Part III: Networks

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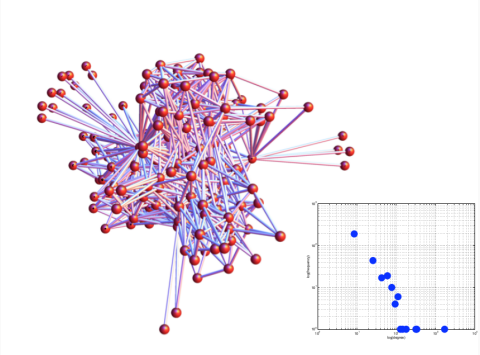
Discussion

Where do you have networks in your business?

Network Quantification

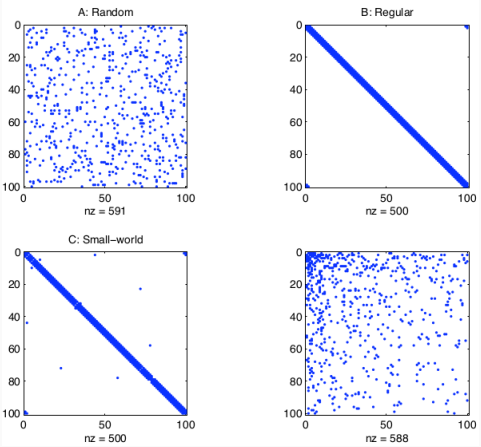
Quantifying Topology of Networks

- Degree sequence
- Degree histogram
- Path-length
- Clustering
- Assortativity
- Betweenness Centrality
- Eigenvalue Centralitiy
- Modularity and Communities
- Hubs and richclubs
- Robustness and fragility
- Motifs and superfamily

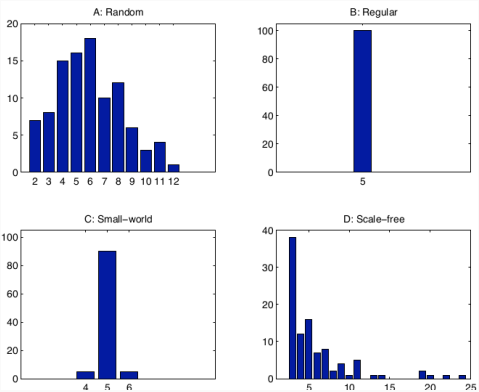


Adjacency matrix

Let A be a binary $N \times N$ matrix with $a_{ij} = (A)_{ij} := 1$ iff there is a link from node-i to node-j (otherwise $a_{ij} = 0$). Generalise to weighted networks with $a_{ij} = w_{ij}$ is the weight, and directed networks with an asymmetric A.
The graph Laplacian L is defined so that $L = A$ except for the diagonal: $(L)_{ii} = -\sum_{j \neq i} a_{ij}$.

