CSci 243 Homework 3

Due: Wednesday, Wed 28 end of day
My name

1. (6 points) Prove by induction that $2^n < n!$, $\forall n \geq 4$, where ! denotes the factorial of the number.

2. (6 points) Prove by induction that $2^n + 1$ is divisible by 3 for all odd integers n.

3. (6 points) Here is an inductive proof that all jelly beens in the world have the same color!
Basis ($n = 1$): We pick one jelly bean and clearly it has the same color as itself.
Inductive hypothesis: Assume that any group of $n = k$ jelly beans have the same color.
Inductive step: Take $k+1$ jelly beans and order them in a line. Consider group A of the first $1, \ldots, k$ jelly beans, and group B of the last $2, \ldots, k+1$ jelly beans. By the inductive hypothesis, all jelly beans in group A have the same color, and all jelly beans in group B have the same color. Because the two groups share the jelly beans $2, \ldots, k$, the colors of the two groups must be the same. Thus any $k+1$ jelly beans have the same color.

   You don't need to have eaten jelly beans to know that the above proposition is false. So what is wrong with the proof?

4. (7 points) Determine whether these statements are true or false (for the empty set, see p.118 of the book). Write a very brief eplanation (less than one line suffices).

   (a) $\emptyset \in \{\emptyset\}$ — characteristics of sets
   (b) $\{\emptyset\} \in \{\{\emptyset\}\}$
   (c) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
   (d) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
   (e) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
   (f) $\{x\} \subseteq \{x\}$
   (g) $\{x\} \in \{x\}$

5. (6 points) Is each of these sets the power set of a set, where a and b are distinct elements? If yes, give the original set.

   (a) $\emptyset$
   (b) $\{\emptyset, \{a\}\}$
   (c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
   (d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

6. (4 points) Show that if $A \subseteq C$ and $B \subseteq D$ then $A \times B \subseteq C \times D$.

— set laws

1

$2^n < n!$ for all $n \geq 4$

base case $n = 4$

$2^4 < 4!$

$2 \cdot 2 \cdot 2 \cdot 2 < 4 \cdot 3 \cdot 2 \cdot 1$

$16 < 24$

IH = assume for any k, $2^k < k!$

IS = want to prove $k+1$

aka $2^{(k+1)} < (k+1)!$

$2^{(k+1)} < (k+1)!$

$\Rightarrow 2^k 2^1 < (k+1) k!$

by I.H. $2^k < k!$

by def. we know $(k+1) > 4$,

and $2 < 4$

left side (2^k) is being multiplied by a number (2) smaller than the number (4) that r. side is being mult. by

if other way around, unsure if inequality would stay the same.

But it isnt

$2^k < k!$

$\Rightarrow 2 < k+1$

so, $2^{k+1} < (k+1)!$

2

prove $2^n + 1$ divis by 3 for all odd int n

base case $n = 1$

$2^1 + 1 = 3$ ✓

IH assume for any odd k, $2^k + 1$ divisible by 3

IS prove for $k+2$ ← next odd #

$2^{(k+2)} + 1$ divis. by 3

should be part of IH

definition of "divisible by 3"

$\exists c,\ 2^{(k)} + 1 = 3c$

want to prove $\exists d,\ 2^{(k+2)} + 1 = 3d$

$2^{k+2} + 1 = (2^k 2^2) + 1$

$= 2^k \cdot 4 + 1$

how do I separate this

I'm stuck

(3)

this is the horses thing from the jan misali video

3. (6 points) Here is an inductive proof that all jelly beens in the world have the same color!
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Inductive step: Take $k+1$ jelly beans and order them in a line. Consider group A of the first $1, \ldots, k$ jelly beans, and group B of the last $2, \ldots, k+1$ jelly beans. By the inductive hypothesis, all jelly beans in group A have the same color, and all jelly beans in group B have the same color. Because the two groups share the jelly beans $2, \ldots, k$, the colors of the two groups must be the same. Thus any $k+1$ jelly beans have the same color.

