Quantitative Modelling of Temperature Options for Volumetric Risk Hedging in Electricity Markets

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Introduction to the Weather Derivatives Market

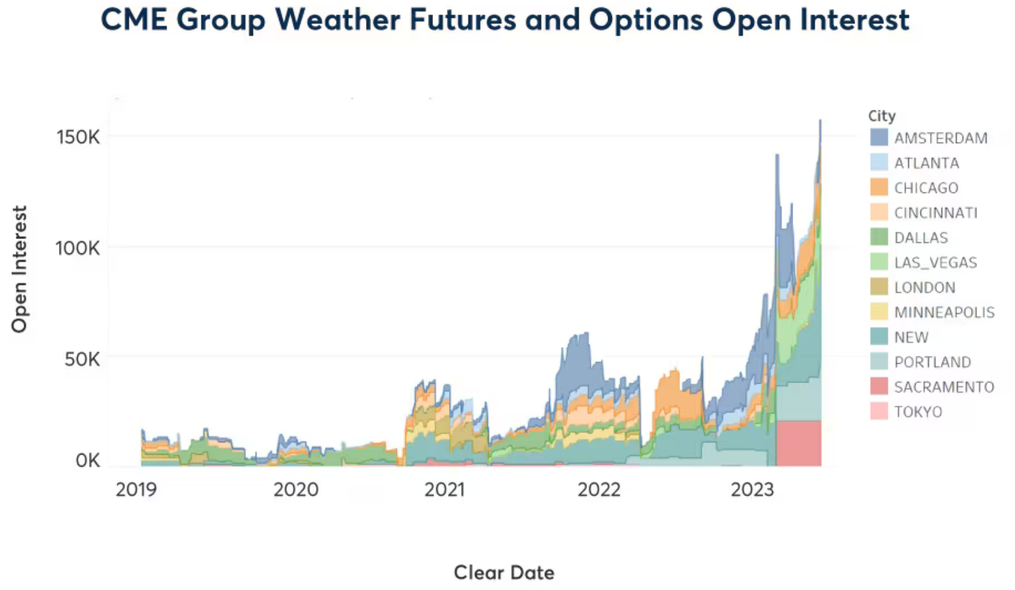
Weather derivatives are financial instruments that can be used by organizations or individuals as part of a risk management strategy to reduce risk associated with adverse or unexpected weather conditions. Just as traditional contingent claims, whose payoffs depend upon the price of some fundamental, a weather derivative has an underlying measure such as rainfall, temperature, humidity, or snowfall.

At the inception of this market the main parties to arrange for, and issue weather derivatives, were energy companies, which after the deregulation of energy markets were exposed to weather risk.

- The first deal took place in 1996 between Aquila Energy and Consolidated Edison in the form of a dual commodity hedge (a weather option embedded in a power contract).
- The first public weather derivative transaction was between Koch Energy and Enron in 1997 in order to transfer the risks of adverse weather. The deal was concerning a temperature index for Milwaukee for the winter of 1997–1998.

In 1999, Chicago Mercantile Exchange (CME) launched the first exchange traded weather derivatives. The CME offered new weather derivatives in various cities in the USA, attracting more participants.

In 2004, the national value of CME weather derivatives was $2.2 billion and grew tenfold to $22 billion through September 2005, with open interest exceeding 300,000 and volume surpassing 630,000 contracts traded [9].


CME Group Weather Futures and Options Open Interest
Open Interest
150K
100K
50K
0K
2019
2020
2021
2022
2023
Clear Date
City
AMSTERDAM
ATLANTA
CHICAGO
CINCINNATI
DALLAS
LAS_VEGAS
LONDON
MINNEAPOLIS
NEW
PORTLAND
SACRAMENTO
TOKYO

Weather-Induced Risk Dynamics in the Economy

Today, weather derivatives are being used for hedging purposes by companies and industries, whose profits can be adversely affected by unseasonal weather or, for speculative purposes, by hedge funds and others interested in capitalizing on those volatile markets.

Rainy or dry and warm or cold periods which are expected to occur frequently can cause large fluctuation on the revenues of a particular company.

