<u>EX</u>: Consider the space R^2 and the two bases:

$$\{e_1, e_2\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \text{ and } \{\bar{e}_1, \bar{e}_2\} = \left\{ \begin{bmatrix} 1/3 \\ -1/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix} \right\}$$

Now let a vector x be represented by $x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ in the $\{e_i\}$ basis.

Find the representation for x in the $\{\bar{e}_i\}$ basis :

First we write down the relationship between the two bases:

$$e_1 = \alpha \bar{e}_1 + \beta \bar{e}_2 = [\bar{e}_1 \quad \bar{e}_2] \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \begin{matrix} \leftarrow b_{11} \\ \leftarrow b_{21} \end{matrix}$$

$$e_2 = \gamma \bar{e}_1 + \delta \bar{e}_2 = [\bar{e}_1 \quad \bar{e}_2] \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \begin{matrix} \leftarrow b_{12} \\ \leftarrow b_{22} \end{matrix}$$

So $B = \begin{bmatrix} \alpha & \gamma \\ \beta & \delta \end{bmatrix}$

Resposta final → x_ = [-2   8]^T