Homework 1

Oscar Carreón Cerda; Mariana González Sañudo; Juan José Merino Zarco

2/5/2021

CONTENTS

1 - Romer 8.1, 8.2, 8.4, 8.5, 8.6

8.1 - Life-cycle saving

An individual who lives in t = {0, T} and has income of $Y_0 + gt$ for $0 \leq t < R$, and 0 for $R \leq t \leq T$. Assume no initial wealth, no interest rate, no uncertainty. Her utility function is given by:

$$U = \int_{t=0}^{T} U\left(C\left(t\right)\right) dt$$

a) What is the individual's lifetime budget constraint?

For R years, this individual earns a yearly income of gt, plus some Y_0 amount. Therefore, consumption over the T years she expects to live should be, at most, equal to her income:

$$\int_{t=0}^{R} \left(Y_o + gt\right) dt \geq \int_{t=0}^{T} C\left(t\right) dt$$

After solving the LHS integral, it boils down to:

$$\int_{t=0}^{R} \left(Y_o + gt\right) dt = (Y_0 * R + \frac{gR^2}{2}) - (Y_0 * (0) + \frac{g(0)^2}{2}) = (Y_0 * R + \frac{gR^2}{2})$$

Hence, the budget constraint is:

$$\rightarrow Y_0 * R + \frac{gR^2}{2} \geq \int_{t=0}^{T} C\left(t\right) dt$$

b) What's her Utility maximizing path of consumption?

Because $U''(*) < 0$, we know there must be a maximum value of consumption that accomplishes Max U. Since interest rate is 0, this amount will be the same for all periods. Therefore, the optimum consumption per period is a constant:

$$\int_{t=0}^{T} C\left(t\right) dt = \int_{t=0}^{T} \overline{C} = T * \overline{C}$$

Moreover, at the maximum all income is spent: the weak inequality in the budget constraint becomes an equality. Thus:

$$T * \overline{C} = R * (Y_0 + \frac{gR}{2})$$

$$\rightarrow \overline{C} = \frac{R}{T} * (Y_0 + \frac{gR}{2}) = C^*(t)$$

The utility-maximizing path of consumption is a constant fraction (1/T) of the life-time income.

c) What is the path of wealth as a function of t?

We know $W_0 = 0$. Now, wealth after each period will follow savings, i.e., a function with two shapes: one from 0 to R and another from R to T.

$$W(t) = \begin{pmatrix} \int_0^t (Y_0 + gs - \overline{C})ds; \forall t \in [0, R) \\ W(R) - \int_R^t \overline{C}ds; \forall t \in [R, T] \end{pmatrix}$$

Consequently, there are two integrals to solve in this problem:
i) for t $\in$ [0,R) (i.e., her working life time):

$$W(t) = \int_0^t (Y_0 + gs - \overline{C})ds$$

$$W(t) = Y_0 t - \frac{g}{2}t^2 - \overline{C}t$$

ii) for t $\in$ [R,T] (i.e., after retirement), the individual has a stock of wealth accumulated from 0 to R, W(R). We rewrite that expression to account for the (known) value of consumption, $\overline{C}$:

$$W(R) = Y_0 R - \frac{g}{2}R^2 - \overline{C}R$$

The first two terms in W(R) are equal to $T\overline{C}$:

$$\overline{C} = \frac{R}{T} * (Y_0 + \frac{gR}{2})$$

Thus making

$$W(R) = T\overline{C} - R\overline{C} = \overline{C}[T - R]$$

Finally, during retirement, this individual will spendthis wealth in consumption at a steady rate of $-\overline{C}$:

$$\int_R^t -\overline{C}ds$$

Putting both expressions together, we have the second leg of this individual's path of wealth:

$$W(t) = \overline{C}[T - R] - \overline{C}[t - R]$$

$$W(t) = \overline{C}[T - t]$$

In summary, for both periods (0 to R and R to T), the path of wealth is:

$$W(t) = \begin{pmatrix} Y_0 t - \frac{g}{2}t^2 - \overline{C}t; \forall t \in [0, R) \\ \overline{C}[T - t]; \forall t \in [R, T] \end{pmatrix}$$

8.2 - Permanent income vs transitory income

The average income of farmers is less than non-farmers', but fluctuates more year-to-year. How does the permanent income hypothesis (PIH) predict that estimated consumption functions for farmers differs from that of non-farmers?

According to the PIH, consumption follows this path:

$$C_i = \alpha + \beta_i + \epsilon_i$$

Because consumption depends on permanent income, we know that β will vary only as the fraction of variance on income that is due to variance on permanent income:

$$\beta = \frac{Var(Y_p)}{Var(Y_p) + Var(Y_t)}$$

Now, we know $Var(Y_t^F) > Var(Y_t^{NF})$, which makes $\beta_i^F < \beta_i^{NF}$: consumption is higher for non-farmers. However, there is also a constant term, α, and we know its value: $\alpha = (1 - \beta) * Y_p$. Because β is higher for non-farmers, α will be higher for farmers, thus making the answer ambiguous: whose consumption is higher?

But we know that non-farmers will consume more on average. Through good years and bad years, their average permanent income is higher than that of farmers'. At some point, unusually, farmers' income may lead to high consumption, but such good years are not expected to stay.

8.4 - Uncertainty, future income, and consumption

In the model of Section 8.2, uncertainty about future income does not affect consumption. Does this mean that the uncertainty does not affect expected lifetime utility?

Expected lifetime utility is given by, $E_1[U] = E_1[\sum_{t=1}^{T}(C_t - \frac{aC_t^2}{2})], a > 0.$

$= \sum_{t=1}^{T}(E(C_t) - \frac{aE(C_t^2)}{2}), a > 0 \ldots$ (1)

Also, $E_1(C_t) = C_1$, $C_t = C_1 + e_t$, where e_t represents uncertainty.

This does not change $E(C_t) = C_1$

$E(C_1 + e_t) = C_1$, with $E(e_t) = 0$

$var(e_t) = \sigma^2$

e_t is independant to C_1

$=> E(C_1 e_t) = E(C_1)E(e_t)$

with $E(e_t) = 0$, $=> C_1 \cdot 0 = 0$

Rewriting equation (1)

$E_1[U] = \sum_{t=1}^{T}(C_1 \frac{a}{2} E_1[(C_1 + e_t)^2])$

$\sum_{t=1}^{T}(C_1 - \frac{a}{2} E_1[C_1^2 + e_t^2 + C_1 e_t])$

$\sum_{t=1}^{T}(C_1 - \frac{a}{2}[E_1(C_1^2) + E(e_t^2) + E(C_1 e_t)])$

$\sum_{t=1}^{T}(C_1 - \frac{a}{2} C_1^2 - \frac{a}{2} E(e_t^2))$

Note that: $var(e_t) = E(e_t^2) - (E(e_t))^2 = E(e_t^2) - 0^2$

$Var(e_t) = E(e_t^2) = \sigma^2$

$U = \sum_{t=1}^{T}(C_1 - \frac{a}{2} C_1^2 - \frac{a}{2}\sigma^2)$

When $e_t = 0, E[e_t] = 0$ and $Var(e_t) = 0$ $U = \sum_{t=1}^{T}(C_1 - \frac{a}{2} C_1^2)$

Therefore, the “U” will be lower for the case with uncertainty.

8.5 - Equilibrium with a CRRA utility function

$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}; \theta > 0$. Assume a constant interest rate which may not be equal to the discount rate, ρ.

a) Euler equation relating current consumption to expected consumption in the following period.

The Euler equation tells us that, at the optimum, marginal utility in period t equals that of period t+1, such that reducing consumption at t and increasing it at t+1, by a small fraction, won't make a difference in utility (overall). Thus:

$$\frac{C_t^{-\theta}}{(1+\rho)^t * (1+\theta)} = \frac{(1+r) * E_t(C_{t+1}^{-}\theta)}{(1+\rho)^{t+1} * (1+\theta)}$$

We clean for C_t in the above expression and reach this:

$$C_t = [E_t(C_{t+1}^{-\theta}) * \frac{1+r}{(1+\rho)}]^{-(1/\theta)}$$

b) Consumption is normally distributed

There is some information to extract from the equation above: $E_t(C_{t+1}^{-\theta})$. Specifically, we know two things about the log of C_{t+1} and we need to include those in our answer. So,

$$E_t(C_{t+1}^{-\theta}) = E_t(e^{-\theta ln(C_{t+1})}) = E_t(e^{-\theta\mu} * e^{\theta^2\sigma^2/2})$$

Notice the θ^2 next to variance. This is because $Var(\theta x) = \theta^2 Var(x)$. Finally, we have the expected valued of consumption in the next period. We can come back to our first equation and do the rest of the work. We begin by applying log to both sides:

$$ln(C_t) = \frac{-1}{\theta}[ln(e^{-\theta\mu} * e^{\theta^2\sigma^2/2}) + ln(1+r) - ln(1+\rho)]$$

Notice: $ln(e^x) = x$, thus:

$$ln(C_t) = \frac{-1}{\theta}[-\theta\mu + \theta^2\sigma^2/2 + ln(1+r) - ln(1+\rho)]$$

Finally:

$$ln(C_t) = \mu + \frac{\theta\sigma^2}{2} + \frac{1}{\theta}[ln(1+\rho) - ln(1+r)]$$

c) Consumption follows a random walk when interest rate and variance are constante over time

In the last expression, μ is equal to $E_t[ln(C_{t+1})]$. Therefore, if we clear for μ and make $a = \frac{\theta\sigma^2}{2} + \frac{1}{\theta}[ln(1+\rho) - ln(1+r)]$ we are left with this:

$$E_t[ln(C_{t+1})] = a + ln(C_t)$$

This expression is compatible with

$$ln(C_{t+1}) = a + ln(C_t) + u_{t+1}$$

provided $E(u_t) = 0$, which should hold for a white noise.

d) How do changes in each of r and σ^2 affect expected consumption growth, $E_t[ln(C_{t+1}) - ln(C_t)]$? Interpret the effect of σ^2 on expected expected consumption growth in light of the discussion of precautionary saving.

Using expected consumption growth

$E_t[ln(C_{t+1})] = \frac{ln(1+r) - ln(1+\rho)}{\theta} + \frac{\theta\sigma^2}{2} + ln(C_t)$

and $ln(C_t) = E_t[ln(C_t)]$

Then,

$E_t[ln(C_{t+1})] - E_t[ln(C_t)] = \frac{ln(1+r) - ln(1+\rho)}{\theta} + \frac{\theta\sigma^2}{2}$

$E_t[ln(C_{t+1}) - ln(C_t)] = \frac{ln(1+r) - ln(1-\rho)}{\theta} + \frac{\theta\sigma^2}{2}$

- If r changes:

$\frac{\partial E_t[ln(C_{t+1}) - ln(C_t)]}{\partial r} = \frac{1}{\theta}\frac{1}{1+r} > 0$, with $\theta > 0, r > 0$

=> An increase in r leads to an increase in expected consumption growth.

