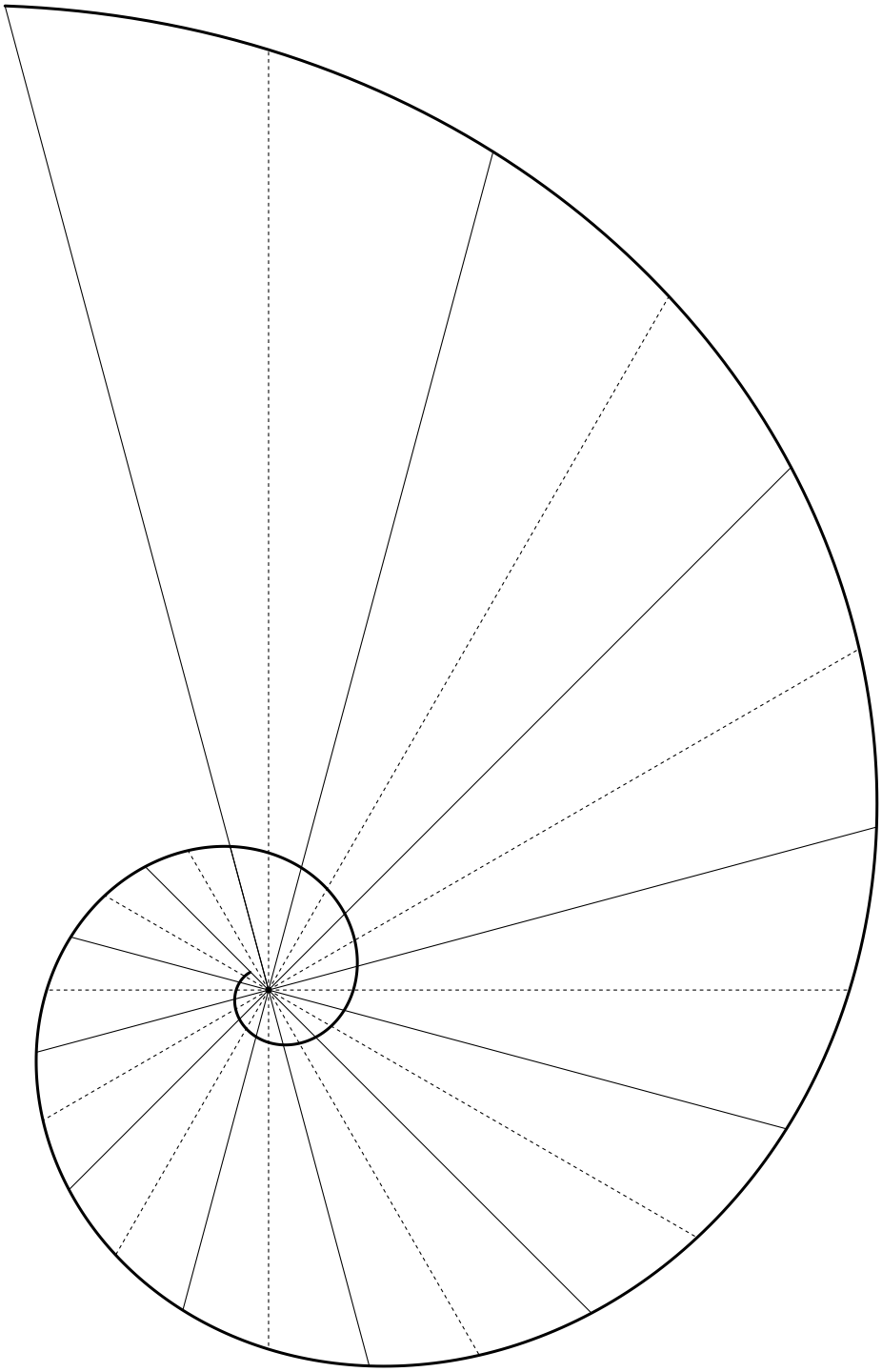
Spira Mirabilis



mmaca

Spira Mirabilis is a teaching aid developed
at the **Catalan Museum of Mathematics**.
https://mmaca.cat/

Feel free to print this document for teaching purposes.

Author: Carlos Luna Mota
Version: 2024/06/10

Spira Mirabilis

Logarithmic spirals are a family of self-similar spiral curves that are characterized by their radius growing in geometric progression as they rotate.

Albrecht Dürer described the logarithmic spirals in 1525, but it was Jacob Bernoulli who called them **Spira Mirabilis**, *«marvelous spirals»*, in 1692, because he was fascinated by their unique mathematical properties.

In this document, we use the notation *«$\lambda \,/\, \alpha$ spiral»* to refer to the particular logarithmic spiral whose radius gets multiplied by λ after rotating an angle α.

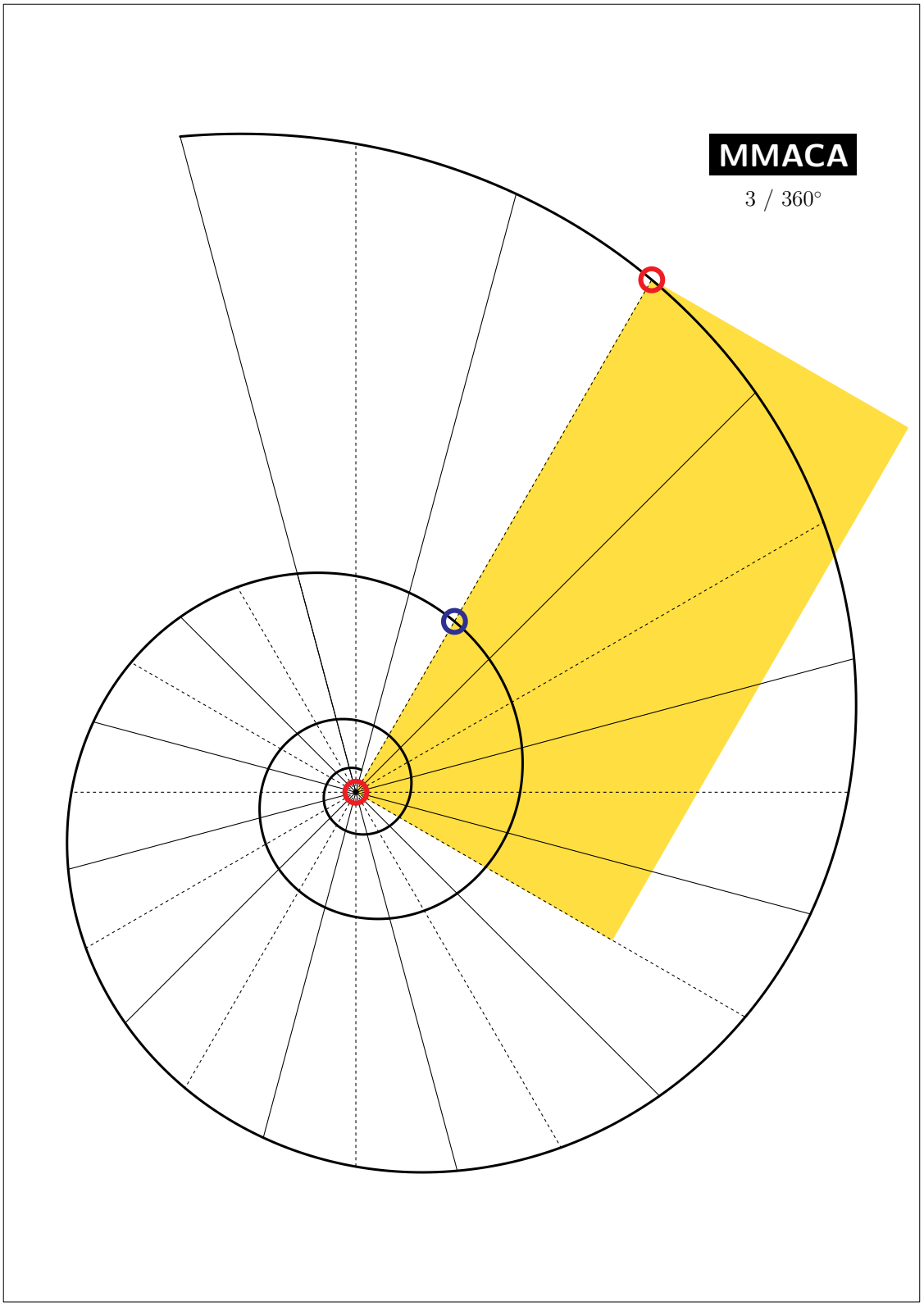
For example, the radius of the *«$2 \,/\, 90^\circ$ spiral»* gets multiplied by 2 every 90°, which means that it will be 16 times longer after a full turn.

Below you can find some usage examples for these curves and a curated set of printable templates that are free to use for educational purposes.

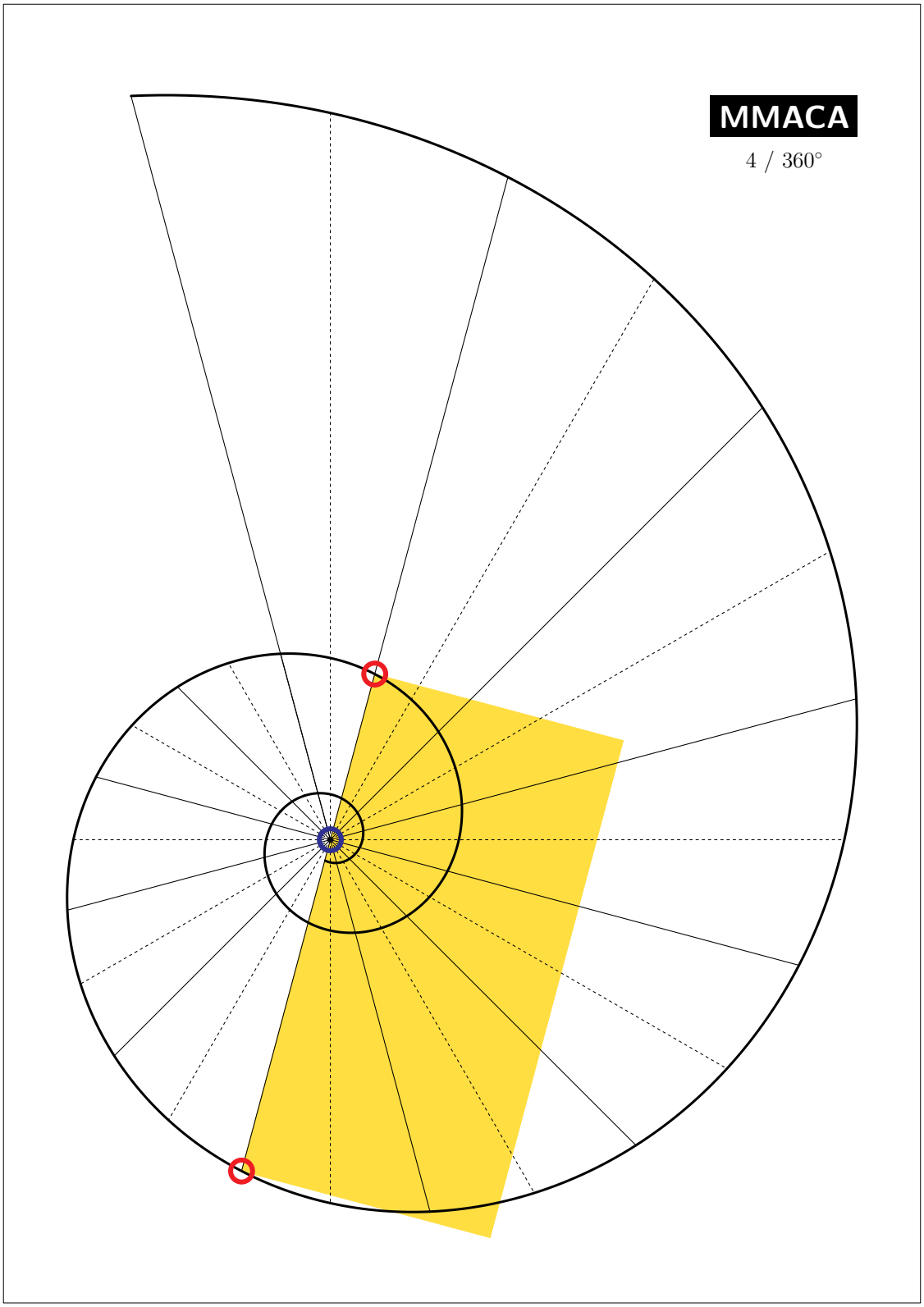
Spira Mirabilis examples

Read the red dots as the input and the blue dot as the output of each exercise

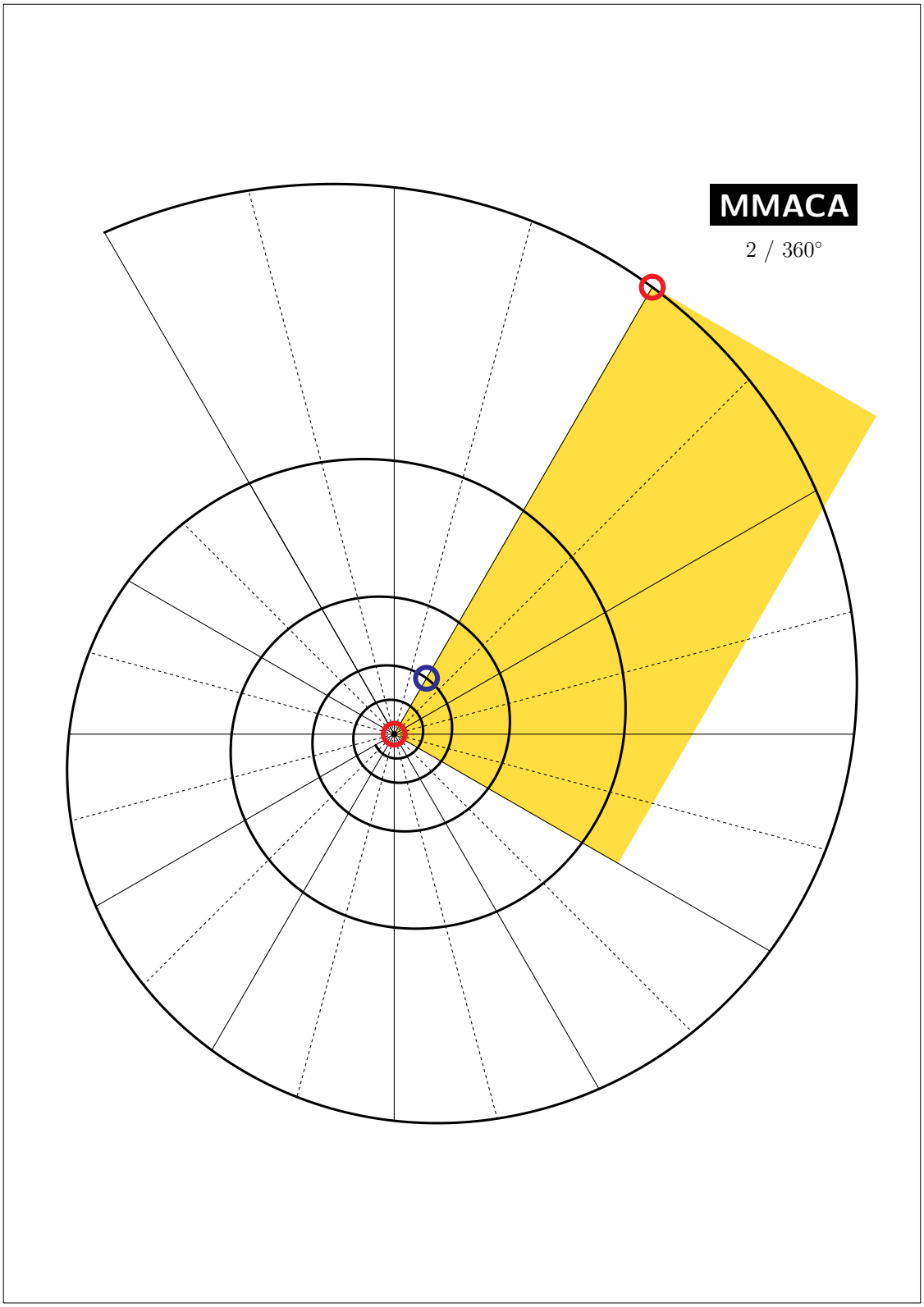
Divide a segment into thirds using the $3 \,/\, 360^\circ$ spiral



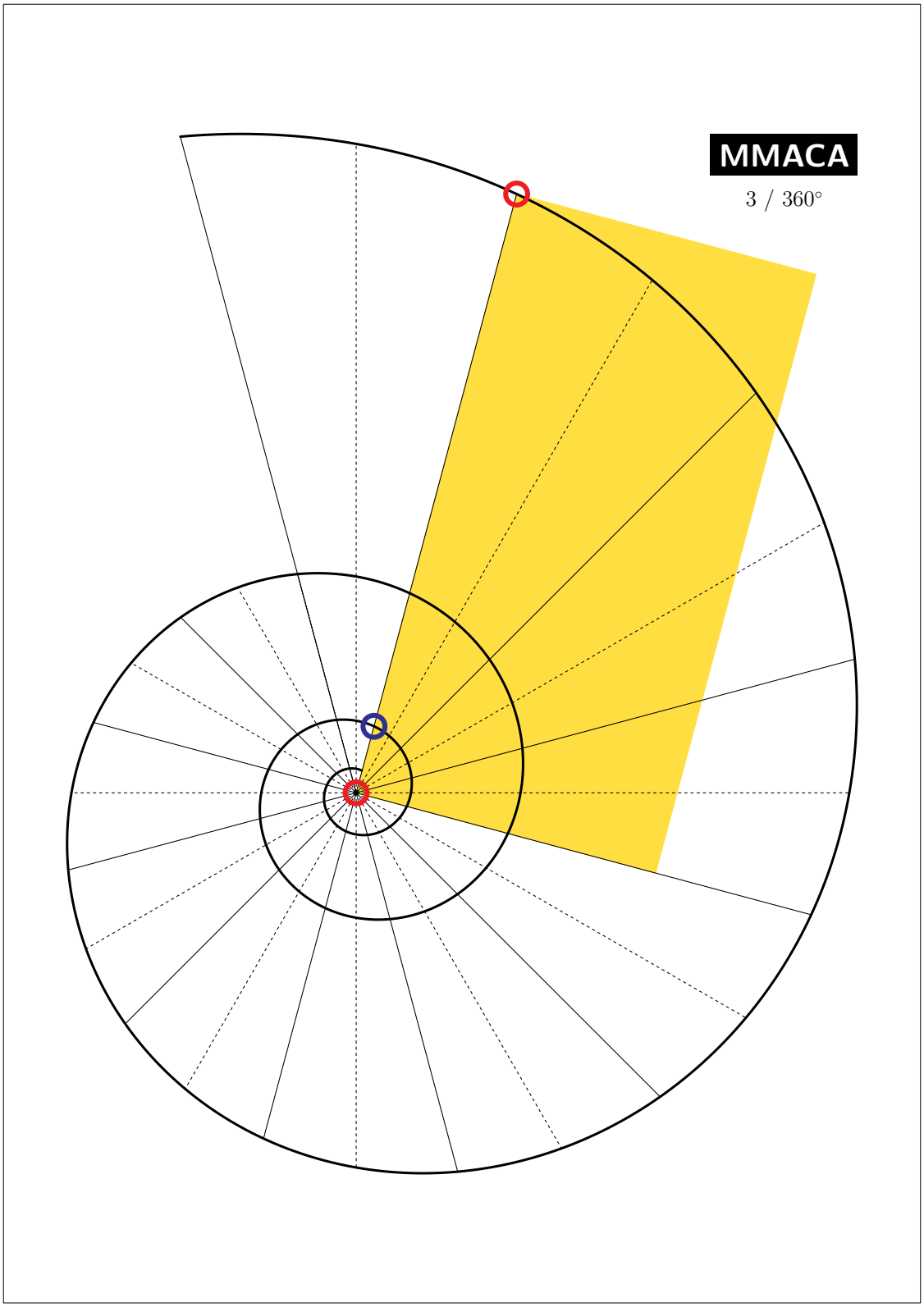
Divide a segment into thirds using the $4 \,/\, 360^\circ$ spiral



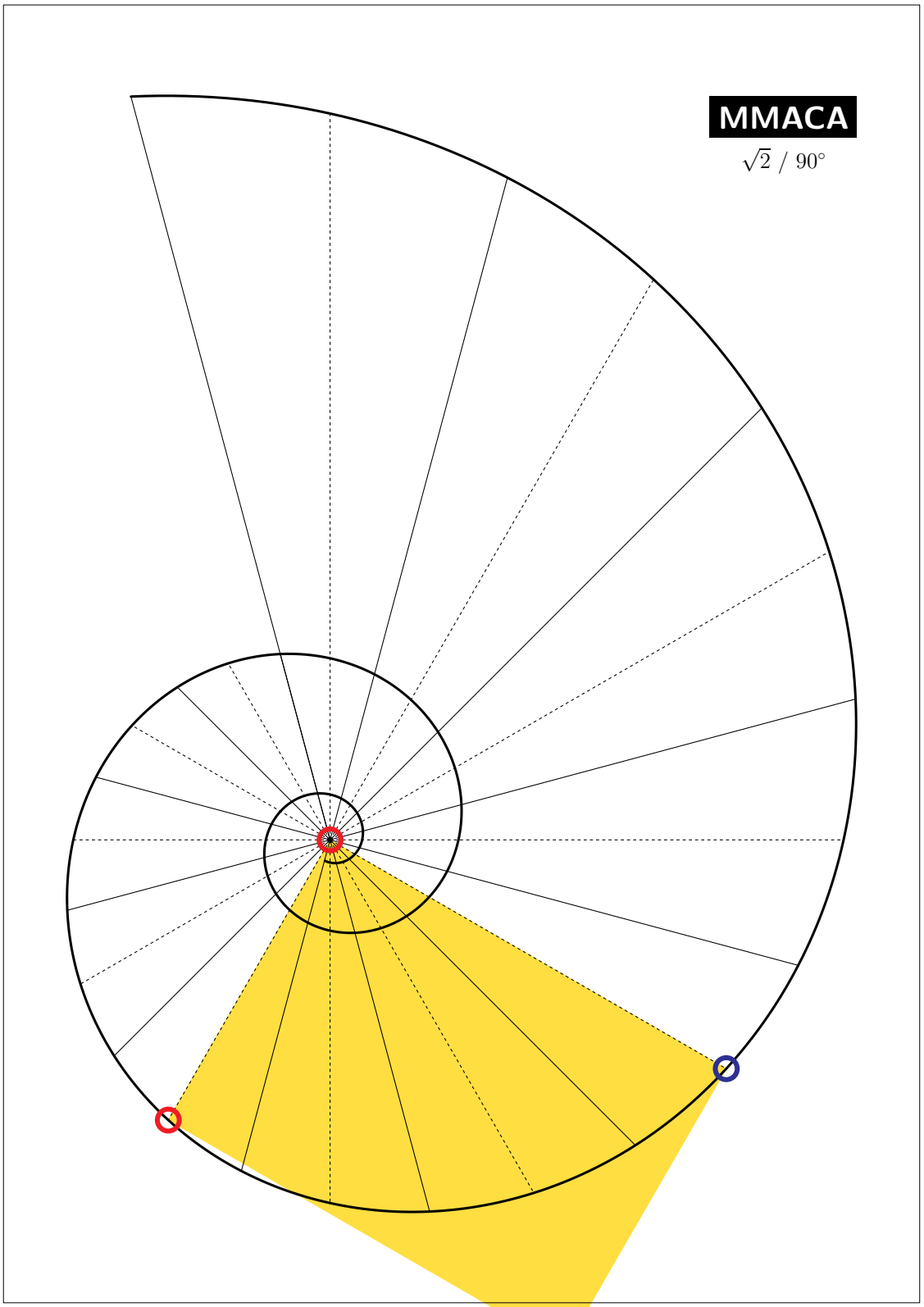
Divide a segment by 8 using the $2 \,/\, 360°$ spiral



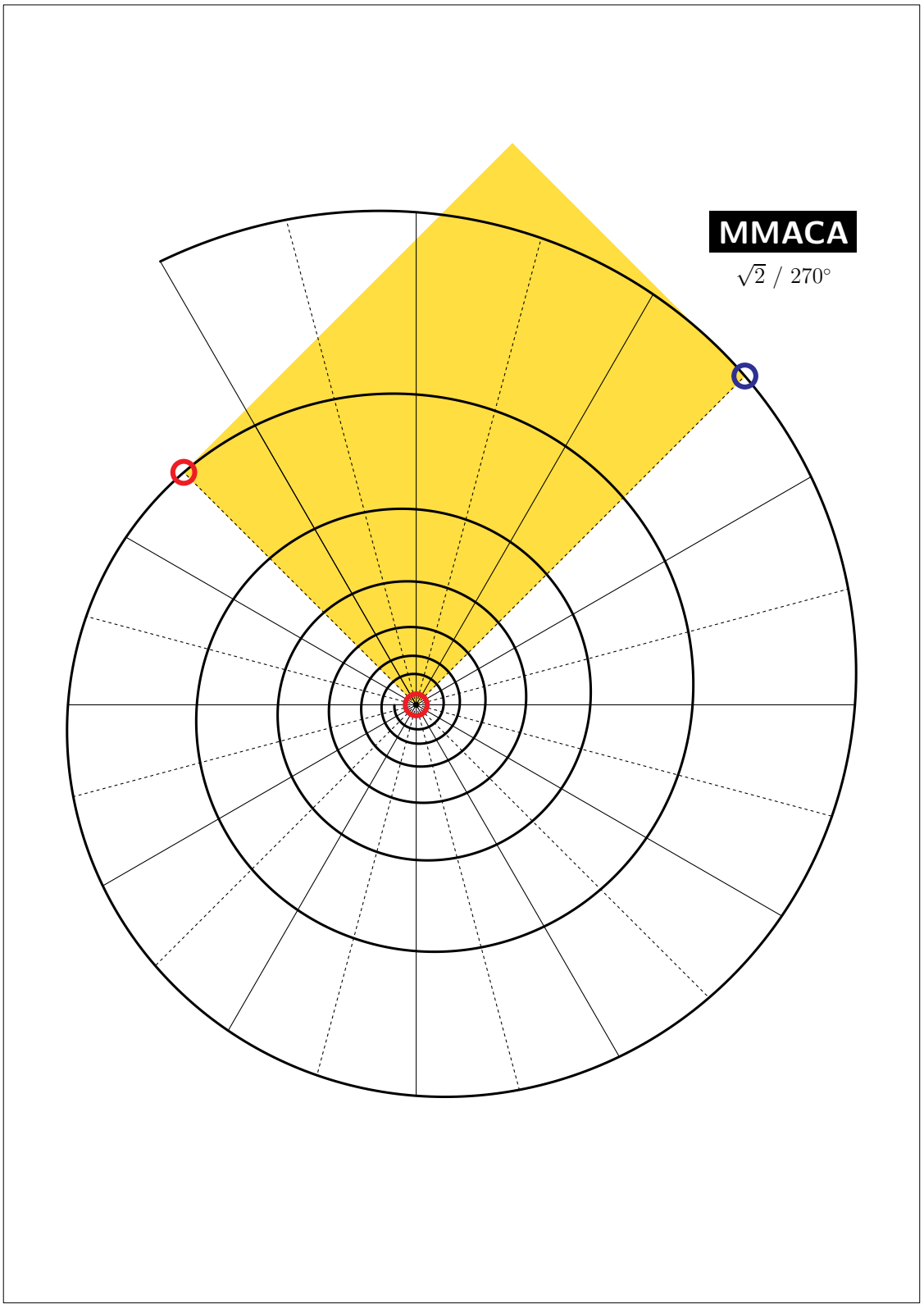
Divide a segment by 9 using the $3\,/\,360°$ spiral



