Linearity of expectation

V: set of random variables.

$\boxed{X : \Omega \to \mathbb{R}}$, $(X, Y) \in V$

- $E[X+Y] = E[X] + E[Y]$

DIM:

$$E[X+Y] = \sum_{\omega} \left((X(\omega) + Y(\omega)) \cdot P(\omega) \right) = \sum_{\omega} X(\omega) P(\omega) + Y(\omega) P(\omega)$$

$$= \sum_{\omega} X(\omega) P(\omega) + \sum_{\omega} Y(\omega) P(\omega) = E[X] + E[Y]$$

- $E[aX] = aE[X]$

DIM:

$$E[aX] = \sum_{\omega} a \cdot X(\omega) \cdot P(\omega) = a \sum X(\omega) \cdot P(\omega) = a(E[X]).$$

(VECTORIAL SPACE STRUCTURE)