Lab #4

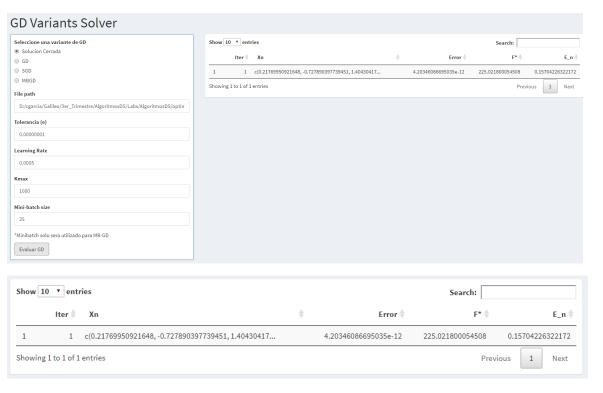
PROBLEMA 1

Parte 0 - Generación de datos

Generar Datos	
Columnas (d)	Data store successfully on path D:/Users/carlos/Desktop/datos.pickle
100	
Observaciones (n)	
1000	
Folder path	
D:/Users/carlos/Desktop/datos.pickle	
Generar datos	

Parte 1 - Solución Cerrada

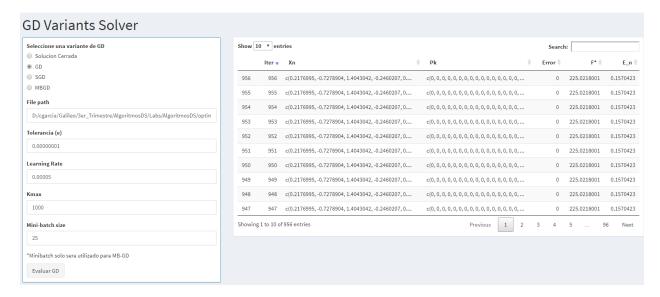
En la práctica no se suele aplicar una solución cerrada para la resolución del problema de mínimos cuadrados ya que para altos volúmenes de datos el cálculo se vuelve computacionalmente demandante.



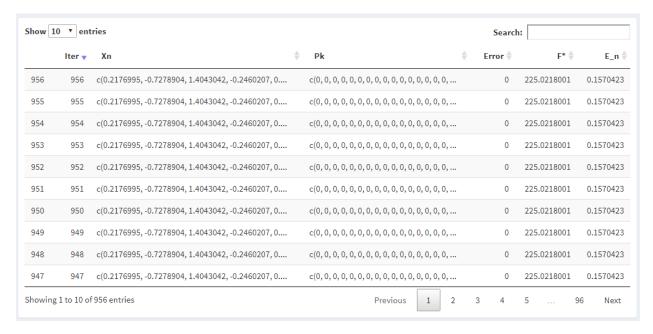
*Nota. La columna E_n representa el error entre X_true y X al computar //x_k - x_true//

Parte 2 - GD

Un step size muy pequeño puede ocasionar un proceso de aprendizaje lento, por lo cual la función objetivo a minimizar requiere de un mayor número de iteraciones para converger, mientras un valor muy grande puede ocasionar que diverja. En este ejercicio el primer valor 0.00005 es muy pequeño por lo cual en la gráfica se observa que su error disminuye más lento que los otros dos. El step size que mejor funciona en esta práctica es el step size mayor con un valor de 0.0007.



Lr = 0.00005

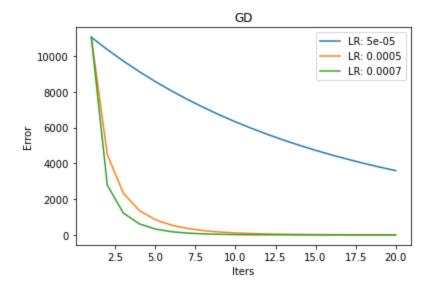


Lr = 0.0005

	Iter 🔻	Xn	\$ Pk	\$ Error 🖣	F* \$	E_n
86	86	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	225.0218001	0.1570423
85	85	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	225.0218001	0.1570423
84	84	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	225.0218001	0.1570423
83	83	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	225.0218001	0.1570423
82	82	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,	0	225.0218001	0.1570423
81	81	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,	0	225.0218001	0.157042
В0	08	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,	0	225.0218001	0.1570423
79	79	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,	1e-7	225.0218001	0.1570423
78	78	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,	1e-7	225.0218001	0.1570423
77	77	c(0.2176995, -0.7278904, 1.4043042, -0.2460207, 0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	1e-7	225.0218001	0.1570423