- If σ^2 changes:

$\frac{\partial E_t[ln(C_{t+1}) - ln(C_t)]}{\partial \sigma^2} = \frac{\theta}{2} > 0$, with $\theta > 0$

=> An increase in σ^2 leads to an increase in expected consumption growth.

To know if the CRRA utility function has the precautionary saving behavior, we need it to have a positive third derivative.

$u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$

$u'(C_t) = C_t^{-\theta}$

$u''(C_t) = -\theta C_t^{-\theta-1}$

$u'''(C_t) = -\theta(-\theta-1)C_t^{-\theta-2} = (\theta^2+\theta)C_t^{-\theta-2} > 0$

Increased uncertainty (measured by σ^2) is an incentive to save on precautionary saving behavior and thus an increase in consumer growth expectations.

8.6 - Excess smoothness

A framework for investigating excess smoothness. Suppose that C_t equals $[\frac{r}{1+r}[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]]$, and that $A_{t+1} = (1+r)(A_t + Y_t - C_t)$

a) Show that these assumptions imply that $E_t[C_{t+1}] = C_t$ (and thus that consumption follows a random walk) and that $\sum_{s=0}^{\infty} \frac{E_t(C_{t+s})}{(1+r)^s} = A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}$.

$C_t = \frac{r}{1+r}[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}] \ldots$ (1)

Replacing in: $A_{t+1} = (1+r)[A_t + Y_t - C_t]$

$A_{t+1} = (1+r)[A_t + Yt - \frac{r}{1+r}[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]$

$A_{t+1} = (1+r)A_t + (1+r)Yt - r[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]$

$A_{t+1} = (1+r)A_t + (1+r)Yt - r[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]$

$A_{t+1} = (1+r)A_t + (1+r)Yt - rA_t - rY_t - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]$

$A_{t+1} = A_t + Yt - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}] \ldots$ (2)

Now, rewriting equation (1) for C_{t+1},

$C_{t+1} = \frac{r}{1+r}[A_{t+1} + \sum_{s=0}^{\infty} \frac{E_{t+1}(Y_{t+1+s})}{(1+r)^s}]$

Replacing (2) in the previous equation

$C_{t+1} = \frac{r}{1+r}[(A_t + Yt - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]) + \sum_{s=0}^{\infty} \frac{E_{t+1}(Y_{t+1+s})}{(1+r)^s}]$

Taking the expected value on both sides

$E_t(C_{t+1}) = E_t(\frac{r}{1+r}[(A_t + Yt - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]) + \sum_{s=0}^{\infty} \frac{E_{t+1}(Y_{t+1+s})}{(1+r)^s}])$

Note that: $E_t(E_{t+1}(x_{t+2})) = E_t(x_{t+2})$ to hold original expectations.

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + Yt - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}] + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+1+s})}{(1+r)^s}]$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + Yt - r[\sum_{s=1}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}] + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+1+s})}{(1+r)^s}]]$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + Yt - r[\frac{E_t(Y_{t+1})}{(1+r)^1} + \frac{E_t(Y_{t+2})}{(1+r)^2} + \frac{E_t(Y_{t+3})}{(1+r)^3} + ...] + [\frac{E_t(Y_{t+1})}{(1+r)^0} + \frac{E_t(Y_{t+2})}{(1+r)^1} + \frac{E_t(Y_{t+3})}{(1+r)^2} + ...]]$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + Yt + (\frac{1}{(1+r)^0})(1 - \frac{r}{1+r})E_t(Y_{t+1}) + (\frac{1}{(1+r)^1})(1 - \frac{r}{1+r})E_t(Y_{t+2}) + (\frac{1}{(1+r)^2})(1 - \frac{r}{1+r})E_t(Y_{t+3}) + ...]$

Note that: $1 - \frac{r}{1+r} = \frac{1+r-r}{1+r} = \frac{1}{1+r}$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + Yt + (\frac{1}{(1+r)^0})(\frac{1}{1+r})E_t(Y_{t+1}) + (\frac{1}{(1+r)^1})(\frac{1}{1+r})E_t(Y_{t+2}) + (\frac{1}{(1+r)^2})(\frac{1}{1+r})E_t(Y_{t+3}) + ...]$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + (\frac{1}{(1+r)^0})E_t(Y_t) + (\frac{1}{(1+r)^0})(\frac{1}{1+r})E_t(Y_{t+1}) + (\frac{1}{(1+r)^1})(\frac{1}{1+r})E_t(Y_{t+2}) + (\frac{1}{(1+r)^2})(\frac{1}{1+r})E_t(Y_{t+3}) + ...]$

$E_t(C_{t+1}) = \frac{r}{1+r}[A_t + \sum_{s=0}^{\infty} \frac{E_t(t_{t+s})}{(1+r)^s}]$

$=> E(C_{t+1}) = C_t$

This means that consumption follows a random walk, so the changes are not predictable.

So we can assume that $E_t(C_{t+s}) = C_t$, with $s \geq 0$

$\sum_{s=0}^{\infty} \frac{E_t(C_{t+s})}{(1+r)^s} = \sum_{s=0}^{\infty} \frac{C_t}{(1+r)^s} = C_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s}$

Considering that it is a convergent serie, because $\frac{1}{1+r} < 1$.

$C_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} = C_t \frac{1}{1-\frac{1}{1+r}} = C_t \frac{1+r}{r}$

Replacing (1) in the previous equation.

$C_t(\frac{1+r}{r}) = \frac{r}{1+r}[A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}]\frac{1+r}{r}$

$=> \sum_{s=0}^{\infty} \frac{E_t(C_{t+s})}{(1+r)^s} = A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}$

b) Suposse that $\Delta Y_t = \Phi \Delta Y_{t-1} + u_t$, where u is white noise. Suppose that Y_t exceeds $E_{t-1}[Y_t]$ by 1 unit (that is, suppose $u_t = 1$). By how much does consumption increase?

Starting from:

$C_t = \frac{r}{1+r}(A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s})$

Taking expected value for the period t-1 on both sides,

$E_{t-1}(C_t) = E_{t-1}(\frac{r}{1+r}(A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}))$

Considering that $E_{t-1}(E_t(Y_{t+s})) = E_{t-1}(Y_{t+s})$,

$E_{t-1}(C_t) = \frac{r}{1+r}(A_t + \sum_{s=0}^{\infty} \frac{E_{t-1}(Y_{t+s})}{(1+r)^s})$

Subtracting $E_{t-1}(C_t)$ from C_t,

$C_t - E_{t-1}(C_t) = \frac{r}{1+r}(A_t + \sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s}) - \frac{r}{1+r}(A_t + \sum_{s=0}^{\infty} \frac{E_{t-1}(Y_{t+s})}{(1+r)^s})$

$= \frac{r}{1+r}(\sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})}{(1+r)^s} - \sum_{s=0}^{\infty} \frac{E_{t-1}(Y_{t+s})}{(1+r)^s})$

$= \frac{r}{1+r}(\sum_{s=0}^{\infty} \frac{E_t(Y_{t+s})-E_{t-1}(Y_{t+s})}{(1+r)^s})$

$= \frac{r}{1+r}((\frac{E_t[Y_t]-E_{t-1}[Y_t]}{(1+r)^0}) + (\frac{E_t[Y_{t+1}]-E_{t-1}[Y_{t+1}]}{(1+r)^1}) + ...)$

Note that:

- $E_t(Y_t) = Y_t$ and $Y_t - E_{t-1}Y_t = 1 = \Delta Y_t$
- $\Delta Y_{t+1} = \Phi \Delta Y_t + u_{t+1} = \Phi(1) + 1 = \Phi + 1 = E_t[Y_{t+1}] - E_{t-1}[Y_{t+1}]$

$=> \frac{E_t[Y_{t+1}]-E_{t-1}[Y_{t+1}]}{(1+r)^1} = \frac{\Phi+1}{1+r}$

- $\Delta Y_{t+2} = \Phi \Delta Y_{t+1} + u_{t+2} = \Phi(\Phi + 1) + 1 = \Phi^2 + \Phi + 1 = E_t[Y_{t+2}] - E_{t-1}[Y_{t+2}]$

$=> \frac{E_t[Y_{t+2}]-E_{t-1}[Y_{t+2}]}{(1+r)^1} = \frac{\Phi^2+\Phi+1}{(1+r)^2}$

So,

$C_t - E_{t-1}(C_t) = \frac{r}{1+r}(1 + \frac{\Phi+1}{1+r} + \frac{\Phi^2+\Phi+1}{(1+r)^2} + ...)$

$= \frac{r}{1+r}((1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + ...) + (\frac{\Phi}{(1+r)^1} + \frac{\Phi}{(1+r)^2} + \frac{\Phi}{(1+r)^3} + ...) + (\frac{\Phi^2}{(1+r)^2} + \frac{\Phi^2}{(1+r)^3} + \frac{\Phi^2}{(1+r)^4} + ...) + ...)$

$= \frac{r}{1+r}((1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + ...) + (\frac{\Phi}{(1+r)^1})(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + ...) + (\frac{\Phi^2}{(1+r)^2})(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + ...) + ...)$

Note that: $(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + ...) = \frac{1+r}{r}$

$= \frac{r}{1+r}((\frac{1+r}{r}) + (\frac{\Phi}{(1+r)^1})(\frac{1+r}{r}) + (\frac{\Phi^2}{(1+r)^2})(\frac{1+r}{r}) + ...)$

$= \frac{r}{1+r}(\frac{1+r}{r})((1) + (\frac{\Phi}{(1+r)^1}) + (\frac{\Phi^2}{(1+r)^2}) + ...)$

$= (1) + (\frac{\Phi}{(1+r)^1}) + (\frac{\Phi^2}{(1+r)^2}) + ...$

If $(\Phi(\frac{1}{1+r})) < 1$, then

$(1) + (\frac{\Phi}{(1+r)^1}) + (\frac{\Phi^2}{(1+r)^2}) + ... = \frac{1}{1-\Phi(\frac{1}{1+r})} = \frac{1+r}{1+r-\Phi}$

$C_t - E_{t-1}(C_t) = \frac{1+r}{1+r-\Phi}$

c) For the case of $\Phi > 0$, which has a larger variance, the innovation in income, u_t , or the innovation in consumption, $C_t - E_{t-1}(C_t)$? Do consumers use saving and borrowing to smooth the path of consumption relative to income in this model? Explain.

