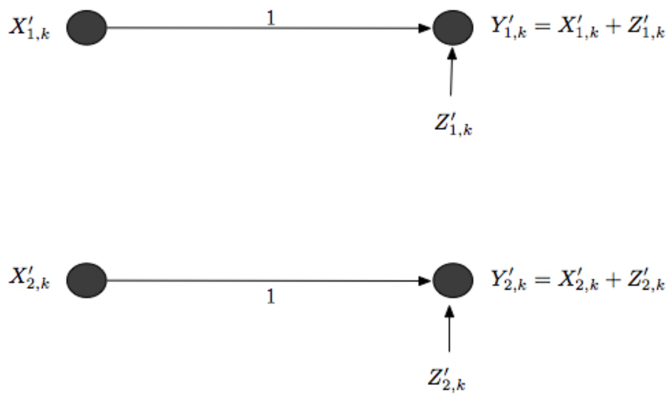



Fig. 6. Gaussian Interference Channel under no interference

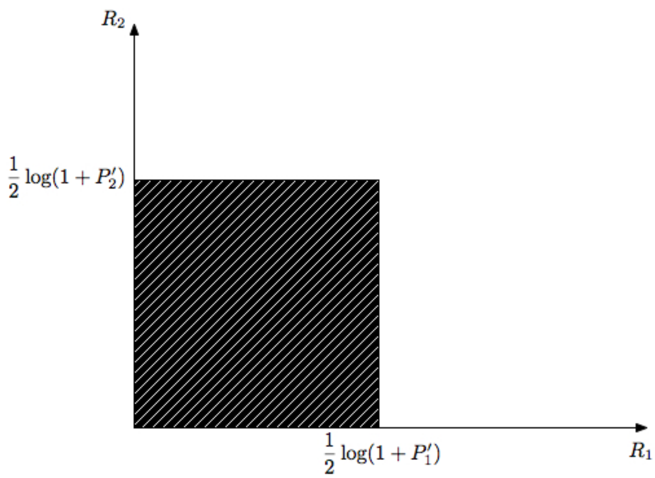



Fig. 7. Capacity of Gaussian Interference Channel with no interference

capacity region for DM-IC is shown below:

$$R_1 \leq I(X_1;Y_1|X_2,Q) \tag{31}$$
$$R_2 \leq I(X_2;Y_2|X_1,Q) \tag{32}$$

Where Q is time sharing random variable. This capacity is achieved using successive interference cancellation and time sharing. The first thing that receiver does is it decodes interfering message and than decodes its own message. This procedure can easily be seen in the context of Gaussian Interference Channel, decoding steps are:

1) Treat intendent message as noise at the receiver 1 and decode $\sqrt{c_{12}}X'_2$ Thus the number of bits, $R_2$, from interfering message decoded at receiver 1 is $R_2 < \frac{1}{2}\log(1+\frac{P'_2}{P'_1+1})$.
2) Subtract decoded interfering signal at the output and decode $X'_1$. $Y_{1new} = Y'_1 - \sqrt{c_{21}}X'_2 = X'_1 + Z'_1$. Thus now maximum achievable rate of intendent message at the receiver 1 is $R_1 \leq \frac{1}{2}\log(1+P'_1)$. This exectly corresponds to the capacity in the case of no interference.
3) Same procedure is perforemed at reciver 2. An maximum achievable rate $R_2$ is $R_2 \leq \frac{1}{2}\log(1+P'_2)$.

### C. *Capacity in the Strong Interference Regime*

Capacity in the strong interference regime for Gaussian Interference channel was obtained by Sato in [6] and the capacity for strong interference case of DM-IC was shown by Costa and Gamal in [8]. Conditions for strong interference in Gaussian Interference Channel is show in eqs.(22),(23). The capacity region of Gaussian Interference Channel is shown below.

$$R_1 \leq \frac{1}{2}\log(1+P'_1) \tag{33}$$
$$R_2 \leq \frac{1}{2}\log(1+P'_2) \tag{34}$$
$$R_1 + R_2 \leq \min(\frac{1}{2}\log(1+P'_1+c_{21}P'_2), \frac{1}{2}\log(1+c_{12}P'_1+P'_2)) \tag{35}$$