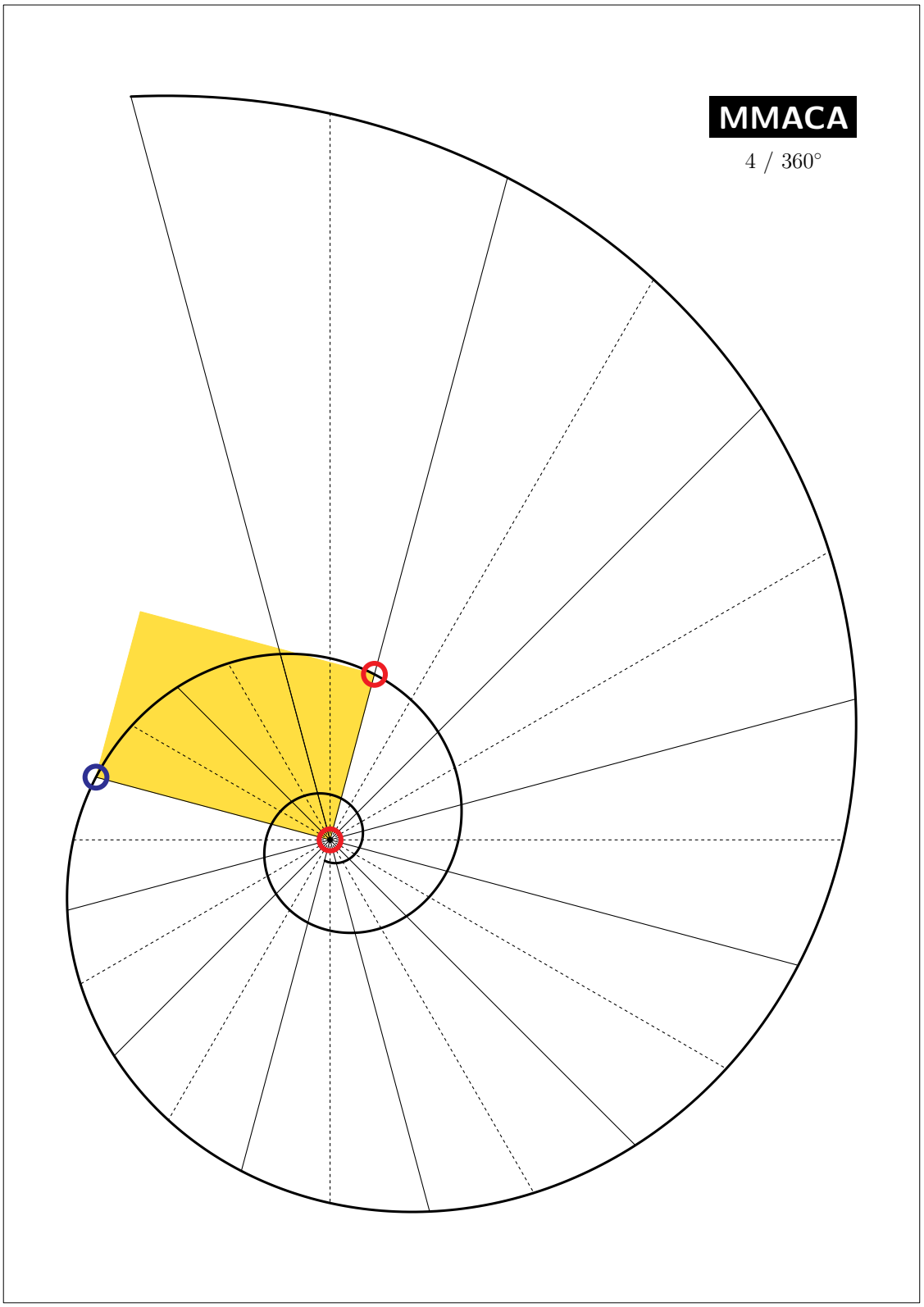
Deduce the ratio of the sides of an A7 sheet of paper using the $\sqrt{2}\,/\,90°$ spiral



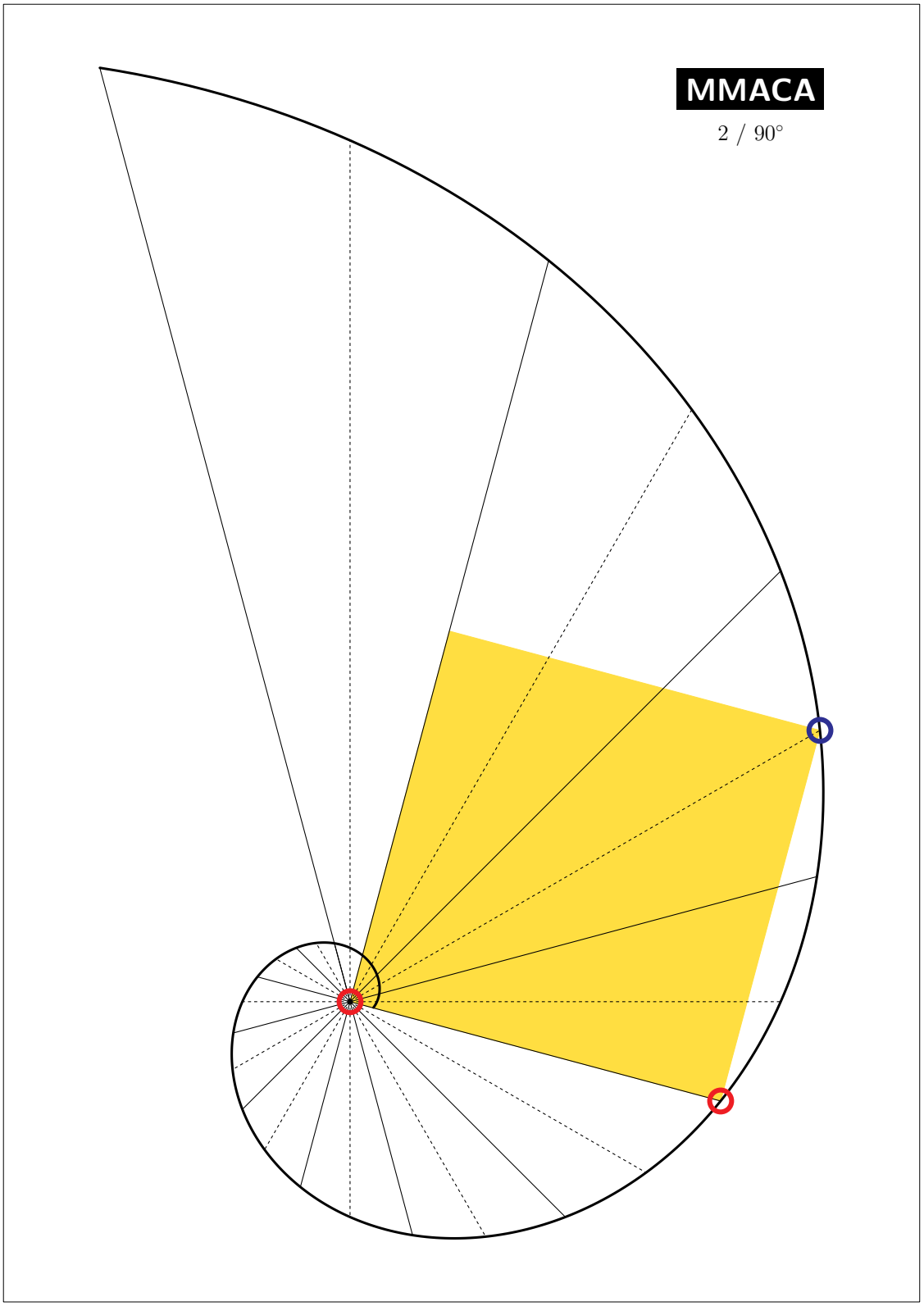
Deduce the ratio of the sides of an A7 sheet of paper using the $\sqrt{2}\,/\,270°$ spiral



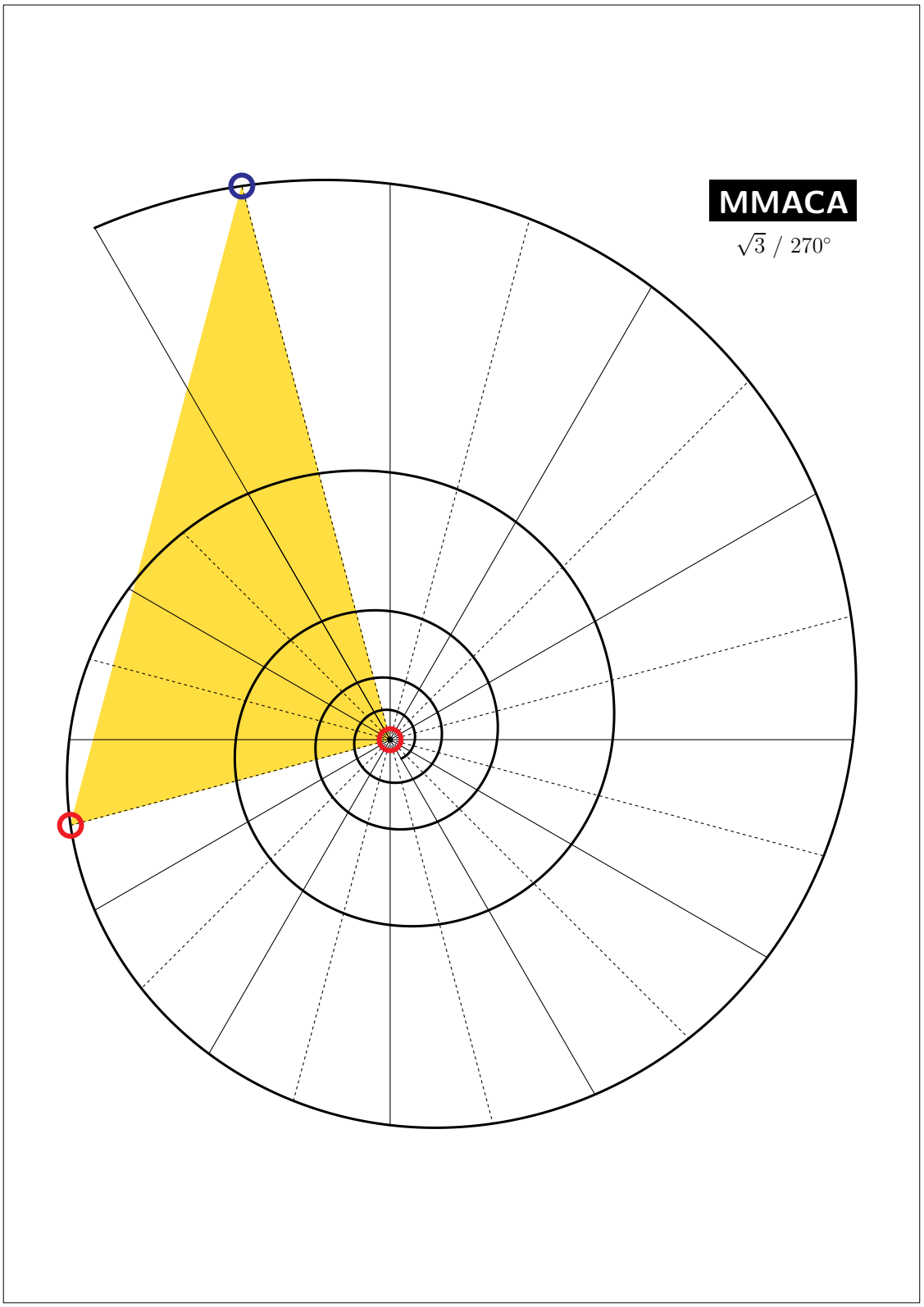
Create a rectangle of $1:\sqrt{2}$ side ratio using the $4\,/\,360°$ spiral



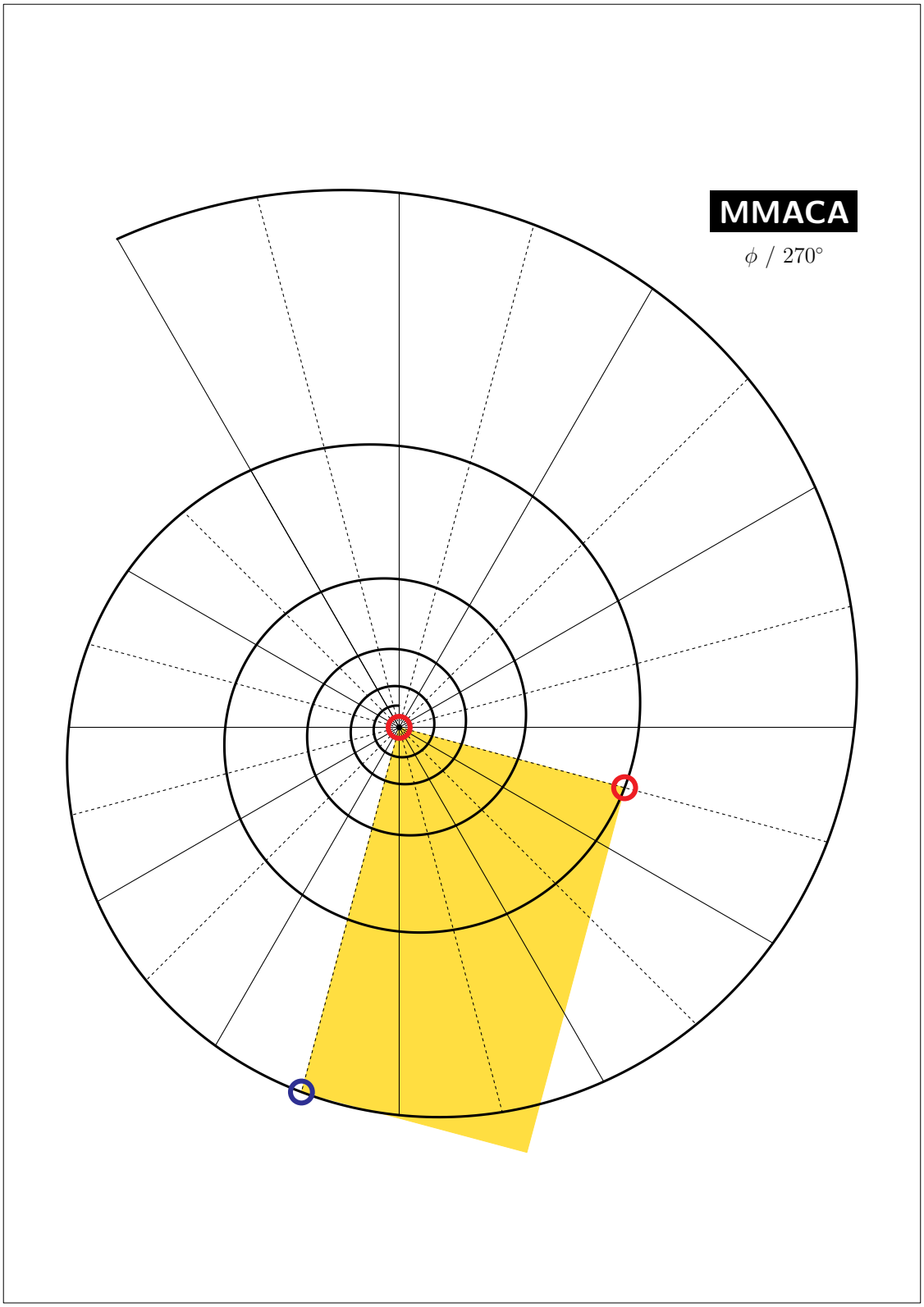
Determine the side / diagonal length ratio of a square using the $2 \,/\, 90°$ spiral



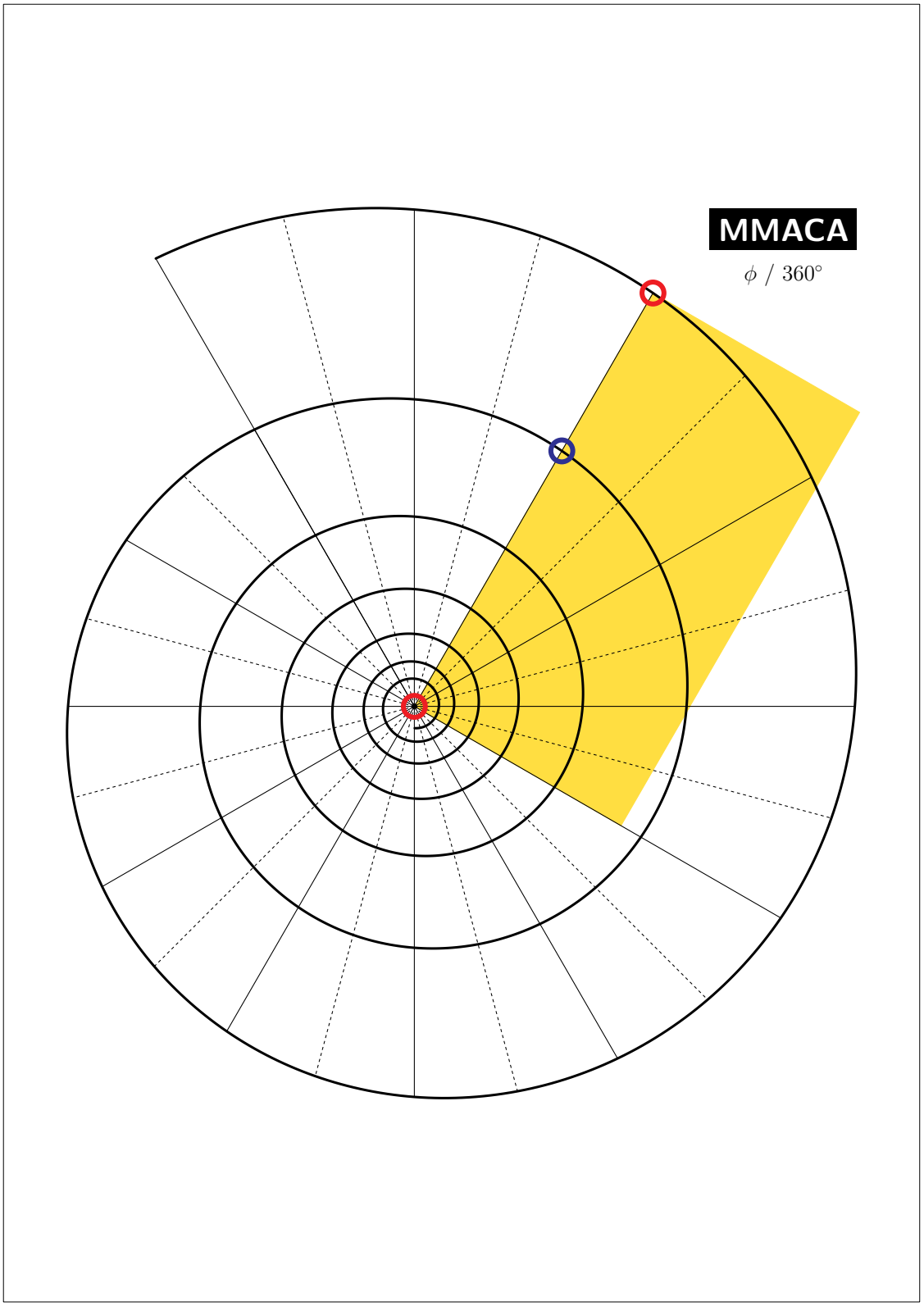
Determine leg length ratio of a $30° - 60° - 90°$ triangle using the $\sqrt{3} \,/\, 270°$ spiral



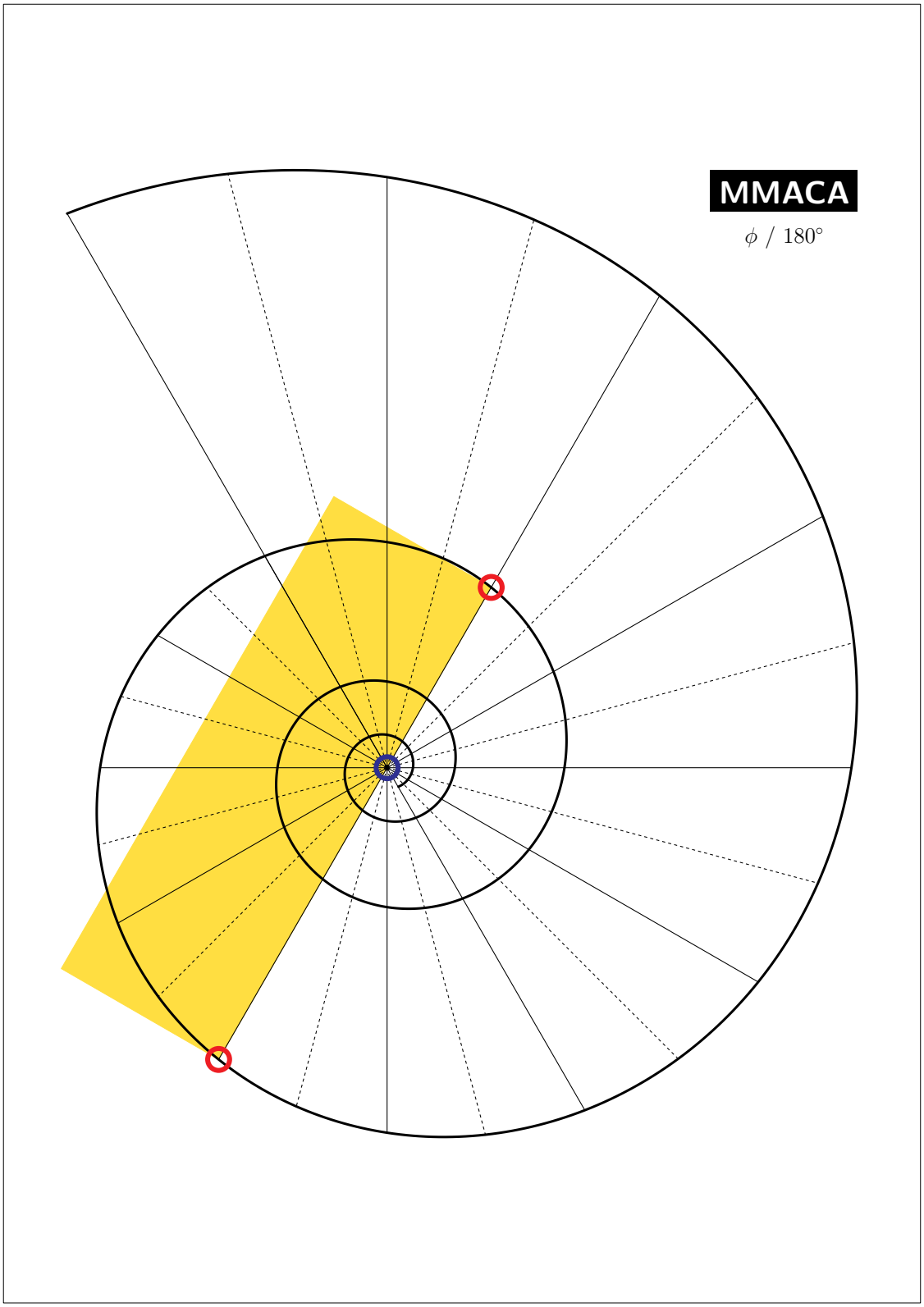
Verify that credit cards are usually golden rectangles using the $\phi \,/\, 270°$ spiral



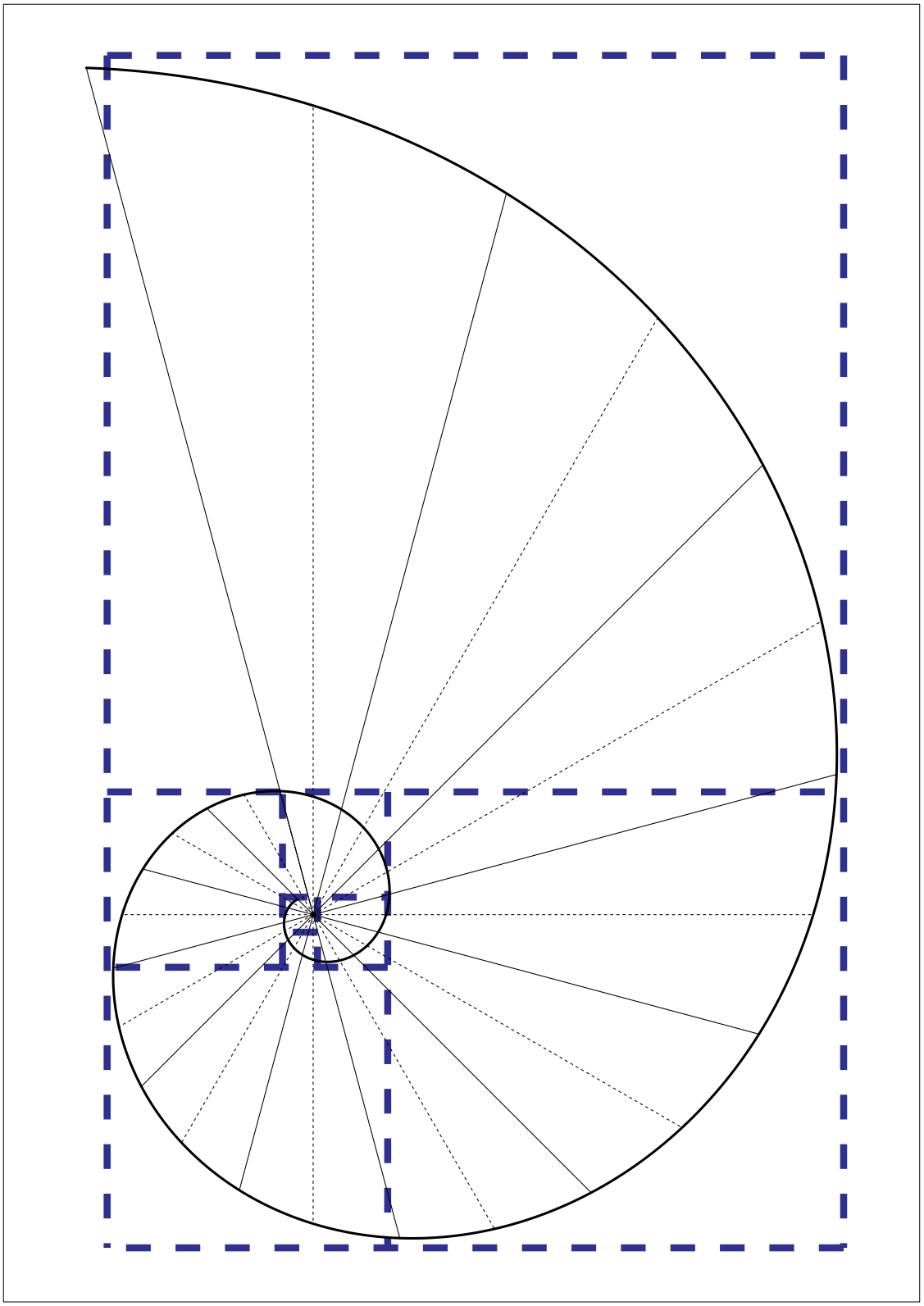
Divide a segment by the golden ratio using the $\phi \,/\, 360°$ spiral