Income innovation => $Var(u_t)$

Consumption innovation $Var(C_t - E_{t-1}(C_t)) = Var(\frac{1+r}{1+r-\Phi}u_t) = (\frac{1+r}{1+r-\Phi})^2 Var(u_t)$

$Var(C_t - E_{t-1}(C_t)) > Var(u_t)$

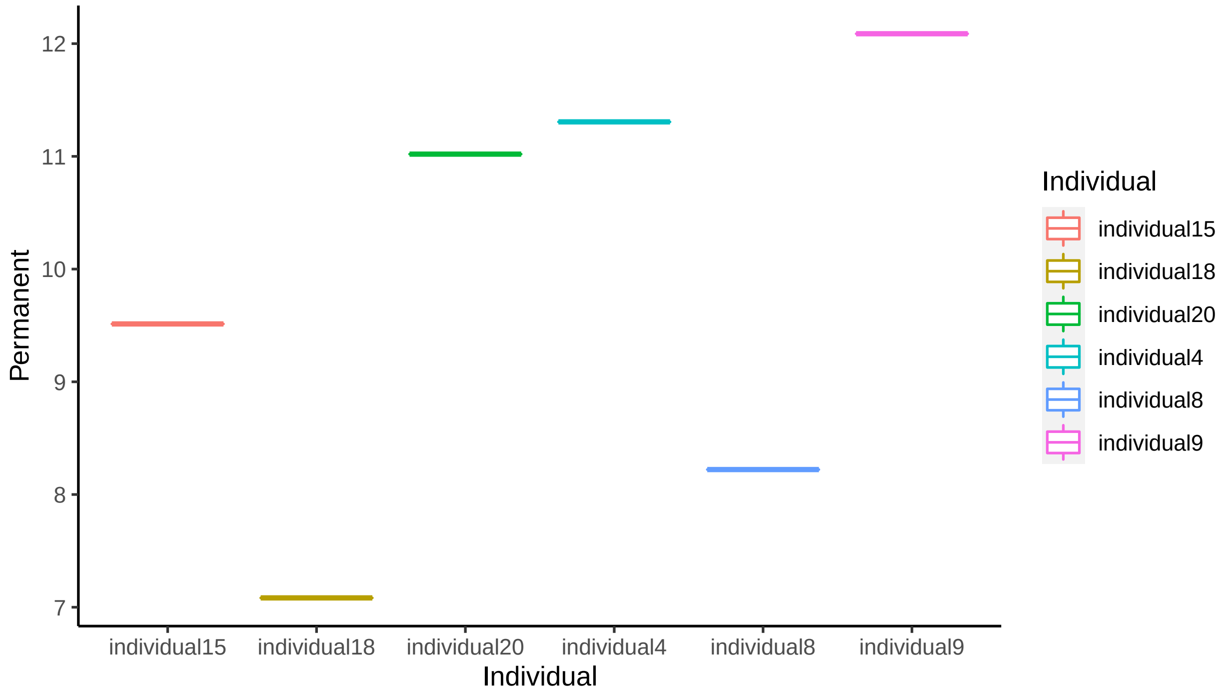
The variance in consumption innovation is greater than the income innovation variance, because when there is an income innovation you expect that in the following periods you have higher incomes.

Consumption follows a random walk, it is not possible to determine whether savings and loans are used to smooth consumption.

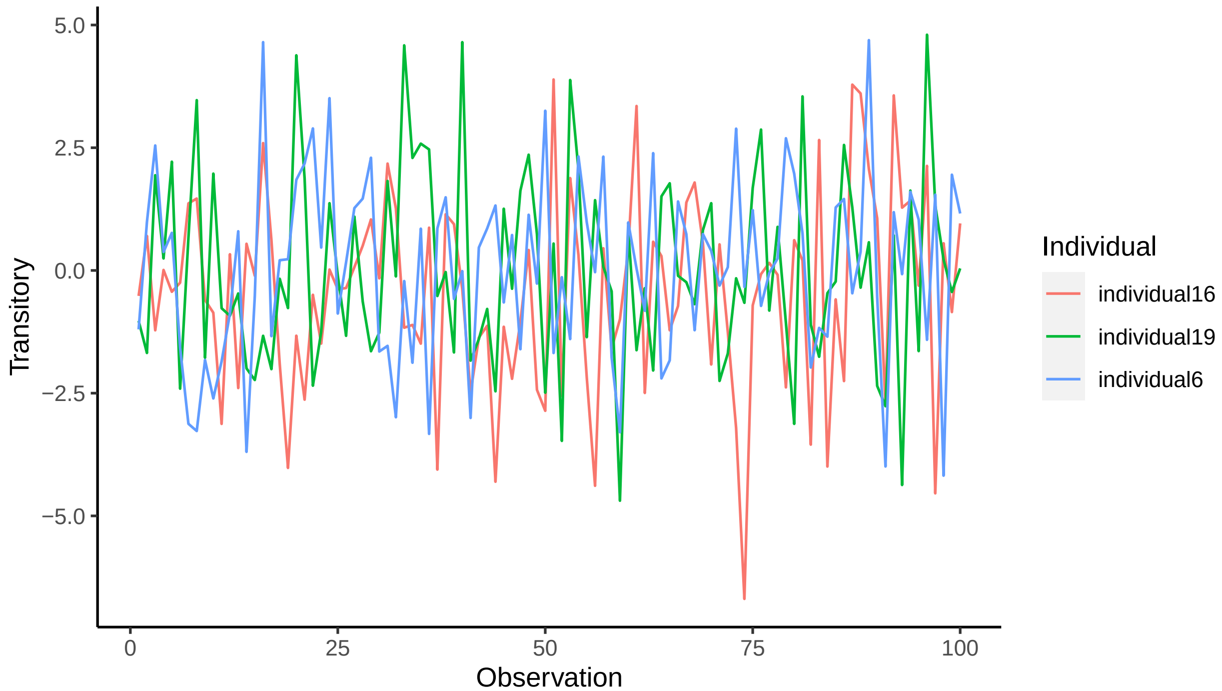
2 - Permanent Income, Transitory Income and Consumption

Simulate vectors for a few individuals with different permanent and transitory incomes, and calculate the relationship between consumption and income:

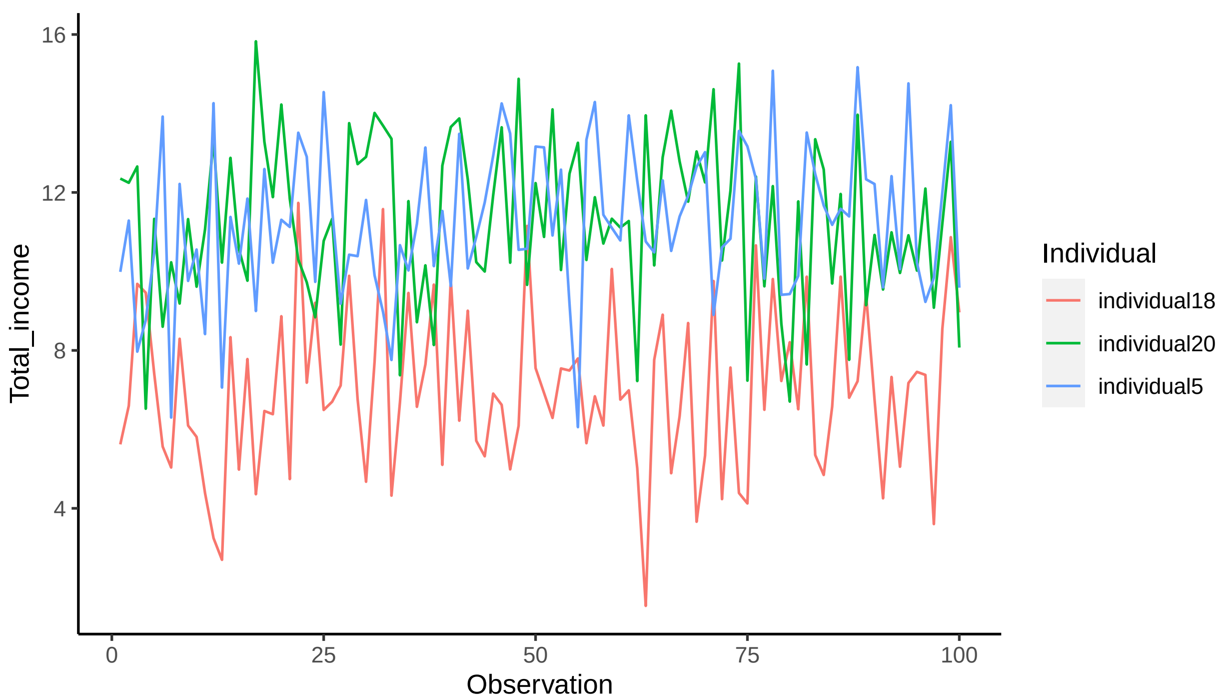
a) vector of 20 random permanent incomes, normally distributed (mean = 10, var > 0)



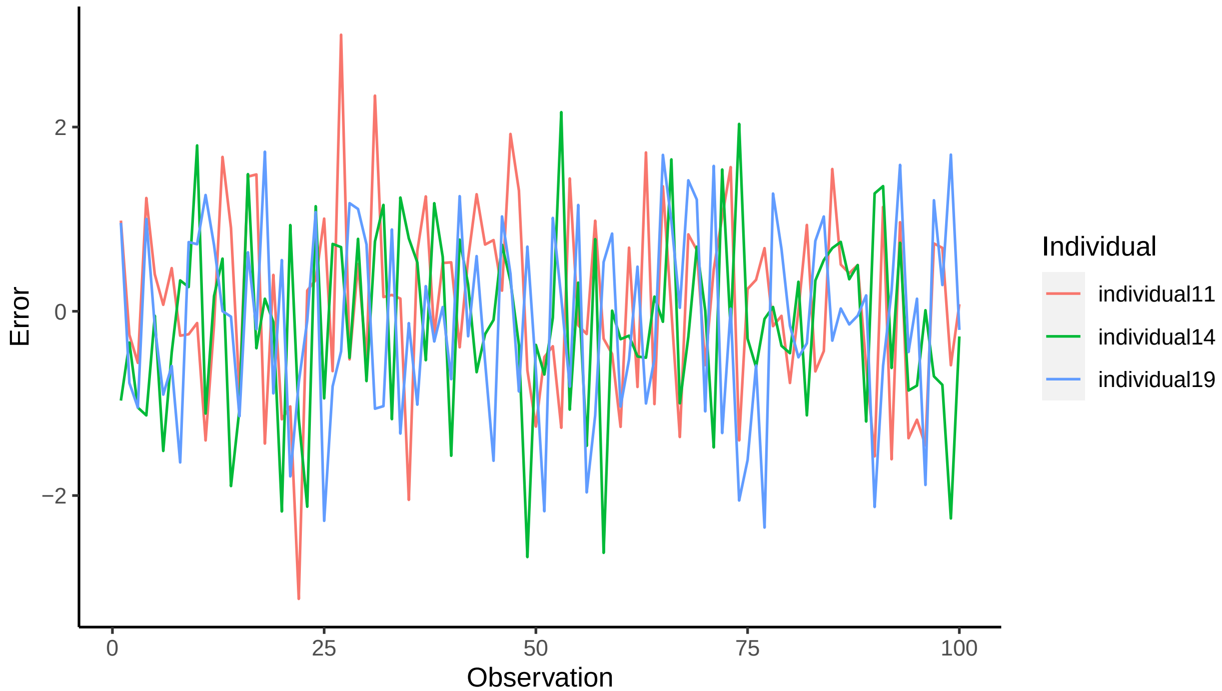
b) 20 random transitory incomes, normally distributed (mean = 0, var >0)