Sato in [6] also proposed that this capacity region is intersection of capacity regions of two MAC channels. Fig.2(a) discrete memoryless MAC channel. This also makes sense intuitively since interference Channel is combination of two MAC channels, MAC1 with output $Y_1^n$ and MAC2 with output $Y_2^n$. The capacity of MAC channel depicted in Fig.2(a) has been established and is shown below:

$$R_1 \leq I(X_1;Y|X_2,Q) \tag{36}$$
$$R_2 \leq I(X_2;Y|X_1,Q) \tag{37}$$
$$R_1 + R_2 \leq (I(X_1,X_2;Y|Q) \tag{38}$$

Fig. 8 show capacity region of MAC Channel. For more information on MAC reader can see [7]. By further extending Sato's ideas Costa and Gamal where able to show capacity for DM-IC in strong interference regime and formally proved that capacity region is intersection of two MAC capacity regions. Capacity of DM-IC is shown below:

$$R_1 \leq I(X_1;Y_1|X_2,Q) \tag{39}$$
$$R_2 \leq I(X_2;Y_2|X_1,Q) \tag{40}$$
$$R_1 + R_2 \leq \min(I(X_1,X_2;Y_1|Q), I(X_1,X_2,Y_2|Q)) \tag{41}$$

Capacity region of DM-IC as interesection of two MAC regions is shown on Fig.9.

## IV. On the Capacity for Weak and Mixed Interference

### A. *Properties of Interference*

Even though, capacity is know in strong and very strong interference cases, in general capacity unknown for weak and mixed interference. This might come as a surprise. However, this is due to our think that interference and noise behave in the same way, even thought, in reality this is not true. Noise, specially Gaussian noise, has no structure to it and can not be interpreted. Interference on the other hand has very well defined structure and can be decoded. This is why, intuitively, when interference is strong more structure can be seen in it, on the other hand when interference is weak it is very difficult to decode its structure and interference starts to resemble noise. The issue of how to deal with interference gets even more

