ECON 21020, PSet 6: Solutions

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([ ] out of 38p) PART I: Heteroschedasticity and the Linear Probability Model

([ ] out of 6p) Q1: Proof of Statements (a)-(g) in Claim 1

$$\text{a) NEED OTHER I THINK} \tag{1}$$

$$\begin{aligned} \text{b) } E[D_i|x] = P(D_i = 1|x) = P(\beta_0 + \beta + 1x \geq u_i|x) &= \beta_0 + \beta_1 x && (2)\\ \text{Since } u_i \sim U[0,1] \text{and } \beta_0 + \beta_1 x &\in [0,1] && (3)\end{aligned}$$

$$\begin{aligned} \text{c) } \sigma^2(x) &= E[D_i|x] - E[D_i|x]^2 && (4)\\ &= (\beta_0 + \beta_1 x) - (\beta_0 + \beta_1 x)^2 && (5)\\ &= \beta_0(1-\beta_0) + \beta_1(1-2\beta_0)x - \beta_1^2 x^2 && (6)\\ & && (7)\end{aligned}$$

$$\begin{aligned} \text{d) let } \epsilon_i &= D_i - E[D_i|x] && (8)\\ \text{then } \epsilon_i &= D_i - \beta_0 - \beta_1 x_i && (9)\\ \text{and } D_i &= \beta_0 + \beta_1 x_i + \epsilon_i && (10)\\ & && (11)\end{aligned}$$

$$\begin{aligned} E[\epsilon_i|x] &= E[D_i - E[D_i|x]|x] && (12)\\ &= E[D_i|x] - E[D_i|x] = 0 && (13)\\ & && (14)\end{aligned}$$

$$\begin{aligned} Var[\epsilon_i] &= E[\epsilon_i^2] - E[\epsilon_i]^2 && (15)\\ &= E[(D_i - E[D_i|x])^2|x] && (16)\\ \text{By def. this is} &= Var[D_i|x] && (17)\\ & && (18)\end{aligned}$$

$$\text{e) let } E[\hat{\beta}_1] = E[\frac{\sum_{i=1}^{n}(x_i-\bar{x})(D_i-\bar{D})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] \tag{19}$$

$$\text{Focusing on numerator } = \sum_{i=1}^{n}(x_i-\bar{x})E[D_i-\bar{D}] \tag{20}$$

$$= \sum_{i=1}^{n}(x_i-\bar{x})(\beta_0+\beta_1 x_i-\beta_0-\beta_1\bar{x}) \tag{21}$$

$$= \sum_{i=1}^{n}(x_i-\bar{x})\beta_1(x_i-\bar{x}) \tag{22}$$

$$= \beta_1\sum_{i=1}^{n}(x_i-\bar{x})^2 \tag{23}$$

$$\text{Brining back the denominator we get } = \beta_1\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{24}$$

$$= \beta_1 \tag{25}$$

$$\tag{26}$$

f) Since LR.5 fails because $Var[\epsilon_i|x] = \sigma^2(x)$ the OLS estimators are not BLUE, specifically because they don't have the minimum va (27)

([ ] out of 5p) Q2: Deep dive into the Asymptotic Properties of FWLS Estimator

([ ] out of 2p) Q2.a

$$E[D_i*|x] = w_i E[D_i|x] = \frac{1}{\sqrt{\sigma^2(x)(\beta_0+\beta_1 x_1+\beta_2 x_2)}}\text{mm} \tag{28}$$

([ ] out of 3p) Q2.b

The only difference between $\hat{\beta}^{FWLS}$ and $\hat{\beta}^{WLS}$ is the use of the estimator $\hat{\sigma}^2$ instead of the true value of σ^2. Since the estimator $\hat{\sigma}^2$ converges to the true value of σ^2 as the same size converges to infinity, therefore it makes sense that $\hat{\beta}^{FWLS}$ would converge to $\hat{\beta}^{WLS}$ and therefore as the same size converges to the mean of $\hat{\beta}^{FWLS}$ converges to the mean of $\hat{\beta}^{WLS}$ and variance of $\hat{\beta}^{FWLS}$ converges to variance of $\hat{\beta}^{WLS}$.

([ ] out of 8p) Q3: Relationship b/w Structural and Reduced-form Parameters

Reduced-form Parameter	Sign	Reason for Sign
λ_0	+	Reason for the sign of Parameter 1
λ_1	-	Reason for the sign of Parameter 2
λ_2	+	Reason for the sign of Parameter 3
λ_3	-	Reason for the sign of Parameter 4
λ_4	+	Reason for the sign of Parameter 5
λ_5	-	Reason for the sign of Parameter 6
λ_6	+	Reason for the sign of Parameter 7
λ_7	+	Reason for the sign of Parameter 7

Table 1: The Reduced-form Parameters of the Labor Force Participation Decision of a Married Woman

([ ] out of 4p) Q4: A Linear-in-parameter Model for the LFP decision of a married female

([ ] out of 2p) Q4.a

$$P(D_i = 1|z) = P(\lambda_0 + \lambda_1 e + \lambda_2 x + \lambda_3 x^2 + \lambda_4 k^{\geq 6} + \lambda_5 k^{<6} + \lambda_6 a + \lambda_7 m > u) \quad = \lambda_0 + \lambda_1 e + \lambda_2 x + \lambda_3 x^2 + \lambda_4 k^{\geq 6} + \lambda_5 k^{<6} + \lambda_6 a + \lambda_7 m \tag{29}$$

([ ] out of 2p) Q4.b

Since both β_0 and $\alpha_{1,0}$ influence labor force participation but their effects cannot be measured independelty of eachother, are they not spereately identified. This is because β_0 represents the baseline wage for women regarless of the eVars and $\alpha_{1,0}$ represents the baseline enjoyment level f leasiure regarless of the eVars. Since there are no data points to find the change in participation from changes in β_0 or $\alpha_{1,0}$, they are not seperately identified.