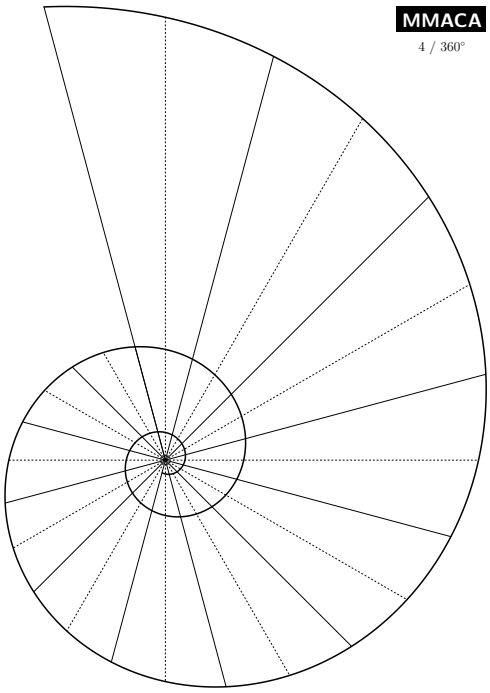
Divide a segment by the golden ratio using the $\phi\,/\,180°$ spiral

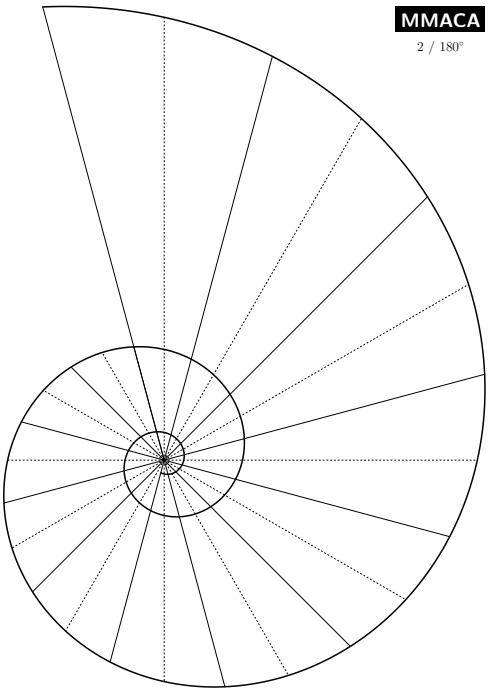


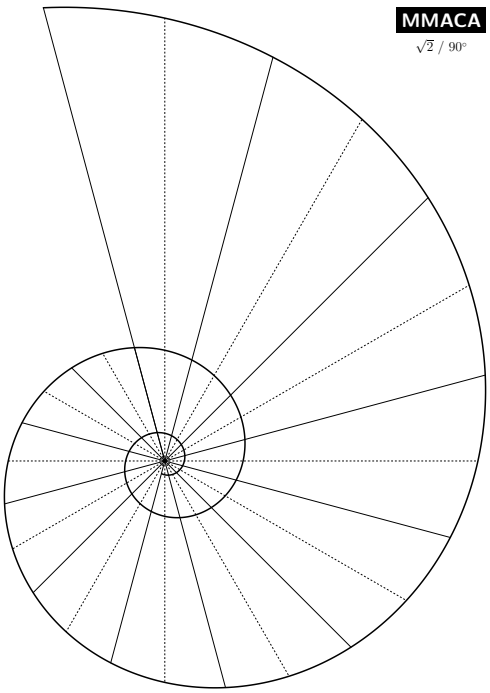
Verify that the Fibonacci spiral is a good approximation of the $\phi \,/\, 90^\circ$ spiral



Explain why the $4\,/\,360°$, the $2\,/\,180°$ and the $\sqrt{2}\,/\,90°$ spirals are equal

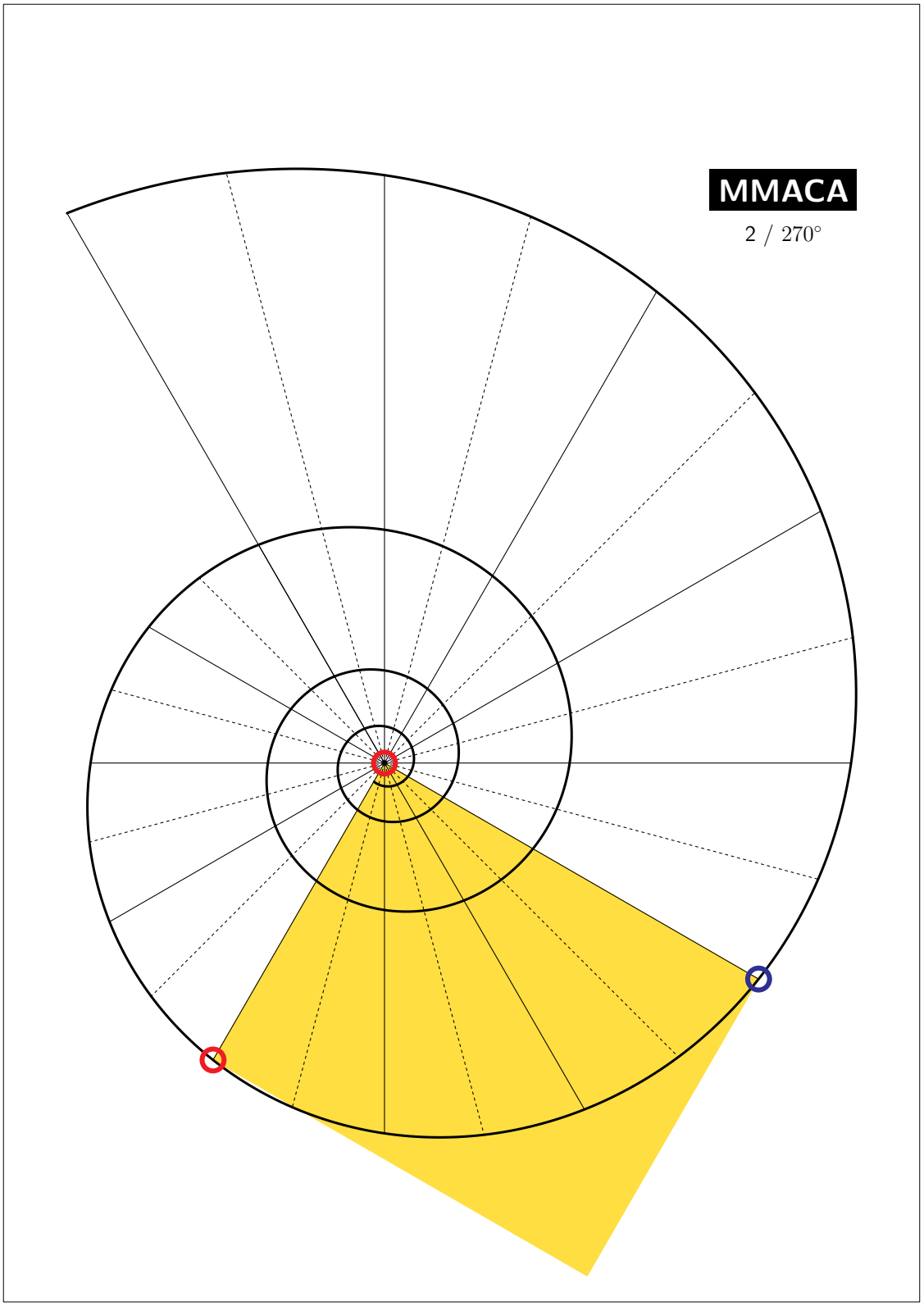






Determine the multiplicative constant λ associated with each of the 24 angles

Explain how the $2 \,/\, 270^\circ$ spiral can be used to solve the «Delian problem»

