"WHAT LIES BEHIND YOU AND WHAT LIES IN FRONT OF YOU, PALES IN COMPARISON TO WHAT LIES INSIDE OF YOU."
— **RALPH WALDO EMERSON**

BAYESIAN STATISTICS

CLASS 2

WHAT DID WE LEARN FROM CLASS 1?

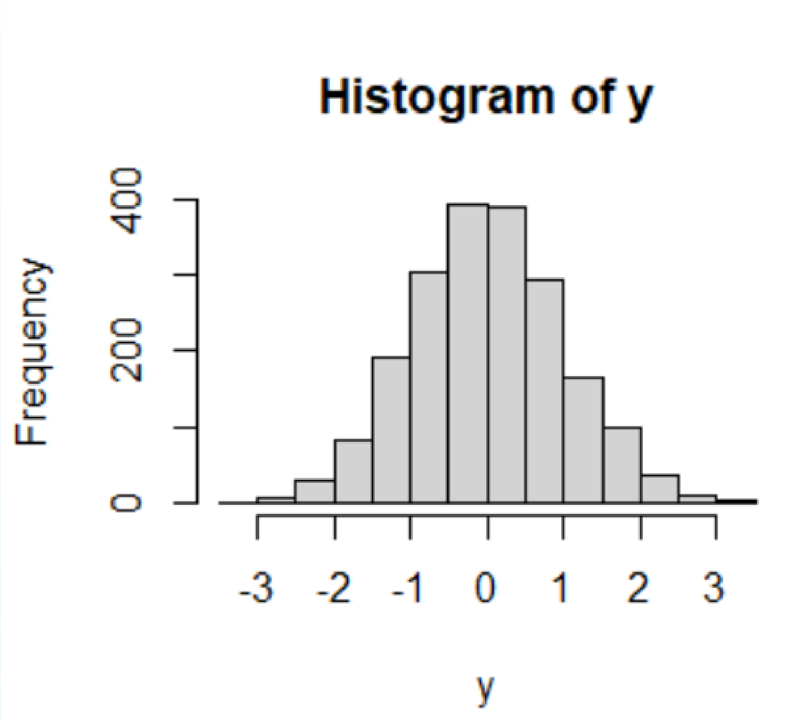
- Terminology: Prior, Sampling distribution, Posterior
- How to define problem (decide sampling distribution of data, define priors for parameters, use Stan to generate posterior distribution of parameters)
- How to use posterior to answer questions about the parameter
- How data (sample size) and prior contribute to the posterior
- Why prior is VERY important when sample size is small

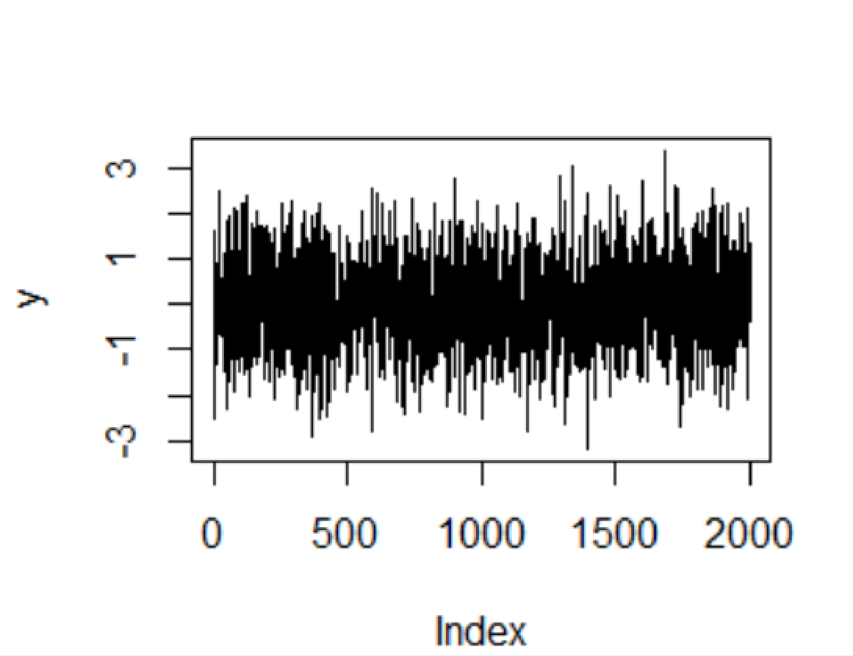
GOALS FOR TODAY

- MCMC - Markov Chain Monte Carlo
  - What it is
  - Has it converged
  - Options to help convergence
- Options in running MCMC to get posterior distribution
- Another in-class example

SIMULATING A DISTRIBUTION

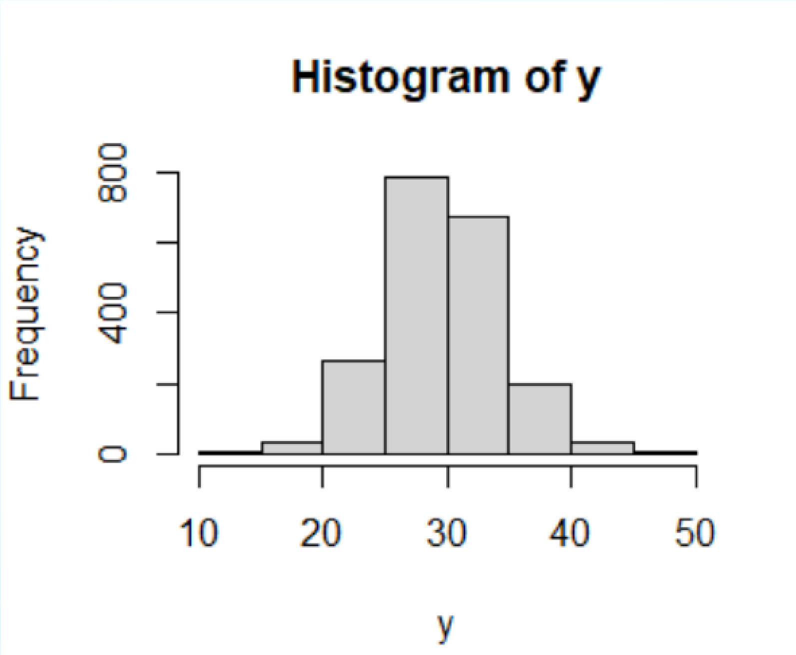
```
y=rnorm(2000)
> hist(y)
> plot(y,type='l')
```

