

Pessimistic Verification for Open Ended Math Questions

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Abstract

The key limitation of the verification performance lies in the ability of error detection. With this intuition we designed several variants of pessimistic verification, which are simple workflows that could significantly improve the verification of open-ended math questions. In pessimistic verification we conducts multiple reviews on a single proof with a special focus on certain parts, and reports false if any one of them finds an error. This simple technique significantly improves the performance across many math verification benchmarks without introducing too much extra budget. Its token efficiency even surpassed extended long-cot in test-time scaling. Self verification and correction are one of the central perspectives of reasoning and intelligence. This enables an agent to constantly refine its own reasoning or actions, and they are also critical for effectively performing long tasks such as mathematical research. We believe pessimistic verification would be especially useful for many related researches.

1 Introduction

Since the release of OpenAI o1 (OpenAI et al., 2024) and DeepSeek-R1 (DeepSeek-AI et al., 2025), reasoning has become one of the most important topics in large language model (LLM) research within both academia and industry. Nevertheless, the scalability of the training recipe behind current large reasoning models (LRMs) is still limited by the requirement of verifiable reward. Even in math, one of the most successful domain of LRM, the absence of a generic verifiable reward still introduces significant challenges to more advanced, open-ended and long-form reasoning tasks (Xu et al., 2025).

One possible solution to this problem is through formal theorem provers such as Lean (Chen et al., 2025; Varambally et al., 2025), which could provide completely reliable verification on math proofs. However, this approach would introduce notable external budget to the AI system and their performance still largely falls behind that of provers in natural language (Dekoninck et al., 2025). Another line of work focuses on leveraging the internal

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capability of the LLM to achieve self evolution (Zuo et al., 2025; Yu et al., 2025; Xu et al., 2025).

We contend that the importance of self-verification capabilities can be reflected in several key dimensions:

- Effective self-verification can substantially enhance the reliability of model outputs and significantly improve overall performance. The performance IMO level math problems can be notably enhanced via a verifier-guided workflow according to Huang & Yang (2025); Luong et al. (2025).
- Existing research indicates that the reliability of single-step task execution strongly influences the duration over which a system can operate dependably, thus introspective abilities may be particularly critical for long-horizon tasks (Kwa et al., 2025).
- We further argue that a general intelligent system should possess intrinsic mechanisms for self-validation, rather than relying exclusively on external ground-truth signals or verification modules.

Intuitively we believe that the key limitation of verification lies in the ability of finding errors in a proof, which is also supported by some recent researches (Pandit et al., 2025). So in this work we introduce three simple workflows which we call **simple pessimistic verification**, **vertical pessimistic verification**, and **progressive pessimistic verification**. These methods imitate the common practice of the review process of math papers, where a paper would be rejected if any one reviewer finds an error in it. They will review a given solution multiple times from different perspectives, and the whole proof will be considered false if any one review finds an error. We have conducted a series of experiments on three datasets, *Hard2Verify* (Pandit et al., 2025), *IMO-GradingBench* (Luong et al., 2025), and our homemade *QiuZhen-Bench*. The former two benchmarks are both contest-level math grading benchmarks with expert annotations. *QiuZhenBench* was collected and curated from S.-T. Yau College Student Mathematics Contest and the doctoral qualifying exams from QiuZhen college, Tsinghua University. These exams covers a wide range of topics in undergraduate-level mathematics and are well-known for their high difficulty. On all benchmarks our methods consistently show impressive improvements in error detection rate and overall f1 score, indicating their effectiveness on proof verifications.

2 Method

2.1 Metrics

In this work, we treat mathematical proof verification as a binary classification problem and focus on the following performance metrics:

- **True Negative Rate (TNR)**: The proportion of detected errors among all erroneous proofs. This is the primary metric for evaluating the model’s ability to identify incorrect proofs.
- **True Positive Rate (TPR)**: The proportion of proofs classified as correct among all truly correct proofs. This helps assess the model’s proof-verification capability.
- **Balanced F1 Score**: The harmonic mean of TPR and TNR, providing a balanced indicator of performance when both false positives and false negatives matter. This is also the primary indicator used in Pandit et al. (2025).

$$\text{Balanced F1} = \frac{2 \cdot \text{TPR} \cdot \text{TNR}}{\text{TPR} + \text{TNR}} \quad (1)$$

2.2 Simple pessimistic verification

A common strategy of enhancing model capability is through majority voting. In majority voting we run the same requests in parallel for multiple times, and choose the majority as

