Práctica 1

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Exercise 1. Find the power set $\mathbf{R^3}$ of $\mathbf{R} = \{(\mathbf{1}, \mathbf{1}), (\mathbf{1}, \mathbf{2}), (\mathbf{2}, \mathbf{3}), (\mathbf{3}, \mathbf{4})\}$. Check your answer with the script `powerrelation.m` and write a LaTeX document with the solution step by step.

The n-th power set of a given binary relation R in a set X is defined recursively in the following manner:

$$R^n = \begin{cases} R & n = 1 \\ (x, y) : \exists z \in X, \ (x, z) \in R^{n-1}, \ (z, y) \in R & n > 1 \end{cases}$$

Using that definition, we can define R^2 and R^3

$$(1, 1) \tag{1}$$

$$(1, 2) \tag{2}$$

$$(2, 3) \tag{3}$$

$$(3, 4) \tag{4}$$

These are the elements in R, using them we define R^2

$$(1, 1)^{(1),(1)} \tag{5}$$

$$(1, 2)^{(1),(2)} \tag{6}$$

$$(1, 3)^{(2),(3)} \tag{7}$$

$$(2, 4)^{(3),(4)} \tag{8}$$

and, finally, these are used to construct R^3:

$$(1, 1)^{(5),(1)} \tag{9}$$

$$(1, 2)^{(5),(2)} \tag{10}$$

$$(1, 3)^{(6),(3)} \tag{11}$$

$$(1, 4)^{(7),(4)} \tag{12}$$

Thusly, $R^3 = \{(1, 1), (1, 2)(1, 3), (1, 4)\}$, as can be checked using the aforementioned script.

Exercise 2. Within the folder "files", find a TEX file in whose content appears the string \usepackage {amsthm, amsmath}. Note: use grep and escape the special characters with \. Complete the proof and answer the question.

The `grep "\usepackage {amsthm, amsmath}" ./*.tex` command is used, and it returns a single file: `mainP.tex`. This is the proof we are asked to complete: Let us consider $L = \{w \in \{a, b\}^* : w$ does not end with $ab\}$. A regular expression generating L is:

$$(a+b)^*(ba)+(a+b)^*(bb)+(a+b)^*(aa) = (a+b)^*((ba)+(bb)+(aa)) = (a+b)^*(b(a+b)+(aa))$$