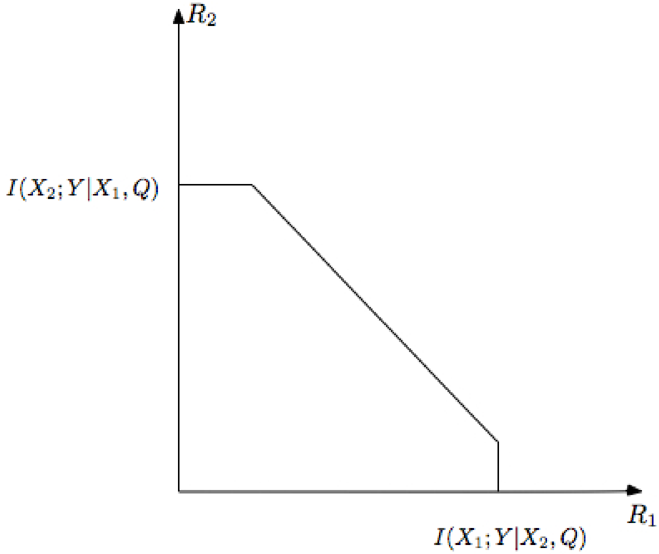



Fig. 8. Capacity Region of MAC

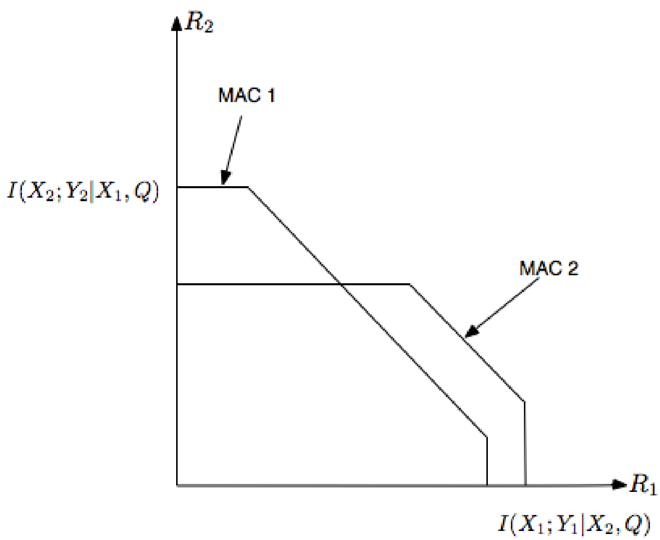



Fig. 9. Capacity Region of DM-IC Channel under Strong Inteference

complicated when interference is mixed or in between strong and week. In this regime it is very difficult to determine which strategy is better: decoding interference or treating interference as noise.

### *B. Inner and Outer Bounds*

Since the capacity of interference channel is unknown in general we have to talk about capacity in terms of outer and inner bounds. Where outer bound is the limit that tells us which rates are not achievable and is the converse part of the capacity proof. Where the inner bound is the limit that tells us which rates are achievable and is the achievebility part of the capacity proof. The goal is to have inner bound and outer bound meet which would give us capacity result. The best know inner bound was derived by Han and Kobayashi in [9] more than 30 years ago. The best know outer bound was derived recently by Etkin, Tse, and Wang [10], in this paper difference between outer and inner bound was shown to be $\frac{1}{2}$ bit. However, before discussing these results we will look at some other inner bounds to get a feel for how interference and other factors effect communications in the interference channel.

## V. Inner Bounds

This section will present five different inner bounds: interference as noise, time devision, time devision with power control, simultaneous decoding and Han and Kobayashi strategy. All the strategies will be presented in the context of the Gaussian Interference Channel while Han and Kobayashi strategy will be presented in the context of DM-IC.

### *A. Treating Interference as Noise*

Treating interference as noise must be the most common way current receivers deal with interference. This strategy might seem optimal however it is not. As was shown earlier in the case of strong interference this strategy is far from being optimal. In fact if this strategy is used in a very strong interference case achievable region would be null($R_1 = R_2 = 0$). Thus the inner bound for treating interference as noise is given below:

$$R_1 < \frac{1}{2}\log(1 + \frac{P_1'}{1 + c_{21}P_2'}) \tag{42}$$
$$R_2 < \frac{1}{2}\log(1 + \frac{P_2'}{1 + c_{12}P_1'}) \tag{43}$$

### *B. Time Division*

Another very intuitive inner bound is achieve using strategy of time division. Where each user only transmits at fraction of a time. The time division variable is defined as $\alpha \in [0, 1]$.The inner bound equations for this strategy are shown below:

$$R_1 < \frac{\alpha}{2}\log(1 + P_1) \tag{44}$$
$$R_2 < \frac{1-\alpha}{2}\log(1 + P_2) \tag{45}$$

### *C. Time Division with Power Control*

An improvement to the time division strategy is using time division plus power control. Since only one transmitter is transmitting at an allocated time slot, the transmitter might as well use more power at that time. Again $\alpha$ is defined in the same way as before. Thus the achievable region is shown below:

$$R_1 < \frac{\alpha}{2}\log(1 + \frac{P_1}{\alpha}) \tag{46}$$
$$R_2 < \frac{1-\alpha}{2}\log(1 + \frac{P_2}{1-\alpha}) \tag{47}$$

### D. Simultaneous Decoding

The simultaneous decoding strategy was mentioned in the case of strong interference and uses the same MAC capacity achieving strategy. Thus the capacity region is shown below:

$$R_1 < \frac{1}{2}\log(1 + P_1') \tag{48}$$

$$R_2 < \frac{1}{2}\log(1 + P_2') \tag{49}$$

$$R_1 + R_2 < \min(\frac{1}{2}\log(1 + P_1' + c_{21}P_2'), \frac{1}{2}\log(1 + c_{12}P_1' + P_2')) \tag{50}$$

This strategy is capacity achieving in strong interference regime only and is sub-optimal in other regimes.

### E. Han and Kobayashi inner bound

This section present Han and Kobayashi Inner Bound and explains basic principles behind this strategy. Han and Kobayashi strategy uses new concept of rate splitting and superposition coding and is the best know inner bound up to date. Superposition coding is very powerful technique used to show inner bounds in may channel. For example supper position coding give a very good inner bound in the case of Broad Cast Channel, [7].

