# INTRODUCTION TO RISK MANAGEMENT

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# INTRODUCTION TO RISK

# Risk Realm

- "Only those who risk going too far can possibly find out how far they can go."
  - T.S. Eliot

- The entirety of risk analysis is a very extensive space.
- Focus is on applied business risk modeling and analysis.
- Examples:
  - Market Risk
  - Operational Risk
  - Credit Risk
  - Liquidity Risk

# Risk vs. Uncertainty

- Risk and uncertainty are related, but different than each other.
- Risk is something that someone bears.
- **Risk** is the outcome of **uncertainty**.

- **Once you have an uncertain event and you can put some distribution to it, you can measure the risk associated with that event.**
- Just because there is uncertainty, there could very well be no risk.
  - Example – Flipping a coin with no care of the outcome.

# Levels of Uncertainty

- There are 3 levels of uncertainty in the world:
  1. The **known** – guaranteed event
  2. The **unknown** – events that carry risk that will be reduced/eliminated over time as the event gets closer.
  3. The **unknowable** – events that carry risk that may not change over time as the event gets closer.

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Risk analysis provides the most value for the unknown factors, but also can handle unknowable factors.

# Dealing with Risk: A Primer

| Name of Project | Cost | Expected Net RETURN | Risk |
|---|---|---|---|
| Project A | $50 | $50 | $25 |
| Project B | $250 | $200 | $100 |
| Project C | $100 | $100 | $10 |

# Dealing with Risk: A Primer

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- Best projects tend to be those with the return for the amount invested.
- Popular extension – Risk Adjusted Return on Capital

# Dealing with Risk: A Primer

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| Project C | $100 | $100 | $10 | 10.0 |

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# Point Estimates

- In the past, most decision makers looked only to single point estimates of a project's profitability.

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- What is the probability on a continuous distribution that these exact values will occur?

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- How much do you trust single point estimates?
- What is the probability on a continuous distribution that these exact values will occur? **ZERO**

# Example of Point Estimate

- Expected Unit Sales (Q): 1500
- Expected Sales Price (P): $10.00
- Expected Cost/Unit (VC): $7.00
- Expected Initial/Fixed Cost (FC): $2,500

- Expected Net Revenue:

$$NR = Q \times (P - VC) - FC$$

$$NR = 1500 \times (\$10 - \$7) - \$2500$$

$$NR = \$2{,}000$$

# DEALING WITH RISK

## SCENARIO ANALYSIS

# Scenario Analysis

- People then started accounting for possible extreme values in their estimation of some of the inputs.
- This introduced the initial idea of risk into these calculations.

# Example of Scenario Analysis

- Expected Unit Sales (Q): 1500 (Most Likely)
  2000 (Best Case)
  500 (Worst Case)
- Expected Sales Price (P): \$10.00
- Expected Cost/Unit (VC): \$7.00
- Expected Initial/Fixed Cost (FC): \$2,500

- Expected Net Revenue:

$$NR = \$2{,}000$$

$$NR\ Range = (-\$1{,}000, \$3{,}500)$$

# Scenario Analysis

- People then started accounting for possible extreme values in their estimation of some of the inputs.
- This introduced the initial idea of risk into these calculations.
- **Outcomes are too variable in this type of analysis.**
- **Doesn’t account for interdependencies.**

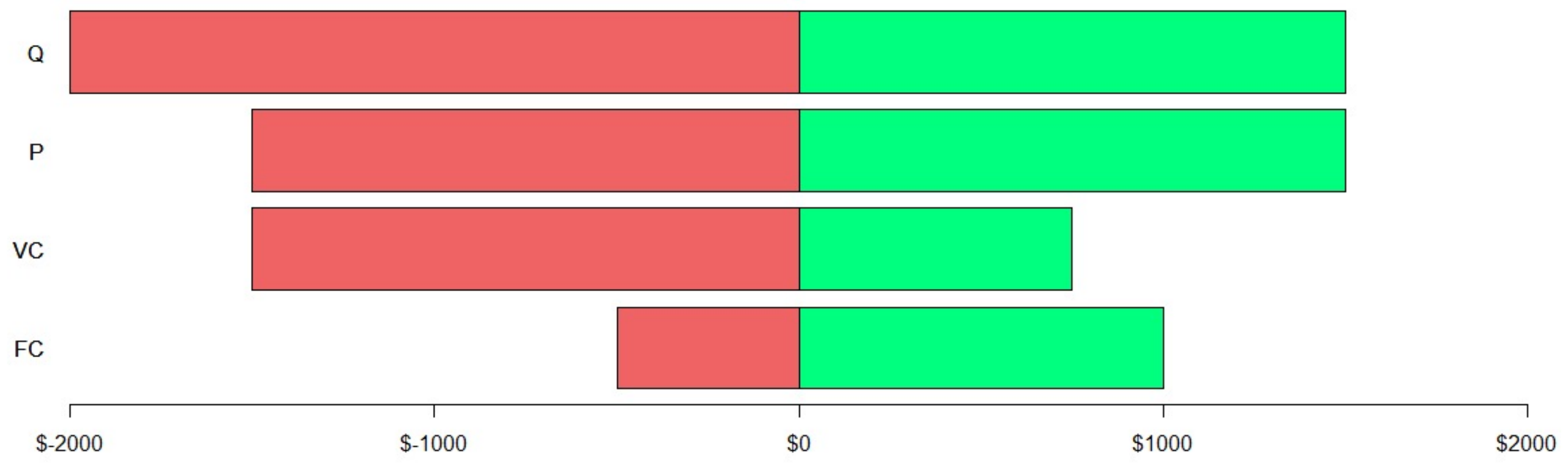
# Scenario Analysis

- Popular Extension – **Tornado analysis** where you look at the best and worst case scenarios for each of the inputs and look at the highest impact.