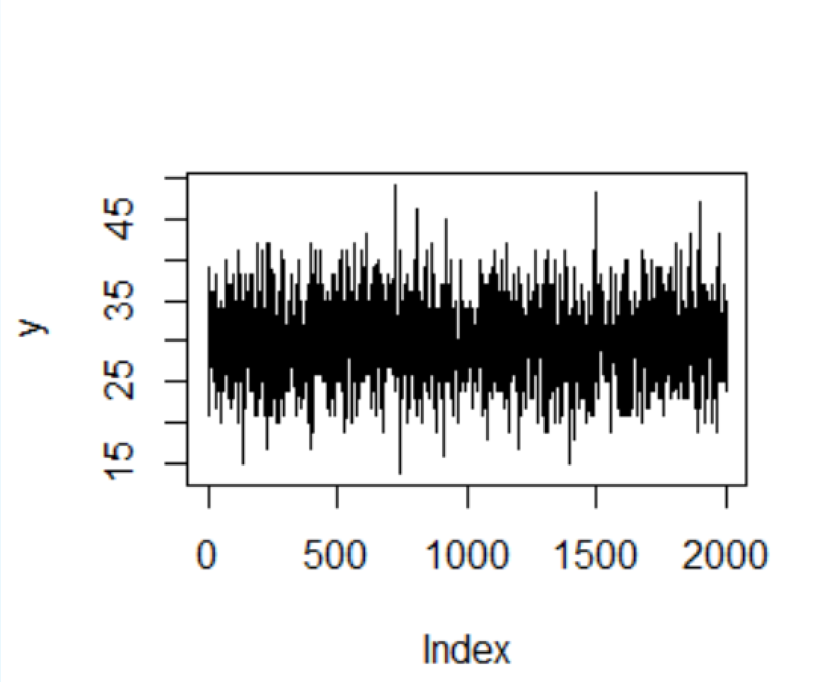




ANOTHER DISTRIBUTION

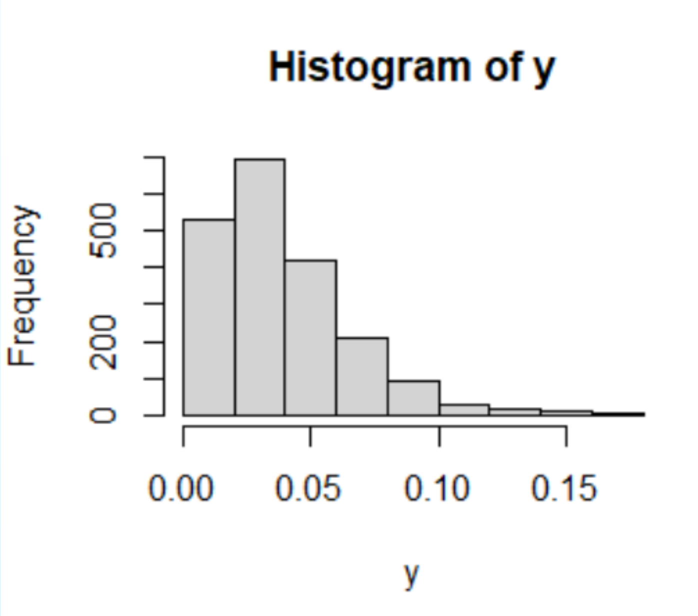
```
> y=rbinom(2000,100,0.3)
> hist(y)
> plot(y,type='l')
```