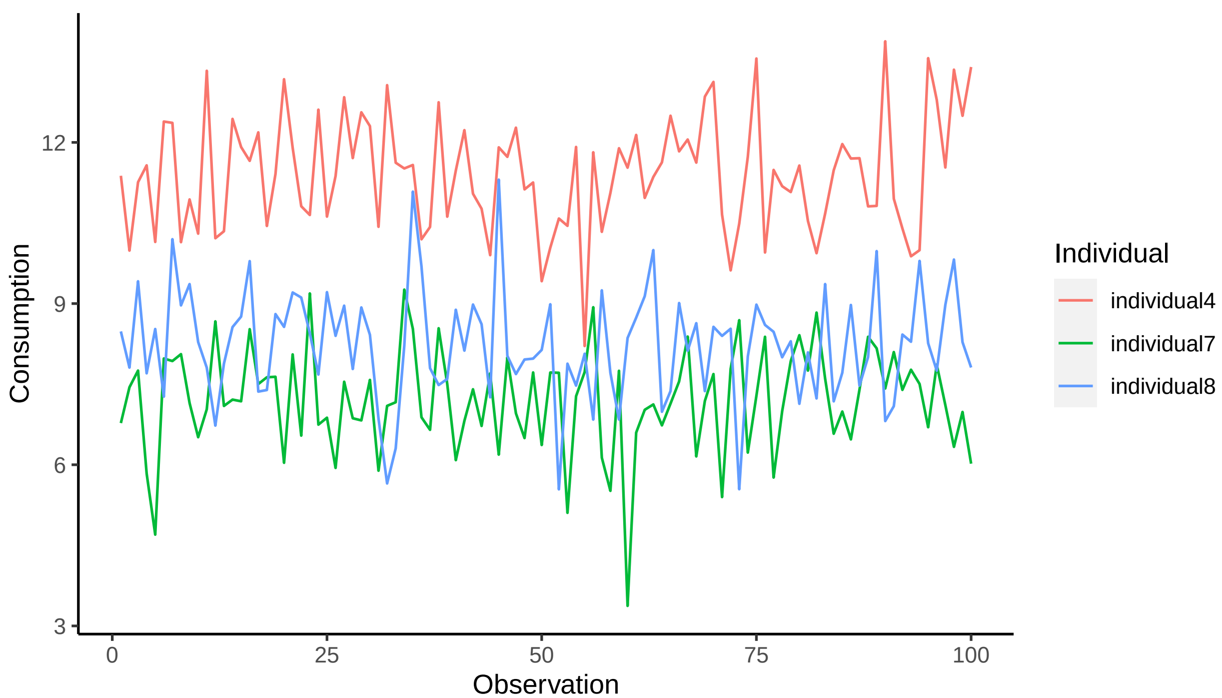
c) Total Income (permanent + transitory):



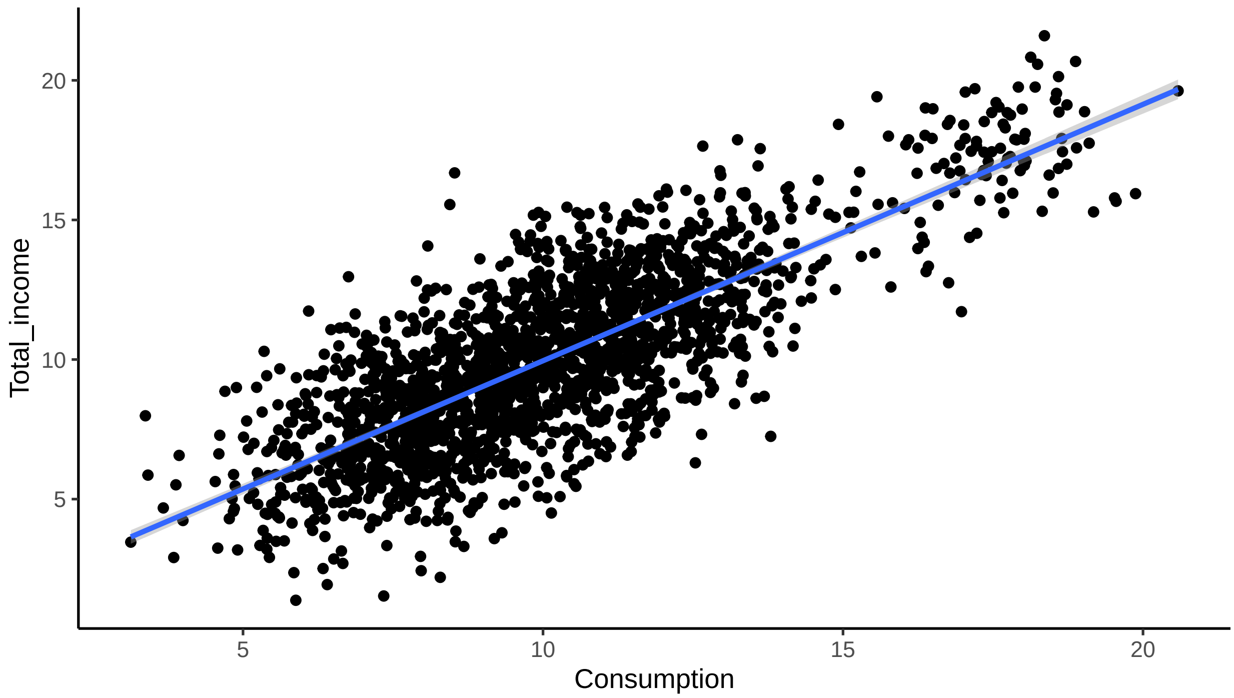
d) 20 errors (white noise, mean = 0, var = 1)



e) Vector of consumption for the individuals, following a specific path:

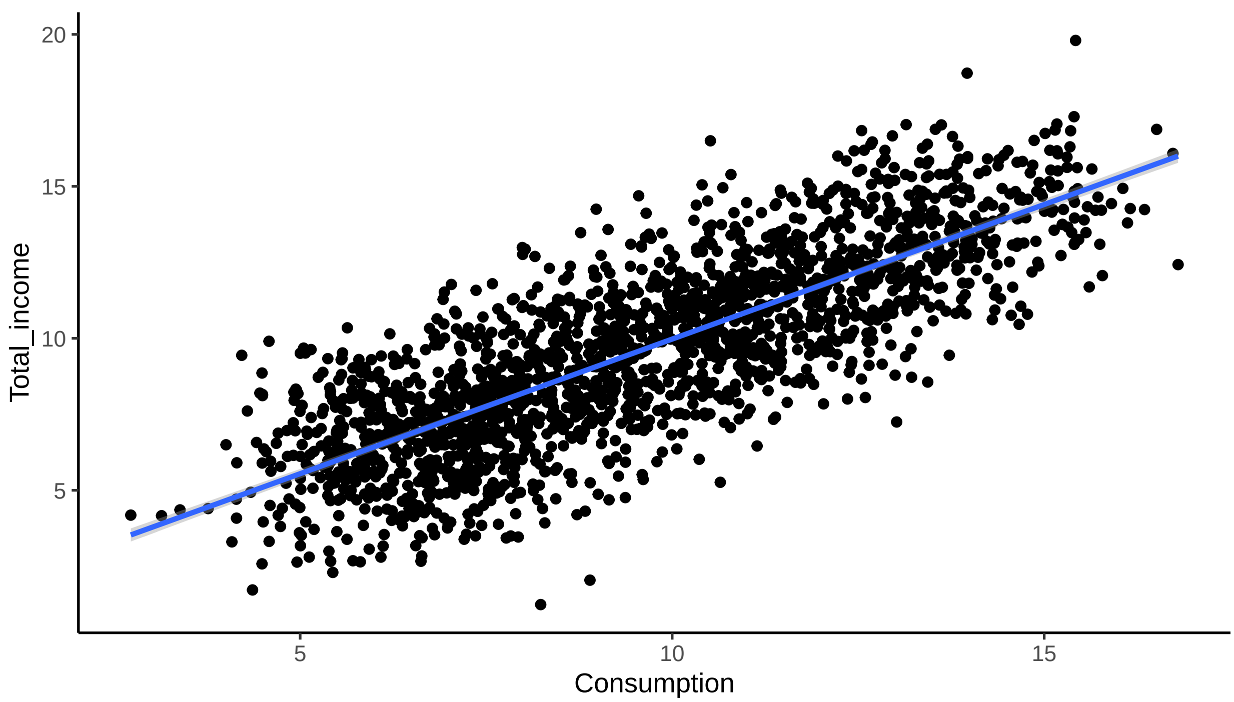


f) Estimate $C_{i,t} = b_0 + b_1 \cdot Y_{i,t} + e_{i,t}$ (regression results reported at bottom)



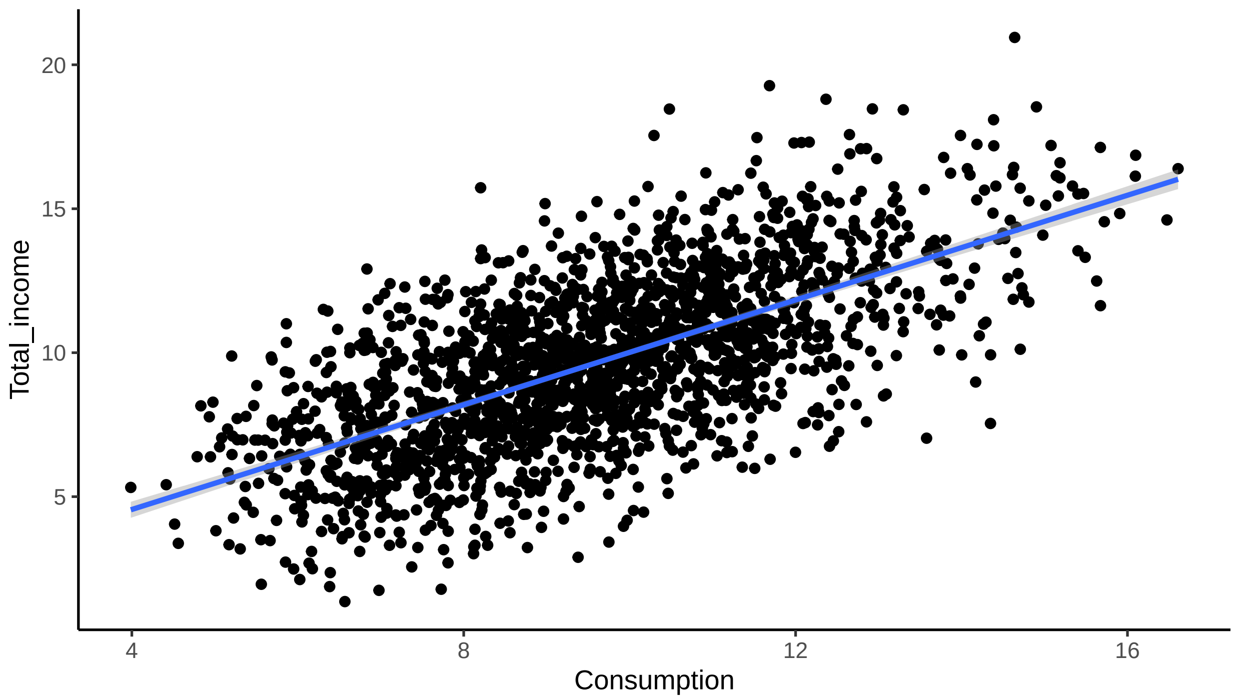
g) Let's repeat the previous exercise but change the variance for all vectors: for permanent income, we change from 6.25 to 9; and for transitory income, from 4 to 3:

Again, we want to estimate the consumption-income relationship (regression results reported at bottom:



h) Let's do it again, with the opposite changes in the variance of our income vectors: permanent income's variance down from 6.25 to 4; for transitory income, it goes up from 4 to 5:

Again, we want to estimate the consumption-income relationship (regression result reported at bottom):



Finally, these are the results of the regression analyses for the 3 levels of variance estimated above:

Table 1: Consumption Income

	Dependent variable:		
	Consumption		
	(1)	(2)	(3)
Total_income	0.667*** (0.010)	0.729*** (0.009)	0.460*** (0.010)
Error	0.990*** (0.031)	1.086*** (0.029)	0.951*** (0.029)
Constant	3.399*** (0.102)	2.635*** (0.093)	5.168*** (0.099)
Observations	2,000	2,000	2,000
R^2	0.741	0.793	0.626
Adjusted R^2	0.741	0.793	0.626
Residual Std. Error (df = 1997)	1.396	1.277	1.297
F Statistic (df = 2; 1997)	2,863.681***	3,830.167***	1,671.559***
Note:			*p<0.1; **p<0.05; ***p<0.01

Comments on the regression analysis:
i) The difference between regression 1 and 3 is small. This is consistent with the permanent income hypothesis: transitory income matters, but has little impact on consumption;
ii) Regression 2, on the other hand, has a constant term half the size of the other two, while its slope is twice that of regression 1 and 3. In this case, variance of permanent income is larger.

3 - Mexico's Aggregate Consumption

We study Mexico's aggregate consumption in the following steps:

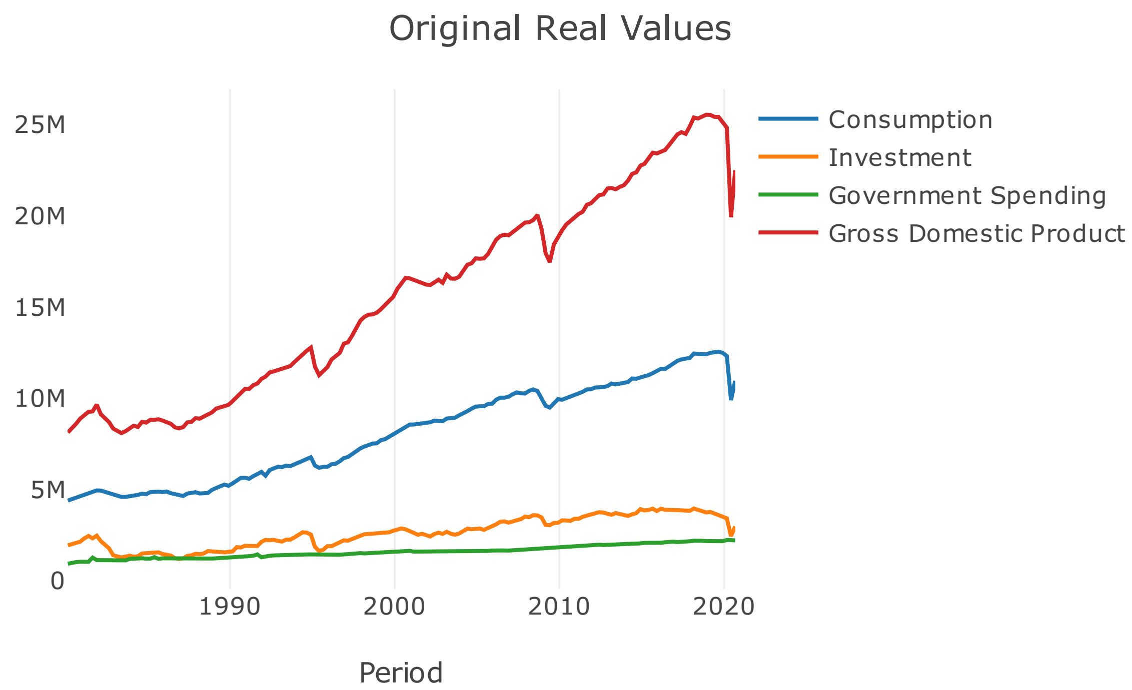
a)

To start this analysis, we obtain data from INEGI about aggregate consumption, aggregate product, aggregate investment, government spending and net exports, between 1980 and 2020's third trimester. The variables used are in real terms.

b)

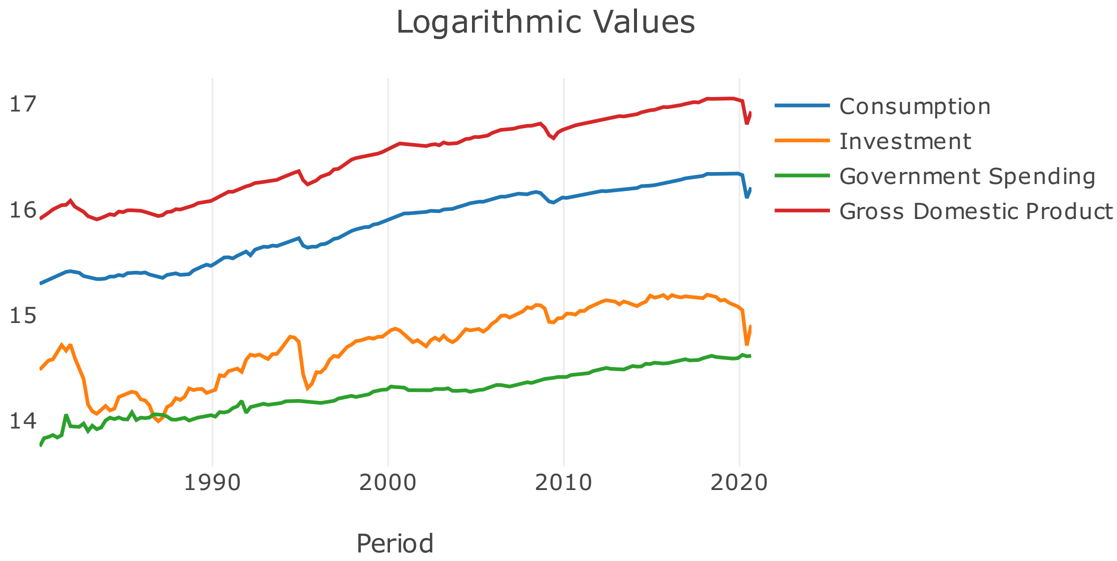
We now graph the time series to gain a better perspective of the variables' behaviour over time.

First, we graph the original values of the variables that always positive.



We can see that all the variables have an upward trend; including government spending, even though its increase is far below.

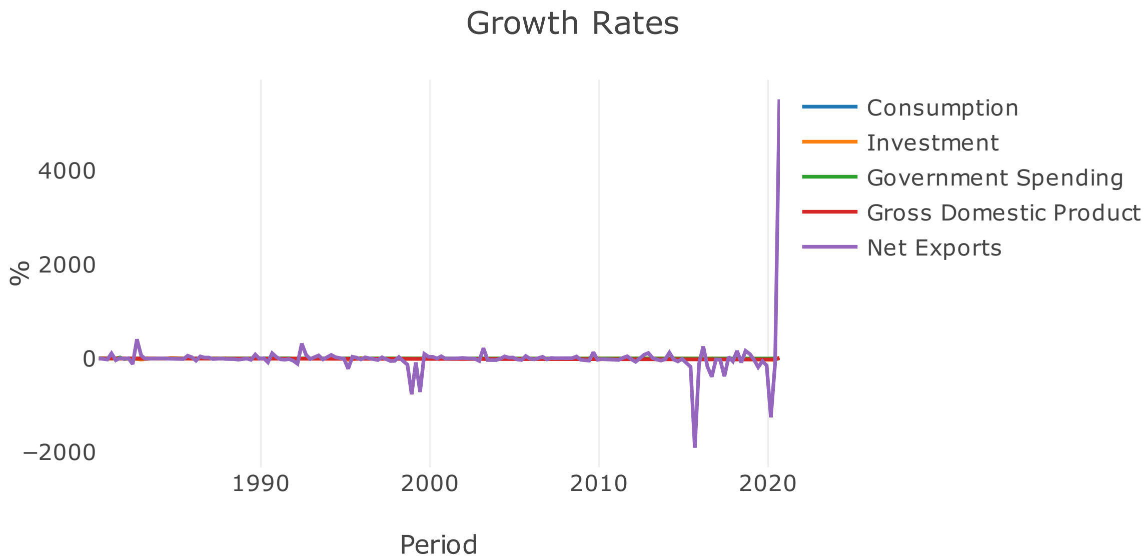
A comparison between the time series taking logarithms of each variable is also relevant.



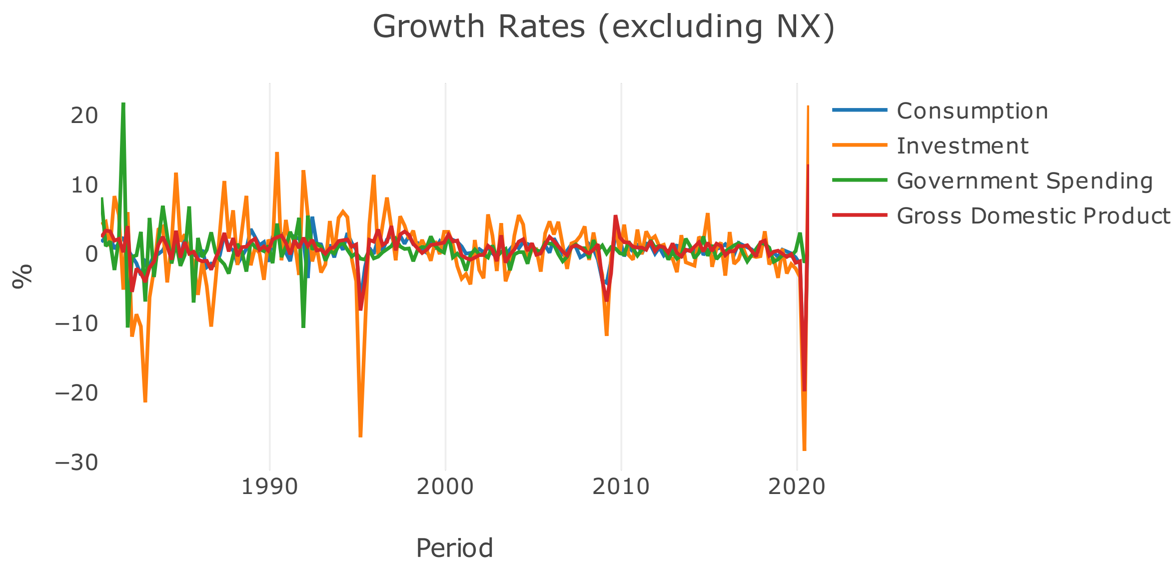
The same upward trend is noticeable, along with a close relation between consumption and GDP.

c)

Obtaining the growth rates, $\%\Delta a_t = (a_t - a_{t-1})/a_{t-1}$, will also give us meaningful information.



