ARCH & GARCH MODELS

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History

- Until 1980's: Econometrics focused almost solely on modeling the mean of the series (actual values of the target variable)
- Mid-1980's to now: Increased focus on volatility, what influences volatility and volatility's effect on the mean values.

- "One of the funny things about the stock market is that every time one person buys, another sells, and both think they are astute."
  - William Feather

Unconditional vs. Conditional Variance

- Unconditional variance is the same standard variance calculation that we have done in the past:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sigma^2 = E\left(x - E(x)\right)^2$$

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$$\sigma^2 = E\left(x - E(x)\right)^2$$

- Conditional variance is the measure of our uncertainty about a variable given a set of information (or data).
  - Heteroscedasticity – variance depends on external factors

$$\sigma^2_{cond} = E(x - E(x|\boldsymbol{I})\ )^2$$

Heteroscedasticity

- Variance depends on external factors.
- Cross-sectional data:

$$Var(\varepsilon_i | \boldsymbol{x}_i) = \sigma_i^2$$

  - Heteroscedasticity is a nuisance we try to avoid or correct.

- Time series data:

$$Var(\varepsilon_t | \boldsymbol{I}_t) = \sigma_t^2$$

  - Heteroscedasticity is of interest, especially in finance, and we desire to model it!

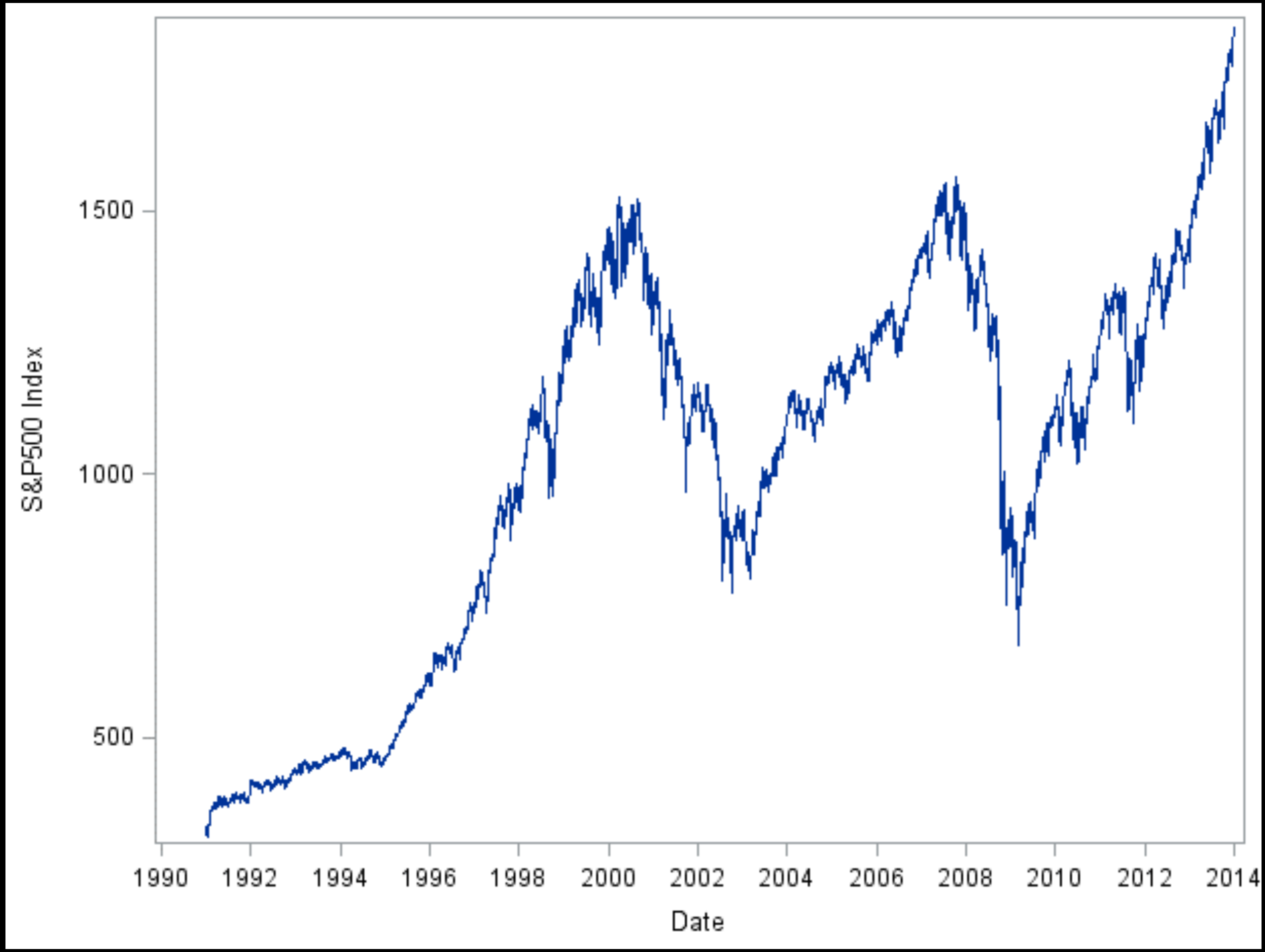
WHY DO WE MODEL VOLATILITY?

Facts About Financial Time Series

- Non-stationarity of prices.
- Mean-reversion of the returns of the series.

Facts About Financial Time Series

- Non-stationarity of prices.