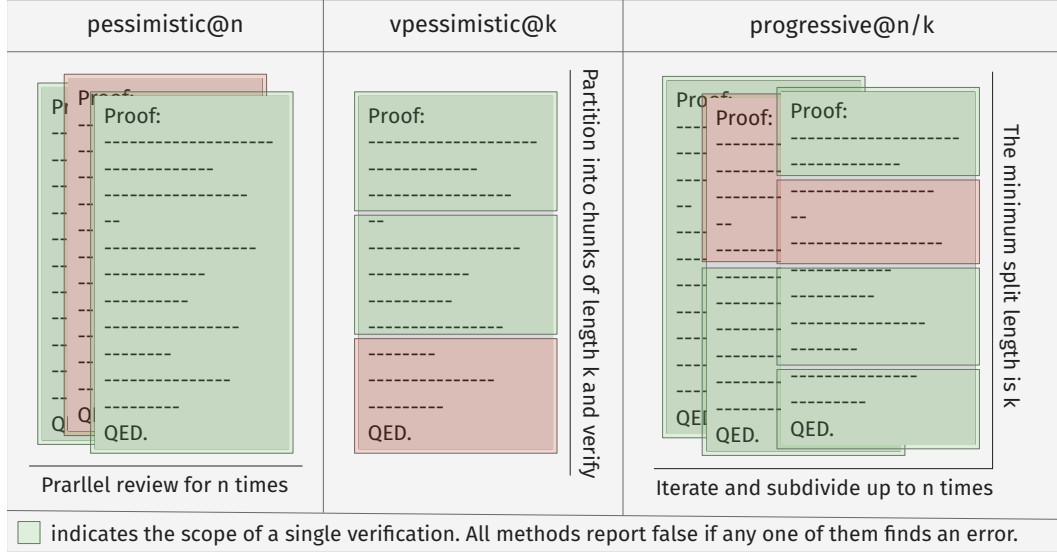


Figure 1: Three variants of pessimistic verification methods in this work. In following experiments they will separately be denoted as “pes@n”, “vp@l”, and “prog@n/l”

final answer. However, this mechanism does not work on verification tasks according to our experiments and some related researches (Pandit et al., 2025).

In simple pessimistic verification, we conduct multiple reviews on a single proof as majority voting, but instead of using the majority as final answer, we will constantly choose the worst verification from these reviews. As shown in Figure 2, this method drastically improves the overall performance of evaluation where majority has almost no effect.

We can roughly understand this phenomenon as follows: since the most difficult part of mathematical proof verification is detecting errors, it is likely that only a small number of evaluations can identify the critical mistakes. In this case, majority voting may actually restrict the model’s ability to uncover potential errors, while pessimistic verification further reinforces this ability.

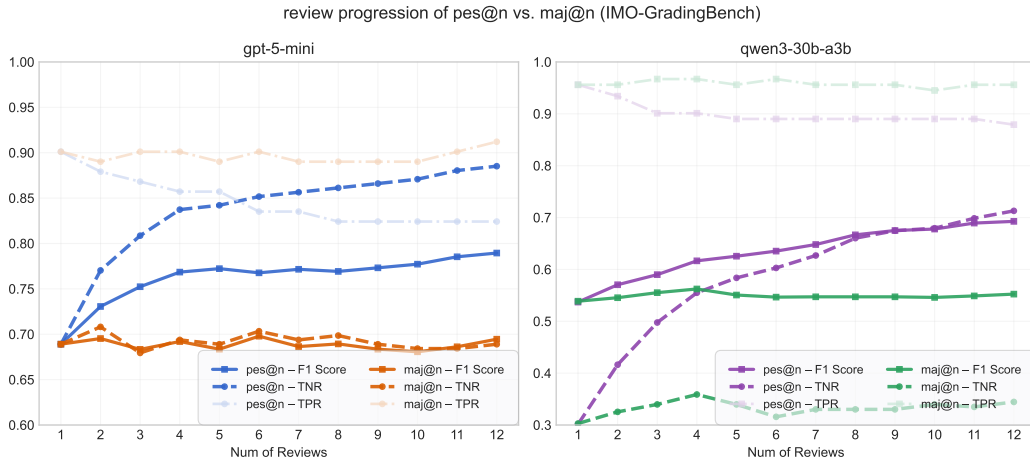


Figure 2: The comparison of simple pessimistic verification and majority voting. The former exhibits steady performance gains as sampling budget increases, whereas the latter shows almost no changes in performance.

2.3 Vertical pessimistic verification

In spite of simply applying multiple reviews on the whole proof, we also explored a pessimistic verification from another dimension. As shown in Figure 3, we adopted a special prompting method and require the LLM to focus on a certain part of the proof, and try looking deep into these contents to find errors. Vertical pessimistic verification adopts a hyperparameter l , it first splits the whole proof into chunks with l lines, and create a series of parallel review tasks for each chunk. Although this method only goes through the proof once, we also witnessed improved performance in error detection and even a higher scaling efficiency compared to simple pessimistic verification.

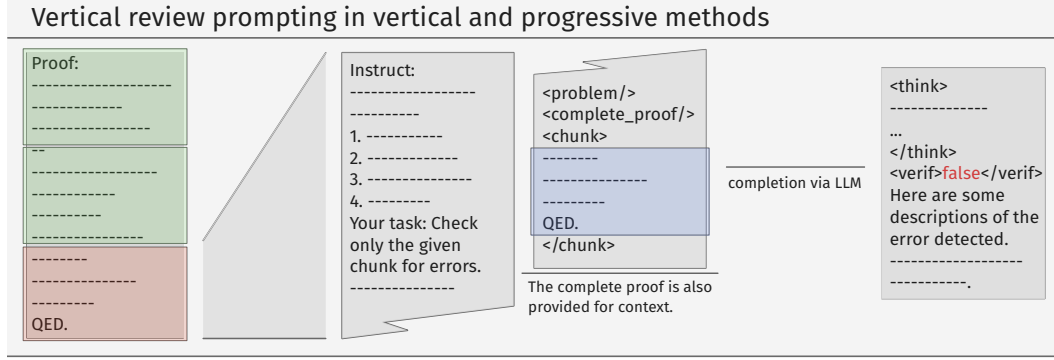


Figure 3: The vertical review prompting method used in vertical pessimistic verification and progressive pessimistic verification.