A company that uses weather derivatives as a part of its hedging strategy can eliminate the risk related to weather. As a result, the volatility of the year-to-year profits will be significantly reduced. First, low volatility in revenues reduces the risk of great losses and bankruptcy. Second, it decreases the volatility in the share price of the company while it increases the share price. Finally, the interest rate that the company can borrow money at is reduced.

It is estimated that nearly 30% of the U.S. economy is directly affected by the weather. An increasing focus on climate-related risks has become a major driver of the demand for financial products offering protection to adverse outcomes related to weather and climate change. As a result of the recommendations of the Task Force on Climate-related Financial Disclosures (TCFD), credit rating agencies, more firms, and other bodies are examining their exposure to climate-related risks. [11]

Hedger	Weather Variable	Risk
Agricultural industry	temperature / precipitation	Significant crop losses due to extreme temperatures or precipitations
Airlines	wind	Cancellation of flights during windy days
Airports	frosts	Higher operational costs
Amusement parks	temperature / precipitation	Fewer visitors during cold or rainy days
Beverage producers	temperature	Lower sales during cool summers
Construction suppliers	temperature / snowfall	Lower sales during severe winters (construction sites shut down)
Construction companies	temperature / precipitation	Delays in meeting schedules during periods of poor weather
Energy-dense industries	temperature	Higher heating/cooling costs during cold winters and hot summers
Energy producers	temperature	Lower sales during warm winters or cool summers
Hotels	temperature / precipitation	Fewer visitors during rainy or cold periods
Hydropower producers	precipitation	Lower revenue during periods of drought
Municipal governments	snowfall	Higher snow removal costs during winters with above-average snowfall
Road maintenance firms	snowfall	Lower revenues during low snowfall winters
Ski resorts	snowfall	Lower revenue during winters with below-average snowfall
Transportation	wind / snowfall	Cancellation of ship services due to wind or buses due to blocked roads

Electricity Markets and Volumetric Risk

Following the deregulation of the energy market in the early 1990s (US), energy companies began to create financial derivatives referencing electricity prices and temperature. These instruments were developed as a means to hedge against the risks associated with excess production and limited consumption of electricity. Temperature exerts a direct impact on the revenues of companies operating in the energy sector, as it directly impacts the volumes of electricity sold, hence the term volumetric risk.

Electricity represents a unique fungible commodity, distinguished by its inability to be efficiently stored. This characteristic, consequently, leads to significant price fluctuations in the wholesale (spot) market.

- The electricity market adheres to the fundamental principles of demand and supply, with its primary objective being to guarantee that demand is met at all times in a manner that is maximally cost-efficient.
- The formation of spot prices in the electricity market is grounded in the concept of marginal cost, which represents the expense incurred by a producer to generate an additional MWh of electricity.

In Italy, GME operates, through IPEX, a forward physical market (MTE), a day-ahead hourly auction market (MGP), a daily products market (MPEG) and an intraday auction market (MI) structered in seven sessions. It also operates, on behalf of Terna, a platform for ancillary services (MSD), which collects the bids and communicates the results as well as a platform for the registration of OTC transactions (PCE).

Quantifying and Modelling Volumetric Risk

The accurate quantification of volumetric risk, is crucial for implementing a successful hedging strategy. Prior research indicates that electricity consumption is, as noted above, predominantly influenced by temperature. Therefore, an electricity producer can effectively hedge by utilizing temperature indexes. A simple model relating temperature, electricity prices and income is hereby presented.

$$Y = a + bP + cT + \varepsilon$$

- Y, represents the monthly profit
- P, represents the average electricity price for the month
- T, represents the relevant variable for the month

A structured framework is proposed in [8] and in [7], where various dummy variables are incorporated to model the daily and the monthly seasonality in electricity demand E_t. The dependent variable is modelled as follows:

$$E_t = c_1 + a_1 t + \beta_1 HDD_t + \gamma_1 CDD_t + \sum_{i=2}^{7} \delta_{1i} W_{it} + \sum_{j=2}^{12} \lambda_{1j} M_{jt} + \omega_1 H_t + \kappa_1 H_{t-1} + \varepsilon_{1t}$$

Hedging Volumetric Risk with Temperature Options

Electricity producers and retailers will try to protect against seasonal temperatures deviations from seasonal averages. [6]