- Expected Unit Sales (Q): 500, 1500, 2000
- Expected Sales Price (P): $9.00, $10.00, $11.00
- Expected Cost/Unit (VC): $6.50, $7.00, $8.00
- Expected Initial/Fixed Cost (FC): $1,500, $2,500, $3,000

# Scenario Analysis

- Popular Extension – **Tornado analysis** where you look at the best and worst case scenarios for each of the inputs and look at the highest impact.



# Sensitivity Analysis

- Sensitivity analysis was the next extension.
  - What will happen if fixed costs increase by $1?
  - What if the variable costs increase by $0.50?
  - What if unit sales increase by 2?
- Captures marginal costs.
- Great at capturing sensitivities.
- What is the probability of different possible outcomes?

# DEALING WITH RISK

## SIMULATION APPROACH

# Monte Carlo Simulation

- Simulation analysis allows us to account for all of the possible changes in all these variables and the possible correlations between them.
- The final output is a **probability distribution of all possible outcomes**.

# Monte Carlo Simulation – R

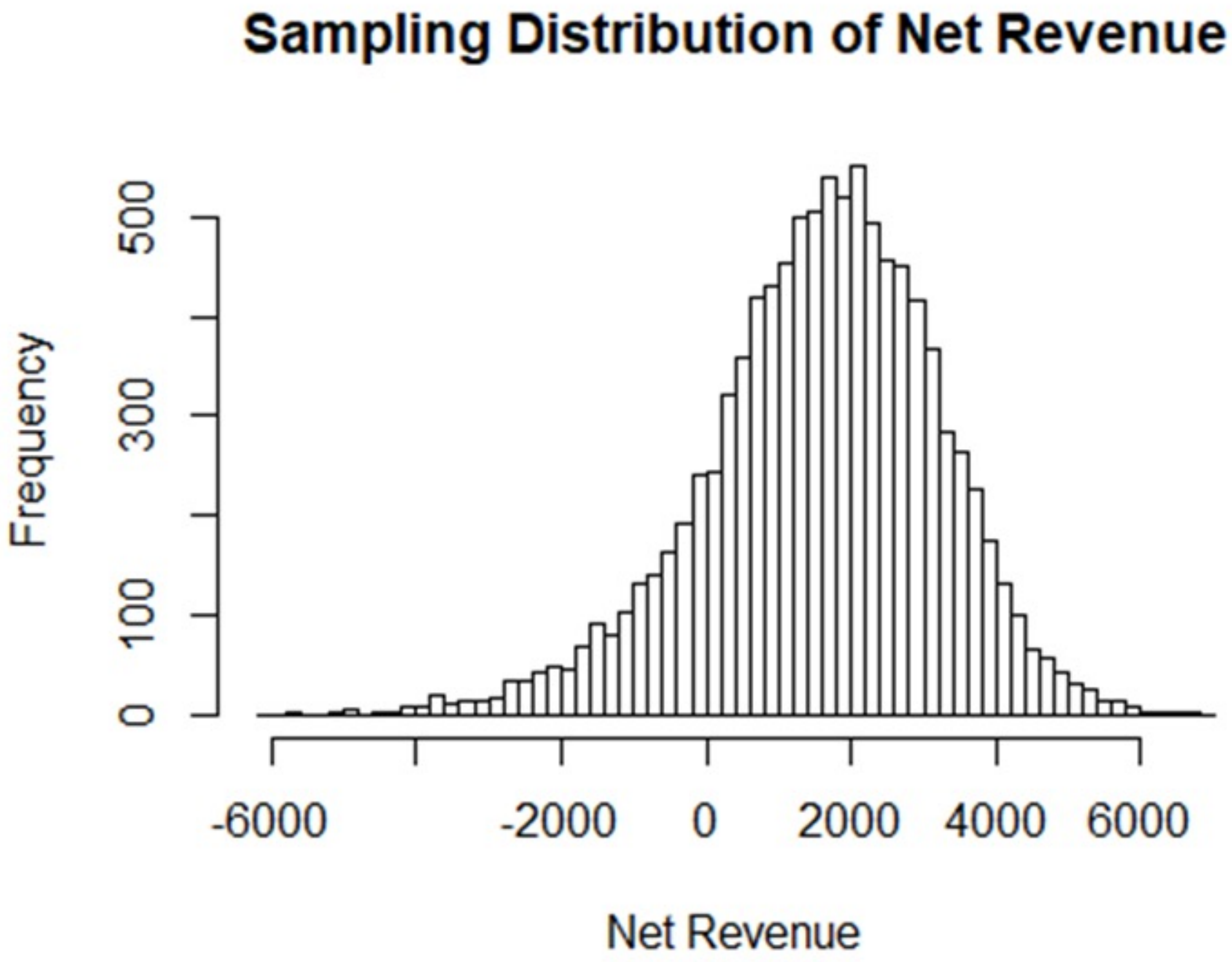
```
simulation.size <- 10000

Units <- rtriangle(simulation.size, a=500, b=2000, c=1500)
Var.Cost <- 1 + 0.004*Units + rnorm(simulation.size, mean=0, sd=sqrt(0.8))
Fixed.Cost <- 2500
Price <- rtriangle(simulation.size, a=8, b=11, c=10)

Net.Revenue <- (Price - Var.Cost)*Units - Fixed.Cost

hist(Net.Revenue, breaks=50, main='Sampling Distribution of Net Revenue',
     xlab='Net Revenue')
```