2.4 Progressive pessimistic verification

Combining the mechanism of both simple and vertical methods we can create a progressive pessimistic verification method. This method starts from the whole proof verification, and then progressively subdivide the proof for up to n times. Each chunk is restricted to contain at least l lines. After this process we can create at most $2^n - 1$ different verification requests on a single proof at different scales. This approach eventually achieved the highest performance under certain sampling budget.

2.5 Pruning in pessimistic verification

The mechanism of pessimistic verification also enables pruning in the process. We can implement serial execution at certain levels and stop subsequent checks once an earlier validation detects an error. This allows us to effectively reduce computational resource consumption without sacrificing performance. However, running in a serial manner also slows down execution speed, so we need to find a balance between speed and cost.

The progressive verification approach naturally supports pruning. It can run each round of verification in order from coarse to fine, filtering out incorrect answers step by step. Therefore, in our experiments, pruning is enabled by default for this method, and other approaches can be applied in a similar way. Pruning is especially useful when most of the examples in the dataset are negative examples.

3 Experiments

3.1 Datasets and settings

In our experiments we primarily use three datasets for evaluation, and we constantly use the same prompt and workflow setting across all these dataset. Here are some descriptions about them:

- **IMO-GradingBench** (Luong et al., 2025). This dataset contains 1,000 human-graded solutions to IMO-level proof problems from IMO-ProofBench (Luong et al., 2025), from which we selected a subset with 300 samples for evaluation. This is a challenging test of fine-grained mathematical proof evaluation.
- **Hard2Verify** (Pandit et al., 2025). Hard2Verify contains 200 challenging problems and solutions from recent math competitions such as IMO and Putnam. The solutions are generated by strong models such as GPT-5 and Gemini 2.5 Pro and annotated by humans.
- **QiuZhen-Bench**. This is a homemade collection of advanced math problems, with questions sourced from challenging university-level math competitions. It serves as a supplement to the previous two elementary math competition problem datasets. We randomly selected a subset of 300 problems, had them answered by GPT-5-mini, and used GPT-5 for labeling. This subset can be used to evaluate the performance of weaker models. You can refer to Appendix A.1 for more details.

All evaluation in our experiments was conducted at the response level, and for IMO-GradingBench, only the responses that obtained fully 7 points are considered correct, otherwise they will all be considered false. And aside from our experiments, we will simply use single pass verification with different reasoning effort setting as the baseline, since we already know that majority voting has almost no effect on evaluation. In all our experiments we set the temperature parameter to 1.0 to ensure the diversity of the response.

3.2 Performance

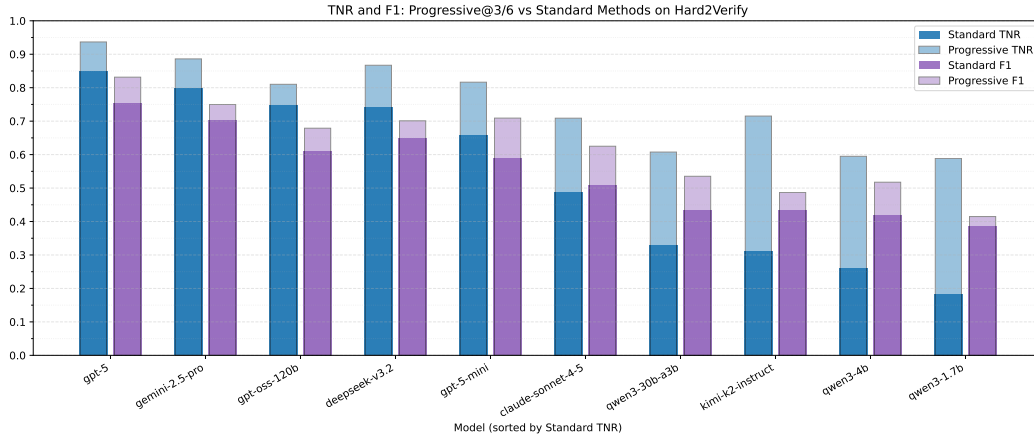


Figure 4: The main results on IMO-GradingBench. Thinking mode is enabled for all models if possible, and the reasoning effort of gpt series model is set to medium. This method constantly improves the classification performance across all tested models.

3.3 Scaling potential of pessimistic verification

3.4 Case study

4 Related work

Using LLM for evaluation or verification tasks is a natural idea. At the early development stage of LLM, some works have already tried this method on traditional tasks in natural language processing, such as text summarization (Liu et al., 2023), dialog generation (Liu et al., 2023), and machine translation (Zheng et al., 2023). This approach has achieved some results, but several problems still remain, such as scoring bias (Li et al., 2025) and self-inconsistency (Halder & Hockenmaier, 2025).

