CSci 243 Homework 4

Due: Wednesday, October 5, end of day

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1. (10 points) For sets A, B, and C, prove that $(B-A)\cup(C-A) = (B\cup C)-A$

   (a) by using set properties and propositional calculus

   (b) by using a membership table

A	B	C	$B-A$	$C-A$	$(B-A)\cup(C-A)$	$B\cup C$	$(B\cup C)-A$
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0
1	1	0	0	0	0	0	0
0	0	1	0	1	0	0	0
1	0	1	0	0	0	0	0
0	1	1	1	1	1	1	1
1	1	1	0	0	0	1	0

2. (5 points) Find these values.

   (a) $\lfloor 1.1 \rfloor$
   $= 1$

   (b) $\lceil 1.1 \rceil$
   $= 2$

   (c) $\lfloor -0.1 \rfloor$
   $= -1$

   (d) $\lceil -0.1 \rceil$
   $= 0$

   (e) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$
   $= 1$

3. (8 points) Determine whether each of these functions from $\mathbb{Z}$ or $\mathbb{Z}\times\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one, onto, both, or neither. Justify your answer.

   (a) $f(n) = 2n-1$
   one-to-one: yes; each value of n corresponds to only one value of $f(n)$.
   onto: no; for $f(n) = 0$, no possible value for n in range $\mathbb{Z}$. n would have to equal $\frac{1}{2}$, so $f(n)$ doesn't map to each value in range $\mathbb{Z}$.

   (b) $f(n) = |n|+1$
   one-to-one: no; some values of $f(n)$ correspond to multiple values of n. ex. $f(1) = 2$, $f(-1) = 2$.
   onto: no; all values of $f(n)$ are greater than zero, not mapping to each value in range $\mathbb{Z}$.

   (c) $f(n) = n^3$
   one-to-one: yes; each value of n corresponds to only one value of $f(n)$.
   onto: yes; $f(n)$ maps to each value in range $\mathbb{Z}$.

(d) $f(n) = \lfloor \frac{n}{2} \rfloor$
one-to-one: no; two values of n correspond to each value of $f(n)$.
onto: yes; $f(n)$ maps to each value in range $\mathbb{Z}$.

4. (12 points) A quick review of algebra. Answer the following:

(a) Simplify: $(q^3 p^{2/3})(\sqrt{p^3} q^{-1}) = q^2 p^{\frac{13}{6}}$

(b) Simplify: $aa^2a^4a^7a^5a^3 = a^{22}$

(c) Simplify to have no x, y in the same factors: $(\log(xy)\log(x/y) = (logx)^2 + (logy)^2$

(d) Add the fractions to produce one fraction: $\frac{2}{x^2+4x+4} + \frac{1}{x+2} = \frac{x+4}{(x+2)(x+2)}$

(e) If $x, y, z, d \in \mathbb{R}$, are the following True or False?

i. $x < y \wedge z < d \rightarrow xz < yd$ False

ii. $x < y \wedge z < d \rightarrow xd < yz$ False

iii. $x > 0 \wedge y > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$ False

iv. $x > 0 \wedge y > 0 \wedge z > 0 \wedge d > 0 \wedge x < y \wedge z < d \rightarrow xd < yz$ False

(f) Solve for x:

i. $10^{3x+5} = 100$
$x = -1$

ii. $3^{x^2}(9^x) = 1/3$
$x = -1$

iii. $\log_x 64 = 2$
$x = 8$

iv. $\log_3(2x+1) = \log_9(x^2)$
$x = -\frac{1}{2}$