RECENT DEVELOPMENTS IN RISK MANAGEMENT

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EXTREME VALUE THEORY

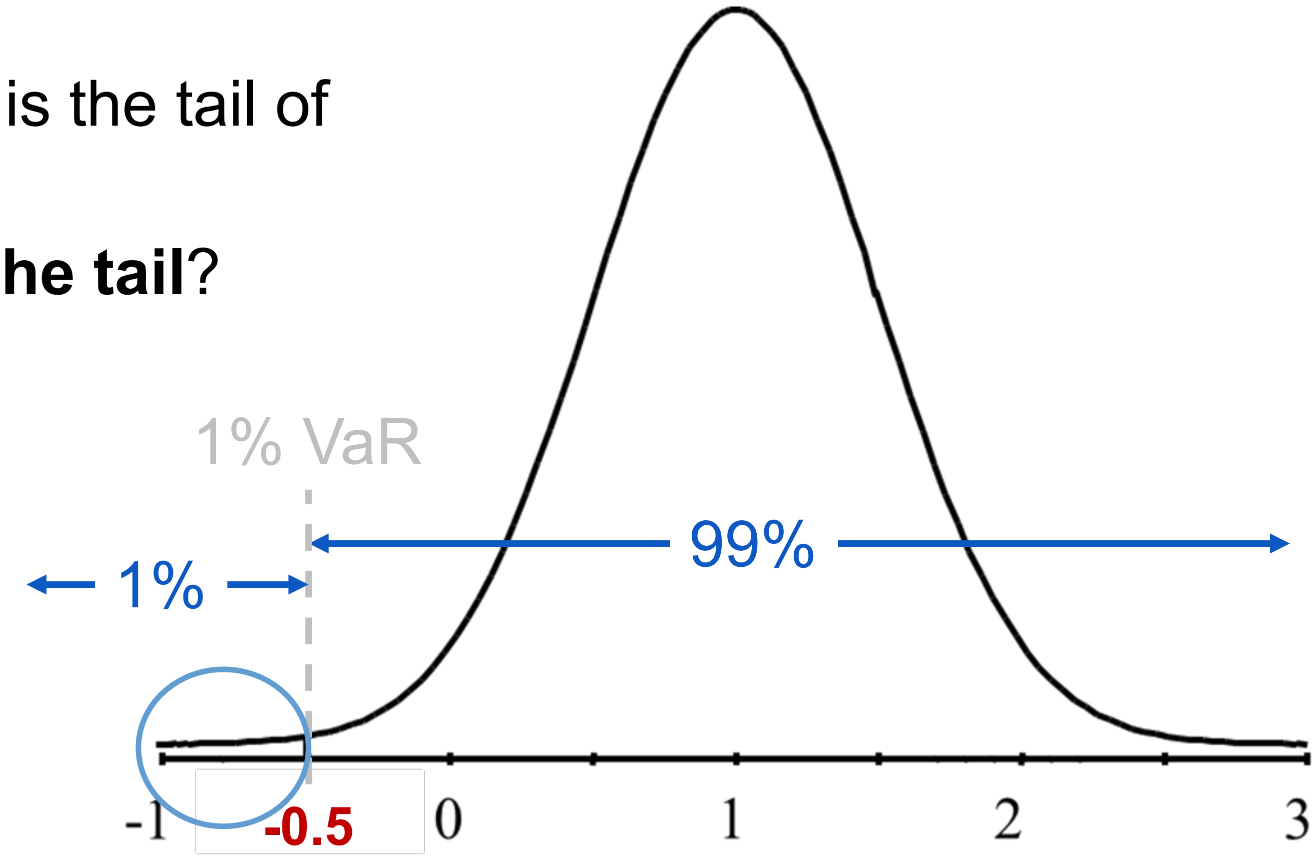
Complications to CVaR / ES

1. CVaR estimates tend to be less stable than VaR for the same confidence level.
   a) Requires a large number of observations to generate a reliable estimate.

2. CVaR is more sensitive to estimation errors than VaR
   a) Depends substantially on the accuracy of the tail model used.

Focus on the Tails

- We try so hard to estimate the full distribution.
- However, our only care is the tail of the distribution.
- Why not just **estimate the tail**?



