3- Utilice la transformada Z unilateral para determinar $y(n)$, $n \geq 0$.

a. $y(n) + \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) = 0 \quad y(-1) = y(-2) = 1$

$$Y^+(z) + \frac{1}{2}[z^{-1}Y^+(z) + y(-1)] - \frac{1}{4}[z^{-2}Y^+(z) + z^{-1}y(-1) + y(-2)] = 0$$

$$Y^+(z) = \frac{\frac{1}{4}z^{-1} - \frac{1}{4}}{1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2}} = \frac{0.154}{1 - 0.31z^{-1}} - \frac{0.404}{1 + 0.81z^{-1}}$$

$$y(n) = [0.154(0.31)^n - 0.404(0.81)^n]u(n)$$

b) $y(n) - 1.5y(n-1) + 0.5y(n-2) = 0 \quad y(-1) = 1,\ y(-2) = 0$

$$Y^+(z) - 1.5[z^{-1}Y^+(z) + 1] + 0.5[z^{-2}Y(z) + z^{-1} + 0] = 0$$

$$Y^+(z) = \frac{1.5 - 0.5z^{-1}}{1 - 1.5z^{-1} + 0.5z^{-2}} = \frac{2}{1 - z^{-1}} - \frac{(.5)}{1 - .05z^{-1}}$$

$$y(n) = [2 - (0.5)(0.5)^n]u(n) = [2 - (0.5)^{n+1}]u(n)$$

c) $y(n) = \frac{1}{2}y(n-1) + x(n) \quad x(n) = (\frac{1}{3})^n u(n) \quad y(-1) = 1$

$$Y^+(z) - 0.5[z^{-1}Y^+(z) + 1] = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{1.5 - \frac{1}{6}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 0.5z^{-1})} = \frac{\frac{7}{2}}{1 - .05z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}}$$

$$y(n) = [\frac{7}{2}(.5)^n - 2(\frac{1}{3})^n]u(n)$$

d)

$$Y^+(z) - \frac{1}{4}[z^{-2}Y^+(z) + 1] = \frac{1}{1 - z^{-1}}$$

$$Y^+(z) = \frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{(1 - z^{-1})(1 - \frac{1}{4}z^{-2})} = \frac{\frac{4}{3}}{1 - z^{-1}} + \frac{-\frac{3}{8}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{7}{24}}{1 + \frac{1}{2}z^{-1}}$$

$$y(n) = [\frac{4}{3} - \frac{3}{8}(\frac{1}{2})^n + \frac{7}{24}(-\frac{1}{2})^n]u(n)$$