CSci 243 Homework 4

Due: Wednesday, October 5, end of day
My name

1. (10 points) For sets A, B, and C, prove that $(B-A)\cup(C-A) = (B\cup C)-A$

   (a) by using set properties and propositional calculus

   (b) by using a membership table

2. (5 points) Find these values.

   (a) $\lfloor 1.1 \rfloor$

   (b) $\lceil 1.1 \rceil$

   (c) $\lfloor -0.1 \rfloor$

   (d) $\lceil -0.1 \rceil$

   (e) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$

3. (8 points) Determine whether each of these functions from $\mathbb{Z}$ or $\mathbb{Z}\times\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one, onto, both, or neither. Justify your answer.

   (a) $f(n) = 2n-1$

   (b) $f(n) = |n|+1$

   (c) $f(n) = n^3$

   (d) $f(n) = \lfloor \frac{n}{2} \rfloor$

4. (12 points) A quick review of algebra. Answer the following:

   (a) Simplify: $(q^3 p^{2/3})(\sqrt{p^3} q^{-1}) =$

   (b) Simplify: $aa^2a^4a^7a^5a^3 =$

   (c) Simplify to have no x, y in the same factors: $(\log(xy)\log(x/y)) =$

   (d) Add the fractions to produce one fraction: $\frac{2}{x^2+4x+4} + \frac{1}{x+2} =$

   (e) If $x, y, z, d \in \mathbb{R}$, are the following True or False?

   F i. $x < y \wedge z < d \rightarrow xz < yd$

   F ii. $x < y \wedge z < d \rightarrow xd < yz$

   F iii. $x > 0 \wedge y > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$

   F iv. $x > 0 \wedge y > 0 \wedge z > 0 \wedge d > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$

   (f) Solve for x:

   i. $10^{3x+5} = 100$

   ii. $3^{x^2}(9^x) = 1/3$

   iii. $\log_x 64 = 2$

   iv. $\log_3(2x+1) = \log_9(x^2)$

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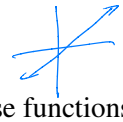
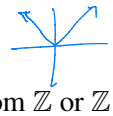
   (e) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$ = $\lfloor \frac{1}{2} + 1 \rfloor$ = 1





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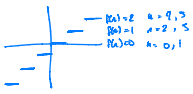

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Organic Chem. Tutor — Logarithms Explained (1:32:52)

$\log_2 4$ = how many 2's do I have = $2^? = 4$? = answer is $\boxed{2}$
to multiply to get 4 ?

$\log_2 8 = 3$
$\log_3 9 = 2$
$\log_4 16 = 2$
$\log_3 27 = 3$
$\log_2 32 = 5$
$\log_5 125 = 3$
$\log_6 36 = 2$
$\log_2 64 = 6$
$\log_3 81 = 4$
$\log_3 1 = 0$ $\quad 3^0 = 1$

$\log(10)$ = when you don't see a base, assume to be 10 = 1
$\log(100) = 2$
$\log(1000) = 3$
$\log(1M) = 6$
$\log(0.1) = -1$
$\log(0.01) = -2$
$\log(0.001) = -3$
$\log(0) =$ ONE can't be zero inside of log()
$\log(-5) =$ ONE can't be negative # inside of log()

$\log_4(16) = 2$
$\log_4\left(\frac{1}{16}\right) = -2$ $\quad 4^{-1} = \frac{1}{4}$ $\quad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\log_{16}(4) = 1/2$ $\quad 16^{1/2} = \sqrt{16} = 4$
$\log_{16}\left(\frac{1}{4}\right) = -1/2$ $\quad 16^{1/2} = \frac{1}{16^{1/2}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

$\log_6 216 = 3$
$\log_6 1/216 = -3$
$\log_{216} 1/6 = -1/3$
$\log_{216} 6 = 1/3$

$\log_a b = \frac{\log_c b}{\log_c a}$ — can change base to whatever

$\log_4 16 = ?$
$\frac{\log 16}{\log 4} =$ (calculator says) 2

$\log_5(x-y) = \frac{\log_a(x-y)}{\log_a 5}$
to convert a log to a natural log use base e
$= \frac{\ln(x-y)}{\ln(5)}$

