Worst-Of Barrier Reverse Convertible

Contents

1 Product Description

1.1 Basic Information on Term Sheet

- Product: Barrier Reverse Convertible
- Issuer: Credit Suisse AG
- Listing: SIX Swiss Exchange Ltd
- Denomination/Issue Price: USD 1,000
- Initial Fixing Date: 10 Nov 2022
- Final Fixing Date: 13 Nov 2023

Underlyings	Exchange	Strike (100%)	Barrier (50%)	Conversion Ratio
Coca-Cola Co. Registered Share	NYSE	60.88	30.44	16.4258
McDonald's Corp Registered Share	NYSE	275.88	137.94	3.6248
Starbucks Corp Registered Share	NASDAQ	96.26	48.13	10.3885

Table 1: Underlyings, Strike and Barrier Prices in USD

1.2 Payoff Structure

- **Definition of Barrier Event**

  If, at any time (continuously observed) on any exchange business day during the period from the Initial Fixing Date (excluding) to the Final Fixing Date (including), the level of any Underlying asset reaches or falls below its designated barrier, the Barrier Event is said to have occurred. In this context, the level of the Underlying asset should be determined based on the unadjusted close price.

- **Interest and Premium Payment**

  Interest 4.88% + Premium 5.37% = 10.25% p.a., quarterly payment.

- **Final Redemption**

  - Case 1: If the barrier has been hit **and** the final level of any underlying is below its strike, the number of worst-performing underlyings specified in the Conversion Ratio column in Table 1 will be delivered. Fractions will be paid in cash.
    Payoff: $min(\frac{S_{it}}{S_{i0}}) \times denomination$ for i = 1, 2, 3
  - Case 2: Otherwise, a cash amount equal to 100% of the Denomination will be delivered.
    Payoff: 100% $\times$ *denomination*

- **Structure:** The product is essentially a combination of long 1-year quarterly coupon note and short worst-of down-and-in European put option.

1.3 Prospects for Profit and Loss

- **Profit Prospects**

  The interest and premium income generated from the product is substantial. The annual coupon rate of 10.25% is significantly higher compared to the prevailing risk-free rate during that period. Additionally, this enhanced coupon payment is guaranteed, ensuring that it will be distributed irrespective of the performance of the underlying assets.

  In addition, the barrier is set at a low level, specifically 50% of the initial strike price. As long as none of the underlying assets have dropped to or below 50% of their respective strikes, the investor will receive 100% of the Denomination at maturity. This feature ensures a decent return, even in a slightly bearish market scenario.

- **Loss Prospects**

  If the barrier has been hit and the price can't revert to a level no lower than its strike at maturity, losses could be tremendous or even a total loss of the initial investment amount.

1.4 Intuition for the Product

The product is categorised as a yield enhancement instrument. It allows investors to pursue higher coupon yields at the cost of higher downside risks. Investors who are unsatisfied with the returns of simple bonds but do not want to take on the high risks of equities are the target buyers of this product.

As the potential loss of the product is floored at the initial investment amount, buyers should have expected that the prices of any underlyings would not plummet dramatically during the contract period. Also, the buyers should have anticipated that the underlyings would not experience a period of significant bullishness so they are willing to give up the opportunity for higher returns as the product's profit is capped. In summary, the target buyers of this product should possess a neutral opinion on the future movement of the stock price, or they should expect that the prices of underlyings would be less volatile during the contract period.

The product was listed in exchange with a minimum initial investment of USD 1,000. Therefore, the investment threshold for retail investors is deemed low. Both institutional and retail investors could invest in the product. However, the retail investors were expected to have a good understanding of the increased risks behind the enhanced yield.

2 Calibration, Simulation and Pricing

In this project, 10,000 paths were used in simulations unless otherwise stated.