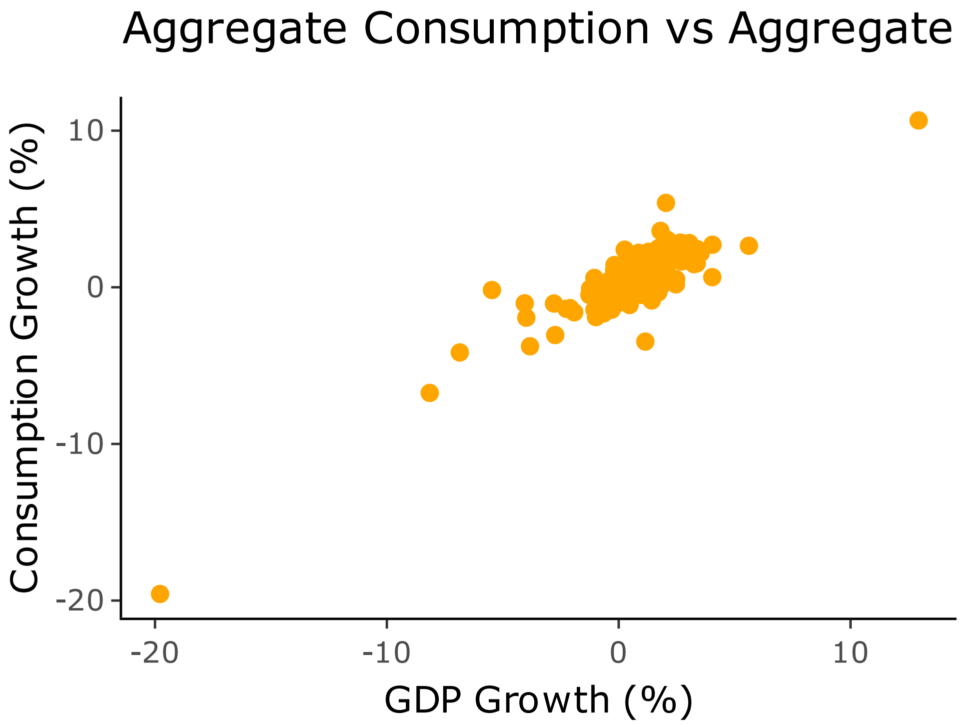
We can observe the net exports' growth rate does not allow to properly compare the rest, due to its much more variable rate. For this reason, we will repeat the graph excluding it.



The variable that has the most extreme peaks and troughs is *Investment* and that it accompanies movements in GDP. We can also notice that, in the last decade, these variables' percentage changes have been way more contained, with exception of 2020, when the COVID pandemic hit.

d)

Our focus will now be on aggregate consumption and GDP changes. To better understand the important relationship between these two variables, we plot them, placing consumption in the ordinate axis.



It is clear that there is a cluster between -4% and 4% of GDP growth and -4% and 5% for consumption growth and, furthermore, the points reveal a linear ascending relationship. In the economic study of a society's behavior, it is intuitive to think that as GDP growth increases, aggregate consumption growth does, too; this is a reason why this relationship is relevant.

e)

Subsequently, we calculate the time series' variance of the two variables to obtain the volatility.

$$var(\Delta\%Y_t, \Delta\%C_t) = \begin{pmatrix} 6.739473 & 5.314731 \\ 5.314731 & 5.313247 \end{pmatrix}$$

f)

The last step of the analysis is to calculate 4 linear models:

(1) $C_t = a + bY_t + \epsilon_t$

(2) $\Delta\%C_t = a + b\Delta\%Y_t + \epsilon_t$

(3) $\Delta\%C_t = a + b\Delta\%Y_{t-1} + \epsilon_t$ y

(4) $c_t = a + by_t + \epsilon_t$, where minuscules reflect logarithms of the original variables.

Table 2:

	Dependent variable:			
	cons	creccons		lcons
	(1)	(2)	(3)	(4)
pib	0.471*** (0.003)			
crecpib		0.789*** (0.032)		
crecpib)			−0.027 (0.076)	
lpib				0.913*** (0.005)
Constant	745,539.600*** (52,116.960)	0.068 (0.086)	0.601*** (0.188)	0.790*** (0.082)
Observations	163	162	161	163
R^2	0.993	0.789	0.001	0.995
Adjusted R^2	0.993	0.787	−0.005	0.995
Residual Std. Error	222,297.100 (df = 161)	1.063 (df = 160)	2.317 (df = 159)	0.024 (df = 161)
F Statistic	22,267.490*** (df = 1; 161)	597.640*** (df = 1; 160)	0.126 (df = 1; 159)	33,715.820*** (df = 1; 161)
Note:				*p<0.1; **p<0.05; ***p<0.01

The results of each model are analyzed separately, in the above order.

(1)

$C_t = a + bY_t + \epsilon_t$

The resulting coefficients are 745,000 for the intercept and 0.47 for the slope:

$C_t = 745,500 - 0.47Y_t + \epsilon_t$

The p-value is very low, consequently, the model is statistically significant, and so are the variables. A large t value means that is is less likely that the reason why the coefficient has a value different from cero is luk, hence, the larger the t value, the better the variable is for this model.

In this case, the t value is large for both the intercept and the aggregate product variable. Additionally, the R^2 of 0.99 indicates that the model explains 99% of the variability of the response data around its mean, so we can conclude that it is a good model.

(2)

$\Delta\%C_t = a + b\Delta\%Y_t + \epsilon_t$

The coefficients (rounded) indicate the following form:

$\Delta\%C_t = 0.07 + 0.79\Delta\%Y_t + \epsilon_t$

Both the model's and the independent variable's p-values are very low, which is good, as it tells us that we have statistical significance; nevertheless, the intercept's p-value is not. The R^2s are high, so the model explains a good part of the variability. The t value is low for the intercept and high for the chosen independent variable.

(3)

$\Delta\%C_t = a + b\Delta\%Y_{t-1} + \epsilon_t$

The coefficiens (rounded) suggest the following form:

$\Delta\%C_t = 0.601 - 0.027\Delta\%Y_{t-1} + \epsilon_t$

The t value of the independent variable is negative, which tells that we cannot trust this variable for explaining consumption. Additionally, the p-values are high, indicating that there is no statistical significance, hence the lagged GDP's growth rate does not have a direct impact in consumption's growth rate.

(4)

$c_t = a + by_t + \epsilon_t$

The following coefficients (rounded) are obtained:

$c_t = 0.79 + 0.913y_t + \epsilon_t$

The p-values indicate that the model is statistically significant, and so are the intercept and the chosen independent variable. The R^2 is high, explaining almost the whole data variability, while the t values are high. This model is addecuate for explaining the relationship between aggregate consumption and aggregate product.

g)

Under the Permanent Income Hypothesis, aggregate income and variations in aggregate income both change aggregate consumption. It involves positive coefficients, smaller than 1, since aggregate consumption is a share of aggregate income.

Regressions 1, 2 and 4 are all statistically significant and indicate a positive relationship between consumption and income in different forms. Regression 3 has a negative coefficient but, as it has no statistical significance, we do not take it into account for analysis.

4 - Mexico's Household Consumption

We study Mexico's household consumption in the following steps:

a)

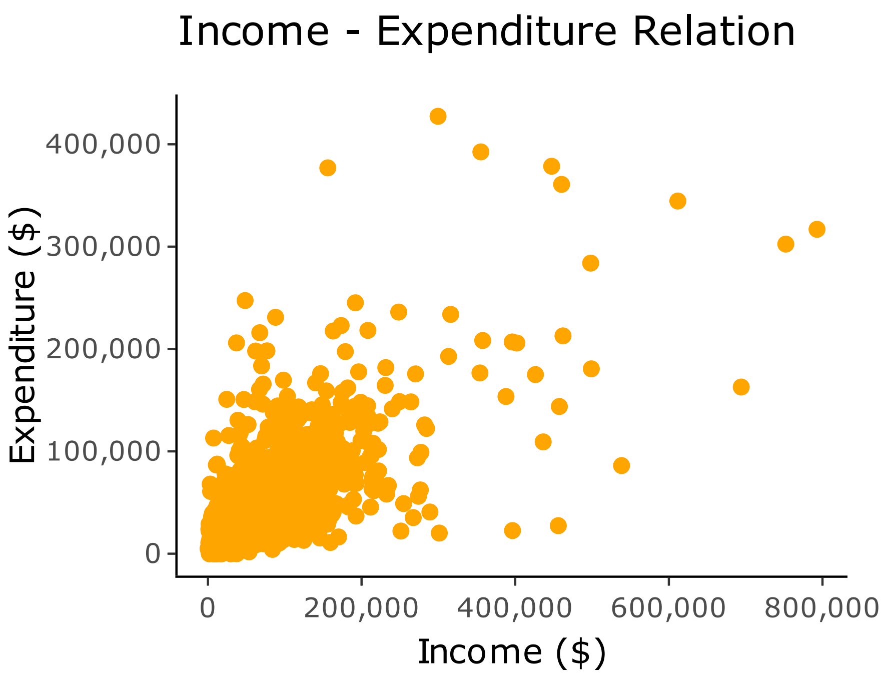
The 2012 ENIGH (National Survey of Household Incomes and Expenditures), available at INEGI website, provides us with the data for this analysis.

Households	*Average Income	*Average Expenditure
31559379	37999.64	25554.87

The number of households in 2012 was 31,559,379, while household average current income was 37,999 pesos per trimester and average monetary expenditure 25,554.87 pesos.

b)

Now, we want to find a relationship between income and expenditure, for what we will uso two different approaches. Plotting the two variables gives us a first picture of the values.



At first glance, we can appreciate some kind of linear relationship.

To continue with our analysis, we start with a simple quotient between expenditure and income. This will tell us the proportion of income that households allocate for expenditure.

Mean	Median	Minimum	Maximum
0.81	0.69	0	33.84

The information in this table gives us a better picture of how this quotient was in 2012 for households in Mexico. In average, households spent 80.58% of their current income, while the median was 68.77%. Both the minimum and the maximum are extreme values; in the case of the minimum, it is due to households reporting no expenditure at all, while the maximum means that households spent 33 times their income, they had contracted debt.

The second approach consists on creating a linear regression, $G = a + bI + \epsilon_t$, obtaining the following:

% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu
% Date and time: lun., feb. 08, 2021 - 10:01:44 p. m.

