

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

My name

1. (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$

$a_n = a_{n-1} + 4$

$1+(-1)^n = 0, 1, 0, 1, 0, 1, \dots$
1 2 3 4 5 6 ...

(b) $a_n = 1 + (-1)^n$

(c) $a_n = (\frac{1}{2})^n$

$4-2$ $8-2$ $12-2$ $16-2$ $20-2$
2 6 10 14 18

$a_n = 1^{n-1}$

2. For string $w = a_1 a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2 a_1$.

- (a) (4 points) What is ϵ^R ? What is $(10110)^R$?

- (b) (4 points) Give a recursive definition of the reversal of a string.

- (c) (6 points) Use structural induction to prove that $(w_1 w_2)^R = w_2^R w_1^R$.

3. (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?

4. (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?

5. An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

ayyhhk...
woyhhk...

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

My name

1. (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$ basis: $a_1 = 4 - 2 = 2$ recursive: $a_n = a_{n-1} + 4$ c. $a_n = (\frac{1}{2})^n$ basis: $a_1 = \frac{1}{2}$ recursive: $a_n = a_{n-1} \cdot \frac{1}{2}$

(b) $a_n = 1 + (-1)^n$ basis: $a_1 = 1 + (-1)^1 = 1 - 1 = 0$ recursive: $a_n = 1 + (-1)^n$ basis: $a_1 = 1 + (-1)^1 = 0$ recursive: $a_n = 1 + (-1)^n$

(c) $a_n = (\frac{1}{2})^n$ basis: $a_1 = \frac{1}{2}$ recursive: $a_n = a_{n-1} \cdot \frac{1}{2}$

2. For string $w = a_1 a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2 a_1$.

(a) (4 points) What is ϵ^R ? What is $(10110)^R$? $\epsilon^R = \text{empty string}^R = \epsilon$ $10110^R = 01101$

(b) (4 points) Give a recursive definition of the reversal of a string. basis: $\epsilon^R = \epsilon$ recursive: $(a_1 a_2 \dots a_n)^R = a_n \dots a_2 a_1$

(c) (6 points) Use structural induction to prove that $(w_1 w_2)^R = w_2^R w_1^R$. basis: $(a_1 a_2)^R = a_2 a_1$ recursive: $(a_1 \dots a_n)^R = a_n \dots a_1$

3. (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?
4. (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?
5. An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

My name

- (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$

(b) $a_n = 1 + (-1)^n$

(c) $a_n = (\frac{1}{2})^n$

- For string $w = a_1a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2a_1$.

(a) (4 points) What is ϵ^R ? What is $(10110)^R$?

(b) (4 points) Give a recursive definition of the reversal of a string.

(c) (6 points) Use structural induction to prove that $(w_1w_2)^R = w_2^Rw_1^R$.

- (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?

- (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?

- An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

$(w_1w_2)^R = w_2^Rw_1^R$
 $(1\ 2)^R = 2^R1^R$
 $2\ 1 = 2\ 1$
 base $w_1 = \epsilon$ $w_2 = \epsilon$
 $(\epsilon\ \epsilon)^R = \epsilon^R\ \epsilon^R$
 $\epsilon\ \epsilon = \epsilon\ \epsilon$
 IH Assume for two strings w_1 and w_2 the equality holds
 $(w_1w_2)^R = w_2^Rw_1^R$
 IS add a character to each string?
 add a character to each string?
 what do I do
 $w_1 = a_1a_2 \dots a_n$ $w_2 = b_1b_2 \dots b_m$
 $(w_1w_2)^R = w_2^Rw_1^R$
 $(a_1a_2 \dots a_nb_1b_2 \dots b_m)^R = b_m^Rb_{m-1}^R \dots b_1^Ra_n^Ra_{n-1}^R \dots a_1^R$
 $w_1^Rw_2^R = a_n^Ra_{n-1}^R \dots a_1^Rb_m^Rb_{m-1}^R \dots b_1^R$
 $w_1^Rw_2^R = w_2^Rw_1^R$

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

****My name****

1. (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$

(b) $a_n = 1 + (-1)^n$

(c) $a_n = \left(\frac{1}{2}\right)^n$

2. For string $w = a_1a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2a_1$.

(a) (4 points) What is ϵ^R ? What is $(10110)^R$?

(b) (4 points) Give a recursive definition of the reversal of a string.

(c) (6 points) Use structural induction to prove that $(w_1w_2)^R = w_2^Rw_1^R$.

3. (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?

4. (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?