2.1 Black-Scholes Model

- **Model Specification**
  We assume that that each stock follows Black-Scholes dynamic, where the correlation matrix and the covariance matrix of three stock are fixed and remain unchanged through out the simulation process.
  Under $\mathbb{Q}$-Measure, we simulate the stock price and compute the price of product based on the model:

$$\frac{dS_{it}}{S_{it}} = rdt + \sigma_i dW_{it}^{\mathbb{Q}}$$

  where r is risk-free rate.
  Solving the SDE, then for each stock we have

$$S_i(t + \Delta t) = S_i(t) \times exp[(r - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i W_{it}^{\mathbb{Q}} \times \sqrt{\Delta t}]$$

  where $(W_{1t}, W_{2t}, W_{3t}) \sim N(0, \Sigma)$.

  In the calculation of value-at-risk and expected shortfall in the next section, it will be changed back to $\mathbb{P}$-Measure

$$\frac{dS_{it}}{S_{it}} = u_i dt + \sigma_i dW_{it}^{\mathbb{P}}$$
$$W^{\mathbb{P}} = W^{\mathbb{Q}}$$

where u_i is the averaged return calculated from historical price for i = 1, 2, 3.
Solving the SDE, then for each stock we have

$$S_i(t + \Delta t) = S_i(t) \times exp[(u_i - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i W_{it}^{\mathbb{P}} \times \sqrt{\Delta t}]$$

where $(W_{1t}, W_{2t}, W_{3t}) \sim N(0, \Sigma)$.

- **Simulation Process**
  - Step 1: We calculate the log return correlation matrix based on historical stock returns and generate W_i which follow follow multinomial normal distribution with mean zero and constant correlation matrix.
  - Step 2: Based on the Black-Scholes dynamic, we simulate 10,000 stock paths. Based on whether they hit the barrier or not, we calculate their terminal payoff.
  - Step 3: After Calculating 10,000 paths' terminal payoff, take average and discount them to the valuation date. The price of convertible is determined by the present value of option payoff and coupons

Finally, we got a price of $1025, which is slightly higher than the actual price $1000.

2.2 Heston Model

- **Model Specification**
We assume that each stock follows the Heston stochastic volatility model under the real-world P-measure as follows:

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{v_t} S_t dW_{1,t}^{\mathbb{P}} \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dW_{2,t}^{\mathbb{P}} \\ \rho dt &= dW_{1,t}^{\mathbb{P}} dW_{2,t}^{\mathbb{P}} \end{aligned}$$

where the variance v_t follows a CIR process. It is mean reversion, and the positive variance is guaranteed under the constraint $2\kappa\theta > \sigma^2$. Then, we use Girsanov Theorem to transfer the wiener process from P-measure to Q-measure:

$$\begin{aligned} dW_{S,t}^{\mathbb{Q}} &= dW_{S,t}^{\mathbb{P}} + \alpha_S dt, \alpha_S = \frac{\mu_{\mathbb{P}} - r}{\sqrt{v_t}} \\ dW_{v,t}^{\mathbb{Q}} &= dW_{v,t}^{\mathbb{P}} + \alpha_v dt, \alpha_v = \frac{\lambda}{\sigma^{\mathbb{P}}}\sqrt{v_t} \end{aligned}$$

Where λ is the variance premium. After changing the measure, we get

$$\begin{aligned} dS_t &= rS_t dt + \sqrt{v_t} S_t dW_{1,t}^{\mathbb{Q}} \\ dv_t &= \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - v_t)dt + \sigma\sqrt{v_t} dW_{2,t}^{\mathbb{Q}} \\ \rho^{\mathbb{Q}} dt &= dW_{1,t}^{\mathbb{Q}} dW_{2,t}^{\mathbb{Q}} \end{aligned}$$

where $\rho^{\mathbb{Q}} = \rho, \kappa^{\mathbb{Q}} = \kappa + \lambda, \theta^{\mathbb{Q}} = \kappa\theta/(\kappa + \lambda)$ and λ is the variance risk premium.

