2020 春哈工大自测试题答案

一、填空题（每小题 1 分，共 4 小题，满分 4 分）

1.2　2. 2π　3. $\frac{1}{18}$　4. e^{-x}

二、选择题（每小题 2 分，共 4 小题，满分 8 分）

ADDB

三、解：对 x 求导，$\frac{\partial z}{\partial x}=f'(u)\frac{\partial u}{\partial x}$

$$\frac{\partial u}{\partial x}=\psi'(u)\frac{\partial u}{\partial x}+p(x)$$

故 $p(y)\frac{\partial z}{\partial x}=f'(u)p(y)\left[\frac{p(x)}{1-\psi'(u)}\right]$

对 y 求导，$\frac{\partial z}{\partial y}=f'(u)\frac{\partial u}{\partial y}$

$$\frac{\partial u}{\partial y}=\psi'(u)\frac{\partial u}{\partial y}-p(y)$$

故 $p(x)\frac{\partial z}{\partial y}=f'(u)p(x)\left[\frac{-p(y)}{1-\psi'(u)}\right]$

所以 $p(y)\frac{\partial z}{\partial x}+p(x)\frac{\partial z}{\partial y}=0$

四、解：$\iint\limits_D \max(y,x^2)d\sigma=\int_{-1}^{1}dx\int_0^{x^2}x^2dy+\int_{-1}^{1}dx\int_{x^2}^{1}ydy$

$$=\frac{2}{5}+\frac{4}{5}=\frac{6}{5}$$

五、解：$\varphi(x)=e^x-x\int_0^x\varphi(u)du+\int_0^x u\varphi(u)du$

两边求导即得

$$\varphi'(x)=e^x-\int_0^x\varphi(u)du-x\varphi(x)+x\varphi(x)=e^x-\int_0^x\varphi(u)du$$

再次求导得 $\varphi''(x)=e^x-\varphi(x)$，于是有

$$\begin{cases}\varphi''(x)+\varphi(x)=e^x\\ \varphi(0)=1,\ \varphi'(0)=1\end{cases}$$

解之，特征方程为 $r^2+1=0$，其特征根 $r_1,\ r_2=\pm i$，设特解 $\varphi^*=Ae^x$ 代入上述微分方程可

得 $A=\frac{1}{2}$，其通解为$\varphi(x)=(c_1\cos x+c_2\sin x)+\frac{1}{2}e^x$，由初始条件得$c_1=c_2=\frac{1}{2}$，从而原方程的特解为$\varphi(x)=\frac{1}{2}(\cos x+\sin x+e^x)$.

六、解： 令$t=x^2+y^2$，则$z=tf(t)$

$$\frac{\partial z}{\partial x}=[f(t)+tf'(t)]2x$$

$$\frac{\partial^2 z}{\partial x^2}=2[f(t)+tf'(t)]+[2f'(t)+tf''(t)]4x^2$$

$$\frac{\partial^2 z}{\partial y^2}=2f(t)+2tf'(t)+[2f'(t)+tf''(t)]4y^2$$

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=4f(t)+12tf'(t)+4t^2f'(t)=0$$

$$\therefore\begin{cases}t^2f''(t)+3tf'(t)+f(t)=0\\ f(1)=0,\ \ f'(1)=0\end{cases}$$

$f(t)=\frac{\ln t}{t}$，　$f'(t)=\frac{1-\ln t}{t^2}=0$，$t=e$.

$1<t<e$　　　$f'(t)>0$

$t>e$　　　　$f'(t)<0$

$\therefore\ \ f_{\max}=f(e)=\frac{1}{e}$