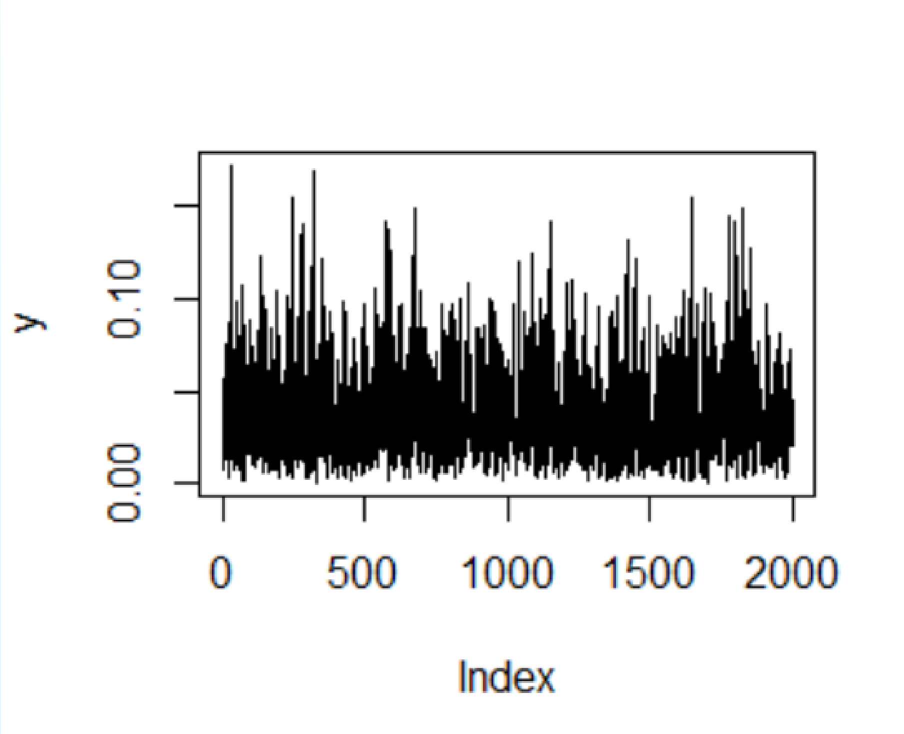




SKEWED DISTRIBUTION

```
> y=rbeta(2000,2,50)
> hist(y)
> plot(y,type='l')
```





MARKOV CHAIN MONTE CARLO

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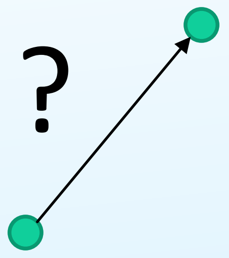
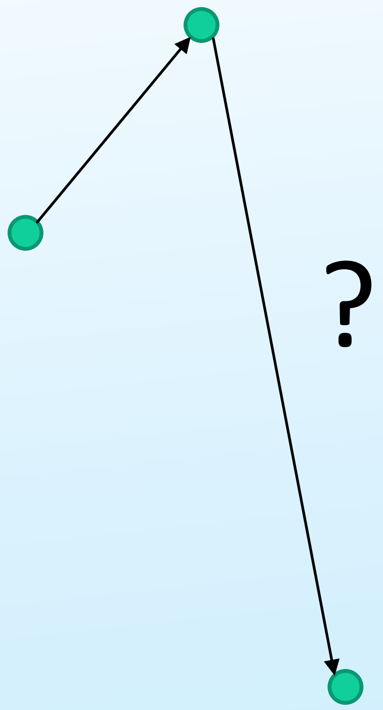
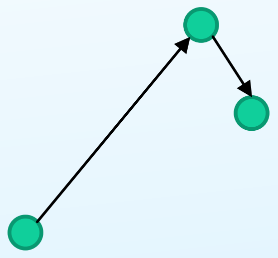