- **Model Calibration**(1)

The parameters to be calibrated are $\Theta = (v_0, \kappa, \theta, \sigma, \rho, \lambda)$.
We implemented the semi-analytical solution for the option price under the Heston model:

$$C(S_0, K, v_0, \tau) = \frac{1}{2}(S_0 - Ke^{-r\tau}) + \frac{1}{\pi}\int_0^{\inf} \Re[e^{-r\tau}\frac{\varphi(\phi - i)}{i\phi K^{i\phi}} - Ke^{-r\tau}\frac{\varphi(\phi)}{i\phi K^{i\phi}}]d\phi$$

where the function $\Re$ is inverse Fourier Transformation and the $\psi(\phi)$ is the characteristic function

$$\varphi(S_0, K, v_0, \tau; \phi) = e^{r\phi i\tau}S^{i\phi}[\frac{1 - ge^{d\tau}}{1 - g}]^{\frac{-2a}{\sigma^2}} exp[\frac{a\tau}{\sigma^2}(b - \rho\sigma\phi i + d) + \frac{v_0}{\sigma^2}(b - \rho\sigma\phi i + d)[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}]],$$

$$d = \sqrt{(\rho\sigma\phi i - b)^2 + \sigma^2(\phi i + \phi^2)}, g = \frac{b - \rho\sigma\phi i + d}{b - \rho\sigma\phi i - d}, a = \kappa\theta, b = \kappa + \lambda$$

Note that the integration in the option price calculation is performed numerically using rectangular integration. Next, we conduct the optimization using the option data and obtain the parameters by minimizing the following loss function:

$$\hat{\Theta} = argmin \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}(C_{MKT}(K_i, \tau_j) - C_{Heston}(S_\tau, K_i, \tau_j, r_j, \Theta))^2$$

where the weight for each square is the Vega weight in our case, serving as a proxy for the trading volume, as the value is bell-shaped and higher near the ATM point. Finally, we obtain the calibration result as follows:

	v_0	κ	θ	σ	ρ	λ
KO	0.0864	6.3472	0.0585	0.7972	-0.5227	0.074
MCD	0.0752	5.1732	0.0531	0.6974	-0.5162	0.1273
SBUX	0.1101	0.7168	0.0649	0.5685	-0.6755	-0.6156

Table 2: Calibrated Heston Parameters

- **Path Simulation**

  We have adapted the Euler Discretization as our simulation scheme:

$$S_{i+1} = S_i exp[(r - \frac{v_i}{2})\Delta t + \sqrt{v_i}\Delta t W^{\mathbb{Q}}_{S,i+1}]$$
$$v_{i+1} = v_i + \kappa(\theta - v_t)\Delta t + \sigma\sqrt{v_i}\Delta t W^{\mathbb{Q}}_{v,i+1}$$

  Consider the correlation between the three indices, we implemented the above formula at each step as follows:

  – Step 1: Generate three multivariate normal with a mean of zero and a correlation matrix derived from historical stock return data for simulating the stock price. Calculate the stock price at the next time point based on the normal random variable and the previously calculated variance using the formula above.

  – Step 2: Generate one standard normal for simulating all three variances. Calculate the variance at the next time point using the formula above. Compute the dW_v for each variance dynamics as $\rho dW_s + \sqrt{1 - \rho^2}dZ$.

The adoption of a single Brownian motion for all the three variance processes is justified by extremely high historical correlation among the variances of underlyings and immaterial historical correlation among the variances and log returns of the different underlyings, as shown in the correlation matrix in Figure 1.

	KO_ret	MCD_ret	SBUX_ret	KO_var	MCD_var	SBUX_var
KO_ret	1.000000	0.636571	0.488424	0.020564	0.027336	0.030125
MCD_ret	0.636571	1.000000	0.608359	0.050527	0.057605	0.060767
SBUX_ret	0.488424	0.608359	1.000000	0.046818	0.042867	0.050459
KO_var	0.020564	0.050527	0.046818	1.000000	0.983414	0.968831
MCD_var	0.027336	0.057605	0.042867	0.983414	1.000000	0.970025
SBUX_var	0.030125	0.060767	0.050459	0.968831	0.970025	1.000000

Figure 1: Historical Correlation Matrix of Log Returns and Variances

We conducted daily simulations with a time step of $\Delta t = \frac{1}{252}$ in our case. Here Figure 2 is an example of the simulation results for 50 KO price paths:

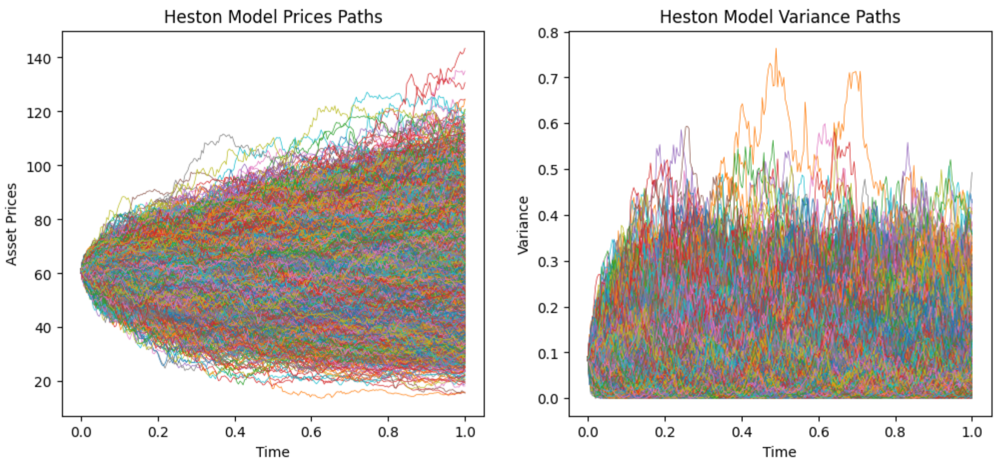

Figure 2: Price and Variance of KO under Heston Model

As a result, we obtained a price of \$67.9 for the Worst of DIP option. In combination with the quarterly coupon note of \$1,055.71, we ultimately arrived at a product price of \$987.81, which is slightly lower than the issue price of \$1000. Additionally, the barrier event occurred 1379 times out of the 10,000 simulations.

3 Risk Analytics

3.1 Risk Measures

Value-at-risk (VaR) and expected shortfall (ES) over the life of the product were computed to quantify the potential downside risks of the product and assess the riskiness of the product under different models.

To compute VaR and ES, the models were firstly changed back to $\mathbb{P}$-measure. Then, the P&L at maturity was simulated under $\mathbb{P}$-measure to derive the VaR and ES. Note that ES was calculated with equal weights on tail losses. The distribution of the simulated P&L and results of the risk measures are summarised in Figure 3 and 4, respectively.

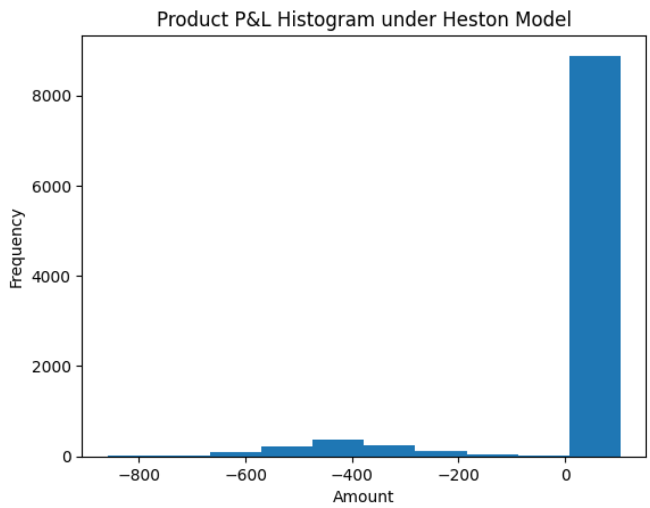

Figure 3: Simulated P&L Distribution

	Heston	Black-Scholes
95% VaR	790.58	612.49
99% VaR	842.08	631.22
97.5% ES	843.68	633.13

Figure 4: VaR and ES over the Product Life

While in 89% of scenarios, investors could receive a positive gain, this gain from the enhanced yield of the coupon note component is capped. Furthermore, the losses in the worst 1% cases could exceed 84% of the initial investment amount, incurring tremendous risks for the investors.