We get the following form: $G = 6,572 + 0.49I + \epsilon_t$

Both the model and the coefficients are statistically significant, as the p-values are very low. We obtain

Table 3:

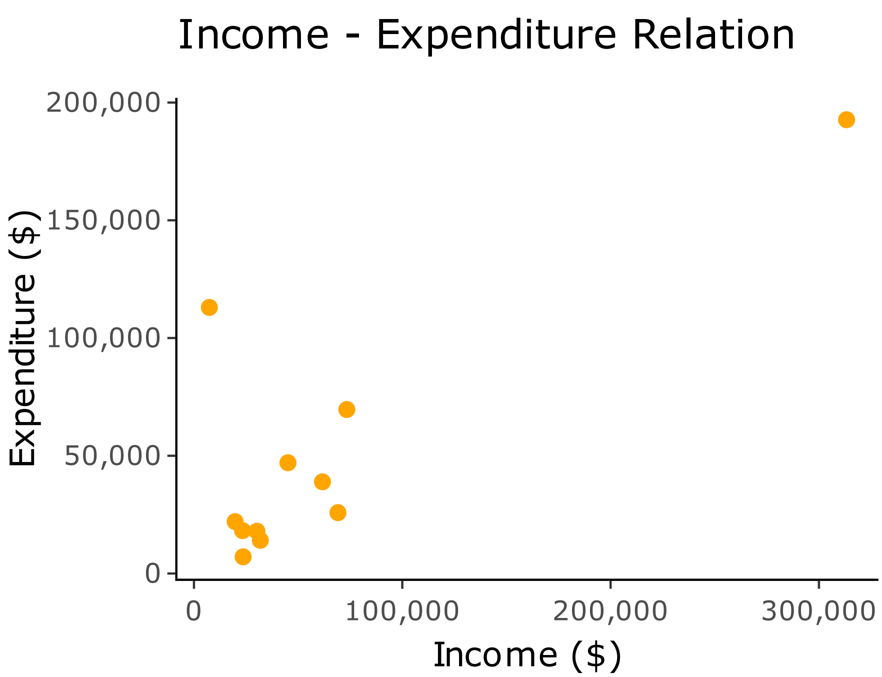
	Dependent variable:
	gasto_mon
ing_cor	0.500***
	(0.004)
Constant	6,572.178***
	(251.467)
Observations	9,002
R^2	0.605
Adjusted R^2	0.605
Residual Std. Error	1,081,985.000 (df = 9000)
F Statistic	13,784.660*** (df = 1; 9000)
Note:	*p<0.1; **p<0.05; ***p<0.01

$R^2 > 0.5$ and very high t values, so we conclude that this model can properly depict the linear relationship between monetary expenditure and current income.

c)

We will now do the same for the specific case of single person households between 40-50 years old, located in Mexico City.

First, we obtain some information about the data.



Plotting expenditure with income reveals some kind of linear relationship.

Mean	Median	Minimum	Maximum
2.29	0.63	0.3	15.32

In 2012, there were 73,998 households with these characteristics. Using the previous method of the quotient, we find that, in average, these households spent more than twice their current income per trimester. This can be due to incurring debt or other phenomenon; it would be interesting to explore how this happens some other time.

We also find that the median is 62.81%, while the maximum is an expenditure of 15.3 times the current income, and the minimum is 30.02%.

This gives us a better picture of the data distribution.

Following the same methodology, we create a linear regression to determine the relationship.

Table 4:

	Dependent variable:
	gasto_mon
ing_cor	0.508***
	(0.147)
Constant	21,037.130
	(14,100.920)
Observations	11
R^2	0.571
Adjusted R^2	0.523
Residual Std. Error	3,042,268.000 (df = 9)
F Statistic	11.972*** (df = 1; 9)
Note:	*p<0.1; **p<0.05; ***p<0.01

We get the following form:

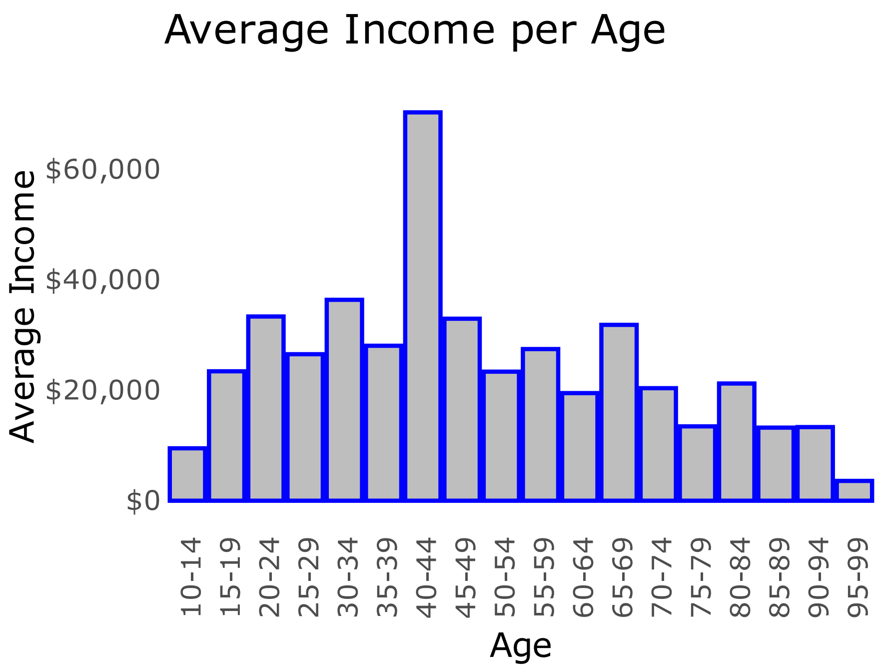
$G = 21,040 + 0.508I + \epsilon_t$

The model's p-value is less than 0.05, being statistically significant; the same for the independent variable. The R^2 is higher than 0.5, which means we can consider this model as good. The t values are not as high as the ones obtained in the first regression.

When comparing this regression with the one previously calculated, we can see that the income's coefficient is almost the same; although, in general, it seems like this model is not as good as the first one.

e)

We now group the data for single person households in age ranges to gain information about the average income.



The age range with the highest average income is 40-44, with 70,330 pesos per trimester. The other ranges are well below, the lowest being 3,590 pesos per trimester for single person households of age 95-99.

f)

As we know, the COVID pandemic has had a great economic impact in Mexico and the world. As it was so unexpected, millions of jobs have been lost, affecting households' income. What we expect to see is a higher consupmtion-income relation, due to the effect of a lower income having to cover the constant level of consumption. Even though 2020 has seen a decrese in consumption, families will have to destine more of their income to cover basic consumption.

5 - The "Equity-Premium Puzzle" for Mexico

Study the The "Equity-Premium Puzzle" for Mexico by following these steps:

a. Get the annual values of IPC, the Price and Quote Index of the Mexican Stock Exchange at least 1990.

b. Calculate your nominal return rate for each year.

Nominal return rate of the IPC = IPC_T = $\frac{IPC_t - IPC_{t-1}}{IPC_{t-1}}$

c. Get the average annual interest rate values of CETES at 7 days, or TIIE, the interbank equilibrium rate and the one-year interest rate, for the period that is available.

Table 5: CETES and TIIE

YEAR	IPC	IPC_T	CT_28	CT_91	CT_182	CT_364	TIIE91	TIIE_28
1,990	570.1417	NA	0.3482	0.3495	0.2994	0.2495	NA	NA
1,991	1,086.0842	0.9049373	0.1929	0.1984	0.1980	0.1977	NA	NA
1,992	1,671.6125	0.5391187	0.1566	0.1593	0.1592	0.1681	NA	NA
1,993	1,856.2417	0.1104498	0.1485	0.1541	0.1544	0.1544	NA	NA
1,994	2,520.6508	0.3579324	0.1404	0.1451	0.1398	0.1373	NA	NA
1,995	2,219.3583	-0.1195296	0.4866	0.4854	0.4165	0.3759	NA	0.5303
1,996	3,163.1750	0.4252656	0.3133	0.3285	0.3366	0.3422	NA	0.3159
1,997	4,442.4225	0.4044188	0.1983	0.2125	0.2187	0.2235	0.2210	0.2189
1,998	4,241.0300	-0.0453339	0.2462	0.2604	0.2154	0.2232	0.2736	0.2690
1,999	5,332.0925	0.2572636	0.2129	0.2226	0.2339	0.2423	0.2450	0.2400
2,000	6,515.8558	0.2220073	0.1527	0.1616	0.1660	0.1694	0.1723	0.1696
2,001	6,119.7058	-0.0607978	0.1126	0.1219	0.1300	0.1358	0.1339	0.1284
2,002	6,517.9900	0.0650822	0.0708	0.0743	0.0807	0.0862	0.0844	0.0815
2,003	7,186.9208	0.1026284	0.0624	0.0653	0.0692	0.0725	0.0717	0.0679
2,004	10,677.2642	0.4856521	0.0684	0.0713	0.0740	0.0780	0.0747	0.0717
2,005	14,458.6100	0.3541493	0.0919	0.0932	0.0928	0.0924	0.0962	0.0961
2,006	21,074.7500	0.4575917	0.0719	0.0729	0.0741	0.0749	0.0769	0.0751
2,007	29,713.7150	0.4099202	0.0719	0.0736	0.0748	0.0759	0.0779	0.0766
2,008	26,859.8992	-0.0960437	0.0768	0.0789	0.0802	0.0812	0.0861	0.0828
2,009	25,306.0258	-0.0578511	0.0539	0.0547	0.0556	0.0577	0.0590	0.0591
2,010	33,285.8883	0.3153345	0.0440	0.0457	0.0468	0.0485	0.0500	0.0491
2,011	36,340.5258	0.0917697	0.0424	0.0435	0.0451	0.0466	0.0486	0.0482
2,012	40,037.1925	0.1017230	0.0424	0.0438	0.0451	0.0462	0.0481	0.0479
2,013	42,060.9658	0.0505473	0.0375	0.0381	0.0390	0.0398	0.0428	0.0428
2,014	42,644.2142	0.0138667	0.0300	0.0312	0.0322	0.0337	0.0352	0.0351
2,015	43,770.9567	0.0264219	0.0298	0.0314	0.0329	0.0353	0.0334	0.0332
2,016	45,901.9133	0.0486843	0.0417	0.0436	0.0452	0.0457	0.0457	0.0447
2,017	48,995.6225	0.0673983	0.0669	0.0688	0.0702	0.0710	0.0712	0.0705
2,018	46,730.6925	-0.0462272	0.0762	0.0783	0.0797	0.0806	0.0805	0.0800
2,019	43,066.3350	-0.0784144	0.0785	0.0794	0.0795	0.0789	0.0827	0.0832
2,020	38,704.0942	-0.1012912	0.0532	0.0533	0.0528	0.0479	0.0566	0.0571

Source:Own elaboration with data from INEGI and BANXICO, 2020

d. Calculate the difference between the return of the CPI and the return on investing in CETES at different timeframes.

