- There are 66 decision variables
  - 60 (10 x 6) are for variable cost…how much it cost to send from warehouse to distribution center (how much should we send from each warehouse to each distribution center?)
  - 6 are binary decision variables (should this location be used for a warehouse?)

Distribution Centers

	Warehouses	Alb	Boise	Dall	Denv	Hous	Okla	Phoe	Salt	SanA	Wich
Y_1	Albuquerque	$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	$x_{1,5}$	$x_{1,6}$	$x_{1,7}$	$x_{1,8}$	$x_{1,9}$	$x_{1,10}$
Y_2	Dallas	$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	$x_{2,4}$	$x_{2,5}$	$x_{2,6}$	$x_{2,7}$	$x_{2,8}$	$x_{2,9}$	$x_{2,10}$
Y_3	Denver	●			●					●	
Y_4	Houston	●			●					●	
Y_5	Pheonix	●			●					●	
Y_6	San Antonio	$x_{6,1}$	$x_{6,2}$	$x_{6,3}$	$x_{6,4}$	$x_{6,5}$	$x_{6,6}$	$x_{6,7}$	$x_{6,8}$	$x_{6,9}$	$x_{6,10}$

Variable cost….depends on quantity being shipped from warehouse to distribution center (6x10)=60

Distribution Centers

Warehouses

		Alb	Boise	Dall	Denv	Hous	Okla	Phoe	Salt	SanA	Wich
Y_1	Albuquerque	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$X_{1,6}$	$X_{1,7}$	$X_{1,8}$	$X_{1,9}$	$X_{1,10}$
Y_2	Dallas	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$X_{2,6}$	$X_{2,7}$	$X_{2,8}$	$X_{2,9}$	$X_{2,10}$
Y_3	Denver	●			●				●		
Y_4	Houston	●			●				●		
Y_5	Pheonix	●			●				●		
Y_6	San Antonio	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	$X_{6,6}$	$X_{6,7}$	$X_{6,8}$	$X_{6,9}$	$X_{6,10}$

Fixed cost (binary variables)…only incur cost if make into warehouse

Distribution Centers

	Warehouses	Alb	Boise	Dall	Denv	Hous	Okla	Phoe	Salt	SanA	Wich
Y_1	Albuquerque	$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	$x_{1,5}$	$x_{1,6}$	$x_{1,7}$	$x_{1,8}$	$x_{1,9}$	$x_{1,10}$
Y_2	Dallas	$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	$x_{2,4}$	$x_{2,5}$	$x_{2,6}$	$x_{2,7}$	$x_{2,8}$	$x_{2,9}$	$x_{2,10}$
Y_3	Denver	●			●				●		
Y_4	Houston	●			●				●		
Y_5	Pheonix	●			●				●		
Y_6	San Antonio	$x_{6,1}$	$x_{6,2}$	$x_{6,3}$	$x_{6,4}$	$x_{6,5}$	$x_{6,6}$	$x_{6,7}$	$x_{6,8}$	$x_{6,9}$	$x_{6,10}$

Objective Function

- Minimize Cost: Variable cost + Fixed Cost

Objective function

- Minimize cost (variable cost and fixed cost)

VARIABLE COST= 0*x[1,1] +47*x[1,2]+32*x[1,3]+22*x[1,4] + 42.5*x[1,5] + 27*x[1,6] + 23*x[1,7] + 30*x[1,8] + 36.5*x[1,9]+29.5*x[1,10]+…..0*x[6,9] +32*x[6,10]

Location	Alb	Boise	Dall	Denv	Hous	Okla	Phoe	Salt	SanA	Wich
Albuquerque	0	47	32	22	42.5	27	23	30	36.5	29.5
Dallas	32	79.5	0	39	12.5	10.5	50	63	13.5	17
Denver	21	42	39	0	51.5	31.5	40.5	24	47.5	26
Houston	42.5	91	12.5	51.5	0	23	58	72	10	31
Pheonix	23	49	50	40.5	58	49	0	32.5	50	52
San Antonio	36.5	83.5	13.5	47.5	10	24	50	66.5	0	32

Objective function

- Minimize cost (variable cost and fixed cost)

