Monte Carlo Simulation in Inventory Management

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***Abstract*— This report attempted to find the optimal policy for an inventory management problem with the use of Monte Carlo simulation. With 95% confidence, the simulation limited the policy space to two policies which should be considered by the manager. Further ways to improve these results were also discussed.**

I. INTRODUCTION

Researchers have identified Mars as one planet which has suitable conditions for humans to settle. In order to find further evidence, a group of researchers have decided to travel to Mars to conduct further experiments. These scientists believe that many of the goods consumed by humans can be produced on Mars. One exception is the 'Supper Apple'. For this reason, the UN has authorised the opening of a store on Mars, it's only purpose is to sell super apples to the researchers.

The challenge for the store manager is as follows: given the probability distribution of the weekly demand for super apples, a fixed order policy must be arranged with NASA before the shop is opened. The store is open Monday-Friday and at the end of each week, the stock is counted and an order is placed according to the fixed policy. The new stock is delivered before the store reopens on Monday. To maximise profits, the policy must minimise the following operational costs. Super apples must be stored in a specially designed warehouse over the weekend. This costs 5 gold coins per weekend. If the store cannot meet the weekly demand, a penalty of 20 gold coins is issued. Finally, if any stock is left over on the last week, the store is charged 10 gold coins per apple to return them to Earth.

II. OPTIMAL POLICY PREDICTION

Given the two weekly operational costs (stock penalty and warehouse cost) differ by a factor of 4, a policy which ensures the stock penalty is minimised should take priority. The secondary consideration is to ensure the stock is not kept unnecessarily high. With this philosophy in mind, the distribution of the demand shows that $P(D \leq 4) = 0.96$ and the mean weekly demand is 3. From this information, I would expect the optimal policy to fall in the range: $3 \leq y \leq 5$ and $3 \leq r \leq 5$, where y represents the amount of stock to order if it falls below, or equal to, the threshold r. The policy $(y, r) = (4, 4)$ would make a good choice, this would ensure the chances of the demand not being met are low.

III. MONTE CARLO FORMULATION

To use MC simulation, the problem must be defined according to (1), where $x \in \Re^n$ represents a random variable with joint distribution $f(\cdot)$. The function $h(\cdot) : \Re^n \to \Re$ acts on the random variable and is parametrized by $\alpha \in \Re^m$. The expected value of the function is represented by $\theta \in \Re$.

$$\theta = E_{f(x)}[h(x; \alpha)] \tag{1}$$

To apply this model to the inventory management problem, the demand for each of the 52 weeks was represented by $x \in D^{52}$. The set D is defined by the set of values the demand can take. The joint distribution of the demand was represented by $f(x)$, where each weekly demand was independently and identically distributed (i.i.d). The total operational cost, over the 52 weeks, was represented by $h(x; \alpha)$ where α represented the parameters of the policy (y, r).

The dynamics of the simulation were represented by (2). The stock delivered over the weekend, before the start of week i, was denoted by y_{i-1}. The stock at the beginning of week i was represented by S_i^B, the stock at the end of week i was represented by S_i^E, and x_i represented the demand for that week. The fixed policy which determined how the stock was updated is defined by (3), where $y, r \in \mathbb{Z}$.

The operational cost, over the 52 weeks, was represented by (4). The weekly warehouse cost, stock penalty and return cost was respectively represented by w_i, p_i and r_i, each defined in the appendix.

$$\begin{aligned} S_i^B &= S_{i-1}^E + y_{i-1} \\ S_i^E &= S_i^B - x_i \end{aligned} \tag{2}$$

$$y_i = \begin{cases} y & S_i^E \leq r \\ 0 & otherwise \end{cases} \tag{3}$$

$$h(x; y, r) = \sum_{i=1}^{52} w_i + p_i + r_i \tag{4}$$

IV. SIMULATIONS

MC simulation samples from the joint distribution of the demand and uses the sample mean to approximate θ; thus, for a given policy (y, r), the expected operational cost can be approximated. For these experiments, 500 samples were drawn from the joint distribution of the demand. The first experiment simulated the expected operational cost for the predicted optimal policy, $(y, r) = (4, 4)$. The second experiment simulated the expected operational cost for the range of policies in the interval $1 \leq y \leq 6$ and $1 \leq r \leq 6$. This range covered the full combination of policies with a maximum stock update of 6.

V. RESULTS

The results of the first MC simulation, for the predicted optimal policy, are illustrated by Fig. 1. The operational cost ranged between 245 and 355 gold coins. It had a sample mean of 293.88 gold coins and a sample variance of 337.05, which corresponded to a standard deviation of 18.36 gold coins. The results of the second MC simulation are illustrated by Fig. 2. The optimal policy was $(y,r) = (4,3)$, which had a sample mean of 280.73 gold coins. The operational cost ranged between 220 and 390 gold coins with a sample variance of 487.53, this corresponded to a standard deviation of 22.08 gold coins.