You don't need to have eaten jelly beans to know that the above proposition is false. So what is wrong with the proof?

Should base case have been comparing 2 jelly beans?
or should I.H. not say $n = k$
↓
Wrong b.c. inductive step does not work for ~~$n = 2$~~ 2 jellybeans
with $n = 2$, / $k+1 = 2$ and $k = 1$, following inductive step
group A has the 1st bean ; same color by IH (or base case)
group B has the 2nd bean ; same color by IH (or base case)
group A + B share beans $2, \ldots, k$; does not work ; A + B don't share any beans

(4)

4. (7 points) Determine whether these statements are true or false (for the empty set, see p.118 of the book). Write a very brief eplanation (less than one line suffices).

(a) $\emptyset \in \{\emptyset\}$ empty set is in empty set
(b) $\{\emptyset\} \in \{\{\emptyset\}\}$ empty set is in set containing empty
(c) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
(d) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
(e) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
(f) $\{x\} \subseteq \{x\}$ is $\{x\}$ a subset of itself?
(g) $\{x\} \in \{x\}$ is $\{x\}$ within itself?

or is it $\emptyset$ in a set containing $\emptyset$
$\emptyset$ in a set containing a set containing $\emptyset$

from notes: "note that $\emptyset \neq \{\emptyset\}$"

(a) $\emptyset \in \{\emptyset\}$
yes elements of $\{\emptyset\}$ are: $\emptyset$

(b) $\{\emptyset\} \in \{\{\emptyset\}\}$
yes elements of $\{\{\emptyset\}\}$ are: $\{\emptyset\}$

(c) yes elements of $\{\emptyset, \{\emptyset\}\}$ are: $\emptyset, \{\emptyset\}$
elements of $\{\emptyset\}$ are: $\emptyset$ → all elements are within S

(d) yes elements of $\{\emptyset, \{\emptyset\}\}$ are: $\emptyset, \{\emptyset\}$
elements of $\{\{\emptyset\}\}$ are: $\{\emptyset\}$ → all elements are within S

(e) yes elements of $\{\{\emptyset\}, \{\emptyset\}\}$ are: $\{\emptyset\}$
elements of $\{\{\emptyset\}\}$ are: $\{\emptyset\}$ → all elements are within S

(f) yes elements of $\{x\}$: x
elements of $\{x\}$: x → all elements are within S

also note using $\subseteq$ symbol, not $\subset$
so doesn't have to be proper subset

(g) no elements of $\{x\}$ are: x
↳ not equal to $\{x\}$

also, Russell's paradox

5. (6 points) Is each of these sets the power set of a set, where a and b are distinct elements? If yes, give the original set.

(a) $\emptyset$

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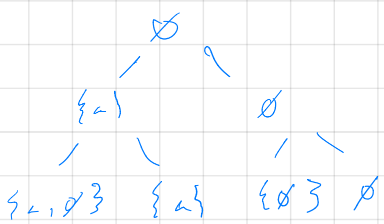
(d) $\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

(a) $\emptyset$ is the power set of none

$p(\emptyset) = \{\emptyset\}$
$p(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
but none $= \emptyset$

(b) $p(\{a\}) = \{\emptyset, \{a\}\}$

(c) none b/c 3 elements



(d) $p(\{a, b\})$

6. (4 points) Show that if $A \subseteq C$ and $B \subseteq D$ then $A \times B \subseteq C \times D$.

if A subset of C
and B subset of D

then $A \times B$ subset of $C \times D$

if every element of A is within C
and every element of B is within D,

then every element of $A \times B$
will also be in $C \times D$

ex. $A = \{a, b\}$ $\quad C = \{a, b, e\}$
$B = \{c, d\}$ $\quad D = \{c, d, f\}$

$A \times B = \{(a,c)(a,d)(b,c)(b,d)\}$

$C \times D = \{(a\ c)(a\ d)(a\ f)(b\ c)(b\ d)(b\ f)(e\ c)(e\ d)(e\ f)\}$

$A \times B$ is a subset of $C \times D$

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