ES728 CONTROLE AVANÇADO – PROF. ELY PAIVA – LISTA EXERCÍCIOS N. 1

(Para entrega, individual, valendo nota, via Moodle – até 15/10)

Example 1. Consider the 2×2 matrix

$$A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

The matrix A has characteristic polynomial λ^2 and hence its only eigenvalue is 0. The eigenvectors for the eigenvalue 0 have the form $[x_2, x_2]^T$ for any $x_2 \neq 0$. Thus the eigenspace for 0 is the one-dimensional span$\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$ which is not enough to span all of $\mathbb{R}^2$. However A^2 is the zero matrix so $A^2\vec{v} = (A - 0I)^2\vec{v} = 0$ for all vectors $\vec{v}$. If we pick $\vec{v}_2$ so that it solves $A\vec{v}_2 = \vec{v}_1$. If we let $\vec{v}_1 = [1, 1]^T$, $\vec{v}_2 = [1, 0]^T$ and $P = [\vec{v}_1, \vec{v}_2]$ we can write $A = PBP^{-1}$ where $B = \begin{bmatrix} \lambda & a \\ 0 & \lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Example 2. ◇ Let

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

The characteristic polynomial of A is $\lambda^2 - 6\lambda + 9 = (\lambda - 3)^2$, so $\lambda = 3$ is the only eigenvalue of A. Next, we compute

$$A - 3I = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \qquad (A - 3I)^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now we choose $\mathbf{v}_2$ to be any vector in $\ker(A - 3I)^2$ that is not in $\ker(A - 3I)$. One such vector is $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. With this choice, we than have $\mathbf{v}_1 = (A - 3I)\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The solution of the system $\frac{d\mathbf{y}}{dt} = A\mathbf{y}$ is therefore

$$\mathbf{y} = c_1 e^{3t}\left(t\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) + c_2 e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Example 3. ◇ Let

$$A = \begin{bmatrix} 5 & 1 & -4 \\ 4 & 3 & -5 \\ 3 & 1 & -2 \end{bmatrix}$$

The characteristic polynomial of A is $p_A(\lambda) = (\lambda - 2)^3$. Since

$$A - 2I = \begin{bmatrix} 3 & 1 & -4 \\ 4 & 1 & -5 \\ 3 & 1 & -4 \end{bmatrix} \xrightarrow[rref]{} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix},$$

the eigenspace $E_2 = \ker(A - 2I)$ has dimension 1. Next, since

$$(A - 2I)^2 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow[rref]{} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

the space $\ker(A - 2I)^2$ has dimension 2. Finally, since

$$(A - 2I)^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

the space $\ker(A-2I)^2$ has dimension 2. Finally, since

$$(A-2I)^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

the space $\ker(A-2I)^3$ has dimension 3, which matches the algebraic multiplicity of the eigenvalue $\lambda = 2$.

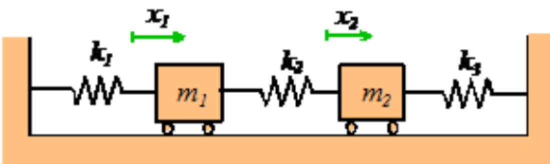
$$\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \qquad \mathbf{y}_1(t) = e^{2t}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\implies \mathbf{v}_2 = (A-2I)\mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} \Rightarrow \qquad \mathbf{y}_2(t) = e^{2t}\left(t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}\right)$$

$$\implies \mathbf{v}_1 = (A-2I)\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{y}_3(t) = e^{2t}\left(\frac{t^2}{2!}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

Ex. 1 – Considere m=k=1 para todos os parâmetros abaixo. Explicite a matriz dinâmica A do sistema, e depois diagonalize essa matriz, através de uma transformação de similaridade.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\frac{d^2}{dt^2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$m_1\frac{d^2x_1}{dt^2} + (k_1+k_2)x_1 - k_2x_2 = 0 \;\Big/\; m_2\frac{d^2x_2}{dt^2} - k_2x_1 + (k_2+k_3)x_2 = 0$$

Ex 2. Encontre os autovetores generalizados e coloque as matrizes na forma bloco-diagonal.

Problems Compute the eigenvalues For the given 2×2 matrices A

(a) $A = \begin{bmatrix} 5 & -1 \\ 0 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} -5/2 & 1/2 \\ -1/2 & -3/2 \end{bmatrix}$

Answers

(a) Repeated eigenvalue $\lambda = 5$, $v_1 = [1, 0]^T$, $v_2 = [0, -1]^T$ $B = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) Repeated eigenvalue $\lambda = -2$, $v_1 = [1/2, 1/2]^T$, $v_2 = [1, 2]^T$ $B = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$ and $P = \begin{bmatrix} 1/2 & 1 \\ 1/2 & 2 \end{bmatrix}$

Ex 3. Transforme para a forma bloco-diagonal

Problems For each $n \times n$ matrix, find a basis of $\mathbb{R}^n$ consisting of generalized eigenvectors.

(a) $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

Answers The answers are not unique, but there is a logic to answers choosen, it is so the super-diagonal entries of something would be one. Careful, these answers were machine generated and not yet checked.

(a) $\lambda = 3$ order $k = 1$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, order $k = 2$ $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and order $k = 3$ $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

(b) $\lambda = 3$ order $k = 1$ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, order $k = 2$ $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and order $k = 1$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Ex. 4

Calcule a Matriz de transição de estados do caso abaixo, usando o teorema de <u>CAYLEY-HAMILTON</u>

$$A = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

Ex. 5

Calcule a Matriz de transição de estados dos dois casos abaixo, usando a <u>transformada Inversa de Laplace</u>

$$A = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Ex. 6

Calcule a Matriz de transição de estados do caso abaixo, considerando a <u>propriedade</u> relacionada à forma diagonal: $f(A) = Tf(\Lambda)T^{-1}$ $\qquad A = T\Lambda T^{-1}$

$$A = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

Ex. 7

Encontre a resposta x(t) do sistema abaixo para uma entrada tipo <u>Impulso</u> e condições iniciais nulas. Indique se a resposta contém todos os modos do sistema.

$$\begin{aligned} \dot{x}_1 &= -5x_1 + 2x_2 + u \\ \dot{x}_2 &= -12x_1 + 5x_2 + 2u \end{aligned}$$

Ex. 8

Encontre a resposta do sistema abaixo x(t) para uma entrada tipo <u>Degrau</u> e com a condição inicial x(0) apresentada.

$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$

a.) Find e^{At}.

b.) Solve for x(t) given that $x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, u(t) = *Unit step input.*