Daily Average Temperature (DAT): $AvgT = (MinT + MaxT)/2$

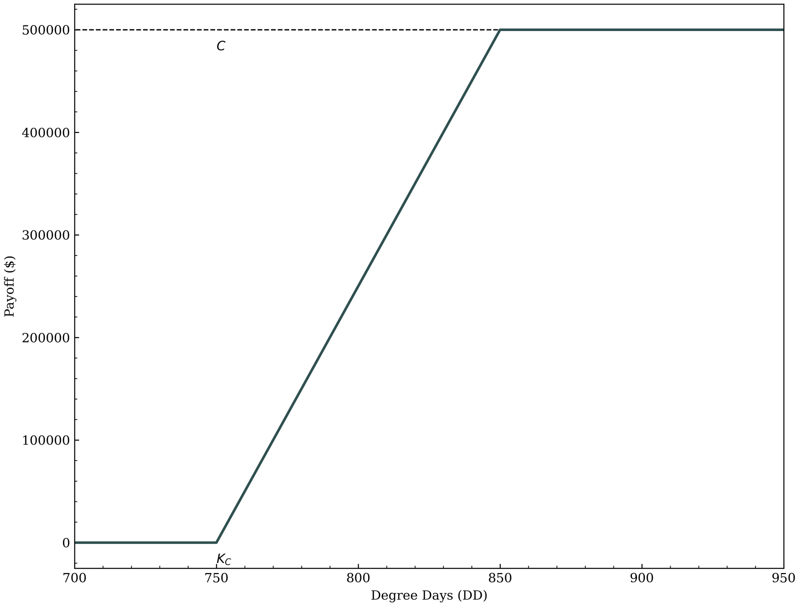
Heating Degree Days (HDD): $HDD_t = (18 - AvgT_t)^+ = \max\{18 - AvgT_t, 0\}$

Cooling Degree Days (CDD): $CDD_t = (AvgT_t - 18)^+ = \max\{AvgT_t - 18, 0\}$

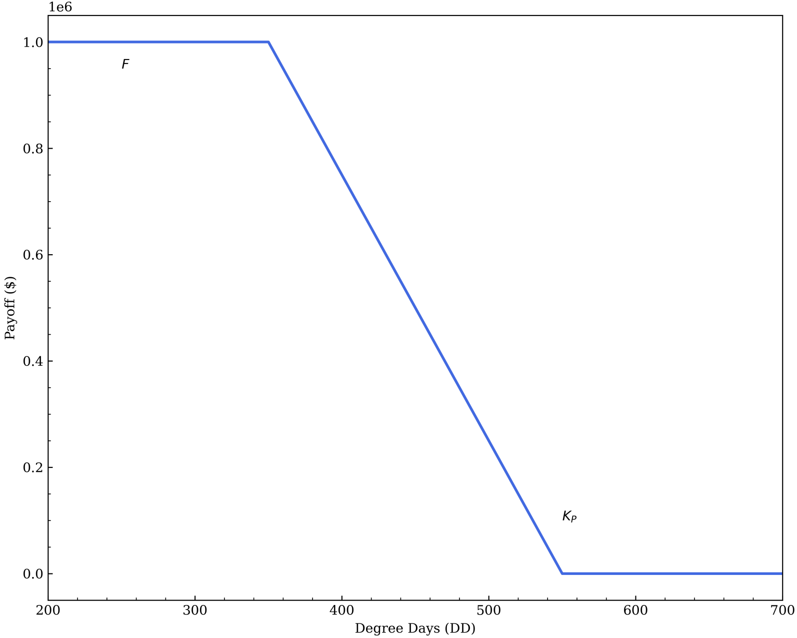
Payoffs

- **Call with a Cap**: $\xi = \min\{\alpha(DD - K)^+, C\}$
- **Put with a Floor**: $\xi = \min\{\alpha(K - DD)^+, F\}$
- **Collar**: $\xi = \min\{\alpha(DD - K_c)^+, C\} - \min\{\beta(K_p - DD)^+, F\}$

