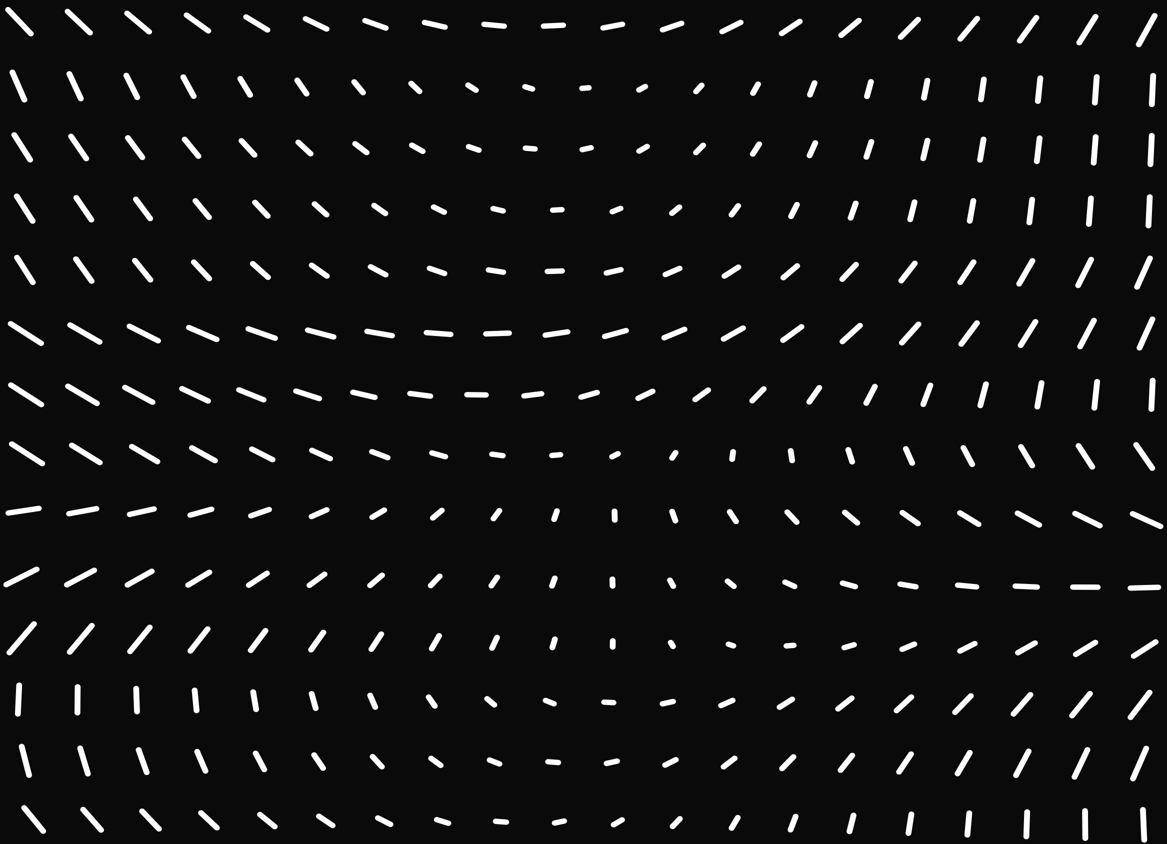
Upg 13



13. ** *Prove the following statements for all functions F and G that can be expressed as a power series in their arguments:*

$$[x_i, G(\mathbf{p})] = i\hbar\frac{\partial G}{\partial p_i}, \quad [p_i, F(\mathbf{x})] = -i\hbar\frac{\partial F}{\partial x_i} \tag{7}$$

We will use these identities when we discuss Ehrenfest's theorem.

$$[X_i, G(P)] = i\hbar \frac{\partial G}{\partial p_i}$$

$$X_i G(P) - G(P) X_i$$

X_i in momentum space

is represented by $-\frac{\hbar}{i}\frac{\partial}{\partial p_i}$

We will use a test function

$\psi(x)$ to evaluate

$$[X_i, G(P)] \longrightarrow$$

$$[x_i, G(p)]\psi$$

$$= (x_i G(p) - x_i G(p))\psi =$$

$$\left[x_i = i\hbar \frac{d}{dp_i} \right]$$

$$\rightarrow i\hbar \frac{d}{dp_i}\left(G(p)\psi\right) - i\hbar G(p) \frac{d}{dp_i}\psi$$

from product rule of derivatives:

$$i\hbar \frac{d}{dp_i} G(p)\psi + i\hbar G(p) \cdot \frac{d}{dp_i}\psi - i\hbar G(p) \frac{d}{dp_i}\psi$$

$$= i\hbar \frac{d}{dp_i} G(p)\psi$$

$$\longrightarrow [x_i, G(p)] = i\hbar \frac{d}{dp_i} G(p)$$

We know evaluate

$[p_i, F(\mathbb{X})]$ with test function $\psi(\mathbb{X})$ and

$p_i = -i\hbar \frac{\partial}{\partial x_i}$ in position space

$[p_i, F(\mathbb{X})]\psi(\mathbb{X}) =$

$= p_i F(\mathbb{X})\psi(\mathbb{X}) - F(\mathbb{X}) p_i (\mathbb{X})\psi(\mathbb{X}) =$

$-i\hbar\left(\frac{\partial}{\partial x_i}\left(F(\mathbb{X})\psi(\mathbb{X})\right) - F(\mathbb{X})\frac{\partial}{\partial x_i}\psi(\mathbb{X})\right) =$

$= \left[\text{Product rule of derivatives}\right] =$

$= -i\hbar\left(\frac{\partial}{\partial x_i} F \cdot \psi + F \frac{\partial}{\partial x_i}\psi - F\frac{\partial}{\partial x_i}\psi\right)$

$$= -i\hbar \frac{\partial}{\partial x_i} F \psi = [P_i , F(\mathbf{x})] \psi$$

$$\rightarrow [P_i , F(\mathbf{x})] = -i\hbar \frac{\partial}{\partial x_i} F \quad \square$$

Thus we have proven the two identifires.