Extreme Value Theory

- Extreme Value Theory (EVT) provides the theoretical foundation for building statistical models **describing extreme events**.
- Used in many fields:
  - Finance
  - Structural Engineering
  - Traffic Prediction
  - Weather Prediction
  - Geological Prediction (Seismic events, flooding, etc.)

Extreme Value Theory

- Extreme Value Theory (EVT) provides the theoretical foundation for building statistical models **describing extreme events**.
- EVT provides the distributions for the following:
  - **Block Maxima (Minima)** – the maximum (or minimum) the variable takes in successive period, for example months or years.
  - **Exceedances** – the values that exceed a certain threshold.

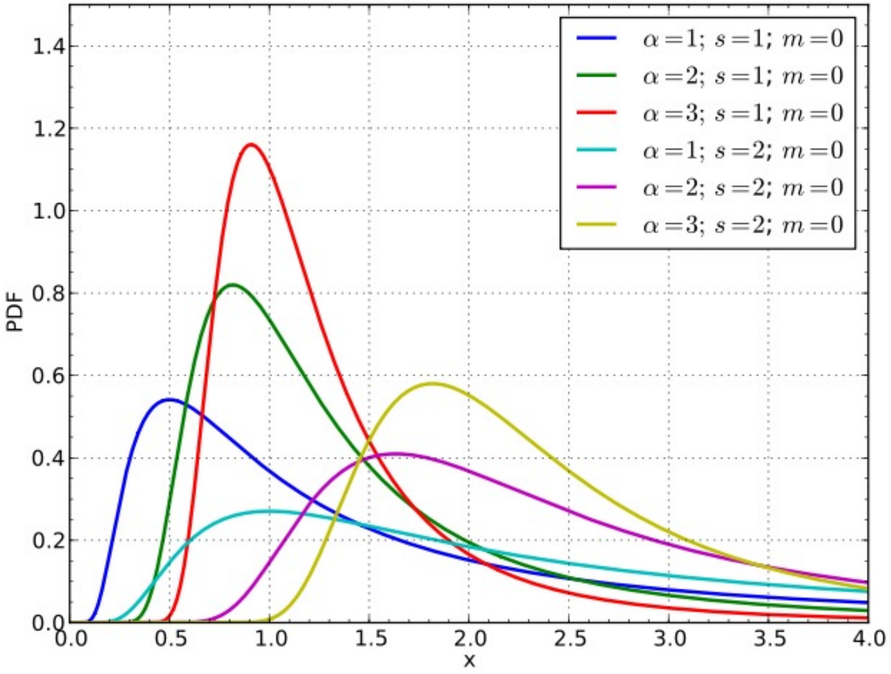
Block Maxima (Minima)

- Trying to build the series of maximum (or minimum) values across time.
  - Example – the highest annual rainfall in Raleigh, NC between 1900 and 2015.

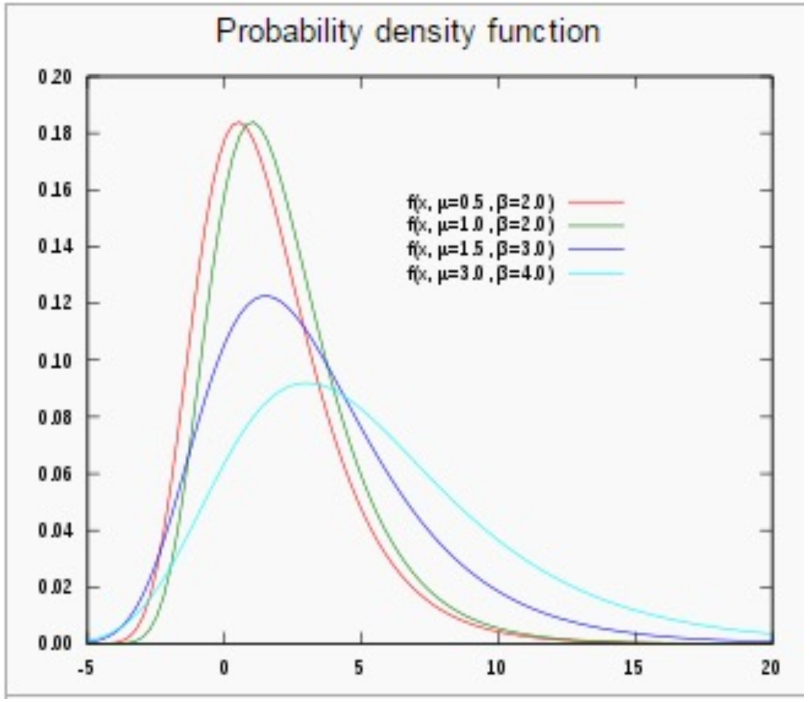
- Popular distributions used (Right Skewed):
  - Fréchet
  - Weibull
  - Gumbel