Facts About Financial Time Series

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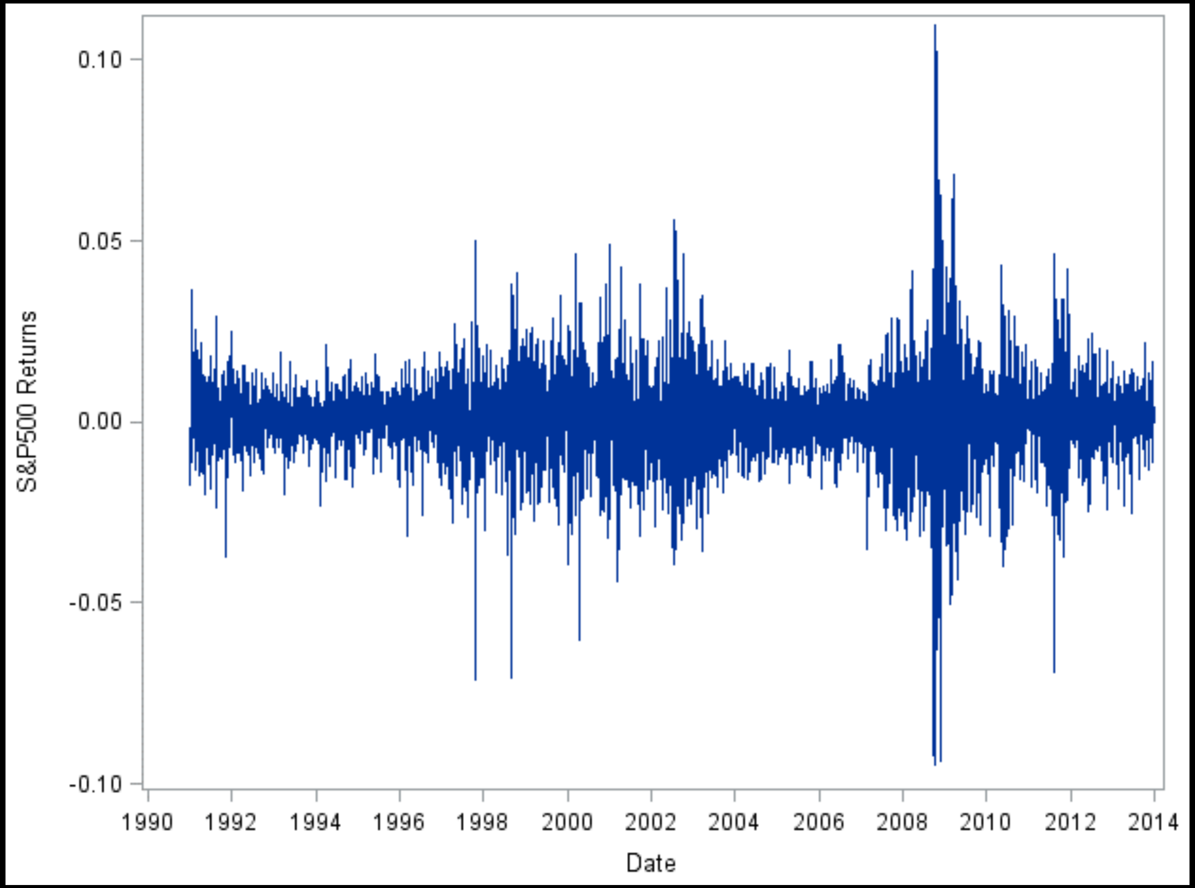
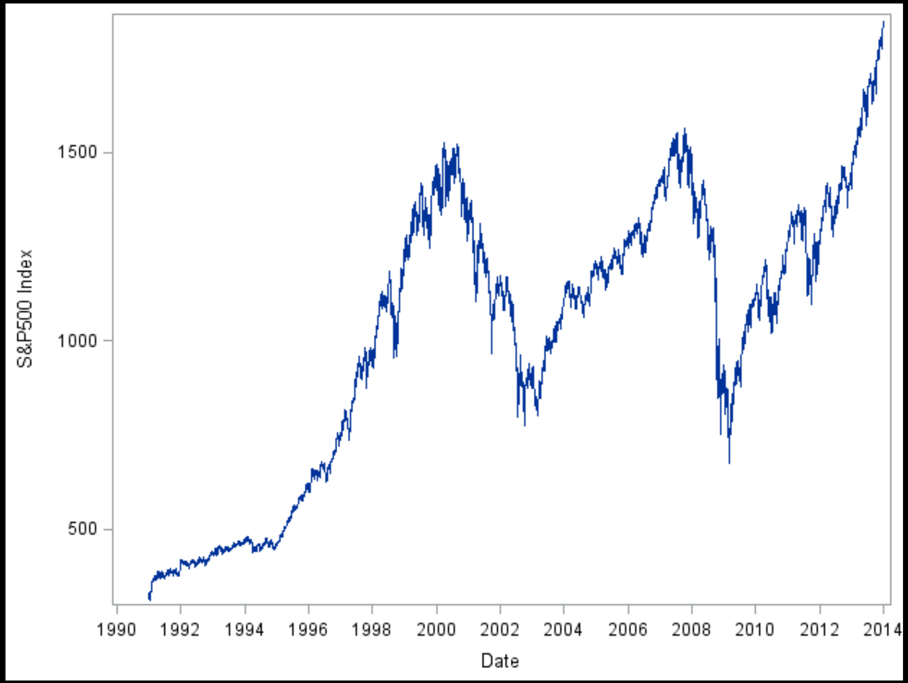
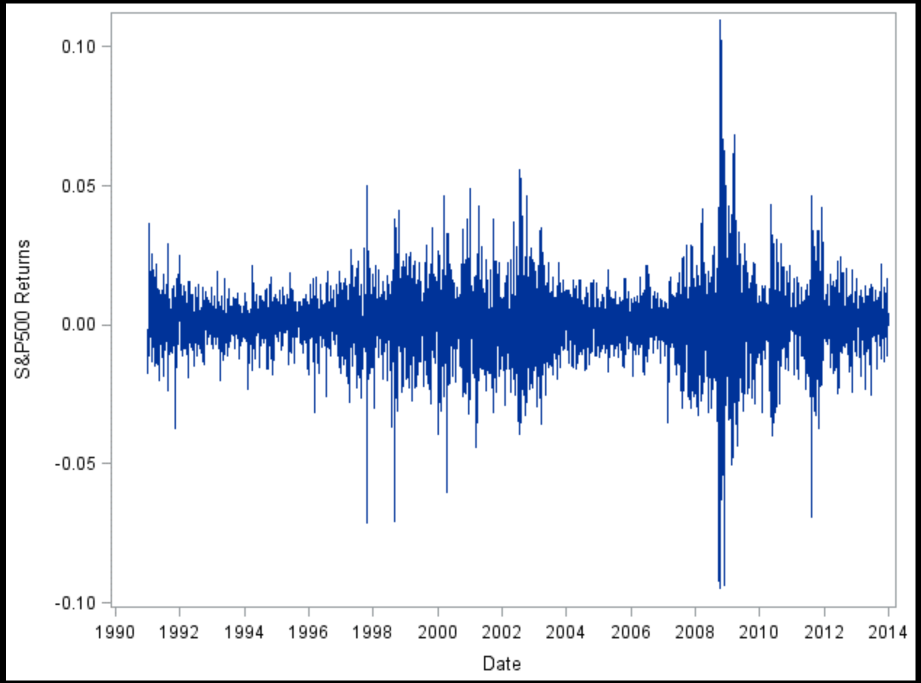


Facts About Financial Time Series





Facts About Financial Time Series

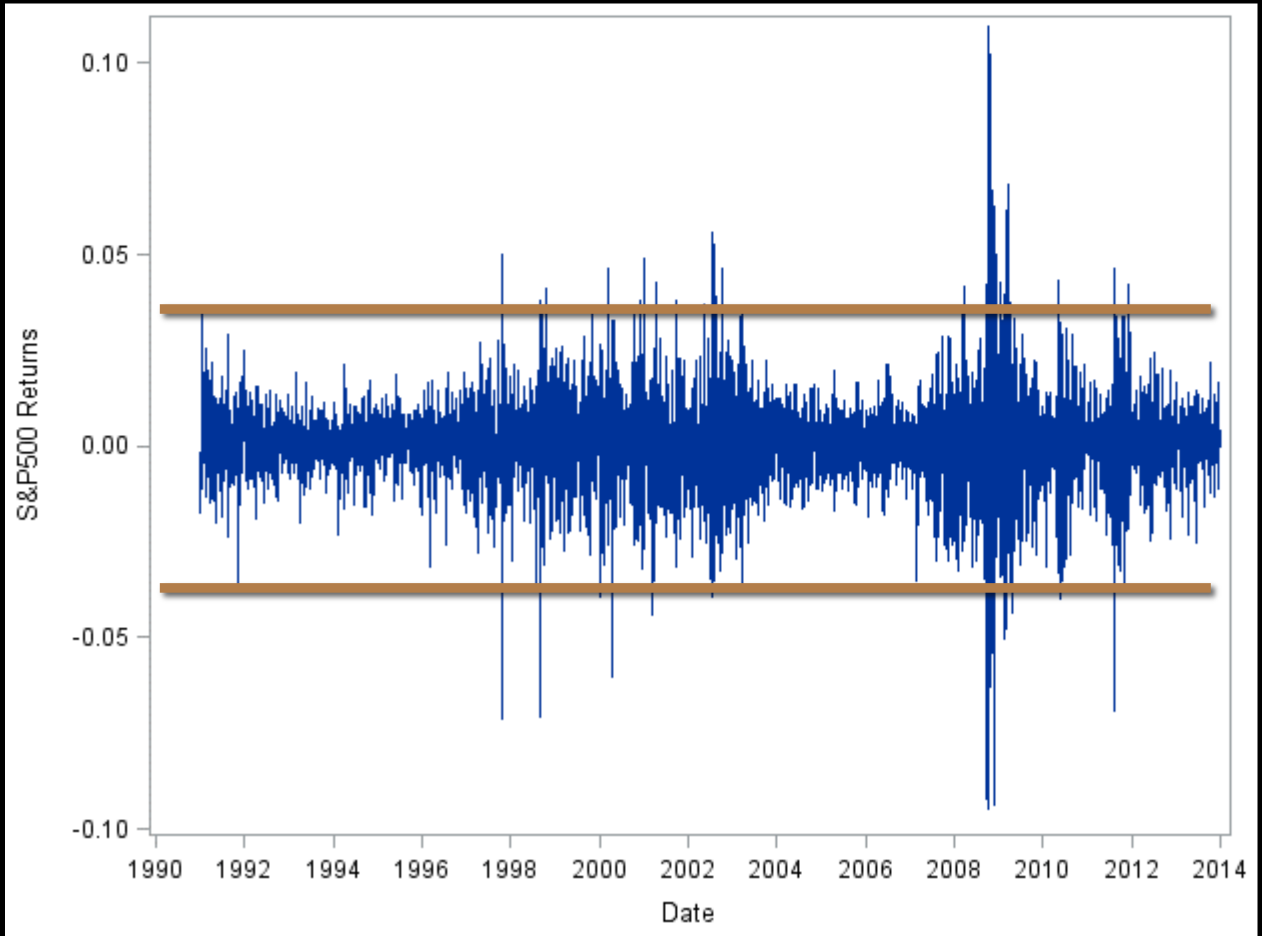
- Non-stationarity of prices.
- Mean-reversion of the returns of the series.

