Minimal covers - Exercises

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Part of the material is taken from the book: Atzeni, Ceri, Paraboschi, Torlone Basi di dati - Modelli e linguaggi di interrogazione , McGraw-Hill, 2002.

Example 1

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- given the following schema:

  $R = (A, B, C, D, E, H)$

  and the following set of functional dependencies:

  $F = \{ AB \rightarrow CD, C \rightarrow E, AB \rightarrow E, ABC \rightarrow D \}$

  find **a** minimal coverage G of F

- to find the minimal cover, we first reduce the dependents to singletons (step 1):

  $F = \{AB \rightarrow C, AB \rightarrow D, C \rightarrow E, AB \rightarrow E, ABC \rightarrow D\}$

Example 1: Step 2

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$F = \{AB \rightarrow C, AB \rightarrow D, C \rightarrow E, AB \rightarrow E, ABC \rightarrow D\}$

- $AB \rightarrow C$: check whether $A \rightarrow C$ is in F^+, i.e., whether C is in $(A)^+_F$ and check whether $B \rightarrow C$ is in F^+, i.e., whether C is in $(B)^+_F$
- $(A)^+_F = \{A\}$ and $(B)^+_F = \{B\}$, so the left hand side of the dependency cannot be reduced
- the same happens for the dependencies $AB \rightarrow D$ and $AB \rightarrow E$
- let us try to reduce $ABC \rightarrow D$; as $AB \rightarrow D$ exists in F, we can eliminate the dependency
- at the end of this step we have the set:

$F = \{AB \rightarrow C, AB \rightarrow D, C \rightarrow E, AB \rightarrow E\}$

Example 1: Step 3

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$F = \{ AB \rightarrow C, AB \rightarrow D, C \rightarrow E, AB \rightarrow E\}$

- we now look for redundant dependencies.
- C is determined by AB only, so if we remove $AB \rightarrow C$, C would no longer be determined by AB with respect to the new set of dependencies
- the same applies to $AB \rightarrow D$
- let us consider $C \rightarrow E$; with respect to the new set of test dependencies $G = \{ AB \rightarrow C, AB \rightarrow D, AB \rightarrow E\}$ we have that $(C)^+_G = \{C\}$ (i.e., E cannot be determined by C), so the dependency cannot be removed
- let us check $AB \rightarrow E$; with respect to $G = \{ AB \rightarrow C, AB \rightarrow D, C \rightarrow E\}$ we have that $(AB)^+_G = \{A,B,C,D,E\}$, therefore $AB \rightarrow E$ can be eliminated
- the **minimal cover of** F is:
  - $G = \{AB \rightarrow C, AB \rightarrow D, C \rightarrow E\}$

Observation

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- it is important to respect the order of steps 2 and 3 as, while in general the result is still correct, there are cases where that is not true
- for example: F= {AB → C, C → B, A → B}

- by performing steps 2 and 3 in the correct order, we have:
- (step 2) $(A)^+_F$= {A, B, C} , $(B)^+_F$ = {B} so the only possible reduction is from AB → C to A → C, so F={A → C, C → B, A → B}
- (step 3) C is only determined by A → C, so it is useless to try to remove this dependency; let's check C → B: we compute $(C)^+_G$ with G={A → C, A → B}, so $(C)^+_G$ = {C}, then the dependency cannot be removed; let's try it with A → B; we compute $(A)^+_G$ with G={A → C, C → B}, so $(A)^+_G$ = {A, C, B} and the dependency can be eliminated

- the minimal cover is $\boxed{G=\{A \rightarrow C, C \rightarrow B\}}$

Observation

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$F = \{AB \to C, C \to B, A \to B\}$

- by performing steps 3 and 2 in the reverse order, we obtain:
- (step 3) C is only determined by $AB \to C$, so it is useless to try to remove this dependency; let's try with $C \to B$: we compute $(C)^+_G$ with $G = \{AB \to C, A \to B\}$, so $(C)^+_G = \{C\}$ and the dependency cannot be removed; let's try with $A \to B$: we compute $(A)^+_G$ with $G = \{AB \to C, C \to B\}$, so $(A)^+_G = \{A\}$ and F remains unchanged
- (step 2) $(A)^+_F = \{A, B, C\}$ and $(B)^+_F = \{B\}$, so the only possible reduction is from $AB \to C$ to $A \to C$, and $F = \{A \to C, C \to B, A \to B\}$

