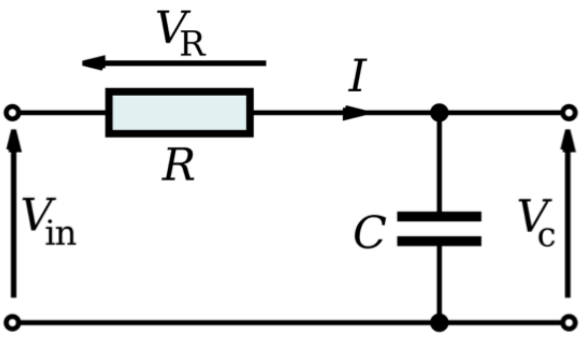
14 – Diagramas polares

Rodrigo Makoto Takumi - RA: 176897
Carlos Augusto Jardim Chiarelli - RA: 165685

Ex. 14.1) Filtro passivo baseado em circuito RC em série ($R = 100\ \Omega$, $C = 100\ \mu F$):

- a) Obtenha a função de transferência do filtro passa-baixas $G_L(s) = V_c(s)/V_s(s)$ e do passa-altas $G_H(s) = V_R(s)/V_s(s)$;
- b) Trace os diagramas de Bode, Nyquist e Nichols e discuta os resultados.



Lei de Kirchhoff:

$$v_s(t) = v_R(t) + v_c(t)$$

Tensão no Resistor:

$$v_R(t) = Ri(t) = R\dot{q}(t)$$

Tensão no Capacitor:

$$v_c(t) = v_o(t) = \frac{1}{C}q(t)$$

$$q(t) = Cv_o(t)$$

$$\frac{v_s(t)}{v_c(t)} = \frac{v_R(t)}{v_c(t)} + 1$$

$$\frac{v_s(t)}{v_c(t)} = \frac{R\dot{q}(t)}{\frac{q(t)}{C}} + 1$$

$$\frac{v_s(t)}{v_c(t)} = \frac{RC\dot{q}(t)}{q(t)} + 1$$

Filtro passa-baixas:

Aplicando a transformada de laplace:

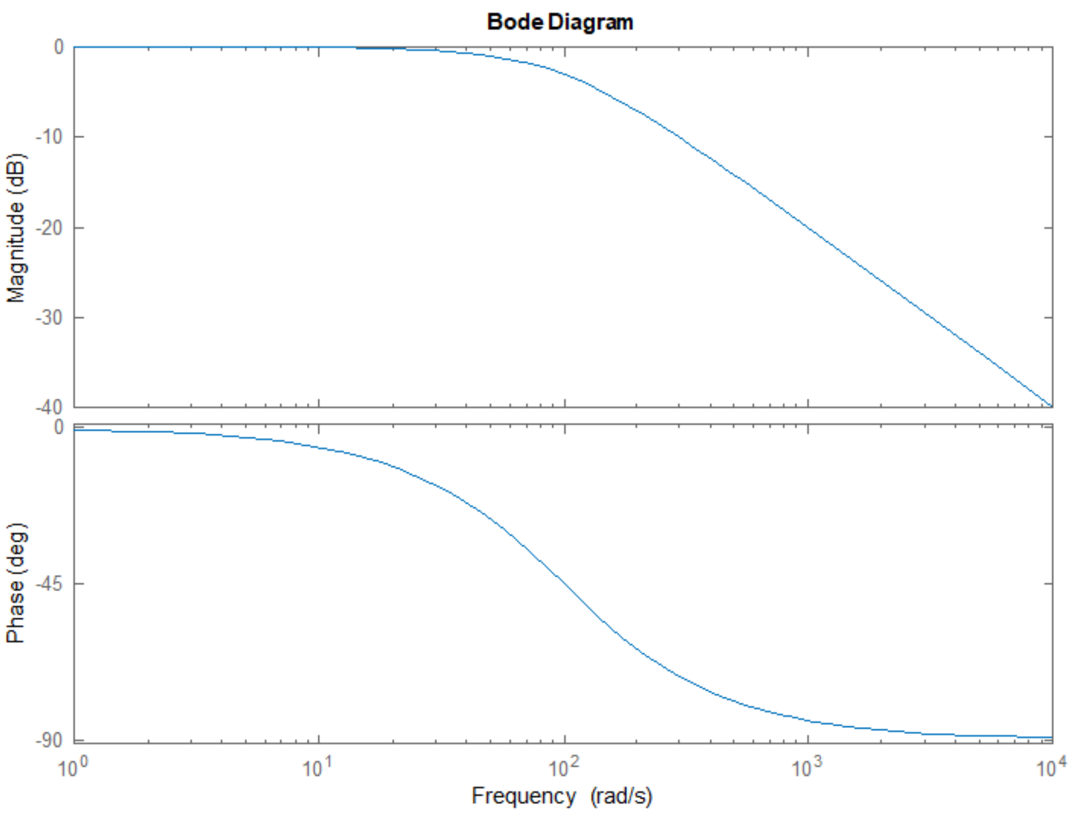
$$G_L(s) = \frac{v_s(s)}{v_c(s)} = RCs + 1$$

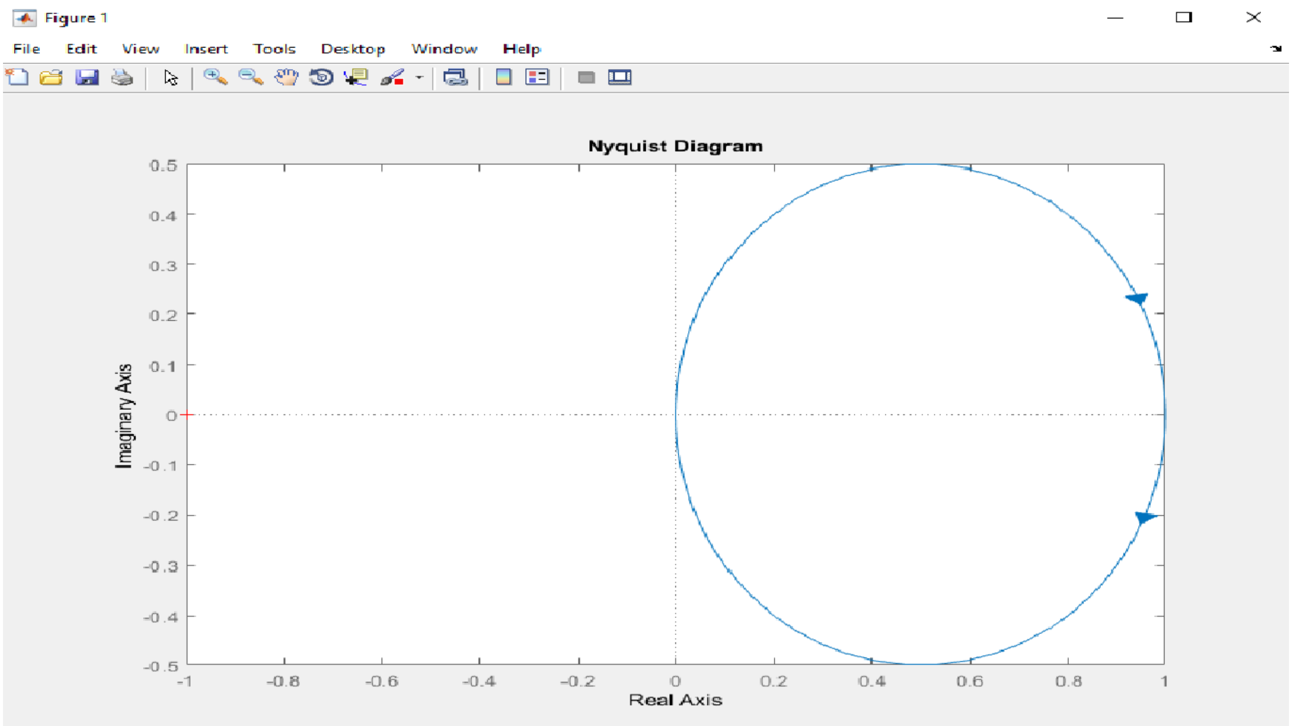
$$G_L(s) = \frac{v_c(s)}{v_s(s)} = \frac{1}{RCs + 1}$$

