Demostración

$$
\begin{aligned}
s_{xy} &= \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}[x_iy_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y}]\right) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}x_iy_i - \sum_{i=1}^{n}x_i\bar{y} - \sum_{i=1}^{n}\bar{x}y_i + \sum_{i=1}^{n}\bar{x}\bar{y}\right) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}x_iy_i - \bar{y}\sum_{i=1}^{n}x_i - \bar{x}\sum_{i=1}^{n}y_i + n\bar{x}\bar{y}\right) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}x_iy_i - n\bar{y}\frac{1}{n}\sum_{i=1}^{n}x_i - n\bar{x}\frac{1}{n}\sum_{i=1}^{n}y_i + n\bar{x}\bar{y}\right) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}x_iy_i - n\bar{y}\bar{x} - n\bar{x}\bar{y} + n\bar{x}\bar{y}\right) \\
&= \frac{1}{n}\left(\sum_{i=1}^{n}x_iy_i - n\bar{x}\bar{y}\right)
\end{aligned}
$$

$\diamond$