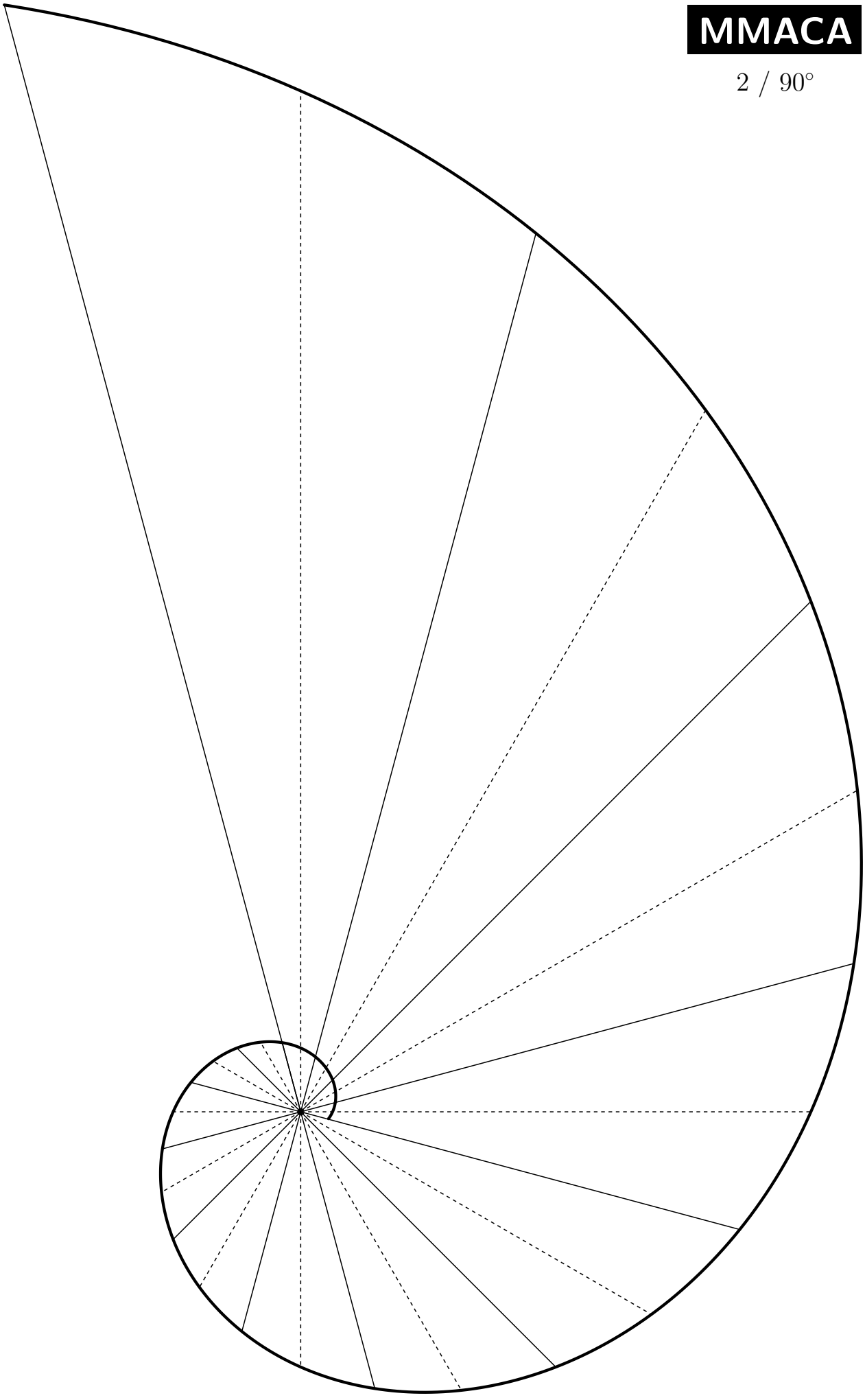


Spira Mirabilis templates

A curated set of printable templates for educational purposes

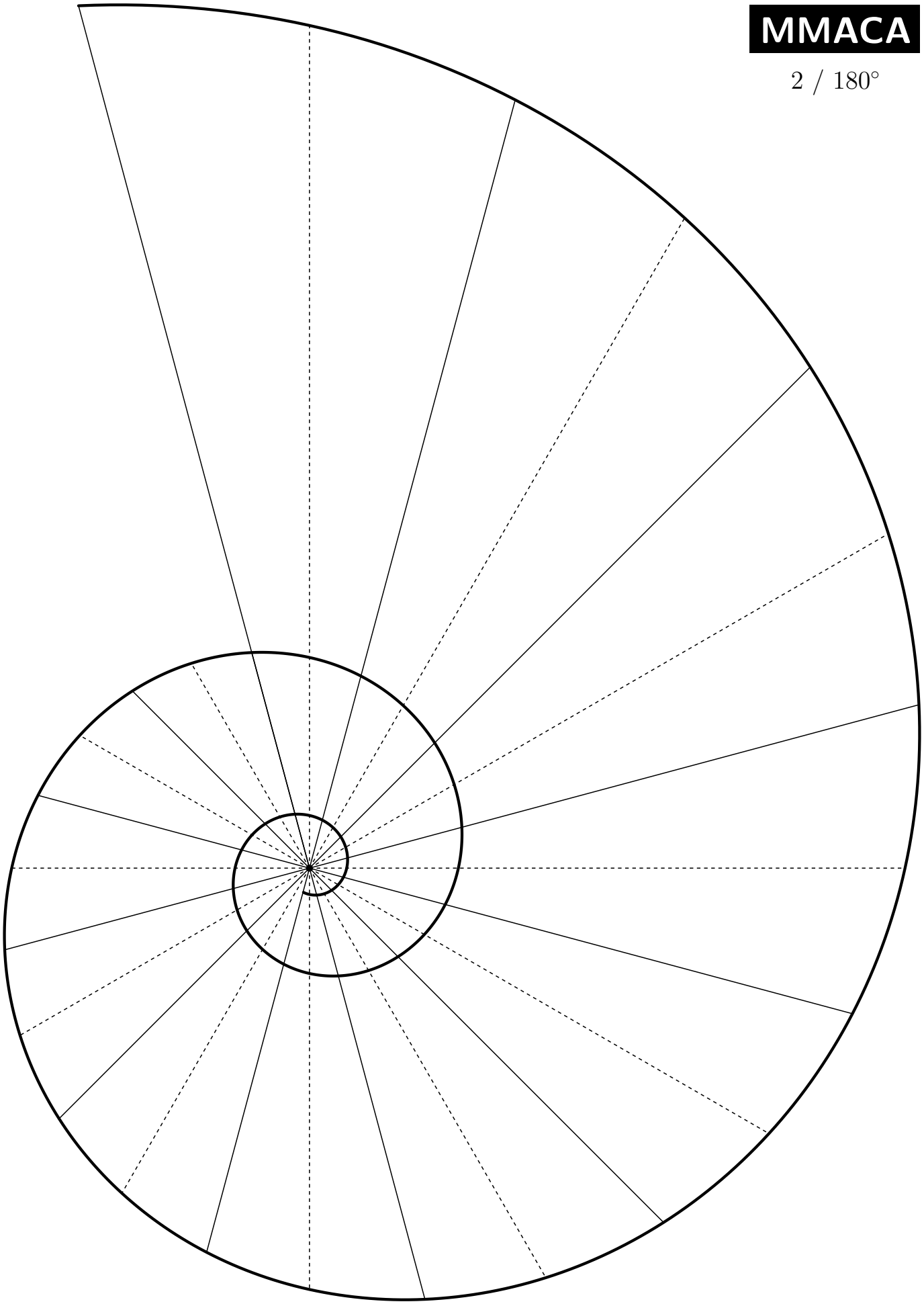
MMACA

2 / 90°



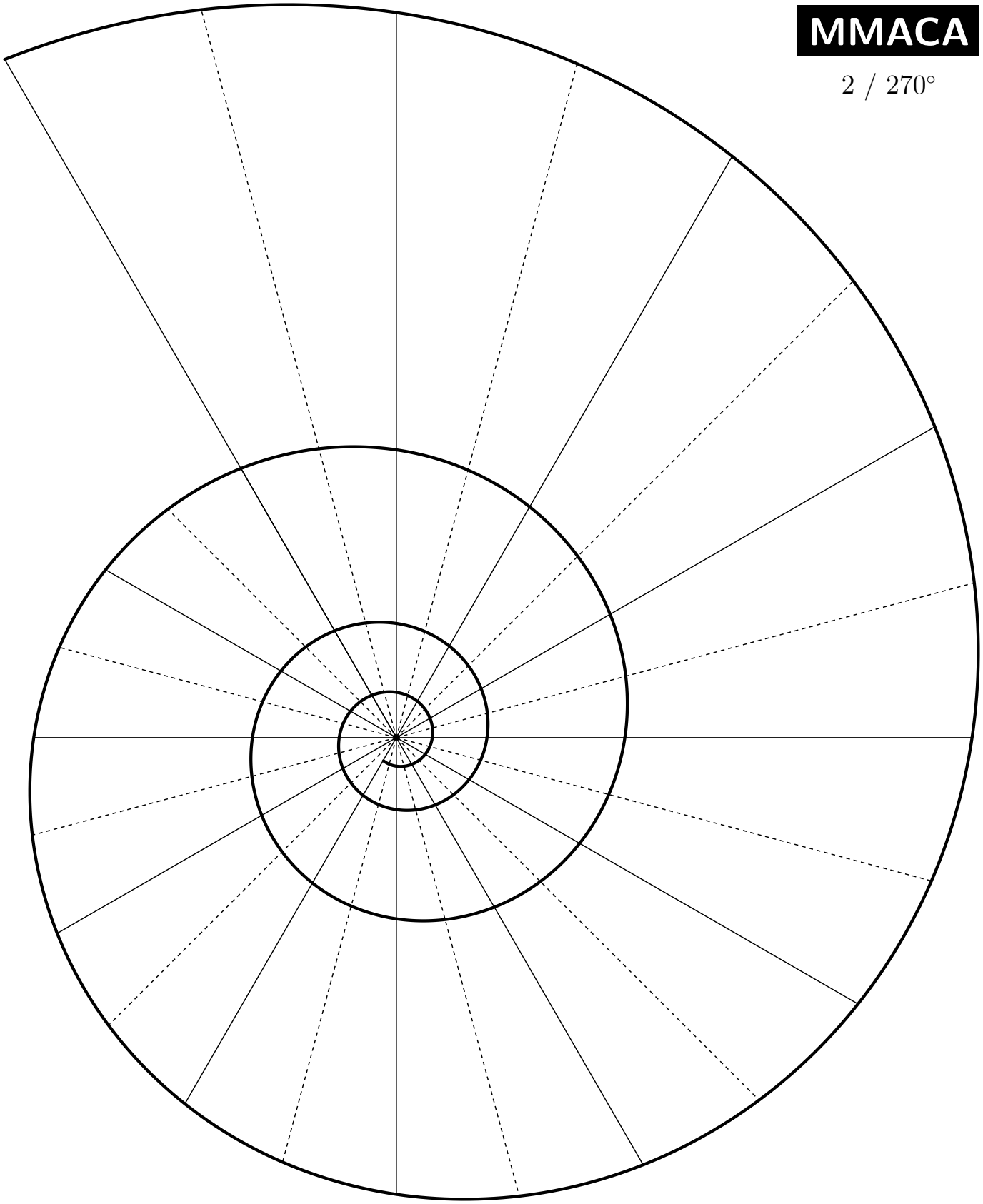
MMACA

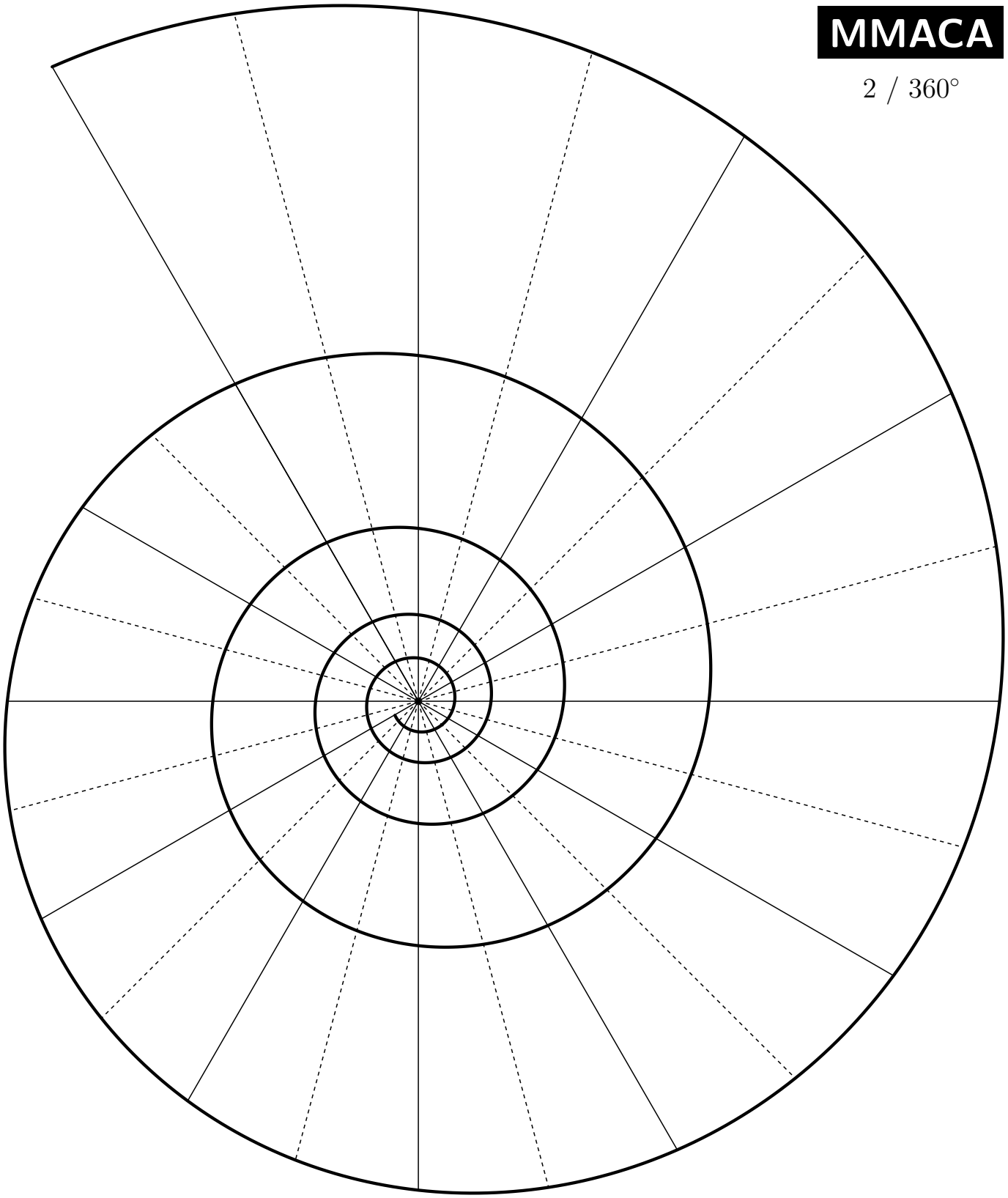
2 / 180°



MMACA

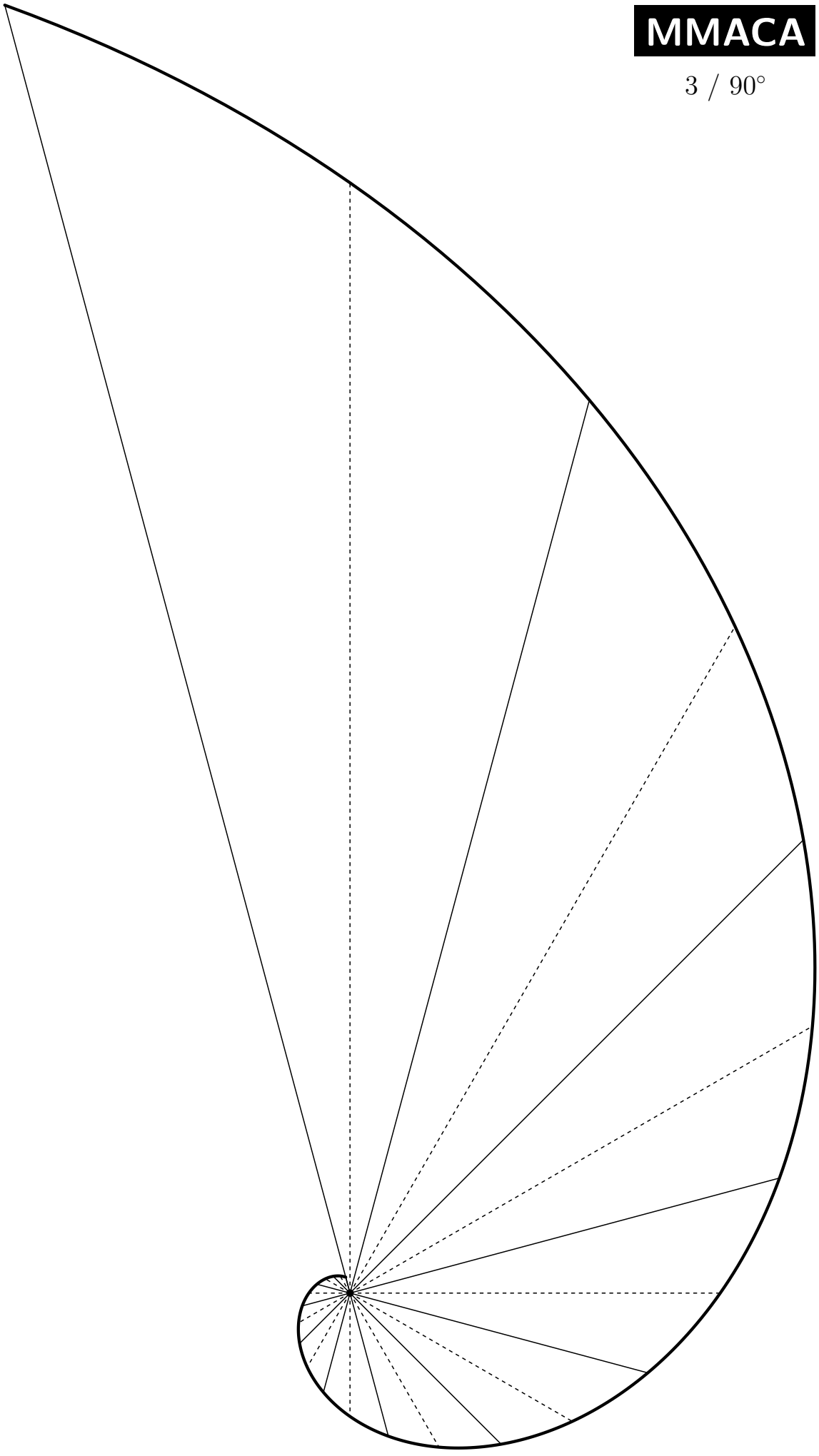
2 / 270°




MMACA
2 / 360°

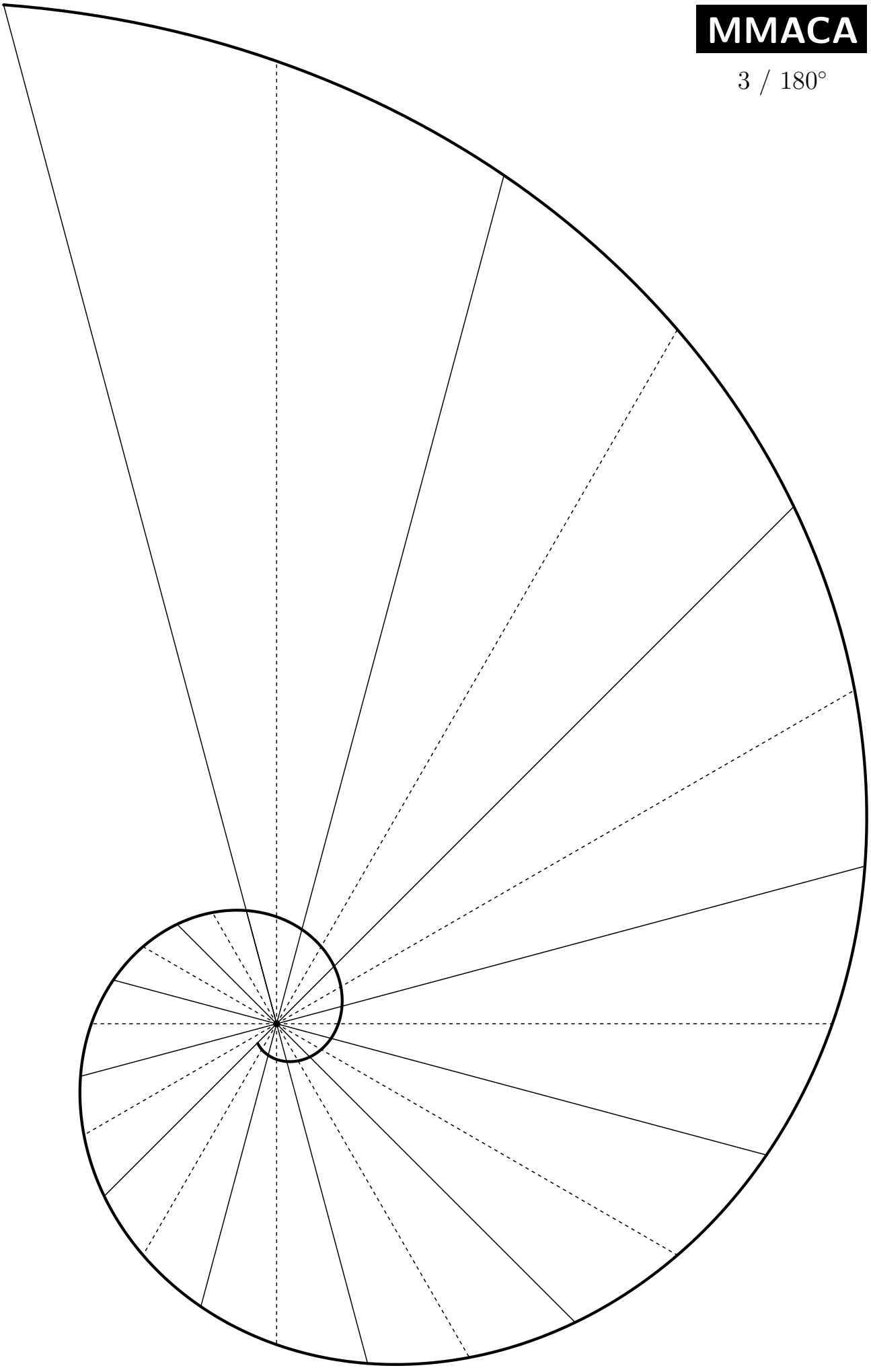
MMACA

3 / 90°



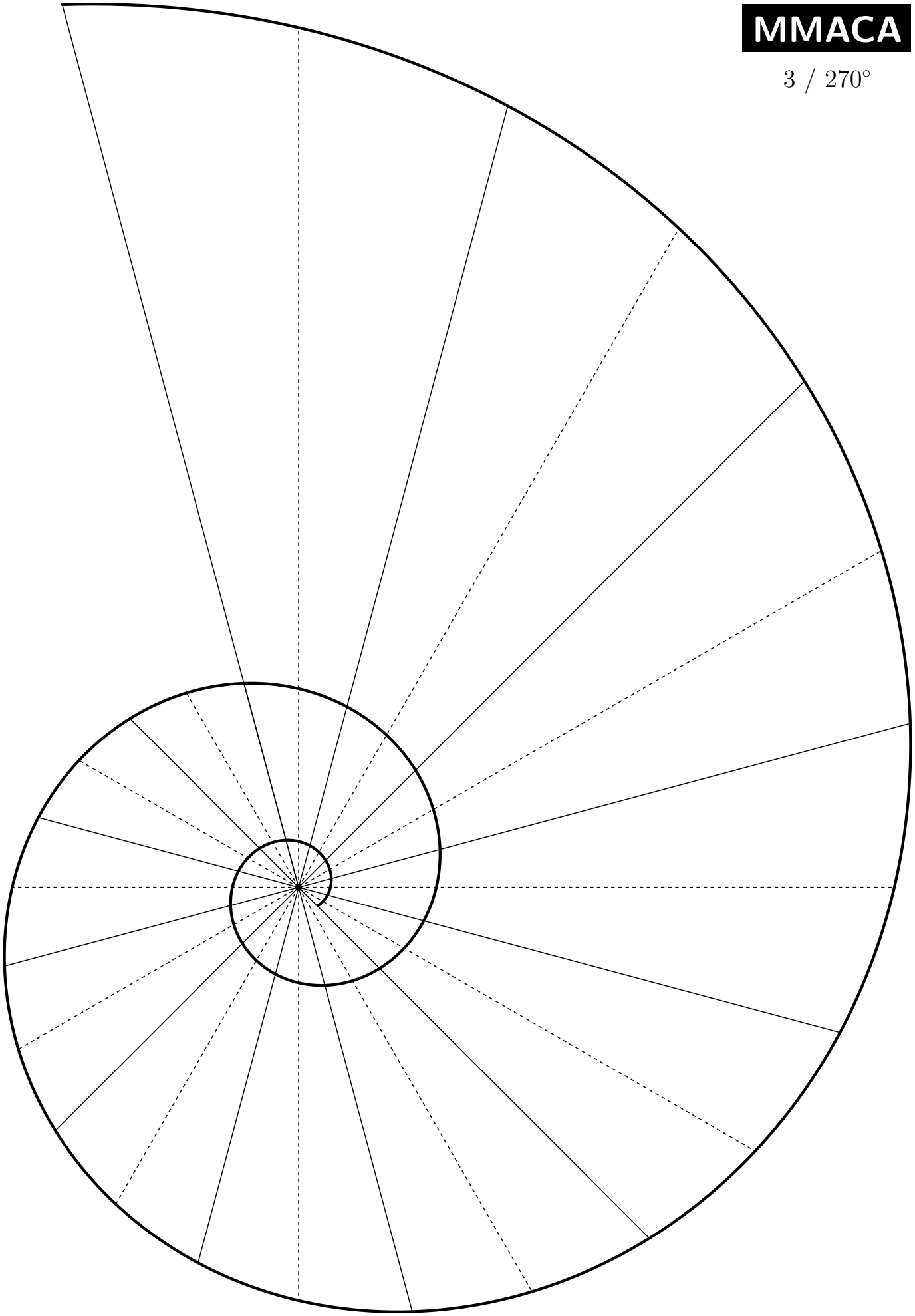
MMACA

3 / 180°



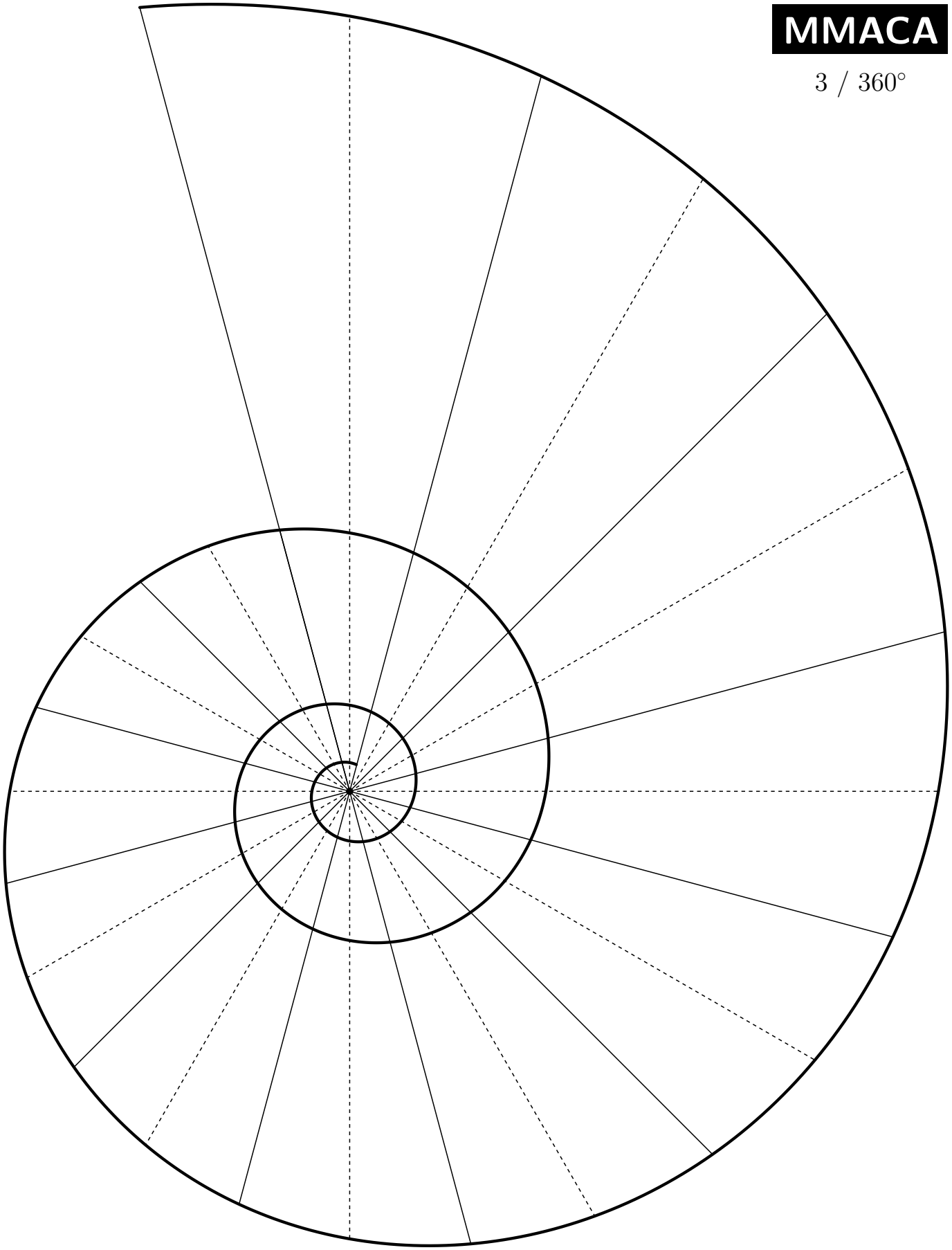
