Cálculo

Ficha 8-B — Dezembro de 2008

Derivação do integral

1. Calcule a derivada da função $\int_1^x \frac{\sqrt{1+t^4}}{t^2}\,dt$, para $x>0$;

2. Determine uma função contínua f tal que
$$\int_0^{x^2} f(t)\,dt = x^3e^x - x^4, \quad \forall x\in\mathbb{R}.$$

3. Estude a monotonia da função $f(x)=\int_0^{x^3} e^{-t^2}dt$.

4. Seja f uma função real de variável real que satisfaz a condição
$$\int_0^x f(t)\,dt = \frac{4}{3}+3x^2+\operatorname{sen}(2x)+\frac{1}{2}\cos(2x).$$
Calcule $f(\frac{\pi}{2})$ e $f'(\frac{\pi}{4})$.

5. Seja f uma função real de variável real definida por
$$f(x)=\int_0^x \frac{1+\operatorname{sen}(t)}{2+t^2}dt.$$
Sem calcular o integral, encontre um polinómio P de grau 2 tal que $P(0)=f(0)$, $P'(0)=f'(0)$ e $P''(0)=f''(0)$.

Áreas planas

6. Em cada alínea, determine a medida da área da região limitada pelas curvas cujas equações são as indicadas:

   (a) $y=0, \quad x=-\ln 2, \quad x=\ln 2, \quad y=\operatorname{senh}(x).$
   (b) $x=0, \quad x=2, \quad x^2+(y-2)^2=4, \quad x^2+(y+2)^2=4;$
   (c) $x=0, \quad x=1, \quad y=3x, \quad y=-x^2+4;$
   (d) $y+x^2=6, \quad y+2x-3=0;$
   (e) $x=-1, \quad y=|x|, \quad y=2x, \quad x=1;$
   (f) $y\le -|x|, \quad y\ge -4 \quad x\le 2;$
   (g) $y-x=6, \quad y-x^3=0, \quad 2y+x=0;$
   (h) $y=-x^3, \quad y=-(4x^2-4x);$
   (i) $y=-x^2+\frac{7}{2}, \quad y=x^2-1.$
   (j) $y=\cos x, \quad y=x+1, \quad x=\pi;$
   (k) $y=\frac{1}{x}, \quad 2x+2y=5;$

(l) $y = \dfrac{4}{x^2}, \quad y = 5 - x^2$;

(m) $x^2 = 12(y-1), \quad x^2 + y^2 = 16$;

7. Indique como recorreria ao cálculo integral para determinar a área de cada uma das seguintes regiões:

(a) $\{(x,y) \in \mathbb{R}^2 : x^2 - 1 \leq y \leq x + 1\}$;

(b) $\{(x,y) \in \mathbb{R}^2 : -1 \leq x \leq 2 \ \text{ e } \ 0 \leq y \leq e^x \ \text{ e } \ 0 \leq y \leq e^{-x}\}$;

(c) $\{(x,y) \in \mathbb{R}^2 : y \geq 0 \ \text{ e } \ y \geq x^2 - 2xy \leq 4\}$.

Soluções da Ficha 8-B

1. $f'(x) = \frac{\sqrt{1+x^4}}{x^2}$

2. $f(x) = \frac{3}{2}\sqrt{x}\,\mathrm{e}^{\sqrt{x}} + \frac{1}{2}x\,\mathrm{e}^{\sqrt{x}} - 2x, \ \forall x \geq 0$

3. Crescente

4. $f\left(\frac{\pi}{2}\right) = 3\pi - 2, \ f'\left(\frac{\pi}{4}\right) = 8$

5. $P(x) = \frac{1}{4}x^2 + \frac{1}{2}x$

6. (a) $2\int_0^{\ln(2)} \operatorname{senh}(x)dx = \frac{1}{2}$

(b) $2\int_0^1 2 - \sqrt{4-x^2}dx = 4 - \sqrt{3} - \frac{2\pi}{3}$

(c) $\int_{-1}^3 (6-x^2) - (3-2x)dx = \frac{32}{3}$

(d) $\int_{-1}^0 -x - 2xdx + \int_0^1 2x - xdx = 2$

(e) $\int_{-4}^0 x - (-4)dx + \int_0^2 -x - (-4)dx = 14$

(f) $\int_{-4}^0 (x+6) - (-\frac{x}{2})dx + \int_0^2 (x+6) - x^3dx = 22$

(g) $\int_0^{-2} (-4x^2 + 4x) - (-x^3)dx = \frac{4}{3}$

(h) $\int_{-3/2}^{3/2} (-x^2 + 7/2) - (x^2 - 1)dx = 9$

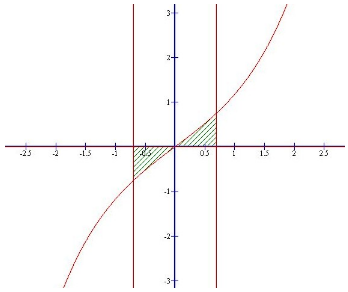
(i) $\int_0^\pi (x+1) - \cos(x)dx = \pi + \frac{\pi^2}{2}$