- **this IS NOT** a minimal cover, as it violates the third property!
- in fact:

$$\{A \to C, C \to B, A \to B\} \equiv \{A \to C, C \to B, A \to B\} - \{A \to B\}$$

Example 2

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- given the following set of functional dependencies:

$F = \{ BC \rightarrow DE, C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow AL \}$

find a minimal cover of F

- we first decompose the dependents of the dependencies:

$$\boxed{F = \{ BC \rightarrow D, BC \rightarrow E, C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow A, BC \rightarrow L \}}$$

Example 2: Step 2

$F = \{ BC \to D, BC \to E, C \to D, B \to D, E \to L, D \to A, BC \to A, BC \to L \}$

$BC \to D$; we should check whether $B \to D$ or $C \to D$ belong to F^+, i.e., whether $D \in (B)^+{}_F$ or $D \in (C)^+{}_F$

we have both $C \to D$ and $B \to D$ in F, so $BC \to D$ is definitely redundan:

$F = \{ BC \to E, C \to D, B \to D, E \to L, D \to A, BC \to A, BC \to L \}$

Example 2: Step 2

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$F = \{ BC \rightarrow E, C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow A, BC \rightarrow L \}$

- $BC \rightarrow E$: we need to check whether $B \rightarrow E$ or $C \rightarrow E$ belong to F^+, i.e., whether $E \in (B)^+_F$ or $E \in (C)^+_F$
- by applying the algorithm we obtain:
- $(B)^+_{FB} = \{B, D, A\}$ and $(C)^+_F = \{C, D, A\}$
- so we can't eliminate any attribute from the determinant
- in fact, it was enough to observe that E only appears to the right of this dependency (**it is functionally determined** only by this **pair of** attributes) and so we cannot insert it into the closures of the individual attributes in any other way
- $BC \rightarrow A$: we already computed the closures of B and C (**F has not changed** or **it would be an equivalent set**), and in both we found A

attention: this time the dependencies $B \rightarrow A$ and $C \rightarrow A$ **are not** in F, so we cannot simply delete $BC \rightarrow A$ but we must replace it with **one of them**

Example 2: Step 2

- let's explore the two **alternative** routes:

so, we **may** end up **with two different minimal covers**!

$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, \mathbf{B \rightarrow A}, BC \rightarrow L \}$

- $BC \rightarrow L$: we need to check whether $B \rightarrow L$ or $C \rightarrow L$ are in F^+, i.e., whether $L \in (B)^+_F$ or $L \in (C)^+_F$
- we already computed $(B)^+_F$ and $(C)^+_F$, and neither of them contains L, so **we cannot** eliminate any attribute from the determinant
- **note**: in this case we needed to check, L is the dependent of **at least another dependency** (so we could insert it via **transitivity**)
- at the end of step 2 we have:

$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, B \rightarrow A, BC \rightarrow L \}$

Example 2: Step 3

$F = \{ BC \rightarrow E,\ C \rightarrow D,\ B \rightarrow D,\ E \rightarrow L,\ D \rightarrow A,\ B \rightarrow A,\ BC \rightarrow L \}$

- E **is determined** by $B \rightarrow C$ only, so we cannot remove $B \rightarrow C$
- $C \rightarrow D$: with respect to the new set $G = \{BC \rightarrow E,\ B \rightarrow D,\ E \rightarrow L,\ D \rightarrow A,\ B \rightarrow A,\ BC \rightarrow L \}$ we have that $(C)^+_G = \{C\}$ and $D \notin (C)^+_G$
- $B \rightarrow D$: with respect to $G = \{BC \rightarrow E,\ C \rightarrow D,\ E \rightarrow L,\ D \rightarrow A,\ B \rightarrow A,\ BC \rightarrow L\}$ we have that $(B)^+_G = \{B, A\}$ and $D \notin (B)^+_G$
- $E \rightarrow L$: with respect to $G = \{ BC \rightarrow E,\ C \rightarrow D,\ B \rightarrow D,\ D \rightarrow A,\ B \rightarrow A,\ BC \rightarrow L \}$ we have that $(E)^+_G = \{E\}$ and $L \notin (E)^+_G$
- $D \rightarrow A$: with respect to $G = \{ BC \rightarrow E,\ C \rightarrow D,\ B \rightarrow D,\ E \rightarrow L,\ B \rightarrow A,\ BC \rightarrow L \}$ we have that $(D)^+_G = \{D\}$ and $A \notin (D)^+_G$
- $B \rightarrow A$: with respect to $G = \{ BC \rightarrow E,\ C \rightarrow D,\ B \rightarrow D,\ E \rightarrow L,\ D \rightarrow A,\ BC \rightarrow L \}$ we have that $(B)^+_G = \{B, D, A\}$ and $A \in (B)^+_G$
- so,