Table 6: IPC difference with CETES and TIIE

YEAR	IPC_C28	IPC_C91	IPC_C182	IPC_C364	IPC_TIIE91	IPC_TIIE_28
1,990	NA	NA	NA	NA	NA	NA
1,991	0.7120373	0.7065373	0.7069373	0.7072373	NA	NA
1,992	0.3825187	0.3798187	0.3799187	0.3710187	NA	NA
1,993	-0.0380502	-0.0436502	-0.0439502	-0.0439502	NA	NA
1,994	0.2175324	0.2128324	0.2181324	0.2206324	NA	NA
1,995	-0.6061296	-0.6049296	-0.5360296	-0.4954296	NA	-0.6498296
1,996	0.1119656	0.0967656	0.0886656	0.0830656	NA	0.1093656
1,997	0.2061188	0.1919188	0.1857188	0.1809188	0.1834188	0.1855188
1,998	-0.2915339	-0.3057339	-0.2607339	-0.2685339	-0.3189339	-0.3143339
1,999	0.0443636	0.0346636	0.0233636	0.0149636	0.0122636	0.0172636
2,000	0.0693073	0.0604073	0.0560073	0.0526073	0.0497073	0.0524073
2,001	-0.1733978	-0.1826978	-0.1907978	-0.1965978	-0.1946978	-0.1891978
2,002	-0.0057178	-0.0092178	-0.0156178	-0.0211178	-0.0193178	-0.0164178
2,003	0.0402284	0.0373284	0.0334284	0.0301284	0.0309284	0.0347284
2,004	0.4172521	0.4143521	0.4116521	0.4076521	0.4109521	0.4139521
2,005	0.2622493	0.2609493	0.2613493	0.2617493	0.2579493	0.2580493
2,006	0.3856917	0.3846917	0.3834917	0.3826917	0.3806917	0.3824917
2,007	0.3380202	0.3363202	0.3351202	0.3340202	0.3320202	0.3333202
2,008	-0.1728437	-0.1749437	-0.1762437	-0.1772437	-0.1821437	-0.1788437
2,009	-0.1117511	-0.1125511	-0.1134511	-0.1155511	-0.1168511	-0.1169511
2,010	0.2713345	0.2696345	0.2685345	0.2668345	0.2653345	0.2662345
2,011	0.0493697	0.0482697	0.0466697	0.0451697	0.0431697	0.0435697
2,012	0.0593230	0.0579230	0.0566230	0.0555230	0.0536230	0.0538230
2,013	0.0130473	0.0124473	0.0115473	0.0107473	0.0077473	0.0077473
2,014	-0.0161333	-0.0173333	-0.0183333	-0.0198333	-0.0213333	-0.0212333
2,015	-0.0033781	-0.0049781	-0.0064781	-0.0088781	-0.0069781	-0.0067781
2,016	0.0069843	0.0050843	0.0034843	0.0029843	0.0029843	0.0039843
2,017	0.0004983	-0.0014017	-0.0028017	-0.0036017	-0.0038017	-0.0031017
2,018	-0.1224272	-0.1245272	-0.1259272	-0.1268272	-0.1267272	-0.1262272
2,019	-0.1569144	-0.1578144	-0.1579144	-0.1573144	-0.1611144	-0.1616144
2,020	-0.1544912	-0.1545912	-0.1540912	-0.1491912	-0.1578912	-0.1583912

Source:Own elaboration with data from INEGI and BANXICO, 2020

Table 7: Change in aggregate consumption and private domestic consumption of annual imported goods

YEAR	Aggregate_consumption	Priv_consumption_imported_goods
1,990	NA	NA
1,991	NA	NA
1,992	NA	NA
1,993	NA	NA
1,994	0.0488	0.2349
1,995	-0.0502	-0.4814
1,996	0.0368	0.2011
1,997	0.0701	0.3407
1,998	0.0553	0.1841
1,999	0.0490	0.1105
2,000	0.0645	0.3536
2,001	0.0164	0.1435
2,002	0.0148	0.0598
2,003	0.0186	0.0770
2,004	0.0389	0.0969
2,005	0.0269	0.1062
2,006	0.0386	0.1126
2,007	0.0241	0.0337
2,008	0.0102	-0.0094
2,009	-0.0492	-0.1743
2,010	0.0339	0.1247
2,011	0.0331	0.0519
2,012	0.0250	0.0649
2,013	0.0157	0.1062
2,014	0.0222	0.0341
2,015	0.0257	0.0722
2,016	0.0350	-0.0084
2,017	0.0280	0.0637
2,018	0.0258	0.0308
2,019	0.0032	0.0313
2,020	-0.1011	-0.0019

Source:Own elaboration with data from INEGI and BANXICO, 2020

e. Calculate the covariance between these differences and the actual growth rate of aggregate consumption in the Mexican economy.

Table 8: Covariance, with consumption of imported goods

Covariance_A	ValueCA
IPC-CETES28d	0.00416250063370709
IPC-CETES91d	0.00407300670208316
IPC-CETES182d	0.00391187849695495
IPC-CETES364d	0.00374499089011734
IPC-TIIE91	0.00231662058498402
IPC-TIIE28	0.00411319436428128

Source:Own elaboration with data from INEGI and BANXICO, 2020

f. Calculate the risk-related aversion value involved by these numbers, given the assumption of a utility an CRRA form.

$\theta = \frac{E[r^i]-E[r^j]}{cov(r^i-r^j,g^c)}$

where:

θ = risk-related aversion

^{i} = IPC

^{j} = Cetes, TIIE

^{r} = rate of return

g^c = real growth in aggregate consumption

Table 9: Risk aversion coefficient, with added consumption

Aversión_A	Valor_A
IPC-CETES28d	6.03775585980154
IPC-CETES91d	5.20925923518026
IPC-CETES182d	5.92088743048937
IPC-CETES364d	6.0284811630956
IPC-TIIE91	12.9678843159408
IPC-TIIE28	2.05283049131273

Source:Own elaboration with data from INEGI and BANXICO, 2020

g. Now calculate the covariance between these differences and the actual growth rate of aggregate consumption of IMPORTED GOODS from the Mexican economy.

Table 10: Covariance, with consumption of imported goods

Covarianza_BI	ValorCI
IPC-CETES28d	0.0198706393057929
IPC-CETES91d	0.0194200578528015
IPC-CETES182d	0.0180073506733143
IPC-CETES364d	0.0169924690493827
IPC-TIIE91	0.00472234963137641
IPC-TIIE28	0.019766118144819

Source:Own elaboration with data from INEGI and BANXICO, 2020

h. Calculate the risk-related aversion value involved in these numbers, given the assumption of a profit with an ARRC shape.

$\theta = \frac{E[r^i]-E[r^j]}{cov(r^i-r^j,g^c)}$

where:

θ = risk-related aversion

^{i} = IPC

^{j} = Cetes, TIIE

^{r} = rate of return

g^c = real growth in private domestic consumption of imported goods

Table 11: Risk aversion ratio, with consumption of imported goods

Aversión_BI	Valor_AI
IPC-CETES28d	1.26478882766826
IPC-CETES91d	1.09254812414048
IPC-CETES182d	1.286240971391
IPC-CETES364d	1.32862428475968
IPC-TIIE91	6.36159329465909
IPC-TIIE28	0.427180023200738

Source:Own elaboration with data from INEGI and BANXICO, 2020

6 - Pricing an Option with Binomial Trees

The binomial trees method is a powerful tool for pricing options financial assets (it can be stocks, commodities, and other derivatives as well). In this exercise, we will price a “call” option on some asset which currently sells at $P_0 = 80$ and a strike price of $K = 76$. We assume the risk-free rate equals the discount rate, $\rho = r = 5$

i) Subjective probabilities

We first make some assumptions about the possible prices of the asset within a year. Thus, we generate random data representing either increases or decreases of this price. Note: 1 year is “next period”.

We chose the uniform distribution (min = 1.05, max = 1.95) because we find it closer to the random walk; downward movements are also assumed to follow a Uni(0.05, 0.95) distribution. This simplification will help us avoid downward movements above 100% and upward movements below 0%.

Moreover, we set the “objective” probability of the price reaching the values simulated above (vectors ‘up’ and ‘down’). This probabilities are computed as the probability of price reaching some value, following the uniform distribution.
Now, let’s think of an alternative world were risk is left out of the equation (i.e., let’s think of a riskless world). Then, the probability of the asset’s price increasing is given by: $subjective = \frac{exp^{\rho} - down}{up - down}$

```
   Alza Objetiva Subjetiva Diferencia
1  1.59      40%     33.2%      7 p.p.
2  1.41    59.6%     64.8%     -5 p.p.
3  1.70    27.5%     56.8%    -29 p.p.
4  1.41    59.6%     62.1%     -3 p.p.
5  1.86    10.4%       51%    -41 p.p.
6  1.41    60.5%     70.9%    -10 p.p.
7  1.73    24.6%     25.4%     -1 p.p.
8  1.17    86.8%     81.7%      5 p.p.
9  1.60    38.6%     36.5%      2 p.p.
10 1.08    96.8%       96%      1 p.p.
11 1.46    54.5%     55.3%     -1 p.p.
12 1.69    28.9%     56.1%    -27 p.p.
13 1.44    56.9%     49.5%      7 p.p.
14 1.14    89.7%     91.1%     -1 p.p.
15 1.64    34.2%     56.7%    -22 p.p.
```

Notice that “Alza” is a factor that multiplies the initial price, $P_0 = 80$. We can see that large values of “Alza” are associated to small probabilities, which seems intuitive.

The two probabilities (objective, subjective) differ because the subjective probability is not accounting for randomness, whereas the objective probability does (assuming the price is a uniformly distributed random variable). Notice that, by ignoring risk we have calculated a subjective probability under which the expected return on the asset equals the risk-free return (5%), always:

```
 [1] 84.10169 84.10169 84.10169 84.10169 84.10169 84.10169 84.10169 84.10169
 [9] 84.10169 84.10169 84.10169 84.10169 84.10169 84.10169 84.10169
```

```
[1] 84.10169
```

According to the "Options, Futures and Other Derivatives" book, by John Hull, the assumption of an investment choice under risk neutrality is of use in pricing the option on an asset because the option is riskier, and it would be close to impossible to find an appropriate discount rate.

ii) Pricing the option

We will now find a price, at present, for the option to buy this asset in a year. Let's think of the possible outcomes of holding this option in our portfolio. In fact, there are only two possible outcomes: either the price is greater than 76 (then the option has a positive value); or the price is less or equal to 76, which makes the option's worth 0. Then, the option pays $80 * Alza - 76$, with some subjective probability and 0 (with the complement of that probability):

```
 [1] 17.00567 24.06814 34.19123 23.03816 36.96886 25.86987 15.83030 14.28834
 [9] 19.05864  9.88089 22.57882 33.20771 19.32619 14.05231 31.37841
```

Finally, these expected payoffs would occur within one year, so we need to calculate its present value.

```
 [1] 16.176292 22.894320 32.523702 21.914577 35.165864 24.608179 15.058243
 [8] 13.591489 18.129141  9.398993 21.477639 31.588148 18.383640 13.366974
[15] 29.848069
```

```
[1] "0.7275  = correlation option price ~ increase underlying price"
```

```
[1] "-0.5211  = correlation option price ~ decrease underlying price"
```

This present value of expected payoffs is the fair price of the option. Notice it is negatively correlated to scenarios of the price going down, and positively correlated with the price going up. This was expected, as the option's payoff is zero in the former case.