CSci 243 Homework 2

Due: Wednesday, Sept 21
My name

1. Using logical identities and laws, show the logic equivalence of

$p \leftrightarrow q$
$\equiv (p \rightarrow q) \land (q \rightarrow p)$
$\equiv (\neg p \lor q) \land (\neg q \lor p)$

   (a) (5 points) $p \leftrightarrow q$ and $(p \land q) \lor (\neg p \land \neg q)$

   (b) (5 points) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

2. (10 points) Prove that if $3n+2$ is even integer then n is even, using

   (a) a proof by contraposition.

   (b) a proof by contradiction.

$(p \land q) \lor \neg(p \lor q)$

   (Hint: see a similar set of proofs in the book).

3. (5 points) Disprove the statement that "for any integer $x > 1$, there are positive integers y, z, such that $x^2 = y^2 + z^2$", by finding a counterexample.

4. (5 points) Prove that there are no positive perfect cubes (i.e., x^3) less than 1000 that are the sum of the cubes of two positive integers (i.e., $\exists x, y, z > 0 (x^3 < 1000 \land x^3 = y^3 + z^3)$).

5. (5 points) Prove that $3 \cdot 10^{100} + 11$ is not a perfect square **OR** $3 \cdot 10^{100} + 12$ is not a perfect square. Is your proof constructive or nonconstructive?

6. (5 points) Given three numbers, prove that at least one pair of them has nonnegative product.

1. Using logical identities and laws, show the logic equivalence of

   (a) (5 points) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

   (b) (5 points) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

①

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

②

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

(a)

① $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

② $\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

~~Same thing oops~~

$\cancel{\equiv (\neg q \to \neg p) \wedge (\neg p \to \neg q)}$

$\cancel{\equiv (q \vee \neg p) \wedge (p \vee \neg q)}$

$\cancel{\equiv (\neg p \vee q) \wedge (p \vee \neg q)}$

~~DeMorgan~~ $\cancel{\equiv \neg(p \wedge \neg q) \wedge \neg(\neg p \wedge q)}$

~~Demorgan?~~ $\cancel{\equiv \neg(\neg(p \wedge \neg q) \vee \neg(\neg p \wedge q))}$

$\cancel{\equiv (p \wedge \neg q) \vee (\neg p \wedge q)}$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$

distribute! $\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

$\equiv F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$

identity $\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

(b) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\neg p \vee q$

$\neg p \leftrightarrow \neg q$

$\equiv$

$(\neg p \vee q)$

$\equiv \neg(p \wedge \neg q)$

1. Using logical identities and laws, show the logic equivalence of

   (a) (5 points) $p \leftrightarrow q$ and $(p \land q) \lor (\neg p \land \neg q)$

   (b) (5 points) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

(a)

① $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

② $\equiv (\neg p \lor q) \land (\neg q \lor p)$

$\equiv (\neg p \lor q) \land (p \lor \neg q)$

distribute! $\equiv (\neg p \land p) \lor (\neg p \land \neg q) \lor (q \land p) \lor (q \land \neg q)$

$\equiv F \lor (\neg p \land \neg q) \lor (q \land p) \lor F$

identity $\equiv (\neg p \land \neg q) \lor (q \land p)$

(b) $p \leftrightarrow q \cancel{\equiv}$

$\neg p \leftrightarrow \neg q$

$\equiv (\neg p \to \neg q) \land (\neg q \to \neg p)$

$\equiv (q \to p) \land (p \to q)$

$\equiv p \leftrightarrow q$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

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TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \land \neg q$
$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

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2. (10 points) Prove that if $3n+2$ is even integer then n is even, using

(a) a proof by contraposition.

(b) a proof by contradiction.

(Hint: see a similar set of proofs in the book).

(a) contraposition

↳ assume q is false & show p is false

↓

assume n is not even

↳ adding $+2$ to an even/odd number — still an even/odd #

so, $3n+2$ shares even/odd nature with $3n$

3 is odd, and assuming n is odd

odd times odd = odd (do I need to explain more?)

therefore $3n$ is odd and $3n+2$ is odd when n is odd

when n is odd, $3n+2$ is odd

$\neg q \rightarrow \neg p$

therefore if $3n+2$ is even then n is even

$\neg q \rightarrow \neg p \equiv p \rightarrow q$

(2b) contradiction: assume p is false, show a contradiction $(r \wedge \neg r)$

assume $3n+2$ is odd

$3n$ is odd b/c shares nature with $3n+2$

odd times odd makes an odd

n has to be odd for $3n$ to be odd

this doesn't show a contradiction or anything

---

assume that both p and $\neg q$ are true

$\underbrace{3n+2 \text{ is even}}$ $\underbrace{n \text{ is odd}}$ → because we wanna prove