- THIS MAKES IT HARD TO GET INFORMATION FROM FORECASTING MARKET!!!

Facts About Financial Time Series

- **Thick tails** – more outliers than what the Normal distribution would suggest.
- **Volatility clustering** – large changes tend to be followed by large changes.
- **Leverage effects** – tendency for changes in stock prices to be negatively correlated with changes in volatility.
- **Non-trading period effects** – information accumulates at a different rate when market is closed as compared to when it is open.
- **Co-movements in volatility** – volatility is positively correlated across assets in a market and even across markets.

Constant Volatility?



Applications

- Estimating the Value at Risk
- Optimizing Allocations of Assets
- Hedging Risk
- Pricing Multiple Assets in an Option

HOW TO MODEL VOLATILITY?

Simple Approaches

How Can We "Model" Variance?

- Need to lay the foundation of how one can model variance over time.
- What is a reasonable model for Y, the actual price?

$$Y_t = \beta_0 + Y_{t-1} + \varepsilon_t$$

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- How do we get returns?

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

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- How do we get returns?

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\beta_0}{Y_{t-1}} + \frac{\varepsilon_t}{Y_{t-1}}$$

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- What is a reasonable model for Y, the actual price?

$$Y_t = \beta_0 + Y_{t-1} + \varepsilon_t$$

- How do we get returns?

Still a Constant

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

Still Normal

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \beta_0^* + \varepsilon_t^*$$

How Can We "Model" Variance?

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- What is a reasonable model for Y, the actual price?

$$Y_t = \beta_0 + Y_{t-1} + \varepsilon_t$$

- How do we get returns?

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

$$r_t = \beta_0^* + \varepsilon_t^*$$

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- How do we get returns?

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

$$r_t = \beta_0^* + \varepsilon_t^*$$

Intercept only model, $\beta_0 = \bar{Y} \approx 0$

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$$Y_t = \beta_0 + Y_{t-1} + \varepsilon_t$$

- How do we get returns?

$$Y_t - Y_{t-1} = \beta_0 + \varepsilon_t$$

$$r_t = \varepsilon_t^*$$

$$r_t \sim N(0, \sigma^2)$$

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$$Y_t = \beta_0 + Y_{t-1} + \varepsilon_t$$

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$$r_t = \varepsilon_t^*$$

$$r_t \sim N(0, \sigma^2)$$

MODEL THIS!

Or…

- What IF we COULD model price?
- How would this change our model?
- All you need to do is model the RESIDUALS!

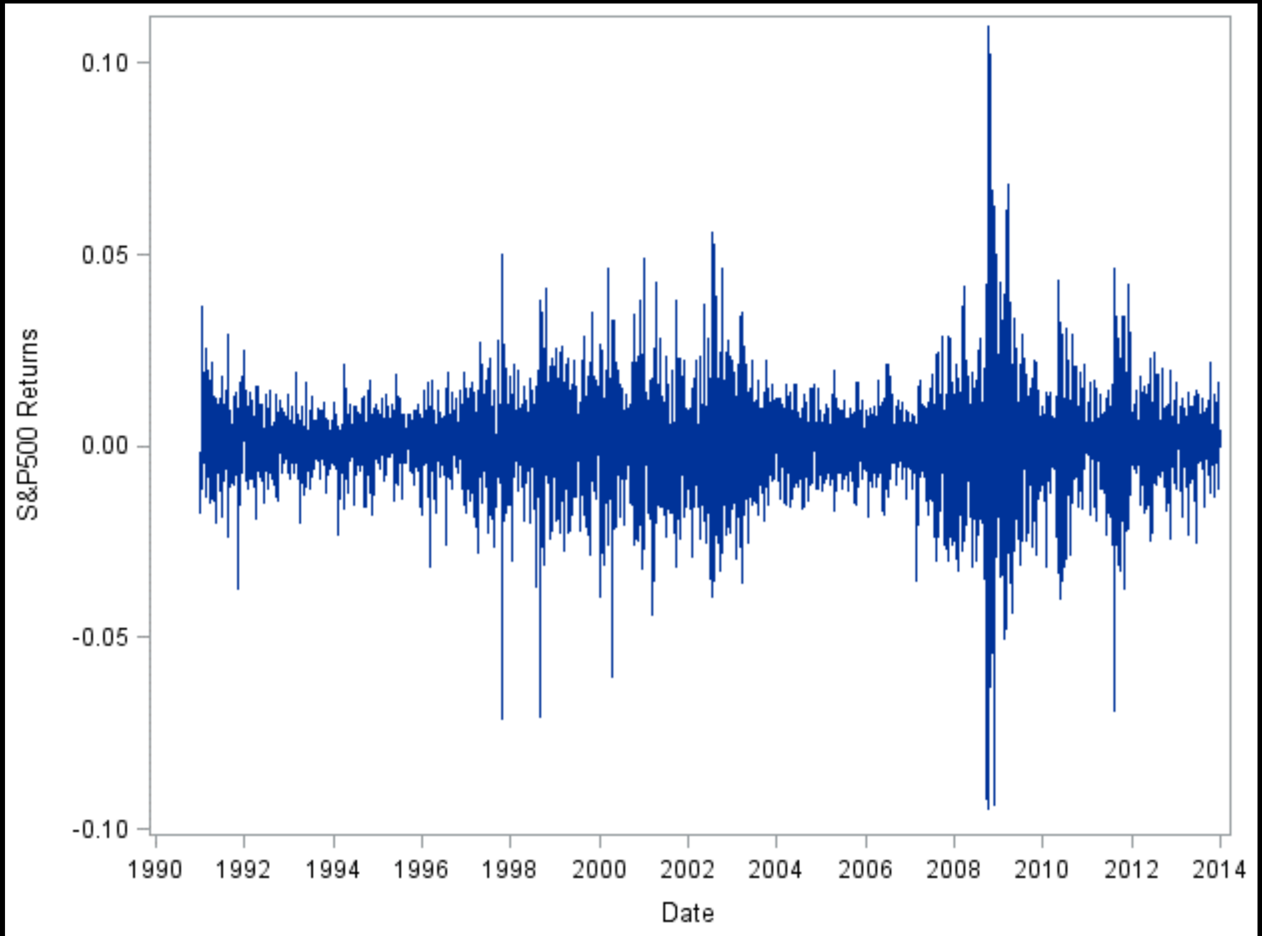
$$Y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

$$Y_t - \left(\hat{\beta}_0 + \hat{\beta}_1 x_{1,t} + \cdots + \hat{\beta}_k x_{k,t}\right) = \hat{\varepsilon}_t$$