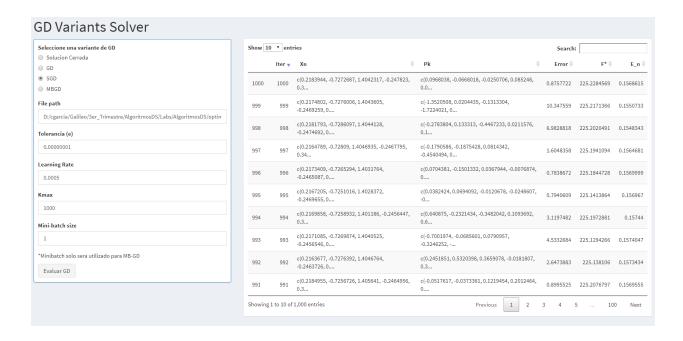
Lr = 0.0007

	Iter 🔻	Xn	\$	Pk	\diamondsuit	Error 🏺	F* 	E_n
58	58	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		0	225.0218001	0.1570423
57	57	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		0	225.0218001	0.1570423
56	56	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0		0	225.0218001	0.1570423
55	55	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		0	225.0218001	0.1570423
54	54	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		0	225.0218001	0.1570423
53	53	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		1e-7	225.0218001	0.1570423
52	52	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,		1e-7	225.0218001	0.1570423
51	51	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0		1e-7	225.0218001	0.1570423
50	50	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1e-07, 0, 0		2e-7	225.0218001	0.1570423
49	49	c(0.2176995, -0.7278904, 1.4043042, -0.2460207,	0	c(0, 0, 0, 0, 0, 0, 0, 0, 1e-07, 0, 0, -1e-07, .		3e-7	225.0218001	0.1570423



Parte 3 - SGD

En el caso de SGD el step size se comporta igual que en lo descrito para GD, sin embargo, es importante tomar en consideración que dado la actualización de los parámetros de forma recurrente que genera este algoritmo, la función objetivo fluctúa y no es monótona decreciente. En general el step size de 0.0005 genera valores más bajos que los demás dado que es menor y evita que la función diverja.



Lr = 0.0005

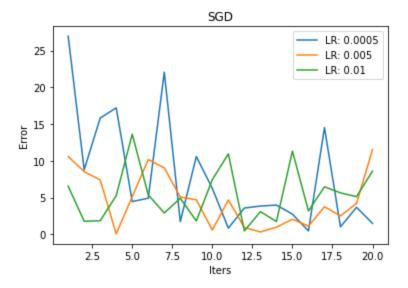
ow 10	enti	ries		Search:		
	lter ▼	Xn	\$ Pk	\$ Error 🏺	F* \$	E_n (
1000	1000	c(0.2183944, -0.7272687, 1.4042317, -0.247823, 0.3	c(0.0968038, -0.0668018, -0.0250706, 0.085248, 0.0	0.8757722	225.2284569	0.1568615
99	999	c(0.2174802, -0.7276006, 1.4043605, -0.2469259, 0	c(-1.3520508, 0.0204435, -0.1313304, -1.7224021, 0	10.347559	225.2171366	0.1550733
98	998	c(0.2181793, -0.7286097, 1.4044128, -0.2474692, 0	c(-0.2783804, 0.133313, -0.4467233, 0.0211576, 0.1	6.9828818	225.2020491	0.1548343
97	997	c(0.2164789, -0.72809, 1.4046935, -0.2467795, 0.34	c(-0.1790586, -0.1875428, 0.0814342, -0.4540494, 0	1.6048358	225.1941094	0.1564681
96	996	c(0.2173409, -0.7265294, 1.4031764, -0.2465087, 0	c(0.0704381, -0.1501332, 0.0367944, -0.0076874, 0	0.7838672	225.1844728	0.1569999

Lr = 0.005

Show 10	0 ▼ ent	ries		Search			
	lter 🔻	Xn	÷	Pk ♦	Error \$	F* \$	E_n ♦
1000	1000	c(0.2422101, -0.7329947, 1.3886038, -0.1770028, 0		c(-0.1701694, 0.0914637, -0.0671831, -0.3044425, 0	2.5583536	305.7202127	0.3276617
999	999	c(0.1897133, -0.7217438, 1.4328917, -0.2701082, 0		c(-1.1404992, 1.0665235, 0.6102068, 0.5368463, -0	6.9621267	285.2180752	0.2914956
998	998	c(0.2275646, -0.7781923, 1.4207747, -0.2485388, 0		c(0.1638462, -0.1076662, 0.6108325, -0.7505592, -0	5.0282458	289.1627278	0.2942787
997	997	c(0.2398791, -0.7268998, 1.3890545, -0.2636472, 0		c(-1.1536714, 0.6279076, -1.5842493, -0.6985853,	13.8786246	285.1985759	0.3022362
996	996	c(0.2267822, -0.7224249, 1.4347496, -0.2189591, 0		c(-0.1349218, 0.0062201, -0.1744518, 0.1302051, -0	1.7929403	293.7402232	0.2860799