The open-ended math problem lies between the math answering problem and other evaluation problems. It lacks simple and direct means of verification, but its correctness is entirely objective. Existing works in this area primarily focus on the alignment of LLM grading and that of humans. Some of them proposed certain agentic workflows that could enhance this ability (Mahdavi et al., 2025a;b). Reinforcement learning on manually annotated data also exhibited effectiveness on the evaluation of mathematical proofs (Dekoninck et al., 2025). However, these methods lack the scalability in further enhancing the performance, and they cannot distinguish performance improvements brought by subjective preference alignment from those resulting from objectively discovering new errors. Some work also highlights the importance of error detection, as this is the key ability that separates strong verifiers from weaker ones (Pandit et al., 2025). Before the release of this work, there is no well-known method that could leverage test-time scaling to obtain better evaluation performance other than scaling long chain of thought (Pandit et al., 2025).

5 Conclusion and discussion

In this work we proposed several variants of pessimistic verification method, which exhibits strong performance and even higher scaling potential than long chain of thought on the evaluation of open-ended math problems. These methods construct multiple different verification queries for a single mathematical proof in different ways, and deems the proof incorrect if any one of these queries determines it to be wrong.

Beyond the existing concrete implementations and results, we believe that the error-centered idea behind pessimistic verification is what truly deserves attention. This approach will naturally make the verification of mathematical problems increasingly stringent, which may also align with the field’s gradual trend toward greater formalization and rigor. It may likewise help guide large language models away from merely computing correct answers and toward generating fully rigorous proofs.

We can also envision several direct applications of pessimistic verification:

- Using pessimistic verification in math or code related workflows can further improve the reliability of the response, especially for long-form tasks.
- This method can further push the capability frontier of state-of-the-art large models, so there is an opportunity to incorporate it into the training pipeline to further raise the upper limit of large models’ abilities in executing verification and rigorous proof tasks.

We are also excited about the future works inspired by our pessimistic verification.

References

- Luoxin Chen, Jinming Gu, Liankai Huang, Wenhao Huang, Zhicheng Jiang, Allan Jie, Xiaoran Jin, Xing Jin, Chenggang Li, Kaijing Ma, Cheng Ren, Jiawei Shen, Wenlei Shi, Tong Sun, He Sun, Jiahui Wang, Siran Wang, Zhihong Wang, Chenrui Wei, Shufa Wei, Yonghui Wu, Yuchen Wu, Yihang Xia, Huajian Xin, Fan Yang, Huaiyuan Ying, Hongyi Yuan, Zheng Yuan, Tianyang Zhan, Chi Zhang, Yue Zhang, Ge Zhang, Tianyun Zhao, Jianqiu Zhao, Yichi Zhou, and Thomas Hanwen Zhu. Seed-Prover: Deep and Broad Reasoning for Automated Theorem Proving, August 2025. URL <http://arxiv.org/abs/2507.23726>. arXiv:2507.23726 [cs].
- DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, Aixin Liu, Bing Xue, Bingxuan Wang, Bochao Wu, Bei Feng, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang, Chong Ruan, Damai Dai, Deli Chen, Dongjie Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, Hanwei Xu, Haocheng Wang, Honghui Ding, Huajian Xin, Huazuo Gao, Hui Qu, Hui Li, Jianzhong Guo, Jiashi Li, Jiawei Wang, Jingchang Chen, Jingyang Yuan, Junjie Qiu, Junlong Li, J. L.