VI. ANALYSIS

The bounds of the confidence interval (CI) with confidence level β were calculated from (5). The sample mean, of the expected operational cost under policy (y,r), was represented by $\hat{\theta}_N$. The corresponding bounds for the standardised normal distribution were given by $z_{\beta/2}$, whilst $\hat{\sigma}_N$ represented the sample standard deviation and N represented the number of samples.

$$P(\hat{\theta}_N - z_{\beta/2}\frac{\hat{\sigma}_N}{\sqrt{N}} \leq \theta_{(y,r)} \leq \hat{\theta}_N + z_{\beta/2}\frac{\hat{\sigma}_N}{\sqrt{N}}) = \beta \qquad (5)$$

For a confidence level of 95%, the bounds for the standardised normal distribution were ± 2. Table 1 shows the lower and upper bounds of the three best performing policies. The CI of the two best performing policies showed that with 95% certainty, the true expected operational cost of policy $(4,3)$ lied within a subset of the lowest costs that the true expected operational cost of policy $(4,2)$. The third best performing policy did not overlap with the policy $(4,3)$. This suggested, one of $(4,3)$ or $(4,2)$ must be optimal, but since the CI of policy $(4,3)$ was narrower than that of $(4,2)$, the sample mean was more informative. Given this information, the optimal policy, according to the MC simulation, was still most likely to be $(4,3)$.

The confidence level could be increased, but this would just widen the interval which the true mean lied in. It would be more useful to maintain the confidence level and reduce the width of the CI. This would help to separate the CI of the two best policies. As shown by 5, the CI could be reduced by decreasing the sample standard deviation $\hat{\sigma}_N$ or increasing the number of samples N (which in turn decreases the sample standard deviation). Run length control (RLC), defined by (6), is used to determine the number of samples needed, M, to give the desired estimation accuracy, ε. When 500 samples were used, the estimation accuracy was 1.97. Doubling the number of samples, without any improvement to the sample standard deviation would result in an improved estimation accuracy of 1.39, which is a 30% reduction in estimation error. Alternatively, if increasing the number of samples is not an option, variance reduction methods such as importance sampling could be employed.

$$M = (\frac{z_{\beta/2}\hat{\sigma}_N}{\varepsilon})^2 \qquad (6)$$

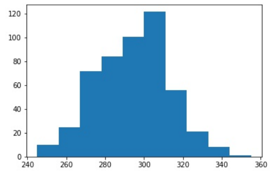

Fig. 1. Histogram of operational cost for policy $(y,r) = (4,4)$

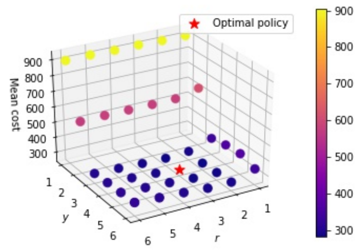

Fig. 2. Distribution of mean operational cost for range of policies

TABLE I

CI FOR THE THREE BEST PERFORMING POLICIES

Policy (y, r)	Lower bound	Upper bound	Sample mean
(4, 3)	278.75	282.70	280.73
(4, 2)	278.76	284.08	281.42
(3, 3)	283.81	287.77	285.79

VII. CONCLUSION

The predicted optimal policy, $(4,4)$, ranked 6^{th} in terms of minimum expected operational cost. However, it is worth noting that the preceding 5 policies all had thresholds in the range 3 to 6. This suggests adjusting the philosophy behind the prediction to ensure the stock rarely runs out whilst further reducing excessive stock produced better results.

With a confidence of 95%, the range of potential polices was reduced to a list of two optimally performing ones. When a confidence of 99% was used, the CI of a larger number of policies overlapped which did not aid the decision making process. This could be improved by increasing the number of samples and/or employing a variance reduction method which would result in narrower CI. Importance sampling would be an easy method to consider as the implementation would be the same as in these experiments. The difference being a different function is to be estimated and a different probability distribution is to be sampled from. The challenge of this approach comes with finding a suitable distribution to sample from. This becomes more challenging when considering a practical problem as it is unlikely the probability distribution of the random variable will be known. This would need to be inferred which would further widen the CI.

APPENDIX

$$w_i = \begin{cases} 5 & S_i^E > 0 \\ 0 & otherwise \end{cases} \tag{7}$$

$$p_i = \begin{cases} 20 & x_i > S_i^B \\ 0 & otherwise \end{cases} \tag{8}$$

$$r_i = \begin{cases} 10S_i^E & i = 52 \\ 0 & otherwise \end{cases} \tag{9}$$