# Monte Carlo Simulation – R



# Monte Carlo Simulation

- Parametric Monte Carlo Simulation
  - Specific distributional parameters are required before a simulation can begin.

- Nonparametric Monte Carlo Simulation
  - Raw historical data is used to estimate the distribution and no distributional parameters are required for the simulation to run.

# KEY RISK MEASURES

# Key Risk Characteristics

- Risk is an uncertainty that affects a system in an unknown fashion and brings great fluctuation in value and outcome.
- **Risk is the outcome of uncertainty** – fluctuations can be measured in a probabilistic sense.
- Risk has a time horizon.
- Risk measurement has to be set against a benchmark.

# Statistics of Risk

- Risk analysis is using some of the “typical” statistical measures.
  - ~~Mean~~
  - Variance
  - Skewness
  - Kurtosis – used for catastrophic, extreme tail events

# Common Risk Measures

- There are some common measures that are used in risk analysis:
  1. Probability of Occurrence
  2. Standard Deviation / Variance / Coefficient of Variation
  3. Semi-standard Deviation
  4. Volatility
  5. Value at Risk (VaR)
  6. Expected Shortfall (ES)

# Common Risk Measures

- **Probability of Occurrence**
  - Examples – Probability of failure of a project, probability of default, migration probabilities, transition matrices.

- **Standard Deviation, Variance, Coefficient of Variation**
  - Two-sided measures
  - Sufficient only under normality or maybe symmetry

# Common Risk Measures

- **Semi-standard Deviation (Downside Risk)**
  - Measure of dispersion for the values falling below the mean.

$$\sigma_{semi} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \min(X_t - \bar{X}, 0)^2}$$

- **Volatility**
  - Standard deviation of an asset's logarithmic returns

$$\sigma_{volatility} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \ln\left(\frac{X_t}{X_{t-1}}\right)^2}$$

# Common Risk Measures

- **Value at Risk – VaR**
  - The amount of money at risk given a particular time period at a particular probability of loss
  - Example – 1 year 99.9% VaR is $10,000
    - There is a 99.9% chance you will lose at least $10,000 in 1 year

# Common Risk Measures

- **Expected Shortfall – ES**
  - The average money given a particular time in the worst q% of the cases
  - Example – 1 year 99.9% ES is $15,000
    - In the worst 0.1% of scenarios, the average amount of money you will lose in one year is $15,000

# VALUE AT RISK

# History of VaR

- Developed in the early 1990’s by JP Morgan
- The *“4:15pm” report*
- JP Morgan launched *RiskMetrics*® (1994)
- VaR has been widely used since that time
- Currently, researchers are looking into more advanced “VaR-like” measures.

# Definition

- The VaR calculation is aimed at making a statement of the following form:
  - We are 99% certain that we will not lose more than $10,000 in the next 3 days.
  - $10,000 is the 3-day 99% VaR

- VaR is the maximum amount at risk to be lost…

  …**over a period of time**…

  …**at a particular level of confidence**.