MMACA

3 / 270°



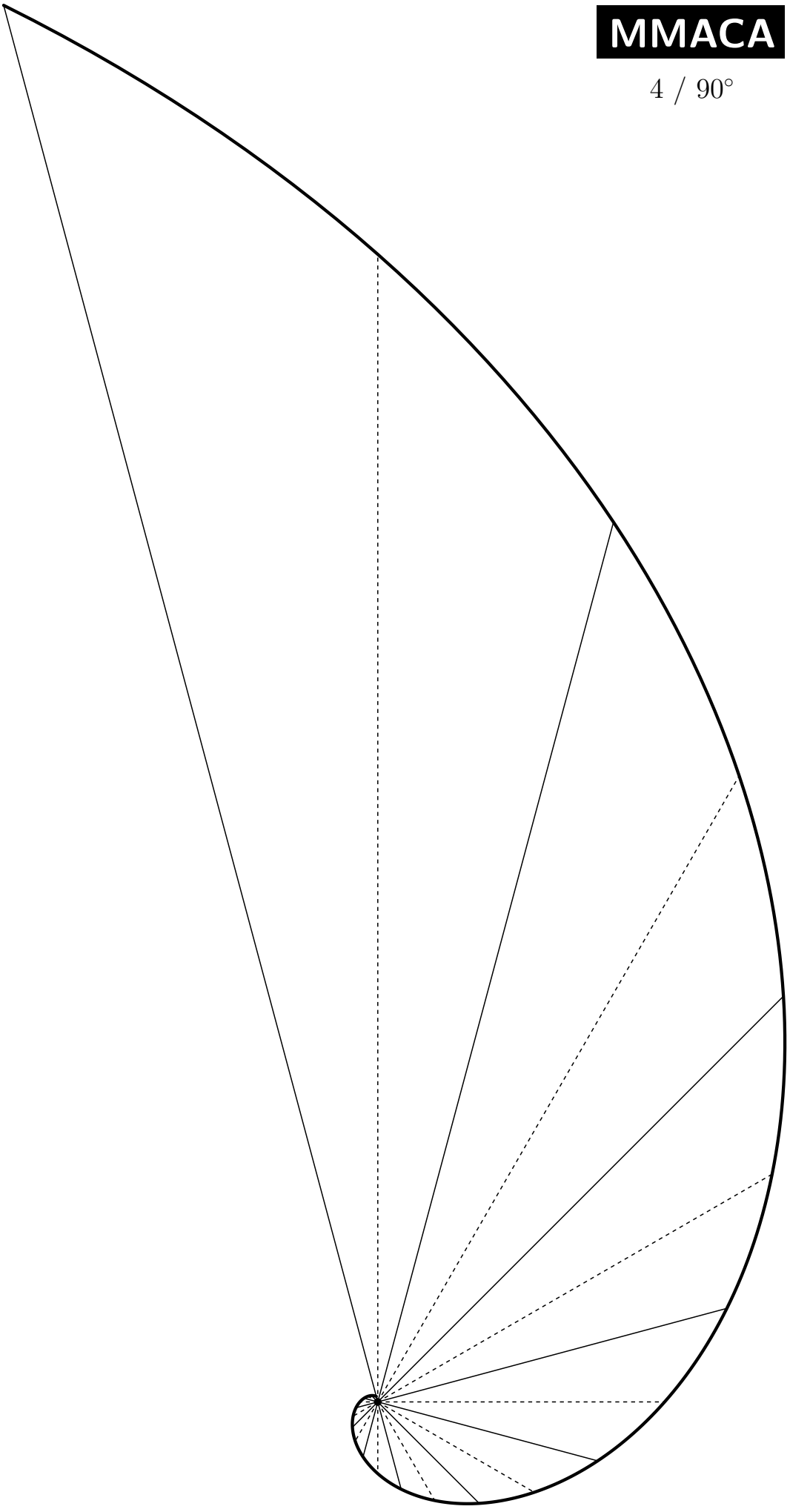
MMACA

3 / 360°



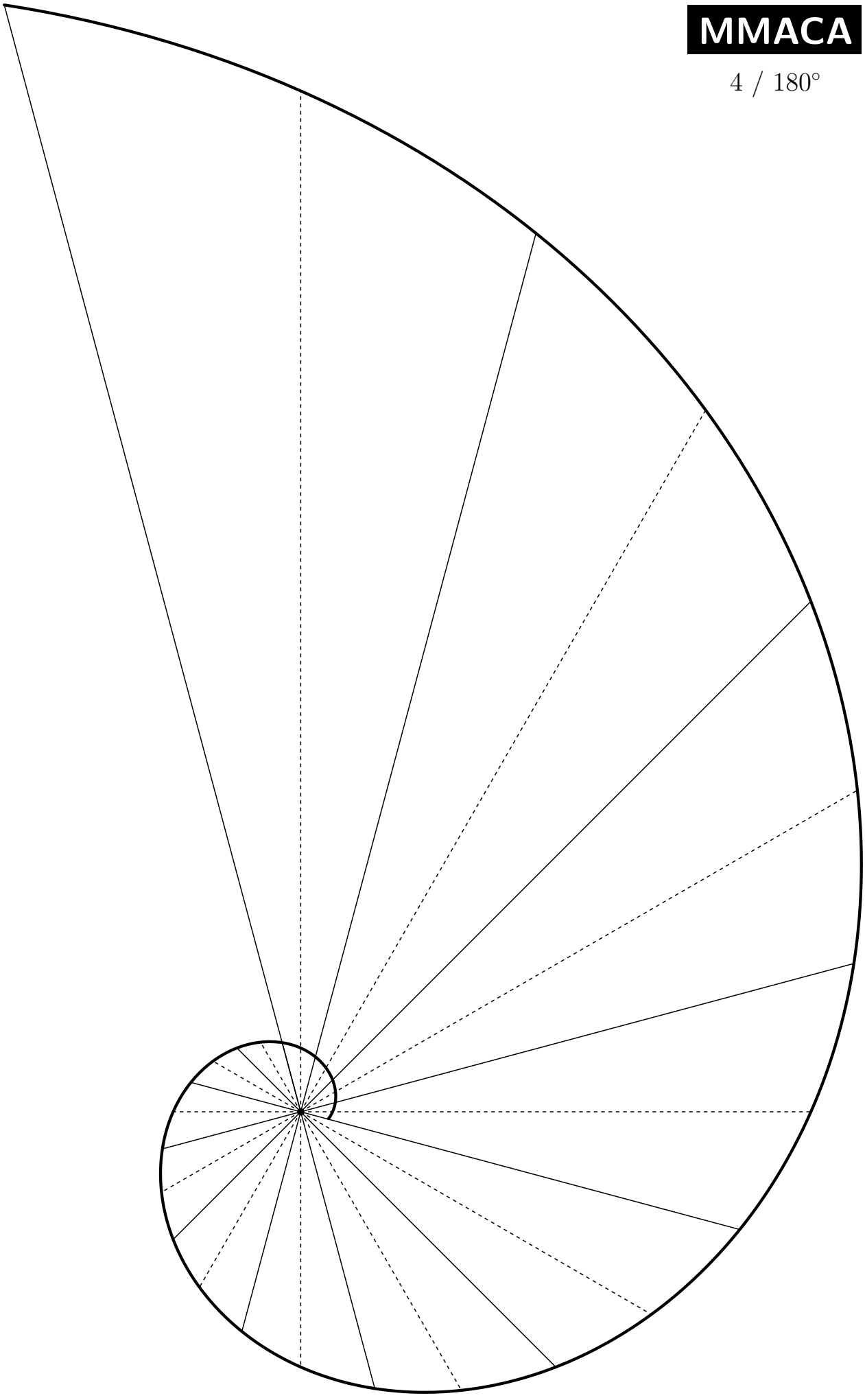
MMACA

4 / 90°



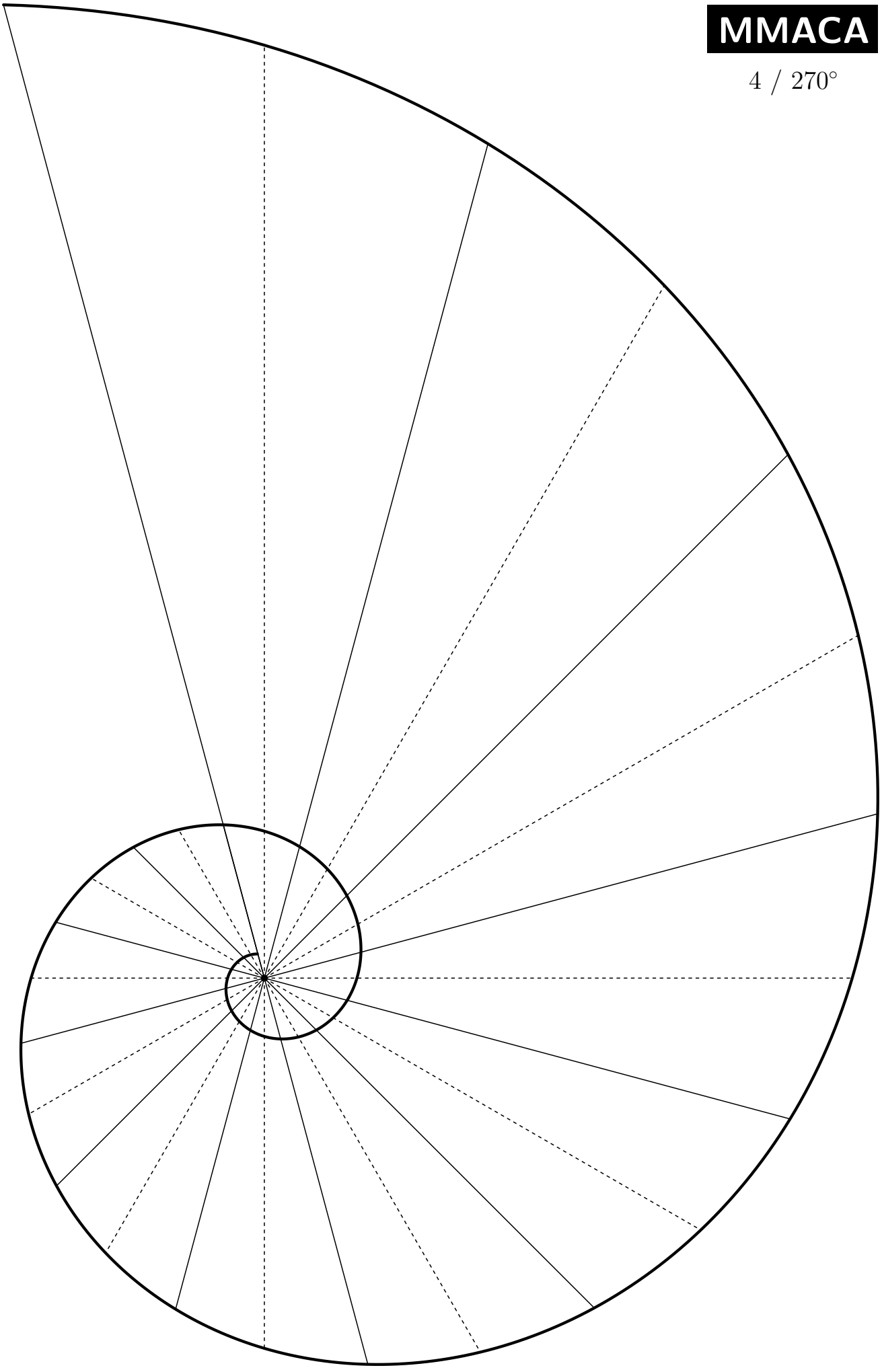
MMACA

4 / 180°



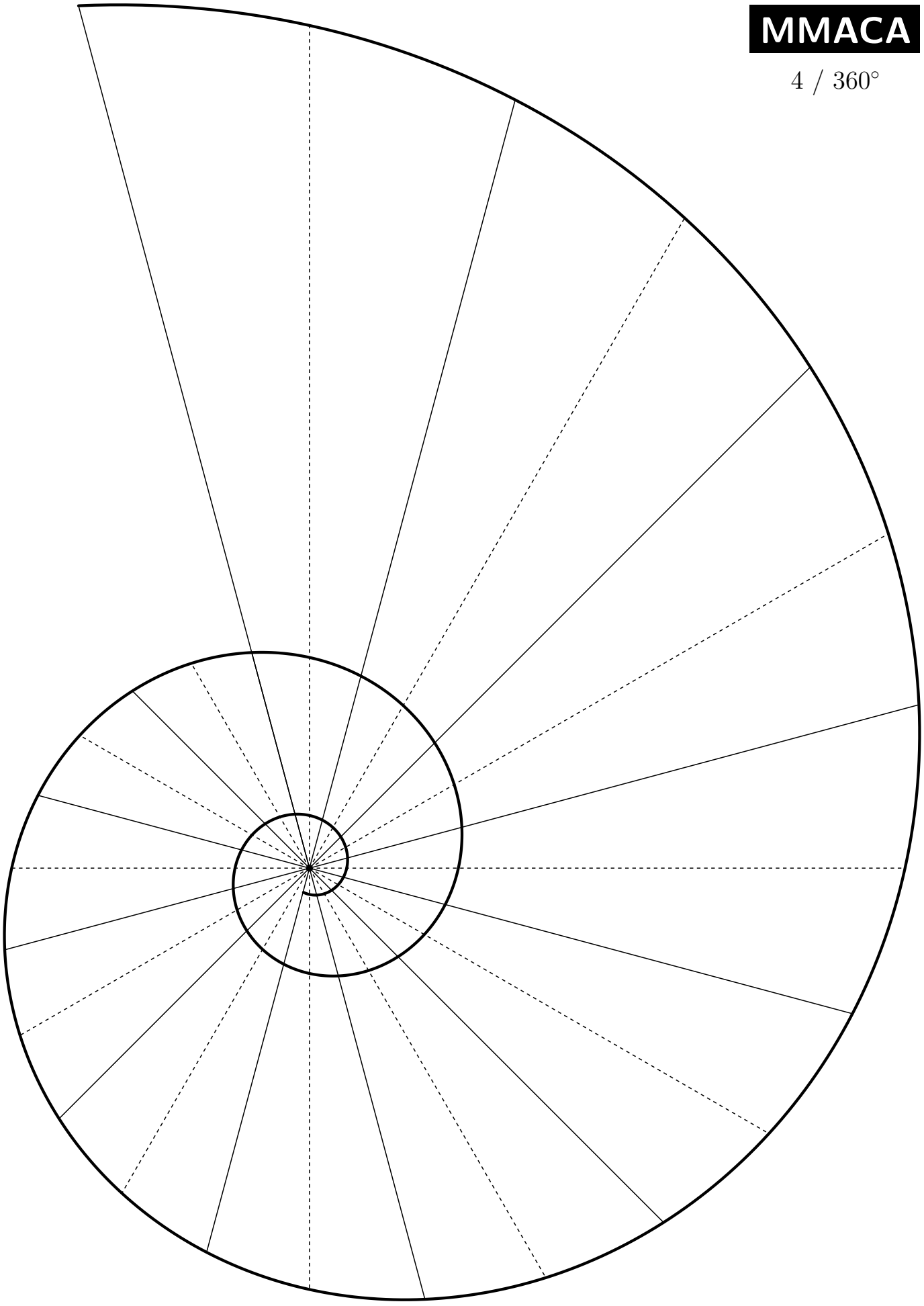
MMACA

4 / 270°



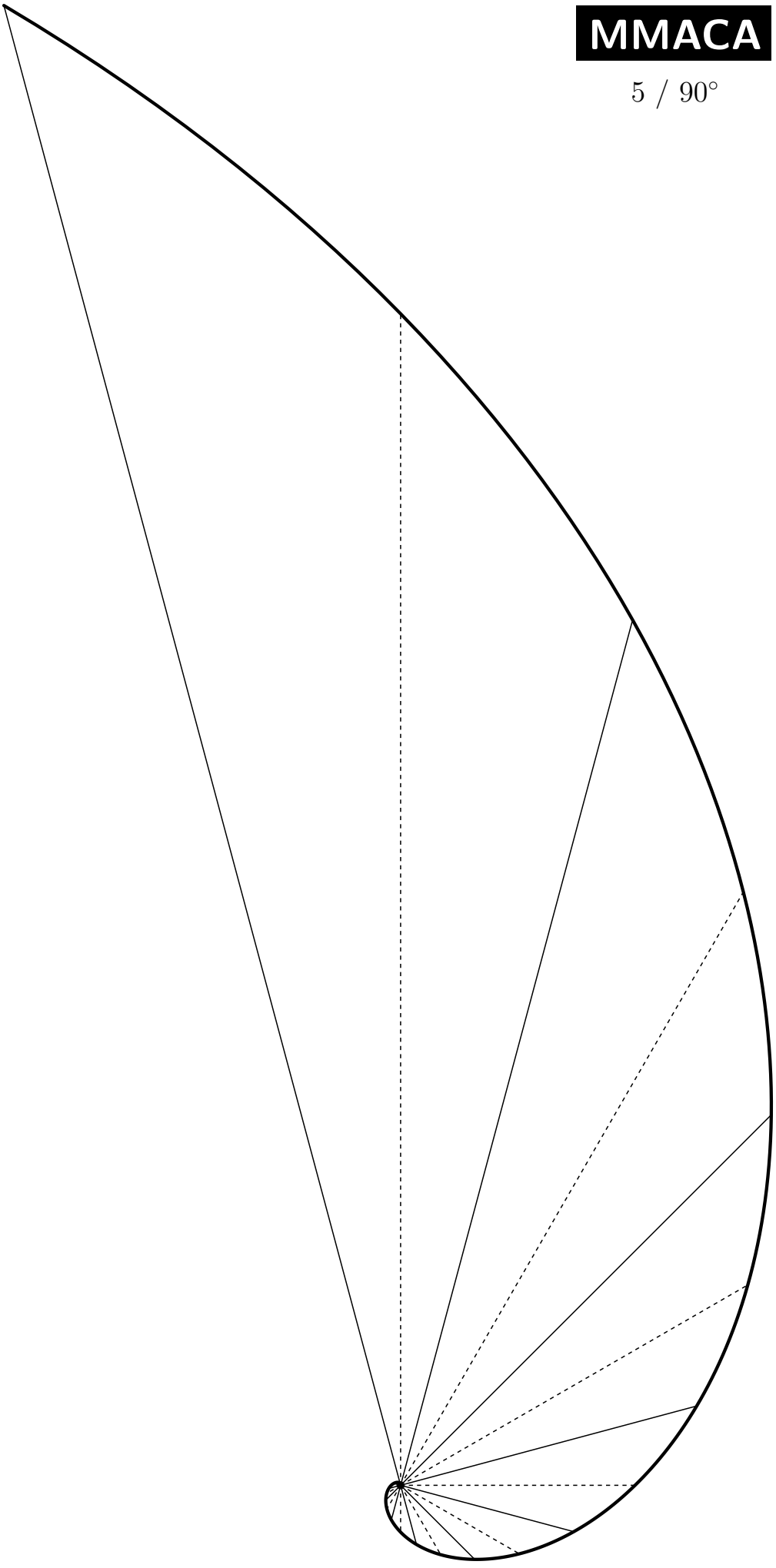
MMACA

4 / 360°



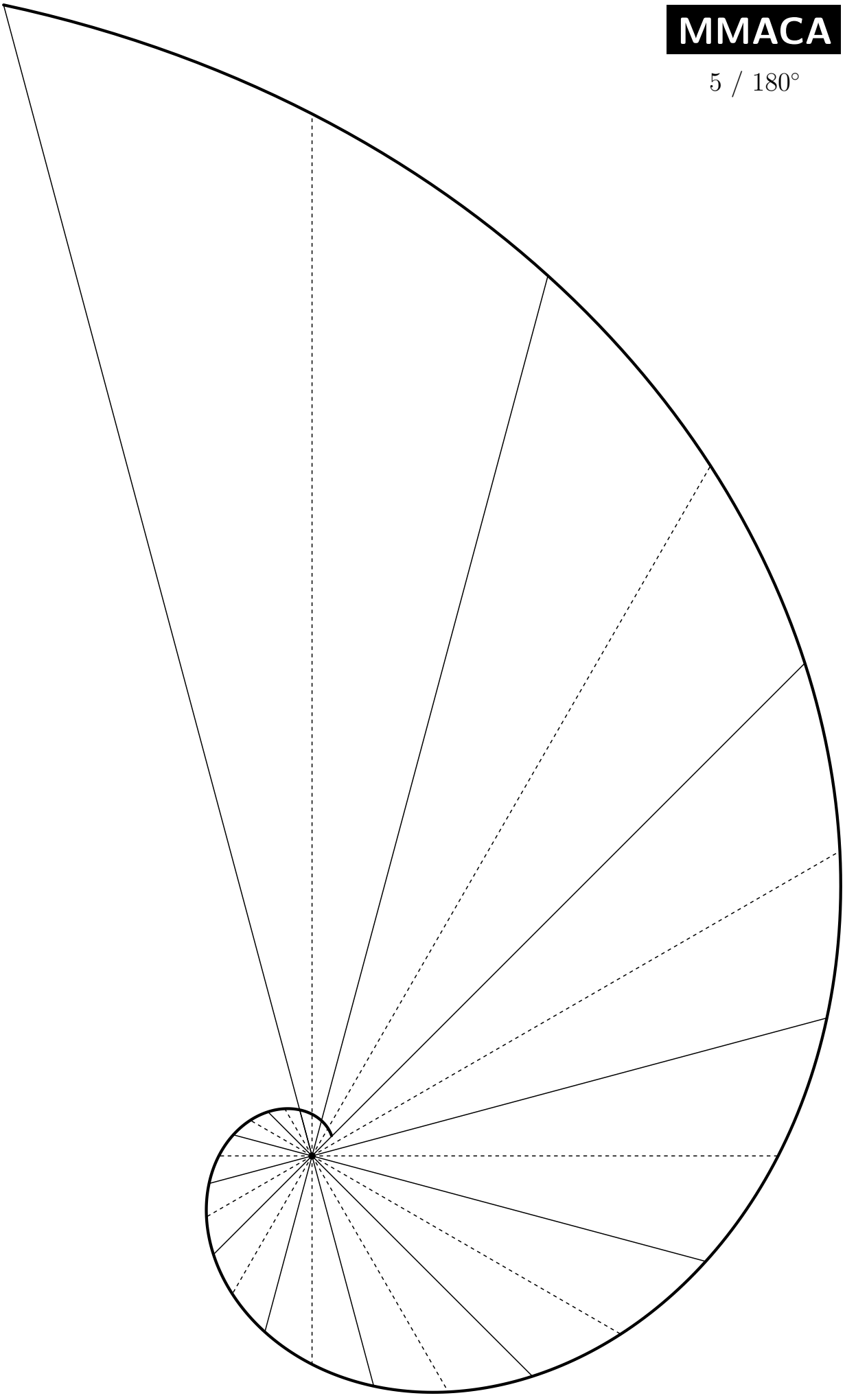
MMACA

5 / 90°



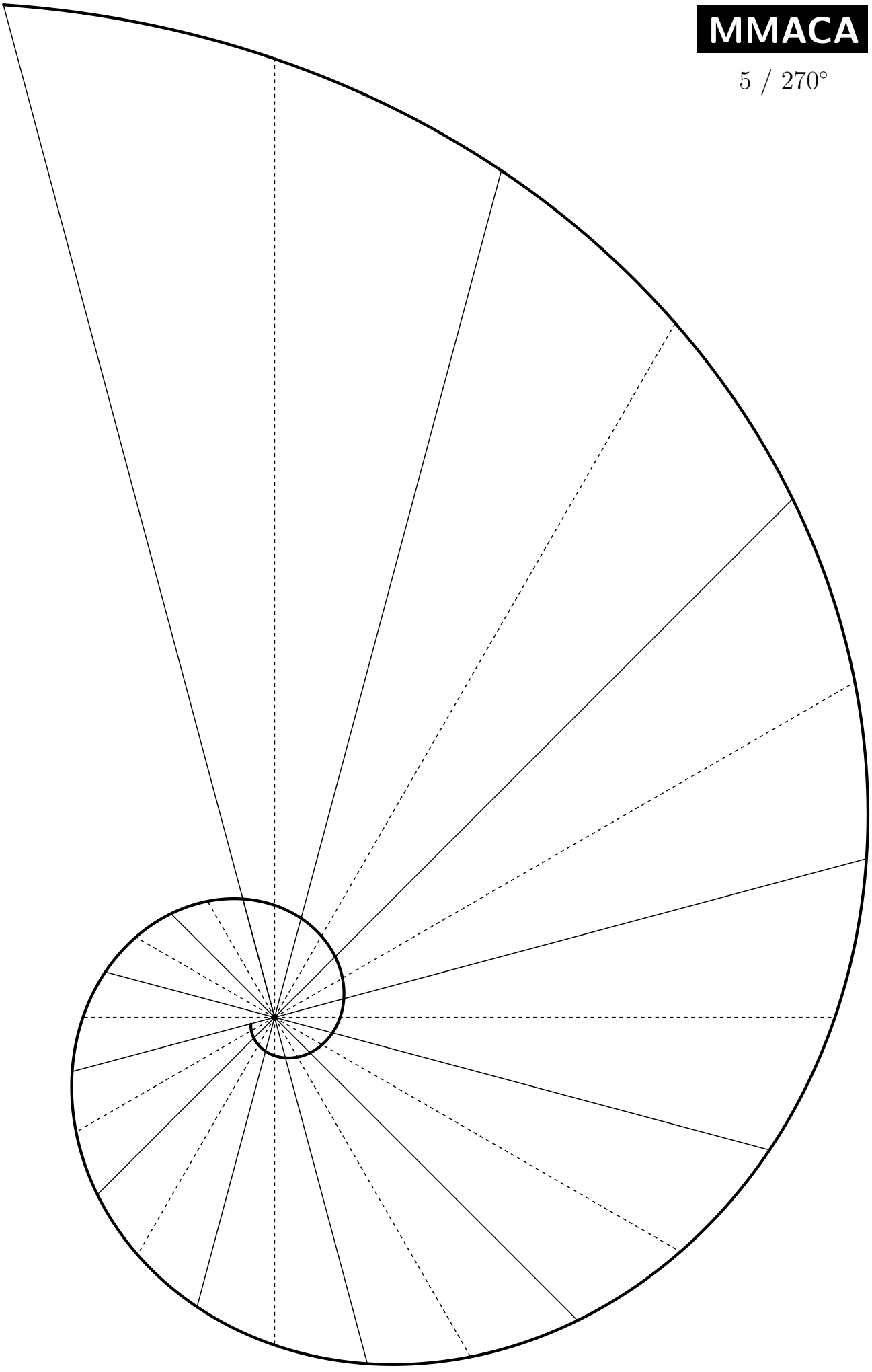
MMACA

5 / 180°



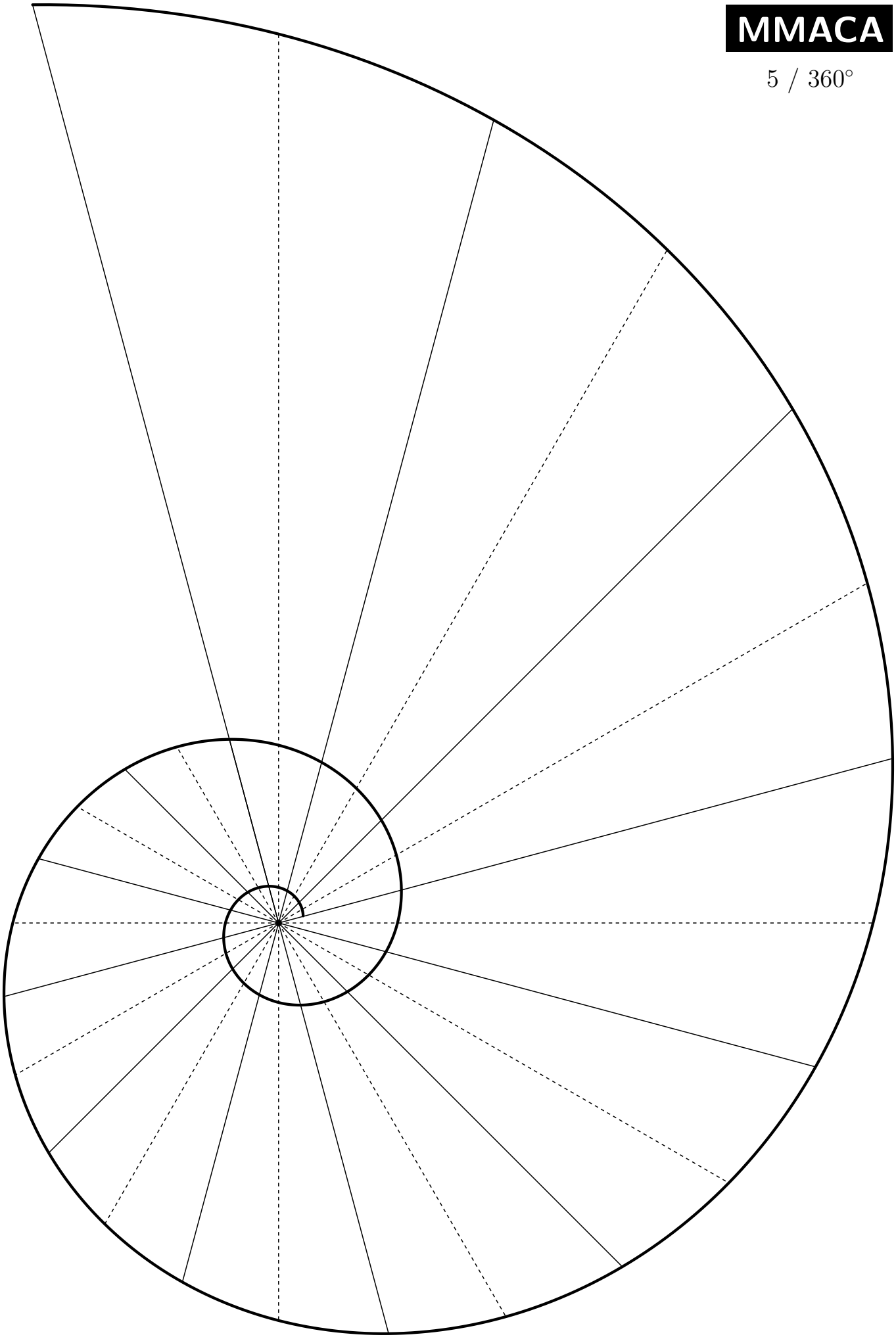
MMACA

5 / 270°



