

# CSci 243 Homework 4

Due: Wednesday, October 5, end of day  
 \*\*My name\*\*

1. (10 points) For sets  $A$ ,  $B$ , and  $C$ , prove that  $(B - A) \cup (C - A) = (B \cup C) - A$

- (a) by using set properties and propositional calculus
- (b) by using a membership table

2. (5 points) Find these values.

- (a)  $\lfloor 1.1 \rfloor$
- (b)  $\lceil 1.1 \rceil$
- (c)  $\lfloor -0.1 \rfloor$
- (d)  $\lceil -0.1 \rceil$
- (e)  $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$

3. (8 points) Determine whether each of these functions from  $\mathbb{Z}$  or  $\mathbb{Z} \times \mathbb{Z}$  to  $\mathbb{Z}$  is one-to-one, onto, both, or neither. Justify your answer.

- (a)  $f(n) = 2n - 1$  a [one-to-one: yes  
onto: for  $f(n) = 0$ , no possible  $n$  in  $\mathbb{Z}$ ;  $n \text{ cont} = \frac{1}{2}$   
 $\hookrightarrow n \text{ has to be } \frac{1}{2} \text{ for } f(n) \text{ to work}$
- (b)  $f(n) = |n| + 1$  b [one-to-one: no!  
onto: no!]
- (c)  $f(n) = n^3$
- (d)  $f(n) = \lfloor \frac{n}{2} \rfloor$

4. (12 points) A quick review of algebra. Answer the following:

- (a) Simplify:  $(q^3 p^{2/3})(\sqrt{p^3} q^{-1}) =$
- (b) Simplify:  $a^2 a^4 a^7 a^5 a^3 = 1+2+4+7+5+3 = 22 = a^{22}$
- (c) Simplify to have no  $x, y$  in the same factors:  $(\log(xy)) \log(x/y) =$
- (d) Add the fractions to produce one fraction:  $\frac{2}{x^2+4x+4} + \frac{1}{x+2} =$
- (e) If  $x, y, z, d \in \mathbb{R}$ , are the following True or False?

$$\begin{aligned}
 & \text{F i. } x < y \wedge z < d \rightarrow xz < yd \quad \text{- } x \text{ less than } y \text{ and } z \text{ less than } d \text{ therefore } xz < yd \\
 & \text{F ii. } x < y \wedge z < d \rightarrow xd < yz \quad \text{e.ii.} \\
 & \text{Fiii. } x > 0 \wedge y > 0 \wedge x < y \wedge z < d \rightarrow xd < yz \quad \text{therefore } xz < yd \\
 & \text{F iv. } x > 0 \wedge y > 0 \wedge z > 0 \wedge d > 0 \wedge x < y \wedge z < d \rightarrow xd < yz \\
 & \text{(f) Solve for } x: \\
 & \text{i. } 10^{3x+5} = 100 \quad 2 < 4 \\
 & \text{ii. } 3^{x^2} (9^x) = 1/3 \quad 3 < 4 \\
 & \text{iii. } \log_x 64 = 2 \quad 2.4 < 3.5 \\
 & \text{iv. } \log_3(2x+1) = \log_9(x^2) \quad 2.5 < 3.4 \\
 & \text{Solve: } x < 4 \rightarrow x^2 < y^2 \quad 6 < 7 \\
 & z < d \rightarrow zd < yd \quad 8 < 100 \\
 & b < 7 \quad 600 < 0.7 \\
 & b < 100 \quad 600 \not< 0.7 \\
 & b < 100 \quad \text{False} \\
 & \text{F. 6th negative number} \\
 & 0 < 2 \\
 & -3 < -2 \\
 & 0 < -4 \\
 & \text{no!}
 \end{aligned}$$

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- (a) by using set properties and propositional calculus
- (b) by using a membership table

2. (5 points) Find these values.