By introducing stochastic volatility and additional correlation structure, the volatility of the entire product was magnified. Especially when the Wiener's process in stock price dynamics took a negative value, the volatility was inclined to rise as the Wiener's processes for price and variance were negatively correlated as indicated by negative ρ in the Heston model. Therefore, the stock price tended to be more volatile when its performance was poor, which contributed to the larger VaR under the Heston model compared with the Black-Scholes model.

3.2 Market Risk

- **Prices of the Underlyings**

  The sensitivity of product price to underlying prices right after the initial fixing was calculated using the central finite difference method. Final results are presented as Dollar Delta as it represents the actual dollar value change in the product price, is more commonly used in risk management as risk exposure, and can provide more information for monitoring and hedging.

  The second-order sensitivity was also computed as Dollar Gamma to capture the curvature risk of the product with respect to underlying prices. As the Dollar Gamma is essentially a second-order derivative with high computational complexity, the estimation of it is less accurate. The low complexity of the Black-Scholes model allowed us to generate 100,000 paths to compute Dollar Gamma, while 10,000 paths were used for the others.

  Results are summarised in Table 3.

  All Dollar Delta are positive while all Dollar Gamma are negative, which is in line with our expectation as the product is embedded with a short position in down-and-in put options. No

	\$ Delta (Heston)	\$ Delta (BS)	\$ Gamma (Heston)	\$ Gamma (BS)
KO	39.30	55.35	-10013.29	-17323.50
MCD	26.90	34.51	-7318.11	-9592.44
SBUX	138.46	147.03	-40902.98	-37863.89

Table 3: Dollar Delta and Dollar Gamma

significant difference between the Heston model and the BS model was observed, supporting the robustness of the Heston model in this project. It was also noted that the product exhibits larger delta and curvature risk exposures to the Starbucks (SBUX) stock price.

- **Interest Rate**
  The interest rate risk of this product arises from both the coupon note and option components simultaneously. Rho was adopted to measure the sensitivity of the option component to interest rates, and the price value of a basis point (PVBP) for the coupon note component. Both the Rho and PVBP were computed with central finite difference. Results are presented in Table 4.

	Heston	Black-Scholes
Rho+PVBP	-0.0607	-0.0751

Table 4: Sensitivity to Interest Rates

The overall sensitivity of the product to interest rate was negative and not very large. This is also in line with our expectation, as the PVBP for the coupon note should be negative, and Rho for the short position in the put option is expected to be positive. Therefore, Rho partially offset PVBP while PVBP dominated the sign as it was larger in magnitude than Rho. Consequently, the rate hike during the life of the product might have had a negative impact on the product price.

- **Volatility**
  The study of different ways of modelling volatility is an important part of this project. Therefore, in risk analytics, we also would like to assess and compare the sensitivity of product price to the volatility of underlying assets, which is measured by Vega. Central finite difference was also employed in the Vega calculation. Results are in Table 5.

	Vega (Heston)	Vega (BS)
KO	-0.1814	-1.7027
MCD	-0.1750	-0.8649
SBUX	-2.8101	-3.4335

Table 5: Sensitivity to Volatility of the Underlyings

Although there is a noticeable discrepancy in Vega under Heston and Black-Scholes models, the order of magnitude of the Vega is consistent. KO and MCD share similar Vega, while the Vega with respect to SBUX is obviously larger. We expect the discrepancy could be partially explained by the stochastic and, more importantly, the mean-reversion nature of volatility in the Heston model. The long-term variance θ calibrated in the Heston model was lower than the initial variance v_0. Therefore, the shifted variance that remained unchanged in the Black-Scholes model throughout the simulation gradually reverted to the long-term variance in the Heston model, causing the significant discrepancy in Vega.