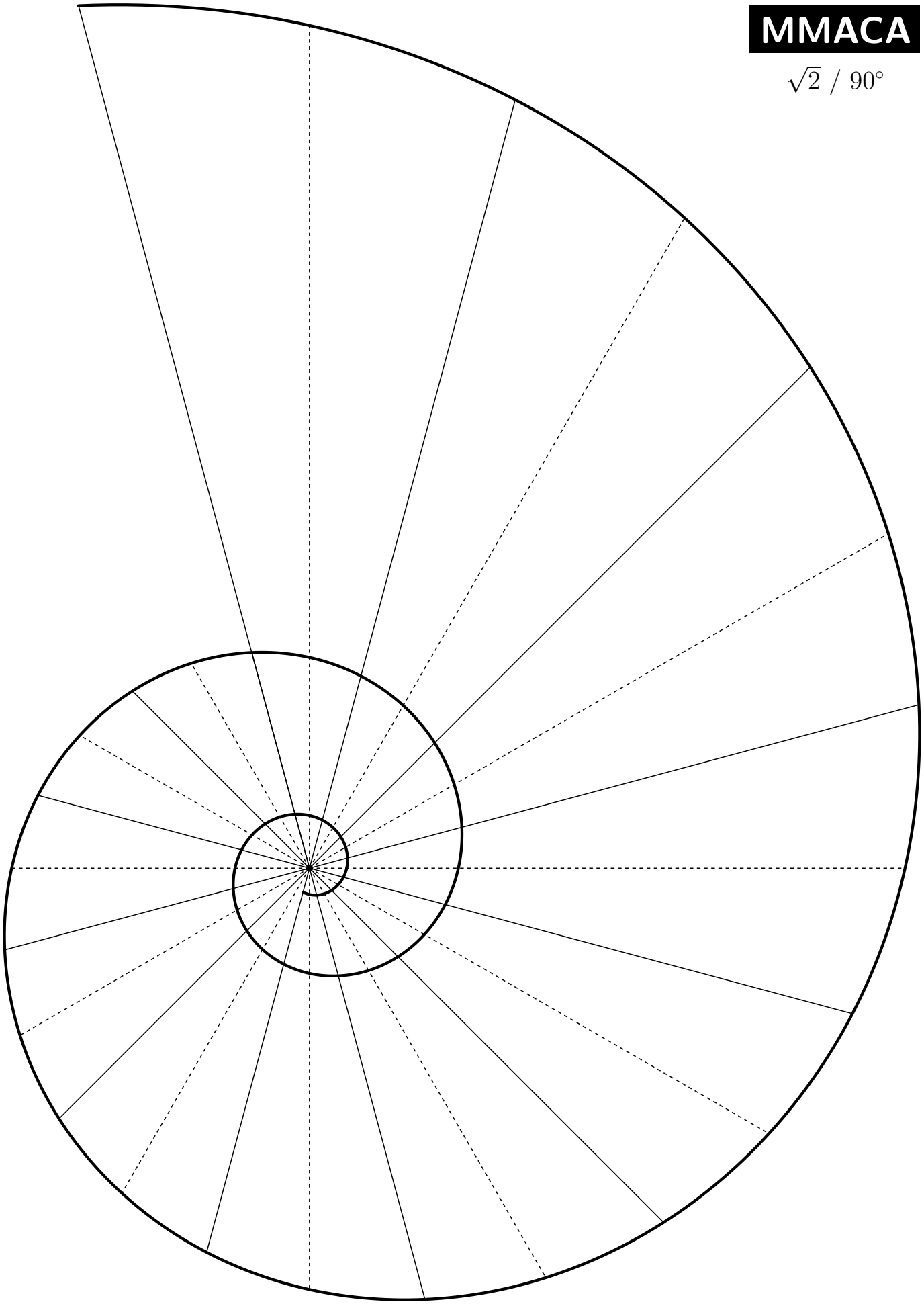
MMACA

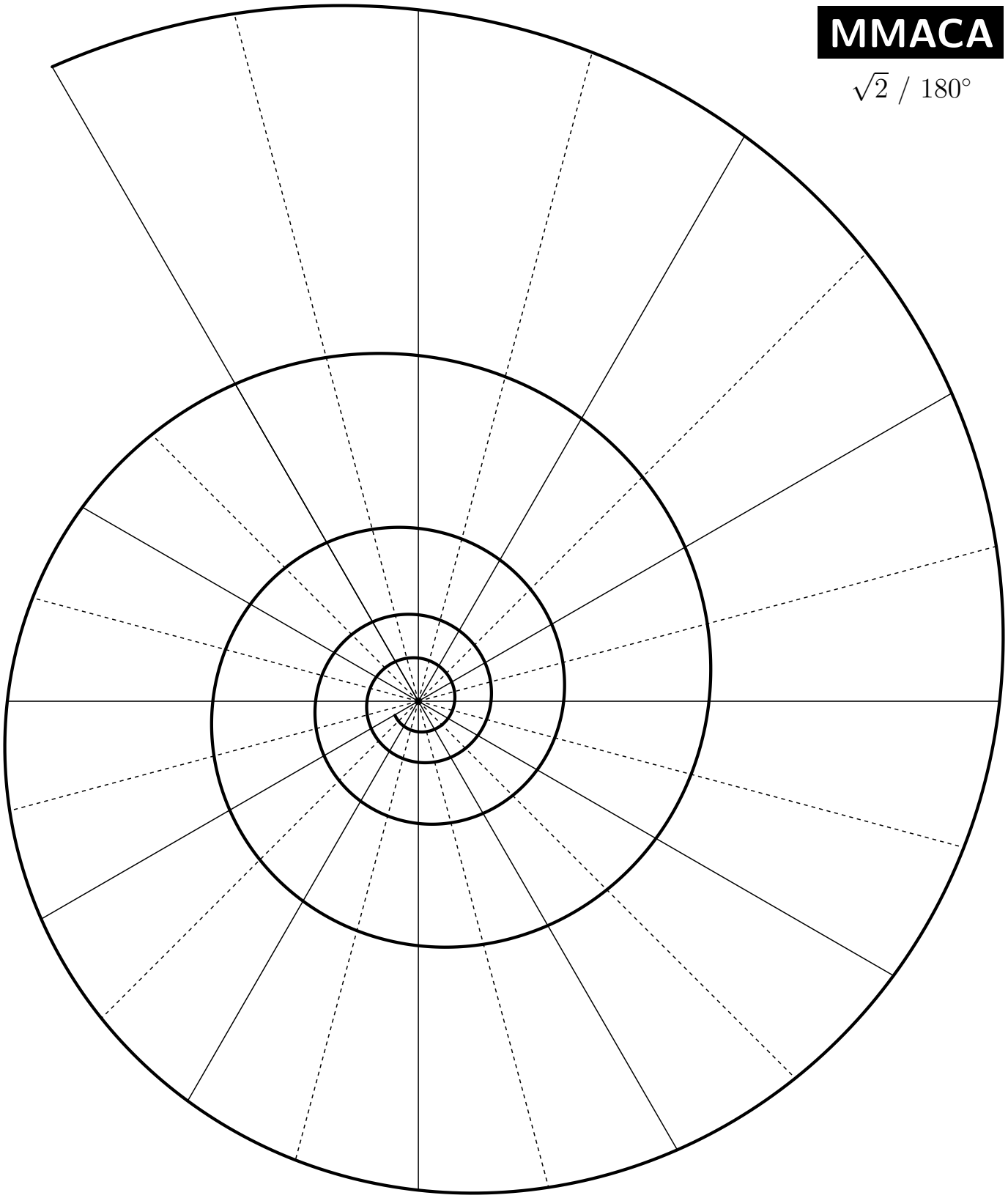
5 / 360°

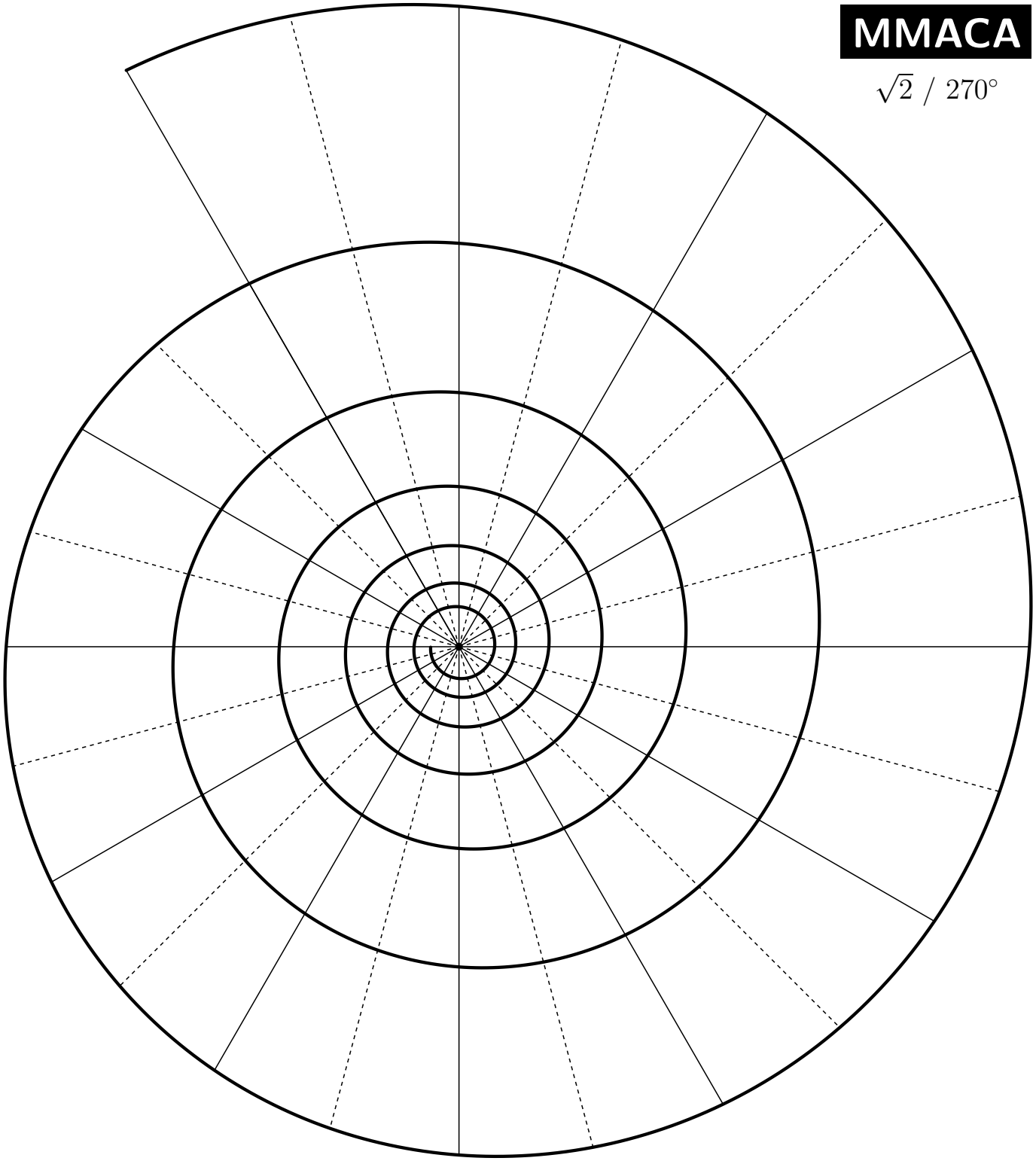


MMACA

$\sqrt{2}$ / $90°$

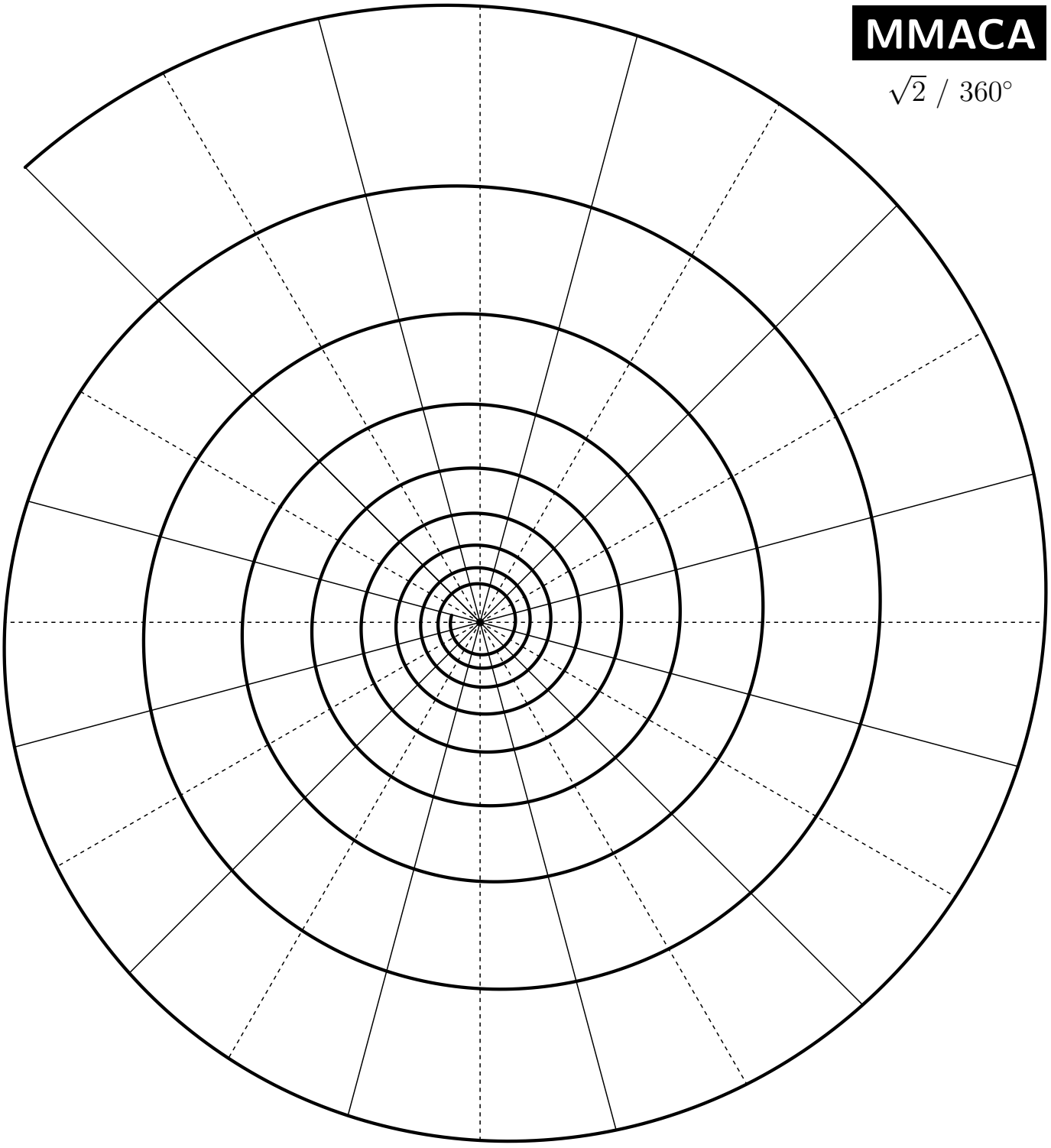



MMACA
$\sqrt{2}$ / 180°


MMACA
$\sqrt{2}$ / 270°

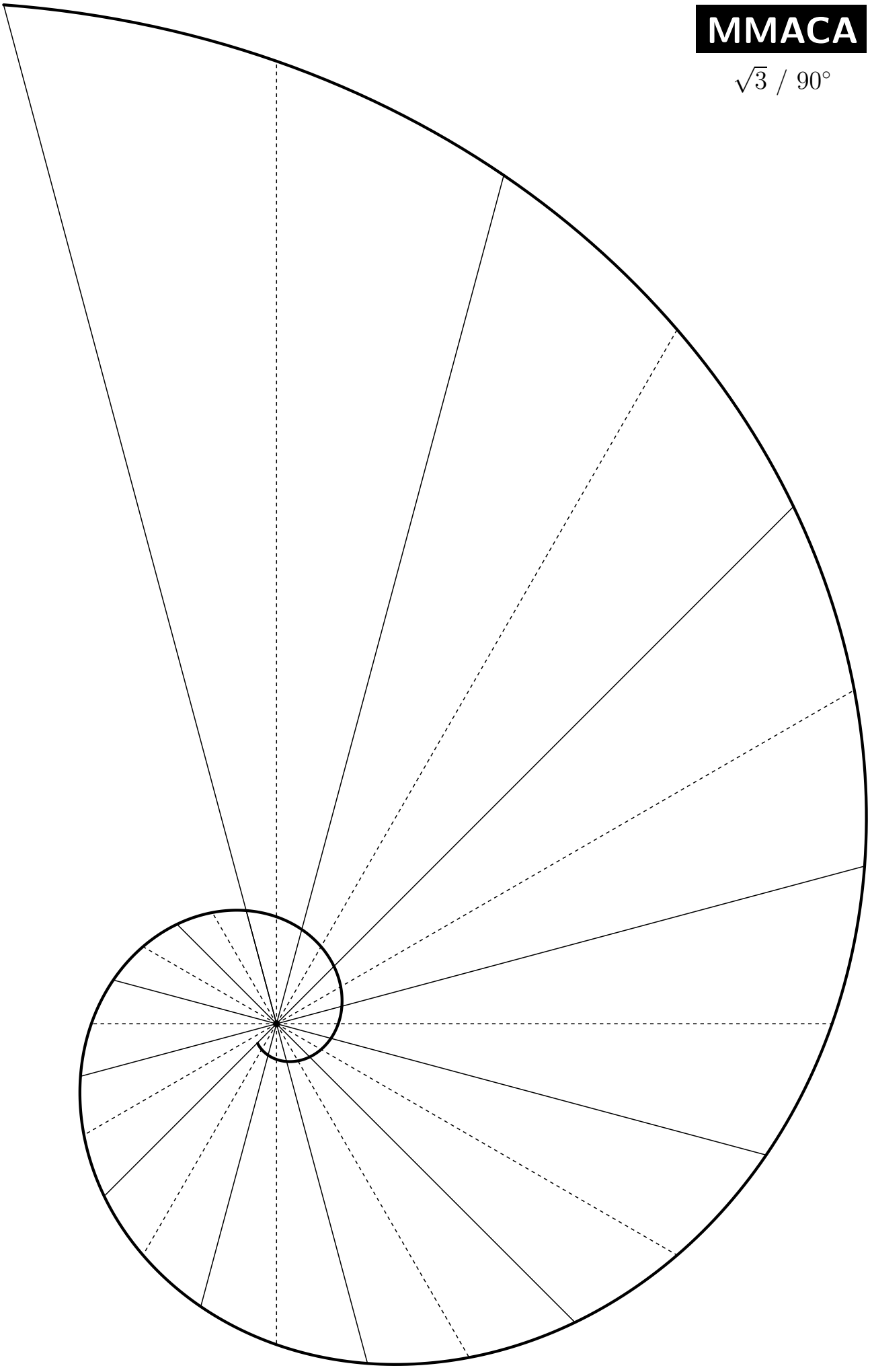
MMACA

$\sqrt{2}$ / 360°



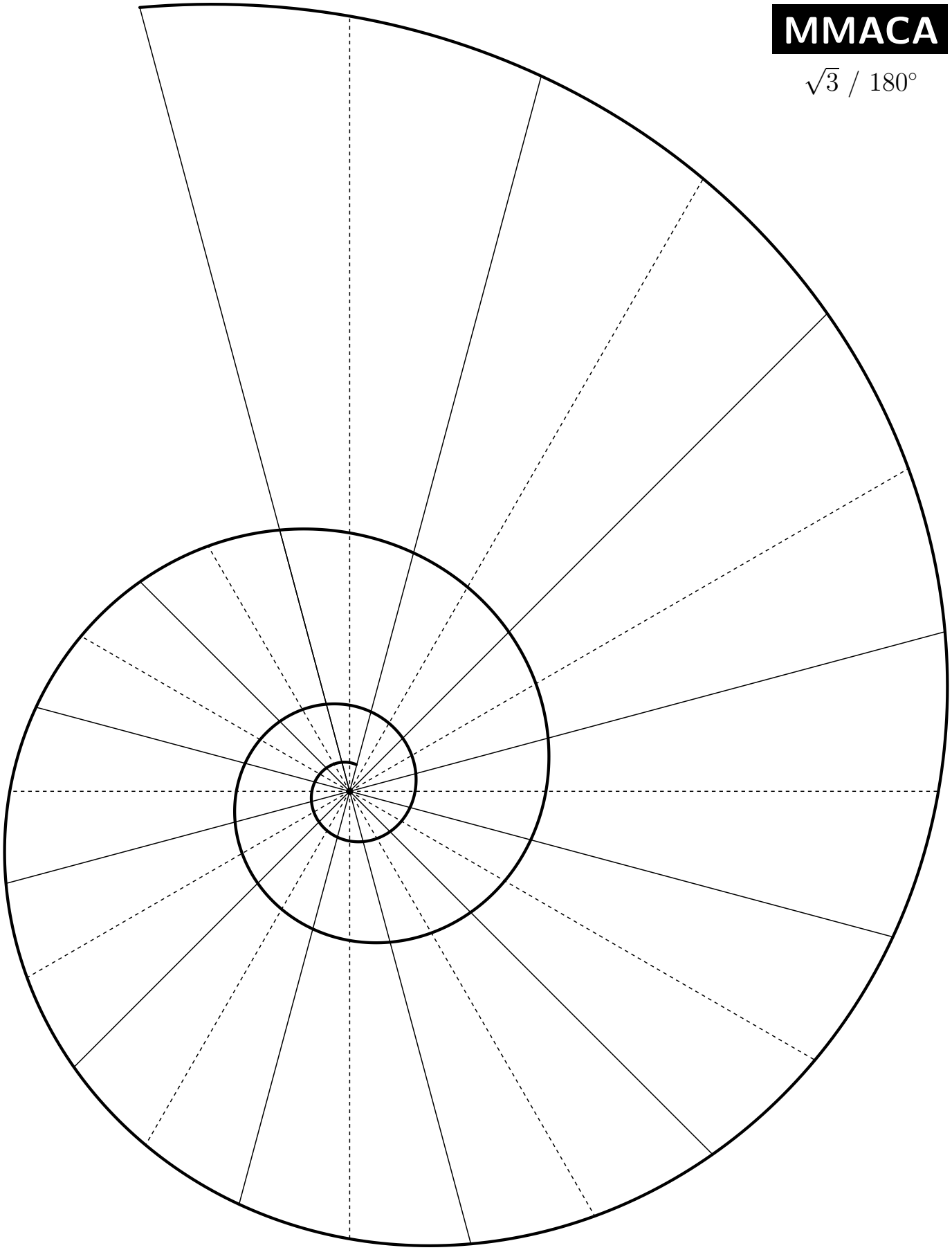
MMACA

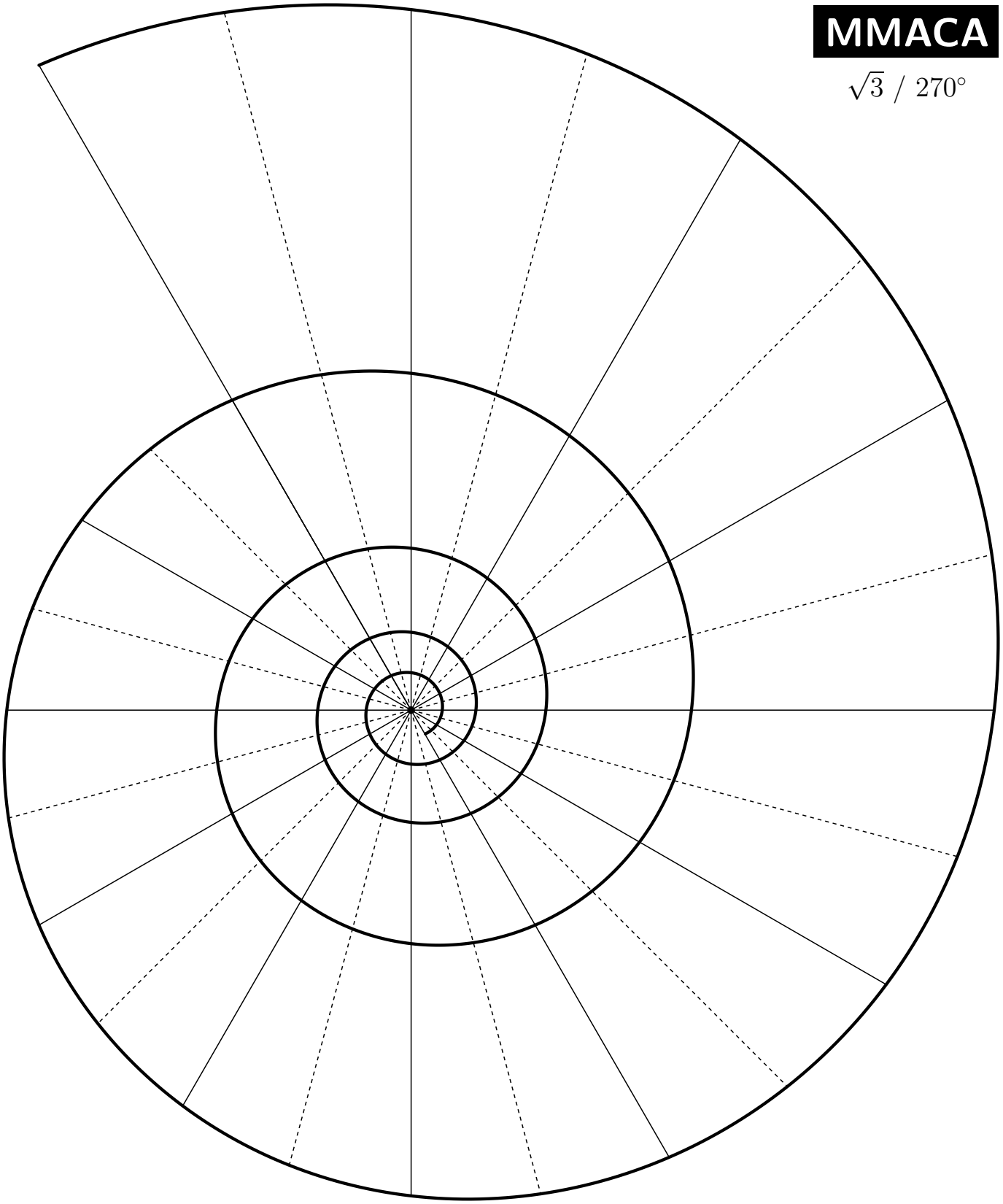
$\sqrt{3}$ / $90°$



MMACA

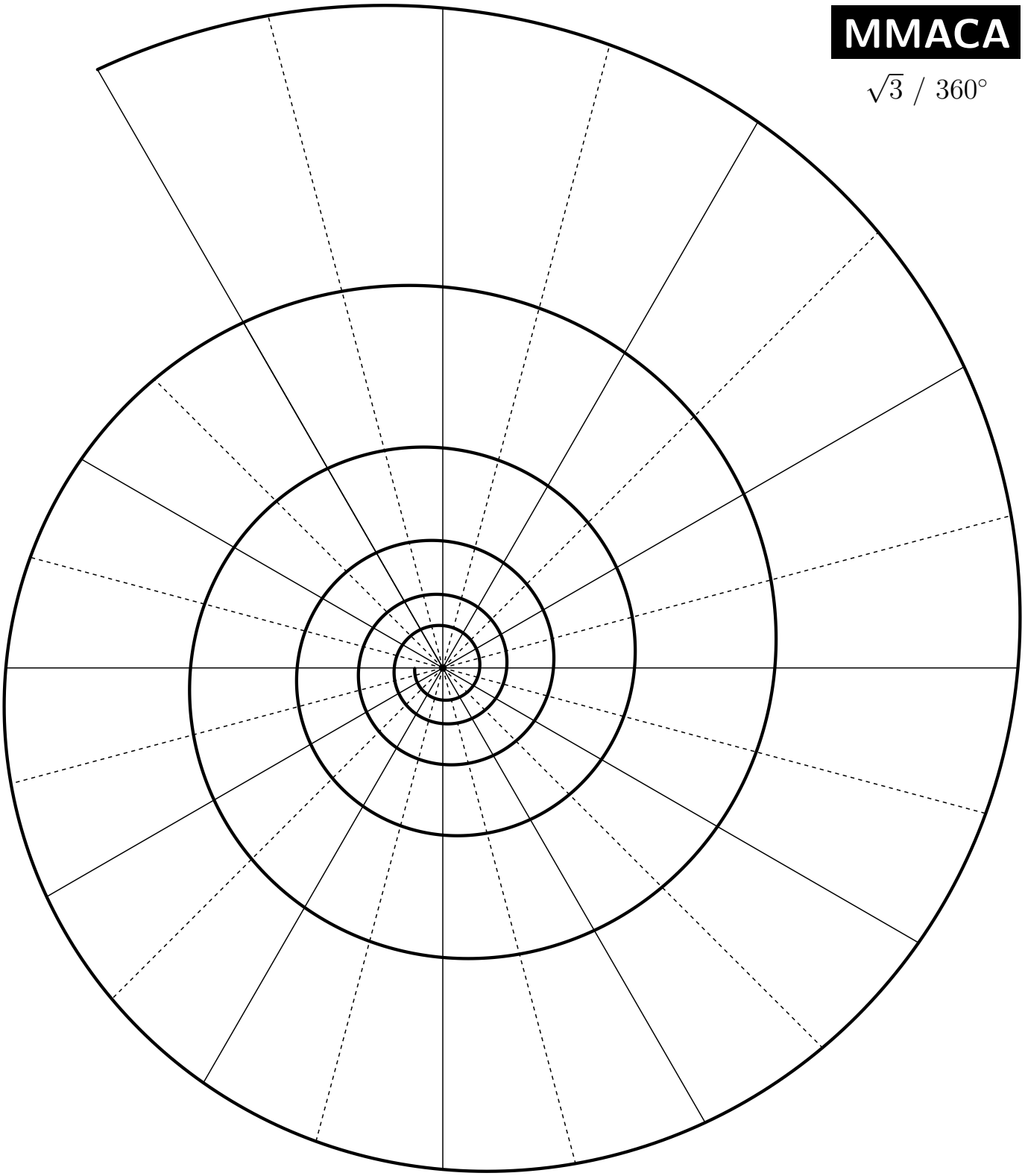
$\sqrt{3}$ / 180°




MMACA