- Cai, Jiaqi Ni, Jian Liang, Jin Chen, Kai Dong, Kai Hu, Kaige Gao, Kang Guan, Kexin Huang, Kuai Yu, Lean Wang, Lecong Zhang, Liang Zhao, Litong Wang, Liyue Zhang, Lei Xu, Leyi Xia, Mingchuan Zhang, Minghua Zhang, Minghui Tang, Meng Li, Miaojun Wang, Mingming Li, Ning Tian, Panpan Huang, Peng Zhang, Qiancheng Wang, Qinyu Chen, Qiushi Du, Ruiqi Ge, Ruisong Zhang, Ruizhe Pan, Runji Wang, R. J. Chen, R. L. Jin, Ruyi Chen, Shanghao Lu, Shangyan Zhou, Shanhuang Chen, Shengfeng Ye, Shiyu Wang, Shuiping Yu, Shunfeng Zhou, Shuting Pan, S. S. Li, Shuang Zhou, Shaoqing Wu, Shengfeng Ye, Tao Yun, Tian Pei, Tianyu Sun, T. Wang, Wangding Zeng, Wanjia Zhao, Wen Liu, Wenfeng Liang, Wenjun Gao, Wenqin Yu, Wentao Zhang, W. L. Xiao, Wei An, Xiaodong Liu, Xiaohan Wang, Xiaokang Chen, Xiaotao Nie, Xin Cheng, Xin Liu, Xin Xie, Xingchao Liu, Xinyu Yang, Xinyuan Li, Xuecheng Su, Xuheng Lin, X. Q. Li, Xiangyue Jin, Xiaojin Shen, Xiaosha Chen, Xiaowen Sun, Xiaoxiang Wang, Xinnan Song, Xinyi Zhou, Xianzu Wang, Xinxia Shan, Y. K. Li, Y. Q. Wang, Y. X. Wei, Yang Zhang, Yanhong Xu, Yao Li, Yao Zhao, Yaofeng Sun, Yaohui Wang, Yi Yu, Yichao Zhang, Yifan Shi, Yiliang Xiong, Ying He, Yishi Piao, Yisong Wang, Yixuan Tan, Yiyang Ma, Yiyuan Liu, Yongqiang Guo, Yuan Ou, Yuduan Wang, Yue Gong, Yuheng Zou, Yujia He, Yunfan Xiong, Yuxiang Luo, Yuxiang You, Yuxuan Liu, Yuyang Zhou, Y. X. Zhu, Yanhong Xu, Yanping Huang, Yaohui Li, Yi Zheng, Yuchen Zhu, Yunxian Ma, Ying Tang, Yukun Zha, Yuting Yan, Z. Z. Ren, Zehui Ren, Zhangli Sha, Zhe Fu, Zhean Xu, Zhenda Xie, Zhengyan Zhang, Zhewen Hao, Zhicheng Ma, Zhigang Yan, Zhiyu Wu, Zihui Gu, Zijia Zhu, Zijun Liu, Zilin Li, Ziwei Xie, Ziyang Song, Zizheng Pan, Zhen Huang, Zhipeng Xu, Zhongyu Zhang, and Zhen Zhang. DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning, January 2025. URL <https://arxiv.org/abs/2501.12948> [cs].
- Jasper Dekoninck, Ivo Petrov, Kristian Minchev, Mislav Balunovic, Martin Vechev, Miroslav Marinov, Maria Drencheva, Lyuba Konova, Milen Shumanov, Kaloyan Tsvetkov, Nikolay Drenchev, Lazar Todorov, Kalina Nikolova, Nikolay Georgiev, Vanesa Kalinkova, and Margulan Ismoldayev. The Open Proof Corpus: A Large-Scale Study of LLM-Generated Mathematical Proofs, June 2025. URL <https://arxiv.org/abs/2506.21621>. arXiv:2506.21621 [cs].
- Rajarshi Haldar and Julia Hockenmaier. Rating Roulette: Self-Inconsistency in LLM-As-A-Judge Frameworks, October 2025. URL <https://arxiv.org/abs/2510.27106>. arXiv:2510.27106 [cs].
- Yichen Huang and Lin F. Yang. Gemini 2.5 Pro Capable of Winning Gold at IMO 2025, July 2025. URL <https://arxiv.org/abs/2507.15855>. arXiv:2507.15855 [cs].
- Thomas Kwa, Ben West, Joel Becker, Amy Deng, Katharyn Garcia, Max Hasin, Sami Jawhar, Megan Kinniment, Nate Rush, Sydney Von Arx, Ryan Bloom, Thomas Broadley, Haoxing Du, Brian Goodrich, Nikola Jurkovic, Luke Harold Miles, Seraphina Nix, Tao Lin, Neev Parikh, David Rein, Lucas Jun Koba Sato, Hjalmar Wijk, Daniel M. Ziegler, Elizabeth Barnes, and Lawrence Chan. Measuring AI Ability to Complete Long Tasks, March 2025. URL <https://arxiv.org/abs/2503.14499>. arXiv:2503.14499 [cs].
- Qingquan Li, Shaoyu Dou, Kailai Shao, Chao Chen, and Haixiang Hu. Evaluating Scoring Bias in LLM-as-a-Judge, August 2025. URL <https://arxiv.org/abs/2506.22316>. arXiv:2506.22316 [cs].
- Yang Liu, Dan Iter, Yichong Xu, Shuohang Wang, Ruochen Xu, and Chenguang Zhu. G-Eval: NLG Evaluation using GPT-4 with Better Human Alignment, May 2023. URL <https://arxiv.org/abs/2303.16634>. arXiv:2303.16634 [cs].
- Thang Luong, Dawsen Hwang, Hoang H. Nguyen, Golnaz Ghiasi, Yuri Chervonyi, Insuk Seo, Junsu Kim, Garrett Bingham, Jonathan Lee, Swaroop Mishra, Alex Zhai, Clara Huiyi Hu, Henryk Michalewski, Jimin Kim, Jeonghyun Ahn, Junhwi Bae, Xingyou Song, Trieu H. Trinh, Quoc V. Le, and Junehyuk Jung. Towards Robust Mathematical Reasoning, November 2025. URL <https://arxiv.org/abs/2511.01846>. arXiv:2511.01846 [cs].
- Hamed Mahdavi, Pouria Mahdavinia, Samira Malek, Pegah Mohammadipour, Alireza Hashemi, Majid Daliri, Alireza Farhadi, Amir Khasahmadi, Niloofar Miresheghallah,

and Vasant Honavar. RefGrader: Automated Grading of Mathematical Competition Proofs using Agentic Workflows, October 2025a. URL <http://arxiv.org/abs/2510.09021>. arXiv:2510.09021 [cs].

Sadegh Mahdavi, Branislav Kisacanin, Shubham Toshniwal, Wei Du, Ivan Moshkov, George Armstrong, Renjie Liao, Christos Thrampoulidis, and Igor Gitman. Scaling Generative Verifiers For Natural Language Mathematical Proof Verification And Selection, November 2025b. URL <http://arxiv.org/abs/2511.13027>. arXiv:2511.13027 [cs].