$\log(A) + \log(B) = \log(AB)$

$\log(A) - \log(B) = \log(A/B)$

$\log A^2 = 2 \log A$ can move exponent to the front

$\log(x) + \log(y) - \log(z) = \log\left(\frac{xy}{z}\right)$

$\log(x) - \log(y) + \log(z) - \log(r) = \log\left(\frac{xz}{yr}\right)$

$2\log x + 3\log y - 4\log z$ = can't combine bc coefficient = use power rule

$= \log x^2 + \log y^3 - \log z^4 = \log\left(\frac{x^2y^3}{z^4}\right)$

$\frac{1}{2}\log x - \frac{1}{3}\log y + \frac{1}{4}\log z = \log\sqrt{x} - \log\sqrt[3]{y} + \log\sqrt[4]{z} = \log\left(\frac{\sqrt{x}\sqrt[4]{z}}{\sqrt[3]{y}}\right)$

$\log\left(\frac{r^2 s^5}{z^6}\right) = 2\log r + 5\log s - 6\log z$

$\log\sqrt[5]{\frac{x^2y}{z^4}} = \log\left(\frac{x^2y}{z^4}\right)^{1/5} = \frac{1}{5}\log\left(\frac{x^2y}{z^4}\right) = \frac{1}{5}\left[\log x^2 + \log y + \log z^4\right] = \frac{2}{5}\log x + \frac{1}{5}\log y + \frac{4}{5}\log z$

$\log\left(\frac{x^2\sqrt{y}}{\sqrt[3]{z^4}}\right) = \log\left(\frac{x^2 y^{1/2}}{z^{4/3}}\right) = 2\log x + \frac{1}{2}\log y - \frac{4}{3}\log z$

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most section 22 min in

$\log_e x = \ln x$

$\ln 1 = 0$

$\ln e = 1$

$\ln e^5 = 5 \ln e = 5(1) = 5$

$e^{\ln 7} = e^{\log_e 7}$ = base equal = 7

$e^{3\ln x} = e^{\ln x^3} = x^3$

$2e^{4\ln y} = 2y^4$

$8^{\log_8 y^3} = y^3$

$x^{(\log_x \ln e)} = x^{\ln e} = x^1 = x$

$\log_7 6 - \log_7 2 = \log_7(6/2) = \log_7 3 = 1$

$\log_2 12 + \log_2 24 - \log_2 9 = \log_2(12 \cdot 24/9) = \log_2(288/9) = \log_2 32 = 5$

$\log_7\left(\frac{1}{7}\right) + \log_7\left(\frac{1}{49}\right) = \log_7\left(\frac{1}{343}\right) = -2$

$\log_2(8) = 3$ $\quad 2^3 = 8$

$\log_3 9 = 2$ $\quad 3^2 = 9$

$\log_a b = c$ $\quad a^c = b$

$\log_2 x = y$ $\quad 2^y = x$

$3^3 = 27$ $\quad \log_3 27 = 3$

$2^4 = 16$ $\quad \log_2 16 = 4$

$3^x = 18$ $\quad \log_3 18 = x$

$\log_2 y = 5 \rightarrow 2^5 = y \rightarrow y = 32$

$\log_2 (x+5) = 3 \rightarrow 2^3 = x+5 \rightarrow x = 3$

$\log_3 (2x-3) = 2 \rightarrow 3^2 = 2x-3 \rightarrow 12 = 2x \rightarrow x = 6$

$\log_2 (x-3) + \log_2 (x-1) = 3$

condense two logs with a single one

$\log_2 ((x-3)(x-1)) = 3$

$2^3 = (x-3)(x-1)$

$8 = x^2 - 4x + 3$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = -1, 5$

☆ need to find if there are extraneous solutions!