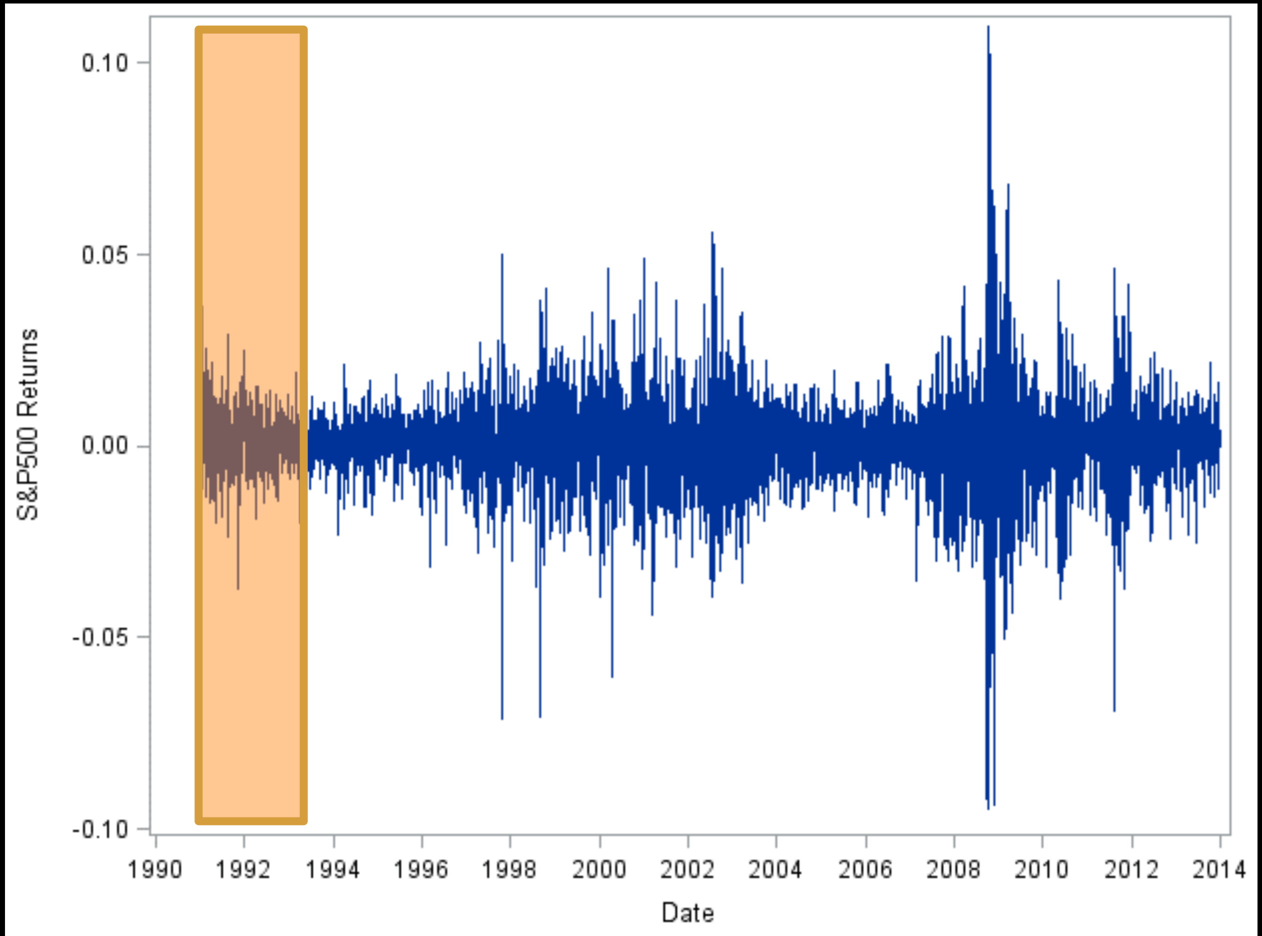
MODEL THIS!

