CSci 243 Homework 8

Due: Wednesday, November 9, end of day
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1. (5 points) Use bubble sort to sort 1, 3, 8, 2, 9, 4, showing the lists obtained at each outer step.

   1 3 8 2 9 4

   1 3 2 8 9 4

   1 3 2 8 4 9

   1 2 3 8 4 9

   1 2 3 4 8 9

2. For each of the following program fragments, give an analysis of the running time. You may use summations to evaluate the running times of nested loops. Use Θ notation (i.e., ignore constants) and show your work.

   (a) (6 points)

   ```
   sum = 0
   for i = 0 to n-1
      for j = 0 to i
         for k = 1 to n/(2^j)          (2^j) means 2 to the power j
            sum ++
   ```

   (b) (6 points)

   ```
   for i = 1 to n
      j=1
      while (j <= i) {
         k=1
         while (k <= n) {
            k=k*2
            sum ++
         }
         j=2*j
      }
   ```

3. (8 points) Suppose that a store offers gift certificates in denominations of \$25 and \$40. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction.

4. (10 points) Consider the dot game we saw in class. Again there are two rows of dots, with n_1 and n_2 dots respectively, and players can remove any number of dots during their turn, but only from one row. However, now, the player who removes the last dot loses.

   If $n_1 = n_2 > 1$, prove that there is a winning strategy for the second player. What happens if $n_1 \neq n_2$?

   Base case: $n_1 = n_2 = 2$

   - P1 will either remove 1 or 2 dots from a row

   - If P1 removes 1 dot from a row, P2 can remove 2 dots from the other row, leaving one dot for

P1 to pick, which means P1 loses.

- If P1 removes 2 dots from a row, P2 can remove 1 dot from the other row, leaving one dot for P1 to pick, which means P1 loses.

Inductive hypothesis: for any number of dots $2 \leq j \leq k$, the 2nd player can choose a winning strategy.

Inductive step: there are $n = k+1$ dots per row

- P1 will remove $1 \leq r \leq k+1$ dots from one row

- If $r = k+1$, then P2 can remove k dots from the other row, leaving one dot for P1 to pick, which means P1 loses.

- If $r = k$, then P2 can remove all dots from the other row, leaving one dot for P1 to pick, which means P1 loses.

- If $r < k$, then P2 can remove r dots from the other row, leaving each row with $2 \leq k+1-r \leq k$ dots, and by I.H. P2 can choose a winning strategy.

If $n_1 \neq n_2$, then the first player can remove $|n_1 - n_2|$ dots to make the rows equal, and then use the strategy above, as they are effectively player 2 now.