Laboratorio 9: Resultante

1.

In[205]:=

```
f = x^2 * y - 3*x*y^2 + x^2 - 3*x*y;
g = x^3*y + x^3 - 4*y^2 - 3*y + 1;

Resultant[f, g, x]
Resultant[f, g, y]
Print["Los polinomios como polinomios en y tienen una raiz en comun."]
```

Out[207]=

$$\left(1-3\,y-4\,y^2\right)\left(1-10\,y^2+7\,y^3+93\,y^4+158\,y^5+108\,y^6+27\,y^7\right)$$

Out[208]=

0

```
Los polinomios como polinomios en y tienen una raiz en comun.
```

2.

In[230]:=

```
f = x*y - 1;
g = x^2 + y^2 - 4;

R = Resultant[f, g, x]

GB = GroebnerBasis[{f, g}, {x, y}, MonomialOrder → Lexicographic];

GB1 = Select[GB, FreeQ[#, x] &]

{R} == GB1
```

Out[232]=

$$1-4\,y^2+y^4$$

Out[234]=

$$\left\{1-4\,y^2+y^4\right\}$$

Out[235]=

True

In[241]:=

```
f = x * y - 1 ;
g = x^2 * y + y^2 - 1;

R = Resultant[f, g, x]
GB = GroebnerBasis[{f, g}, {x, y}, MonomialOrder → Lexicographic];
GB1 = Select[GB, FreeQ[#, x] &]
Print["Claramente el ideal generado por R no contiene el elemento de GB1."]
```

Out[243]=

$y - y^2 + y^4$

Out[245]=

$\{1 - y + y^3\}$

Claramente el ideal generado por R no contiene el elemento de GB1.

3.

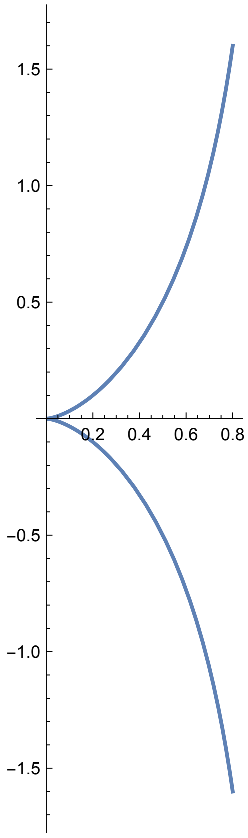
In[285]:=

```
ParametricPlot[{t^2 / (1 + t^2), t^3 / (1 + t^2)}, {t, -2., 2.}]
f = (1 + t^2) * x - t^2 ;
g = (1 + t^2) * y - t^3;

R = Resultant[f, g, t]

GB = GroebnerBasis[{f, g}, {t, x, y}, MonomialOrder → Lexicographic];
P = Select[GB, FreeQ[#, t] &][[1]]
ContourPlot[P == 0, {x, -2., 2.}, {y, -2., 2.}]
```

Out[285]=



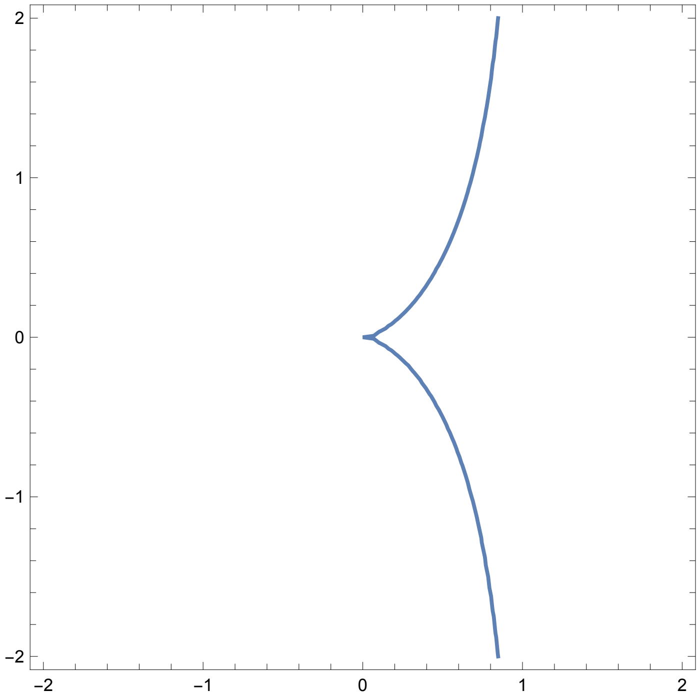
Out[288]=

$x^3 - y^2 + x\, y^2$

Out[290]=

$x^3 - y^2 + x\, y^2$

Out[291]=



4.

```
AddAlgebraic[f_, g_ , var_] := Module[{R, P, S, x1, x2, k},
   R = Resultant[f, g /. {var → z - var}, var] ;
   P = R /. {z → var} ;
   S = FactorList[P] ;
   x1 = Root[f, 1];
   x2 = Root[g, 1];
   For[k = 1, k ≤ Length[S], k++,
    If[
      Simplify[S〚k〛〚1〛 /. {var → x1 + x2}] === 0 ,
      Return[S〚k〛〚1〛];
     ];
   ];
   Return[$Failed]
  ];
ProdAlgebraic[f_, g_, var_] := Module[{deg, Q, R, P, S, x1, x2, k},
   deg = Exponent[g, var];
   Q = g /. {var → z / var};
   R = Resultant[f, var^deg * Q, var] ;
   P = R /. {z → var};
   S = FactorList[P];
   x1 = Root[f, 1];
   x2 = Root[g, 1];
   For[k = 1, k ≤ Length[S], k++,
    If[
      Simplify[S〚k〛〚1〛 /. {var → x1 * x2}] === 0,
      Return[S〚k〛〚1〛];
     ];
   ];
   Return[$Failed]
  ];
```

$\sqrt{2} + \sqrt[3]{2}$

In[376]:=

```
AddAlgebraic[x^2 - 2, x^3 - 2, x]
```

Out[376]=

$-4 - 24\,x + 12\,x^2 - 4\,x^3 - 6\,x^4 + x^6$

$\sqrt{2} \cdot \sqrt[7]{1 - \sqrt{2}}$

In[383]:=

```
ProdAlgebraic[x^2 - 2, (x^7 - 1)^2 - 2, x]
```

$\{\{1, 1\}, \{128 - 32\,x^7 + x^{14}, 1\}, \{128 + 32\,x^7 + x^{14}, 1\}\}$

Out[383]=

$128 - 32\,x^7 + x^{14}$

$\sqrt{2} + \sqrt[3]{2} + 1$

In[384]:=

```
AddAlgebraic[AddAlgebraic[x^2 - 2, x^3 - 2, x], x - 1, x]
```

Out[384]=

$$31 - 42\,x + 3\,x^2 + 9\,x^4 - 6\,x^5 + x^6$$