$F = \{ BC \rightarrow E,\ C \rightarrow D,\ B \rightarrow D,\ E \rightarrow L,\ D \rightarrow A,$

Example 2: Step 3

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$F = \{ BC \rightarrow E\ , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow L \}$

- $BC \rightarrow L$: with respect to $G = \{ BC \rightarrow E\ , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A\}$ we have that $(BC)^+_G = \{B, C, E, D, A, L\}$ and $L \in (BC)^+_G$

- the minimal cover F is:

$F = \{ BC \rightarrow E\ , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A\}$

Example 2: Step 2 (second alternative)

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$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, \mathbf{C \rightarrow A}, BC \rightarrow L \}$

- $BC \rightarrow L$: we need to check whether $B \rightarrow L$ or $C \rightarrow L$ are in F^+, i.e., whether $L \in (B)^+_F$ or $L \in (C)^+_F$
- we already computed the closures of B and C, and checked that in neither of them we find the attribute L, so **we cannot** eliminate anything from the determinant

- at the end of step 2 in this execution we have:

$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, C \rightarrow A, BC \rightarrow L \}$

Example 2: Step 3 (second alternative)

$F = \{ BC \to E , C \to D, B \to D, E \to L, D \to A, C \to A, BC \to L \}$

- E **is determined** by BC only, so if we remove $BC \to E$, then E we would no longer be able to be added to the closure of BC with respect to the new set of dependencies
- $C \to D$: with respect to $G = \{ BC \to E , B \to D, E \to L, D \to A, C \to A, BC \to L \}$ we have that $(C)^+_G = \{C, A\}$ and $D \notin (C)^+_G$
- $B \to D$: with respect to $G = \{ BC \to E , C \to D, E \to L, D \to A, C \to A, BC \to L \}$ we have that $(B)^+_G = \{B\}$ and $D \notin (B)^+_G$
- $E \to L$: with respect to $G = \{ BC \to E , C \to D, B \to D, D \to A, C \to A, BC \to L \}$ we have that $(E)^+_G = \{E\}$ and $L \notin (E)^+_G$

Example 2: Step 3 (second alternative)

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$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, C \rightarrow A, BC \rightarrow L \}$

- $D \rightarrow A$: with respect to $G = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, C \rightarrow A, BC \rightarrow L \}$ we have that $(D)^+_G = \{D\}$ and $A \notin (D)^+_G$
- $C \rightarrow A$: with respect to $G = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow L \}$ we have that $(C)^+_G = \{C, D, A\}$ and $A \in (C)^+_G$
- $BC \rightarrow L$: with respect to $G = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A\}$ we have that $(BC)^+_G = \{B, C, E, D, A, L \}$ and $L \in (BC)^+_G$
- in this case we obtain the same **minimal cover**:

$F = \{ BC \rightarrow E , C \rightarrow D, B \rightarrow D, E \rightarrow L, D \rightarrow A, BC \rightarrow L \}$

Example 3

- given the following set of functional dependencies:

  $F = \{ AB \rightarrow C, A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$

  find a minimal coverage of F

- there is no need to decompose the dependents in F, as they **are already singletons**

Example 3: Step 2

$F = \{AB \to C, A \to E, E \to D, D \to C, B \to A\}$

- we check whether the dependency $AB \to C$ has redundant attributes in the determinant
- so, we check whether $C \in (A)^+_F$ or $C \in (B)^+_F$.
- $(A)^+_F = \{A, E, D, C\}$ and $(B)^+_F = \{B, A, E, D, C\}$
- so, $AB \to C$ can be replaced either by $A \to C$ **or** by $B \to C$

- at the end of step 2 we have:

$F = \{\mathbf{A \to C}, A \to E, E \to D, D \to C, B \to A\}$

or:

$F = \{\mathbf{B \to C}, A \to E, E \to D, D \to C, B \to A\}$

Example 3: Step 3

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$F = \{\mathbf{A \rightarrow C}, A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$