- (a)  $\lfloor 1.1 \rfloor = 1$
- (b)  $\lceil 1.1 \rceil = 2$
- (c)  $\lfloor -0.1 \rfloor = -1$
- (d)  $\lceil -0.1 \rceil = 0$
- (e)  $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor = \lfloor \frac{1}{2} + 1 \rfloor = 1$



3. (8 points) Determine whether each of these functions from  $\mathbb{Z}$  or  $\mathbb{Z} \times \mathbb{Z}$  to  $\mathbb{Z}$  is one-to-one, onto, both, or neither. Justify your answer.

- (a)  $f(n) = 2n - 1$       c. [one-to-one: yes?  
onto: yes!  $\mathbb{Z}$  map  $\mathbb{Z}$ ]  
d. [one-to-one: no, 2 values map to each value f(a)]
- (b)  $f(n) = |n| + 1$       e. [onto: yes!  
one-to-one: no, 2 values map to each value f(a)]
- (c)  $f(n) = n^3$       f. [onto: yes!  
one-to-one: yes!]
- (d)  $f(n) = \lfloor \frac{n}{2} \rfloor$       g. [onto: yes!  
one-to-one: no, 2 values map to each value f(a)]

4. (12 points) A quick review of algebra. Answer the following:

- (a) Simplify:  $(q^3 p^{2/3})(\sqrt{p^3} q^{-1}) =$
- (b) Simplify:  $aa^2 a^4 a^7 a^5 a^3 =$
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- (d) Add the fractions to produce one fraction:  $\frac{2}{x^2+4x+4} + \frac{1}{x+2} =$
- (e) If  $x, y, z, d \in \mathbb{R}$ , are the following True or False?

- i.  $x < y \wedge z < d \rightarrow xz < yd$
- ii.  $x < y \wedge z < d \rightarrow xd < yz$
- iii.  $x > 0 \wedge y > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$
- iv.  $x > 0 \wedge y > 0 \wedge z > 0 \wedge d > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$

- (f) Solve for  $x$ :

- i.  $10^{3x+5} = 100$
- ii.  $3^{x^2}(9^x) = 1/3$
- iii.  $\log_x 64 = 2$
- iv.  $\log_3(2x+1) = \log_9(x^2)$

$$\begin{aligned} 10^{3x+5} &= 100 \\ \log_{10} 10^{3x+5} &= \log_{10} 100 \\ 3x+5 &= 2 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

$x = -1$

$$\begin{aligned} \log_3(2x+1) &= 2 \\ 2x+1 &= 3^2 \\ 2x+1 &= 9 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

$x = 4$

$$\begin{aligned} \log_3(2x+1) &= \log_9(x^2) \\ \log_3(2x+1) &= \log_3(x^2) \\ (2x+1)^2 &= x^2 \\ 4x^2 + 4x + 1 &= x^2 \\ 3x^2 + 4x + 1 &= 0 \\ (3x+1)(x+1) &= 0 \\ x &= -\frac{1}{3} \end{aligned}$$

$x = -\frac{1}{3}$

$$\begin{aligned} \log_x 64 &= 2 \\ x^2 &= 64 \\ x &= \pm 8 \end{aligned}$$

$x = 8$

$$\begin{aligned} \log_3(2x+1) &= \log_9(x^2) \\ \log_3(2x+1) &= \log_3(x^2) \\ (2x+1)^2 &= x^2 \\ 4x^2 + 4x + 1 &= x^2 \\ 3x^2 + 4x + 1 &= 0 \\ (3x+1)(x+1) &= 0 \\ x &= -\frac{1}{3} \end{aligned}$$

$x = -\frac{1}{3}$

$$(4a) \quad \boxed{a^{22}}$$

$$\begin{aligned} &q^7 \cdot p^{2/3} \cdot \sqrt{p^3} \cdot q^{-1} \\ &= q^7 \cdot p^{2/3} \cdot p^{3/2} \cdot p^{-1} = \boxed{q^2 \cdot p^{13/6}} \\ &\frac{2}{3} + \frac{3}{2} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6} = 2\frac{1}{6} \end{aligned}$$