$$\hat{\varepsilon}_t \sim N(0, \sigma^2)$$

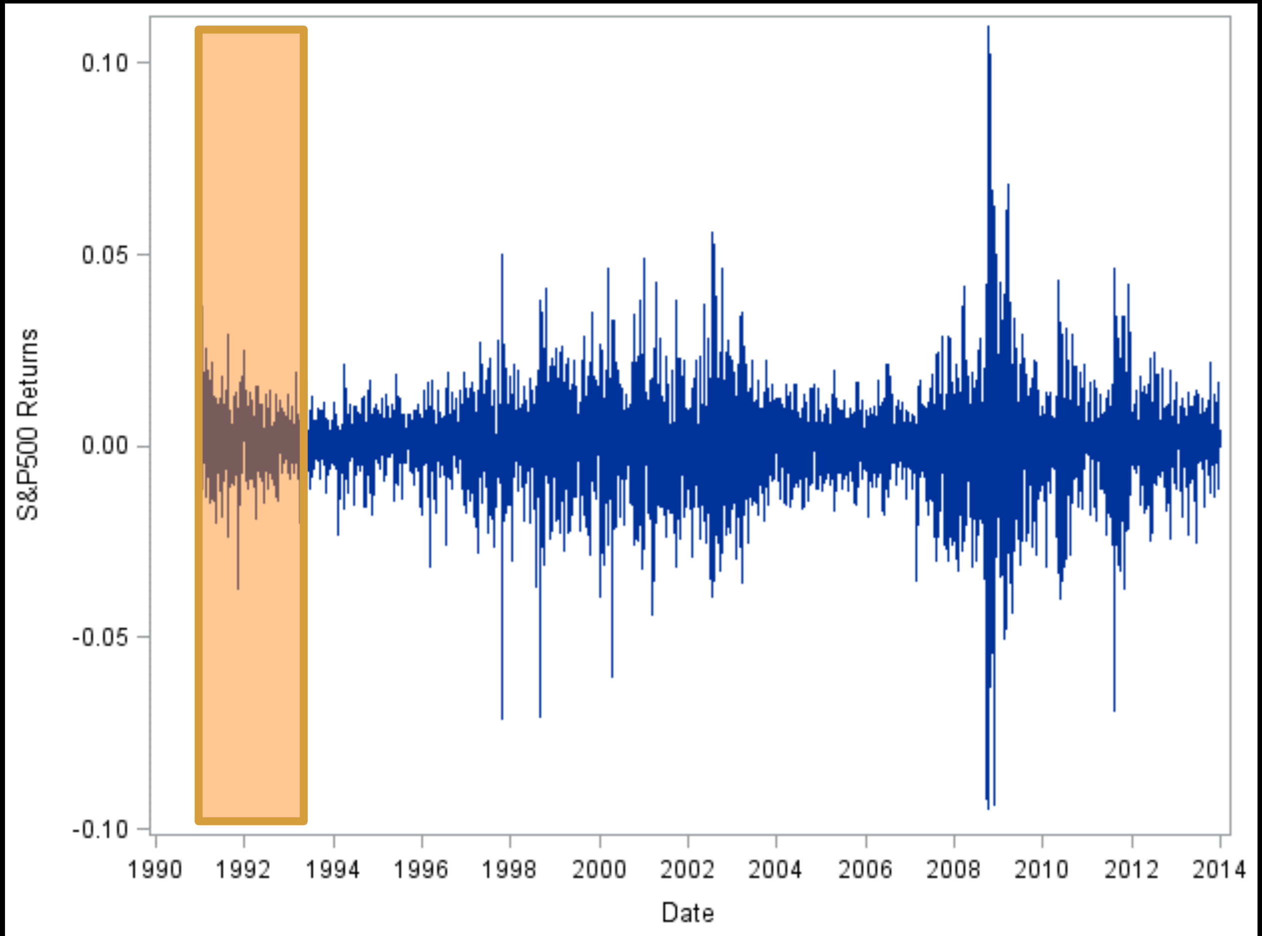
Rolling Window Calculation



Rolling Window Calculation



Rolling Window Calculation



Weighting Time Periods

- Why not weight more recent observations heavier than previous ones?
- Exponential Smoothing Models

$$\sigma_{t+1}^2 = \omega r_t^2 + (1-\omega)\hat{\sigma}_t^2$$

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Volatility Tomorrow

"Actual" Today

Estimated Today

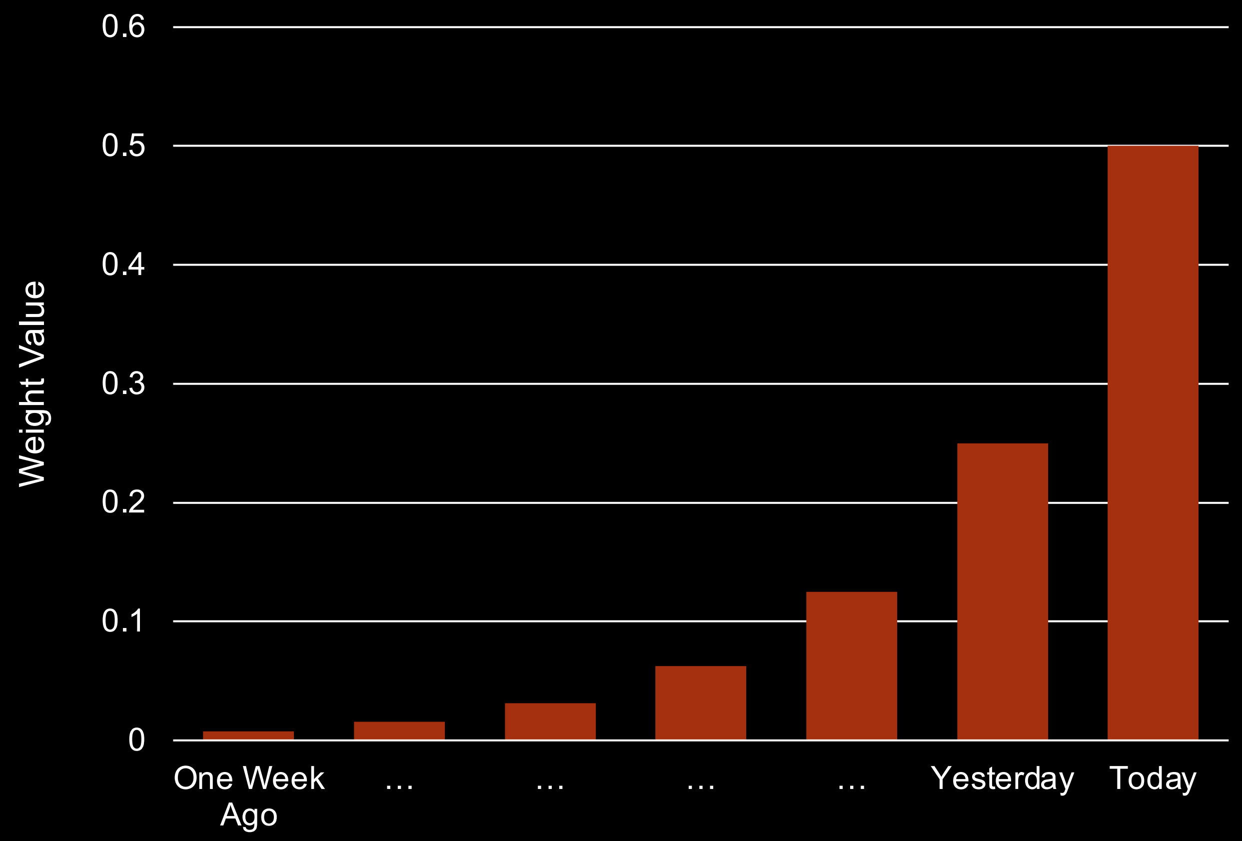
Weighting Time Periods

- Why not weight more recent observations heavier than previous ones?
- Exponential Smoothing Models

$$\sigma_{t+1}^2 = \omega r_t^2 + (1-\omega)\hat{\sigma}_t^2$$

Weight

Weighting Time Periods – $\omega = 0.5$



HOW TO MODEL VOLATILITY?

Not as Simple Approaches

Time Series Framework

- Weighted average of the volatility
  - Higher weights on the recent past
  - Small but non-zero weights on the distant past

- Choose weights with “ARIMA-like” approaches.

AutoRegressive Conditional Heteroscedasticity (ARCH) Models

- Autoregressive time series approach to modeling volatility.
- Trying to account for time dependency and persistence.

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2$$

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$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 r_{t-1}^2 + \cdots + \alpha_q r_{t-q+1}^2$$

"Actual" Today

"Actual" Yesterday

"Actual" (q-1) Days Ago

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"Actual" Today

"Actual" Yesterday

"Actual" (q-1) Days Ago

- Variances need to be positive, $\alpha_0 > 0$, $\alpha_i \geq 0$
- Model is a stationary one: $\sum_{i=1}^{q} \alpha_i < 1$

AutoRegressive Conditional Heteroscedasticity (ARCH) Models

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$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 r_{t-1}^2 + \cdots + \alpha_q r_{t-q+1}^2$$

Real World Data
Needs LARGE q!

Generalized ARCH (GARCH) Models

- Generalize the ARCH model
  - Similar to autoregressive (AR) model extending to the autoregressive moving average model (ARMA)

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \beta_1 \hat{\sigma}_t^2$$

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$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \beta_1 \hat{\sigma}_t^2$$

Volatility Tomorrow

"Actual" Today

Forecasted Today

GARCH(1,1) Model: Restrictions

- Given that σ_t^2 is a variance, it needs to be positive:
  - $\alpha_0 > 0$
  - $\alpha_1 > 0,\ \beta_1 > 0$

- Stationary model:
  - $0 < \alpha_1 + \beta_1 < 1$

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$$\sigma_{t+1}^2 = \alpha_0 + \sum_{i=0}^{q} \alpha_i r_{t-i}^2 + \sum_{j=0}^{p} \beta_j \hat{\sigma}_{t-j}^2$$

Generalized ARCH (GARCH) Models

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$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \beta_1 \hat{\sigma}_t^2$$

Real World Data "Typically" Only
Needs One of Each

Interpretations

- If we need to “force” the constraints about α_0, α_1, and β_1 its possible that our model is not appropriate and some other GARCH-type model should be used.