- $A \rightarrow C$: we compute $(A)^+_G$ with respect to $G=\{A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$
- we obtain $(A)^+_G = \{A, E, D, \mathbf{C}\}$ so the dependency **can be eliminated**

$F = \{A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$

- no attribute appears as dependent of **more than one** dependency, so it cannot be obtained by transitivity, i.e., none of the remaining dependencies can be removed!
- the **minimal cover** of the initial set F is:

$F = \{A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$

Example 3: Step 3 (second alternative)

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- we check the second alternative:

$$\boxed{F = \{\mathbf{B \rightarrow C}, A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}}$$

- $B \rightarrow C$: we compute $(B)^+_G$ with respect to $G=\{A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}$
- we obtain $(B)^+_G = \{B, A, E, D, C\}$ so the dependency **can be eliminated**

- we obtained the same **minimal cover**:

$$\boxed{F = \{A \rightarrow E, E \rightarrow D, D \rightarrow C, B \rightarrow A\}}$$

Example 4

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- given the following set of functional dependencies:

  $$F = \{ A \rightarrow B, A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$$

  find a minimal coverage of F

- both determinants and dependents are singletons, so let's we go straight to step 3

Example 4: Step 3

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$F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$

- $A \rightarrow B$: $G=\{A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$ and $(A)^{+}_{G} = \{A, C, D, B\}$ which contains B, so we can eliminate the dependency

$F = \{A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$

- $C \rightarrow D$: $G = \{A \rightarrow C, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$ and $(C)^{+}_{G} = \{C, A\}$ which does not contain D, so the dependency **cannot be eliminated**
- $B \rightarrow D$: $G = \{A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A\}$ and $(B)^{+}_{G} = \{B\}$ which does not contain D so the dependency **cannot be eliminated**
- **a minimal cover is:**

$F = \{A \rightarrow C, C \rightarrow D, D \rightarrow B, C \rightarrow A, B \rightarrow D\}$

Example 4: Step 3 (second alternative)

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$$\boxed{F = \{A \to B, A \to C, C \to D, D \to B, C \to A, B \to D\}}$$

- $C \to D$: $G=\{A \to B, A \to C, D \to B, C \to A, B \to D\}$ and $(C)^+_G = \{C, A, B, D\}$ which contains D, so we can eliminate the dependency

$$\boxed{F = \{A \to B, A \to C, D \to B, C \to A, B \to D\}}$$

- if an attribute appears in a dependent of a single dependency, that dependency cannot be eliminated
- $A \to B$: $G = \{ A \to C, D \to B, C \to A, B \to D \}$ and $(A)^+_G = \{A, C\}$ which does not contain B, so the dependency **cannot be eliminated**
- $D \to B$: $G = \{ A \to B, A \to C, C \to A, B \to D \}$ and $(D)^+_G = \{D\}$ which does not contain B, so the dependency **cannot be eliminated**
- **a second possible minimal cover is:**

$$\boxed{F = \{A \to B, A \to C, D \to B, C \to A, B \to D\}}$$

- **this time the two minimal covers are <u>different</u>**

Example 4: check

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- we obtained two different minimal covers, with the same cardinality but different dependencies:

$$\boxed{F = \{ A \rightarrow C,\ C \rightarrow D,\ D \rightarrow B,\ C \rightarrow A,\ B \rightarrow D\}}$$

$$\boxed{F = \{A \rightarrow B,\ A \rightarrow C,\ D \rightarrow B,\ C \rightarrow A,\ B \rightarrow D\}}$$

- we check that the two sets of dependencies are equivalent (individually, we know that they are both equivalent to initial !) i.e. that $F1 \equiv F2$, i.e., that $F1^+ = F2^+$, i.e., that $F1^+ \subseteq F2^+$ and $F2^+ \subseteq F1^+$, i.e., $F1 \subseteq F2^+$ and $F2 \subseteq F1^+$
- $A \rightarrow C$, $D \rightarrow B$, $C \rightarrow A$, $B \rightarrow D$ are in both sets, so it is unnecessary to check them
- F1 contains $C \rightarrow D$ which does not belong to F2, so we check if $C \rightarrow D$ belongs to $F2^+$, i.e., if D is in $(C)^+_{F2}$ and $(C)^+_{F2} = \{C, A, B, D\}$ then it is ok
- F2 contains $A \rightarrow B$ which does not belong to F1, so we check if $A \rightarrow B$ belongs to $F1^+$, i.e., if B is in $(A)^+_{F1}$ and $(A)^+_{F1} = \{A, C, D, B\}$ then it is ok
- as we expected (as it should be!) F1 and F2 are equivalent