OpenAI, Aaron Jaech, Adam Kalai, Adam Lerer, Adam Richardson, Ahmed El-Kishky, Aiden Low, Alec Helyar, Aleksander Madry, Alex Beutel, Alex Carney, Alex Iftimie, Alex Karpenko, Alex Tachard Passos, Alexander Neitz, Alexander Prokofiev, Alexander Wei, Allison Tam, Ally Bennett, Ananya Kumar, Andre Saraiva, Andrea Vallone, Andrew Duberstein, Andrew Kondrich, Andrey Mishchenko, Andy Applebaum, Angela Jiang, Ashvin Nair, Barret Zoph, Behrooz Ghorbani, Ben Rossen, Benjamin Sokolowsky, Boaz Barak, Bob McGrew, Borys Minaiev, Botao Hao, Bowen Baker, Brandon Houghton, Brandon McKinzie, Brydon Eastman, Camillo Lugaresi, Cary Bassin, Cary Hudson, Chak Ming Li, Charles de Bourcy, Chelsea Voss, Chen Shen, Chong Zhang, Chris Koch, Chris Orsinger, Christopher Hesse, Claudia Fischer, Clive Chan, Dan Roberts, Daniel Kappler, Daniel Levy, Daniel Selsam, David Dohan, David Farhi, David Mely, David Robinson, Dimitris Tsipras, Doug Li, Dragos Oprica, Eben Freeman, Eddie Zhang, Edmund Wong, Elizabeth Proehl, Enoch Cheung, Eric Mitchell, Eric Wallace, Erik Ritter, Evan Mays, Fan Wang, Felipe Petroski Such, Filippo Raso, Florencia Leoni, Foivos Tsimpourlas, Francis Song, Fred von Lohmann, Freddie Sulit, Geoff Salmon, Giambattista Parascandolo, Gildas Chabot, Grace Zhao, Greg Brockman, Guillaume Leclerc, Hadi Salman, Haiming Bao, Hao Sheng, Hart Andrin, Hessam Bagherinezhad, Hongyu Ren, Hunter Lightman, Hyung Won Chung, Ian Kivlichan, Ian O’Connell, Ian Osband, Ignasi Clavera Gilaberte, Ilge Akkaya, Ilya Kostrikov, Ilya Sutskever, Irina Kofman, Jakub Pachocki, James Lennon, Jason Wei, Jean Harb, Jerry Twore, Jiacheng Feng, Jiahui Yu, Jiayi Weng, Jie Tang, Jieqi Yu, Joaquin Quiñero Candela, Joe Palermo, Joel Parish, Johannes Heidecke, John Hallman, John Rizzo, Jonathan Gordon, Jonathan Uesato, Jonathan Ward, Joost Huizinga, Julie Wang, Kai Chen, Kai Xiao, Karan Singhal, Karina Nguyen, Karl Cobbe, Katy Shi, Kayla Wood, Kendra Rimbach, Keren Gu-Lemberg, Kevin Liu, Kevin Lu, Kevin Stone, Kevin Yu, Lama Ahmad, Lauren Yang, Leo Liu, Leon Maksin, Leyton Ho, Liam Fedus, Lilian Weng, Linden Li, Lindsay McCallum, Lindsey Held, Lorenz Kuhn, Lukas Kondraciuk, Lukasz Kaiser, Luke Metz, Madelaine Boyd, Maja Trebacz, Manas Joglekar, Mark Chen, Marko Tintor, Mason Meyer, Matt Jones, Matt Kaufer, Max Schwarzer, Meghan Shah, Mehmet Yatbaz, Melody Y. Guan, Mengyuan Xu, Mengyuan Yan, Mia Glaese, Mianna Chen, Michael Lampe, Michael Malek, Michele Wang, Michelle Fradin, Mike McClay, Mikhail Pavlov, Miles Wang, Mingxuan Wang, Mira Murati, Mo Bavarian, Mostafa Rohaninejad, Nat McAleese, Neil Chowdhury, Neil Chowdhury, Nick Ryder, Nikolas Tezak, Noam Brown, Ofir Nachum, Oleg Boiko, Oleg Murk, Olivia Watkins, Patrick Chao, Paul Ashbourne, Pavel Izmailov, Peter Zhokhov, Rachel Dias, Rahul Arora, Randall Lin, Rapha Gontijo Lopes, Raz Gaon, Reah Miyara, Reimar Leike, Renny Hwang, Rhythm Garg, Robin Brown, Roshan James, Rui Shu, Ryan Cheu, Ryan Greene, Saachi Jain, Sam Altman, Sam Toizer, Sam Toyer, Samuel Miserendino, Sandhini Agarwal, Santiago Hernandez, Sasha Baker, Scott McKinney, Scottie Yan, Shengjia Zhao, Shengli Hu, Shibani Santurkar, Shraman Ray Chaudhuri, Shuyuan Zhang, Siyuan Fu, Spencer Papay, Steph Lin, Suchir Balaji, Suvansh Sanjeev, Szymon Sidor, Tal Broda, Aidan Clark, Tao Wang, Taylor Gordon, Ted Sanders, Tejal Patwardhan, Thibault Sottiaux, Thomas Degry, Thomas Dimson, Tianhao Zheng, Timur Garipov, Tom Stasi, Trapit Bansal, Trevor Creech, Troy Peterson, Tyna Eloundou, Valerie Qi, Vineet Kosaraju, Vinnie Monaco, Vitchyr Pong, Vlad Fomenko, Weiye Zheng, Wenda Zhou, Wes McCabe, Wojciech Zaremba, Yann Dubois, Yinghai Lu, Yining Chen, Young Cha, Yu Bai, Yuchen He, Yuchen Zhang, Yunyun Wang, Zheng Shao, and Zhuohan Li. OpenAI o1 System Card, December 2024. URL <http://arxiv.org/abs/2412.16720>. arXiv:2412.16720 [cs].

