Laboratorio 6: Eliminación de Variables

1.

In[18]:=
```
CalculatePoints[S_] := Module[{P, Rz, Ry, Rx, Q, l, t, solutions},
  P = Eliminate[S, {x, y}];
  Rz = Solve[P, {z}];
  If[Rz == {}, Return[{}]];
  Q = S //. Rz;
  P = Table[Eliminate[Q[[i]], {x}], {i, 1, Length[Q]}];
  Ry = Table[Solve[P[[i]], {y}], {i, 1, Length[Q]}];
  For[l = 0, l < Length[Q], l++,
   If[Ry[[l]] == {},
     Ry = Delete[Ry, l];
     Rz = Delete[Rz, l];
    ];
  ];
  Q = Table[S[[i]] //. Ry[[i]], {i, 1, Length[Q]}];
  Rx = Table[
    Table[
     Solve[Q[[i]][[j]], {x}],
     {j, 1, Length[Q[[i]]]}
    ],
    {i, 1, Length[Q]}
   ];
  For[l = 0, l < Length[Rx], l++,
   For[t = 0, t < Length[Rx[[l]]], t++,
    If[Rx[[l]][[t]] == {},
      Rx[[l]] = Delete[Rx[[l]], t];
      Ry[[l]] = Delete[Ry[[l]], t];
     ];
   ];
   If[Rx[[l]] == {},
    Rx = Delete[Rx, l];
    Ry = Delete[Ry, l];
    Rz = Delete[Rz, l];
   ];
  ];
  solutions = Flatten[
    Table[
     Table[
```

```
      Table[
       {x, y, z} /. {
         x → Rx⟦i⟧⟦j⟧⟦k, 1, 2⟧,
         y → Ry⟦i⟧⟦j, 1, 2⟧,
         z → Rz⟦i, 1, 2⟧
        },
       {k, 1, Length[Rx⟦i⟧⟦j⟧]}
      ],
      {j, 1, Length[Rx⟦i⟧]}
     ],
     {i, 1, Length[Rx]}
    ],
    2
   ];
  solutions
 ]
```

In[20]:=
```
CalculatePoints[{
  x^2 + y^2 + z^2 - 1 == 0,
  x^2 + y^2 + z^2 - 2 * x == 0,
  2 * x - 3 * y - z == 0
 }]
```

Out[20]=

$$\left\{\left\{-\frac{\sqrt{169-6\sqrt{26}-200\,z^2}}{10\sqrt{2}},\ \frac{1}{20}\left(6+\sqrt{26}\right),\ \frac{1}{20}\left(2-3\sqrt{26}\right)\right\},\right.$$

$$\left\{\frac{\sqrt{169-6\sqrt{26}-200\,z^2}}{10\sqrt{2}},\ \frac{1}{20}\left(6+\sqrt{26}\right),\ \frac{1}{20}\left(2-3\sqrt{26}\right)\right\},$$

$$\left\{\frac{1}{10}\left(10-\frac{\sqrt{169+6\sqrt{26}-200\,z^2}}{\sqrt{2}}\right),\ \frac{1}{20}\left(6-\sqrt{26}\right),\ \frac{1}{20}\left(2+3\sqrt{26}\right)\right\},$$

$$\left.\left\{\frac{1}{10}\left(10+\frac{\sqrt{169+6\sqrt{26}-200\,z^2}}{\sqrt{2}}\right),\ \frac{1}{20}\left(6-\sqrt{26}\right),\ \frac{1}{20}\left(2+3\sqrt{26}\right)\right\}\right\}$$

In[21]:=
```
CalculatePoints[{
  x^2 * y - z^3 == 0,
  2 * x * y - 4 * z - 1 == 0,
  z - y^2 == 0,
  x - 4 * z * y == 0
 }]
```

Out[21]=

$$\{\}$$

2.

In[48]:=
```
PuntosCriticos2D[f_] := Module[{S, P, Ry, Rx, l, solutions},
   S = Grad[f, {x, y}];
   P = Eliminate[Table[S[[i]] == 0, {i, 1, Length[S]}], {x}];
   Ry = Solve[P, {y}];
   S = S //. Ry;
   Rx = Table[Solve[S[[i]] == 0, {x}], {i, 1, Length[S]}];
   For[l = 1, l < Length[S], l++,
    If[Rx[[l]] == {},
      Rx = Delete[Rx, l];
      Ry = Delete[Ry, l];
     ];
   ];
   solutions = Flatten[
     Table[
      Table[
       {x, y} /. {
         x → Rx[[i]][[j, 1, 2]],
         y → Ry[[i, 1, 2]]
        },
       {j, 1, Length[Rx[[i]]]}
      ],
      {i, 1, Length[Rx]}
     ],
     1];
   solutions
  ];
```