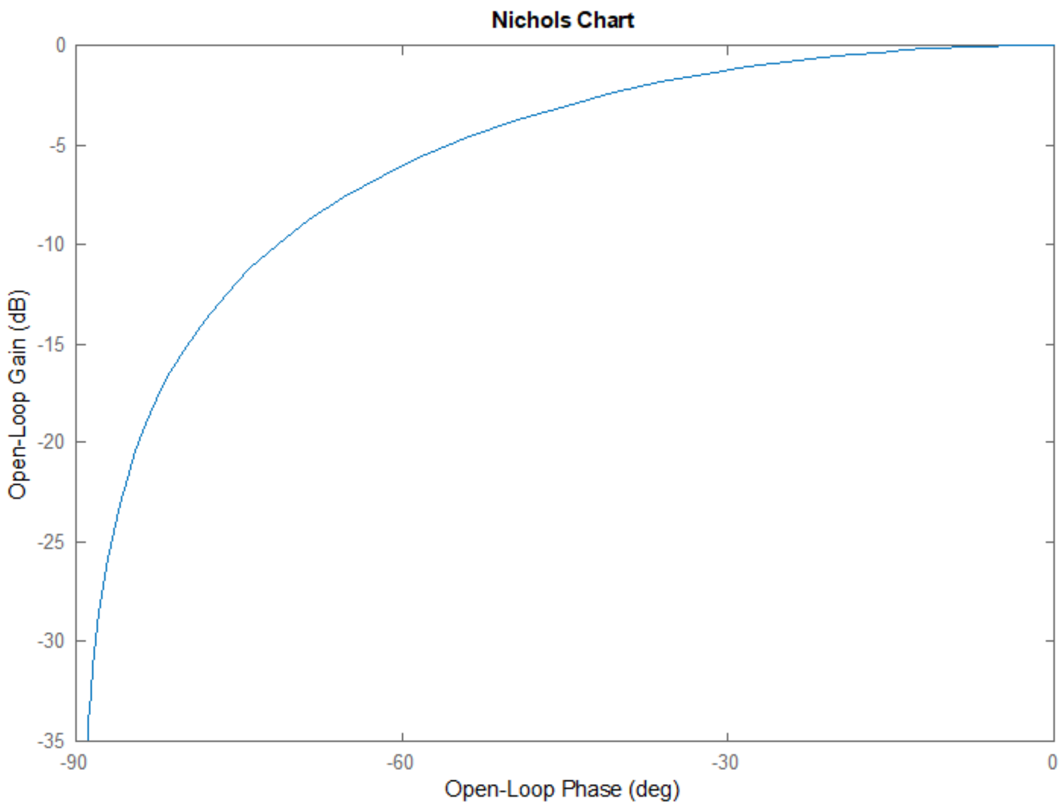
$$M_H(w) = \frac{wRC}{\sqrt{1 + (wRC)^2}}$$

$$\phi_H(w) = \tan^{-1}(\frac{1}{wRC})$$

- Em ω = 0, M = 0 e ϕ = 0°;
- Em ω = 1/RC = 100rad/s, M=1/$\sqrt{2}$ e ϕ = –45°;
- Em $\omega \to \infty$, $M \to$ 1 e $\phi \to$ -90°.







Código:

```
G = tf( [1], [0.01, 1] )
bode(G)
nyquist(G)
nichols(G)
```

Filtro passa-altas:

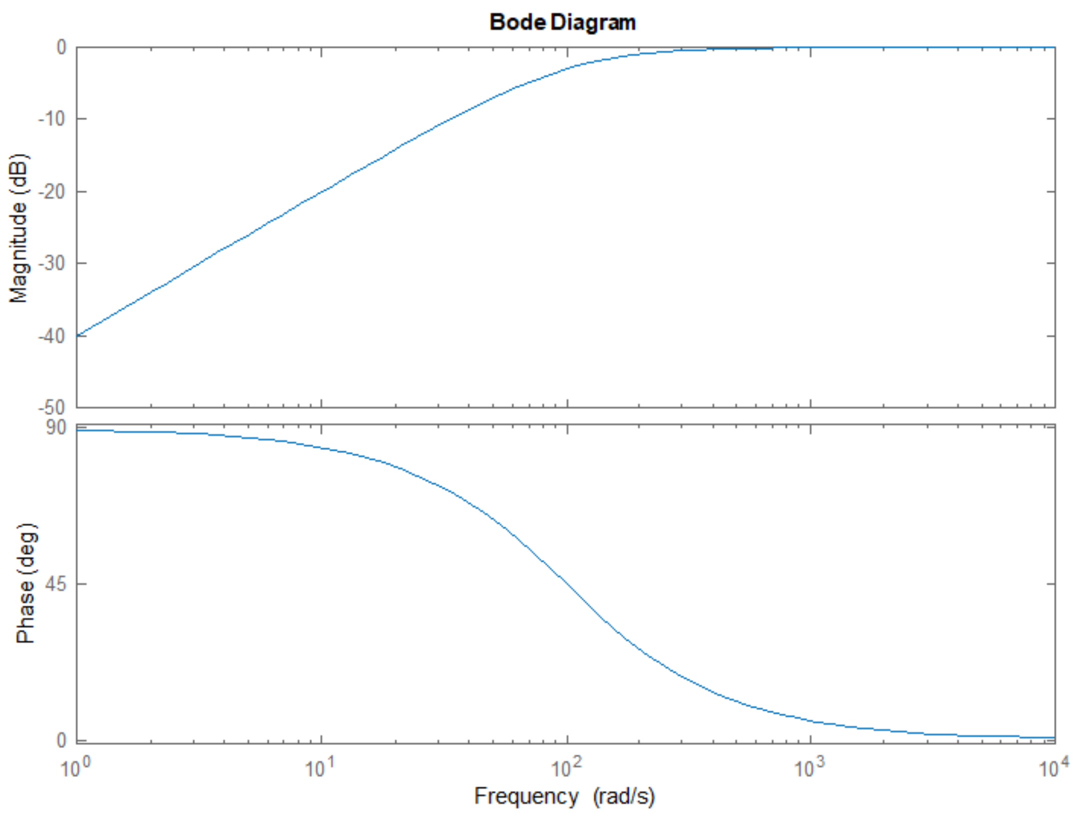
Aplicando a transformada de laplace:

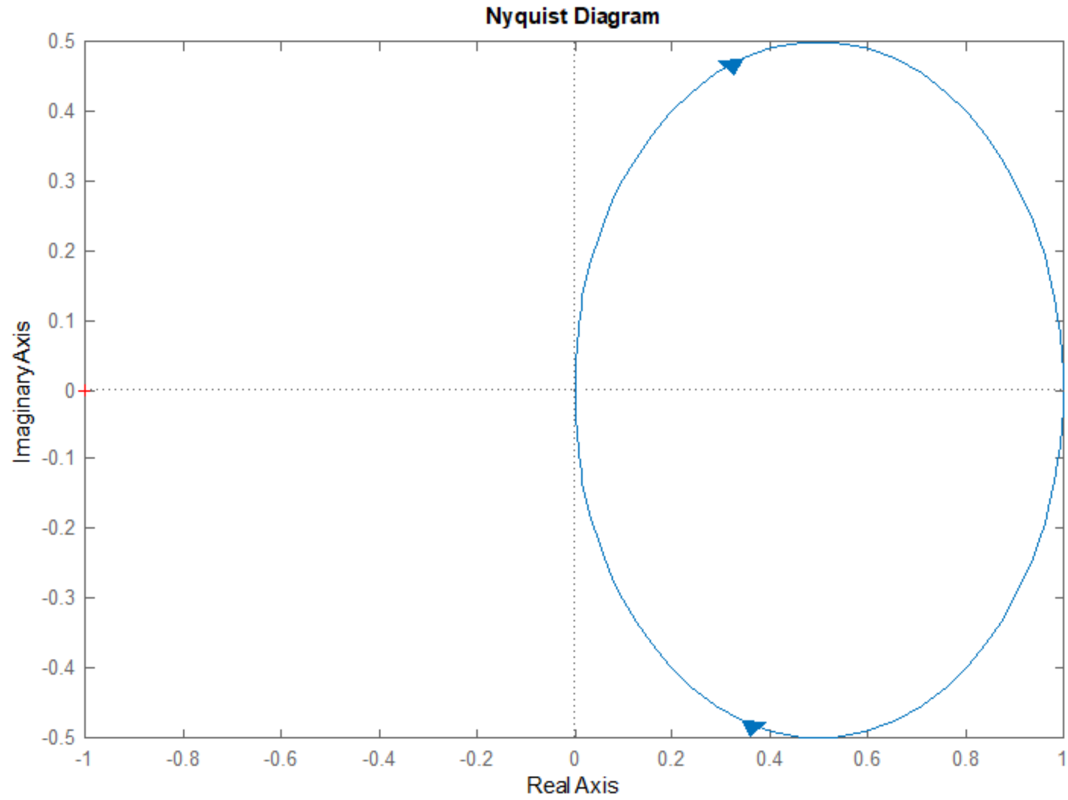
$$G_H(s) = \frac{v_R(s)}{v_s(s)} = \frac{RCs}{RCs + 1}$$

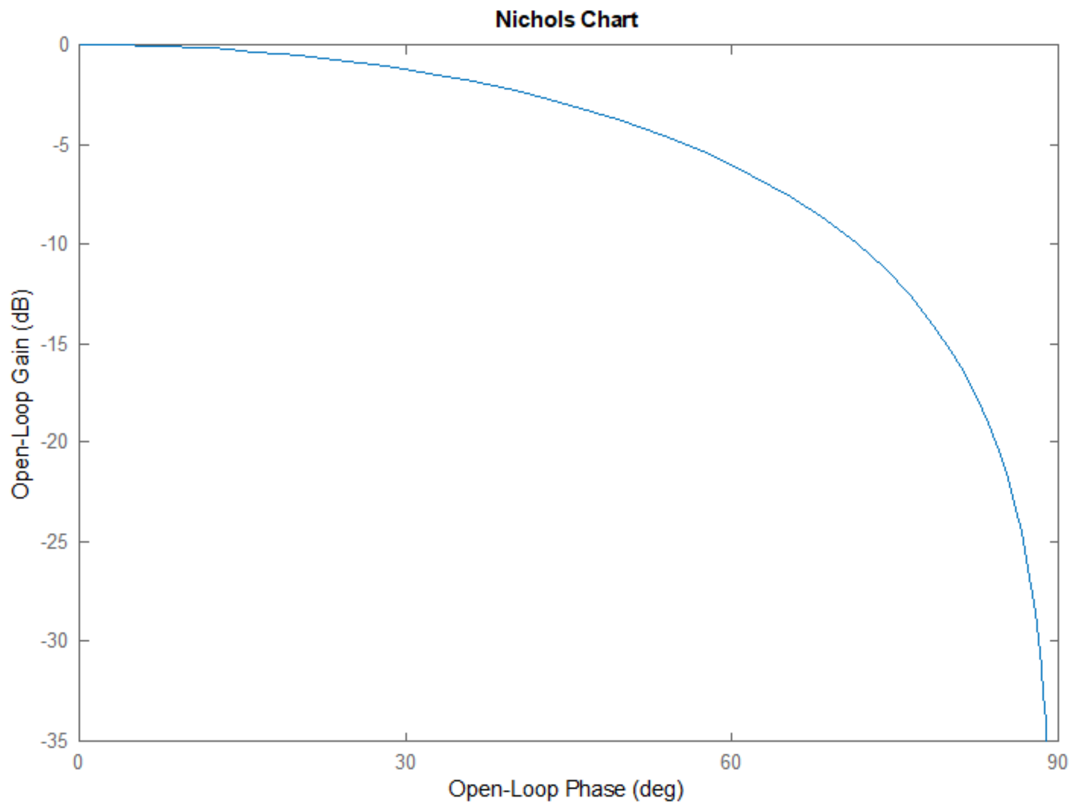
$$M_H(w) = \frac{wRC}{\sqrt{1 + (wRC)^2}}$$

$$\phi_H(w) = \tan^{-1}(\frac{1}{wRC})$$

- Em $\omega = 0$, $M = 0$ e $\phi = 90°$;
- Em $\omega = 1/RC = 100$rad/s, $M = 1/\sqrt{2}$ e ϕ=45°;
- Em $\omega \to \infty$, $M \to 1$ e $\phi \to 0°$.