Example 5

- given the following set of functional dependencies:

  $F = \{ AB \rightarrow C, AD \rightarrow BC, AC \rightarrow B, B \rightarrow D \}$

  find a minimal coverage of F

- we decompose the dependents:

  $F = \{ AB \rightarrow C, AD \rightarrow B, AD \rightarrow C, AC \rightarrow B, B \rightarrow D \}$

- we compute the closures of the attributes that appear in the determinants to see if they can be reduced:
- $(A)^+_F = \{A\}$ $(B)^+_F = \{BD\}$ $(C)^+_F = \{C\}$ $(D)^+_F = \{D\}$

- step 2 leaves F unchanged

Example 5: Step 3

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$F = \{ AB \rightarrow C, AD \rightarrow B, AD \rightarrow C, AC \rightarrow B, B \rightarrow D \}$

- $AB \rightarrow C$: $G=\{AD \rightarrow B, AD \rightarrow C, AC \rightarrow B, B \rightarrow D\}$ and $(AB)^+_G = \{A, B, D, C\}$ which contains C, so we can eliminate the dependency
- $AD \rightarrow B$> $G = \{AD \rightarrow C, AC \rightarrow B, B \rightarrow D\}$ and $(AD)^+_G = \{A, D, C, B\}$ which contains B, so we can eliminate the dependency
- if an attribute appears in the dependent a single dependency, that dependency cannot be deleted
- **a minimal coverage is:**

$F = \{AD \rightarrow C, AC \rightarrow B, B \rightarrow D\}$

Example 5: Step 3 (second alternative)

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$F = \{ AB \rightarrow C, AD \rightarrow B, AD \rightarrow C, AC \rightarrow B, B \rightarrow D \}$

- we reverse the order of the dependencies
- $B \rightarrow D$: $G=\{AB \rightarrow C, AD \rightarrow B, AD \rightarrow C, AC \rightarrow B\}$ and $(B)^+_G = \{B\}$ which **does not contain D,** so **we cannot eliminate the dependency**
- $AC \rightarrow B$: $G = \{AB \rightarrow C, AD \rightarrow B, AD \rightarrow C, B \rightarrow D\}$ and $(AC)^+_G = \{A, C\}$ which **does not contain B, so we cannot eliminate the dependency**
- $AD \rightarrow C$: $G = \{AB \rightarrow C, AD \rightarrow B, AC \rightarrow B, B \rightarrow D\}$ and $(AD)^+_G = \{A, D, B, C\}$ which **contains C, so we can eliminate the dependency**
- $AD \rightarrow B$: $G = \{AB \rightarrow C, AC \rightarrow B, B \rightarrow D\}$ and $(AD)^+_G = \{A, D\}$ which **does not contain B, so we cannot eliminate the dependency**
- if an attribute appears in the dependent of a single dependency that dependency cannot be deleted
- **another minimal cover is:**

$F = \{AB \rightarrow C, AD \rightarrow B, AC \rightarrow B, B \rightarrow D\}$

Example 5: check

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- we obtained two **different** minimal covers, with **different** cardinality:

$$\boxed{F1 = \{AD \rightarrow C, AC \rightarrow B, B \rightarrow D\}}$$

$$\boxed{F2 = \{AB \rightarrow C, AD \rightarrow B, AC \rightarrow B, B \rightarrow D\}}$$

- let us verify that the two sets of dependencies are equivalent
- the dependencies $AC \rightarrow B$ and $B \rightarrow D$ are in both sets, and therefore in their closures, so there is no point in checking them
- F1 contains $AD \rightarrow C$ which is not in F2, so we check if $AD \rightarrow C$ is in $F2^+$, i.e., if C is in $(AD)^+_{F2}$ and $(AD)^+_{F2}$ =\{A, D, B, C\} then it is ok

  F2 contains $AB \rightarrow C$ which is not in F1, so we check if $AB \rightarrow C$ is in $F1^+$, i.e., if C is in $(AB)^+_{F1}$ and $(AB)^+_{F1}$ =\{A, B, D, C\} then it is ok

  F2 contains $AD \rightarrow B$ which is not in F1, so we check if $AD \rightarrow B$ is not in $F1^+$, i.e., if B is in $(AD)^+_{F1}$ and $(AD)^+_{F1}$ = \{A, D, C, B\} then it is ok