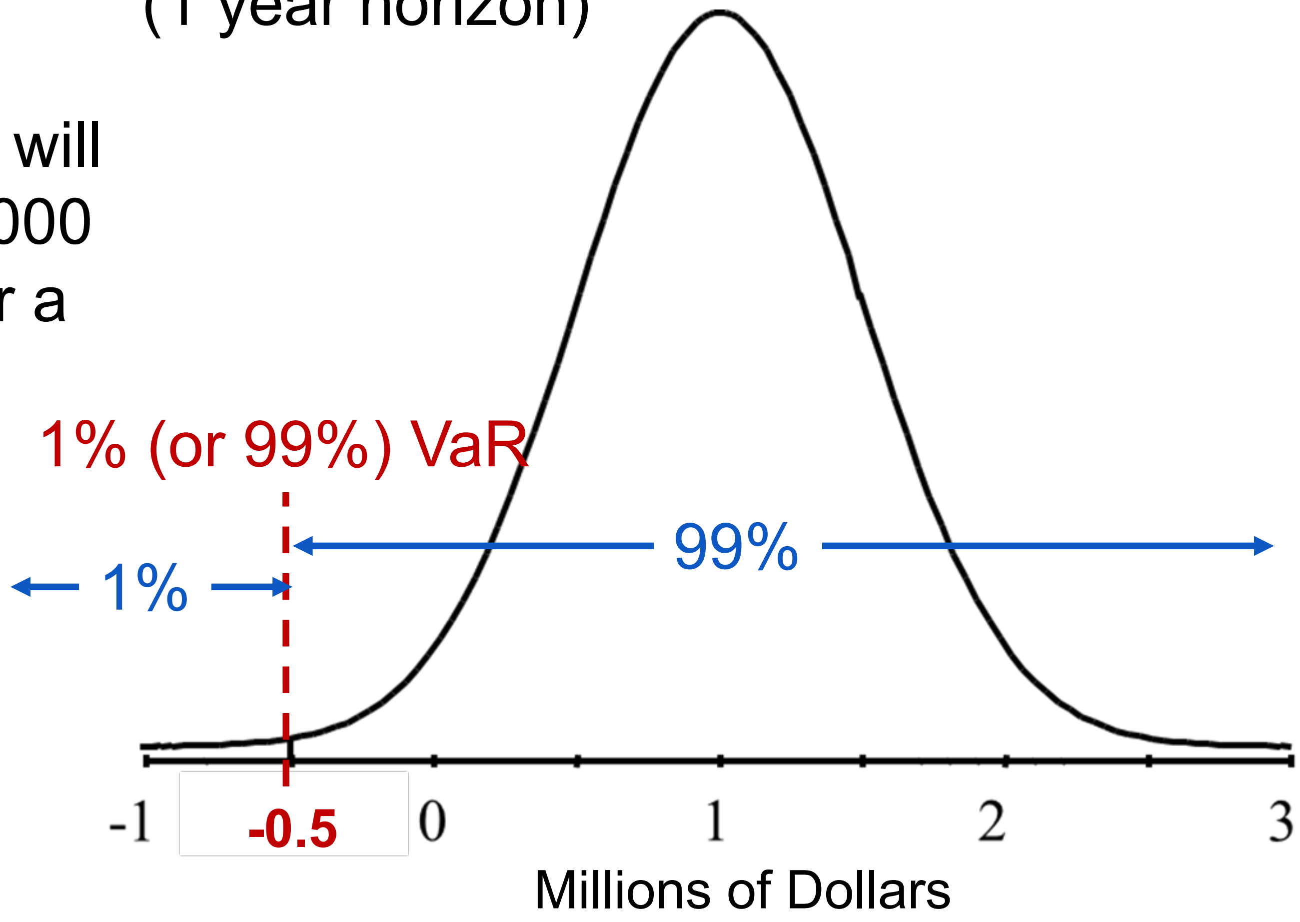
# Focus on the Tail

- The Value at Risk is associated with a percentile (quantile) of a distribution.
- Focused on the tail of the distribution.

# Visualizing VaR – Left Tail

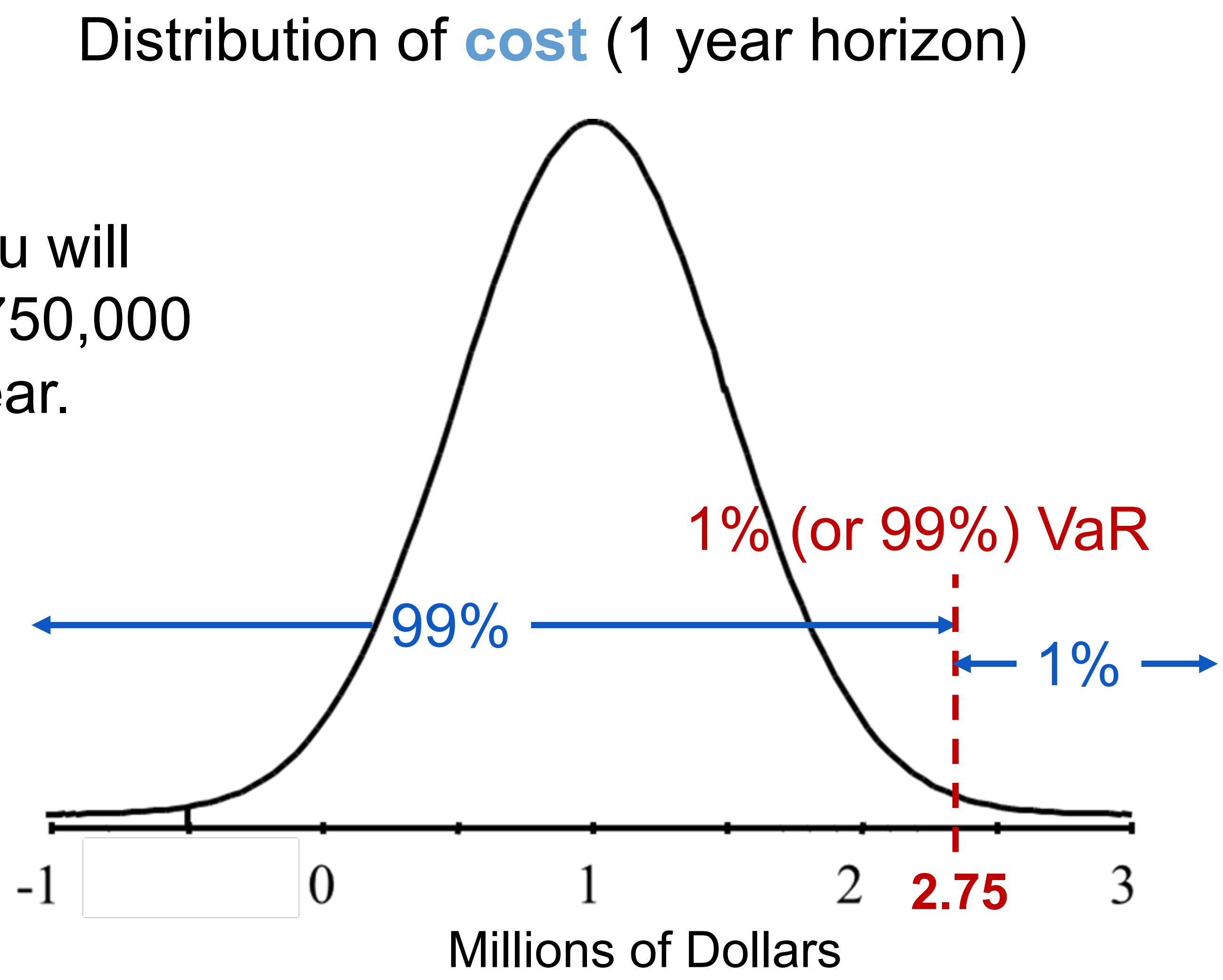
With 99% probability, you will not lose more than $500,000 by holding the portfolio for a year