Código:

```
G = tf( [0.01, 0], [0.01, 1] )
bode(G)
nyquist(G)
nichols(G)
```

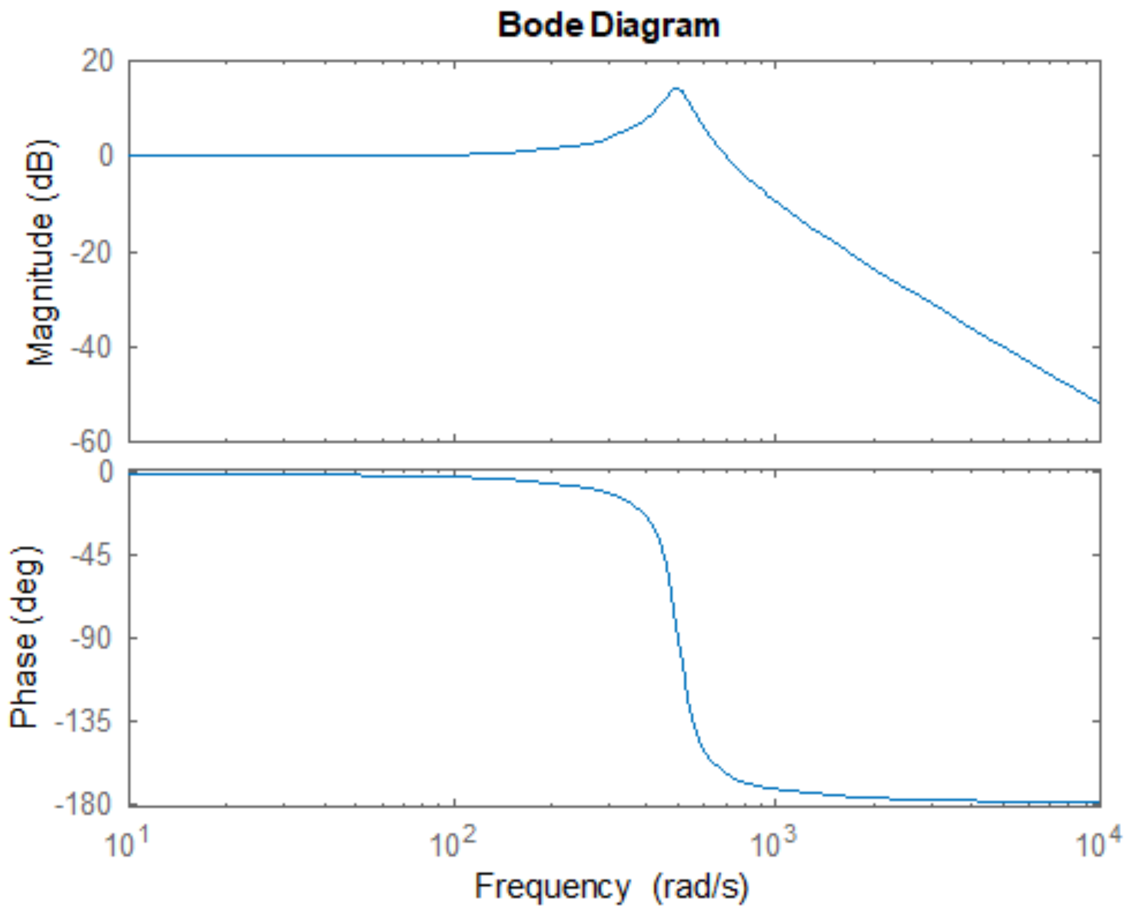
Ex. 14.2) Considere um sistema de segunda ordem com $\omega_n = 500$ rad/s e $\xi = 0.1$. Plote os diagramas de Bode, Nyquist e Nichols e discuta os resultados obtidos.

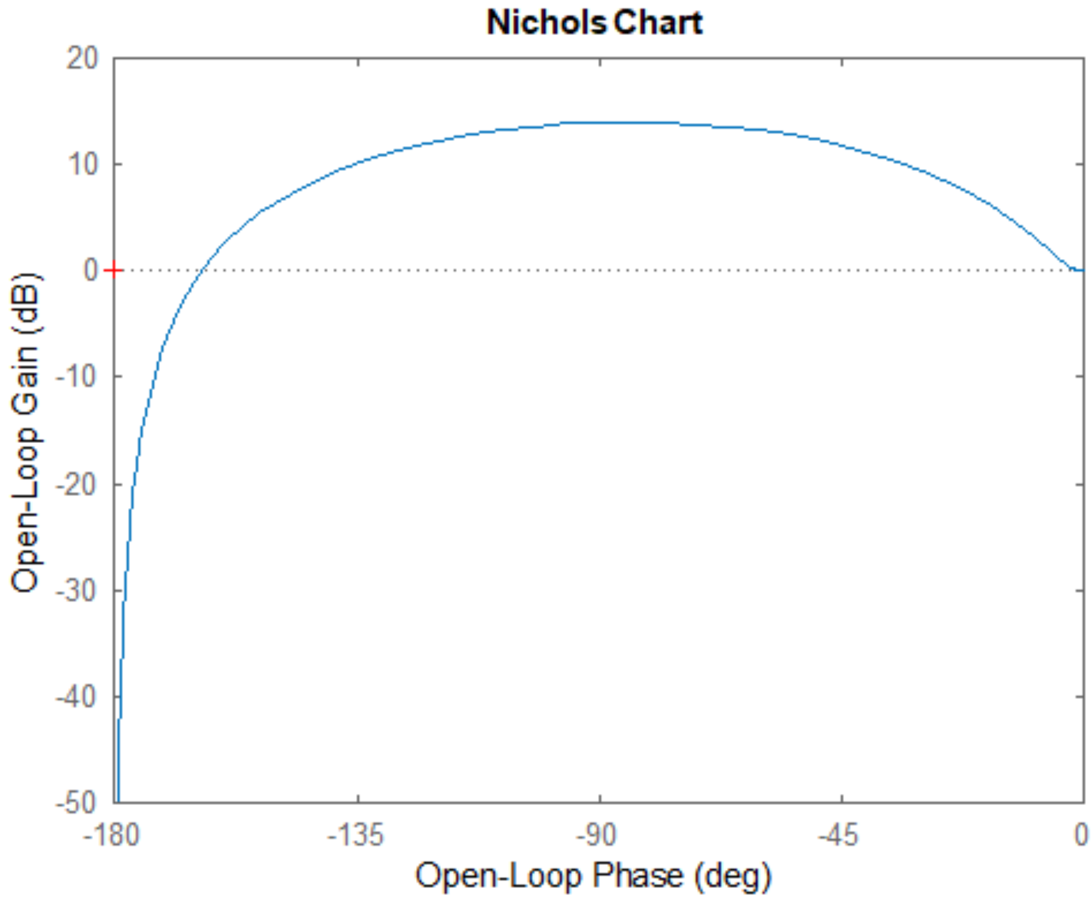
$$G_s = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

