CSci 243 Homework 8

Due: Wednesday, November 9, end of day
My name

1. (5 points) Use bubble sort to sort 1, 3, 8, 2, 9, 4, showing the lists obtained at each outer step. gross but ok

2. For each of the following program fragments, give an analysis of the running time. You may use summations to evaluate the running times of nested loops. Use Θ notation (i.e., ignore constants) and show your work. gross idk

   (a) (6 points)

   ```
   sum = 0
   for i = 0 to n-1
      for j = 0 to i
         for k = 1 to n/(2^j)          (2^j) means 2 to the power j
            sum ++
   ```

   (b) (6 points)

   ```
   for i = 1 to n
      j=1
      while (j <= i) {
         k=1
         while (k <= n) {
            k=k*2
            sum ++
         }
         j=2*j
      }
   ```

3. (8 points) Suppose that a store offers gift certificates in denominations of \$25 and \$40. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction. bitch ok I'll watch a TT vid or smth

4. (10 points) Consider the dot game we saw in class. Again there are two rows of dots, with n_1 and n_2 dots respectively, and players can remove any number of dots during their turn, but only from one row. However, now, the player who removes the last dot loses.

   If $n_1 = n_2 > 1$, prove that there is a winning strategy for the second player. What happens if $n_1 \neq n_2$?

   I wasn't paying attention in class but ok probably super cool and ez

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   (2^j) means 2 to the power j

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         sum ++
      }
      j=2*j
   }
```

1 3 8 2 9 4
good

1 3 8 2 9 4
good

1 3 8 2 9 4

1 3 2 8 9 4
good

1 3 2 8 9 4
bad

1 3 2 8 4 9

1 3 8 2 9 4
1 3 2 8 9 4
1 3 2 8 4 9
1 2 3 8 4 9
1 2 3 4 8 9

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To prove $P(n)$ true for integer $n \geq 1$

we ok

$(P(1) \wedge (\forall k \geq 1, P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1))) \rightarrow \forall n P(n)$

Basis step: verify that $P(1)$ is true

```
for i = 1 to n
   j=1
   while (j <= i) {
      k=1
      while (k <= n) {
         k=k*2
         sum ++
      }
      j=2*j
   }
```

25
40
50
65
75
80
90
105
125
100
115
130
145
160

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what am I supposed to prove???

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base $n_1 = n_2 = 2$

player 1 will remove 1 or 2 dots from a row

if remove 1 dot, p2 can remove 2 dots from other row + win

if remove 2 dot, p2 can remove 1 dot from remaining row + win

win

IH for all $2 \leq j \leq k$ p2 can win

IS $n = k+1$

p1 removes $1 \le r \le k+1$ dots from a row

· x 0 · x k+1 ↳ if $r = k+1$, p2 can remove k dots from other row, leaving 1 → win

· x 1 · x k+1 ↳ if $r = k$, p2 can remove all dots from other row, leaving 1 → win

~~↳ if $r = k-1$, p2 can remove r dots from other row, leaving lose case~~ → win

· x ≥2 · x k+1 ↳ if $r < k$, p2 can remove r dots from other row, leaving IH → win

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