5. An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

method isPalindrome (w)

if w.length == 1 or w.length == 0
return true

else if w.length > 1

left = 0

right = w.length - 1

while left < right

if left != right

return false

left += 1

right -= 1

performance: $\frac{1}{2}n$

= $O(n)$

each step compares 2 elements

recursive!

method isPalindrome (w)

if w.length == 0 or w.length == 1

return true

if w.length == 2

return (w[0] == w[1])

if w.length > 2

outer_matching = w[0] == w[w.length - 1]

sub_string = w with first and last characters removed

return (outer_matching and isPalindrome (sub_string))

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

My name

1. (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$

(b) $a_n = 1 + (-1)^n$

(c) $a_n = (\frac{1}{2})^n$

2. For string $w = a_1a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2a_1$.

(a) (4 points) What is ϵ^R ? What is $(10110)^R$?

(b) (4 points) Give a recursive definition of the reversal of a string.

(c) (6 points) Use structural induction to prove that $(w_1w_2)^R = w_2^Rw_1^R$.

3. (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?

4. (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?

5. An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

~~FindMax ([a₁, ..., a_n])~~
~~max ← a₁~~
~~for i in range (n)~~
~~if a_i > max~~
~~max ← a_i~~
~~return max~~

complexity O(n)
looks at each element

method FindMax (a₁, ..., a_n)
 if n == 2
 return max (a₁, a₂)
 else
 return max (a_n, FindMax (a₁, ..., a_{n-1}))

recursive !!

CSci 243 Homework 9

Due: Wednesday, November 16, end of day

My name

1. (6 points) Give a recursive definition for each of the following sequences $\{a_n\}$ for $n = 1, 2, 3, \dots$

(a) $a_n = 4n - 2$

(b) $a_n = 1 + (-1)^n$

(c) $a_n = (\frac{1}{2})^n$

2. For string $w = a_1 a_2 \dots a_n$, the reversal of the string is defined as $w^R = a_n \dots a_2 a_1$.

(a) (4 points) What is ϵ^R ? What is $(10110)^R$?

(b) (4 points) Give a recursive definition of the reversal of a string.

(c) (6 points) Use structural induction to prove that $(w_1 w_2)^R = w_2^R w_1^R$.

3. (10 points) A palindrome is a string that reads the same forward and backward, i.e., $w = w^R$. Give a recursive algorithm in pseudocode that checks whether a given string w is a palindrome. What is the time complexity of your algorithm?

4. (10 points) Give a recursive algorithm in pseudocode that finds the maximum number among n integers. What is the time complexity of your algorithm?

5. An employee joined a company in 2015 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 4% of the salary of the previous year.

- (3 points) Set up a recurrence relation for the salary of this employee n years after 2015.
- (4 points) Find an explicit formula for the salary of this employee n years after 2015.
- (3 points) What will the salary of this employee be in 2030?

② $a_0 = 50,000$
 $a_n = a_{n-1} \cdot 1.04 + 1000$

⑤ $a_n = 50,000 \times 1.04^n + \underbrace{1000 \times 1.04^{n-1} + 1000 \times 1.04^{n-2} + \dots + 1000 \times 1.04^1 + 1000}_{\text{geometric sequence}}$
 $= 50,000 \times 1.04^n + \frac{1000 \times 1.04 (1 - 1.04^{n-1})}{1 - 1.04} + 1000$
 $= 50,000 \times 1.04^n + \frac{1000 \times 1.04 (1 - 1.04^{n-1})}{-0.04} + 1000$
 $= 50,000 \times 1.04^n + \frac{1040 (1 - 1.04^{n-1})}{-0.04} + 1000$

$a_1 = 50,000 \times 1.04 + 1000$
 $a_2 = a_1 \times 1.04 + 1000$
 $= (50,000 \times 1.04 + 1000) \times 1.04 + 1000$
 $= 50,000 \times 1.04^2 + 1000 \times 1.04 + 1000 + 1000$
 $a_3 = (50,000 \times 1.04^2 + 1000 \times 1.04 + 1000) \times 1.04 + 1000$
 $= (50,000 \times 1.04^3 + 1000 \times 1.04^2 + 1000 \times 1.04 + 1000)$

⑥ $a_{15} = 50,000 \times 1.04^{15} + \frac{1000 \times 1.04 (1 - 1.04^{14})}{-0.04} + 1000$
 $= 50,000 \times 1.8009 + \frac{1000 \times 1.04 (1 - 1.7317)}{-0.04} + 1000$
 $= 90047.175 + \frac{1040 (-0.7317)}{-0.04} + 1000$
 $= 90047.175 + \frac{-760.968}{-0.04} + 1000$
 $= 90047.175 + 19,024.2 + 1000$
 $= 110,071.375$