Esercizio 2.
Determinare le soluzioni del seguente sistema di equazioni congruenziali

$$\begin{cases} 12387^{8525}x \equiv 1(10) \\ 13x + 7 \equiv 0(12) \end{cases}$$

Soluzione.

$$\begin{cases} 12387^{8525} x \equiv 1(10) \\ 13x + 7 \equiv 0(12) \end{cases} \qquad a^{\phi(n)} \equiv 1(n)$$

$\phi(10)$ $\qquad 10 \qquad 5 \cdot 2$

$\phi(10) = 5 - 5^0 \cdot 2 - 2^0 = 4$

$$\begin{cases} \left(\cancel{\left(12387^4\right)^{2131}} \cdot 12387\right) x \equiv 1(10) \\ 13x + 7 \equiv 0(12) \end{cases}$$

$$\begin{cases} 12387x \equiv 1(10) \\ 13x \equiv -7(12) \end{cases} \rightarrow \begin{cases} x \equiv 3(10) \\ x \equiv 5(12) \end{cases}$$

$$\begin{cases} x \equiv 3(5) \\ \cancel{x \equiv 3(2)} \\ x \equiv 5(4) \\ x \equiv 5(3) \end{cases} \rightarrow \begin{cases} x \equiv 3(5) \\ x \equiv 1(4) \\ x \equiv 2(3) \end{cases}$$

$$\begin{cases} x \equiv 3(5) \\ x \equiv 1(4) \\ x \equiv 2(3) \end{cases}$$

$x = 5t + 3$

$5t \equiv +2(4)$

$t = 4u + 2$

$x = 20u + 13$

$20u + 13 = 2(3)$

$20u = 1(3)$

$u = 2(3)$

$u = 3w + 2$

$x = 60w + 40 + 13 \rightsquigarrow x \equiv 53(60).$

3. Risolvere, se possibile, il sistema

$$\begin{cases} \cancel{3}^{\,1}x \equiv \cancel{9}^{\,3} \bmod \cancel{21}^{\,7} \\ 2x \equiv 3 \bmod 5 \end{cases}$$

$$\begin{cases} x \equiv 3 \bmod (7) \\ 2x \equiv 3 \bmod (5) \end{cases} \rightsquigarrow \begin{cases} x \equiv 3 \bmod (7) \\ x \equiv 4 \bmod (5) \end{cases}$$

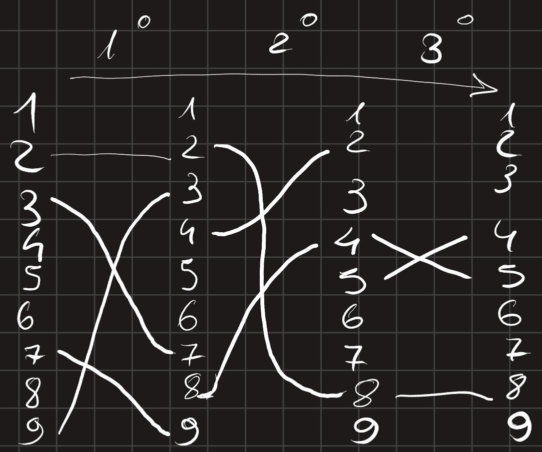
$$x = 7t + 3$$
$$7t + 3 \equiv 4 \bmod (5)$$
$$7t \equiv 1 \bmod (5)$$
$$t \equiv 3 \bmod (5)$$
$$t = 5u + 3$$
$$x = 35u + 21 + 3$$
$$x = 35u + 24 \rightsquigarrow x \equiv 24 \bmod 35$$

Esercizio 4. Sia $\sigma = (13564) \in S_9$. Sia $\tau = (45)(842)(793) \in S_9$.
Determinare σ^{-1} e τ^{-1}.
Determinare $\tau\sigma\tau^{-1}$.
Determinare (se esiste) $\tau \in S_9$ tale che $\beta = \tau\alpha\tau^{-1}$ con:

- $\alpha = (4657)(98123)$ $\beta = (5746)(123)(89)$:
- $\alpha = (1357)$ $\beta = (2468)$

$$\sigma = (13564) \in S_9$$
$$\tau = (45)(842)(793) \in S_9$$
$$\sigma^{-1} = (14653)$$
$$\tau =$$

$$\nexists\, \tau \in S_9 \mid \beta = \tau\alpha\tau^{-1}$$



$$\tau = (12)(34)(56)(78)$$
$$\tau$$

$$\tau = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 8 & 7 & 2 & 4 & 6 & 9 & 5 & 3 \end{bmatrix}$$

$$\tau = (2854)(379)$$

$$\tau^{-1} = (2\ 4\ 5\ 8)(3\ 9\ 7) \qquad \sigma = (1\ 4\ 3\ 2)(7\ 9)$$

$$\sigma = (1\ 3\ 5\ 6\ 4) \qquad \beta = (1\ 4)(1\ 2)(8\ 7)(5\ 3)$$

$$\tau\sigma\tau^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 8 & 7 & 2 & 4 & 6 & 9 & 5 & 3 \end{bmatrix} \circ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 2 & 5 & 1 & 6 & 4 & 7 & 8 & 9 \end{bmatrix} \circ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 9 & 5 & 8 & 6 & 3 & 2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 3 & 6 & 5 & 2 & 4 & 8 & 9 \end{bmatrix} \qquad \tau\sigma\tau^{-1} = (1\ 7\ 4\ 6\ 2)$$

$$\tau\sigma\tau^{-1} = (1\ 7\ 4\ 6\ 2).$$

$$\alpha = (4\ 6\ 5\ 7)(9\ 8\ 1\ 2\ 3)$$

$$\beta = (5\ 7\ 4\ 6)(1\ 2\ 3)(8\ 9)$$

$$\alpha = (4\ 6)(4\ 5)(4\ 7)(9\ 8)(9\ 1)(9\ 2)(9\ 3)$$

$$\beta = (5\ 7)(5\ 4)(5\ 6)(1\ 2)(1\ 3)(8\ 9)$$

Esercizio 2. Utilizzando un opportuno sviluppo di Laplace, calcolare

$$\det \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & -1 \\ 1 & 1 & -1 & 1 \\ 0 & 2 & 0 & -1 \end{vmatrix}$$

e determinare per quali valori di k la matrice è invertibile.

Matrice invertibile se $\det(A) \neq 0$

$$\sum_{k=1}^{n} (-1)^{i+k} a_{ik} \det(A(i,k))$$

$$+1 \cdot 1 \cdot \det \begin{pmatrix} 1 & k & -1 \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{pmatrix}$$

$$+2 \det \begin{pmatrix} k & -1 \\ -1 & 1 \end{pmatrix} -1 \det \begin{pmatrix} 1 & k \\ 1 & -1 \end{pmatrix} = 0$$

$$2 \cdot ((k)\cdot(1)) + (1)) - 1$$

$$2 \cdot (k+1) - 1 (1 \cdot -1)(1 \cdot k) = 0$$

$$2k + 2 - 1(-1) k = 0$$

$$2k + 2 + 1k$$

$$3k = -2 \Rightarrow k = \frac{-2}{3}$$

$$k \neq -\frac{2}{3}$$