Distribution of change in portfolio's **value** (1 year horizon)



# Visualizing VaR – Right Tail

With 99% probability, you will not incur more than $2,750,000 in costs over the next year.

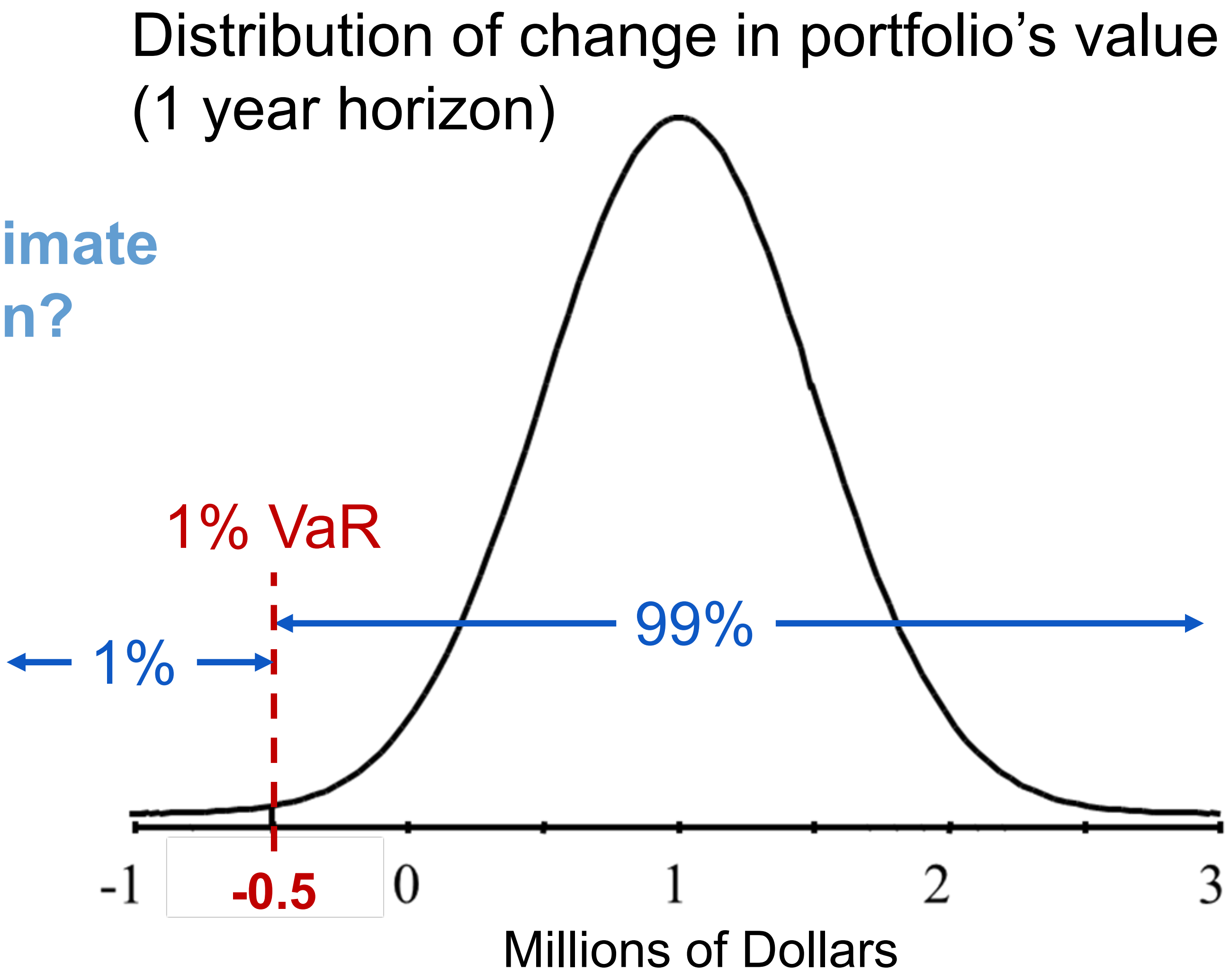


# VaR Estimation

- Main Steps:
  1. Identify the variable of interest (asset value, portfolio value, credit losses, insurance claims, etc.)
  2. Identify the key risk factors that impact the variable of interest (assets prices, interest rates, duration, volatility, default probabilities, etc.)
  3. Perform deviations in the risk factors to calculate the impact in the variable of interest

# Visualizing VaR – Left Tail

**How do we estimate this distribution?**



# VaR Estimation

- How do we estimate this distribution?