$\sqrt{3}$ / 270°

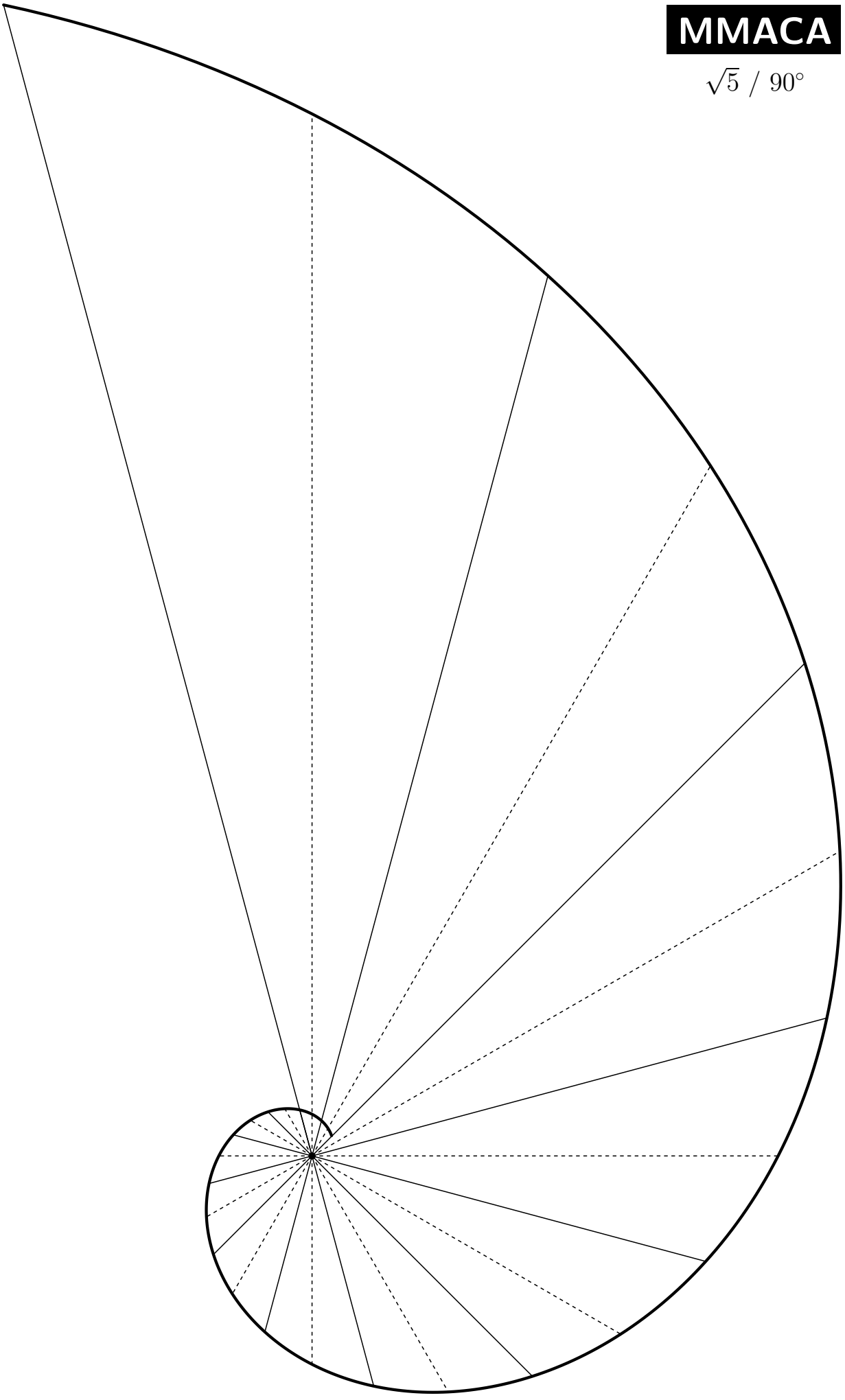
MMACA

$\sqrt{3}$ / 360°



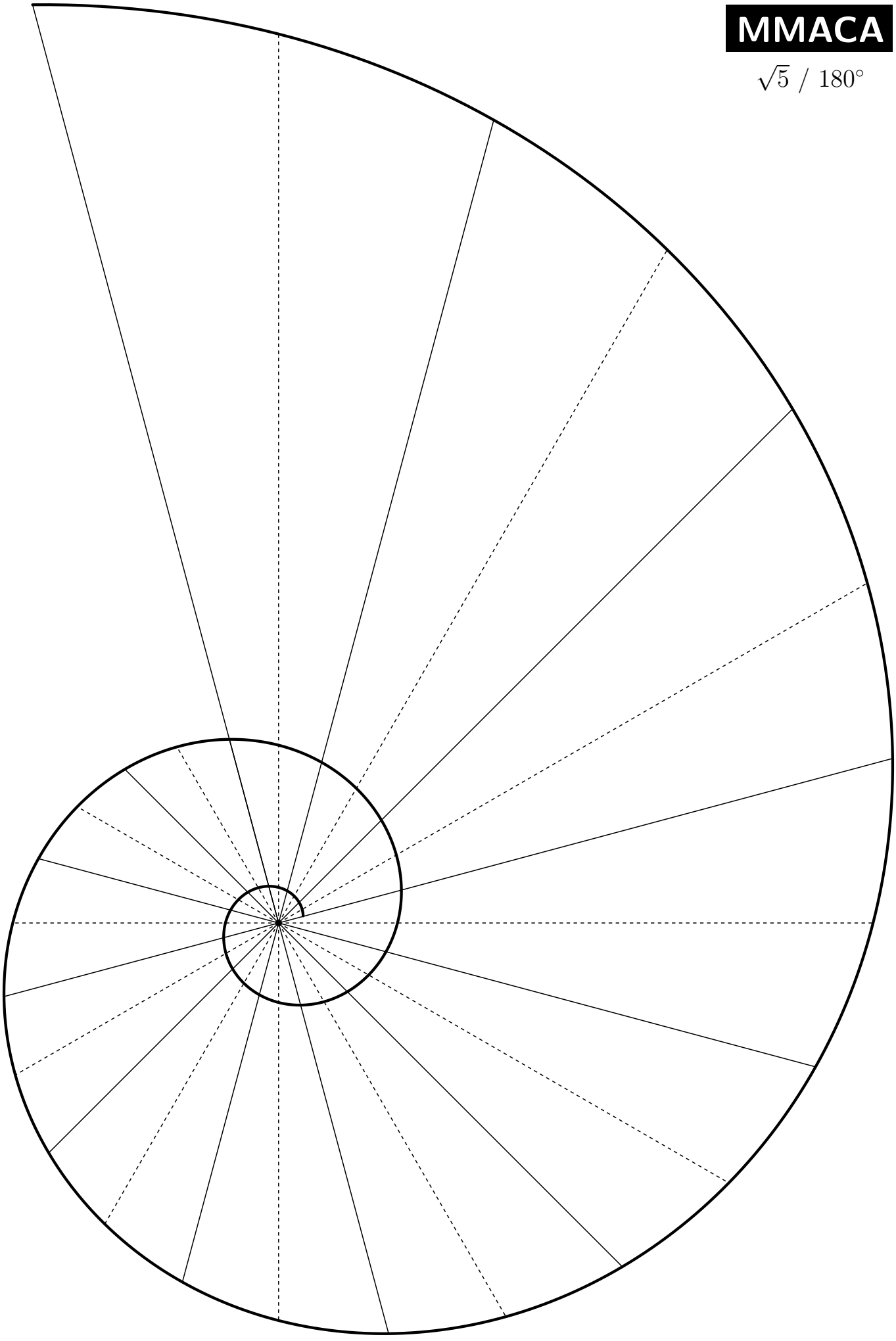
MMACA

$\sqrt{5}$ / $90°$



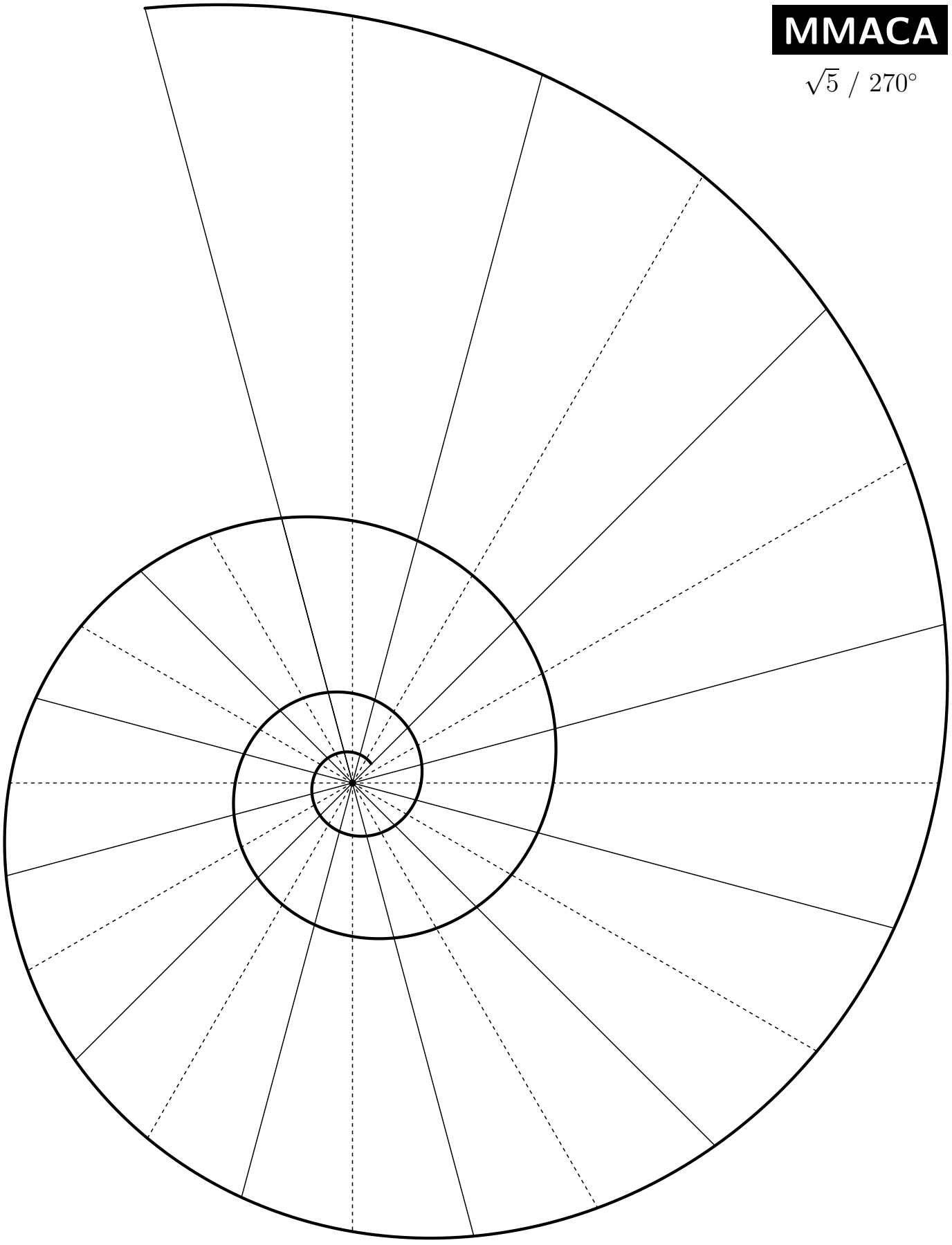
MMACA

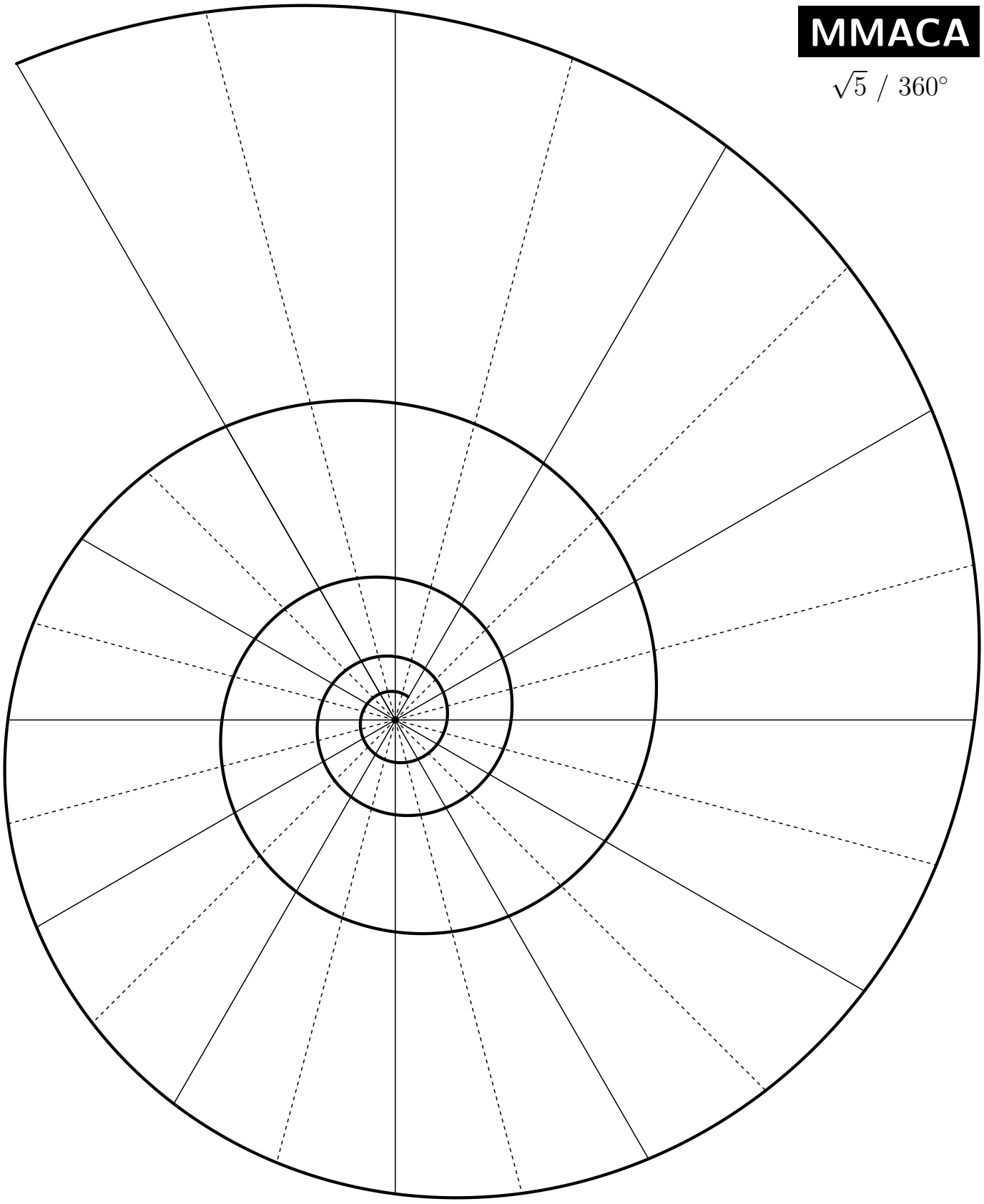
$\sqrt{5}$ / 180°



MMACA

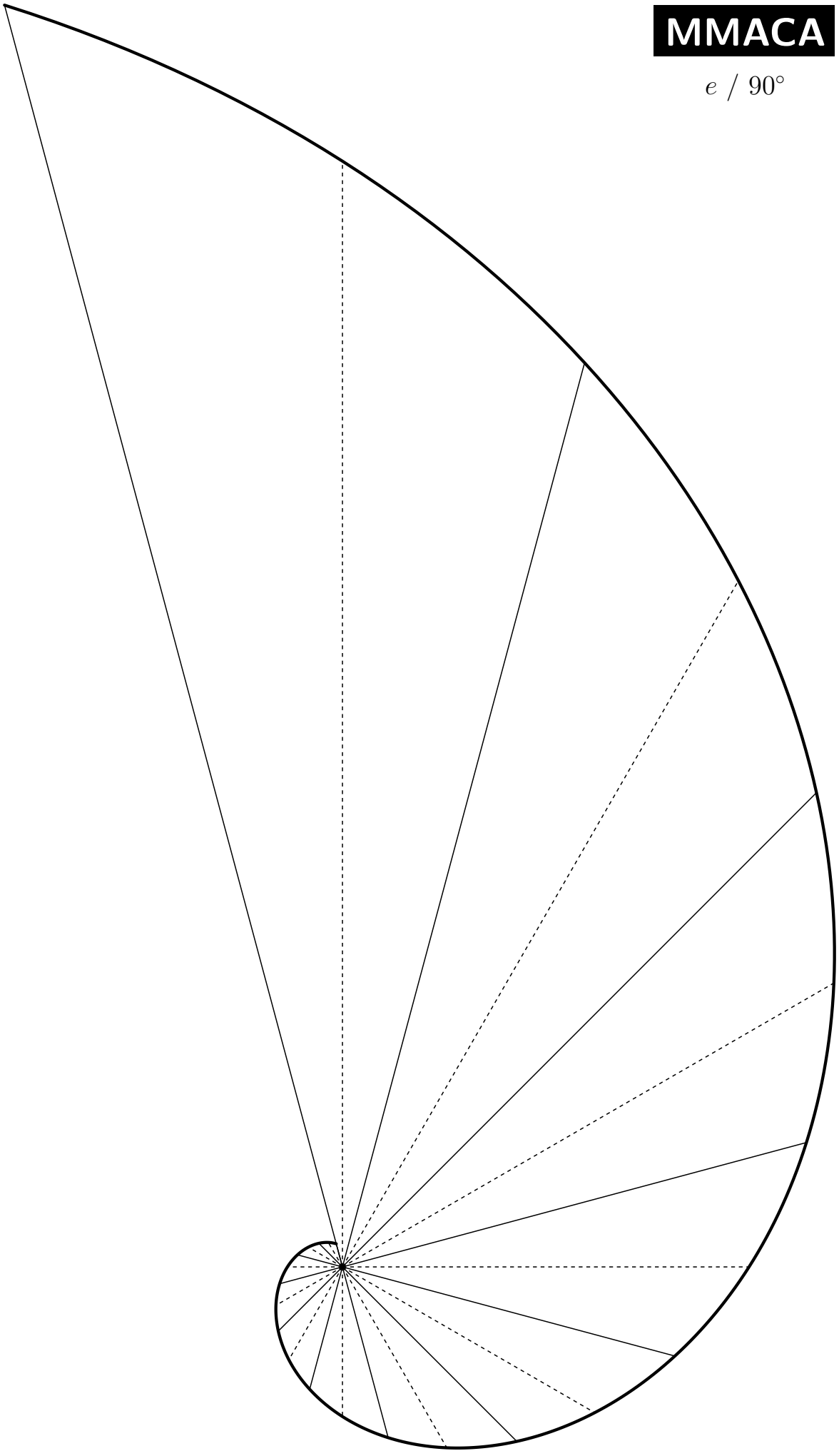
$\sqrt{5}$ / 270°




MMACA
$\sqrt{5}$ / 360°

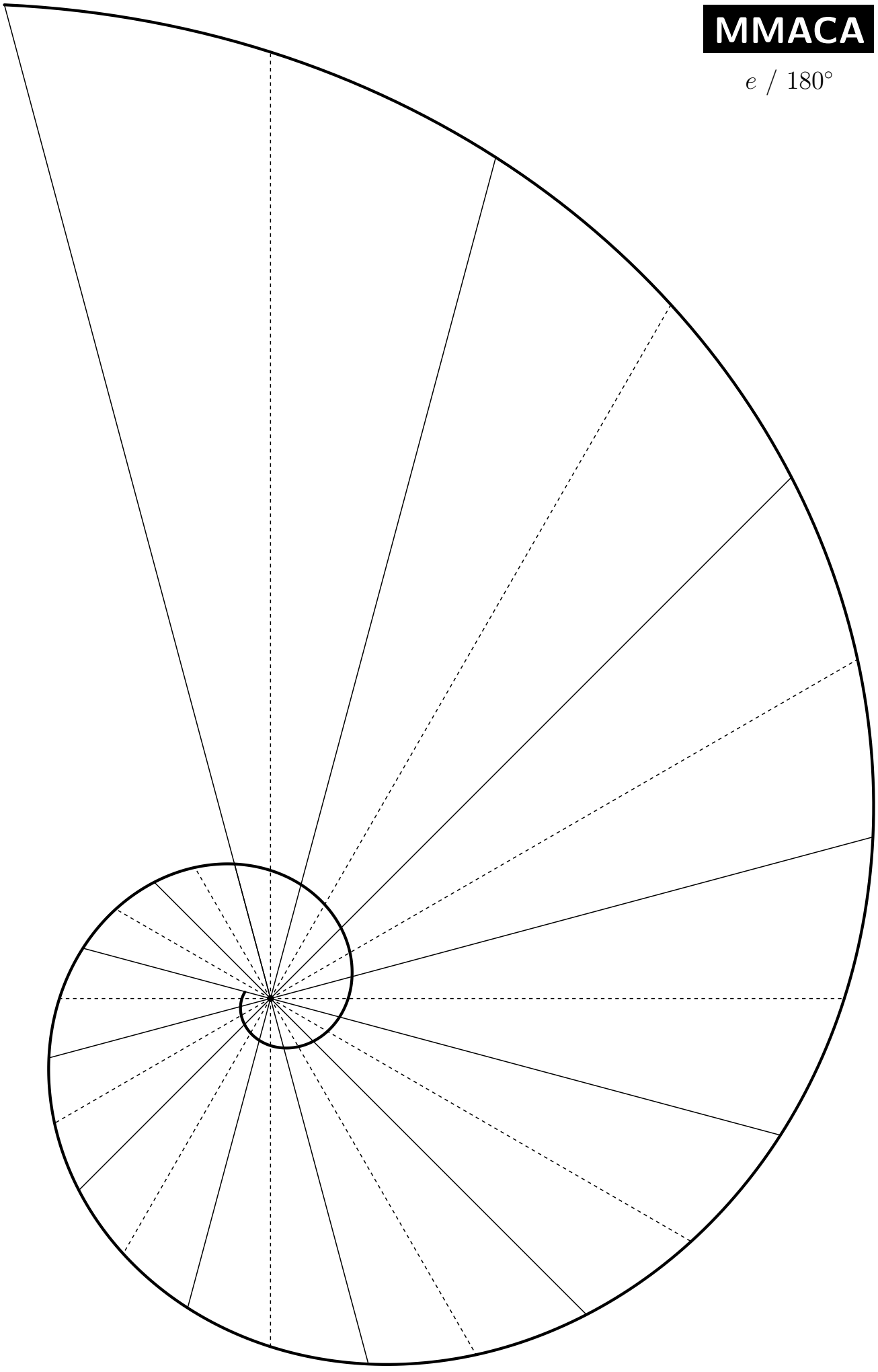
MMACA

e / 90°



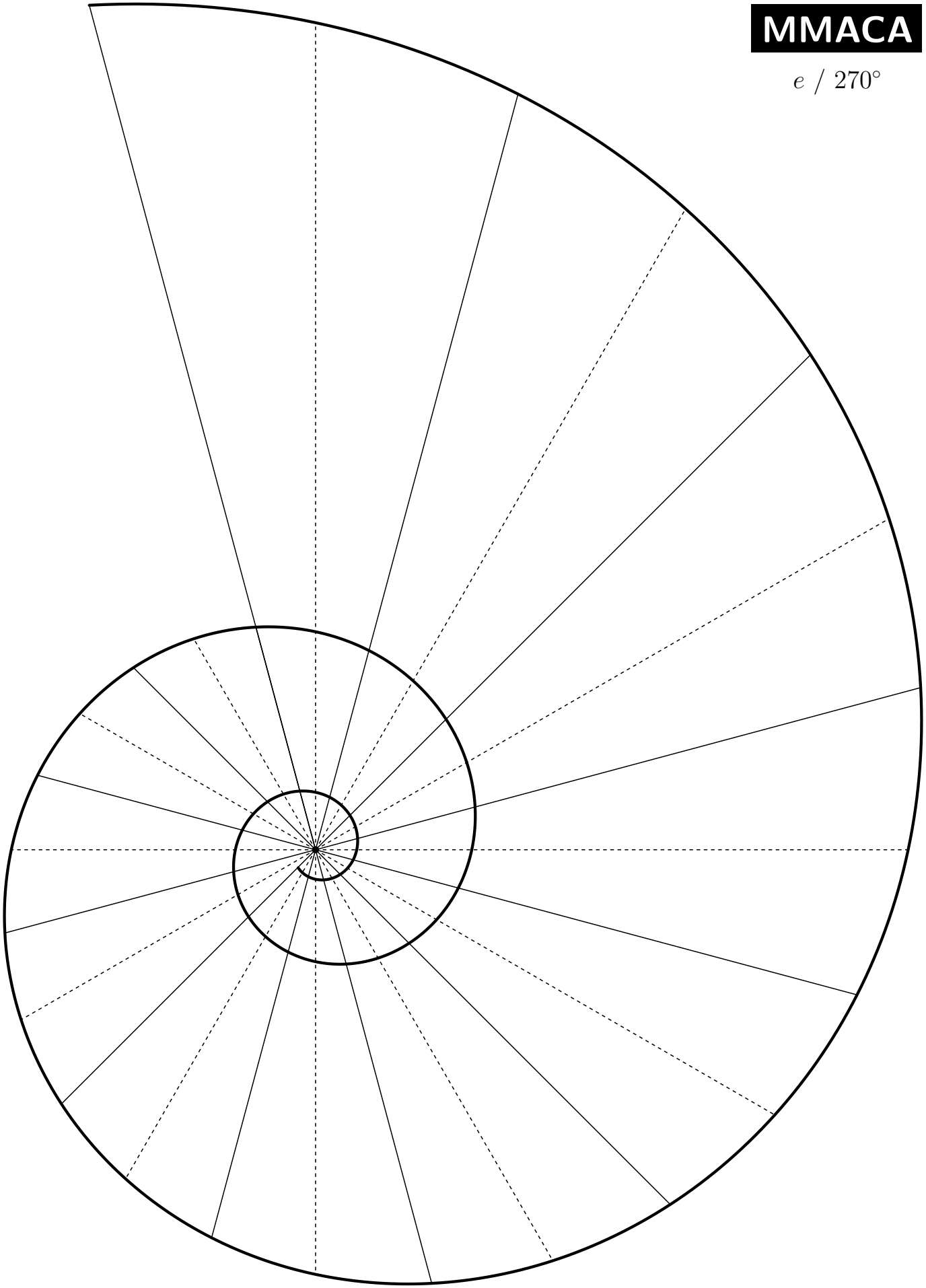
MMACA

e / 180°



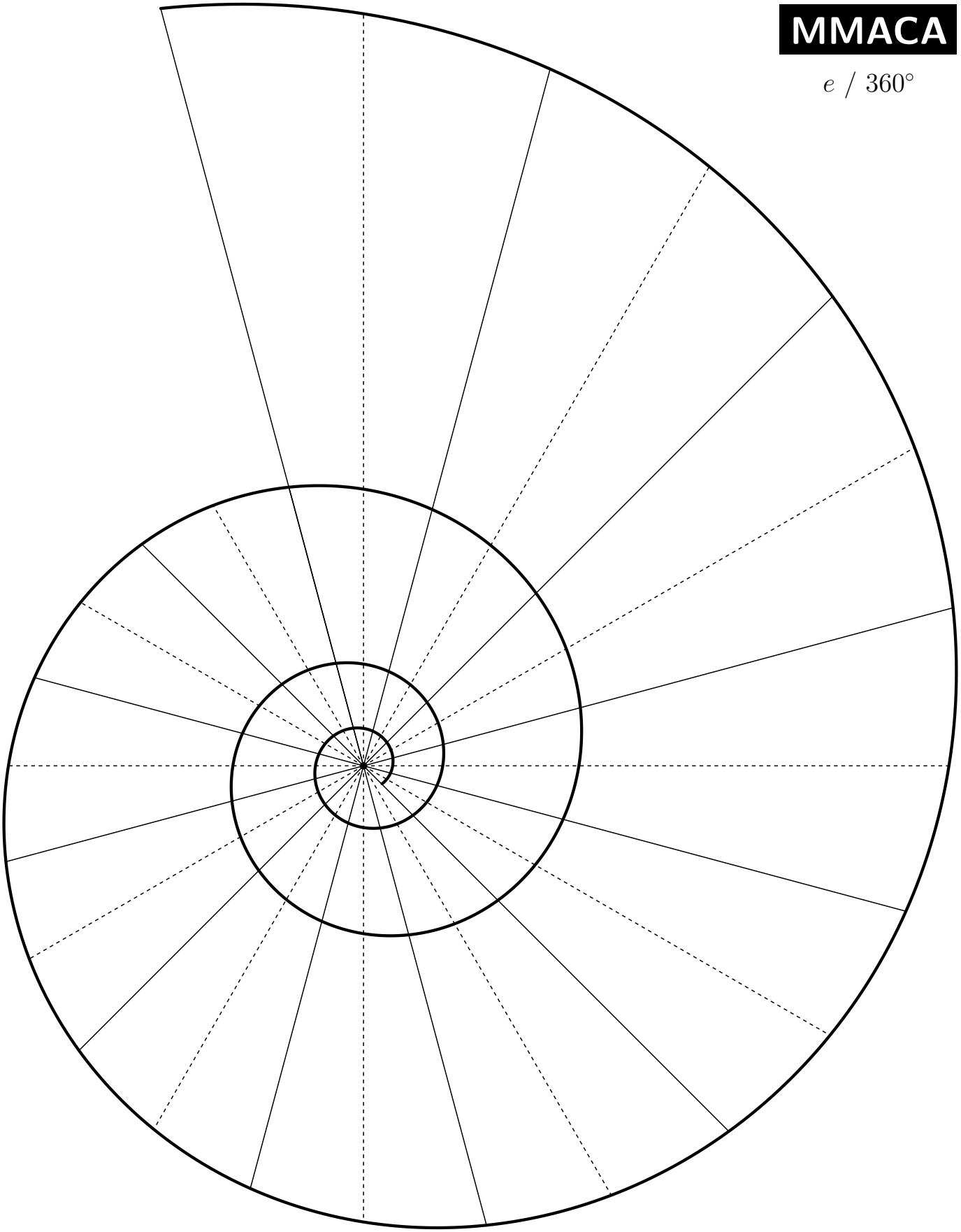