(j) $\int_{1/2}^2 (\frac{5}{2} - x) - \frac{1}{x}dx = -2\ln(2) + \frac{15}{8}$

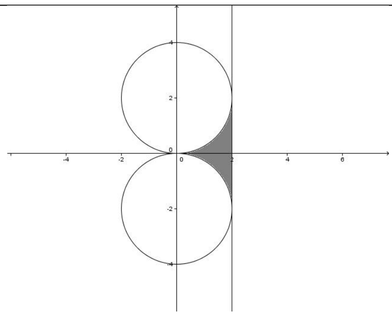
(k) $2\int_1^2 (5-x^2) - \frac{4}{x^2}dx = \frac{4}{3}$

(l) $\int_{-2\sqrt{3}}^{2\sqrt{3}} \sqrt{16-x^2} - (\frac{x^2}{12} + 1)dx = \frac{16\pi - 4\sqrt{3}}{3}$

Correspondendo às seguintes regiões:



(a)



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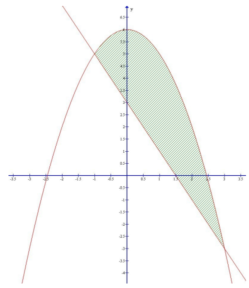

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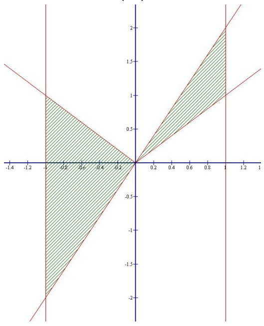

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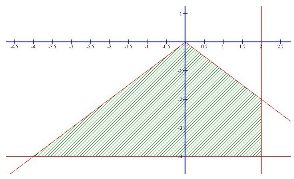

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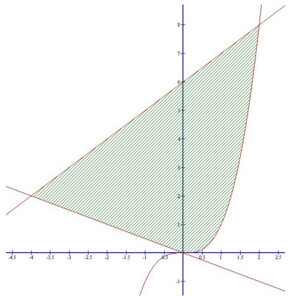

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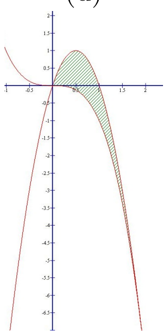

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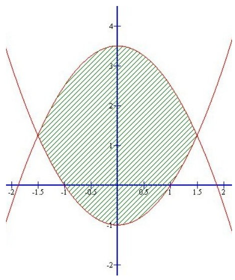

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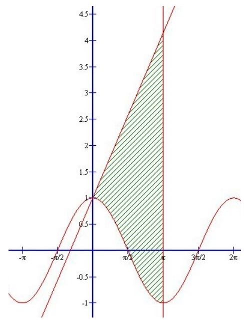

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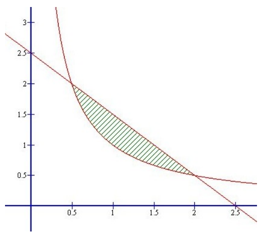

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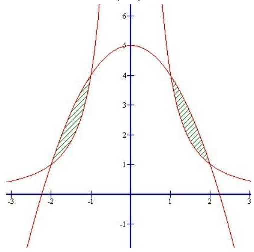
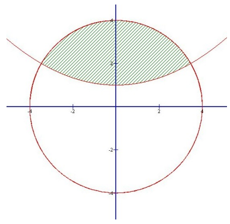


(l)

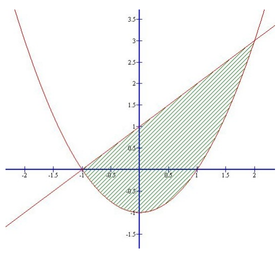


(m)

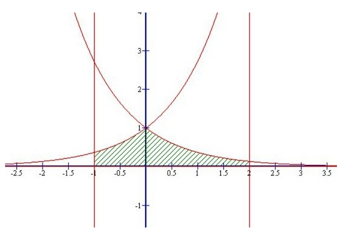
7. (a) $\int_{-1}^{2}(x+1)-(x^2-1)dx$

   (b) $\int_{-1}^{0} \mathrm{e}^{x}\, dx + \int_{0}^{2} \mathrm{e}^{-x}\, dx$

   (c) $$\int_{-2}^{4-2\sqrt{5}} \frac{x^2-4}{2x}dx + \int_{4-2\sqrt{5}}^{2} \frac{x^2}{1+2x} + \int_{2}^{42\sqrt{5}} \frac{x^2}{1+2x} - \frac{x^2-4}{2x}dx$$



(a)



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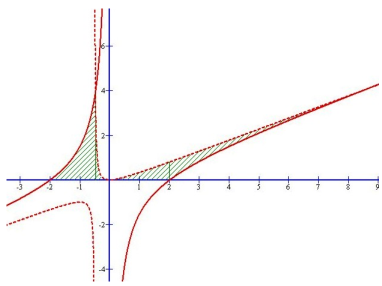

(c)