Problemas impares Carlos Gallegos

1.- $z = \ln(x^2+y^2)$; $x = t^2$, $y = t^{-2}$

$$\frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt}$$

$$= \frac{2x}{x^2+y^2}\cdot 2t + \frac{2y}{x^2+y^2}\cdot -2t^{-3}$$

$$= \frac{4xt}{x^2+y^2} - \frac{4yt^{-3}}{x^2+y^2} = \boxed{\frac{4xt - 4yt^{-3}}{x^2+y^2}}$$

3.- $z = \cos(3x+4y)$; $x = 2t\ \frac{\pi}{2}$; $y = -t\ \frac{\pi}{4}\ t=\pi$

$$\frac{dz}{dt} = -3\operatorname{Sen}(3x+4y)\cdot 2 - 4\operatorname{Sen}(3x+4y)\cdot -1$$

$$= -6\operatorname{sen}(3x+4y) + 4\operatorname{Sen}(3x+4y)$$

$$= -2\operatorname{Sen}(3x+4y) = -2\operatorname{Sen}(2\pi+\tfrac{\pi}{2}) + 4(-\pi-\tfrac{\pi}{2})$$

$$= -2\operatorname{Sen}\left(\frac{15\pi}{2} - \frac{10\pi}{2}\right) = -2\operatorname{Sen}\left(-\frac{5}{2}\pi\right)$$

$$= -2$$

$5.- \rho = \frac{r}{2s+t} \quad r = u^2 \quad s = \frac{1}{u^2} \quad t = \sqrt{u}$

$$\frac{d\rho}{du} = \frac{d\rho}{dr} \cdot \frac{dr}{du} + \frac{d\rho}{ds} \cdot \frac{ds}{du} + \frac{d\rho}{dt} \cdot \frac{dt}{du}$$

$$= \frac{1}{2s+t} \cdot 2u - \frac{2r}{(2s+t)^2} \cdot \frac{-2}{u^3} - \frac{r}{(2s+t)^2} \cdot \frac{1}{2\sqrt{u}}$$

$$= \frac{2u}{2s+t} + \frac{4r}{u^3(2s+t)^2} - \frac{r}{2\sqrt{u}(2s+t)^2}$$

$7.- z = e^{xy^2}; \quad x = u^3; \quad y = u - v^2;$

$$\frac{dz}{du} = y^2 e^{xy^2} \cdot 3u^2 + 2y e^{xy^2} \cdot 1$$

$$\frac{dz}{du} = 3u^2 y^2 e^{xy^2} + 2xy e^{xy^2}$$

$$\frac{dz}{dv} = y^2 e^{xy^2} \cdot 0 + 2xy e^{xy^2} \cdot -2v = \boxed{-4vxy e^{xy^2}}$$

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9.- $Z = 4x - 5y^2 \quad x = u^4 - 8v^3 \quad y = (2v - v)^2$

$$\frac{dz}{du} = 4 \cdot 4u^3 - 10y \cdot 4(2u-v)$$

$$= \boxed{16u^3 - 40y(2u-v)}$$

$$\frac{dz}{dv} = 4 \cdot -24v^2 - 10y \cdot -2(2u-v)$$

$$\boxed{= -96v^2 + 20(2u-v)}$$

11.- $w = (u^2 + v^2)^{3/2} \quad v = e^{-t} \operatorname{Sen}\theta \quad v = e^{t} \operatorname{Cos}\theta$

$$\frac{dw}{dt} = 3u \ (u + v^2)^{1/2} \cdot -e^{-t} \operatorname{Sen}\theta + 3v (u^2+v^2)^{1/2} \cdot e^{-t} \operatorname{Cos}\theta$$

$$= -3u(u^2+v^2)^{1/2} e^{-t} \operatorname{Sen}\theta - 3v(v^2+v^2)^{1/2} e^{-t} \operatorname{Cos}\theta$$

$$\frac{dw}{d\theta} = 3u (u+v^2)^{1/2} e^{-t} \operatorname{Cos}\theta + 3v(u^2+v^2)^{1/2} e^{-t}(-\operatorname{sen}\theta)$$

$$= 3u(u^2+v^2)^{1/2} e^{-t} \operatorname{Cos}\theta - 3v(u^2+v^2)^{1/2} e^{-t} \operatorname{Sen}\theta$$

13.- $R = rs^2 + t^4$ $r = ve^{v^2}$, $s = ve^{-u^2}$, $t = e^{u^2v^2}$

$$\frac{dR}{du} = \frac{dR}{dr} \cdot \frac{dr}{d\theta} + \frac{dR}{ds} \cdot \frac{ds}{du} + \frac{dR}{dt} \cdot \frac{dt}{du}$$

$$= s^2 + 4 \cdot e^{v^2} + 2rst^4 uve^{-u^2} + 8rs^2t^3 uv^2 e^{u^2v^2}$$

$$\frac{dR}{dv} = s^2 + 4 \cdot 2uve^{v^2} + 2rst^4 \cdot e^{-u^2} + 4rs^2t^3 \cdot 2u^2ve^{u^2v^2}$$

$$= 2s^2 + 4uve^{v^2} + 2rst^4e^{-u^2} + 8rs^2t^3 u^2ve^{u^2v^2}$$

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$$15 = w = \sqrt{x^2+y^2} \quad x = \ln(rs+tu) \quad y = \frac{t}{u}\cosh rs$$

$$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt} + \frac{dw}{dy} \cdot \frac{dy}{dt}$$

$$= \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{u}{(rs+tu)} + \frac{y}{\sqrt{x^2+y^2}} \cdot \frac{\cosh(rs)}{u}$$

$$= \frac{xu}{\sqrt{x^2+y^2}(rs+tu)} + \frac{y\cosh rs}{u\sqrt{x^2+y^2}}$$

$$\frac{dw}{dr} = \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{s}{rs+tu} + \frac{y}{\sqrt{x^2+y^2}} \cdot \frac{st\cosh(s)}{u}$$

$$= \frac{xs}{\sqrt{x^2+y^2}(rs+tu)} + \frac{sty\cosh(rs)}{u\sqrt{x^2+y^2}}$$

$$\frac{dw}{du} = \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{t}{rs+tu} + \frac{y}{\sqrt{x^2+y^2}} \cdot \frac{t\cosh(rs)}{u^2\sqrt{x^2+y^2}}$$

$$= \frac{xt}{\sqrt{x^2+y^2}(rs+tu)} + \frac{ty\cosh(rs)}{u^2\sqrt{x^2+y^2}}$$