Horton–Strahler number and centrality measures

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Abstract

Cristian: Agregar un abstract acá.

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1 Introduction

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2 Preliminaries

Graphs. A graph is a pair $G = (V, E)$ where V is a finite set and $E \subseteq V \times V$ is a finite relation. Each pair of vertex let be (v_1, v_2) , $v_1, v_2 \in V$ this set of vertex its called edges and can be directed or not, so the edges $\in E$. A path in a graph is a sequence of non-repeated nodes connected through edges present in a graph, if there exist a path between two vertex, this vertex are called connected. An example, let be $x, y \in V$ a path can be expressed so $\{x, x_1, x_2, \cdots, y_1, y_2, \cdots, y_n\}$ with $x, x_i, y_i, y \in V$

Trees. A tree is an undirected graph in which any two vertices are connected by only and only one path. A rooted tree is refereed as a tree with a vertex who serves as the "root" of the tree, being a references to the others vertices in the tree. A neighborhood of a vertex v is denoted as $N_G(v)$, and contains all vertex (u) if $u, v \in V$ then $u \in N_G(v)$.

Centrality. The centrality of a graph is defined as a assignation of a number to nodes within a graph corresponding to their network position, the application of a centrality to a graph indicates the importance of the vertex in the graph. There exist a different number of centrality measures, so the importance of a vertex changes from centrality measure to another, also, a centrality measure has to be any function $C : VG \rightarrow R$ that assigns a score $C(v, G)$ to v depending on its graph G. An example of a centrality measure would be the one who was previously defined in the paper associated, such as, F-subgraph centrality (denoted by $C_F(v, G)$):

$$C_F(v, G) := \log(|F(v, G)|) \quad (1)$$

Cristian: Agrega estas referencias [2] y [3]. También agrega esta sobre Strahler number [1] y esta otra [4]

Strahler number. In mathematics, the Strahler number or Horton–Strahler number of a mathematical tree is a numerical measure of its branching complexity. One may assign a Strahler number to all nodes of a tree, in bottom-up order, as follows:
If the node is a leaf (has no children), its Strahler number is one.
If the node has one child with Strahler number i, and all other children have Strahler numbers less than i, then the Strahler number of the node is i again.

If the node has two or more children with Strahler number i, and no children with greater number, then the Strahler number of the node is i + 1.

3 Main results

We can observe that the Strahler number is similar to a centrality measure, in a sense which, is a function who takes a node an assigns a score (with certain conditions), that's why the co-relation between Strahler number is visible. We can define the centrality measure using Strahler number in which sense, a higher Strahler number means higher centrality, it's important to remember that a potential function tell us if a centrality measure is able to root trees, in other words, if a centrality measure admits a potential function, it will roots trees if and only the potential function is simetric (not necessarily works backwards). Its valid to inquire if this centrality roots trees, we can approach this by finding a a potential function such as:

$$f_{S_N}(v,T) = \begin{cases} 1 & \text{If v is a leaf} \\ \\ i & \text{If all childs } (\forall u) \text{ of v have } f_{S_N}(u,T) = i \\ \\ i+1 & \text{If at least one child } u^* \text{ have } f_{S_N}(u^*,T) = i+1 \\ & \text{and all the others child have at most i} \end{cases} \tag{2}$$

We can see now if this function is symmetric knowing that the concept of the Strahler number, is to represent a directed graph and measure the centrality to different contexts. We can make a counter-example for this and observe that if we consider:

Here goes the contra-example

Knowing that this admits a potential function f and f is not symmetrical, by theorem it will not root trees, in other words, we can not find the root of tree in all cases with this centrality. If this centrality does not roots trees we can explore other things like if this potential function in a monoids version, such thing can be drafted like:

$$(\mathbb{R}_{\geq 1}, \max, 1)\ l(x) = x + 1 \tag{3}$$

If this centrality can not root trees this means, we can not find a root for a tree T, which can be a problem in diferent aplications.

Possible Aplications

Knowing this centrality does not root tress, in the problem of finding the root of a vector river in such way that, we can not find the **origin** of the river in all cases using this centrality.

4 Conclusions

References

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