Intersection Priority Auctions With Sequencing

Carlin Liao and Stephen D. Boyles

2020 INFORMS Annual Meeting

This writeup serves to supplement my INFORMS 2020 presentation on intersection priority auctions with sequencing. Due to time constraints, the presentation focuses on the vehicle sequencing portions of the auction mechanism, whereas this write gives a full description of the entire mechanism.

If you have any questions, contact me at carlin.liao@utexas.edu.

1 Desiderata

To estimate the potential impacts of efficient intersection auctions, our mechanism is designed to

- support intersections of any configuration and any turn restrictions,
- take into account vehicles with heterogeneous characteristics,
- support realistic and varied vehicle kinematics in the 2D plane,
- be reasonably optimal and strategy-proof,
- be computationally tractable in real-time, and
- ensure, as much as reasonably possible given the above constraints, to not unduly advantage any eligible set of vehicles.

2 Priority auction basics

At an intersection, individual vehicles have the self-interested goal of moving through the intersection as fast as possible. This is complicated by the vehicles in front of them in their incoming lane, as well as vehicles coming into the intersection from opposing incoming lanes. This makes the scarce item being bid on time in the intersection conflict area; all incoming lanes are competing for use of this area as soon as possible.

To decide which lane or lanes get to enter the intersection, we run an intersection priority auction. All vehicles in each incoming lane bid for the right to have their leading vehicle(s) move through the intersection conflict area. Because different movements take different amounts of time, the bids are the values of time of each vehicle so they can be used to quantify the value of any movement for any vehicle.

3 Required information

To run the auction, we need to have or assume information about the intersection and the vehicles entering the intersection. To this end, we define an intersection as consisting of

- a conflict area,
- incoming lanes,
- outgoing lanes,
- with each incoming lane is connected to one *or more* outgoing lanes,
- whose IO connections have fixed trajectories.

For each incoming lane ℓ,

- demand per lane is Poisson(μ_ℓ)
- average value of time (VOT) $\overline{\nu}_\ell$

Vehicles are assumed to be

- fully autonomous
- self-interested

and have

- variable length
- variable width
- variable approach speeds
- fixed acceleration rate
- fixed braking rate
- fixed VOT ν

4 Bid calculation

Recall that all vehicles in a lane would benefit if their leading vehicle(s) proceed through the intersection. Thus, all vehicles in an incoming lane contribute to the bid for time in the intersection, making the total bid for the movement of the leader of lane ℓ, b_ℓ,

$$b_\ell = \nu_\ell = \sum_{i \in \ell} \nu_i,$$

where ν_i is the VOT of vehicle i. This is also equal to the effective VOT of the entire lane ν_ℓ.

If we want to let multiple vehicles from a lane move through the intersection at once (more on this in the next section), we need to calculate the effective bid of all vehicles in the lane as well. As you progress down the lane, the ith vehicle in a sequence has a supporting VOT from trailing vehicles of

$$b_i = \sum_{i}^{n_\ell} \nu_i$$

i.e., the bid decreases as you progress in the sequence because the vehicles in front are no longer contributing to the bid. That makes the effective bid of the sequence up to vehicle i

$$b_\ell^i = \frac{\sum_{k=1}^{i} b_k t_k}{\sum_{k=1}^{i} t_k}$$

where t_k is the marginal additional time needed for the kth vehicle to exit the intersection. Notice that $b_\ell^1 > b_\ell^2 > \ldots$; in other words, the longer a sequence, the lower its effective bid, just like with their supporting VOTs.

5 Deciding the winner(s)

The winners are the lanes whose leaders have the highest sum bid of nonconflicting movements. By allowing multiple lanes' leaders to move at once, we can make more efficient use of the limited time available in the conflict region.

We can further increase throughput to approach that observed by more traditional autonomous intersection control schemes by allowing multiple vehicles in each lane to proceed at once. However, simulating conflicts is very computationally expensive, so we limit it to sequences of vehicles in a lane with the same movement, so we don't need to simulate mix and matched sequences of movements with different start times.

A second major caveat about sequences is that, from the perspective of competing lanes, letting a sequence of vehicles win is strictly worse for them than only the first vehicle winning; more consecutive vehicles coming through means more time until the competing lanes' next chance to win an auction. Further recall that the effective bid of each vehicle in a sequence decreases the longer the sequence is. For these reasons, sequences will only be allowed to win an auction if their lane would win the auction without sequencing and if replacing the VOT of the lane's unsequenced movement with the lower VOT of the sequenced movement still wins the auction.

6 Payment calculation

To determine an individual vehicle's payment for an individual auction, we approximate the externality of its victory, as in Vickrey-Clarke-Groves (VCG) auctions, and make that its payment for winning the auction. This serves to better avoid incentivizing unintended behavior and encourage truthful reporting from vehicles of their value of time.

The externality of a winning vehicle i is the difference between the total value of the current scenario and if i wasn't in the auction. To find this, we consider 3 sets of lanes:

- the actual winners Ω;
- the first losers E, the lanes that would have won; and
- everyone else $L \setminus (E \cup \Omega)$; the lanes that lose the auction either way.

Recall that we've assumed that for each for each lane ℓ, new arrivals

1. enter at an average rate per time unit of $\overline{\mu}_\ell$ and
2. have an average VOT $\overline{\nu}_\ell$.

By winning, i

1. forces every first loser in E to wait the winning movement time t_ρ and suffer new bidders,
2. benefits every actual winner Ω by saving them the first loser's movement time t_E and from the arrival of new bidders, and
3. causes every other lane $L \setminus (E \cup \Omega)$ to wait the difference $t_\rho - t_E$ longer (or skip this time, if the difference is negative).

Thus the estimated externality χ_i of vehicle i is

$$\chi_i = \sum_{\epsilon \in E} \left(\nu_\epsilon + \overline{\nu}_\epsilon \overline{\mu}_\epsilon \frac{t_\rho}{2} \right) t_\rho - \sum_{\omega \in \Omega \setminus i} \left(\nu_\omega + \overline{\nu}_\omega \overline{\mu}_\omega \frac{t_E}{2} \right) t_E$$
$$+ \sum_{\ell \in L \setminus (E \cup \Omega)} \left(\nu_\ell + \overline{\nu}_\ell \overline{\mu}_\ell \frac{t_\rho - t_E}{2} \right) (t_\rho - t_E)$$

Note that χ_i can be negative if $t_\rho < t_E$ because vehicles aside from the first losers benefit if a short movement wins instead of a long one. This can cause unintended consequences, as we don't want vehicles to be able to get paid by winning a priority auction or this would incentivize a lot of induced vehicle trips aiming to farm the system for payouts, but discarding negative externalities would effectively discount the cost of longer movements in relative terms.

To approximately reconcile these competing priorities, we instruct the intersection to only collect payments after the vehicle has experienced all of its auctions in the intersection and exited. At this point, we'll sum its payments across all its auctions and then floor them so i pays

$$p_i = \max\left\{0, \sum_{\alpha \in \mathcal{A}} \chi_i^\alpha\right\}$$

where $\mathcal{A}$ is the set of all auctions i won and χ_i^α the externality of i winning auction α. This aggregation reduces the skew of flooring on the auction mechanism.

As a final note, to be clear, our auction design is not a proper VCG auction and the externality calculation we use is an estimate, as it requires removing a winning vehicle and rerunning the auction without it. This is not a realistic scenario due to the physical configuration of an intersection, but it suffices for the purposes of our mechanism.