$\log_2 (x-3) + \log_2 (x-1)$

plug in 5

$\log_2 (2) + \log_2 (4)$ ✓

plug in -1

$\log_2 (-4) + \log_2 (-2)$ ✗ can't have negatives in logs

$x = 5$

$\log_3 (2x+1) + \log_3 (x+8) = 3$

$3^3 = (2x+1)(x+8)$

$27 = 2x^2 + 17x + 8$

$0 = 2x^2 + 17x - 19$ multiply to -38 add to 17 $= 19, -2$

$0 = 2x^2 - 2x + 19x - 19$

$0 = 2x(x-1) + 19(x-1)$

$0 = (2x+19)(x-1)$

$x = 1, -19/2$

plug in $-19/2$, will get negatives in logs

$x = 1$

check our work

$\log_3 (9) + \log_3 (9) = 3$

$1 + 2 = 3$ ✓

$\log_2 (x+5) = \log_2 (3x-9)$

same base + since log so inside should be equal

$x+5 = 3x-9 \rightarrow 14 = 2x \rightarrow x = 7$

$\ln x = 5 \qquad e^5 = x$

$\ln x + 3 \ln x = 0$

$\rightarrow 4 \ln x = 0$

$\ln x = 0 \qquad e^0 = x \qquad x = 1$

$\ln (x-3) = 2 \qquad e^2 = (x-3)$

$x = e^2 + 3$

$8^{x+4} = 16^{2x-3}$ want to convert bases to a common base $2^3 = 8$ $2^4 = 16$

$(2^3)^{x+4} = (2^4)^{2x-3}$

$2^{3x+12} = 2^{8x-12}$ bases are equal, so can set exponents equal

$3x+12 = 8x-12$

$24 = 5x$

$x = 24/5$

$9^{2x-1} = 27^{3x+6}$

$(3^2)^{2x-1} = (3^3)^{3x+6}$

$3^{4x-2} = 3^{9x+18}$

$4x-2 = 9x+18$

$-5x = 20$

$x = -4$

$2^x = 7$ $2^2=4$ $2^3=8$ x is somewhere between 2 & 3

$\ln 2^x = \ln 7$ ← taking ln of both sides because have to be ln's, just easy log

$\frac{x \ln 2}{\ln 2} = \frac{\ln 7}{\ln 2}$

$x = \frac{\ln 7}{\ln 2}$ need calculator at this point

≈ 2.807355

$e^{x+2} = 8$ using ln here for same like e is present

$\ln e^{x+2} = \ln 8$

$(x+2)\ln e = \ln 8$ $\ln e = 1$

$x+2 = \ln 8$

$x = \ln 8 - 2$

$f(x) = \ln(2x-6)$

$y = \ln(2x-6)$

$2x-6 > 0$ have to be >0 no negatives

$2x > 6$

$x > 3$

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   (a) by using set properties and propositional calculus

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could prove that

$(B-A)\cup(C-A) \subseteq (B\cup C)-A$

and $(B\cup C)-A \subseteq (B-A)\cup(C-A)$

$(B\cup C)-A$

Proof 3. In a membership table (like a truth table), 1s and 0s are used to specify if an arbitrary element x is in a set expression or not. The rows in the table are all possible membership combinations of x. If the columns for the two expressions are identical then the expressions are equal.

A	B	$\overline{A}$	$\overline{B}$	$A\cup B$	$\overline{A\cup B}$	$\overline{A}\cap\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

(b) membership table aka a truth table basically

A	B	C	B−A	C−A	B−A ∪ C−A
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	1	0	0
1	1	0	0	0	0
0	0	1	0	1	0
1	0	1	0	0	0
0	1	1	1	1	1
1	1	1	0	0	0

B−A

↳ 0−0 means

x not in either = 0

↳ 0−1 means

x not in B

x is in A

taking away x from B = 0

↳ 1−0

x is in B

x not in A

x is still in B = 1

↳ 1−1

x is in B

x not in A

finally, order sense cancels out = 0

I looked it up this is right

0
0
0
1

//

A	B	C	B∪C	(B∪C)−A
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
1	1	0	0	0
0	0	1	0	0
1	0	1	0	0
0	1	1	1	1
1	1	1	1	0

(a)