- 3 Main Approaches
  1. Delta-Normal or Variance-Covariance Approach
  2. Historical Simulation (variety of approaches)
  3. Monte Carlo Simulation

# EXPECTED SHORTFALL

# Drawbacks of VaR – Magnitude

- VaR ignores the distribution of a portfolio's return beyond its VaR.
- Example:
  - The 99.9% VaR for an investment in stock A is $100K. The 99.9% VaR for an investment in stock B is $100K.
  - Are you indifferent between the two?

# Drawbacks of VaR – Magnitude

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- Example:
  - The 99.9% VaR for an investment in stock A is \$100K. The 99.9% VaR for an investment in stock B is \$100K.
  - Are you indifferent between the two?

- Stock A: The loss can be up to \$250K
- Stock B: The loss can be up to \$950K
- **VaR ignores the magnitude of the worst returns.**

# Drawbacks of VaR - Diversification

- Under non-normality, VaR may not capture diversification.
- VaR fails to satisfy the **subadditivity property**.

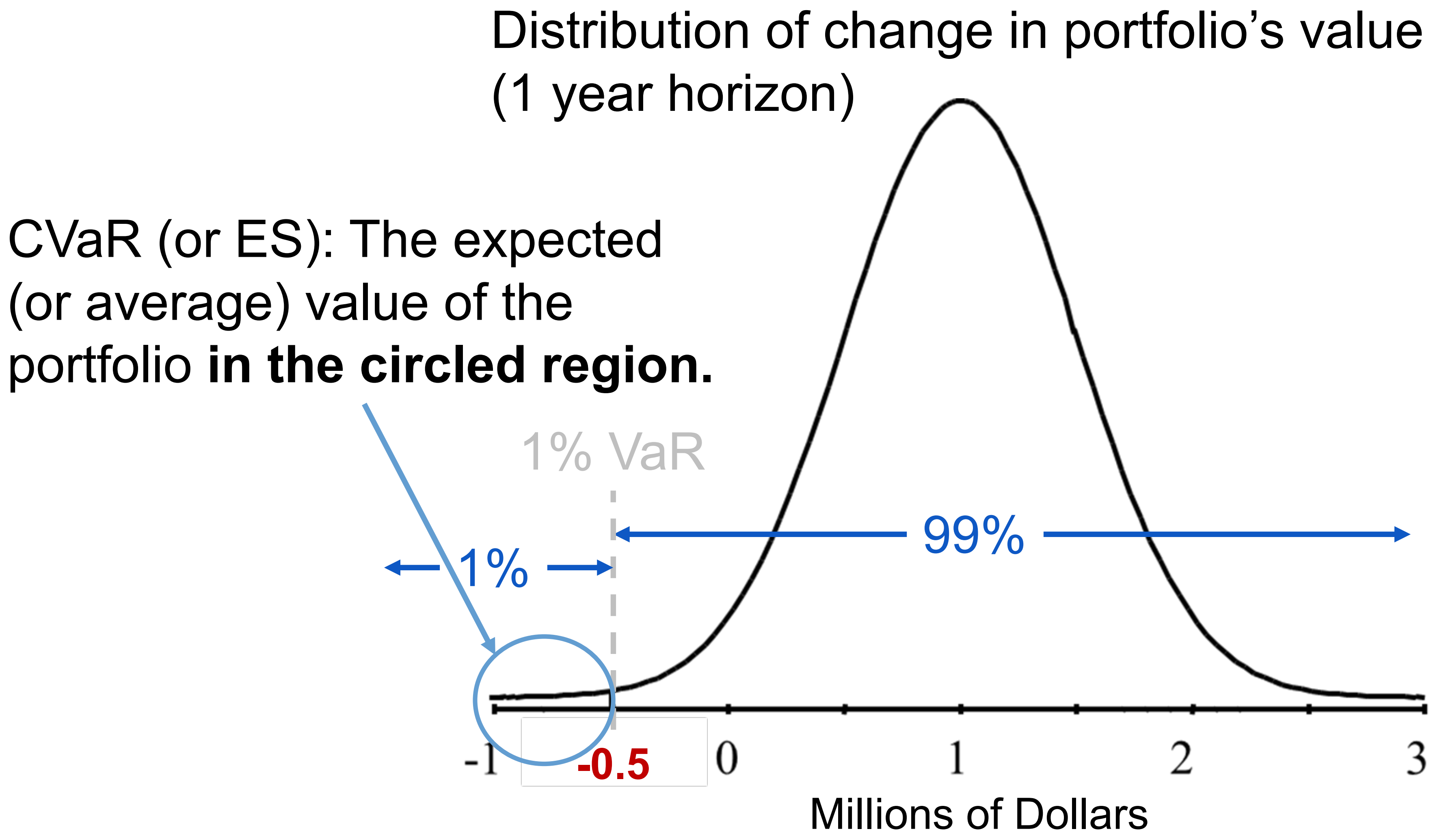
$$Risk(A+B) \leq Risk(A) + Risk(B)$$

- The VaR of a portfolio with two securities may be larger than the sum of the VaR's of the securities in the portfolio.