MMACA

e / 270°



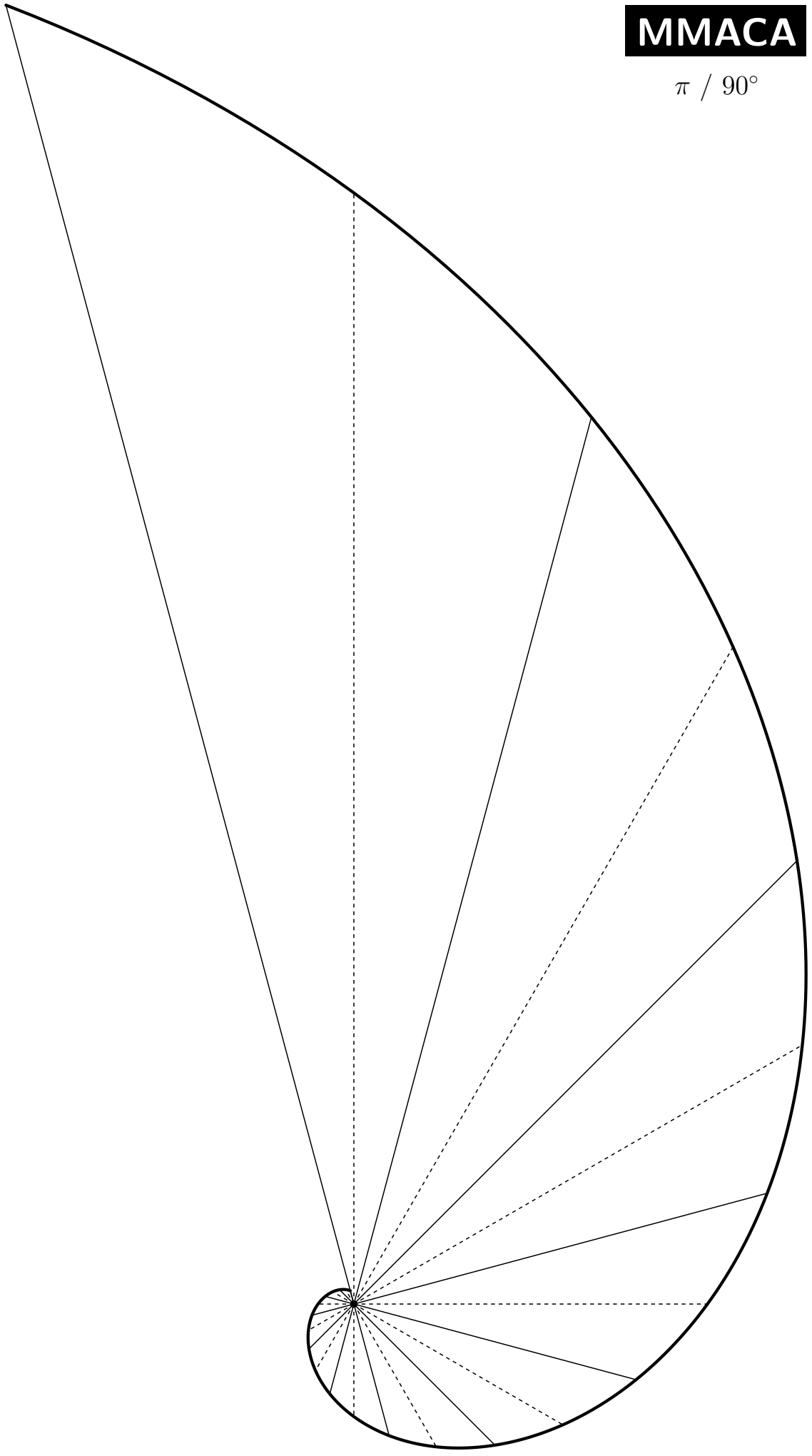
MMACA

e / 360°



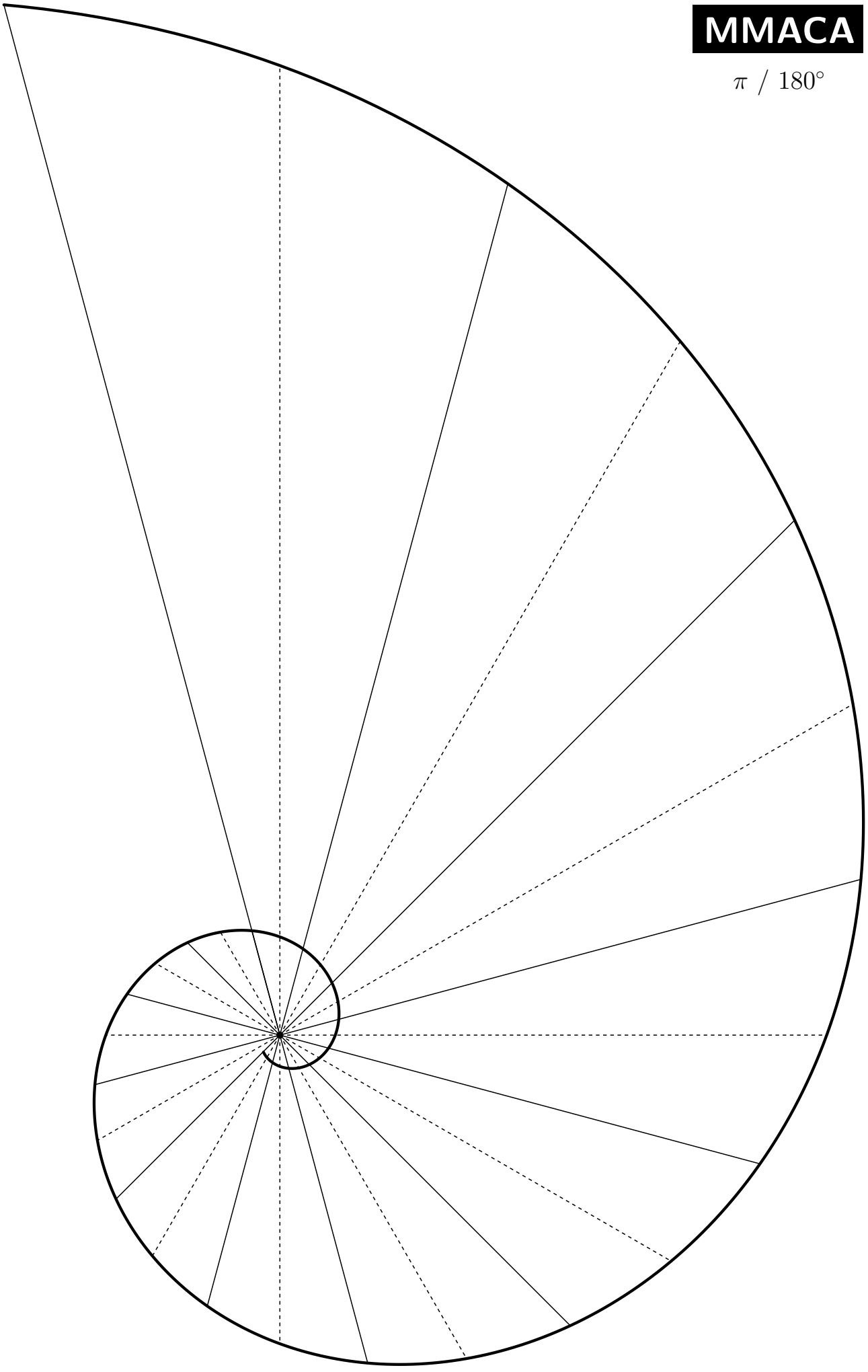
MMACA

π / 90°



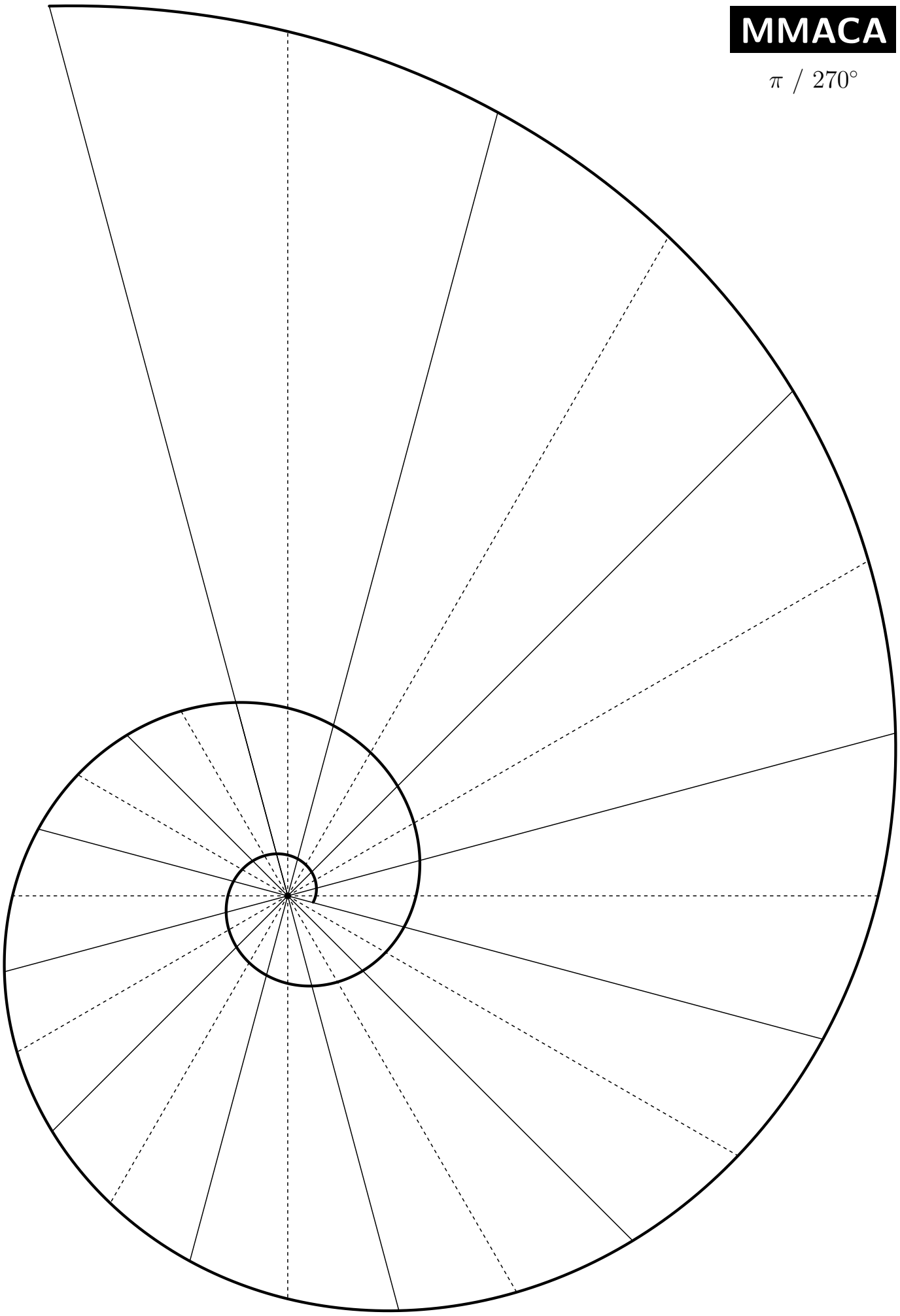
MMACA

π / 180°



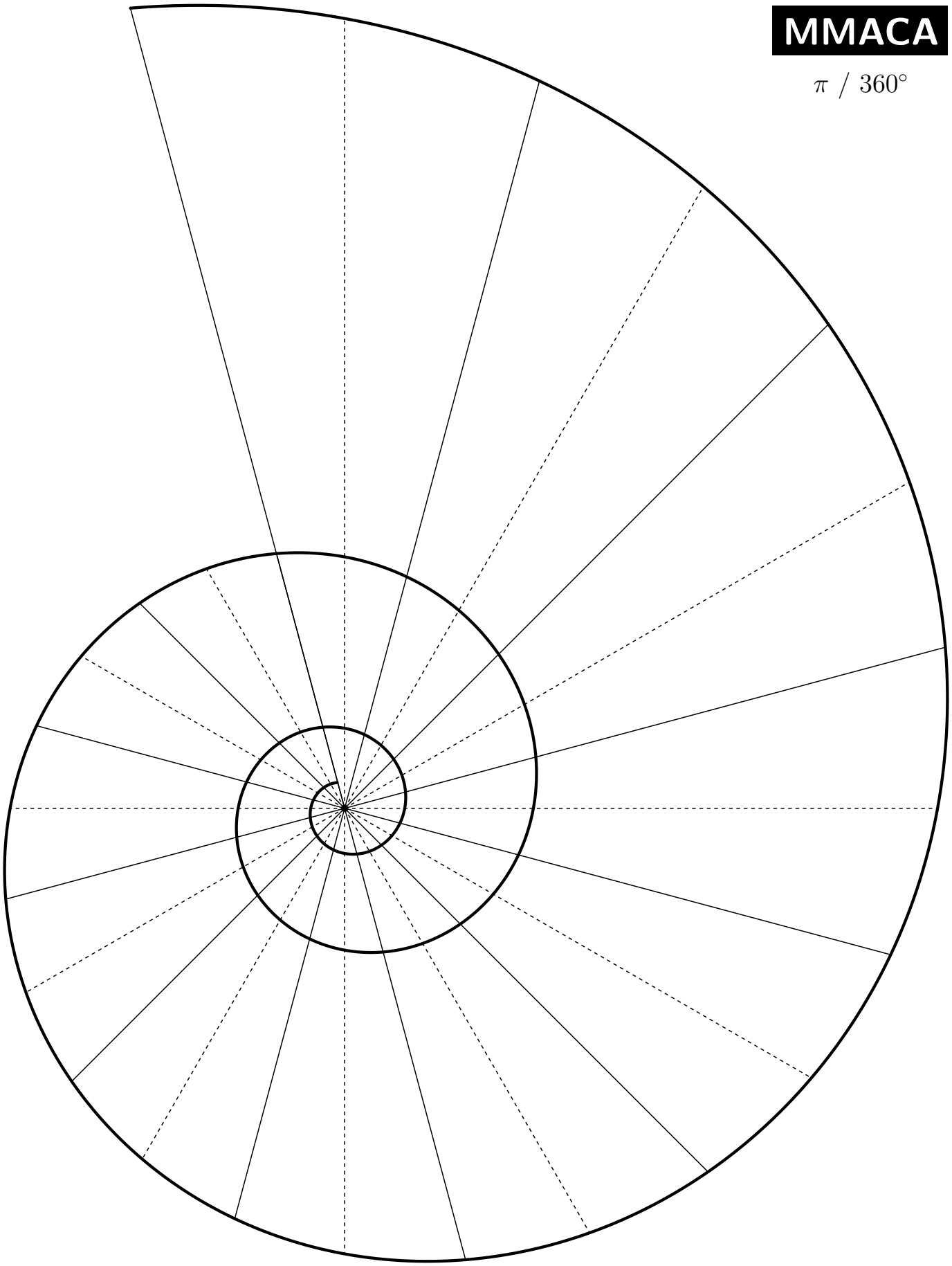
MMACA

π / 270°



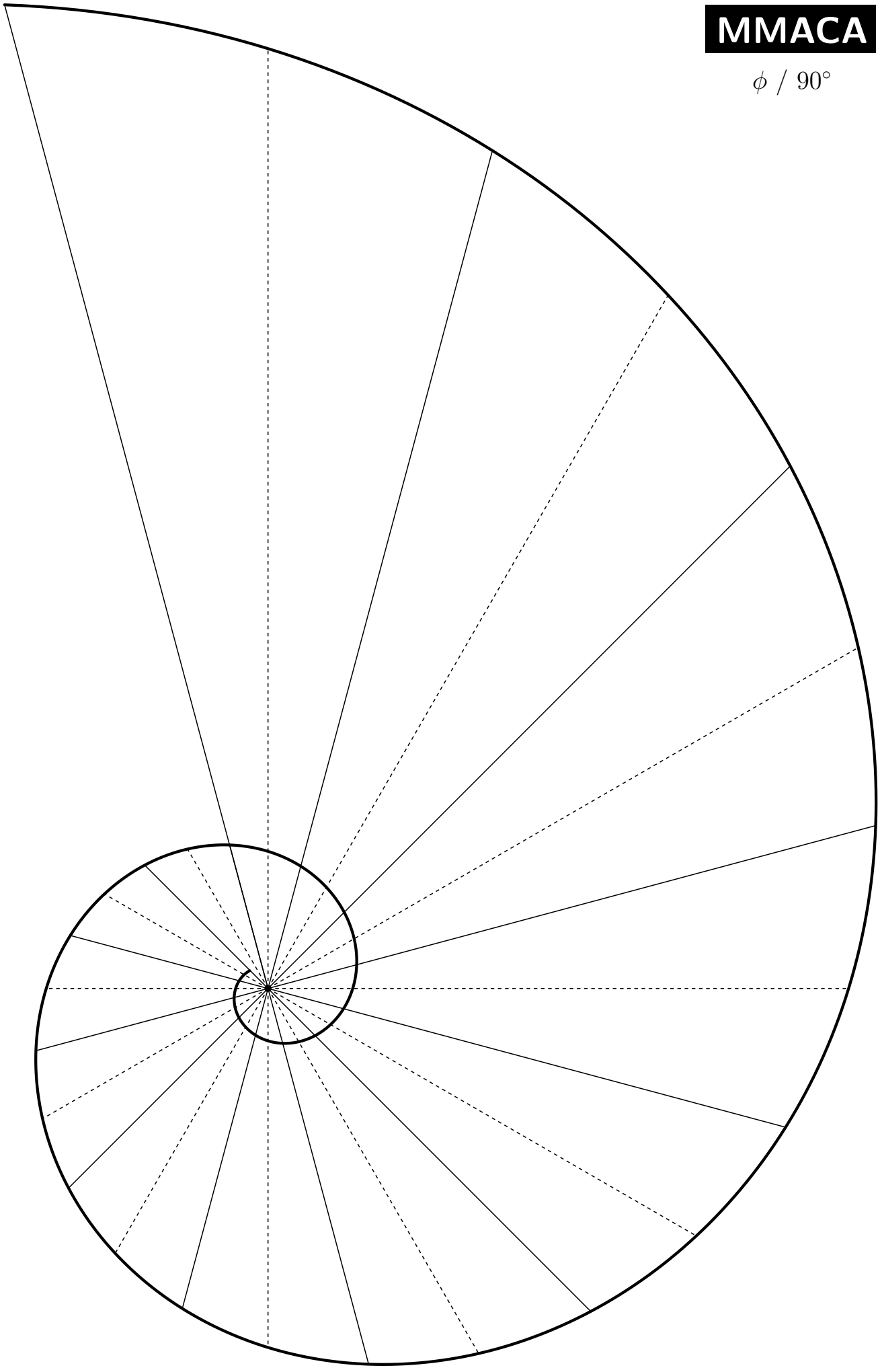
MMACA

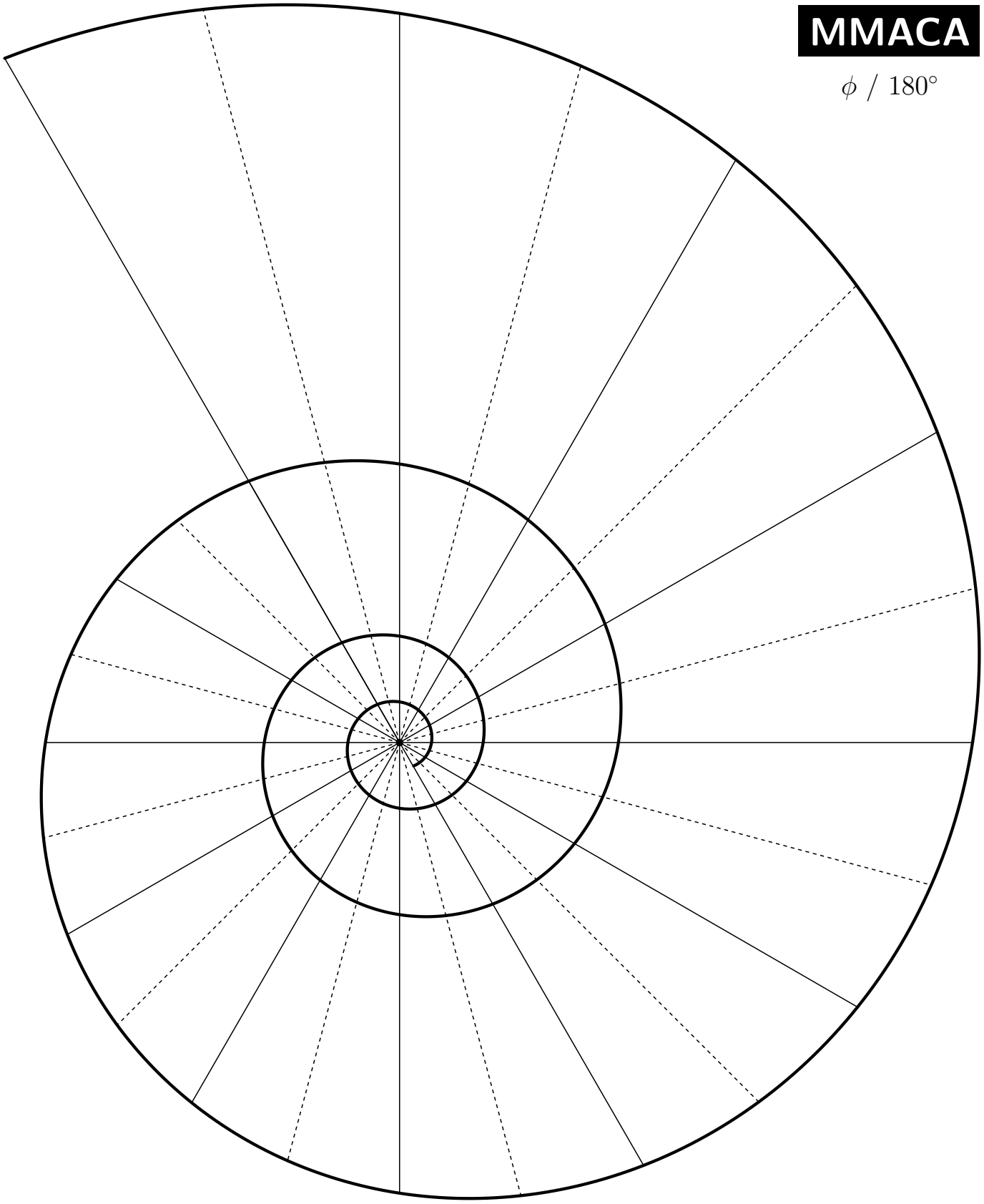
π / 360°

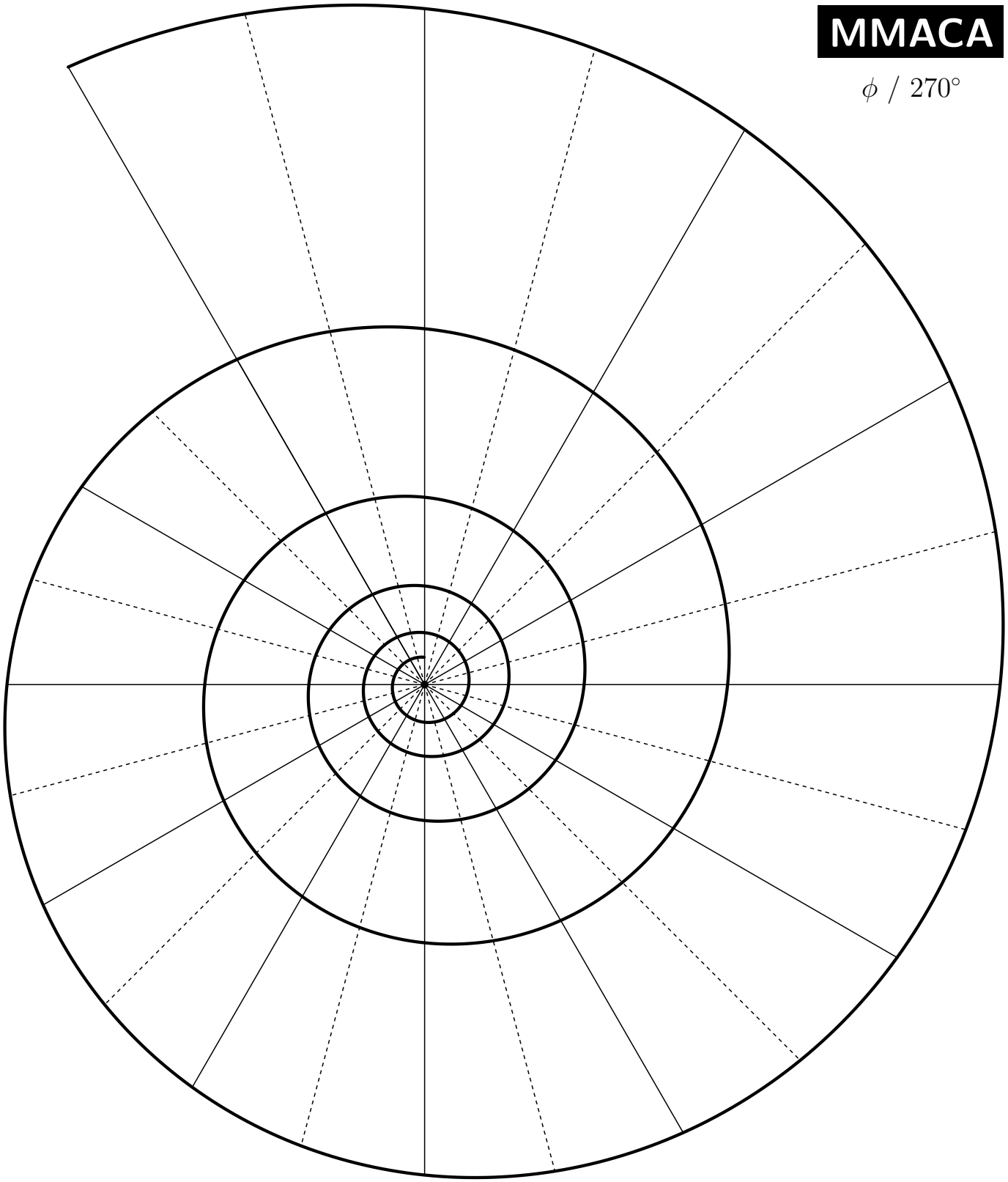


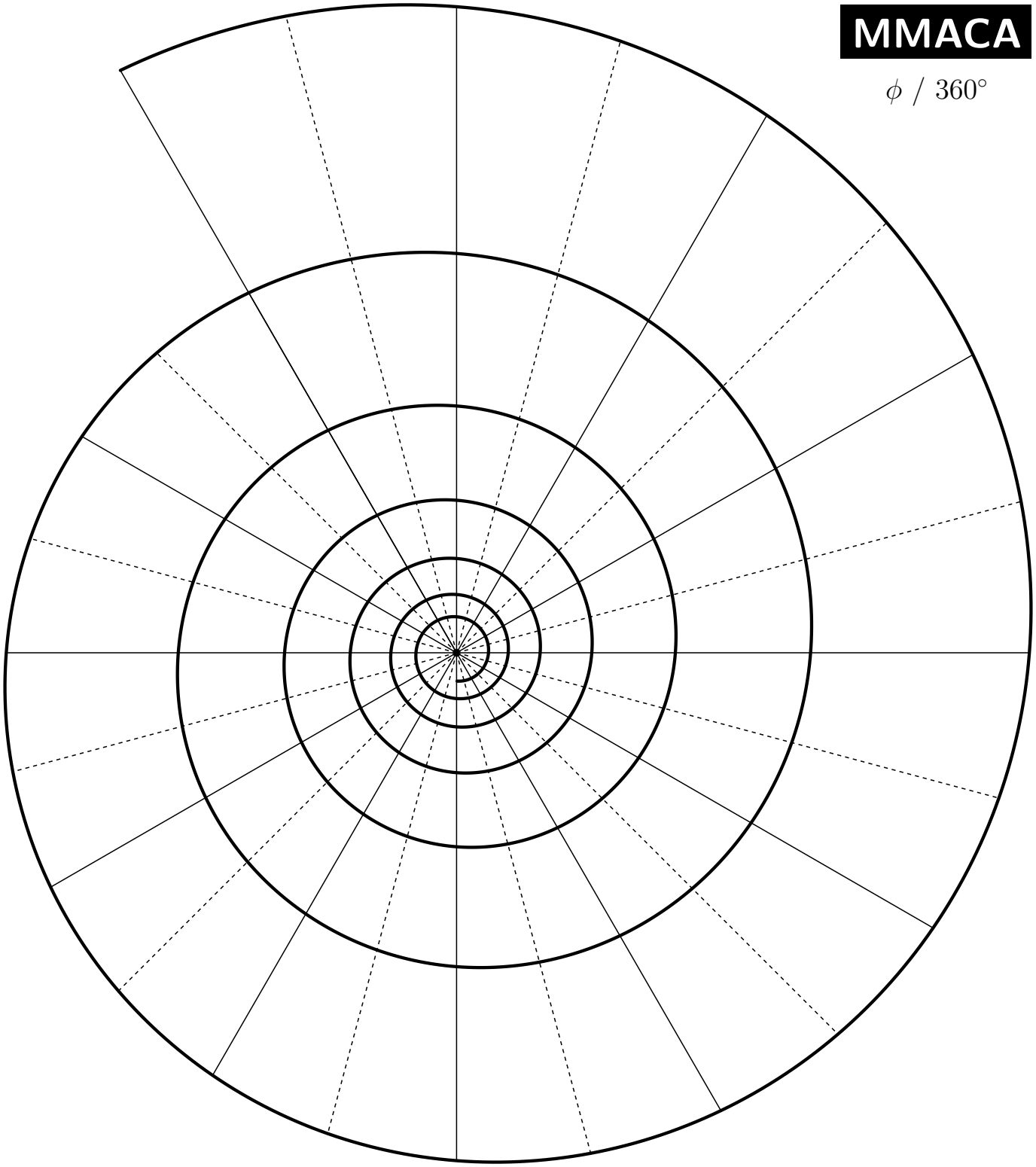
MMACA

ϕ / $90°$




MMACA
ϕ / 180°


MMACA
ϕ / 270°


MMACA
ϕ / 360°