*1) Definitions:* A $(2^{nR_1}, 2^{nR_2}, n)$ codes for an interference channel with independent message sets $\mathcal{W}_1 \in \{1, 2, ...., 2^{nR_1}$ and $\mathcal{W}_1 \in \{1, 2, ...., 2^{nR_2}\}$ with encodign functions$X_1 : \mathcal{W}_1 \to \mathcal{X}_n^1$ and $X_2 : \mathcal{W}_2 \to \mathcal{X}_n^2$, and with two decoding functions $g_1 : \mathcal{Y}^n \to \hat{W}_1$ and $g_2 : \mathcal{Y}^n \to \hat{W}_2$. The average probability of error is:

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{(w_1,w_2)\in\mathcal{W}_1\times\mathcal{W}_2} Pr(\hat{W}_1 \neq W_1 \text{ or } \hat{W}_2 \neq W_2 | w_1, w_2 \text{ sent}) \tag{51}$$

The rate pait $(R_1, R_2)$ is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with$P_e^n \to 0$

*2) Private and Common message:* In Han Kobayashi strategy message $W_1$ is split into to messages $W_{11}, W_{12}$. Where $W_{11}$ is referred to as private message intended only for receiver 1 and $W_{12}$ is referred to as common message and is intended for both receivers. This message splitting is intendent for partial interference cancellation where private message is treated as noise at cross link receiver. Rates $R_{11}$ and $R_{12}$, $R_1 = R_{11} + R_{12}$ are message rate for private and common messages respectively. Superposition coding is used where common message is represented by auxiliary random variable $U_1$. Moreover, we randomly and generate $2^{nR_{12}}$ sequences of $u_1^n(w_{12})$ to represent common message set $\mathcal{W}_{12}$. Now for each $u_1^n(w_1)$ we randomly generate $2^{R_{11}}$ sequences of $x_1^n(w_{11}, w_{12})$. This is how supper position coding is performed. Notice that the resulted number of sequences is $2^{nR_{11}} \times 2^{nR_{12}} = 2^{nR_1}$ which is exactly the cardiaiality of $W_1$. Now $x(w_{11}, w_{12})$ is transmitted. Notice that $u_1^n(w_1)$ is not being transmitted it's only used for code generation and therefore is called auxiliary random variable. $u_1^n(w_1)$ is also referred to as a cloud center since $x_1^n(w_{11}, w_{12})$ are generated around it. The exact same procedure is performed at the second transmitter on message $W_2$.

*3) Decoding:* In Han and Kobayashi strategy joint typicality decoder is used to find unique pair of messages $\hat{w_{11}}, \hat{w_{12}}, \hat{m}_{21}$ and declares and error if no such sequence is found. Now at receiver 1 we have four message $W_{11}, W_{12}$ and $W_{21}, W_{22}$. Thus Receiver 1 simultaneously decodes messages $W_{12}$,$W_{21}$,$W_{11}$ and treats private message $W_{22}$ as noise. Similarly Receiver 2 simultaneously decodes messages $W_{12}$,$W_{21}$,$W_{22}$ and treats private message $W_{11}$ as noise.

*4) Region:* Han and Kobayashi region is shown bellow $Q$ is again a time sharing parameter.

$$R_1 < I(X_1; Y_1 | U_2, Q) \tag{52}$$

$$R_2 < I(X_2; Y_2 | U_1, Q) \tag{53}$$

$$R_1 + R_2 < I(X_1, U_2; Y_1 | Q) + I(X_2; Y_2 | U_1, U_2, Q) \tag{54}$$

$$R_1 + R_2 < I(X_1, Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2, Q) \tag{55}$$