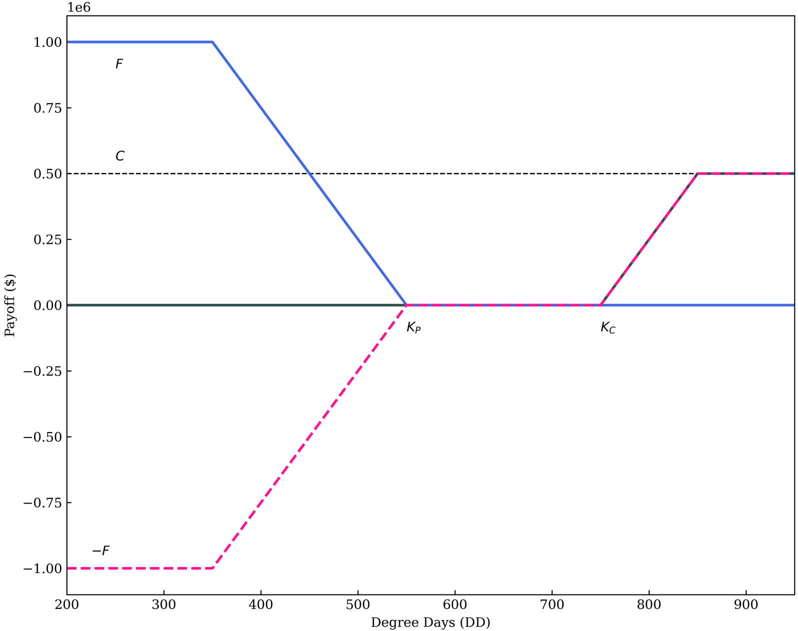
Call with a Cap



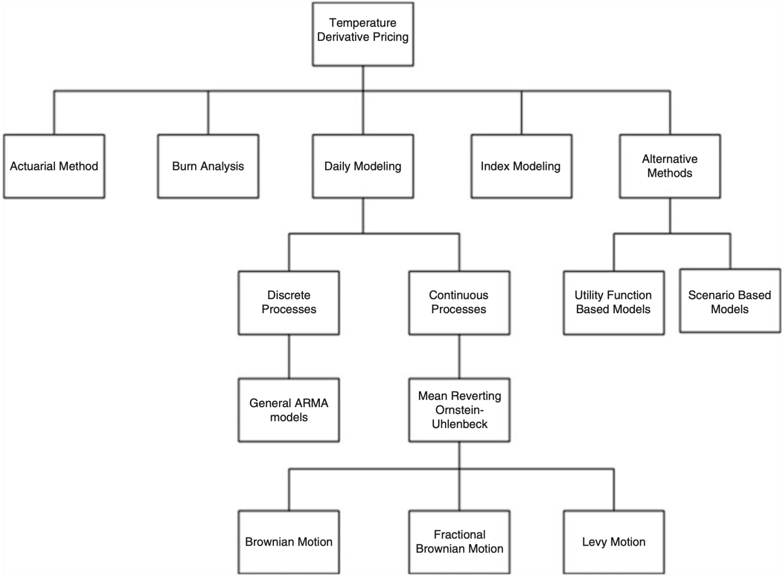
Put with a Floor



Collar


1e6
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Payoff ($)
F
C
$-F$
K_P
K_C
200
300
400
500
600
700
800
900
Degree Days (DD)

Weather Derivatives Pricing Approaches: An Overview



Introducing an SDE for Daily Average Temperature Dynamics (1)

$$dT(t) = \left[\frac{dS(t)}{dt} + \kappa\left(T(t) - S(t)\right)\right] dt + \sigma(t) dW(t) \tag{1}$$

Where,

- κ, represents the speed of mean reversion
- $S(t)$, represents a deterministic function describing trend and seasonal evolution of DATs
- $\sigma(t)$, represents a deterministic function describing seasonality in volatility
- $dW(t)$, represents a Brownian increment

The deterministic functions $S(t)$ and $\sigma(t)$ are hereby defined. ($\omega = 2\pi/365.25$)

$$S(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \sum_{i=1}^{I_1} a_i \cos(\omega i t + \rho) + \sum_{j=1}^{J_2} b_j \sin(\omega j t + \varphi) \tag{2}$$

$$\sigma^2(t) = c + \sum_{i=1}^{I_2} d_i \cos(\omega i t) + \sum_{j=1}^{J_2} e_j \sin(\omega j t) \tag{3}$$