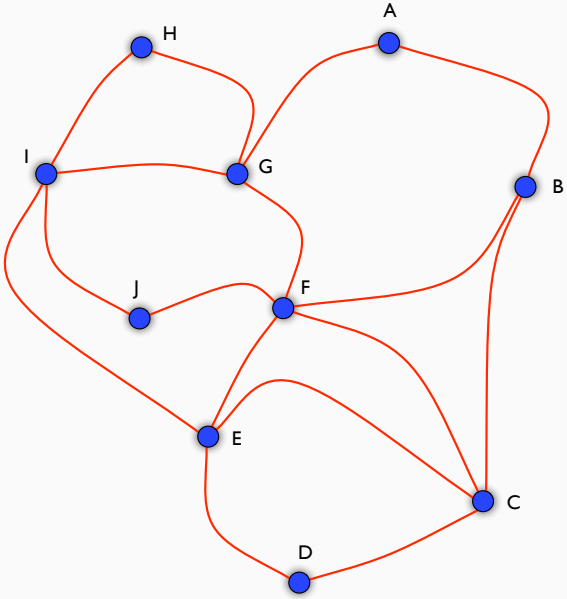


Degree distribution

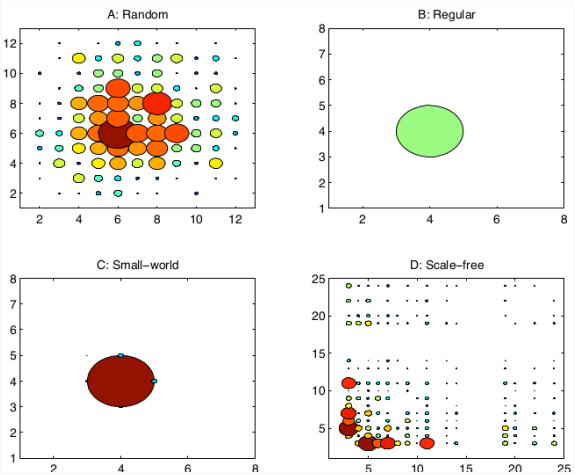


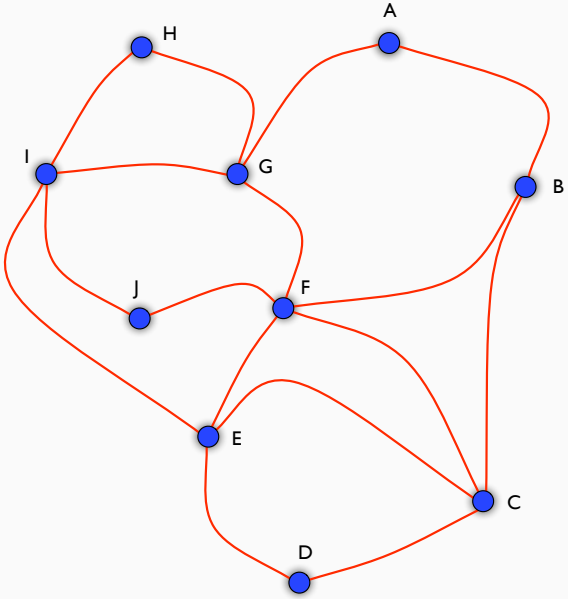
Compute the number of nodes n_k (or the probability p_k) with degree k.

- *Path-length:* The path-length is the shortest path (number of edges traversed) between two nodes.
- *Diameter:* The maximum path-length

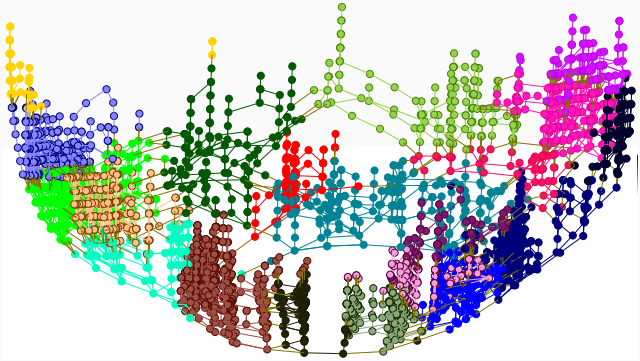


- *Clustering:* The number of triangles — the probability of neighbours being neighbours.
- *Assortativity:* The linear (Pearson) correlation between pairs of nodes with a give property





- *Betweenness Centrality:* The number (fraction) of shorter paths passing through a given node
- *Eigenvalue Centrality:* (AKA: Google's PageRank) Eigenvalue decomposition of the Laplacian



- *Communities* should have more links between members than between communities.
- *Modularity* Q measures this:

$$Q = \frac{1}{2m} \sum_{i,j} \left(a_{ij} - \frac{k_i k_j}{2m} \right) \delta(i,j).$$

- *Hubs* are nodes with the highest degree
- *Rich-club* is the connection between the hub nodes and the tendency of hub nodes to be connected to one another
- *Giant component:* The property that most of the nodes are connected (directly or indirectly) to one another.
- *Robustness:* The ability of a network to maintain it's giant component even after random removal of a relatively large number of edges (or, equivalently, nodes).
- *Fragility:* The corresponding inability of a network to maintain the rich club under targeted removal of edges (or nodes)
- *Motifs:* The structure of interconnection in sub-graphs of particular size k.
- *Super-familiy:* The relative frequency of all such sub-graphs for fixed k.

Pythonification

Discussion

Why would *you* need to know any of this stuff for your business?

Discussion

Why would *you* need to know any of this stuff for your business?

Exercise

Everything we've discussed here is implemented and easily usable within the `networkx` package. Refer to the notbook

References

Sources

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