Laboratorio 8: operaciones con ideales

1.

In[17]:=
```
IdealSuma[A_, B_] := Union[A, B];
IdealProducto[A_, B_] := Flatten[Outer[Times, A, B]];
IdealInterseccion[A_, B_, vars_] := Module[{H, GB, solution},
   H = Union[
     Table[T * A[[j]] , {j, 1, Length[A] }],
     Table[(1 - T) * B[[j]], {j, 1, Length[B]}]
    ];
   GB = GroebnerBasis[H, Join[{T}, vars], MonomialOrder → Lexicographic];
   solution = Select[GB, FreeQ[#, T] &];
   solution
  ];
```

In[20]:=
```
A = {x^2, y^3, z^3};
B = {x^2};
IdealSuma[A, B]
IdealProducto[A, B]
IdealInterseccion[A, B, {x, y, z}]
```

Out[22]=

$\{x^2, y^3, z^3\}$

Out[23]=

$\{x^4, x^2 y^3, x^2 z^3\}$

Out[24]=

$\{x^2\}$

In[25]:=
```
B = {x^3};
IdealSuma[A, B]
IdealProducto[A, B]
IdealInterseccion[A, B, {x, y, z}]
```

Out[26]=

$\{x^2, x^3, y^3, z^3\}$

Out[27]=

$\{x^5, x^3 y^3, x^3 z^3\}$

Out[28]=

$\{x^3\}$

In[29]:=
```
B = {x, y} ;
IdealSuma[A, B]
IdealProducto[A, B]
IdealInterseccion[A, B, {x, y, z}]
```

Out[30]=
$\{x, x^2, y, y^3, z^3\}$

Out[31]=
$\{x^3, x^2 y, x y^3, y^4, x z^3, y z^3\}$

Out[32]=
$\{y z^3, y^3, x z^3, x^2\}$

In[33]:=
```
B = {x, y, z};
IdealSuma[A, B]
IdealProducto[A, B]
IdealInterseccion[A, B, {x, y, z}]
```

Out[34]=
$\{x, x^2, y, y^3, z, z^3\}$

Out[35]=
$\{x^3, x^2 y, x^2 z, x y^3, y^4, y^3 z, x z^3, y z^3, z^4\}$

Out[36]=
$\{z^3, y^3, x^2\}$

2. La funcion comprueba si f pertenece al radical de J, en caso de que si, devuelve la potencia mínima del elemento que pertenece al ideal, en caso de que no devuelve 0.

In[130]:=
```
RadicalMembership[f_, J_, vars_] := Module[{GB, r, n},
   GB = GroebnerBasis[Join[J, {1 - T * f}],
     Join[{T}, vars], MonomialOrder → DegreeReverseLexicographic];
   r =
    PolynomialReduce[1, GB, MonomialOrder → DegreeReverseLexicographic][[2]];
   If[r == 0,
    n = 1;
    GB = GroebnerBasis[J, vars, MonomialOrder → DegreeReverseLexicographic];
    r =
     PolynomialReduce[f, GB, MonomialOrder → DegreeReverseLexicographic][[2]];
    While[r =!= 0,
     n++;
     r = PolynomialReduce[f^n,
        GB, MonomialOrder → DegreeReverseLexicographic][[2]];
    ];
    Return[n]
    ,
    Return[0]
   ];
  ];
```

In[131]:=

```
J = {x^3, y^3, x*y*(x + y)};
f = x + y;
RadicalMembership[f, J, {x, y}]
```

Out[133]=

3

In[134]:=

```
J = {x + z, x^2*y, x - z^2};
f = x^2 + 3 * x * y;
RadicalMembership[f, J, {x, y, z}]
```

Out[136]=

0

3.

In[100]:=

```
J = {x^5 - 2*x^4 + 2 x^2, x^5 - x^4 - 2*x^3};
GB = GroebnerBasis[J, {x}]
```

Out[101]=

$\{x^2\}$

Por lo que claramente el radical de J es (x).

4.

In[158]:=

```
f = x^5 - 2*x^3*y^2 + x*y^4 - x^4 * z^2 + 2*x^2*y^2*z^2 - y^4*z^2;
g = x^4 + 2 x^3*z^2 - x^2*y^2 + x^2*z^4 - 2*x*y^2*z^2 - y^2*z^4;
```

In[160]:=

```
A = IdealInterseccion[{f}, {g}, {x, y, z}]
```

Out[160]=

$\{x^7 - 2 x^5 y^2 + x^3 y^4 + x^6 z^2 - 2 x^4 y^2 z^2 + x^2 y^4 z^2 - x^5 z^4 + 2 x^3 y^2 z^4 - x y^4 z^4 - x^4 z^6 + 2 x^2 y^2 z^6 - y^4 z^6\}$

In[161]:=

```
J = IdealProducto[{f}, {g}];
GB = GroebnerBasis[J, {x, y, z}, MonomialOrder → DegreeReverseLexicographic]
Factor[GB[[1]]]
```

Out[162]=

$\{-x^9 + 3 x^7 y^2 - 3 x^5 y^4 + x^3 y^6 - x^8 z^2 + 3 x^6 y^2 z^2 - 3 x^4 y^4 z^2 + x^2 y^6 z^2 + x^7 z^4 - 3 x^5 y^2 z^4 + 3 x^3 y^4 z^4 - x y^6 z^4 + x^6 z^6 - 3 x^4 y^2 z^6 + 3 x^2 y^4 z^6 - y^6 z^6\}$

Out[163]=

$-(x - y)^3 (x + y)^3 (x - z^2) (x + z^2)^2$

In[164]:=

```
RadJ = {x * (x - y) * (x + y) * (x + 3*y) * (x + z^2)};
```

In[196]:=

```
MCD = 1 ;
F = FactorList[RadJ⟦1⟧];
For[i = 1, i ≤ Length[F], i++,
  If[And[
     PolynomialReduce[f, {F⟦i⟧⟦1⟧}, {x, y, z}]⟦2⟧ === 0,
     PolynomialReduce[g, {F⟦i⟧⟦1⟧}, {x, y , z}]⟦2⟧ === 0
    ],
    MCD = MCD * F⟦i⟧⟦1⟧;
   ];
 ];
MCD
```

Out[199]=

$(x - y)\ (x + y)$