$$(4c) \quad \log(xy) = \log x + \log y$$

$$\log(x/y) = \log x - \log y$$

$$(\log x + \log y)(\log x - \log y)$$

$$= (\log x)^2 + \log x \log y - \log x \log y - (\log y)^2$$

$$= (\log x)^2 + (\log y)^2$$

~~$$\begin{aligned} &= 2 \log x + 2 \log y \\ &= 2 (\log x + \log y) \end{aligned}$$~~

cancel ab  
thus  
simplification  
 $\log x^2$   
 $\neq (\log x)(\log x)$

$$\begin{aligned} &\frac{2}{(x+2)(x+4)} + \frac{1}{(x+2)} \\ &\frac{2}{(x+2)(x+4)} + \frac{1}{(x+2)} \end{aligned}$$

$$\begin{aligned} &\frac{2}{(x+2)(x+4)} + \frac{(x+2)}{(x+2)(x+4)} = \frac{2+x+2}{(x+2)(x+4)} = \boxed{\frac{x+4}{(x+2)(x+4)}} \end{aligned}$$

Organic Chem. Tutor - Logarithms Explained (1:32:52)

$\log_2 4$  = how many 2's do I have =  $2^x = 4$ ? = answer is  $\boxed{2}$   
to multiply to get 4?

$$\log_2 8 = 3$$

$$\log_2 9 = 2$$

$$\log_2 16 = 2$$

$$\log_2 27 = 3$$

$$\log_2 32 = 5$$

$$\log_2 128 = 7$$

$$\log_2 36 = 2$$

$$\log_2 124 \approx 6$$

$$\log_2 28 \approx 4$$

$$\log_2 1 = 0 \quad 2^0 = 1$$

$\log(10) =$  when you don't see a base, assume it to be 10 = 1

$$\log(100) = 2$$

$$\log(1000) = 3$$

$$\log(10000) \approx 4$$

$$\log(0.1) = -1$$

$$\log(0.01) = -2$$

$$\log(0.001) \approx -3$$

$\log(2) \Rightarrow$  ONE count less than the value of  $\log(1)$

$\log(-5) \Rightarrow$  ONE count less negative than the value of  $\log(1)$

$$\log_2(16) = 4$$

$$\log_2\left(\frac{1}{16}\right) = -4 \quad 4^{-1} = \frac{1}{4} \quad 4^{-2} = \frac{1}{16} = \frac{1}{4^2}$$

$$\log_{10}(4) = \frac{1}{2} \quad 10^{1/2} = \sqrt{10} > 4$$

$$\log_{10}\left(\frac{1}{4}\right) = -\frac{1}{2} \quad 10^{-1/2} = \frac{1}{\sqrt{10}} = \frac{1}{4}$$

$$\log_2(216) \approx 8$$

$$\log_2\left(\frac{1}{216}\right) = -8$$

$$\log_{10}\left(\frac{1}{6}\right) = -0.5$$

$$\log_{10}(6) = 0.5$$

$$\log_a b = \frac{\log_b b}{\log_a a} \text{ or convert base to whatever}$$

$$\log_a b = ?$$

$$\frac{\log_b b}{\log_a a} = (\text{calculator says})^2$$

$$\log_s(x-y) = \frac{\log(x-y)}{\log s}$$

do convert - log to a natural log, use base e

$$= \frac{\ln(x-y)}{\ln(s)}$$

$$\log(A) + \log(B) = \log(AB)$$

$$\log(A) - \log(B) = \log(A/B)$$

$$\log A^2 = 2 \log A \quad \text{can move exponent to the front}$$

$$\log(x) + \log(y) - \log(z) = \log\left(\frac{xy}{z}\right)$$

$$\log(x) - \log(y) + \log(z) - \log(r) = \log\left(\frac{xz}{yr}\right)$$

$$2\log x + 3\log y - 4\log z = \text{can't cancel bc unknown} = \text{use power rule}$$

$$= \log x^2 + \log y^3 - \log z^4 = \log\left(\frac{x^2y^3}{z^4}\right)$$

$$\frac{1}{2}\log x - \frac{1}{3}\log y + \frac{1}{4}\log z = \log x^{1/2} - \log y^{1/3} + \log z^{1/4} = \log\left(\frac{x^{1/2}y^{-1/3}z^{1/4}}{1}\right)$$