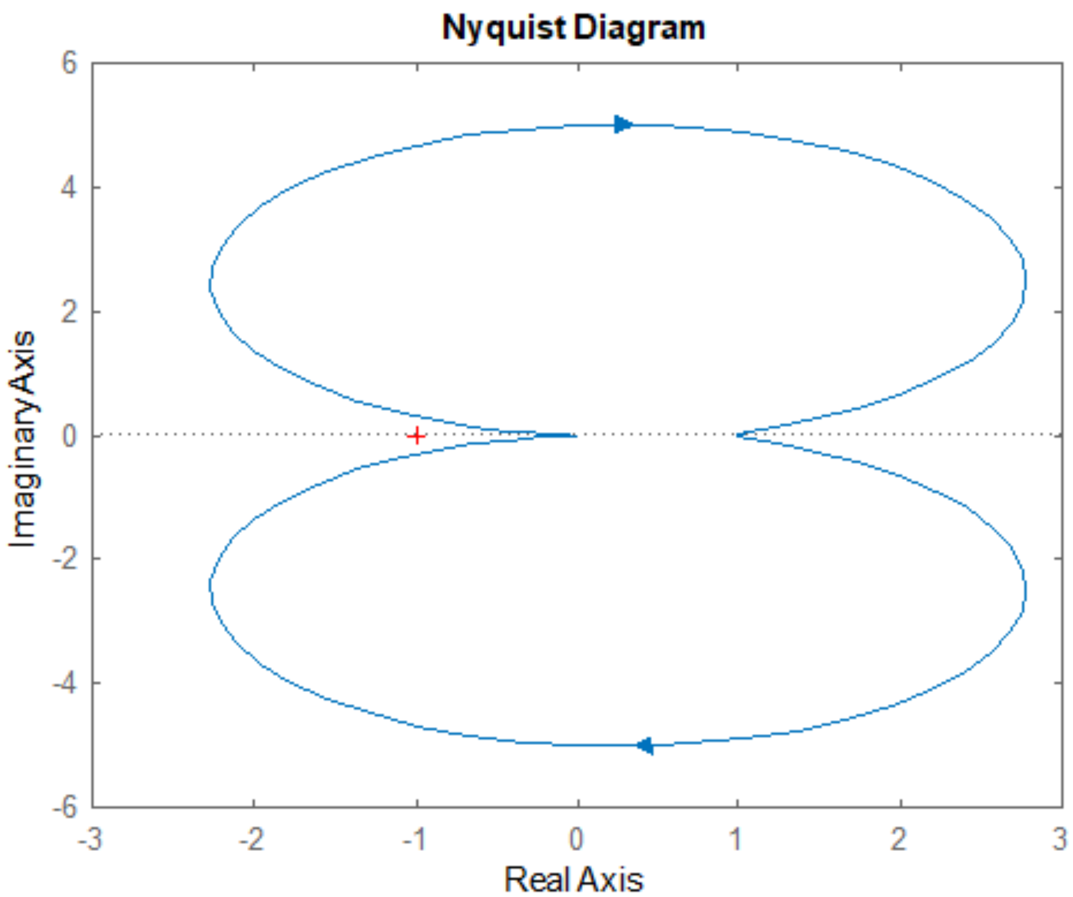
Sabendo que ω_n= 500 e ξ = 0.1, temos:

$$G_s = \frac{25 * 10^4}{s^2 + 100s + 25 * 10^4}$$

- $\sigma = \xi\omega_n$ = 50;
- $\omega_d = \omega_n\sqrt{(1 - 2 * \xi^2)}$ = 497.97 rad/s;
- s = $-\sigma \pm j\omega d$ = −50 ± 497.97j;
- $\omega_R = \omega_n\sqrt{(1 - 2 * \xi^2)}$= 494.97 rad/s.




Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
20
10
0
-10
-20
-30
-40
-50
-180
-135
-90
-45
0


Nyquist Diagram
Imaginary Axis
Real Axis
6
4
2
0
-2
-4
-6
-3
-2
-1
0
1
2
3

Código:

```
G = tf( [250000],[1, 100, 250000] )
bode(G)
nyquist(G)
nichols(G)
```

Ex. 14.3) Analise a resposta de um motor de corrente contínua sem carga considerando as plantas com saída de velocidade e de posição.

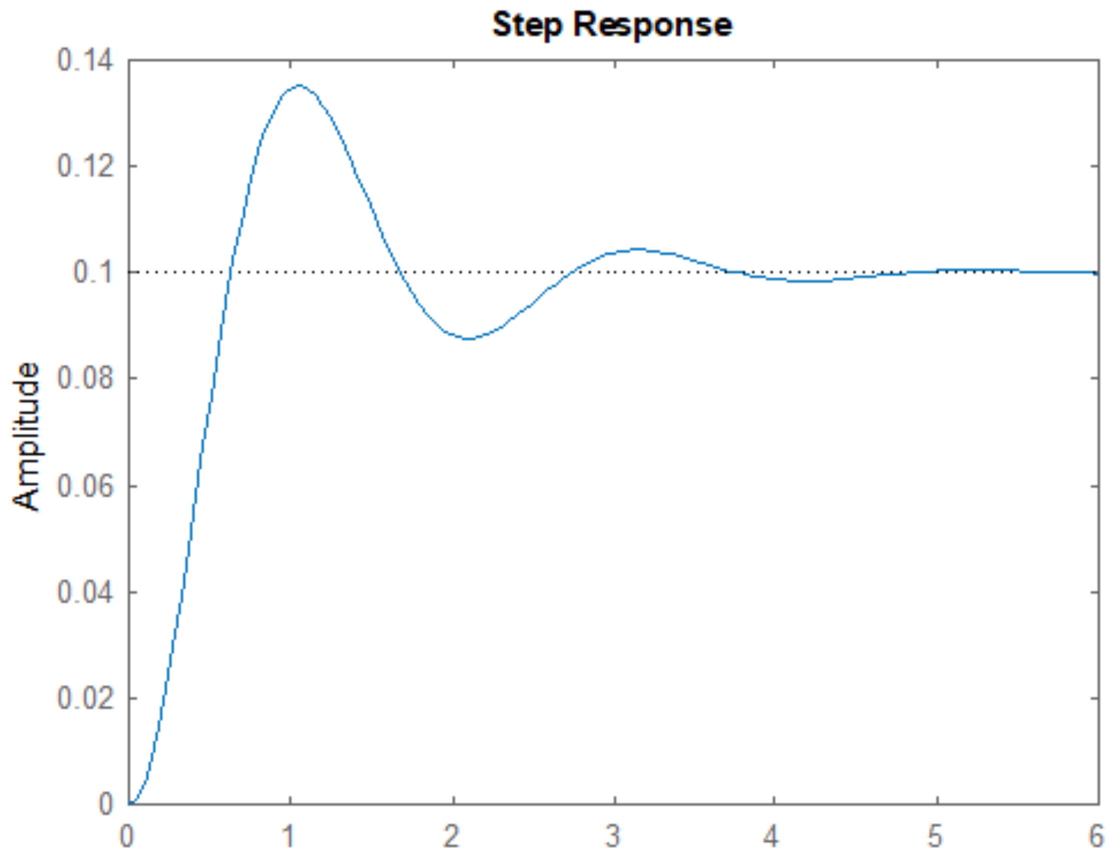
- Dados do motor (SI):
  - $R = 0.2\ \Omega$;
  - $L = 0.01$ H;
  - $J = 1 \times 10^{-5}$;
  - $B = 0.2$;
  - $k = 10$.

