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1) $$f(x) = \frac{3x^2 + x - 1}{x^2 - x + 1}$$

$$D(f) = \mathbb{R} \Rightarrow f'(x) = \frac{3 \cdot 2x + 1}{2x - 1}$$

$$f'(x) = \frac{(3 \cdot 2x + 1) \cdot (x^2 - x + 1) - (3x^2 + x - 1) \cdot (2x - 1)}{(x^2 - x + 1)^2}$$

$$f'(x) = \frac{\cancel{6x^3} - 5x^2 + 5x + \cancel{1} - \cancel{6x^3} + x^2 + 3x - \cancel{1}}{(x^2 - x + 1)^2}$$

$$f'(x) = \frac{-4x^2 + 8x}{(x^2 - x + 1)^2}$$

sendo

$D(f') \; \mathbb{R}$ logo $f'(x) = 0$

$$\therefore \frac{-4x^2 + 8x}{(x^2 - x + 1)^2} \Rightarrow -4x^2 + 8x = 0 \cdot (x^2 - x + 1)^2$$

$$-4x^2 + 8x = 0 \; /(-4)$$

$$x^2 - 2x = 0$$

somando por $(-1)^2$:

(I)
$$x^2 - 2x + (-1)^2 = (-1)^2 + 0$$
$$x^2 - 2x + (-1)^2 = 1$$
$$(x-1)^2 = 1$$
$$\sqrt{(x-1)^2} = \pm\sqrt{1}$$
$$x_1 - 1 = \sqrt{1}$$
$$\boxed{x_1 = 2 \text{ (ponto máx)}}$$

(II)
$$x_2 - 1 = -\sqrt{1}$$
$$x_2 - 1 = -1$$
$$x_2 = 0 \text{ (ponto mínimo)}$$

* graficamente:

+ + + + (0) - - - - (2) + + + +

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2) I $V = 2000\pi \ dm^3$

$$V(r,h) = \pi r^2 \cdot h \Rightarrow \cancel{\pi} r^2 \cdot h = 2000\cancel{\pi}$$

$$h = \frac{2000}{r^2} \Rightarrow h = \frac{2000}{10^2} \Rightarrow \boxed{h = 20\,dm}$$

II $A(r,h) = 2\pi r^2 + 2\pi r \cdot h$

$$A(r,h) = 2\pi(r^2 + r \cdot h)$$

$$A(r,h) = 2\pi\left(r^2 + r \cdot \frac{2000}{r^2}\right)$$

$$A(r) = 2\pi \cdot \left(r^2 + \frac{2000}{r}\right)$$

$$\therefore \boxed{A(r) = 2\pi \cdot \left(\frac{r^3 + 2000}{r}\right)}$$

III $A'(r) = 2\pi \cdot \left(\frac{3r^2 \cdot r - (r^3 + 2000) \cdot 1}{r^2}\right)$

$$A'(r) = 2\pi \cdot \left(\frac{3r^3 - r^3 - 2000}{r^2}\right)$$

$$A'(r) = 2\pi \cdot \left(\frac{2r^3 - 2000}{r^2}\right) \quad \therefore A'(r) = 0$$

$$\therefore 2r^3 - 2000 = 0 \Rightarrow r^3 = 1000 \Rightarrow \boxed{r = 10\,dm}$$

IV Mínimo:

$$A_{mín} = A(r) = 2\pi \cdot \left(\frac{10^3 + 2000}{10}\right)$$

$$\therefore A_{mín} = 2\pi \cdot \left(\frac{1000 + 2000}{10}\right) \Rightarrow \boxed{A_{mín} = 600\pi \ dm^2}$$