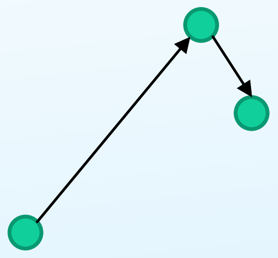
MARKOV CHAIN MONTE CARLO



MARKOV CHAIN MONTE CARLO



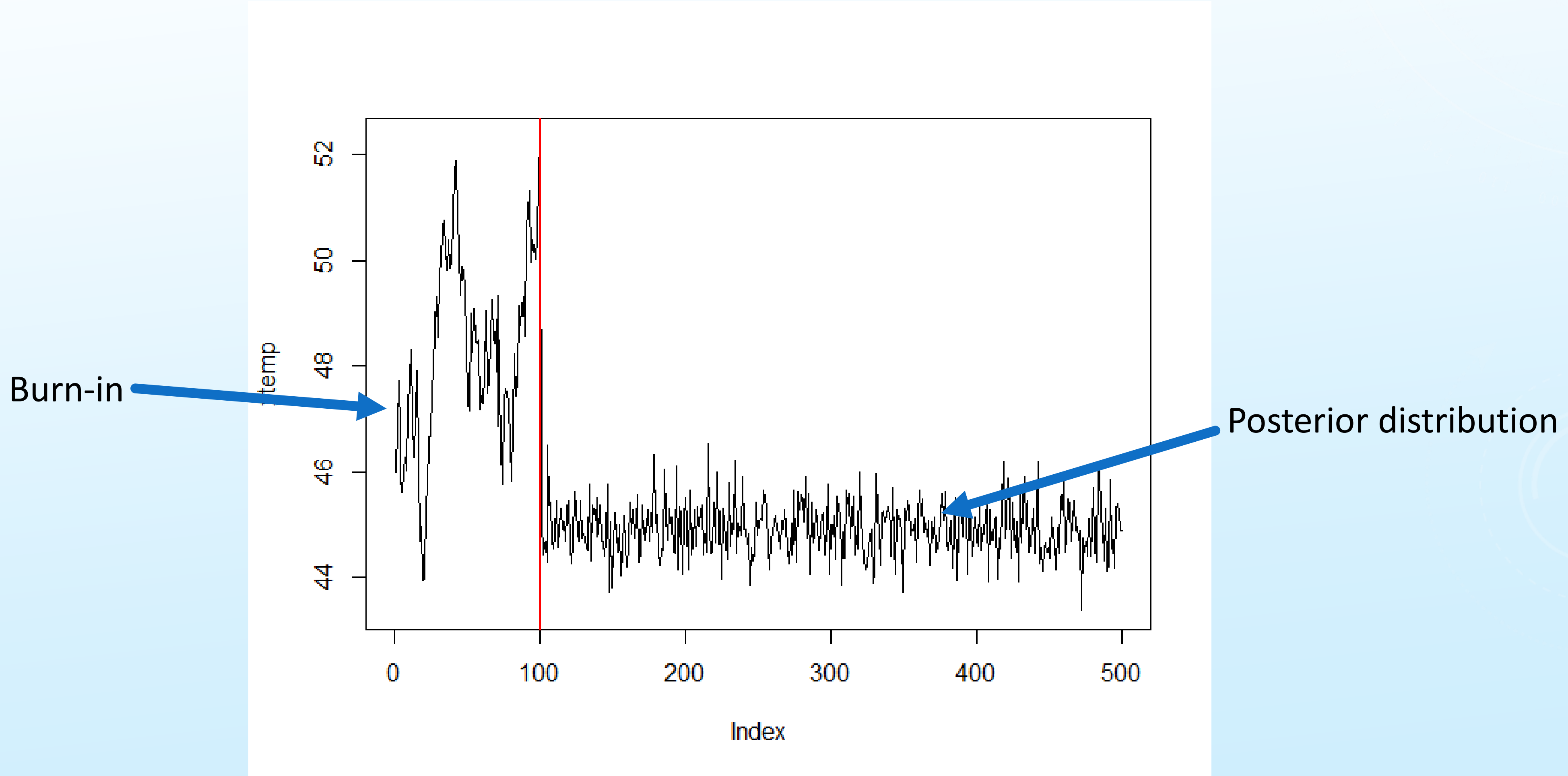
MARKOV CHAIN MONTE CARLO



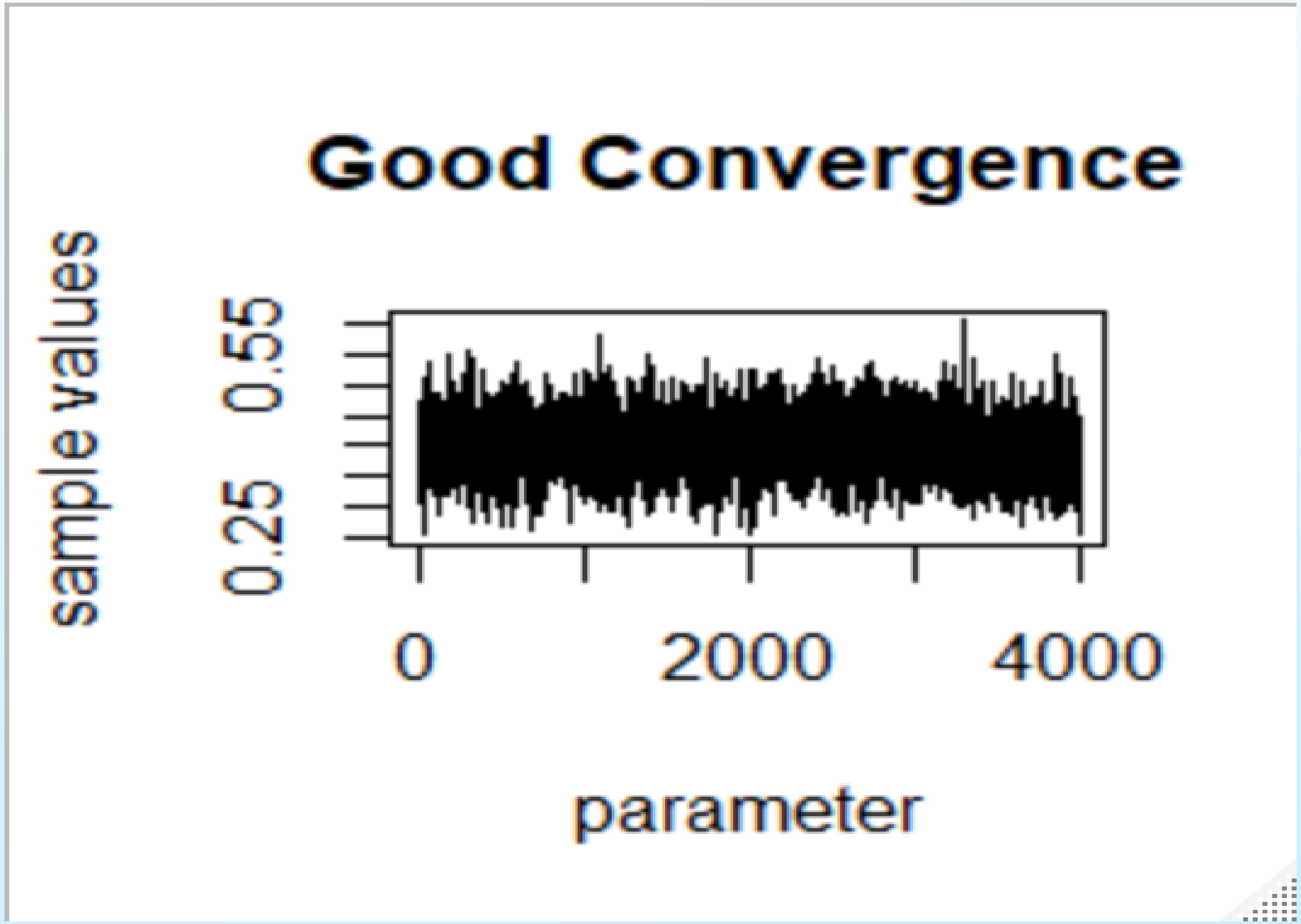
Stan uses the Hamiltonian Monte Carlo method for its Markov Chain and its adaptive variant the no U-turn sampler (NUTS). For more details, see https://mc-stan.org/docs/2_19/reference-manual/hamiltonian-monte-carlo.html