Block Maxima (Minima)

Frechet



Gumbel



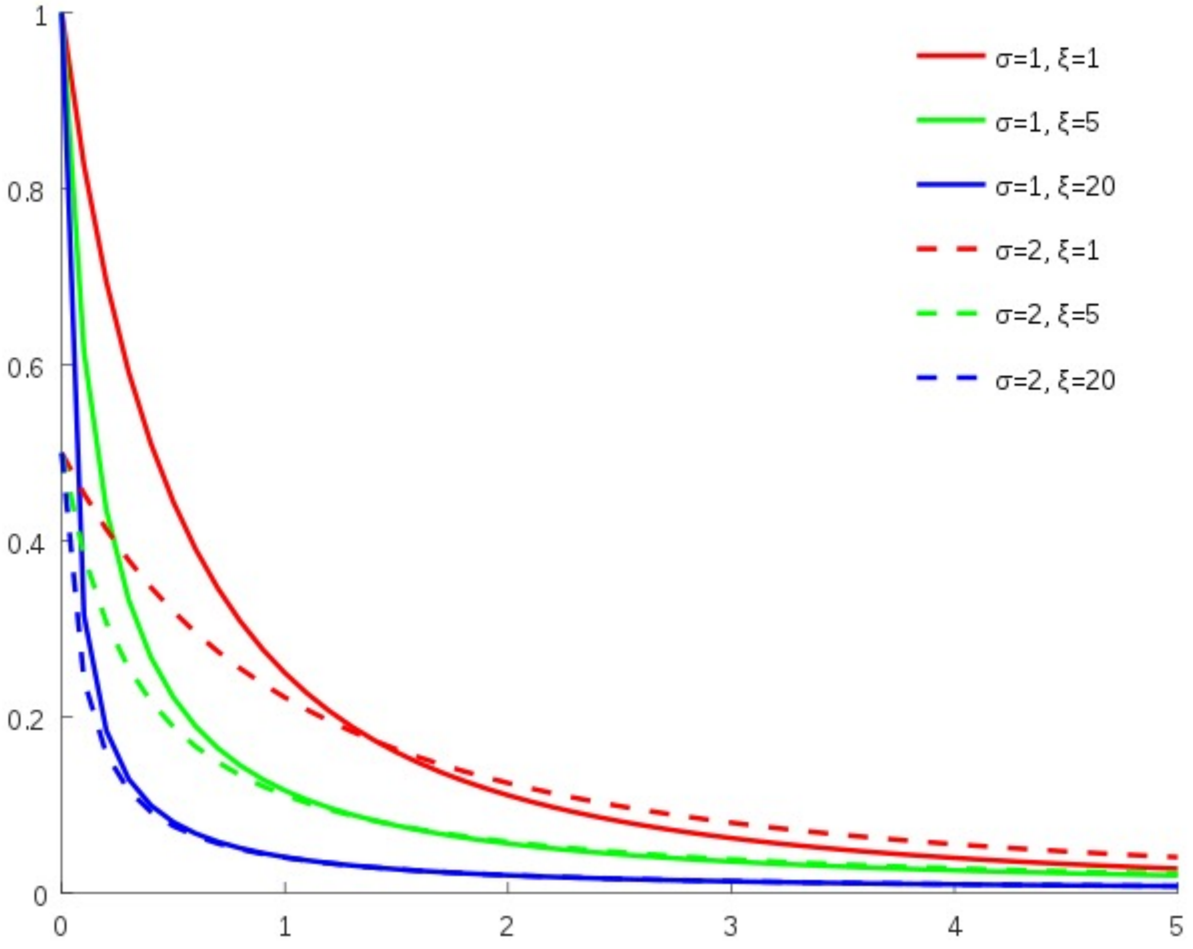
Exceedances

- Trying to understand the distribution of values that exceed a certain threshold.
- Instead of isolating the tail of an overall distribution (limiting the values) we are trying to build a distribution for the tail events themselves.
- Popular distribution:
  - Generalized Pareto