$$\log\left(\frac{x^2y^5}{z^6}\right) = 2\log x + 5\log y - 6\log z$$

$$\log\sqrt[3]{\frac{xy^2}{z^4}} = \log\left(\frac{xy^2}{z^4}\right)^{1/3} = \frac{1}{3}\log\left(\frac{xy^2}{z^4}\right) = \frac{1}{3}[ \log x + \log y^2 - \log z^4 ] = \frac{1}{3}\log x + \frac{2}{3}\log y + \frac{4}{3}\log z$$

$$\log\left(\frac{x^2y^4z^5}{x^3y^2z^4}\right) = \log\left(\frac{yz^4}{x}\right) = 2\log x + \frac{1}{2}\log y - \frac{4}{3}\log z$$

ex 2 work standard 22 min in

$$\log_e e = \ln e$$

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln e^5 = 5 \ln e = 5(1) = 5$$

$$e^{\ln 7} = e^{\ln e^7} = \text{base equal} = 7.$$

$$e^{3\ln 2} = e^{\ln 2^3} = 2^3$$

$$2e^{\ln 4} = 2 \cdot 4$$

$$8e^{\ln 8} = 8^2$$

$$g^{\ln g \ln g+3} = g^3$$

$$g^{\ln g \ln g+3} = g^3+6$$

$$\log_2 6 = \log_2 2 + \log_2 3 = \log_2 (2 \cdot 3) = \log_2 3 = 1.$$

$$\log_2 12 + \log_2 24 - \log_2 4 = \log_2 (12 \cdot 24 / 4) = \log_2 (288/4) = \log_2 72 = 5$$

$$\log_2 (1/8) + \log_2 (1/64) = \log_2 (1/64) = -2$$

$$\log_2 (8) = 3 \quad 2^3 = 8$$

$$\log_3 9 = 2 \quad 3^2 = 9$$

$$\log_a b = c \quad a^c = b$$

$$\log_2 x = 8 \quad 2^8 = x$$

$$3^3 = 27 \quad \log_3 27 = 3$$

$$2^4 = 16 \quad \log_2 16 = 4$$

$$3^x = 18 \quad \log_3 18 = x$$

$$\log_2 4^x = 5 \rightarrow 2^5 = 4^x \rightarrow 32 = 4^x$$

$$\log_2(x+5) = 3 \rightarrow 2^3 = x+5 \rightarrow x = 3$$

$$\log_3(2x-3) = 2 \rightarrow 3^2 = 2x-3 \rightarrow 12 = 2x \rightarrow x = 6$$

$$\log_2(x-3) + \log_2(x-1) = 3$$

condense two log into a single one

$$\log_2((x-3)(x-1)) = 3$$

$$2^3 = (x-3)(x-1)$$

$$8 = x^2 - 4x + 3$$

$$0 = x^2 - 4x - 5$$

$$0 = (x-5)(x+1)$$

$$x = -1, 5$$

\* need to check & here are extraneous solutions!

$$\log_2(x-3) + \log_2(x-1)$$

$$\log_2(-5)$$

$$\log_2(2) + \log_2(4) \checkmark$$

$$\log_2(-1)$$

$$\log_2(-4) + \log_2(-2) \times \text{cut have negative in log!}$$

$$x = 5$$

$$\log_2(2x+1) + \log_2(x+8) = 3$$

$$2^3 = (2x+1)(x+8)$$

$$8 = 2x^2 + 17x + 8$$

$$0 = 2x^2 + 17x - 16 \quad \begin{matrix} \text{multiply by } -1 \\ \rightarrow 2x^2 + 17x - 16 \end{matrix}$$

