A deep learning method for high dimensional PDE's

An application to mean field games

by

Carlos Daniel Contreras Quiroz

Advisor: Mauricio Junca

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Master in
Mathematics

at the
Universidad de los Andes
2023

Thesis Title

Thesis Subtitle

Author Name

Abstract

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Acknowledgements

Contents

Chapter 1

Introduction

Chapter 2

Backward stochastic differential equations and PDEs

When addressing deterministic optimal control problems of dynamical systems, there are two approaches, one involving Bellman's dynamic programming principle, and the other relying on the Pontryagin's maximum principle. The former approach leads to a partial differential equation, the Hamilton-Jacobi-Bell equation, to be solved for the value function and the optimal control of the process. The latter leads to a system of ordinary differential equations, one equation forward in time for the state and one backward in time for its adjoint.

The stochastic version of these problems is solved by methods analogous to those of the deterministic case. However, there are issues with desirable mathematical properties of solutions when we state them extending directly the ones proposed by deterministic methods. That is the case of the stochastic version of the Pontryagin's maximum principle, in which the backward differential equation cannot be stated directly as an SDE with terminal condition, as the solution is not guaranteed to be adapted to the filtration generated by the brownian motion.

The theory of backward stochastic differential equations (BSDEs) emerged in Bismut's [1] early work, and later generalized by Pardoux and Peng[2], as an attempt to formalize the application of the stochastic maximum principle. Here we give an introduction and compilation of results about them based on [3, 4, 5, 6], including its relation with a certain class of nonlinear parabolic partial differential equations, which will be the main tool for the method explained in the following chapters.

2.1 Backward stochastic differential equations

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space

Definition 2.1.1 *Una fola*

2.2 Nonlinear Feynman-Kac formula

Chapter 3

The Deep BSDE method

Chapter 4

Mean field games and crowd motion

Chapter 5

An application

Chapter 6

Conclusion

Appendix A

Neural Networks

Bibliography

[1]   Jean-Michel Bismut. "Conjugate convex functions in optimal stochastic control". en. In: *Journal of Mathematical Analysis and Applications* 44.2 (Nov. 1973), pp. 384–404. ISSN: 0022-247X. DOI: `10.1016/0022-247X(73)90066-8`. URL: `https://www.sciencedirect.com/science/article/pii/0022247X73900668` (visited on 02/21/2023).

[2]   E. Pardoux and S. G. Peng. "Adapted solution of a backward stochastic differential equation". en. In: *Systems & Control Letters* 14.1 (Jan. 1990), pp. 55–61. ISSN: 0167-6911. DOI: `10.1016/0167-6911(90)90082-6`. URL: `https://www.sciencedirect.com/science/article/pii/0167691190900826` (visited on 02/21/2023).

[3]   Jianfeng Zhang. *Backward Stochastic Differential Equations*. en. Vol. 86. Probability Theory and Stochastic Modelling. New York, NY: Springer New York, 2017. ISBN: 978-1-4939-7254-8 978-1-4939-7256-2. DOI: `10.1007/978-1-4939-7256-2`. URL: `http://link.springer.com/10.1007/978-1-4939-7256-2` (visited on 02/15/2023).

[4]   Etienne Pardoux and Aurel Rșcanu. *Stochastic Differential Equations, Backward SDEs, Partial Differential Equations*. en. Vol. 69. Stochastic Modelling and Applied Probability. Cham: Springer International Publishing, 2014. ISBN: 978-3-319-05713-2 978-3-319-05714-9. DOI: `10.1007/978-3-319-05714-9`. URL: `https://link.springer.com/10.1007/978-3-319-05714-9` (visited on 02/15/2023).

[5]   Ricardo Romo Romero. "Maestro en ciencias con especialidad en probabilidad y estadística". es. In: ().

[6]   Nizar Touzi. *Optimal Stochastic Control, Stochastic Target Problems, and Backward SDE*. en. Vol. 29. Fields Institute Monographs. New York, NY: Springer New York, 2013. ISBN: 978-1-4614-4285-1 978-1-4614-4286-8. DOI: `10.1007/978-1-4614-4286-8`. URL: `https://link.springer.com/10.1007/978-1-4614-4286-8` (visited on 02/15/2023).