Exp of Binomial random variable

$\Omega = \{0,1\}^n$, this variable count the occurrencies of 1

for $(n=2)$ $X = \begin{cases} 0 & \text{if } w = 00 \\ 1 & \text{if } w = 01 \text{ or } 10 \\ 2 & \text{if } w = 11 \end{cases}$

$$E[X] = \sum_{k=0}^{n} \underbrace{k}_{K} \underbrace{\left[\binom{n}{k} p^k (1-p)^{n-k}\right]}_{\mathbb{P}(\{K\})} = (n \cdot p)$$

We are counting the occurrencies of 1

DIM:

$$X_i = \begin{cases} 1 & \text{if the } i° \text{ launch is head} \\ 0 & \text{if the } i° \text{ launch is Tail} \end{cases}$$

We can say:

$$E[X_i] = 1 \cdot p + 0 \cdot (1 \cdot p) = p$$

$$\Rightarrow \quad E[X] = \sum_{i=1}^{n} E[X_i] = p \cdot n \quad \blacksquare$$

Variance of Binomial random variable

general observation.

$$V(X) = E(X^2) - (E(X))^2$$

Dim:

$$V(X) = \sum_{y \in Im(X)} (y - E(X))^2 \cdot \mathbb{P}(\{y\})$$

$$\sum_{y \in Im(X)} \left(y^2 + (E(X))^2 - 2\, E(X)\right) \cdot \mathbb{P}(\{y\})$$

$$\sum_{x \in Im(X)} y^2 \mathbb{P}\{y\} + (E(X))^2 \cdot \mathbb{P}(\{y\}) - 2y \cdot E(X) \cdot \mathbb{P}(\{y\})$$

$$\underbrace{\sum_{y \in IM(X)} y^2 \, \mathbb{P}(\{y\})} + \underbrace{\sum_{y \in IM(X)} (E(X))^2 \cdot \mathbb{P}(\{y\})} \underbrace{-2 \sum_{y \in IM(X)} y \, E(X) \, \mathbb{P}(\{y\})}$$

(VARIANZA DEI QUADRATI)

$$-2\, E(X) \cdot E(X)$$

$$E(X^2) \quad + \quad (E(X))^2 \quad -2(E(X))^2$$

$$V(X) = E(X^2) - (E(X))^2 \quad \blacksquare$$

CALCOLIAMO $E(X^2)$ per la variabile aleatoria di Bernulli.

$$E(X^2) = \sum_{k=0}^{n} k^2 \, \mathbb{P}(\{k\}) =$$

[...]