- **Specific Risk from Basket**
  - **Cross Gamma Effect**
    The delta of one underlying asset, acting as the Worst-Of, increases in absolute value. Concurrently, the delta of the other underlying assets decreases in absolute value. In general, the higher the probability that an underlying asset finishes as the Worst-Of, the more the greeks will be allocated to that specific underlying asset. Consequently, the option will exhibit a higher vega to the volatilities of the underlyings that perform poorly.
  - **Correlation risk**
    Cega is defined as $Cega_{(i,j)} = \frac{\partial V}{\partial \rho_{i,j}}$, and we could easily see from the portfolio variance formula $\sigma^2_{Ptf} = \sum_{1 \le i \le n} w_i^2 \sigma_i^2 + 2 \sum_{1 \le i < j \le n} w_i w_j \rho_{i,j} \sigma_i \sigma_j$ that, an increase in correlation implies an increase in the overall basket volatility. Results are in Table 6.

	Cega (Heston)	Cega (BS)
KO&MCD	0.0549	0.0680
KO&SBUX	0.0561	0.1160
MCD&SBUX	0.1971	0.0216

Table 6: Sensitivity to Correlation

3.3 Credit Risk

Credit Suisse (CS) has collapsed and been acquired by UBS. However, as the valuation date in this project is the initial fixing date of the product, we evaluated the credit risk in retrospect.

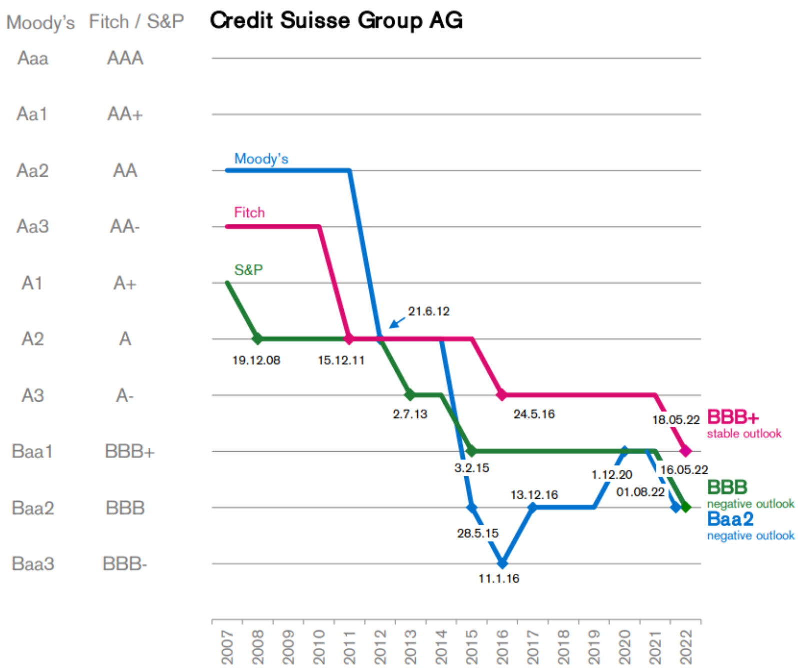

Figure 5: Credit Suisse Credit Ratings Development

It is noteworthy that the credit ratings of the CS group were lowered in mid-2022 after six years of stable ratings of A- by Fitch and BBB+ by S&P (2). A study completed by Fitch in 2022 revealed that the 1-year cumulative probability of default of a BBB global financial institution was estimated at 0.11% (3), which was very low. However, given the development of the credit ratings of the CS group, the investors should have paid attention to the fact that the ratings did not show improvement after a 6-year long period but even deteriorated in mid-2022, before the issuance of the product. Although it was definitely not a significant signal of default, it should have reminded the investors to adjust their expectations of the credit risk.

3.4 Liquidity Risk

While some issuers of reverse convertibles would try to maintain second markets for the products, these markets are still very limited. Investors should expect to be exposed to high trading liquidity risk of the product and think carefully if they would like to trade the product in the market in future.