Shrey Pandit, Austin Xu, Xuan-Phi Nguyen, Yifei Ming, Caiming Xiong, and Shafiq Joty. Hard2Verify: A Step-Level Verification Benchmark for Open-Ended Frontier Math, October 2025. URL <http://arxiv.org/abs/2510.13744>. arXiv:2510.13744 [cs].

Sumanth Varambally, Thomas Voice, Yanchao Sun, Zhifeng Chen, Rose Yu, and Ke Ye. Hilbert: Recursively Building Formal Proofs with Informal Reasoning, September 2025. URL <http://arxiv.org/abs/2509.22819>. arXiv:2509.22819 [cs].

Yifei Xu, Tusher Chakraborty, Srinagesh Sharma, Leonardo Nunes, Emre Kiciman, Songwu Lu, and Ranveer Chandra. Direct Reasoning Optimization: LLMs Can Reward And Refine Their Own Reasoning for Open-Ended Tasks, June 2025. URL <http://arxiv.org/abs/2506.13351>. arXiv:2506.13351 [cs].

Tianyu Yu, Bo Ji, Shouli Wang, Shu Yao, Zefan Wang, Ganqu Cui, Lifan Yuan, Ning Ding, Yuan Yao, Zhiyuan Liu, Maosong Sun, and Tat-Seng Chua. RLPR: Extrapolating RLVR to General Domains without Verifiers, June 2025. URL <http://arxiv.org/abs/2506.18254>. arXiv:2506.18254 [cs].

Lianmin Zheng, Wei-Lin Chiang, Ying Sheng, Siyuan Zhuang, Zhanghao Wu, Yonghao Zhuang, Zi Lin, Zhuohan Li, Dacheng Li, Eric P. Xing, Hao Zhang, Joseph E. Gonzalez, and Ion Stoica. Judging LLM-as-a-Judge with MT-Bench and Chatbot Arena, December 2023. URL <http://arxiv.org/abs/2306.05685>. arXiv:2306.05685 [cs].

Yuxin Zuo, Kaiyan Zhang, Shang Qu, Li Sheng, Xuekai Zhu, Biqing Qi, Youbang Sun, Ganqu Cui, Ning Ding, and Bowen Zhou. TTRL: Test-Time Reinforcement Learning, April 2025. URL <http://arxiv.org/abs/2504.16084>. arXiv:2504.16084 [cs].

A Appendix

A.1 Detail in QiuZhen-Bench

QiuZhen-Bench is a dataset covering a wide range of topics in advanced math, including analysis, algebra, machine learning theory, geometry, topology, probability, statistics and theoretical physics. We have collected 143 exam papers from S.-T. Yau College Student Mathematics Contest since 2010, and the doctoral qualifying exams from QiuZhen college, Tsinghua University since 2023, and required GPT-5 to extract and reformat problems directly from the PDF files. Manual spot checks did not reveal any errors in these extracted contents, indicating the strong capability of GPT-5 in doing this task. We firstly divide the exam papers into pages, and for each page we used the following prompt for extraction:

system prompt

You are a meticulous exam parsing assistant.
Extract ALL distinct problems from the provided text chunks.
Return ONLY valid JSON (no commentary). Focus on problem statements; exclude answers and solutions.

user prompt

Task: Parse the following exam/contest page images to extract problem statements.
Requirements:

- Identify each separate problem with its number or index if present.
- Preserve math using Markdown and LaTeX (inline \dots , display \dots).
- Do not infer content; only use what appears in the images.
- Only include problems written in English; skip non-English (e.g., Chinese).
- If a problem is mixed-language, include only the English statement; otherwise skip.
- If a problem spans multiple pages in this chunk, include the full statement.

Output strictly as JSON array of objects with keys:

- problem_number: string (e.g., '1', 'II-3', 'A.1'; use 'unknown' if missing)
- markdown_statement: string (problem text in Markdown; no solution)
- section: string|null (e.g., 'Analysis', 'Geometry', or null)
- tags: array of strings (optional keywords, may be empty)

- source_pages: array of integers (page numbers within this PDF chunk)
Return ONLY JSON.

fSource: {source.rel_path}\nPages: {chunk.start_page}-{chunk.end_page}

The problems that spans across two pages is extracted by part and combined in the following processing logic. This results in a dataset containing 1,054 problems in total. Here we provide some samples from this dataset:

Yau contest 2015, applied math

Let G be graph of a social network, where for each pair of members there is either no connection, or a positive or a negative one. An unbalanced cycle in G is a cycle which have odd number of negative edges. Traversing along such a cycle with social rules such as friend of enemy are enemy would result in having a negative relation of one with himself! A resigning in G at a vertex v of G is to switch the type (positive or negative) of all edges incident to v . Question: Show that one can switch all edge of G into positive edges using a sequence resigning if and only if there is no unbalanced cycle in G .

