

$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m} + \sum_{i=0}^N \left[\frac{K}{2} (u_{i+1} - u_i)^2 + \frac{\alpha}{3} (u_{i+1} - u_i)^3 \right]$$

$$\frac{dp}{dt} = - \frac{\partial \mathcal{H}}{\partial q_i}$$

$$\frac{\partial}{\partial q_i} \mathcal{H} = \frac{\partial}{\partial q_i} \left(\sum_{i=0}^N \left[\frac{K}{2} (u_{i+1} - u_i)^2 + \frac{\alpha}{3} (u_{i+1} - u_i)^3 \right] \right)$$

$$= \frac{\partial}{\partial q_i} \left(\frac{K}{2} \left((u_{i+1} - u_i)^2 + (u_i - u_{i-1})^2 \right) + \frac{\alpha}{3} \left((u_{i+1} - u_i)^3 + (u_i - u_{i-1})^3 \right) \right)$$

$$= K \left((u_{i+1} - u_i) + (u_i - u_{i-1}) \right) + \alpha \left((u_{i+1} - u_i)^2 + (u_i - u_{i-1})^2 \right)$$

$$\Rightarrow \dot{p} = ma = m\ddot{u}$$

$$\Rightarrow m\ddot{u}_i = K(x_{i+1} + x_{i-1} - 2x_i) \left[1 + \alpha(x_{i+1} - x_{i-1}) \right]$$