Introducing an SDE for Daily Average Temperature Dynamics (2)

Let $s < t$, then the SDE admits the following solution:

$$T(t) = [T(s) - S(s)]\, e^{-\kappa(t-s)} + S(t) + \int_s^t e^{-\kappa(t-u)}\sigma(u)dW(u) \tag{4}$$

Since $W(u)$ represents Brownian motion and $\sigma(u)$ a deterministic function of time, the random variable $\int_s^t e^{-\kappa(t-u)}\sigma(u)dW(u)$ is normally distributed with mean zero and variance $\int_s^t e^{-2\kappa(t-u)}\sigma^2(u)du$.

Consequently, $T(t)$ (conditioned on the filtration $\mathcal{F}(s)$) follows a normal distribution, characterised by:

$$\mathbb{E}\,[T(t)\,|\,\mathcal{F}(s)] = (T(s) - S(s))\, e^{-\kappa(t-s)} + S(t) \tag{5}$$

$$\mathbb{V}\,[T(t)\,|\,\mathcal{F}(s)] = \int_s^t e^{-2\kappa(t-u)}\sigma^2(u)du \tag{6}$$

In the referenced papers, [4] and [1] both leverage Brownian motion as the process driving randomness. Diverging from this approach, [3] implements randomness through a Lévy process following a generalized hyperbolic distribution $dL(t)$. Whereas, [2] distinctively utilizes Fractional Brownian motion $dB^H(t)$. Additionally, [5] adopts a utility-function based model.

SDE Parameter Estimation: Dissecting DAT Time-Series

Estimating the model's parameters requires recasting the continuous-time dynamics of DAT into a time-series model. Setting $s = t - 1$, the solution of the SDE presented in (4) and the variance presented in (6) can be reformulated as follows:

$$T(t) = [T(t-1) - S(t-1)]\, e^{-\kappa} + S(t) + \int_{t-1}^{t} e^{-\kappa(t-u)} \sigma(u) dW(u) \tag{7}$$

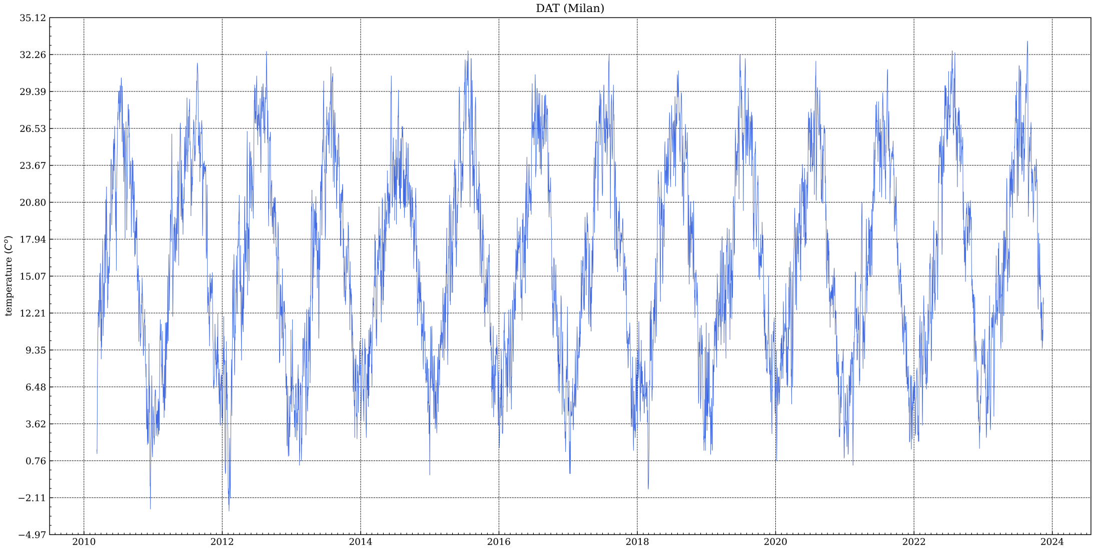
$$s^2(t) = \mathbb{V}\left[T(t) \,|\, \mathcal{F}(t-1)\right] = \int_{t-1}^{t} e^{-2\kappa(t-u)} \sigma^2(u) du \tag{8}$$