QiuZhen qualifying 2024 spring, theoretical physics

$\Pi_{\mu\nu}(q)$ via dimensional regularization (35 points)
At one-loop, the photon propagator in QED receives the correction

$$i\Pi_{\mu\nu}(q) = - \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left(i\gamma^\nu \frac{i}{\not{p} + \not{q} - m} i\gamma^\mu \frac{i}{\not{p} - m} \right),$$

where γ_μ is the gamma matrix and $\not{p} = p^\mu \gamma_\mu$.

i) 10' Show that this expression can be written as

$$i\Pi_{\mu\nu}(q) = -i \int \frac{d^4 p}{(2\pi)^4} \frac{N_{\mu\nu}}{D},$$

where

$$N_{\mu\nu} = \text{tr} [\gamma_\nu (\not{p} + \not{q} + m) \gamma_\mu (\not{p} - m)], \quad \frac{1}{D} = \int_0^1 d\alpha \frac{1}{D},$$

with

$$D = [l^2 + \alpha(1 - \alpha)q^2 - m^2 + i\epsilon]^2, \quad \text{where } l = p + \alpha q.$$

ii) 10' Evaluate $N_{\mu\nu}$ and shift momentum integration from p to l as above.

iii) 10' At the integration step over l , perform dimensional regularization by promoting to a d -dimensional integral. Derive the expression

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - \eta_{\mu\nu} q^2) \Pi(q^2),$$

when taking the limit $d \rightarrow 4$.

iv) 5' Why does this result ensure gauge invariance?

Yau contest 2019, analysis individual

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary. Prove that, for all $p > 1$ and $1 \leq q < \infty$, for all $f \in L^p(\Omega)$, there exists a unique $u \in H_0^1(\Omega)$, such that

$$\Delta u = |u|^{q-1}u + f \quad \text{in } \Omega.$$

QiuZhen qualifying 2025 fall, algebra

Suppose $r \leq n$ are positive integers. Let X be the subset of $M(n \times n, \mathbb{C})$ consisting of complex $n \times n$ matrices with rank at most r . Prove that X is Zariski closed. Find the number of irreducible components of X , and calculate the Krull dimension of each irreducible component of X .

A.2 Prompt template

In our experiments, we use the same evaluation prompt template across all these datasets. For standard, majority voting, and simple pessimistic verification, we use this single-pass verification prompt:

system prompt: single pass

You are an assistant highly proficient in mathematics.
The user will provide a math problem together with its proposed solution, and your task is to verify the correctness of that solution according to the given instruction.

user prompt: single pass

Here is a math problem and a candidate solution of it, and you need to verify the correctness of this solution. Please check each of the following:

1. The provided content is indeed a math problem and its corresponding solution, rather than unrelated material supplied by mistake.
2. The solution actually derives the conclusion required by the original problem.
3. Every step of calculation and formula derivation in the solution is correct.
4. The hypotheses (conditions) and conclusions of any theorems used are correctly matched and applied.
5. The solution relies only on the conditions given in the problem and does not introduce any additional assumptions to obtain the conclusion.

Consistency and error-severity policy (important):

- If only minor, easily fixable issues exist (e.g., small algebraic slips later corrected, notational typos, superficial formatting), treat the solution as correct overall but briefly note such issues.
- If there is any critical error that undermines correctness (e.g., invalid step, wrong theorem usage without required conditions, uncorrected calculation error leading to a wrong result), treat the solution as incorrect.

Response requirements: If the solution is correct overall (possibly with minor issues), reply with ``<verification>true</verification>`` and briefly list minor issues if any.

If the solution is incorrect, reply with ``<verification>false</verification>`` followed by a concise description of the most harmful error.

Do not include any restatement of the entire solution or problem.

`<problem>{problem}</problem>`

`<answer>{solution}</answer>`

The first step of progressive pessimistic verifier also adopts the single pass prompt, and for the following verification of subdivisions, we will use the following chunk verification prompt. This prompt is also used in vertical pessimistic verification in our experiments.

system prompt: chunk verification

You are an assistant highly proficient in mathematics.
The user will provide a math problem together with its proposed solution, and your task is to verify the correctness of that solution according to the given instruction.

user prompt: chunk verification

We provide the original problem and the complete proposed solution for full context

Then we provide a specific chunk from the solution for focused checking.
Your task: Check ONLY the given chunk for errors while considering the overall context.

Checklist:

1. The chunk's reasoning and calculations adhere to mathematical correctness.
2. Any theorems used in the chunk match their hypotheses and conclusions.
3. The chunk does not rely on assumptions not justified by the problem or earlier proven steps.

Consistency and error-severity policy (important):

- If only minor, easily fixable issues exist (e.g., small algebraic slips later corrected, notational typos, superficial formatting), treat the chunk as correct overall but briefly note such issues.
- If there is any critical error that undermines correctness in this chunk (e.g., invalid step, wrong theorem usage without required conditions), treat the chunk as incorrect.

Response requirements: If the chunk is correct overall (possibly with minor issues), reply with ``<verification>true</verification>`` and briefly list minor issues if any.

If the chunk is incorrect, reply with ``<verification>false</verification>`` followed by a concise description of the most harmful error in the proof that you found in the chunk.

`f<problem>{problem}</problem>`

`f<full_answer>{full_proof}</full_answer>`

`f<chunk_index>{idx}</chunk_index>`

`f<chunk>{chunk}</chunk>`

A.3 Original data in experiments

A.4 More detailed case studies