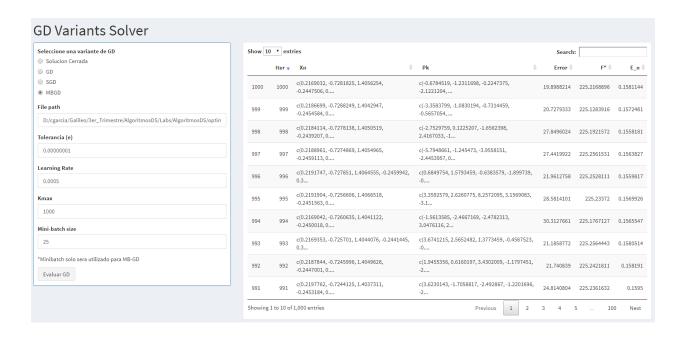
Lr = 0.01

now 10	enti	ries		Search:				
	lter ▼	Xn	è	Pk \$		Error \$	F* 	E_n (
1000	1000	c(0.1978909, -0.720702, 1.395652, -0.2636069, 0.25		c(-0.1350846, -0.4886863, -1.1012594, -0.9593585,	8.	.3867901	491.9861265	0.5226502
999	999	c(0.3421366, -0.7307005, 1.3863443, -0.2584764, 0		c(-0.0097062, 0.0112132, 0.0557695, -0.0476386, -0	(0.267624	428.9500685	0.4405968
998	998	c(0.2446594, -0.7872959, 1.4074952, -0.2220766, 0		c(0.3209135, -0.1455852, 0.0495347, 0.2471216, -0	2	.5558305	536.9191302	0.5610361
997	997	c(0.18037, -0.7767065, 1.4899819, -0.2278221, 0.29		c(-0.9123182, -2.3549744, -2.51389, -0.4375693, 0	17.	.1981523	426.7547064	0.4845017
996	996	c(0.2034179, -0.7136672, 1.4052034, -0.2766779, 0		c(-0.4250318, -0.3176067, -0.7037151, -0.6835359,	6	.3899399	484.1466833	0.5097531

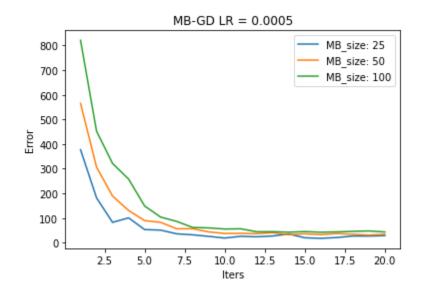


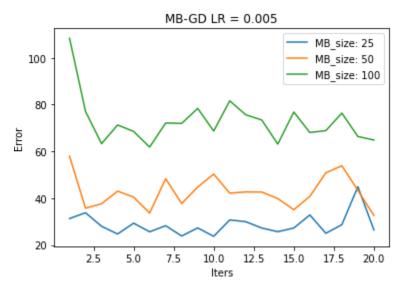
Parte 4 - MBGD

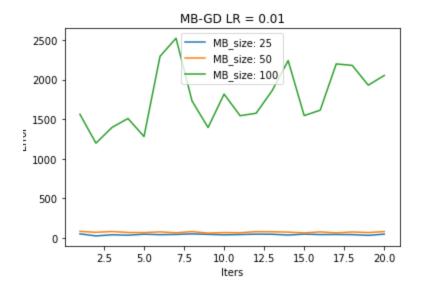
- Un mini batch size muy grande generar mayor error en el conjunto de datos, por lo cual el mini batch de 25 es el que presenta mejor desempeño
- Los mejores resultados se obtienen con un step size de 0.0005 y un mini batch size de 25, esto dado que presenta el menor error de todos los valores evaluados.