Since Brownian motion has independent Gaussian increments of the form $W(t + \Delta t) - W(t) \sim \mathcal{N}(0, \Delta t)$, equation (7) is discretised as follows: (where $\varepsilon(t) \sim \mathcal{N}(0, 1)$)

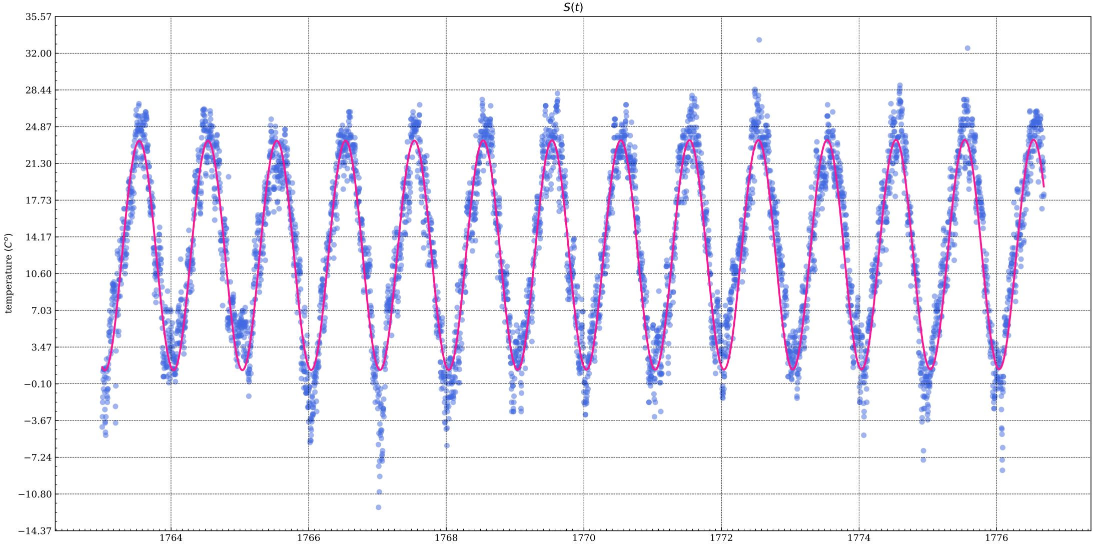
$$T(t) = [T(t-1) - S(t-1)]\, e^{-\kappa} + S(t) + s(t)\, \varepsilon(t) \tag{9}$$

The latter implies that the continuous time SDE has an $AR(1)$ discrete time representation. This becomes evident upon applying the following transformations: Let $\tilde{T}(t) = T(t) - S(t)$, set $\alpha = e^{-\kappa}$, and define $\tilde{\varepsilon}(t) = s(t)\, \varepsilon(t)$. This results in the following expression:

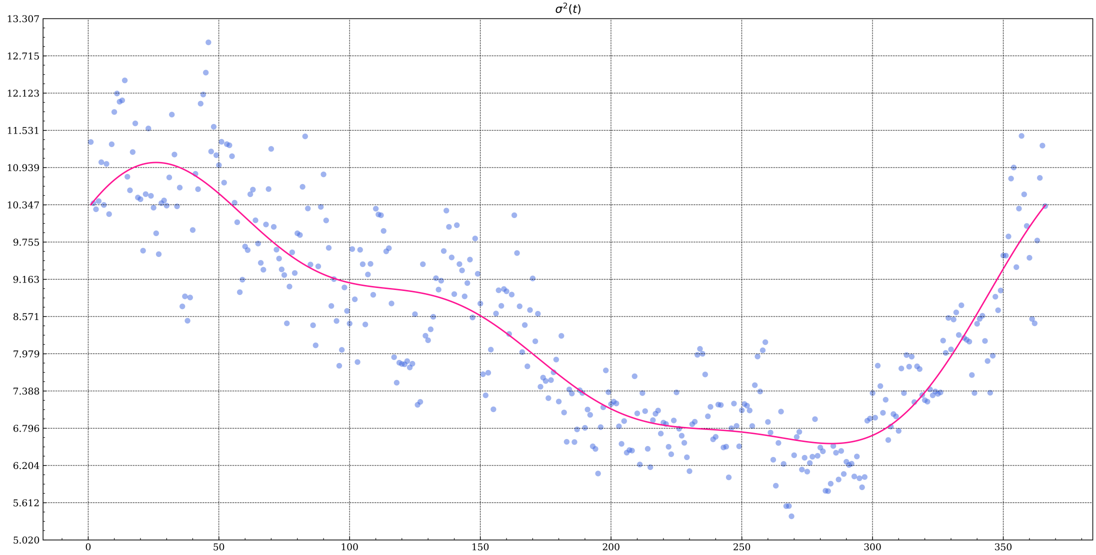
$$\tilde{T}(t) = \alpha \tilde{T}(t-1) + \tilde{\varepsilon} \tag{10}$$