In Python: Stan, PyMC3

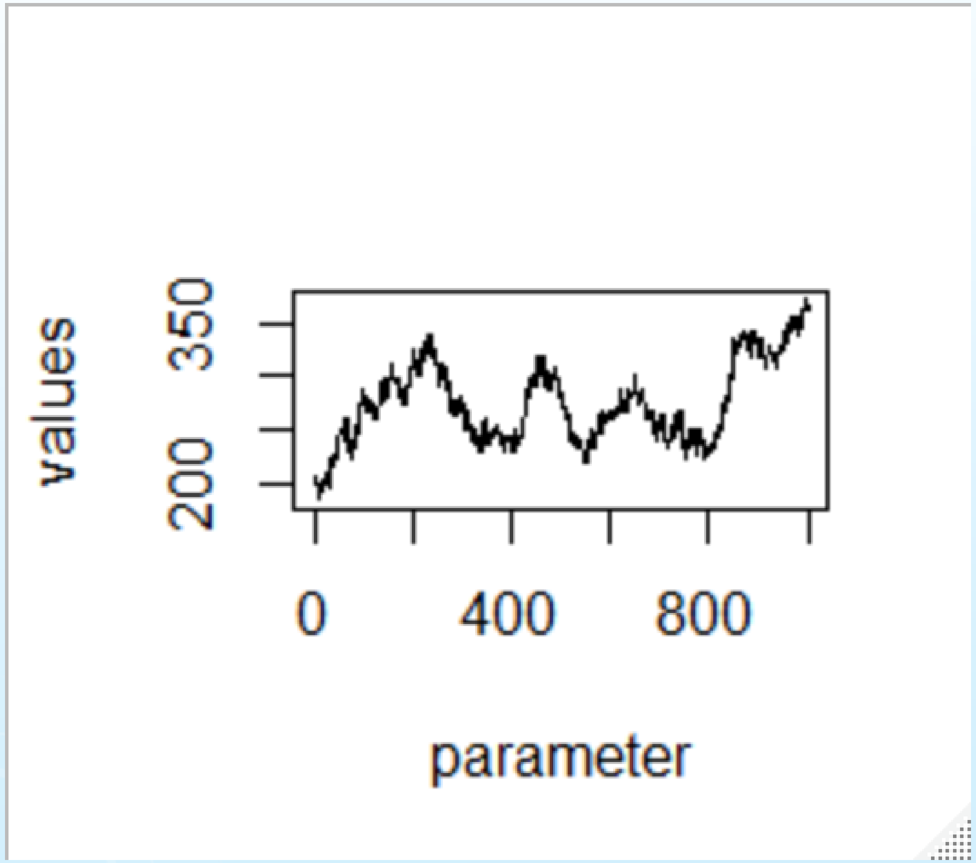
MCMC

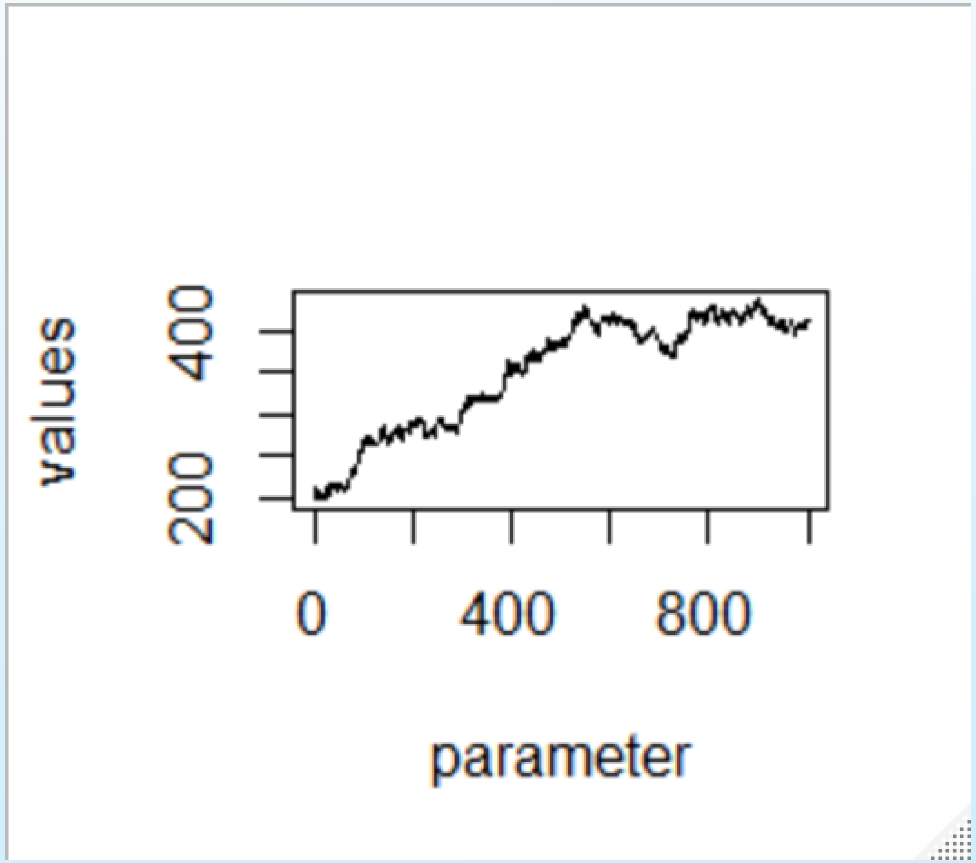

Burn-in
Posterior distribution
temp
52
50
48
46
44
0
100
200
300
400
500
Index

CONVERGENCE



NONCONVERGENCE





FIXES

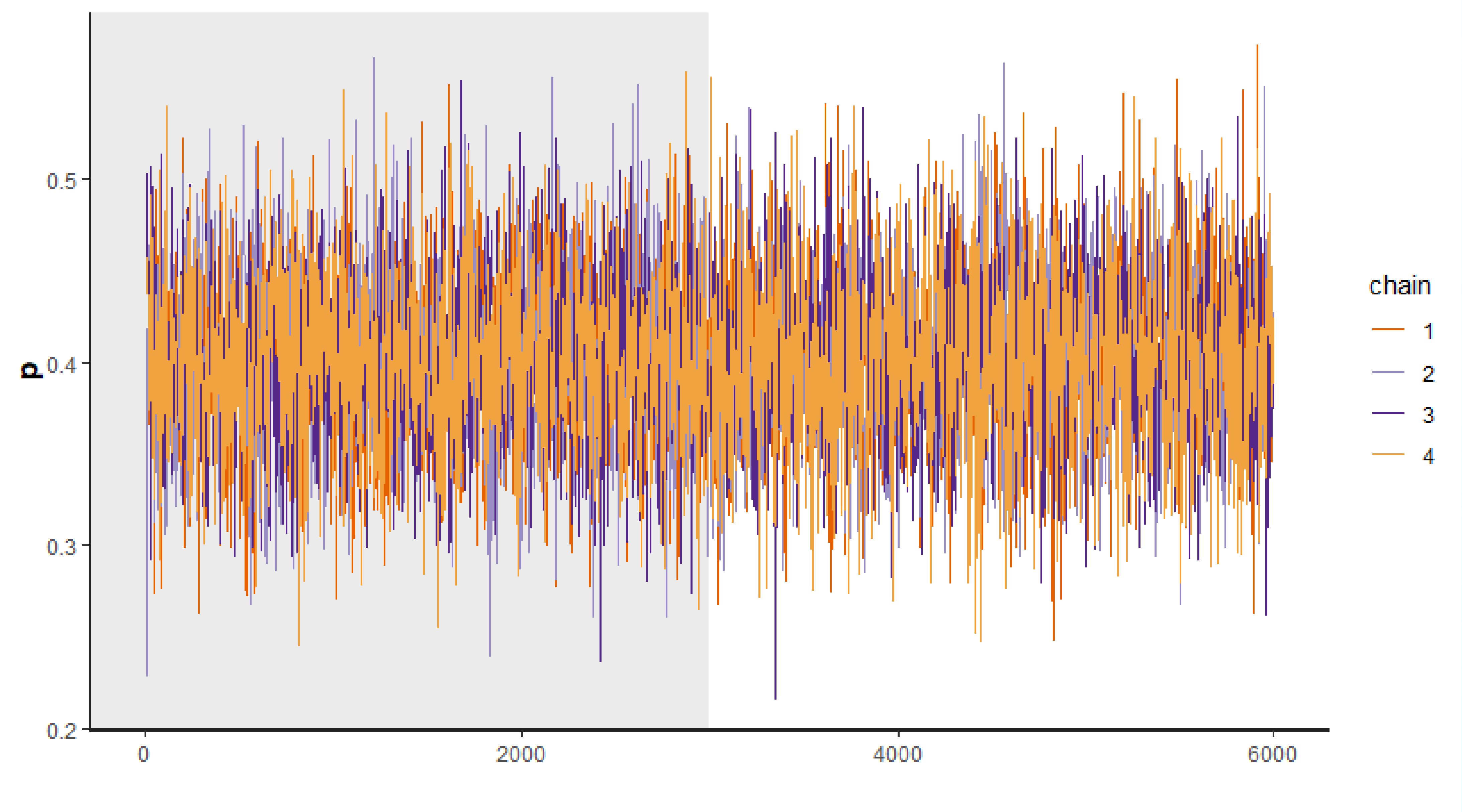
- Improper posterior or bad prior
    - Fix: New prior distribution
- Hasn't converged yet
    - Let the chain run longer
- Chain continues to increase
    - Potentially a bad starting point…provide a new starting point (or change prior)
- Too much autocorrelation in chain
    - Thin the chain

OPTIONS FOR STAN CODE

```
binom.stan=stan(model_code = ex1,data=binom.data,seed=98763,
chains = 4,      # number of Markov chains
warmup = 3000,   # number of warmup iterations per chain
iter = 6000,     # total number of iterations per chain
refresh = 0,     # no progress shown
thin=3,                    # will 'thin' the chains.help with autocorrelated posterior samples
init=0.3                   #specify initial values)