now 10	entr	ries		Search:			
	Iter 🔻	Xn	\$ Pk	÷	Error 🏺	F* \$	E_n
1000	1000	c(0.2169032, -0.7281825, 1.4056254, -0.2447506, 0	c(-0.6784519, -1.2311698, -0.2247375, -2.1221204,		19.8988214	225.2168896	0.1581144
999	999	c(0.2186699, -0.7288249, 1.4042947, -0.2454584, 0	c(-3.3583799, -1.0830194, -0.7314459, -0.5657054,		20.7279333	225.1283916	0.157248
998	998	c(0.2184114, -0.7278138, 1.4050519, -0.2439207, 0	c(-2.7529759, 0.1225207, -1.6562398, 2.4167033, -1		27.8496024	225.1921572	0.155818
997	997	c(0.2188961, -0.7274869, 1.4054965, -0.2459113, 0	c(-5.7948661, -1.245473, -3.9558151, -2.4453957, 0		27.4419922	225.2561531	0.156382
996	996	c(0.2191747, -0.727851, 1.4064555, -0.2459942, 0.3	c(0.6849754, 1.5793459, -0.6383579, -1.899739, -0		21.9612758	225.2528111	0.155981







Parte 5 – Comparación

Es posible observar que los algoritmos tienen un desempeño similar con respecto a la optimización de la función objetivo. Sin embargo, la diferencia en su comparación se resaltar a nivel de número de iteraciones entre los cuales la aplicación de la solución cerrada, así como el método clásico de GD obtienen el mejor desempeño en cuento a tiempo y resultado. A este análisis es necesario agregarle las virtudes que pueden tener los otros algoritmos ya que no utilizan todos los datos (MBGD) o bien pueden ser usados para online learning (SGD).

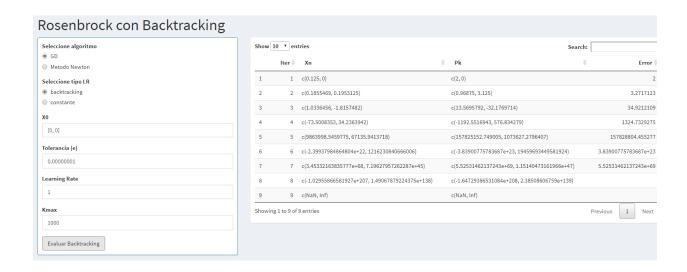
Método	LR	Mini Batch size	Valor óptimo (f [*])	No. Iteraciones	Error (x_k, x_true)
Cerrado	ı	-	225.02	1	0.157
GD	0.0007	1000	225.02	58	0.157
SGD	0.0005	1	225.18	1000	0.156
MB-GD	0.0005	25	225.19	1000	0.158

PROBLEMA 2

Parte 1 – GD con Backtracking Line Search

X ₀	Error Backtracking	Error LR constante
(0, 0)	Inf	0.69
(0.6, 0.6)	0.17	0.22
(-0.5, 1)	0.77	2.18
(-1.2, 1)	2.09	2.11

Usando un step size constante de 0.0005 es posible notar que backtracking funciona mejor para todos los puntos iniciales con excepción del punto (0, 0). Adicionalmente la selección del step size constante fue a través de experimentación y evaluación de resultados pues valores mayores al utilizado tienden a divergir, mientras que backtracking determina el step size de forma automática según el algoritmo.



 $X_0 = [0, 0]$

ow :	v 10 v entries Search:						
	Iter 🔻	Xn	\$	Pk \$	Error		
9	9	c(NaN, Inf)		c(NaN, Inf)			
	8	c(-1.02955866581927e+207, 1.49067879224375e+138)		c(-1.64729386531084e+208, 2.38508606759e+139)	1.64729386531084e+208		
,	7	c(3.45332163835777e+68, 7.19627957262287e+45)		c(5.52531462137243e+69, 1.15140473161966e+47)	5.52531462137243e+69		
	6	c(-2.39937984864804e+22, 1216230840666006)		c(-3.83900775783687e+23, 19459693449581924)	3.83900775783687e+23		

$X_0 = [0.6, 0.6]$

how 10 🔻 entri	es			Search:	
	lter ▼	Xn	♦ Pk	\$	Error 🔷
1000	1000	c(0.8379667, 0.7014747)	c(0.084957	77, 0.1427852)	0.1661488
999	999	c(0.8378838, 0.7013353)	c(0.085013	37, 0.142865)	0.166246
998	998	c(0.8378007, 0.7011957)	c(0.085069	96, 0.142945)	0.1663433
997	997	c(0.8377177, 0.7010562)	c(0.085125	56, 0.143025)	0.1664407