DAT (Milan)
temperature (C^o)
35.12
32.26
29.39
26.53
23.67
20.80
17.94
15.07
12.21
9.35
6.48
3.62
0.76
−2.11
−4.97
2010
2012
2014
2016
2018
2020
2022
2024



$S(t)$
35.57
32.00
28.44
24.87
21.30
17.73
14.17
10.60
7.03
3.47
−0.10
−3.67
−7.24
−10.80
−14.37
temperature (C^o)
1764
1766
1768
1770
1772
1774
1776



$\sigma^2(t)$
13.307
12.715
12.123
11.531
10.939
10.347
9.755
9.163
8.571
7.979
7.388
6.796
6.204
5.612
5.020
0
50
100
150
200
250
300
350

Pricing Implications of Market Incompleteness

In a complete market, it is possible to derive a unique risk-neutral probability measure. Changing the measure from $\mathbb{P}$ to $\mathbb{Q}$ is aimed at turning the process, discounted by the numeraire, into a martingale, thereby enabling the pricing of contingent claims under the risk-neutral measure as the discounted expectation of their future payoffs. However the market for temperature derivatives is incomplete in the sense that the underlying cannot be stored nor traded. Moreover the market is relatively illiquid [10].

In incomplete markets, the lack of a unique arbitrage-free price stems from the existence of multiple equivalent martingale measures. Therefore pricing temperature options requires taking into account investors' risk preferences. A common approach in commodity and interest-rate markets involves singling out an equivalent martingale measure by calibrating the model to observed market prices.

Changing from the physical measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$ under Brownian dynamics is performed by means of the Girsanov Theorem. Under this transformed measure, the dynamics of the process are represented as follows:

$$dT(t) = \left[\frac{dS(t)}{dt} + \kappa\left(T(t) - S(t)\right) - \lambda(t)\sigma(t)\right] dt + \sigma(t) dB(t) \tag{11}$$

With $dB(t)$ being the Brownian motion under $\mathbb{Q}$.

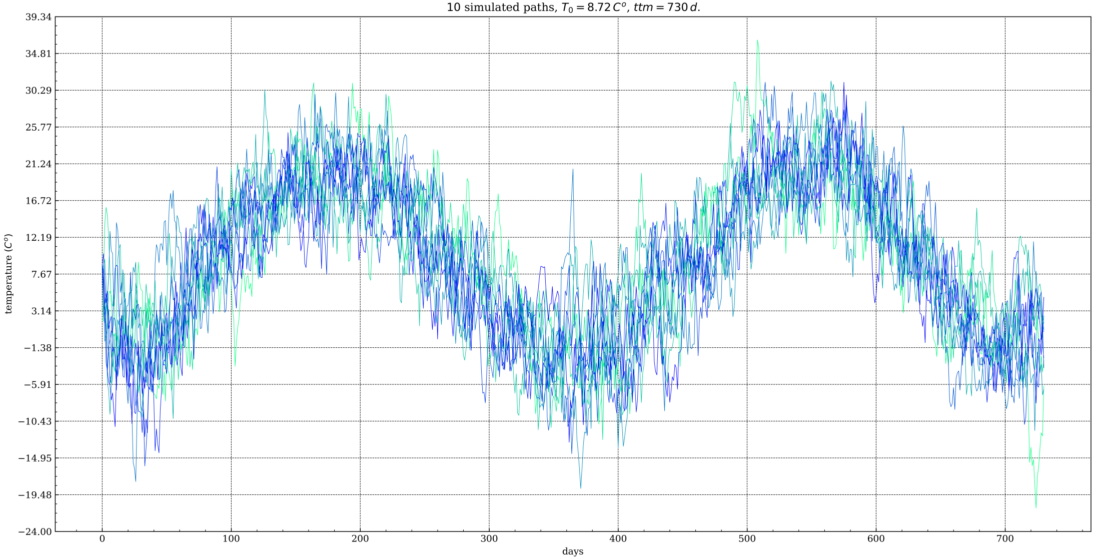
Monte Carlo Method and Market Price of Risk Sensitivity