$3n+2$ is even $\rightarrow$ n is even

so say that

$3n+2$ is even $\wedge$ n is odd

and show that it is not true

$3n$ is even, b/c same nature as $3n+2$

odd times even makes even

↓ ↓

3 times n

so, $\underbrace{n \text{ is even}}$

q

$(\neg q \wedge q)$ contradiction

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counter example

3 $\boxed{x = 2}$

$x^2 = 4$

$y^2 + z^2$

smallest is $y = 1$ $z = 1$

$y^2 + z^2 = 2$

~~next is $y = 1$ $z = 2$~~

~~$y^2 + z^2 = 1 + 4 = 5$~~

already passed 4

$0, 1, 4, 9, 16$

→ no combo of 2 of these makes 3

$y=0$	$z=0$	$y^2+z^2 = 0$
$y=0$	$z=1$	$= 1$
$y=1$	$z=1$	$= 2$
$y=0$	$z=2$	$= 4$
$y=1$	$z=2$	$= 5$

→ no way to make 3

$y = 2$ $z = 0$

$y^2 + z^2 = 4$

so let's make $x = 3$

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4

perfect cubes less than 1000

1, 8, 27, 64, 125, 216, 343, 512, 729
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9

goal: combine two of these to make another one

$x_n - x_{n-1}$		m_n
729 - 512	=	217
512 - 343	=	169
343 - 216	=	127
216 - 125	=	91
125 - 64	=	61
64 - 27	=	37
27 - 8	=	19
8 - 1	=	7

$x_{n-1} + x_{n-2}$
512 + 343 = 855
343 + 216 =
216 + 125 =
125 + 64 =
64 + 27 =
27 + 8 =
8 + 1 =

$x_{n-2} + x_{n-3}$	=	m_n	<	x_n
343 + 216	=	559	<	729
216 + 125	=	341	<	512
125 + 64	=	189	<	343
64 + 27	=	91	<	216
27 + 8	=	35	<	125
8 + 1	=	9	<	64
		N/A		25
		N/A		1

$m_n > x_{n-2} + x_{n-3}$

one of the two cubes we pick, one has to be x_{n-1}, because x_{n-2} combined with anything smaller is always less than x_n

why?

because $x_n > x_{n-2} + x_{n-3}$ already. which mean any smaller combos are $< x_n$ as well.

(4)

1, 8, 27, 64, 125, 216, 343, 512, 729

x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9

goal: combine two of these to make another one

$x_{n-2} + x_{n-3}$	=	l_n	<	x_n
343 + 216	=	559	<	729
216 + 125	=	341	<	512
125 + 64	=	189	<	343
64 + 27	=	91	<	216
27 + 8	=	35	<	125
8 + 1	=	9	<	64
		N/A		25
		N/A		1

→

$x_n - x_{n-1}$		m_n
729 − 512	=	217
512 − 343	=	169
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125 − 64	=	61
64 − 27	=	37
27 − 8	=	19
8 − 1	=	7

here we take the second and third squares below the square we want to sum to (x_{n-2}, x_{n-3}) and see that their sum (l_n) is always less than the square we aimed for (x_n)

this shows that any combinations of only the squares below x_{n-1} will be smaller than x_n.

therefore, x_{n-1} has to be one of the two squares used in the sum.

here we take the square we want to sum to (x_n) and subtract the next square of a lower value than x_n (x_{n-1})

we see that the differences (m_n) are not equal to any of the other squares of positive integers smaller than x_n.

Because we know that the perfect square x_{n-1} has to be one of the numbers used in the sum, and none of the numbers m_n seen here are also perfect squares, there are no possible sums of two perfect squares such that the sum is a perfect square less than 1000.

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5

$3 \cdot 10^{10^{10}}$

if it was a perfect square

$\sqrt{3 \cdot 10^{100} + 12}$ = an integer

$\sqrt{3(10^{100}+4)}$ = an integer

$\sqrt{3}\sqrt{10^{100}+4}$ contradiction

$\sqrt{3}$ is irrational
whether $\sqrt{10^{100}+4}$ is rational or irrational, doesnt matter

an irrational $\times$ rational = irrational
or irrational $\times$ irrational = irrational

integers cannot be irrational

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6. x y z

prove $xy > 0$ or $yz > 0$ or $xz > 0$

if x, y, z all > 0
all products of pairs are > 0

if one is < 0
the product of the other 2 is > 0

if two are < 0
their product is > 0

if all three are < 0
all product pairs are $>$