CSci 243 Homework 1

Due: Wednesday, Sep 14, end of day
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1. Construct a truth table for each of these compound propositions.

   (a) (3 points) $(p \vee \neg q) \rightarrow q$

p	q	$(p \bigvee \neg q)$	$(p \bigvee \neg q) \rightarrow q$
F	F	T	F
F	T	F	T
T	F	T	F
T	T	T	T

   (b) (7 points) $(p \leftrightarrow q) \rightarrow (r \leftrightarrow s)$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \rightarrow (r \leftrightarrow s)$
F	F	F	F	T	T	T
F	F	F	T	T	F	F
F	F	T	F	T	F	F
F	F	T	T	T	T	T
F	T	F	F	F	T	T
F	T	F	T	F	F	T
F	T	T	F	F	F	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	F	T	F	F	T
T	F	T	F	F	F	T
T	F	T	T	F	T	T
T	T	F	F	T	T	T
T	T	F	T	T	F	F
T	T	T	F	T	F	F
T	T	T	T	T	T	T

2. Understanding quantified predicates.

   (a) (3 points) English to quantified predicates: Use predicates, quantifiers, logical and mathematical operators to express statement "The absolute value of the sum of two integers does not exceed the sum of the absolute value of these integers".

   define a, b as integers:
   $|a+b| \leq |a|+|b|$

   (b) (7 points) Give the truth value of each of these statement if the domain of all variables consists of all real numbers. Explain very briefly.

      i. $\forall x \exists y (x^2 = y)$

      T; there is no real number x that cannot be expressed as a real number y after being squared.

ii. $\exists x \forall y(xy = 0)$

T; when $x = 0$ any value of y still causes xy to equal 1.

iii. $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$

T; $y = \frac{1}{x}$

iv. $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$

F; there is no one value of x that makes $xy = 1$ for all y.

v. $\forall x \exists y(x + y = 1)$

T; $y = 1 - x$

vi. $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$

F; there are cases (such as $x = 4$) where there is no value of y that satisfies both $x + y = 2$ and $2x - y = 1$.

vii. $\forall x \forall y \exists z(z = (x + y)/2)$

T; the sum of two real numbers divided by 2 is always a real number.

3. (10 points) Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside a quantifier or an expression involving logical operators.

(a) $\neg \exists x \exists y P(x, y)$

$\equiv \forall x \forall y \neg P(x, y)$

(b) $\neg \forall x \exists y P(x, y)$

$\equiv \exists x \forall y \neg P(x, y)$

(c) $\neg \exists y(Q(y) \wedge \forall x \neg R(x, y))$

$\equiv \forall y \neg Q(y) \bigvee \exists x R(x, y)$

(d) $\neg(\exists x R(x, y) \vee \forall x S(x, y))$

$\equiv \forall x \neg R(x, y) \bigwedge \exists x \neg S(x, y)$

(e) $\neg \exists y(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

$$\equiv \forall y(\exists x \forall z \neg T(x,y,z)) \bigwedge (\forall x \exists z \neg U(x,y,z))$$