COST= 0*x[1,1] +47*x[1,2]+32*x[1,3]+22*x[1,4] + 42.5*x[1,5] + 27*x[1,6] + 23*x[1,7] + 30*x[1,8] + 36.5*x[1,9]+29.5*x[1,10]+…..0*x[6,9] +32*x[6,10]

+**140000*Y1 +150000*Y2 + 100000*Y3 + 110000*Y4 +125000*Y5 + 120000*Y6 (fixed cost)**

Center / Warehouse	Volume	Capacity	Cost
Albuquerque (W)	3,200	16,000	$140,000
Boise	2,500		
Dallas (W)	6,800	20,000	$150,000
Denver (W)	4,000	10,000	$100,000
Houston (W)	9,600	10,000	$110,000
Oklahoma City	3,500		
Phoenix (W)	5,000	12,000	$125,000
Salt Lake City	1,800		
San Antonio (W)	7,400	10,000	$120,000
Wichita	2,700		

Subscript i will denote warehouses and subscript j will denote distribution centers

Now for formula….

- Cost matrix denoted by $C_{i,j}$ corresponds to decision variable $x_{i,j}$
- Warehouse Cost denoted by WC_i corresponds to decision variable y_i

(notice that subscripts match…very important!!)

Now for formula….

- Cost matrix denoted by Ci,j corresponds to decision variable xi,j
- Warehouse Cost denoted by WC_i corresponds to decision variable yi

(notice that subscripts match…very important!!)

OBJECTIVE FUNCTION:

$\Sigma_i\Sigma_j C_{i,j} * x_{i,j} + \Sigma_i WC_i * Y_i$

First constraint:

WAREHOUSE CAPACITY:

- Warehouse 1: $x_{1,1}+x_{1,2}+x_{1,3}+x_{1,4}+x_{1,5}+x_{1,6}+x_{1,7}+x_{1,8}+x_{1,9}+x_{1,10}$

First constraint:

WAREHOUSE CAPACITY:

- Warehouse 1: $x_{1,1}+x_{1,2}+x_{1,3}+x_{1,4}+x_{1,5}+x_{1,6}+x_{1,7}+x_{1,8}+x_{1,9}+x_{1,10}$
- $\Sigma_j x_{1,j} <= Capacity_1$

$Capacity_1$ is from the Capacity vector

First constraint:

For i 1..6: $\Sigma_j x_{i,j}$ <= Capacity_i

Second constraint:

VOLUME FOR DISTRIBUTION CENTER

- Distribution center 1: $x_{1,1} + x_{2,1} + x_{3,1}+x_{4,1}+x_{5,1}+x_{6,1}$

(remember, distribution centers are the ‘j’ ‘s

Second constraint:

VOLUME FOR DISTRIBUTION CENTER

- For j 1..10: $\Sigma_i \; x_{i,j}$ = Demand_j

(Demand_j is from the Demand vector)

LINKING CONSTRAINT:

- Formula from earlier $x<=Max(x)*y$
- In this situation, x is the amount from that warehouse, so from warehouse 1:

$$x_{1,1}+x_{1,2}+x_{1,3}+x_{1,4}+x_{1,5}+x_{1,6}+x_{1,7}+x_{1,8}+x_{1,9}+x_{1,10} <=16000*Y_1$$

LINKING CONSTAINT:

- Formula from earlier $x<=Max(x)*y$
- In this situation, x is the amount from that warehouse, so from warehouse 1:

$x1,1+x1,2+x1,3+x1,4+x1,5+x1,6+x1,7+x1,8+x1,9+x1,10 <=Capacity_1*Y_1$

Formula: $\Sigma_j x_{1,j} <=Capacity_1*Y_1$

LINKING CONSTRAINT:

For i 1..6: $\Sigma_j x_{i,j} <= Capacity_i * Y_i$

Putting everything together

- Objective function: Minimize $\Sigma_i \Sigma_j C_{i,j} * x_{i,j} + \Sigma_{iW} C_i * Y_i$
- Subject to:

  For i 1..6: $\Sigma_j x_{i,j} <= \text{Capacity}_i$

  For j 1..10: $\Sigma_i\ x_{i,j} = \text{Demand}_j$

  For i 1..6: $\Sigma_j x_{i,j} <= \text{Capacity}_i * Y_i$

If you wrote out each statement individually, you would have a total of 6 + 10 +6 = 22 statements

You could also combine constraint 1 and constraint 3 (the linking constraint also constrains the maximum amount at each warehouse…this would give 16 statements….both ways are correct

```
m = Model("Levinson Foods Company")
# Warehouse demand in thousands of units
demand = [3200, 2500, 6800, 4000, 9600, 3500, 5000, 1800, 7400, 2700]

# Plant capacity in thousands of units
capacity = [16000, 20000, 10000, 10000, 12000, 10000]

# Fixed costs for each plantfixed
Costs = [140000, 150000, 100000, 110000, 125000, 120000]

# Transportation costs per thousand units
transCosts = [[0,     47,   32,   22, 42.5,   27,   23,   30, 36.5, 29.5],
              [32,   79.5,    0,   39, 12.5, 10.5,   50,   63, 13.5,   17],
              [21,     42,   39,    0, 51.5, 31.5, 40.5,   24, 47.5,   26],
              [42.5,   91, 12.5, 51.5,    0,   23,   58,   72,   10,   31],
              [23,     49,   50, 40.5,   58,   49,    0, 32.5,   50,   52],
              [36.5, 83.5, 13.5, 47.5,   10,   24,   50, 66.5,    0,   32]]

# Range of plants and warehouses
plants = len(demand) #should be 10
warehouses = len(capacity) #should be 6

# Add Variables (66 total)
x = {} #transportation amounts (continuous)
y = {} #warehouse binary
```

```
#binary for if a warehouse is open (1-6)
for i in range(warehouses):
  y[i] = m.addVar(vtype=GRB.BINARY, name='y%d' % i)

#variable for every warehouse/plant transportation amount (1-60)
for i in range(warehouses):
  for j in range(plants):
    x[(i,j)] = m.addVar(vtype=GRB.CONTINUOUS, lb=0, name='t%d,%d' % (i,j))

# Add Constraints
for i in range(warehouses):
  m.addConstr(quicksum(x[(i,j)] for j in range(plants)) - y[i]*capacity[i] <= 0)

#Plant with binding constraint for demand (10 constraints)
for j in range(plants):
  m.addConstr(quicksum(x[(i,j)] for i in range(warehouses)) == demand[j])

m.setObjective(quicksum(y[i]*fixedCosts[i] + quicksum(x[(i,j)]*transCosts[i][j]
for j in range(plants)) for i in range(warehouses)), GRB.MINIMIZE)

# Optimize
m.optimize()
```

y0: 0
y1: 1
y2: 1
y3: 1
y4: 1
y5: 0

Albuquerque	Dallas	Denver	Houston	Pheonix	San Antonio
## t0,0: 0	## t1,0: 0	## t2,0: 1700	## t3,0: 0	## t4,0: 1500	## t5,0: 0
## t0,1: 0	## t1,1: 0	## t2,1: 2500	## t3,1: 0	## t4,1: 0	## t5,1: 0
## t0,2: 0	## t1,2: 6800	## t2,2: 0	## t3,2: 0	## t4,2: 0	## t5,2: 0
## t0,3: 0	## t1,3: 0	## t2,3: 4000	## t3,3: 0	## t4,3: 0	## t5,3: 0
## t0,4: 0	## t1,4: 0	## t2,4: 0	## t3,4: 9600	## t4,4: 0	## t5,4: 0
## t0,5: 0	## t1,5: 3500	## t2,5: 0	## t3,5: 0	## t4,5: 0	## t5,5: 0
## t0,6: 0	## t1,6: 0	## t2,6: 0	## t3,6: 0	## t4,6: 5000	## t5,6: 0
## t0,7: 0	## t1,7: 0	## t2,7: 1800	## t3,7: 0	## t4,7: 0	## t5,7: 0
## t0,8: 0	## t1,8: 7000	## t2,8: 0	## t3,8: 400	## t4,8: 0	## t5,8: 0
## t0,9: 0	## t1,9: 2700	## t2,9: 0	## t3,9: 0	## t4,9: 0	## t5,9: 0