CSci 243 Homework 6

Due: Monday, October 24, end of day
My name

1. (10 points) Consider the following recursive definition for any $n \geq 1$:
   Let $S_1(n) = \sum_{i=1}^{n} i = n(n+1)/2$.
   Let $S_{k+1}(n) = \sum_{i=1}^{n} i^{k+1}$, for any $k \geq 1$.
   Prove the recurrence in terms of k: $S_{k+1}(n) = nS_k(n) - \sum_{i=1}^{n-1} S_k(i)$.
   Hint: Prove first that $S_{k+1}(n) = \sum_{j=1}^{n} \sum_{i=j}^{n} i^k$. Then, writing this as a table you can consider the row sums as the difference between two sums. Summing for each row will give you the result.

2. (10 points) Consider the series $\sum_{k=2}^{2n+1} \frac{2}{k^2-1}$. Write the series as a telescoping series and prove that

$$\sum_{k=2}^{2n+1} \frac{2}{k^2-1} = \frac{3}{2} - \frac{1}{2n+1} - \frac{1}{2n+2}.$$

   Hint: write out at least the first six terms and the last two terms, and group them in pairs of two.

3. (10 points) Arrange the functions

$$3^n, 2^n, n2^n, n^{30}, (\log n)^3, \sqrt{n}\log^2 n, n\log n, \sqrt{n!}, n^{29} + n^{27}, n^{2\sqrt{n}}$$

   into increasing order of growth rates.

   $\cancel{(\log n)^3}$ $\sqrt{n}\log^2 n$ $(\log n)^3$? $n\log n$ $n^{2\sqrt{n}}$ $[n^{29} + n^{27}]$ n^{30} $n2^n$ 2^n 3^n $\sqrt{n!}$?

4. (10 points) To solve a particular problem you have access to two algorithms. The execution time of the first algorithm can be given as a function of the input size n as $f(n) = n^{1.5}\log^2 n$. The execution time of the second algorithm is similarly: $g(n) = n^2$. Which algorithm is faster asymptotically? Is this algorithm faster for small n? Find the minimum problem size n needed so that the fastest asymptotic algorithm becomes faster than the other one. Hint: limit your search in powers of 2. You may use calculators to help you but the answer must self contained.