Generalized Pareto

- Named after Italian engineer and economist Vilfredo Pareto.
- Came into popularity with the “Pareto Principle” – more commonly known as the “80-20 Rule.”
- Pareto noted in 1896 that 80% of the land of Italy was owned by 20% of the population.
- Richard Koch authored the book *The 80/20 Principle* to iilustrate some common applications.

Exceedances



Extreme Value Theory

- **Application to CVaR –** more accurate estimates of CVaR, but the math is VERY complicated.
- Need to use maximum likelihood estimation to find which generalized Pareto distribution fits our data the best.
- Choose ξ and β to maximize:

$$\sum_{i=1}^{n_u} \ln\left[\frac{1}{\beta}\left(1+\frac{\xi(v_i-u)}{\beta}\right)^{-1/\xi-1}\right]$$

Extreme Value Theory

- **Application to CVaR –** more accurate estimates of CVaR, but the math is VERY complicated.
- **VaR Calculation:**

$$\text{VaR} = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{n_u}(1-q)\right]^{-\xi} - 1\right\}$$

- **CVaR Calculation:**

$$\text{ES} = \frac{\text{VaR} + \beta - \xi u}{1-\xi}$$

EVT: Two Position Portfolio

- $200,000 invested in MSFT & $100,000 in Apple today.
- You have 500 observations on both returns.
- Calculate the portfolio's value using each one of the historical daily returns:

$$\$200{,}000 \times R_M + \$100{,}000 \times R_A$$

- Using the tail (typically 5%) of the 500 observations, estimate the generalized Pareto distribution parameters.
- Estimate VaR and ES from these.

Getting Stock Data – R

```r
tickers = c("AAPL", "MSFT")

getSymbols(tickers)

stocks <- cbind(last(AAPL[,4], '500 days'), last(MSFT[,4], '500 days'))
```

```
##            AAPL.Close MSFT.Close
## 2018-01-12     177.09      89.60
## 2018-01-16     176.19      88.35
## 2018-01-17     179.10      90.14
## 2018-01-18     179.26      90.10
## 2018-01-19     178.46      90.00
## 2018-01-22     177.00      91.61
```

⋮

Manipulating Stock Data – R

```
stocks$msft_r <- ROC(stocks$MSFT.Close)
stocks$aapl_r <- ROC(stocks$AAPL.Close)
```

```
##            AAPL.Close MSFT.Close        msft_r        aapl_r
## 2018-01-12     177.09      89.60            NA            NA
## 2018-01-16     176.19      88.35 -0.0140491215 -0.0050950858
## 2018-01-17     179.10      90.14  0.0200578300  0.0163813730
## 2018-01-18     179.26      90.10 -0.0004438638  0.0008928955
## 2018-01-19     178.46      90.00 -0.0011104721 -0.0044727123
## 2018-01-22     177.00      91.61  0.0177307766 -0.0082147931
```

Two Position Portfolio – R

```
pareto.fit <-gpdFit(as.numeric(stocks$port_v[-1])*-1, type = c("mle"))
tailRisk(pareto.fit, prob = 0.99)
```

```
##      Prob      VaR      ES
## 95% 0.99 13214.64 15120.4
```

```
dollar(tailRisk(pareto.fit, prob = 0.99)[,2]*-1)
```

```
## [1] "$-13,214.64“
```

```
dollar(tailRisk(pareto.fit, prob = 0.99)[,3]*-1)
```

```
## [1] "$-15,120.40"
```