In[64]:=
```
f = (x^2 + y^2 - 4) * (x^2 + y^2 - 1) + (x - 3 / 2)^2 + (y - 3 / 2)^2;
P = PuntosCriticos2D[f]
```

Out[65]=

$$\left\{\left\{-\frac{1}{2}, -\frac{1}{2}\right\}, \left\{-\sqrt{\frac{-5+2\sqrt{13}}{-2+2\sqrt{13}}}, \frac{1}{4}\left(1-\sqrt{13}\right)\right\}, \left\{\sqrt{\frac{5+2\sqrt{13}}{2+2\sqrt{13}}}, \frac{1}{4}\left(1+\sqrt{13}\right)\right\}\right\}$$

In[67]:=
```
For[i = 1, i ≤ Length[P], i++,
  Print[P[[i]]];
  h = Grad[Grad[f, {x, y}], {x, y}] //. {x → P[[i]][[1]], y → P[[i]][[2]]};
  If[PositiveDefiniteMatrixQ[h],
   Print["Mínimo"],
   If[NegativeDefiniteMatrixQ[h],
     Print["Máximo"],
     Print["Punto Silla"]
    ];
  ];
 ];
```

$$\left\{-\frac{1}{2}, -\frac{1}{2}\right\}$$

Máximo

$$\left\{-\sqrt{\frac{-5+2\sqrt{13}}{-2+2\sqrt{13}}}, \frac{1}{4}\left(1-\sqrt{13}\right)\right\}$$

Punto Silla

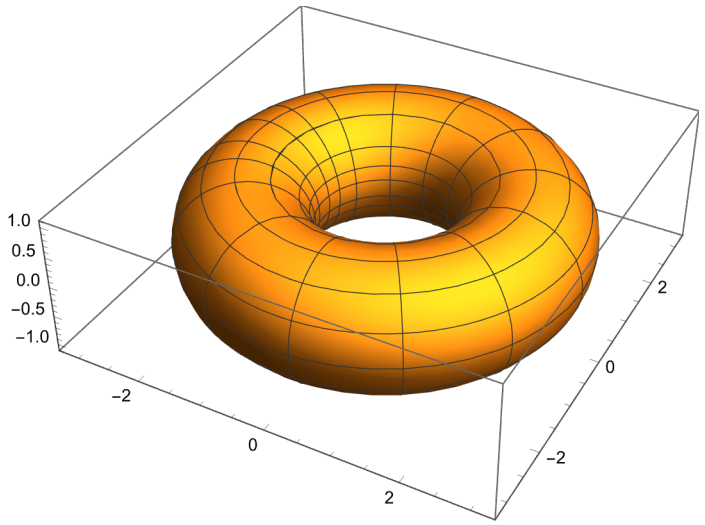
$$\left\{\sqrt{\frac{5+2\sqrt{13}}{2+2\sqrt{13}}}, \frac{1}{4}\left(1+\sqrt{13}\right)\right\}$$

Mínimo

3.

In[70]:=
```
ParametricPlot3D[{(2 + Cos[t]) * Cos[u], (2 + Cos[t]) * Sin[u], Sin[t]},
 {t, -4., 4.}, {u, -4., 4.}]
```

Out[70]=



In[80]:=
```
S = {
    x == (2 + a) * c,
    y == (2 + a) * d,
    z == b,
    a^2 + b^2 - 1 == 0,
    c^2 + d^2 - 1 == 0
   };
GB = GroebnerBasis[S, {a, b, c, d, x, y, z}, MonomialOrder → Lexicographic]
P = Eliminate[Table[GB[[i]] == 0, {i, 1, Length[GB]}], {a, b, c, d}]
```

Out[81]=

$$\{9 - 10x^2 + x^4 - 10y^2 + 2x^2y^2 + y^4 + 6z^2 + 2x^2z^2 + 2y^2z^2 + z^4,$$
$$12d - 13y + x^2y + y^3 + 4dz^2 + yz^2, 4dx^2 - 3y - x^2y + 4dy^2 - y^3 - yz^2,$$
$$12c - 13x + x^3 + xy^2 + 4cz^2 + xz^2, -dx + cy,$$
$$-3 + 4cx - x^2 + 4dy - y^2 - z^2, -1 + c^2 + d^2, b - z, 5 + 4a - x^2 - y^2 - z^2\}$$

Out[82]=

$$9 - 10x^2 + x^4 - 10y^2 + 2x^2y^2 + y^4 + 6z^2 + 2x^2z^2 + 2y^2z^2 + z^4 == 0$$