$X_0 = [-0.5, 1]$

Show 10 v entries				Search:	
	lter 🔻	Xn	♦ Pk	\$	Error 🔷
1000	1000	c(0.4695584, 0.2178303)	c(0.5634576, 0.5314685)		0.77456
999	999	c(0.4690082, 0.2173113)	c(0.5646658, 0.5319816)		0.7757911
998	998	c(0.4684567, 0.2167918)	c(0.5658786, 0.5324949)		0.777026

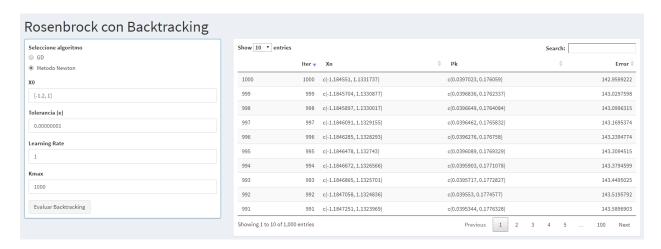
$X_0 = [-1.2, 1]$

Show 10 v entries				Search:	
	Iter 🔻	Xn	\$ Pk	\$	Error \$
1000	1000	c(-0.5320881, 0.2907745)	c(1.4332242, -1.5318962)		2.0978174
999	999	c(-0.532788, 0.2915225)	c(1.4318908, -1.5324489)		2.0973104
998	998	c(-0.5334871, 0.2922708)	c(1.4305596, -1.5329986)		2.0968036
997	997	c(-0.5341856, 0.2930193)	c(1.4292308, -1.5335454)		2.0962972
996	996	c(-0.5348835, 0.2937681)	c(1.4279042, -1.5340892)		2.0957911

Parte 2 – Método de Newton con Backtracking Line Search

X ₀	Error Backtracking	Iteraciones Backtracking	Error LR constante	Iteracions LR constante
(0, 0)	0	344	0	3
(0.6, 0.6)	4.03	3000	0	6
(-0.5, 1)	11.21	3000	0	6
(-1.2, 1)	53.83	3000	7	0

En el caso del método de Newton este algoritmo se beneficia de tener un step size constante en contraste de calcularlo a través de backtracking, pues el número de iteraciones es menor al tener un step size constante de 1 mientras con backtracking solo en el primer punto inicial logra converger a 0.



 $X_0 = [0, 0]$

Show 10 v entries			Search:	
	lter ▼ Xn	♦ Pk	\$	Error 🏺
344	344 c(1, 1)	c(0,0)		0
343	343 c(1, 1)	c(0,0)		0
342	342 c(1, 1)	c(0,0)		0
341	341 c(1, 1)	c(0,0)		0

$X_0 = [0.6, 0.6]$

how 10 v entries				Search:	
	Iter 🔻	Xn	♦ Pk	\$	Error 🆫
3000	3000	c(0.3574143, 0.1405122)	c(-0.4127199, -0.3081366)		4.0282712
2999	2999	c(0.3578173, 0.1408131)	c(-0.4117917, -0.3078161)		4.0322458
2998	2998	c(0.3582195, 0.1411137)	c(-0.4108664, -0.3074956)		4.0362241
2997	2997	c(0.3586207, 0.141414)	c(-0.4099441, -0.307175)		4.0402062

$X_0 = [-0.5, 1]$

Show 10 v entries			Search:
	lter ▼ Xn	♦ Pk	
3000	3000 c(-0.7026381, 0.5336	865) c(-0.2430244, 0	0.3013758) 11.2052606
2999	2999 c(-0.7024008, 0.5333	922) c(-0.2427191, 0	0.3007926) 11.2162356
2998	2998 c(-0.7021638, 0.5330	984) c(-0.2424142, 0	0.3002104) 11.2272213
2997	2997 c(-0.701927, 0.53280	52) c(-0.2421097, 0	0.2996293) 11.2382177
2996	2996 c(-0.7016906, 0.5325	126) c(-0.2418058, 0	0.2990492) 11.2492248

$X_0 = [-1.2, 1]$

how 10 v entries				Search:	
	lter ▼	Xn	\$ Pk	\$	Error \$
3000	3000	c(-1.1207279, 1.1543755)	c(0.0993755, -0.1210504)		53.8283272
2999	2999	c(-1.1207764, 1.1544346)	c(0.0993316, -0.1209117)		53.8546214
2998	2998	c(-1.1208249, 1.1544936)	c(0.0992876, -0.1207731)		53.8809284
2997	2997	c(-1.1208734, 1.1545526)	c(0.0992437, -0.1206345)		53.9072483
2996	2996	c(-1.1209218, 1.1546115)	c(0.0991997, -0.1204959)		53.933581