Part II: Time series prediction

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Prediction

Prediction

- Given a history $\{\ldots, x_{t-4}, x_{t-3}, x_{t-2}, x_{t-1}, x_t\}$, what is the future $\{x_{t+1}, x_{t+2}, x_{t+3}, \ldots\}$?
- And, how good are these predictions?
- 5, -3, 2, -1, 1, 0, 1, 1, 2, 3 — what is next?
- Build $F(\ldots, x_{t-4}, x_{t-3}, x_{t-2}, x_{t-1}, x_t) \approx x_{t+1}$

Discussion

When would you need prediction in your industry? Prediction of *what*?

Linear autoregressive models

- $F(x_{t-d+1}, x_{t-d+2}, x_{t-1}) = a_1 x_{t-1} + a_2 x_{t-2} + \ldots a_d x_{t-d} \approx x_{t+1}$
- *or* $F(X) = X.A \approx y$ where X is a matrix and A and y are vectors:

$$\begin{bmatrix} x_1 & x_2 & \ldots & x_d \\ x_2 & x_3 & \ldots & x_{d+1} \\ \vdots & \vdots & & \vdots \\ x_{n-d+1} & x_{n-d+2} & \ldots & x_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_d \end{bmatrix} \approx \begin{bmatrix} x_{d+1} \\ x_{d+2} \\ \vdots \\ x_{n+1} \end{bmatrix}$$

- I.x. $\hat{A} = X^{-1} y$
- Only question that remains is how to choose d (and whether models like this are useful at all)

Local modelling — Weather forecasting

1. Construct a library of past observations and the past-futures of those past observations. (I.e. at time x_t the corresponding future was f_t).
2. For each new observation x find past similar observations and use their futures to predict the true future
   `(last time it was cloudy, cold and windy it also rained)`

- Also called *machine learning/deep learning/artificial intelligence*.
- Generalisation involve building more complex (or probabilistic) predictions by combining multiple similar points in the past

Nonlinear global models

1. let $v_t = (x_t, x_{t-\ell_1}, x_{t-\ell_2}, x_{t-\ell_3}, \ldots x_{t-\ell_d}$
2. choose nonlinear functions ϕ_i of the data
3. build a nonlinear $F(x_t, x_{t-1}, x_{t-2}, \ldots) = \sum_{i=1}^{M} \lambda_i \phi_i(v_t)$
4. then this can be written as matrices $\Phi\Lambda = y$ and $\Lambda = \Phi^{-1}y$
5. *but* we now need to make good choices of ϕ_i and choose M

Pythonification

Exercises

- Do this.
- Which prediction schemes work best?
- When and why?

References

Sources

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