EVT: Two Position Portfolio

- Parameter estimates:

$$\xi = -0.387 \quad \beta = 4929.92$$

- VaR = $-\$13,214.64$, CVaR = $-\$15,120.40$

Comparison Across Techniques

Technique	Value at Risk	Expected Shortfall (CVaR)
Delta-Normal	$-\$10,351.69$	$-\$11,859.56$
Historical (Common)	$-\$12,024.55$	$-\$14,813.05$
Historical (Stressed)	$-\$18,639.23$	$-\$24,484.47$
Historical (Weighted)	$-\$13,329.83$	$-\$16,128.95$
Monte Carlo	$-\$8,234.93$	$-\$9,269.63$
Extreme Value Theory	$-\$13,214.64$	$-\$15,120.40$

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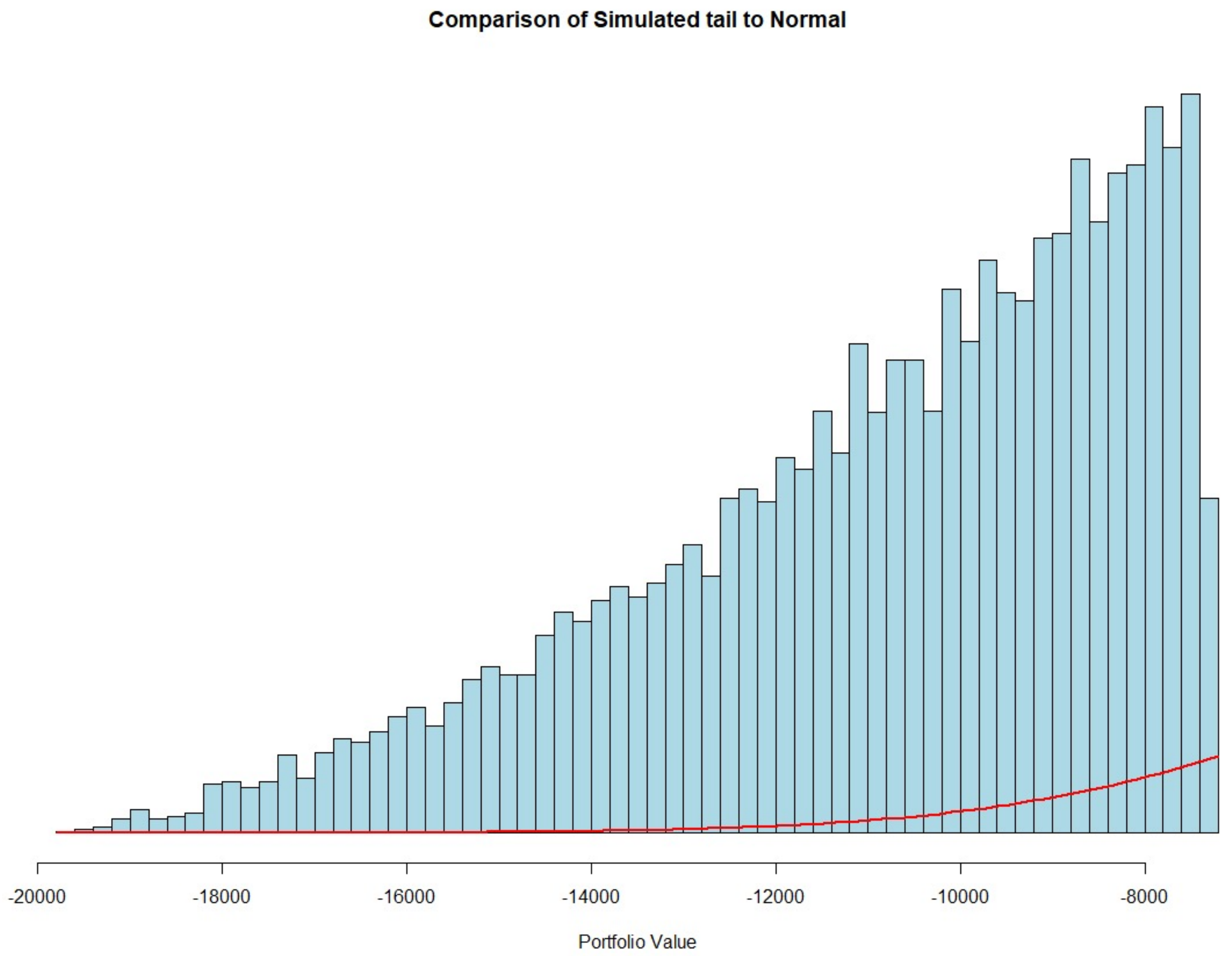
Two Position Portfolio – R

```
hist(-1*rgpd(n = 10000, mu = pareto.fit@parameter$u,
             beta = pareto.fit@fit$par.ests[2],
             xi = pareto.fit@fit$par.ests[1]),
     breaks = 50, main = "Comparison of Simulated tail to Normal",
     xlab = "Portfolio Value", freq = FALSE, yaxt = "n", ylab = "",
     col = "lightblue")

curve(dnorm(x, mean(stocks$port_v, na.rm = TRUE),
            sd(stocks$port_v, na.rm = TRUE)),
      add = TRUE, col = 'red', lwd = 2)
```