3.5 Model Risk

Model risk arises when models are introduced for pricing and analytics. Black-Scholes, Local Volatility, and Heston models all have assumptions and limitations. Especially for complex models like the Heston model, the quality of calibration heavily depends on the initial guess, parameter constraints, weights of market observation, and optimisation techniques. Errors could also accumulate through the discretisation and simulation scheme. Therefore, comparisons between results from different models should be drawn to mitigate model risk.

3.6 Risk from a Issuer's Perspective(4)

From a sell-side perspective, the risk faced by the trader is equivalent to being long a DIP option. Delta hedging is employed to manage this risk. Usually we have a pool of assets that could possibly hedge against each other, however, here we only investigate hedging the delta of this simple DIP option.

In a long position in DIP, the trader buys stocks to hedge the short position in delta. However, when the lagging stock price breaches the 50% barrier level, the DIP option rapidly transforms from a non-option to a 50% in-the-money put, resulting in a significantly large delta change at the barrier. An example is shown in Figure 6.

As the stock price approaches the barrier level, the trader accumulates an excessive number of shares. Once the stock price breaches the barrier, the trader must sell any excess shares. However, selling shares at the barrier could further push down the share price as it is already decreasing, resulting in potential losses for the trader. Therefore, the trader may sell the excess shares below the barrier level, incurring a loss on the sale.

To compensate for this risk, the trader applies a shift to the initial barrier when pricing the DIP option. For example, the barrier could be set at (50-x)%, making the option cheaper for the trader to long. The intuition of the adjustment is that the trader has enough room to sell the excess shares without incurring losses.

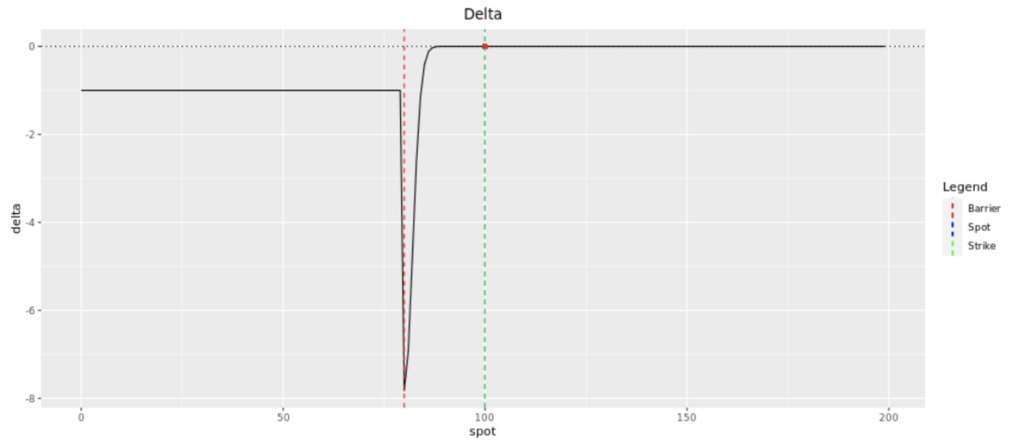

Figure 6: Delta of a 80% DIP Option

In the case of a 50% barrier, the Heston result suggests a price of \$987.71. However, by adjusting the barrier to 46%, the price is reduced to \$999.5, which brings it closer to the issue price of \$1000.

Bibliography

[1] QuantPy. (2022). Heston Model Calibration in the "Real" World with Python - S&P500 Index Options [Video]. YouTube https://www.youtube.com/watch?v=Jy4_AVEyO0w

[2] Credit Suisse. (2023). Credit Suisse credit ratings development. Retrieved from: https://www.credit-suisse.com/media/assets/about-us/docs/investor-relations/debt-investors/ratings-credit-reports/credit-suisse-ratings-history-220801.pdf

[3] Fitch Ratings. (2022). 2021 Transition and Default Studies. Retrieved from: https://assets.fitchratings.com/downloadFile?slug=corporate-finance%2F2021-transition-default-studies-31-03-2022&reportType=report&sfReport=false

[4] Maxime de Bellefroid. (2022). The Derivatives Academy. Retrieved from: https://bookdown.org/maxime_debellefroid/MyBook/