CSci 243 Homework 1

Due: Wednesday, Sep 14, end of day
My name

1. Construct a truth table for each of these compound propositions.

   (a) (3 points) $(p \vee \neg q) \rightarrow q$

   (b) (7 points) $(p \leftrightarrow q) \rightarrow (r \leftrightarrow s)$

2. Understanding quantified predicates.

   (a) (3 points) English to quantified predicates: Use predicates, quantifiers, logical and mathematical operators to express statement "The absolute value of the sum of two integers does not exceed the sum of the absolute value of these integers". $|a+b| \leq |a| + |b|$

   (b) (7 points) Give the truth value of each of these statement if the domain of all variables consists of all real numbers. Explain very briefly.

      i. $\forall x \exists y (x^2 = y)$
      ii. $\exists x \forall y (xy = 0)$
      iii. $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$
      iv. $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$
      v. $\forall x \exists y (x + y = 1)$
      vi. $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$
      vii. $\forall x \forall y \exists z (z = (x+y)/2)$

3. (5/30, 10 points) Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside a quantifier or an expression involving logical operators.

   (a) $\neg \exists x \exists y P(x, y)$

   (b) $\neg \forall x \exists y P(x, y)$

   (c) $\neg \exists y (Q(y) \wedge \forall x \neg R(x, y))$

   (d) $\neg (\exists x R(x, y) \vee \forall x S(x, y))$

   (e) $\neg \exists y (\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

①

p	q	$(p \vee \neg q)$	$(p \vee \neg q) \to q$
F	F	T	F
F	T	F	T
T	F	T	F
T	T	T	T

p	q	r	s	$p \Leftrightarrow q$	$r \Leftrightarrow s$	$(p \Leftrightarrow q) \to (r \Leftrightarrow s)$
F	F	F	F	T	T	T
F	F	F	T	T	F	F
F	F	T	F	T	F	F
F	F	T	T	T	T	T
F	T	F	F	F	T	T
F	T	F	T	F	F	T
F	T	T	F	F	F	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	F	T	F	F	T
T	F	T	F	F	F	T
T	F	T	T	F	T	T
T	T	F	F	T	T	T
T	T	F	T	T	F	F
T	T	T	F	T	F	F
T	T	T	T	T	T	T

(2)

T i. $\forall x \exists y (x^2 = y)$
T ii. $\exists x \forall y (xy = 0)$
T iii. $\forall x (x \neq 0 \to \exists y (xy = 1))$
F iv. $\exists x \forall y (y \neq 0 \to xy = 1)$
T v. $\forall x \exists y (x + y = 1)$
F vi. $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$
F vii. $\forall x \forall y \exists z (z = (x + y)/2)$

(a) $|a+b| \leq |a| + |b|$

(b)

(i) for all x, there exists a y such that $x^2 = y$
T, there is no real number value of x that is not a real number y after being squared

(ii) T, when $x=0$ all values of y still cause xy to equal 0

(iii) for all x, $x \neq 0$, there exists a y such that $xy = 1$
T, $y = \frac{1}{x}$

(iv) there exists an x such that for all y $(y \neq 0)$, $xy = 1$
F, there's no one value of x that makes this true for all y

(v) for all x, there exists a y such that $(x+y=1)$
T, $y = 1-x$

(vi) for all x, there exists a y such that $x+y = 2$ AND $2x - y = 1$

$x + y = 2$
$2x - y = 1$

$2x = 1 + y$
$2x + 2y = 4$
$1 + y + 2y = 4$
$3y = 3$
$y = 1$
uhh now what

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}$$

$R_2 = R_2 - 2R_1 \Rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & -3 \end{pmatrix}$

$-\frac{1}{3}R_2 \Rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$

$R_1 = R_1 - R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$\begin{cases} x = 1 \\ y = 1 \end{cases}$ uhh what do I do with this

$x = 4$
$y = -2$

$x + y = 2$
$4 - 2 = 2$

$2x - y = 1$
$8 - 2 \neq 1$

$$\boxed{\begin{gathered} x = 1000 \\ y = -998 \\ x+y = 2 \\ 2x - y = 1 \\ 2000 + 998 \neq 1 \end{gathered}}$$

F

ok I just had to prove it false w/ an example

(vii) for all x for all y there exists a z such that $z = \frac{x+y}{2}$
F, if $x + y = 0$, $z = \frac{0}{2}$ → ~~not a real number~~
no it is a real number

3. (5/30, 10 points) Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside a quantifier or an expression involving logical operators.

(a) $\neg\exists x\exists y P(x,y)$

(b) $\neg\forall x\exists y P(x,y)$

(c) $\neg\exists y(Q(y)\wedge\forall x\neg R(x,y))$

(d) $\neg(\exists x R(x,y)\vee\forall x S(x,y))$

(e) $\neg\exists y(\forall x\exists z T(x,y,z)\vee\exists x\forall z U(x,y,z))$

implication equivalences

$p \to q \equiv \neg q \to \neg p \equiv \neg p \vee q$

double negative law: $\neg(\neg p) \equiv p$

negation law: $p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

de morgan's law: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

de morgan's law w/ quantifiers $\neg\forall x P(x) \equiv \exists x(\neg P(x))$

$\neg\exists x P(x) \equiv \forall x(\neg P(x))$

in a sequence of quantifiers, apply De Morgans from out to in

$\neg\forall x\forall y P(x,y) \equiv \exists x \neg(\forall y P(x,y))$

$\equiv \exists x\exists y \neg P(x,y)$

(a) $\neg\exists x\exists y P(x,y)$

$\equiv \forall x \neg(\exists y P(x,y))$

$\equiv \forall x\forall y \neg P(x,y)$

(b) $\neg\forall x\exists y P(x,y)$

$\equiv \exists x\forall y \neg P(x,y)$

(c) $\neg\exists y(Q(y)) \wedge \forall x\neg R(x,y))$

$\equiv \forall y \neg(Q(y)) \wedge \forall x\neg R(x,y))$

$\equiv \forall y(\neg Q(y) \vee \neg(\forall x \neg R(x,y)))$

$\equiv \forall y \neg Q(y) \vee \exists x R(x,y)$

(d) $\neg(\exists x R(x,y) \vee \forall x S(x,y))$

$\equiv \neg(\exists x R(x,y)) \wedge \neg(\forall x S(x,y))$

$\equiv \forall x \neg R(x,y) \wedge \exists x \neg S(x,y)$

(e) $\neg\exists y(\forall x\exists z T(x,y,z) \vee \exists x\forall z U(x,y,z))$

$\equiv \forall y \neg(\forall x\exists z T(x,y,z) \vee \exists x\forall z U(x,y,z))$

$\equiv \forall y \neg(\forall x\exists z T(x,y,z)) \wedge \neg(\exists x\forall z U(x,y,z))$

$\equiv \forall y (\exists x\forall z \neg T(x,y,z)) \wedge (\forall x\exists z \neg U(x,y,z))$