Two Position Portfolio – R



EXTENSIONS TO VALUE AT RISK

VaR “like” Extensions

- There have been many additions to Value at Risk calculations:
  1. Delta – Gamma VaR
  2. Delta – Gamma Monte Carlo Simulation
  3. Incremental VaR
  4. Component VaR
  5. Backtesting VaR

Delta – Gamma Approximation

- Improve the Delta – Normal approach by taking into account higher derivatives than the first derivative only.
- In the expansion of the derivatives we can look at the first two:

$$dV = \frac{\partial V}{\partial RF} \cdot dRF + \frac{1}{2} \cdot \frac{\partial^2 V}{\partial RF^2} \cdot dRF^2 + \cdots$$

$$dV = \Delta \cdot dRF + \frac{1}{2} \cdot \Gamma \cdot dRF^2$$

Delta – Gamma Approximation

- Improve the Delta – Normal approach by taking into account higher derivatives than the first derivative only.
- In the expansion of the derivatives we can look at the first two:

$$dV = \Delta \cdot dRF + \frac{1}{2} \cdot \Gamma \cdot dRF^2$$

- Essentially takes the variance component into account in building the Gamma portion of the equation.
- Delta – Normal typically **underestimates** VaR when assumptions do not hold.
- Delta – Gamma tries to correct for this, but is computationally more intensive.

Delta – Gamma Monte Carlo

- The Normality assumption might still be unreliable.
- The Delta – Gamma Monte Carlo simulation technique simulates the risk factor before using the second order Taylor expansion to create simulated movements in the portfolio's value.
- Just like with regular simulation, the VaR is calculated using the simulated distribution of the portfolio.

Delta – Normal vs. Delta – Gamma

- If you have a large number of things to evaluate in a portfolio with few options, the Delta – Normal is fast and efficient enough.
- If you have only a few sources of risk and substantial complexity, the Delta – Gamma is better.
- If you have a large number of risk factors and substantial complexity, the traditional Monte Carlo simulation is still best.

Marginal VaR

- How much will the VaR change if we invest one more dollar in a position?
- In other words, the first derivative of VaR with respect to the weight of a certain position in the portfolio.
- Assuming Normality:

$$\Delta VaR_i = \alpha \times \frac{\mathrm{Cov}(R_i, R_p)}{\sigma_p}$$

Incremental VaR

- The change in VaR due to the addition of a new position in the portfolio.
- Used whenever an institution wants to evaluate the effect of a proposed trade or change in the portfolio of interest.
- How is this different than Marginal VaR?
  - The change can be significant (not marginal – unit of one)
  - The change can be in a vector of positions, not just one.

Incremental VaR

- Trickier to calculate as you now need to calculate VaR under both portfolios and take their difference.
- You can do a Taylor series expansion to get an approximation:

$$IVaR = (\Delta VaR)^T \cdot c$$

- ΔVaR is a vector of marginal VaR's
- c is a vector of additional exposures in our portfolio
  - Example – $10,000 in bonds and $20,000 in options
- The approximation is quicker to calculate then taking the difference of the two correct calculations, but is not as accurate.

Component VaR

- Decompose VaR into its basic components in a way that takes into account the diversification of the portfolio.
- It is a "partition" of the portfolio VaR that indicates the change of VaR if a given component was deleted.
- The Component VaR is defined in terms of marginal VaR:

$$\text{Component } VaR = \Delta VaR_i \times (\$ \text{ value of component i})$$

- The sum of all component VaR's is the total portfolio VaR.