- The parameter α_i measures the reaction of conditional volatility to market shocks.
  - Large values (above 0.1) imply volatility is very sensitive to market events.
- The parameter β_i measures the persistence in conditional volatility.
  - Large values (above 0.9) imply volatility takes a long time to die out following a crisis in the market.

Interpretations

- The $(\alpha_i + \beta_i)$ determines the rate of convergence of the conditional volatility to the long term average level.
  - Large values (above 0.99) imply the terms structure of the volatility forecasts from the GARCH model is relatively flat.

- The constant α_0 together with $(\alpha_i + \beta_i)$ determines the level of the long term average volatility (the unconditional variance in the GARCH model).
  - The larger the value, the higher the long term volatility in the market.

TESTING FOR ARCH EFFECTS

Testing for ARCH Effects

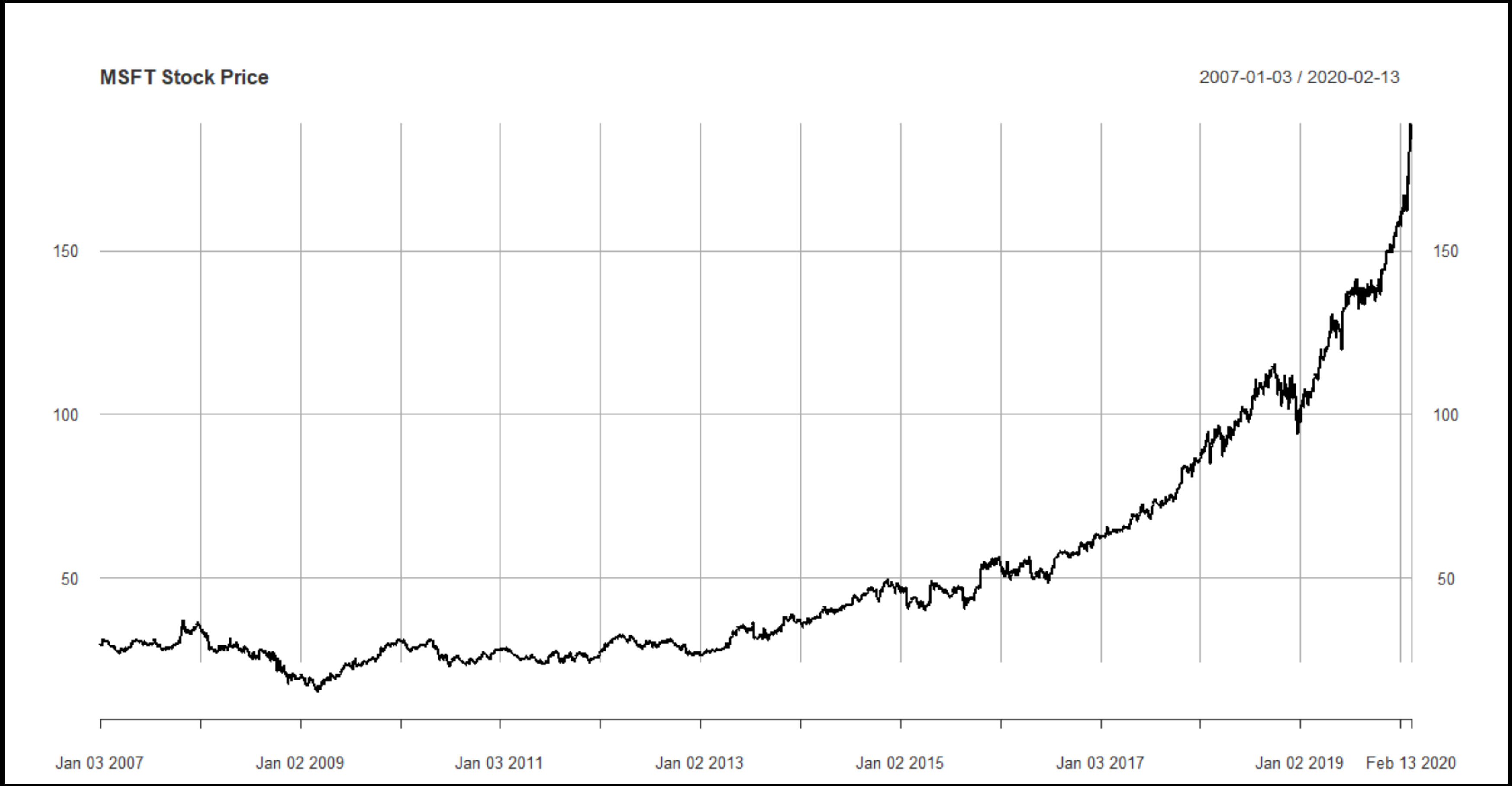
- Just like in time series where we test for autocorrelations, we can also test for different ARCH effects, which are similar to autocorrelations across the squared residuals.
- There are two common tests for ARCH effects:
  - Lagrange Multiplier (LM) test
  - Portmanteau Q test

Testing for ARCH Effects

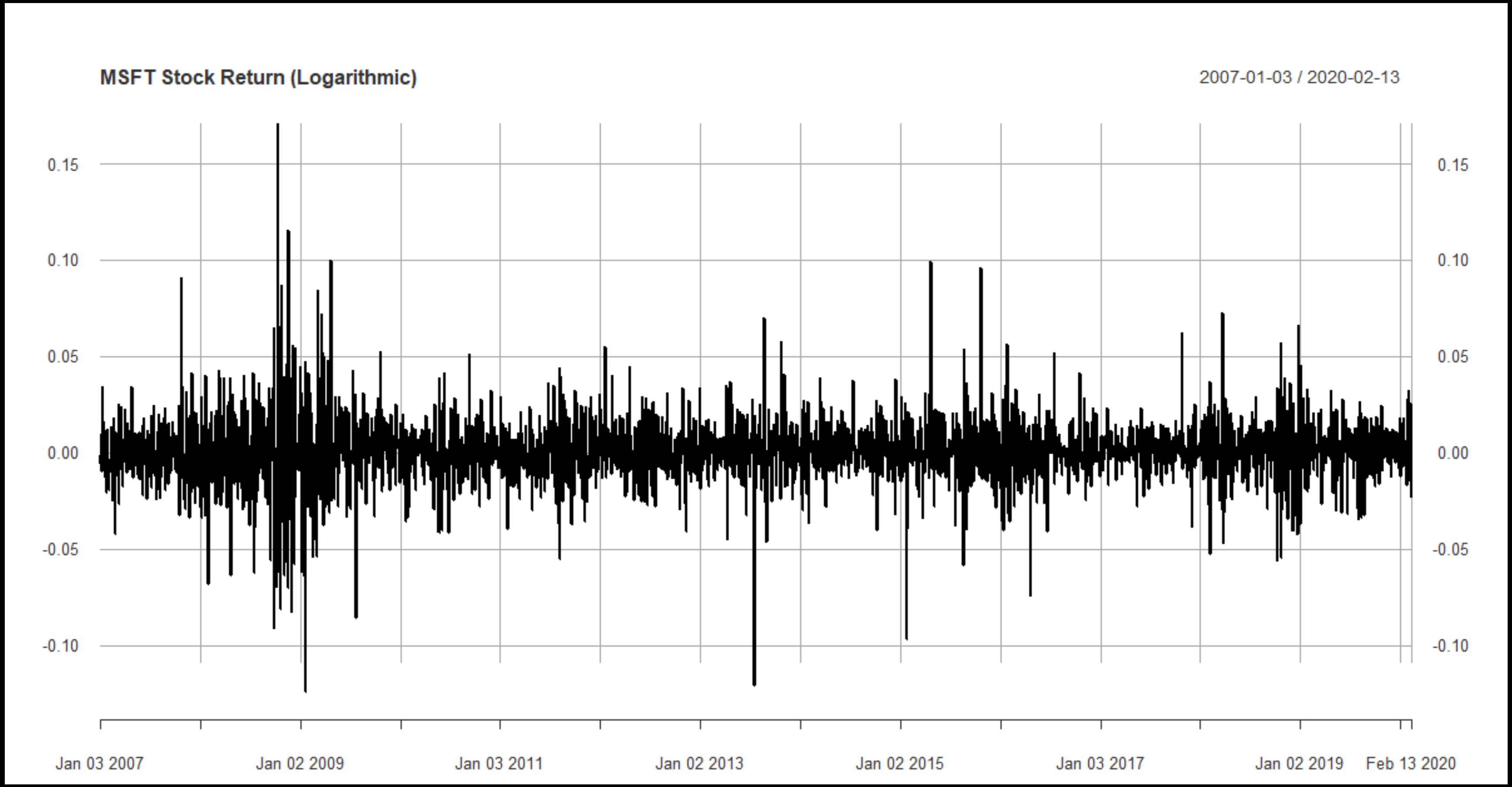
- Just like in time series where we test for autocorrelations, we can also test for different ARCH effects, which are similar to autocorrelations across the squared residuals.
- There are two common tests for ARCH effects:
  - Lagrange Multiplier (LM) test
  - Portmanteau Q test
- Null hypothesis for both tests is the same:

$$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$$

Microsoft Stock Price



Microsoft Returns



Test for ARCH Effects – R

```
arch.test(arima(stocks$msft_r[-1], order = c(0,0,0)), output = TRUE)
```

```
## ARCH heteroscedasticity test for residuals
## alternative: heteroscedastic
##
## Portmanteau-Q test:
##      order   PQ p.value
## [1,]     4  232       0
## [2,]     8  466       0
## [3,]    12  733       0
## [4,]    16  800       0
## [5,]    20  928       0
## [6,]    24 1020       0
## Lagrange-Multiplier test:
##      order   LM p.value
## [1,]     4 5414       0
## [2,]     8 2098       0
## [3,]    12 1192       0
## [4,]    16  881       0
## [5,]    20  670       0
## [6,]    24  553       0
```

GARCH Model – R

```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = "sGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = "norm")

GARCH.N <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(GARCH.N)
```

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GARCH Model – R

```
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega   0.000016    0.000003   4.5519    5e-06
## alpha1  0.130732    0.016838   7.7641    0e+00
## beta1   0.820882    0.017057  48.1267    0e+00
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## omega   0.000016    0.000016  0.98677 0.323755
## alpha1  0.130732    0.067788  1.92854 0.053788
## beta1   0.820882    0.054966 14.93442 0.000000
##
## LogLikelihood : 9792.06
```

GARCH Model – R

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## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
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## Conditional Variance Dynamics
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## beta1   0.820882    0.054966 14.93442 0.000000
##
## LogLikelihood : 9792.06
```

GARCH Model – R

```
## Information Criteria
## ------------------------------------
##
## Akaike       -5.4764
## Bayes        -5.4712
## Shibata      -5.4764
## Hannan-Quinn -5.4745
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic  p-value
## Lag[1]                      6.953 0.008366
## Lag[2*(p+q)+(p+q)-1][2]     7.053 0.011538
## Lag[4*(p+q)+(p+q)-1][5]     9.089 0.015711
## d.o.f=0
## H0 : No serial correlation
```

EXTENSIONS TO ARCH/GARCH MODELING

Extensions to GARCH Framework

- What if the distribution is not Normal?
- What if the underlying distribution is not symmetric?
- What if the variance actually affected the value of the return directly?

Extensions to GARCH Framework

- What if the distribution is not Normal?
  - Test Normality with Jarque-Berra (J-B) test.
  - Null hypothesis is that the standardized residuals follow the Normal distribution.
  - Test follows χ^2_2.
- What if the underlying distribution is not symmetric?
- What if the variance actually affected the value of the return directly?

J-B Test of Normality – R

```
jarque.bera.test(stocks msft_r[-1])
##
##  Jarque Bera Test
##
## data:  stocks$msft_r[-1]
## X-squared = 14712, df = 2, p-value < 2.2e-16
```

Extensions to GARCH Framework

- What if the distribution is not Normal?
  - Bollerslev (1986) developed the t-GARCH model that has an underlying t-distribution instead of a Normal distribution.
- What if the underlying distribution is not symmetric?
- What if the variance actually affected the value of the return directly?

t-GARCH – R

```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = "sGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = “std")

GARCH.t <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(GARCH.t)
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega   0.000007    0.000007   1.0679 0.285551
## alpha1  0.097683    0.024250   4.0282 0.000056
## beta1   0.884110    0.019481  45.3827 0.000000
## shape   4.410255    0.566263   7.7884 0.000000
```

Normal GARCH or t-GARCH

- Formal test for Normality.
- Compare using Information criteria.
- Formal test for d.f. of t-distribution.
  - Likelihood Ratio (LR) test to see if inverse of d.f. ≈ 0

Extensions to GARCH Framework

- What if the distribution is not Normal?
- What if the underlying distribution is not symmetric?
  - Nelson (1991) developed the EGARCH model to account for the **leverage effect** in certain data sets.
  - The leverage effect is when variance increases more when a return is negative compared to when a return is positive.
- What if the variance actually affected the value of the return directly?

EGARCH – R

```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = “EGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = “norm")

EGARCH <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(EGARCH)
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega  -0.341831    0.115771  -2.9526 0.003151
## alpha1 -0.047571    0.013574  -3.5045 0.000457
## beta1   0.956837    0.014323  66.8021 0.000000
## gamma1  0.191023    0.047182   4.0486 0.000052
```

QGARCH Model

- An alternative model to capture the asymmetries and leverage effects.
- Introduces λ into model.

$$\sigma_t^2 = \alpha_0 + \alpha_1 (r_t - \lambda)^2 + \beta_1 \hat{\sigma}_{t-1}^2$$

Skewed GARCH – R

```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = “sGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = “snorm")

Skew.GARCH.N <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(Skew.GARCH.N)
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega   0.000016    0.000004   3.8136 0.000137
## alpha1  0.129639    0.018839   6.8814 0.000000
## beta1   0.822103    0.018491  44.4595 0.000000
## skew    0.988332    0.018228  54.2216 0.000000
```

Skewed GARCH – R

```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = “sGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = “sstd")