```

Creates four chains; each chain has 6000 values, however, only every 3rd value is taken (now down to 2000 per chain that is useful); first 3000 (well, actually only 1000 since we are thinning) is burn-in meaning it is not used

End result will have a total of 4000 posterior values (1000 from each chain) traceplot(binom.stan, inc_warmup = TRUE)



DIAGNOSTICS

```
print(binom.stan, pars=c("p", "lp__"), probs=c(.1,.5,.9))
```

	mean	se_mean	sd	10%	50%	90%	n_eff	Rhat
p	0.40	0.00	0.05	0.34	0.40	0.46	3268	1
lp__	-69.23	0.01	0.72	-70.10	-68.96	-68.74	3290	1

Log posterior value (unnormalized)

Want these to be close to 1 (this means convergence); if greater than 1.1, then there could potentially be a problem (Rhat or potential scale reduction)

DIAGNOSTICS

```
print(binom.stan, pars=c("p", "lp__"), probs=c(.1,.5,.9))
```

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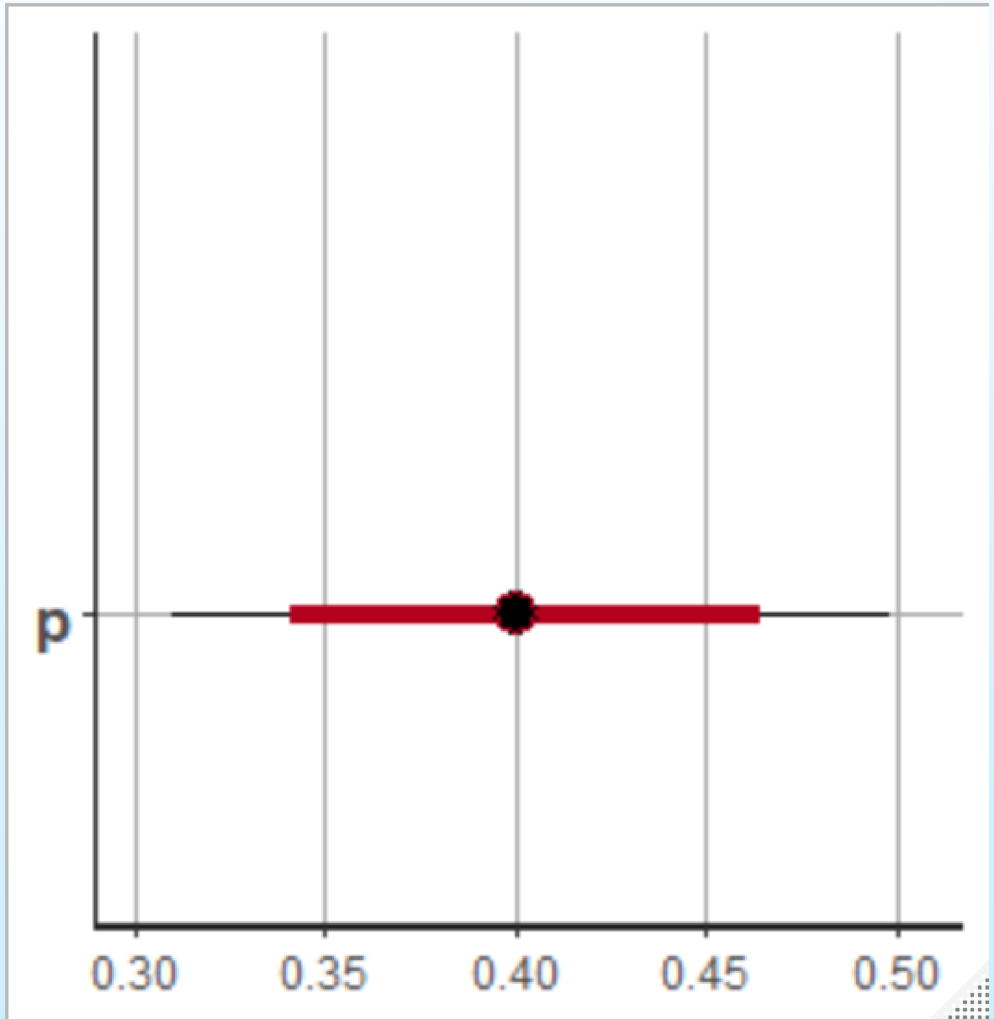
Log posterior value (unnormalized)

