## A deep learning method for high dimensional PDE's

An application to mean field games

by

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### Thesis Title

Thesis Subtitle

#### **Author Name**

#### Abstract

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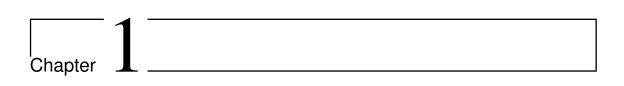
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# Acknowledgements

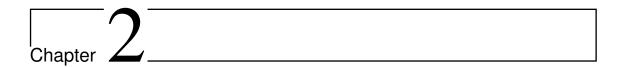
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Introduction



# Backward stochastic differential equations and PDEs

When addressing deterministic optimal control problems of dynamical systems, there are two approaches, one involving Bellman's dynamic programming principle, and the other relying on the Pontryagin's maximum principle. The former approach leads to a partial differential equation, the Hamilton-Jacobi-Bell equation, to be solved for the value function and the optimal control of the process. The latter leads to a system of ordinary differential equations, one equation forward in time for the state and one backward in time for its adjoint.

The stochastic version of these problems is solved by methods analogous to those of the deterministic case. However, there are issues with desirable mathematical properties of solutions when we state them extending directly the ones proposed by deterministic methods. That is the case of the stochastic version of the Pontryagin's maximum principle, in which the backward differential equation cannot be stated directly as an SDE with terminal condition, as the solution is not guaranteed to be adapted to the filtration generated by the brownian motion.

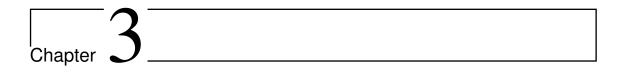
The theory of backward stochastic differential equations (BSDEs) emerged in Bismut's [1] early work, and later generalized by Pardoux and Peng[2], as an attempt to formalize the application of the stochastic maximum principle. Here we give an introduction and compilation of results about them based on [3, 4, 5, 6], including its relation with a certain class of nonlinear parabolic partial differential equations, which will be the main tool for the method explained in the following chapters.

### 2.1 Backward stochastic differential equations

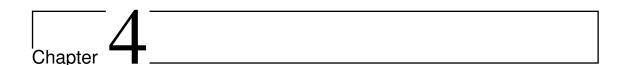
Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a probability space

**Definition 2.1.1** Una fola sapo

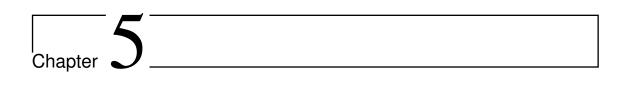
2.2 Nonlinear Feynman-Kac formula



The Deep BSDE method



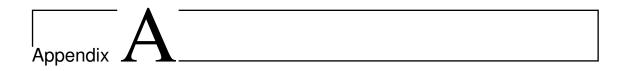
Mean field games and crowd motion



An application



Conclusion



Neural Networks

### Bibliography

- [1] Jean-Michel Bismut. "Conjugate convex functions in optimal stochastic control". en. In: Journal of Mathematical Analysis and Applications 44.2 (Nov. 1973), pp. 384-404. ISSN: 0022-247X. DOI: 10.1016/0022-247X(73)90066-8. URL: https://www.sciencedirect.com/science/article/pii/0022247X73900668 (visited on 02/21/2023).
- [2] E. Pardoux and S. G. Peng. "Adapted solution of a backward stochastic differential equation". en. In: Systems & Control Letters 14.1 (Jan. 1990), pp. 55-61. ISSN: 0167-6911. DOI: 10.1016/0167-6911(90)90082-6. URL: https://www.sciencedirect.com/science/article/pii/0167691190900826 (visited on 02/21/2023).
- [3] Jianfeng Zhang. Backward Stochastic Differential Equations. en. Vol. 86. Probability Theory and Stochastic Modelling. New York, NY: Springer New York, 2017. ISBN: 978-1-4939-7254-8 978-1-4939-7256-2. DOI: 10.1007/978-1-4939-7256-2. URL: http://link.springer.com/10.1007/978-1-4939-7256-2 (visited on 02/15/2023).
- [4] Etienne Pardoux and Aurel R şcanu. Stochastic Differential Equations, Backward SDEs, Partial Differential Equations. en. Vol. 69. Stochastic Modelling and Applied Probability. Cham: Springer International Publishing, 2014. ISBN: 978-3-319-05713-2 978-3-319-05714-9. DOI: 10.1007/978-3-319-05714-9. URL: https://link.springer.com/10.1007/978-3-319-05714-9 (visited on 02/15/2023).
- [5] Ricardo Romo Romero. "Maestro en ciencias con especialidad en probabilidad y estadística". es. In: ().
- [6] Nizar Touzi. Optimal Stochastic Control, Stochastic Target Problems, and Backward SDE. en. Vol. 29. Fields Institute Monographs. New York, NY: Springer New York, 2013. ISBN: 978-1-4614-4285-1 978-1-4614-4286-8. DOI: 10.1007/978-1-4614-4286-8. URL: https://link.springer.com/10.1007/978-1-4614-4286-8 (visited on 02/15/2023).