Skew.GARCH.t <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(Skew.GARCH.t)
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega   0.000007    0.000011  0.65615 0.511727
## alpha1  0.098467    0.039750  2.47716 0.013243
## beta1   0.883623    0.027956 31.60720 0.000000
## skew    0.967612    0.019717 49.07482 0.000000
## shape   4.382270    0.858443  5.10491 0.000000
```

Extensions to GARCH Framework

- What if the distribution is not Normal?
- What if the underlying distribution is not symmetric?
- What if the variance actually affected the value of the return directly?
  - Referred to as GARCH-M models, where M stands for mean.

Exponentially Weighted Moving Average

- The Exponentially Weighted Moving Average (EWMA) model can be considered a “special” GARCH(1,1) model where:
  - $\alpha_0 = 0$
  - $(\alpha_1 + \beta_1) = 1$
  - RiskMetrics database (J.P. Morgan, 1994) has been using this methodology to forecast volatility.
  - RiskMetrics sets $\beta_1 = 0.94$ since this is the value that seems to produce the best out of sample forecasts.

Exponentially Weighted Moving Average

- The Exponentially Weighted Moving Average (EWMA) model can be considered a "special" GARCH(1,1) model where:
  - $\alpha_0 = 0$
  - $(\alpha_1 + \beta_1) = 1$

Nonstationary (called IGARCH)

  - RiskMetrics database (J.P. Morgan, 1994) has been using this methodology to forecast volatility.
  - RiskMetrics sets $\beta_1 = 0.94$ since this is the value that seems to produce the best out of sample forecasts.

EWMA – R

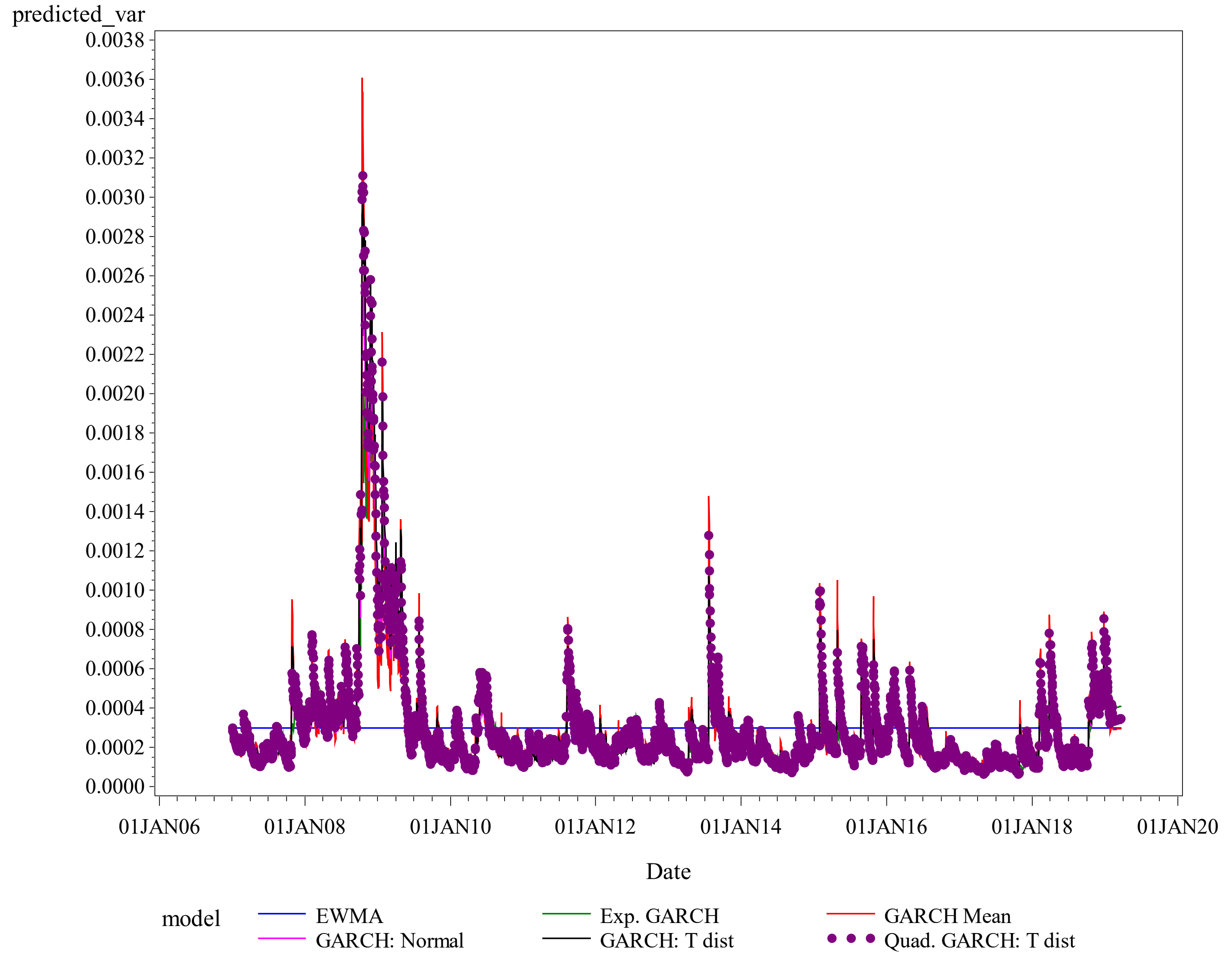
```
spec = ugarchspec(mean.model = list(include.mean = FALSE,
                                    armaOrder = c(0,0)),
                  variance.model = list(model = “iGARCH",
                                        garchOrder = c(1,1)),
                  distribution.model = “norm")

EWMA <- ugarchfit(data = stocks$msft_r[-1], spec = spec)

show(EWMA)
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## omega    0.00001    0.000001   10.111        0
## alpha1   0.17215    0.013743   12.526        0
## beta1    0.82785          NA       NA       NA
```

Comparison of Predictions



Many, Many, Many GARCH Models

- AARCH
- ADCC-GARCH
- AGARCH
- ANN-ARCH
- ANST-GARCH
- APARCH
- ARCH-M
- ARCH-SM
- ATGARCH
- Aug-GARCH
- AVGARCH
- B-GARCH
- BEKK-GARCH
- CCC-GARCH
- Censored-GARCH
- CGARCH
- COGARCH
- CorrARCH
- DAGARCH
- DCC-GARCH
- Diag MGARCH
- DTARCH
- DVEC-GARCH
- EGARCH
- EVT-GARCH
- F-ARCH
- FDCC-GARCH
- FGARCH
- FIAPARCH
- FIEGARCH
- FIGARCH
- FIREGARCH
- Flex-GARCH
- GAARCH
- GARCH-Delta
- GARCH Diffusion
- GARCH-EAR
- GARCH-Gamma
- GARCH-M
- GARCHS
- GARCHSK
- GARCH-t
- GARCH-X
- GARCHX
- GARJI
- GDCC-GARCH
- GED-GARCH
- GJR-GARCH
- GO-GARCH
- GQARCH
- GQTARCH
- HARCH
- HGARCH
- HYGARCH
- IGARCH
- LARCH
- Latent GARCH
- Level GARCH
- LGARCH
- LMGARCH
- Log-GARCH
- MAR-ARCH
- MARCH
- Matrix EGARCH
- MGARCH
- Mixture GARCH
- MS-GARCH
- MV-GARCH
- NAGARCH
- NGARCH
- NL-GARCH
- NM-GARCH
- OGARCH
- PARCH
- PC-GARCH
- PGARCH
- PNP-GARCH
- QARCH
- QTARCH
- REGARCH
- RGARCH
- Robust GARCH
- Root GARCH
- RS-GARCH
- Robust DCC-GARCH
- SGARCH
- S-GARCH
- Sign-GARCH
- SPARCH
- Spline-GARCH
- SQR-GARCH
- STARCH
- Stdev-ARCH
- STGARCH
- Structural GARCH
- Strong GARCH
- SWARCH
- TGARCH
- t-GARCH
- Tobit-GARCH
- TS-GARCH
- UGARCH
- VCC-GARCH
- VGARCH
- VSGARCH
- Weak GARCH
- ZARCH

Many, Many, Many GARCH Models

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Every day, self-proclaimed stock market "experts" tell us why the market just went up or down, as if they really knew. So where were they yesterday?!?

- Anonymous