([ ] out of 2p) Q5: Estimate by OLS a LPM for a Married Female's LFP

Script and Output

Commentary

([ ] out of 7p) Q6: Estimate by FWLS a LPM for a Married Female's LFP

Script and Output

([ ] out of 2p) Q7: Verify FWLS Estimate

Script and Output

([ ] out of 2p) Q8: Compare OLS and FWLS Estimates

Script and Output

Commentary

([ ] out of 2p) Q9: Complete Statements

Commentary

([ ] out of 21p) PART II: Three Distributions derived from the Normal Distribution

([ ] out of 5p) Q10: The Normal Distribution

$$\text{a) } Y \sim N(a + b\mu, b^2\sigma^2) \tag{30}$$

$$\text{b) } Y \sim N(\frac{\mu}{c}, \frac{\sigma^2}{c^2}) \tag{31}$$

$$\text{c) } Y \sim N(a + b\mu_1 + c\mu_2, b^2\sigma^2 + c^2\sigma^2 + 2bc\sigma_{1,2}) \tag{32}$$

$$\text{d) } Y \sim N(\mu, \frac{\sigma^2}{n}) \tag{33}$$

$$\text{e) } Y \sim N(0, 1) \tag{34}$$

$$\tag{35}$$

([ ] out of 4p) Q11: Distributions Derived from the Normal Distribution

$$\text{a) } Y \sim \sigma_2\chi_1^2 \tag{36}$$

$$\text{b) } Y \sim \sigma_1^2\chi_1^2 + \sigma_2^2\chi_1^2 \tag{37}$$

$$\text{c) } Y \sim t_q \text{d) } Y \sim F_{(1,1)} \tag{38}$$

([ ] out of 4p) Q12: The Standardized Sample Average for a Normal RS Has a t of Student Distribution

$$\text{We know} \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \sim N(0,1) and \frac{S^2}{\sigma^2} has chi-squared distribution with n-1 degrees of freedom and since \bar{X} and S^2 and independent, \frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} i \tag{39}$$

([ ] out of 1p) Q13: Support of the Chi-squared Distribution

Script and output

Commentary

([ ] out of 1p) Q14: Practice with `lower.tail` argument

Script and output

Commentary

([ ] out of 4p) Q15: Student's t Distribution Converges to Standard Normal

Script and output

Commentary

([ ] out of 2p) Q16 Relationship between F and Student t Distributions

Script and output

Commentary

([ ] out of 41p) PART III: Hypothesis Testing in the MLRM

([ ] out of 0p) Q17: Load data and estimate log wage model

Script and output

([ ] out of 12p) Q18: $H_0 : \beta_{\texttt{exper}} = 0$ versus $H_0 : \beta_{\texttt{exper}} \neq 0$

([ ] out of 1p) Q18.a

We test that the percentage change in hourly wage caused by an extra year of labor market experience is zero.

([ ] out of 3p) Q18.b

([ ] out of 1p) Q18.c

Script and output

([ ] out of 1p) Q18.d

Script and output

([ ] out of 2p) Q18.e Critical Value Approach

Script and Output

([ ] out of 2p) Q18.f P-value Approach

Script and Output

([ ] out of 2p) Q19: Definition of critical value and p-value

([ ] out of 1p) Q19.a

([ ] out of 1p) Q19.b

([ ] out of 2p) Q20: Test $H_0 : \beta_{\texttt{exper}} = 0.007$ versus $H_0 : \beta_{\texttt{exper}} \neq 0.007$ with PV Approach

([ ] out of 1p) Q20.a

Script and output

([ ] out of 1p) Q20.b

Script and output

([ ] out of 6p) Q21: Use `car::linearHypothesis()` to test simple linear hypotheses

([ ] out of 4p) Q21.a Test $H_0 : \beta_{\texttt{exper}} = 0$ versus $H_0 : \beta_{\texttt{exper}} \neq 0$

([ ] out of 2p) Q21.a.i Use Syntax A of Function `car::linearHypothesis()`

Script and output

Commentary

([ ] out of 2p) Q21.a.ii Syntax B of function `car::linearHypothesis()`

Script and output

Commentary

([ ] out of 2p) Q21.b Test $H_0 : \beta_{\texttt{exper}} = 0.007$ versus $H_0 : \beta_{\texttt{exper}} \neq 0.007$

Script and output

([ ] out of 8p) Q22 Test One-sided Hypothesis $H_0 : \beta_{\texttt{exper}} \leq 0$ versus $H_1 : \beta_{\texttt{exper}} > 0$

([ ] out of 1p) Q22.a

([ ] out of 2p) Q22.b

([ ] out of 3p) Q22.c

Script and output

Commentary

([ ] out of 2p) Q22.d

Script and output

Commentary

([ ] out of 11p) Q23 Test $H_0 : \beta_{\texttt{exper}} = \beta_{\texttt{tenure}}$ versus $H_1 : \beta_{\texttt{exper}} \neq \beta_{\texttt{tenure}}$

([ ] out of 1p) Q23.a Interpretation of the Hypothesis

([ ] out of 2p) Q23.b Run the test using `car::linearHypothesis()`

Script and output

Commentary

([ ] out of 4p) Q23.c Run the test using the Reparametrization Approach

Script and output

Commentary

([ ] out of 4p) Q23.d Run the test using the Model Reformulation Approach

Script and output

Commentary