$$0 = 2x(x-1) + 16(x-1)$$

$$0 = (2x+16)(x-1)$$

$$x = -8, 1$$

$$\text{plug in } -8 \text{, will get negative in log!}$$

$$x = 1$$

check our work

$$\log_2(3) + \log_2(2) = 3$$

$$1 + 2 = 3 \checkmark$$

$$\log_2(x+5) = \log_2(3x-9)$$

same base & same log so neither should be equal

$$x+5 = 3x-9 \rightarrow 14 = 2x \rightarrow x = 7$$

$$\ln x = 5 \quad e^5 = x$$

$$\ln x + 3 \ln x = 0$$

$$\rightarrow 4 \ln x = 0$$

$$\ln x = 0 \quad e^0 = x \quad x = 1$$

$$\ln(x-3) = 2 \quad e^2 = (x-3)$$

$$x = e^2 + 3$$

$$8^{2x+4} = 16^{2x-3}$$

want to convert both to a common base

$$(2^3)^{2x+4} = (2^4)^{2x-3}$$

$$2^{3(2x+4)} = 2^{4(2x-3)}$$

$2^7 = 8$        $2^8 = 16$

$$2^{7x+12} = 2^{8x-12}$$

base on eqn, we can set exponents equal

$$3x+12 = 8x-12$$

$$24 = 5x$$

$$x = \frac{24}{5}$$

$$x = 4.8$$

$$9^{2x+1} = 27^{3x+6}$$

$$(3^2)^{2x+1} = (3^3)^{3x+6}$$

$$3^{4x+2} = 3^{9x+18}$$

$$4x+2 = 9x+18$$

$$-5x = 16$$

$$x = -4$$

$$2^x = 7$$

$2^3=8$        $x$  is somewhere between 2 and 3

$$\ln 2^x = \ln 7$$

← taking ln of both sides  
base must be ln just as log base 10

$$\frac{x \ln 2}{\ln 2} = \frac{\ln 7}{\ln 2}$$

$$x = \frac{\ln 7}{\ln 2}$$

use calculator at this point

$$\approx 2.80735$$

$$e^{x+2} = 8$$

using ln here for sum rule x is present

$$\ln e^{x+2} = \ln 8$$

$$(x+2) \ln e = \ln 8$$

$\ln e = 1$

$$x+2 = \ln 8$$

$$x = \ln 8 - 2$$

$$p(x) = \ln(2x-6)$$

$$y = \ln(2x-6)$$

- $2x-6 > 0$       ln to be in real numbers
- $2x_> 6$
- $x > 3$

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- (a) by using set properties and propositional calculus  
 (b) by using a membership table

could prove that  
 $(B - A) \cup (C - A) \subseteq (B \cup C) - A$   
 and  $(B \cup C) - A \subseteq (B - A) \cup (C - A)$   
 $(B \cup C) - A$

Proof 3. In a membership table (like a truth table),

1s and 0s are used to specify if an arbitrary element  $x$  is in a set expression or not. The rows in the table are all possible membership combinations of  $x$ . If the columns for the two expressions are identical then the expressions are equal.

⑥ Membership Table aka a truth table (basically)

$A$	$B$	$C$	$B - A$	$C - A$	$(B - A) \cup (C - A)$
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	1	0	0
1	1	0	0	0	0
0	0	1	0	1	0
1	0	1	0	0	0
0	1	1	1	1	1
1	1	1	0	0	0

$B - A$

$\hookrightarrow 0 - 0 \text{ means}$   
 $x \text{ not in } A = 0$

$\hookrightarrow 0 - 1 \text{ means}$   
 $x \text{ not in } B$   
 $x \text{ is in } A$   
 $\text{deleting } x \text{ from } B = 0$

$\hookrightarrow 1 - 0$   
 $x \text{ is in } B$   
 $x \text{ not in } A$   
 $x \text{ is still in } B = 1$

$\hookrightarrow 1 - 1$   
 $x \text{ is in } B$   
 $x \text{ not in } A$   
 $\text{Finally, both } x \text{ cancel out} = 0$

I looked it up this is right

0  
0  
0

//

$A$	$B$	$C$	$B \cup C$	$(B \cup C) - A$
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
1	1	0	0	0
0	0	1	0	0
1	0	1	0	0
0	1	1	1	1
1	1	1	1	0

⑦