Effective sample size…bigger is better (more than 1000 is fine). If too small, would recommend trying to thin chains.

PROBABILITY INTERVAL

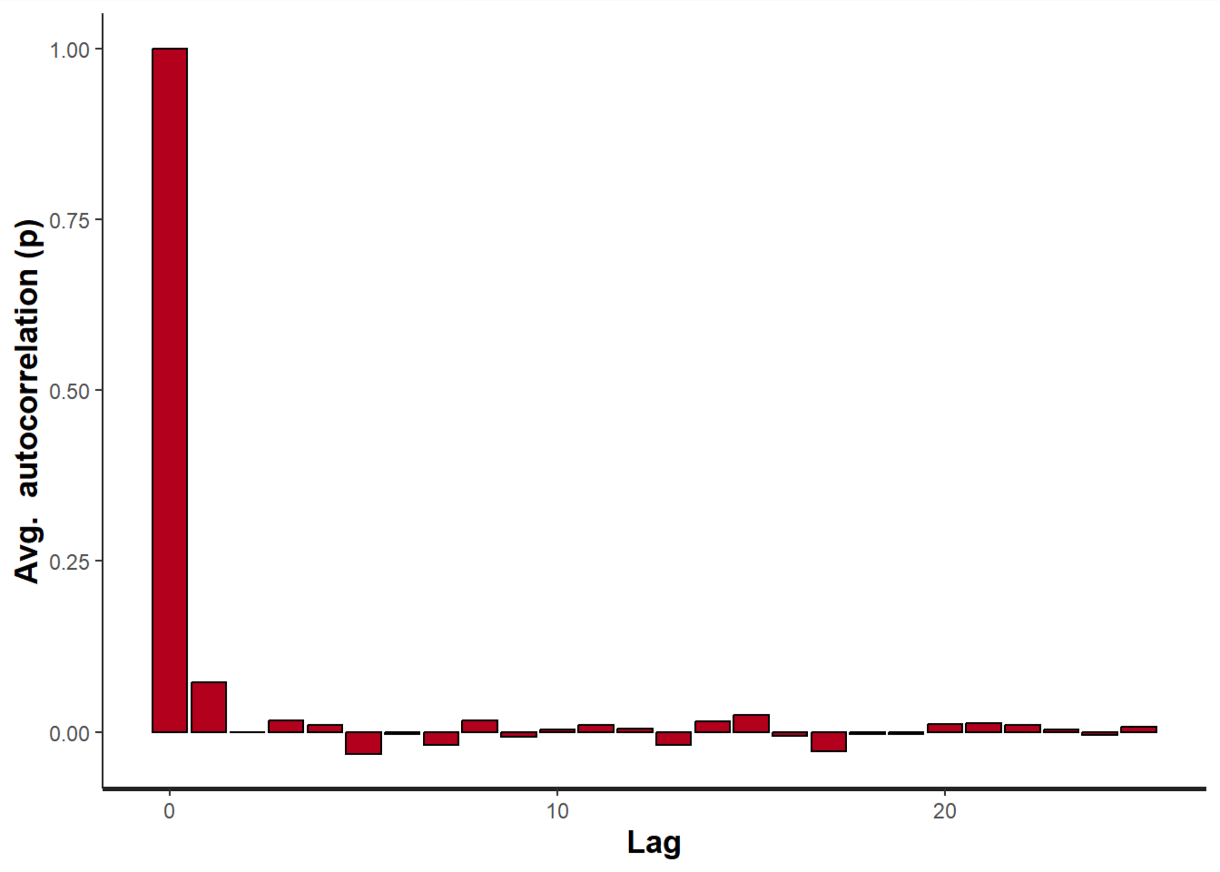
plot(binom.stan)



Inner is 80% probability interval
Outer is 95% probability interval

CHECK AUTOCORRELATION

stan_ac(binom.stan,pars=c("p"))



ANOTHER POTENTIAL WARNING

Warning: There were 2 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help. See http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

Fix: add this line to your options…

```
control=list(adapt_delta=0.9)
```

ESTIMATE VALUE AT RISK

- Recall from Simulation and Risk calculating Value at Risk (VaR)
- Say we are interested in estimating VaR for Apple stock (AAPL) in Rate of Change (ROC) for one day
- If we assume Rate of Change (R_t) follows a Normal distribution with mean μ and standard deviation σ
- We then need to assign a distribution to μ and σ^2
  - Assume μ is distributed as Normal(0,100)
  - Assume σ^2 is distributed as Inv-Gamma(0.001,0.001)
- Once we get posterior for μ and σ^2, we can use this to get the 1st quantile