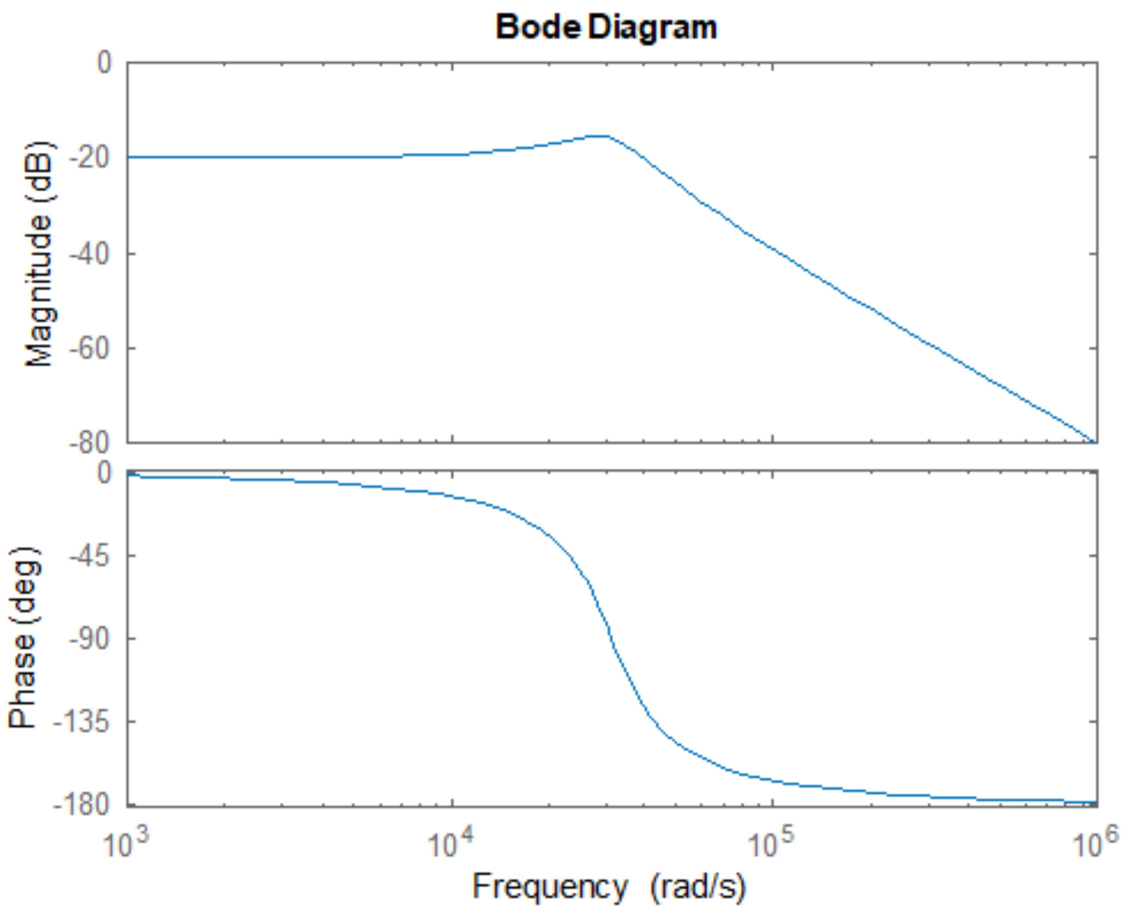
Função de transferência – velocidade:

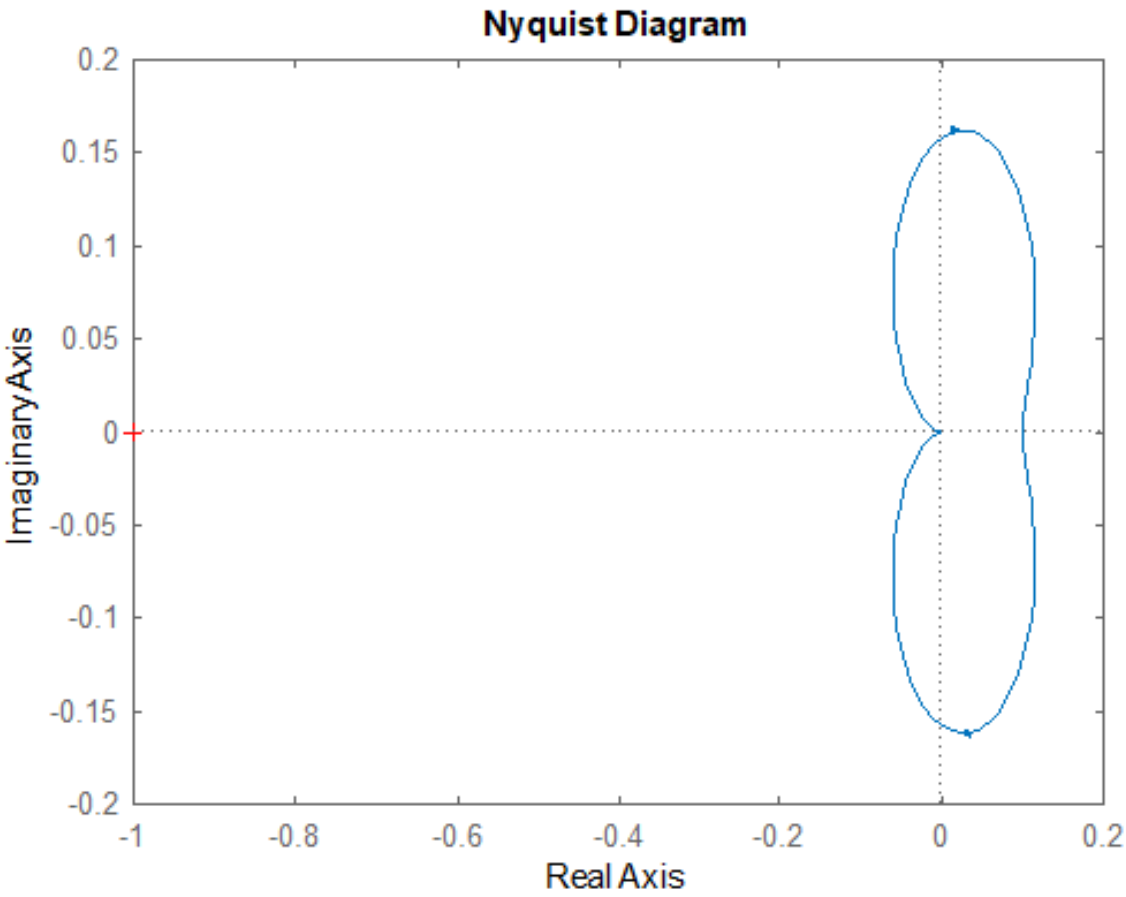
• Tipo 0:

$$G_\omega(s) = \frac{\omega(s)}{V(s)} = \frac{k}{(sL + R)(sJ + B) + k^2}$$

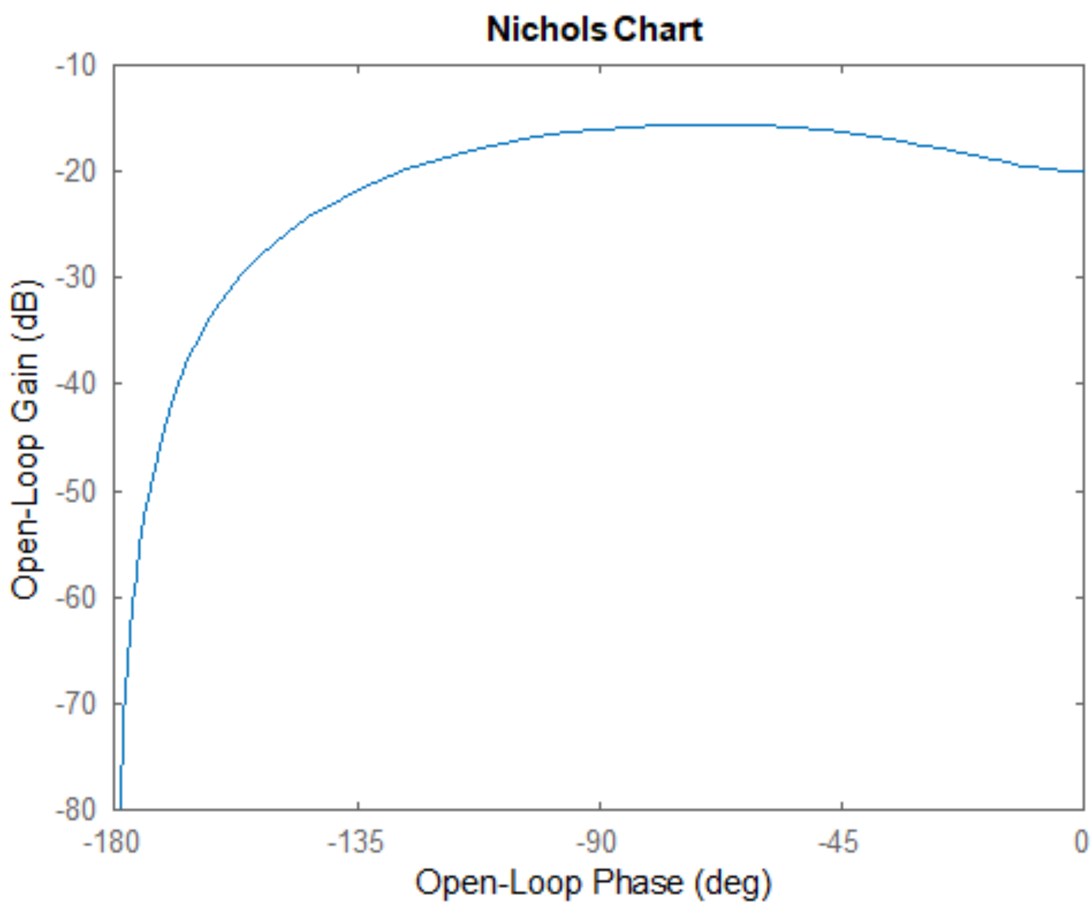
$$G_\omega(s) = \frac{10}{10^{-7}s^2 + 2 * 10^{-3}s + 100}$$


Step Response
Amplitude
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
Phase (deg)
0
-45
-90
-135
-180
10^3
10^4
10^5
10^6
Frequency (rad/s)



Nyquist Diagram
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
Imaginary Axis
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
Real Axis



Nichols Chart
-10
-20
-30
-40
-50
-60
-70
-80
Open-Loop Gain (dB)
-180
-135
-90
-45
0
Open-Loop Phase (deg)

Código:

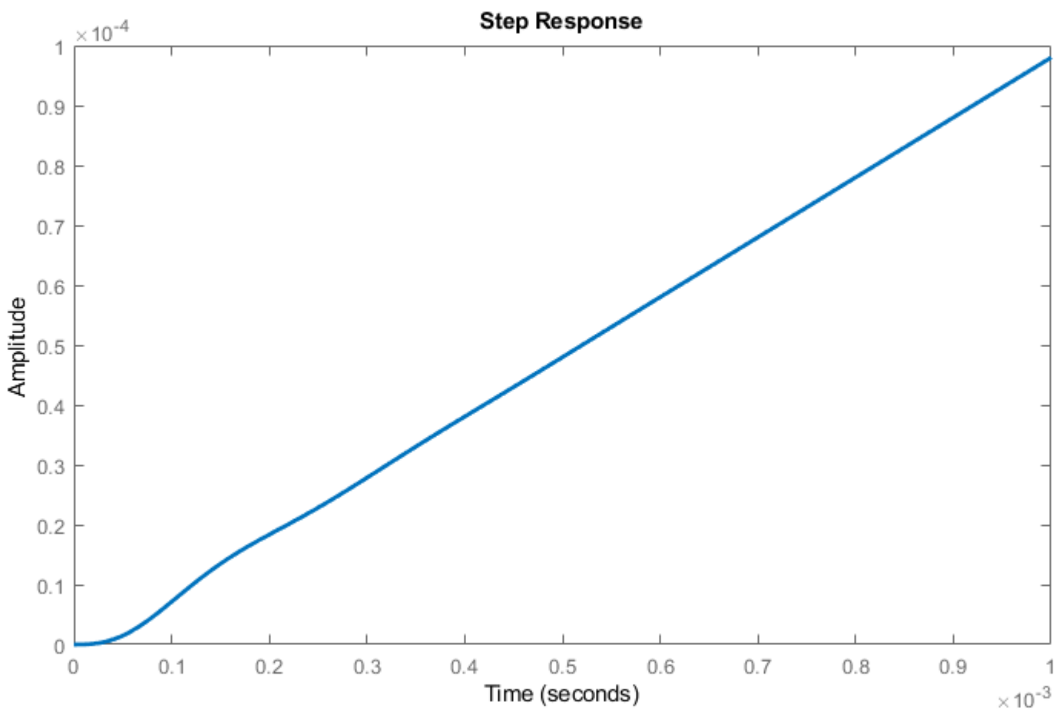
```
G = tf( [10], [1e-7, 2*1e-3, 100] )
bode(G)
nyquist(G)
nichols(G)
```

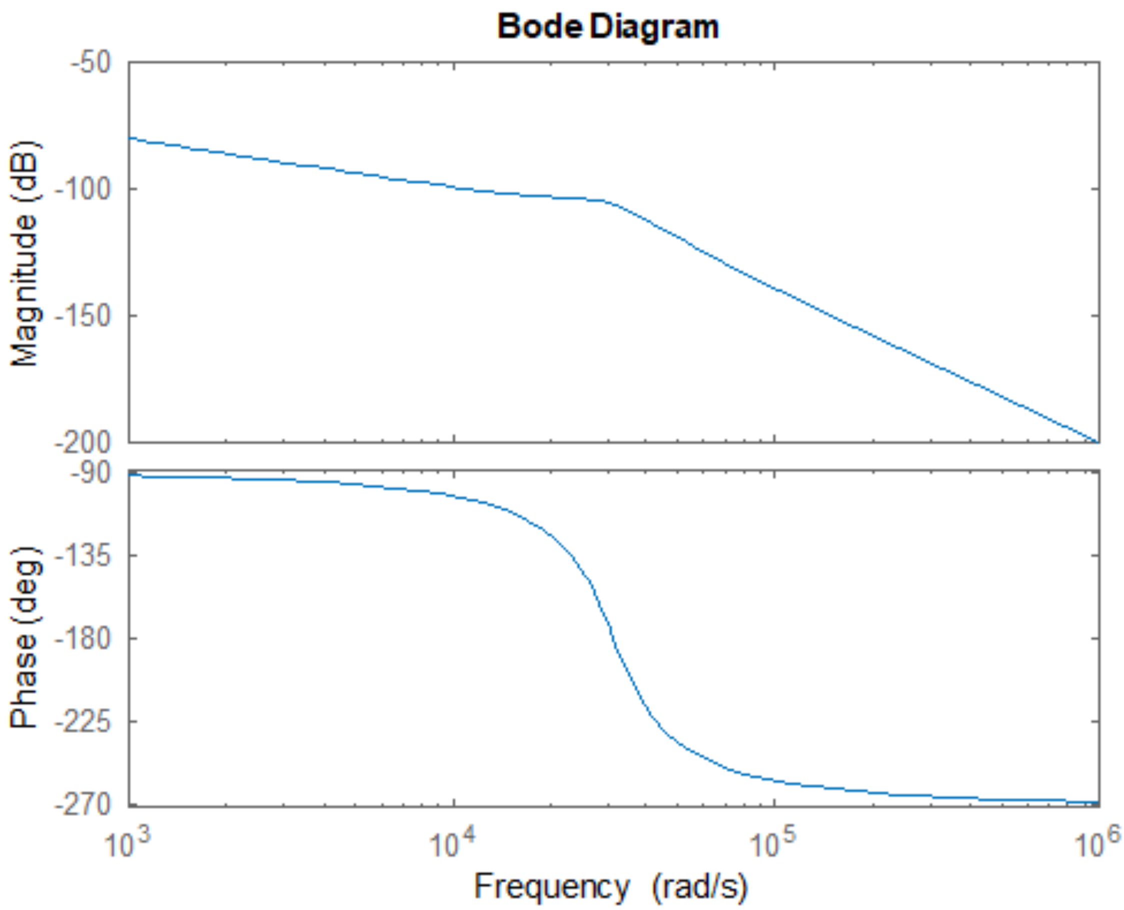
Função de transferência – posição:

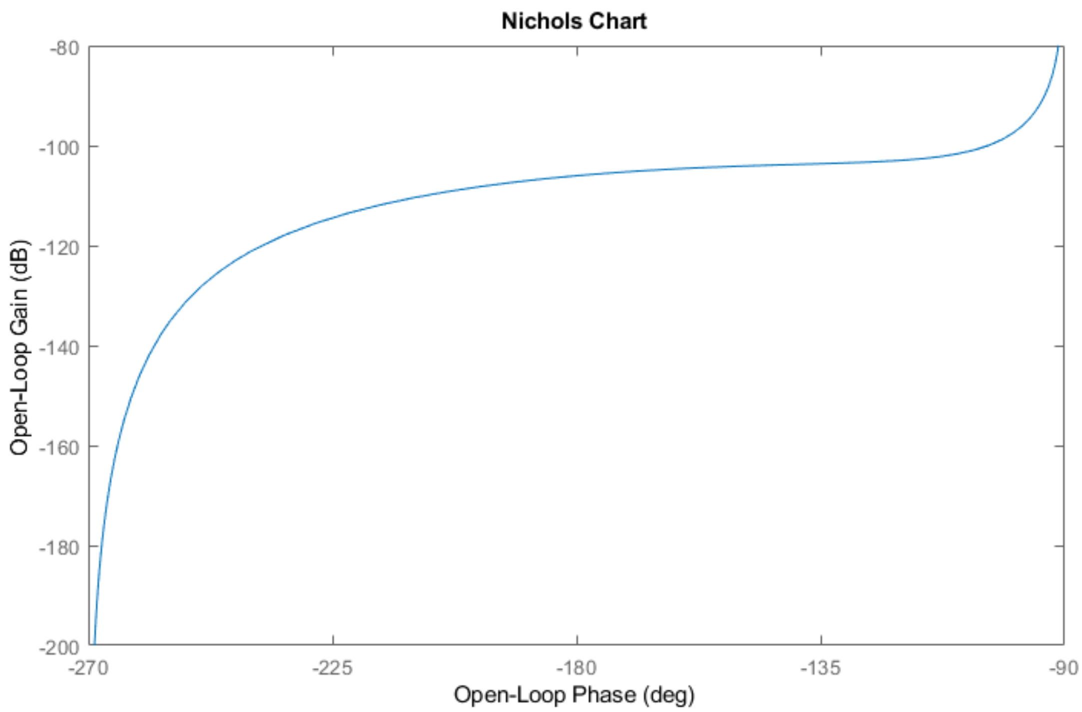
• Tipo 1:

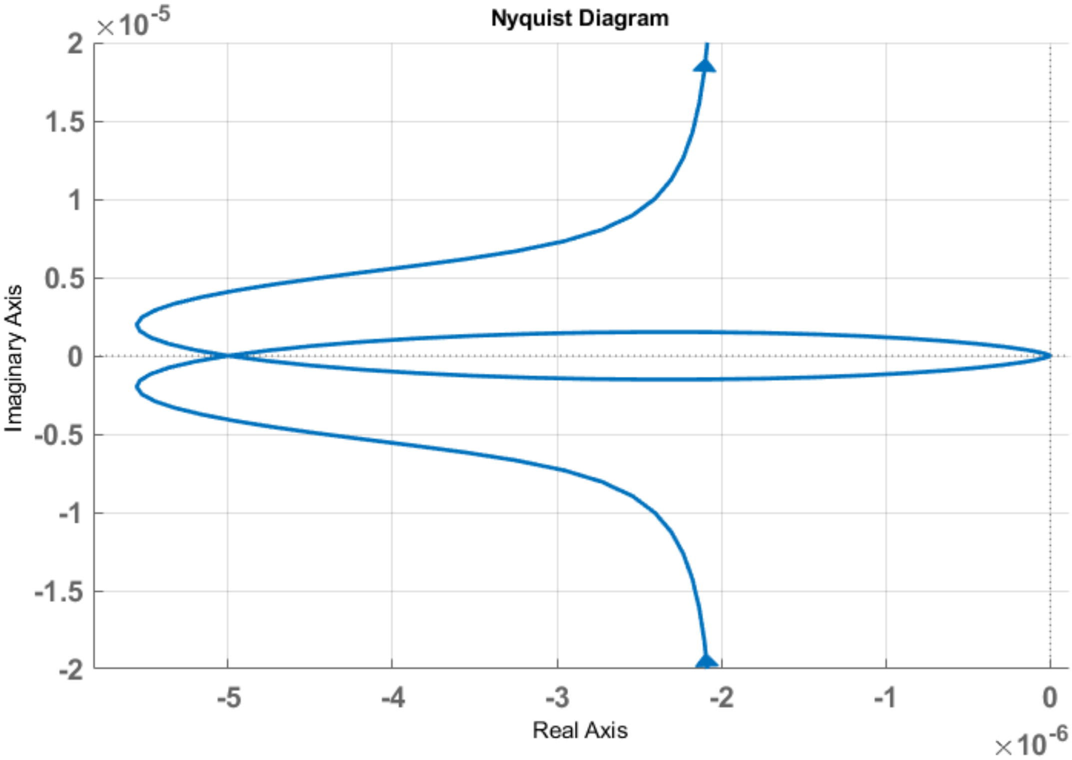
$$G_\theta(s) = \frac{\theta(s)}{V(s)} = \frac{k}{s(sL+R)(sJ+B)+k^2}$$

$$G_\theta(s) = \frac{10}{10^{-7}s^3 + 2 * 10^{-3}s^2 + 100s}$$




Bode Diagram
Magnitude (dB)
-50
-100
-150
-200
Phase (deg)
-90
-135
-180
-225
-270
10^3
10^4
10^5
10^6
Frequency (rad/s)


Nichols Chart
Open-Loop Gain (dB)
-80
-100
-120
-140
-160
-180
-200
-270
-225
-180
-135
-90
Open-Loop Phase (deg)



Código:

```
G = tf( [10], [1e-7, 2*1e-3, 100, 0] )
bode(G)
nyquist(G)
nichols(G)
```