$$R_1 + R_2 < I(X_1, U_2; Y_1 | U_1, Q) + I(X_2, U_1; Y_2 | U_2, Q) \tag{56}$$

$$2R_1 + R_2 < I(X_1, U_2; Y_1 | Q) + I(X_1; Y_1 | U_1, U_2, Q) \tag{57}$$

$$+ I(X_2, U_1; Y_2 | U_2, Q) \tag{58}$$

$$R_1 + 2R_2 < I(X_2, U_1; Y_2 | Q) + I(X_2; Y_2 | U_1, U_2, Q) \tag{59}$$

$$+ I(X_1, U_2; Y_1 | U_1, Q) \tag{60}$$

The derivation of this inner bound is quite complicated and is well treated in, [11]. Original inner bound derived bay Han and Kobayashi was more involved and complicate. The inequalities where in terms of $R_{ip}$ and $R_{ic}$ and $R_i$, however using FourierMotzkin elimination, a powerful tool from linear algebra, the inner bound was simplified to the one seen above. Only until recently it was unknown how close does this strategy get to the capacity. In the next section we will discuss these results.

## VI. Outer bound

The best know outer bound for Gaussian Interference Channel was derived very recently by Etkin, Tse and Wang in [10]. Their result improved on previous outer bounds, it was also shown that the difference between Han and Kobayashi inner bound and this new outer bound is with in 1 bit. In this section we will go thought this very important result, even thought the paper develops this new result for all classes of Gaussian Interference Channels we will only concentrate on symmetric case. It is very important to note that the new bound is for the sum rate meaning, $R_1 + R$. This is comes from the fact that capacity of symmetric channels is obtained by maximizing $R_1 + R_2$ and is referred to as symmetric capacity.

### A. More about Symmetric Channel

Even thought symmetric channel was already introduced, in this section we presented in more detail. In a symmetric channel as mentioned before cross channel gains $c_{12} = c_{21}$, Fig.4, are the same and direct link gains are also the same. Also we assume that input power constrains are also the same, thus $P_1 = P_2 = P$. From this it implies that signals to noise ratios are also the same $SNR_1 = SNR_2 = SNR$. Here we also introduce concept that sometimes an familiar which is interference to noise ratio $INR$. For example $INR_1 = \frac{c_{12}P_2}{N_1} = c_{12}P_2$, this is interfrence to noise ratio at receiver, similarly for receiver 2, $INR_2 = \frac{c_{21}P_1}{N_2} = c_{21}P_2$. Due to symmetry $INR_1 = INR_2 = INR$.

### *B. Interference Limited Scenario vs. Noise limited scenario*

Interference limited scenario is a case when $INR > 1$ meaning interference power is much greater than noise power. Noise limited scenario is a case when interference power is much smaller than noise power, meaning $INR < 1$. The case of $INR < 1$ is less interesting because interference is less than noise at is reasonable to treat it as noise. Therefore, case of $INR > 1$ is looked at.

### *C. Simple Han and Kobayashi strategy*

Etkin, Tse and Wang used very simple Han and Kobayashi strategy to show their result. Basics of Han and Kobayshi strategy relie no rate splitting between private and common message. We define $INR_p$ as interference to noise ratio due to private message. In this strategy $INR_p = 1$, thus private message arrives at the same power as noise. There are many reasons to do this. Since private messages are not decoded and their treated as noise at cross link receiver it makes sense to send private message at the level of the noise. Another reason is that more power can be allocated to common message. Since we want to decode common message at the cross link decoder, where it is seen as interference, it is good to make this interference very strong for better decoding(discussion on why strong inference is better was done in earlier). Thus in this strategy $R_{1p} = R_{2p}$ and $R_{1c} = R_{2c}$. Decoding orders is the following:

1) Decode common messages first This implies that private messages are treated as noise at first.
2) Decode desired private message while treating the other private message as noise

### *D. New parameter Interference Level*

Even though symmetric model reduces channel complexity and gets rid of many parameters, there are stil other parameters which make this model complicated. Thus a new parameter that can track a model would be very useful. Etkin, Tse and Wang come up with such a parameter, called interference level $\alpha$ defined as following.

$$\alpha = \frac{\log INR}{\log SNR} \tag{61}$$

Thus alpha compares strength of interference comparing to the strength of the signal in the logarithmic domain.