DATA

```
library(quantmod)
tickers = c("AAPL")
getSymbols(tickers)
AAPL$aapl_r <- ROC(AAPL$AAPL.Close)
AAPL <- cbind(tail(AAPL$AAPL.Close, 500), tail(AAPL$aapl_r, 500))
```

STAN CODE

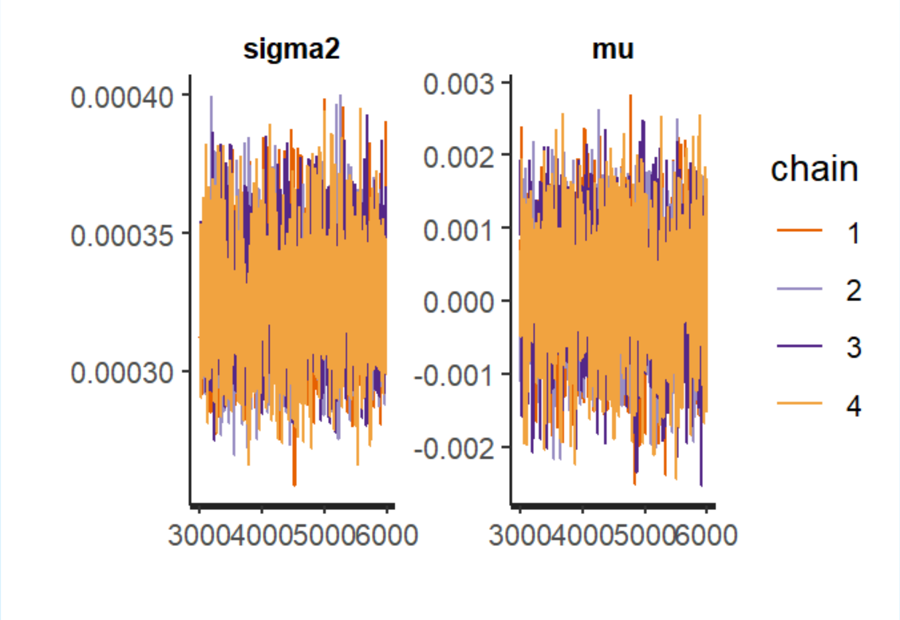
```
ex2 <- “
data {
	int <lower=0> n;
	real y[n];
}
parameters {
	real <lower=0> sigma2;
	real mu;
}
model {
	mu ~ normal(0,100);
	y ~ normal(mu,sigma);
sigma2 ~ inv_gamma(0.001,0.001);
}
"
```

RUN STAN CODE

```
stock.data=list(n=nrow(stocks), y=as.vector(stocks$aapl_r))

stock.stan=stan(model_code = ex2,data=stock.data,seed=15893, chains = 4, warmup = 3000, iter = 6000,
refresh = 0, thin=3)
```

CHECK CONVERGENCE



traceplot(stock.stan,inc_warmup=F)

print(stock.stan, pars=c("mu", "sigma2"), probs=c(.1,.5,.9))

	mean	n_eff	Rhat
mu	0	3995	1
sigma2	0	2986	1

*** Values are VERY small!

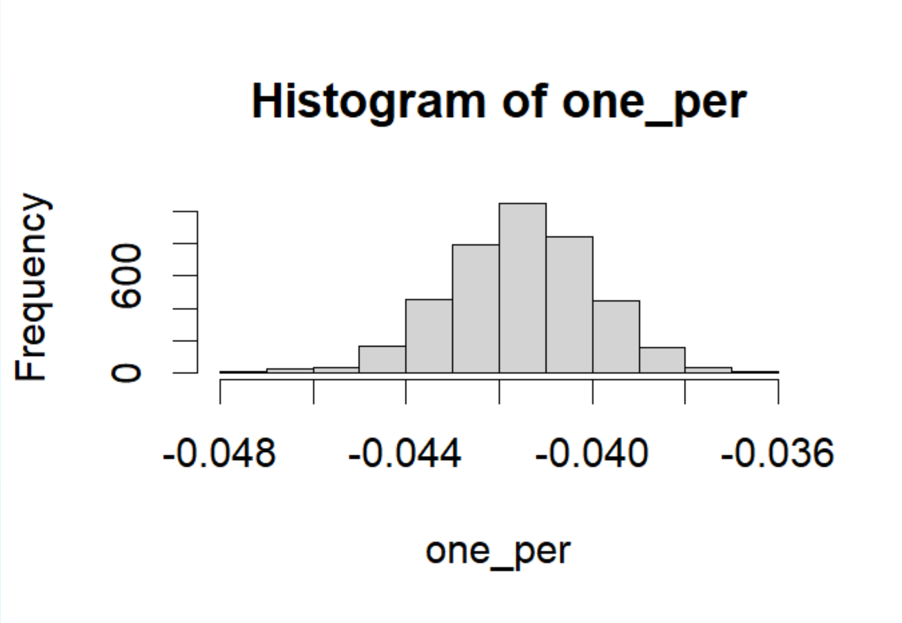
USING POSTERIOR INFORMATION:

Posterior samples

```
post.params=extract(stock.stan)
new.mu=post.params$mu
new.sigma2=post.params$sigma2
head(cbind(new.mu,new.sigma2))
          new.mu   new.sigma2
[1,] -0.0003626559 0.0003317394
[2,] -0.0004120245 0.0003196366
[3,] -0.0008917359 0.0003210392
[4,]  0.0003888528 0.0003505453
[5,]  0.0003630797 0.0003195843
[6,] -0.0001102442 0.0003167621
```

VaR ROC at 99% confidence

- What we need is the 1st percentile for ROC:
- one_per=qnorm(0.01,new.mu,sqrt(new.sigma2))



USE THIS INFO FOR IN-CLASS ASSIGNMENT