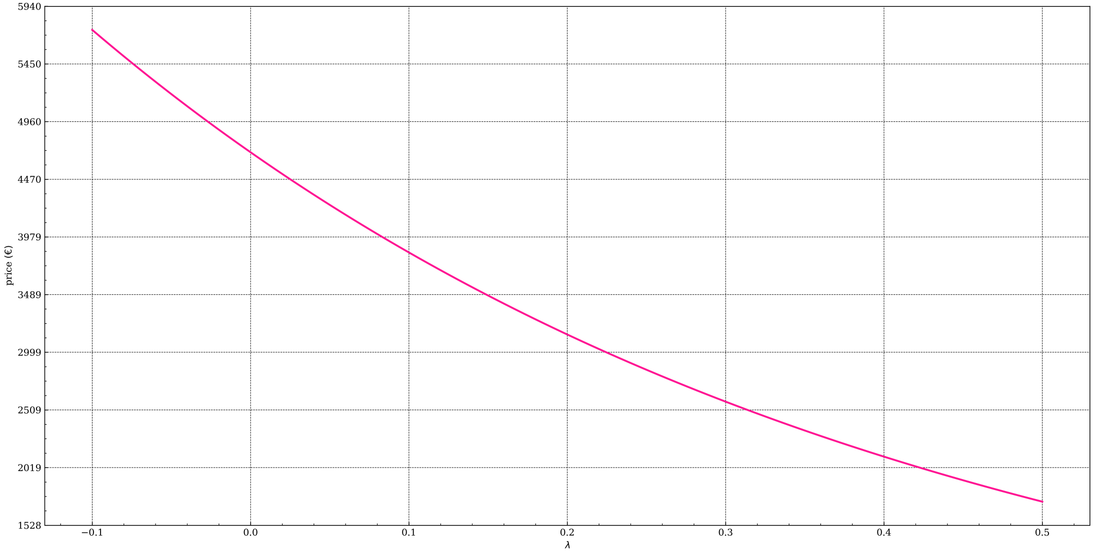
Market incompleteness, the absence of a liquid market in temperature options (in the case examined the absence of a market itself) and of a closed form solution prompted for option price estimation by means of Monte Carlo method.

The approach adopted entails simulating multiple paths for the daily average temperature process under $\mathbb{P}$. For each simulated path, the option's payoff function is computed, the resulting payoffs are then averaged to obtain an estimate of the expected payoff at maturity. The market price of risk capturing the extra compensation required by market participants to hold a risky security is incorporated into the estimation by adding it to the risk-free rate to determine the discount factor for the expected payoff. In the absence of a precise proxy for the market price of risk, the option price is analyzed in relation to it.

Results of this of the approach described above for a collar with price steps $\alpha = 50$ € and $\beta = 50$ €, call strike $K_c = 3300$, put strike $K_p = 3000$, cap $C = 5000$, floor $F = 10000$, and time to maturity $ttm = 730$ days are reported in the slides to follow.

$M = 200000$ paths are simulated starting from an observed daily average temperature of $T_0 = 8.72\, C^o$ (Stazione Milano v.Brera h.1200 15/12/2023), the risk free rate is assumed constant at $r = 3.1\%$.


10 simulated paths, $T_0 = 8.72\,C^o$, $ttm = 730\,d$.
39.34
34.81
30.29
25.77
21.24
16.72
12.19
7.67
3.14
−1.38
−5.91
−10.43
−14.95
−19.48
−24.00
temperature (C^o)
0
100
200
300
400
500
600
700
days


5940
5450
4960
4470
3979
3489
2999
2509
2019
1528
price (€)
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
λ

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Thank you for listening

The End ...