### *E. Summary of New and Old Outer Bounds*

The outer bounds are derived in the converse proof of channel capacity. There are man bounding techniques that there were used on Gaussian Interference Channel. The goal this bounding techniques to tell after which point we can not to better in terms of communication rates.

*1) No interference Bound:* The very first outer bound that can be shown is by using interference channel under no interference described in section III-A. It is clear that under no interference is best case scenario and it is impossible to do better than that. However, this outer bound is very loose and thus very impractical.

*2) Genie-aided interference channel:* Another way to find outer bounds is to use the genie-aided approach. In this approach, for example, we can give to one or both of the receivers part of or all the message sent by interfering transmitter. Thus receiver can use this side information to decode it's own message This of course is vey good scenario and is impossible to do better than this. Because, the information is given "magically" this approach is called genie-aided.

### *F. Genie used in the new outer bound*

The side information that was used by Etkin, Tse and Wang is very simple and innovative at the same time. Let $s_1$ be side information given to receiver 1 and $s_2$ be side information given to receiver 2. $s_1$ and $s_2$ are defined below:

$$s_1 = \sqrt{c_{12}}X_1 + Z_2 \tag{62}$$

$$s_2 = \sqrt{c_{21}}X_2 + Z_1 \tag{63}$$

Thus this genie gives side information that contains interfering message subject to channel gain plus noise. However, what is interesting about this approach is that genie provides information about noise at the other receiver which might seem very meaningless. The new derived outer bound for sum rate capacity is shown below, since $R_1 = R_2$ we refer to them as $R$

$$R_u < \log(1 + INR + \frac{SNR}{1 + INR}) \tag{64}$$

$$R_u < \frac{1}{2}\log(1 + SNR) + \frac{1}{2}\log(1 + \frac{SNR}{1 + INR}) \tag{65}$$

### *G. Capacity to within 1 bit*

It also was show in this paper that new outer bound is within 1 bit from capacity. This was done in the following fashion. Since capacity is achieved when inner and outer bound meet with each other, if one subtracts outer bound from inner bound the result would be the gap between the two bouns. Since capacity is somewhere in between, the width of the gap will be a good indicator how good inner and outer bounds are. This was done by Etkin, Tse and Wang by subtractin symmetric rate achieved by Han and Kobayshis strategy from their newly derived upper bound, $R_u - R_{HK}$. Their results show that this difference is alwasy less or equal to one $R_u - R_{HK} \leq 1$. It was also shown that as SNR goes to infinity in many differece regimes this difference approaches 0.

### *H. Further simplifications*

Now we can used previously defined $\alpha$ to simplify and show this results in more intuitive way. Lets first define a new measure of efficiency of communication over the channel. This

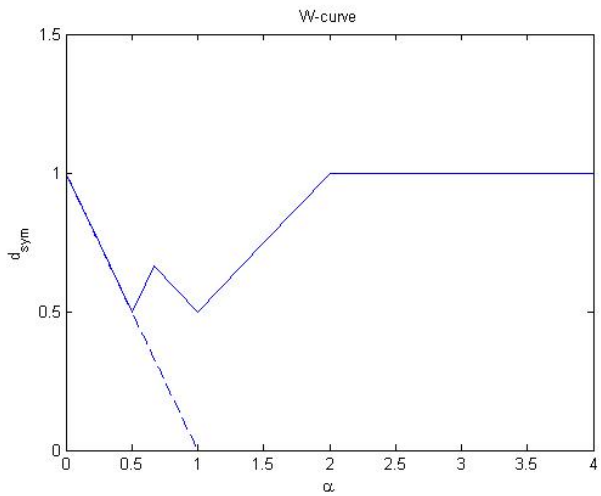



Fig. 10. W curve. Generalized degree of freedom vs $\alpha$

measure is called generalized degree of freedom and is defined as following:

$$d_{sym}(\alpha) = \lim_{SNR,INR\to\infty;\frac{\log(SNR)}{log(INR)}=\alpha} \frac{C_{sym}(INR,SNR)}{C_{awgn}(SNR)} \tag{66}$$