# VaR Alternative – CVaR

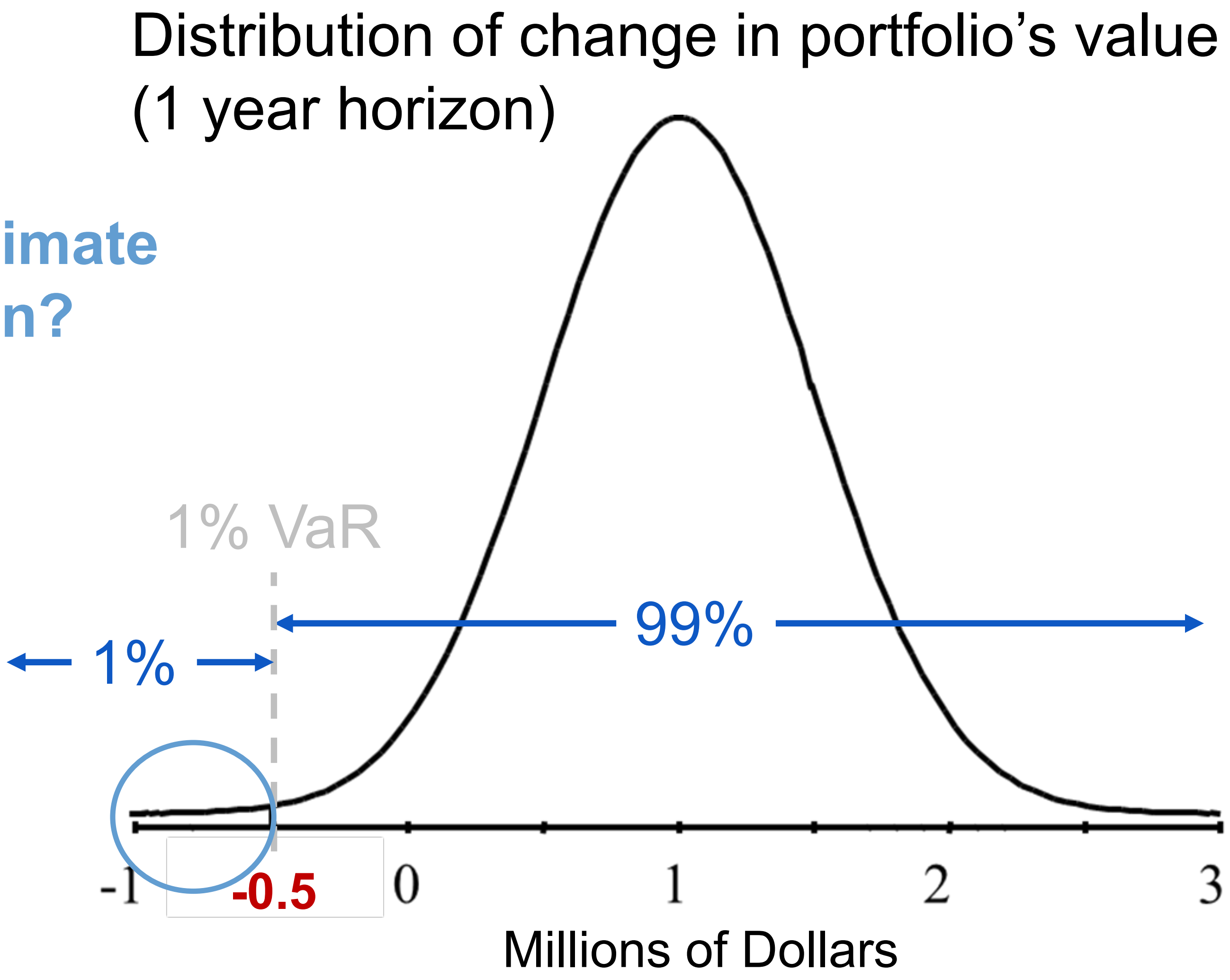
- The **Conditional Value at Risk** (**CVaR**) or **Expected Shortfall** (**ES**) is a measure that doesn't have the two drawbacks of the VaR.
- Given a confidence level and a time horizon, a portfolio's CVaR is the **expected loss** one suffers given that a "bad" event occurs.
- The CVaR is a conditional expectation.
- If my loss exceeds the VaR level, what should I expect it to be equal to?

# Visualizing CVaR (ES) – Left Tail



# Visualizing CVaR (ES) – Left Tail

**How do we estimate this distribution?**



# CVaR (ES) Estimation

- How do we estimate this distribution?