Where $C_{awgn}(SNR)$ is the capacity of point to point Gaussian channel and $C_{sym}(INR,SNR)$ symmetric capacity of Gaussian interference channel. Notice that degree of freedom is a function of $\alpha$ Since capacity point to point channel is the best case scenario comparing rates at other interference levels is can be very insightful. Thus for example at $\alpha = 0$ when there is no interference $d_{sym}(0) = 1$. Now using result of Etkin, Tse and Wang, $d_{sym}(\alpha)$ is shown below for different values of $\alpha$:

$$d_{sym} = 1 - \alpha, \qquad \text{for } 0 \le \alpha \le \frac{1}{2} \tag{67}$$

$$d_{sym} = \alpha, \qquad \text{for } \frac{1}{2} \le \alpha \le \frac{2}{3} \tag{68}$$

$$d_{sym} = 1 - \frac{\alpha}{2}, \qquad \text{for } \frac{2}{3} \le \alpha \le 1 \tag{69}$$

$$d_{sym} = \frac{\alpha}{2}, \qquad \text{for } 1 \le \alpha \le 2 \tag{70}$$

$$d_{sym} = 1, \qquad \text{for } 2 \le \alpha \le 1 \tag{71}$$

If we represent this graphicly we get a famous W-curve shown on Fig10. Importance of W-curve lies in the fact that it shows how capacity is changing with interference, this means that W-curve can serve as very good benchmark for design of systems under different levels of interference.

### I. *Summary of the W-curve*

First regime from $0 \le \alpha \le 0.5$ is called weak interference regime. Notice that capacity in this regime decreases as interference increases. The reason for this is that interference is not strong enough to be decode. Therefore in this regime it is optimal to treat interference as noise. And all message would be private.

Notice a dashed line that continues on going after regime. This line is NOT a part of W-curve. However, this line shows what would happen if inference is treated as noise. Naturally, if interference is treated as noise at very strong interference capacity is 0.

Second regime $\frac{1}{2} \le \alpha \le \frac{2}{3}$. Notice that in this regime capacity starts to increase as $\alpha$ increases. The reason for this is that now interference is strong enough and common messages can be used. This means part of the inference can be decoded via common messages.

Regime three $\frac{2}{3} \le \alpha \le 1$ is might seem strange because capacity start to decrease again. This behavior is due to the fact that increase in interference outperforms gains due to the use of common an private messages. Thus interference reduces capacity in this regime.

Regime four $1 \le \alpha \le 2$ is a strong interference regime and capacity is well know in this regime. In this regime all information is common. And increase in capacity is expect since from previous sections we now that capacity increases with interference.

Regime five $1 \le \alpha \le 2$ is a very strong interference regime. This result is exactly what we would expect since at very strong interference capacity is the same as at no interference.

## VII. Conclusion

This paper briefly summarized main results on the capacity of Interference Channel. The r main results to remember are Han and Koabayashi inner bound as well as Etkin, Tse and Wang outer bound. Another importan thing paper was trying to emphasises is how to deal with interference. Interference effects on capacity can be seen from W-curve, which is also encouraged to be familiar with.

## References


[1] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. IT-24, no. 1, pp. 60–70, Jan. 1978.

[2] C. E. Shannon, "Two-way communications channels," in *4th Berkeley Symp. Math. Stat. Prob.*, Chicago, IL, Jun. 1961, pp. 611–644.

[3] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *IEEE J. Select. Areas Commun.*, 2008.

[4] T. Hunter, A. Hedayat, M. Janani, and A. Nostratinia, "Coded cooperation with space-time transmission and iterative decoding," in *WNCG Wireless*, Oct. 2003.

[5] G. Kramer, "Review of rate regions for interference channels," in *International Zurich Seminar*, 2006.

[6] H. Sato, "The capacity of Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 6, pp. 786–788, Nov. 1981.

[7] T. Cover and J. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, 1991.

[8] A. El Gamal and M. Costa, "The capacity region of a class of deterministic interference channels," *IEEE Trans. Inf. Theory*, vol. 28, no. 2, pp. 343–346, Mar. 1982.

[9] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, Jan. 1981.

[10] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[11] H. Chong, M. Motani, H. Garg, and H. El Gamal, "On the han-kobayashi region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 3188–3195, Jul. 2008.