- 3 Main Approaches
  1. Delta-Normal or Variance-Covariance Approach
  2. Historical Simulation (variety of approaches)
  3. Monte Carlo Simulation

# CALCULATING RETURNS

# Returns on Assets

- A lot of the calculations we will be making in this course will revolve around calculating the returns on assets.
- There are 2 main methods for calculating returns:
  1. Arithmetic Return
  2. Geometric Return

# Basic Notation

- Here is the basic notation needed to calculate returns:
  - Return $(r_t)$ – return at a period *t* (holding an asset from period *t-1* to period *t*)
  - Price $(P_t)$ – price at a given time period *t*
  - Lag Price $(P_{t-1})$ – price a time period *t-1*
  - Dividend $(D_t)$ – dividend payment at time period *t*

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  - Dividend ($D_t$) – dividend payment at time period *t*
    - For small time periods we typically ignore dividend (set equal to 0)
    - Equivalently: $P_t$ is the price of an asset where dividends are fully reinvested (and thus reflected in $P_t$ itself)

# Arithmetic Return

$$r_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$$

$D_t = 0$

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

- If $r_1 = 5\%$ and $r_2 = -5\%$, what is the total return of the two days?

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- If $r_1 = 5\%$ and $r_2 = -5\%$, what is the total return of the two days? **NOT ZERO!**

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- How do we get $r_{0,2}$ as a function of $r_1$ and $r_2$?

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$$r_{0,2} = \frac{P_2 - P_0}{P_0} = \cdots = \frac{P_1}{P_0} r_2 + r_1 \neq r_2 + r_1$$

# Arithmetic Return

- If $r_1 = 5\%$ and $r_2 = -5\%$, what is the total return of the two days?
- How do we get $r_{0,2}$ as a function of $r_1$ and $r_2$?

$$P_0 = 1 \qquad P_1 = 1.05$$

$$r_1 = 5\%$$

# Arithmetic Return

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$$P_0 = 1 \qquad P_1 = 1.05 \qquad P_2 = 0.9975$$

$$r_1 = 5\% \qquad r_2 = -5\%$$

# Arithmetic Return

- If $r_1 = 5\%$ and $r_2 = -5\%$, what is the total return of the two days?
- How do we get $r_{0,2}$ as a function of $r_1$ and $r_2$?

$$P_0 = 1 \qquad P_1 = 1.05 \qquad P_2 = 0.9975$$

$$r_1 = 5\% \qquad r_2 = -5\%$$

$$r_{0,2} = \frac{P_2 - P_0}{P_0} = \frac{0.9975 - 1}{1} = -0.0025 = -0.25\%$$

# Geometric Return

$$R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right)$$

$$D_t = 0$$

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1})$$

- If $R_1 = 5\%$ and $R_2 = -5\%$, what is the total return of the two days?
- How do we get $R_{0,2}$ as a function of $R_1$ and $R_2$?

$$R_{0,2} = \ln\left(\frac{P_2}{P_0}\right) = \ln\left(\frac{P_2}{P_1} \times \frac{P_1}{P_0}\right) = \ln\left(\frac{P_2}{P_1}\right) + \ln\left(\frac{P_1}{P_0}\right) = R_2 + R_1$$

# Geometric Return

- What if we took the same prices and measured returns geometrically instead?
- How do we get $R_{0,2}$ as a function of $R_1$ and $R_2$?

$$P_0 = 1 \qquad P_1 = 1.05$$

$$R_1 = 4.88\%$$

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$$P_0 = 1 \qquad P_1 = 1.05 \qquad P_2 = 0.9975$$

$$R_1 = 4.88\% \qquad R_2 = -5.13\%$$

$$R_{0,2} = \ln\left(\frac{P_2}{P_0}\right) = -0.25\% = 4.88\% - 5.13\%$$

# Mathematical Relation

- What is the difference between the two?

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln\left(\frac{P_t - P_{t-1}}{P_{t-1}} + 1\right) = \ln(1 + r_t)$$

$$= r_t - \frac{r_t^2}{2} + \frac{r_t^3}{3} - \cdots \approx r_t \text{ when } r_t \text{ small}$$

- For a typical **daily** return, the difference between $R_t$ and $r_t$ is very close to 0.

# Mathematical Relation

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$$r_{0,2} = \frac{P_2 - P_0}{P_0} = \frac{0.9975 - 1}{1} = -0.0025 = \mathbf{-0.25\%}$$

$$R_{0,2} = \ln\left(\frac{P_2}{P_0}\right) = \mathbf{-0.250313\%}$$

# Mathematical Relation

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$$r_{0,2} = \frac{P_2 - P_0}{P_0} = \frac{0.9975 - 1}{1} = -0.0025 = \mathbf{-0.25\%}$$

$$R_{0,2} = \ln\left(\frac{P_2}{P_0}\right) = \mathbf{-0.250313\%}$$

**VERY CLOSE!**

# Empirical Relation (Google Inc.)

| Date | Close | Arithmetic Return | Geometric Return |
|---|---|---|---|
| 12/20/2019 | $1,351.22 | - | - |
| 12/23/2019 | $1,350.63 | -0.044% | -0.044% |
| 12/24/2019 | $1,344.43 | -0.460% | -0.459% |
| 12/26/2019 | $1,362.47 | 1.333% | 1.342% |
| 12/27/2019 | $1,354.64 | -0.576% | -0.575% |
| 12/30/2019 | $1,339.71 | -1.108% | -1.102% |
| 12/31/2019 | $1,339.39 | -0.024% | -0.024% |
| 1/2/2020 | $1,366.38 | 1.995% | 2.015% |